

# Recommendation based on Deferred Acceptance Algorithm

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August 6, 2023

## 1 Introduction

The existing recommendation systems mainly have the following problems: (1) Did not take into account mutual interest; (2) Recommend popular users too much; (3) Stability is not achieved. For the first problem, users may be recommended to users who are not interested in them at all, which is a waste of time and leads to inefficiency. For the second problem, this unbalanced situation also causes inefficiency because popular users will not have enough time to process all the recommendations while unpopular users get too few recommendations. These two problems are addressed by reciprocal recommendation introduced by [1].

The third problem leads to the following situation: Suppose an employee finds a job in the recommendation platform, but a few months later he finds another job more appealing in the same platform and then transfers to that new job. There is an efficiency loss here. But the existing recommendation system neglects this problem.

The third problem is known as violating the stability condition of matching. In matching theory, the Deferred Acceptance algorithm is introduced to produce stable matching. In fact, the idea of the DA algorithm also takes care of the first and the second problems. It also requires mutual interest and also naturally limits the number of recommendations. Then, how can we implement the idea of the DA algorithm in the recommendation system?

This is far from the first research that introduces the DA algorithm into recommendation systems. [2], for example, utilize the DA algorithm. However, the authors focus on protecting the privacy of users. The reason is that in their recommendation system, job candidates have to reveal their preference to the systems about the firms to the system in order for the DA algorithm to work. At the same time, asking job candidates to list all firms is not realistic because of the limited time.

Such difficulties are addressed when the system has calculated preference scores. The scores can be seen as the estimated utility of a user when matched with another user. It is worth noticing that since the scores represent the preference, it is ordinal rather than cardinal. That means using the scores for two different users should be avoided.

## 2 Setup

Let  $\mathcal{C}$  be the set of job candidates,  $\mathcal{J}$  be the set of job posts. Suppose that  $|\mathcal{C}| < \infty$  and  $|\mathcal{J}| < \infty$ . Each candidate  $c \in \mathcal{C}$  has a preference over the set of jobs  $\mathcal{J}$ , and each job post  $j \in \mathcal{J}$  has a preference over the set of candidates  $\mathcal{C}$ . Let  $p_{c,j} \in [0, 1]$  be the estimated preference of candidates  $c \in \mathcal{C}$  to employer  $j \in \mathcal{J}$ , and let  $p_{j,c} \in [0, 1]$  be that of  $j$  to  $c$ . The platform chooses a rank list of employers sent to each candidate. Let  $\Sigma_{|\mathcal{J}|}$  be the set of all possible ranked lists of employer  $\mathcal{J}$ . A deterministic ranking policy  $\sigma : \mathcal{C} \rightarrow \Sigma_{|\mathcal{J}|}$  is a mapping from each job candidate  $c \in \mathcal{C}$  to a ranked list of employers. The probability with which the candidate  $c$  applies to an employer  $j$  is

$$\mathbb{P}(c \text{ applies to } j | \sigma(c)) = v(\text{rank}(j | \sigma(c))) \cdot p_{c,j}$$

where  $\text{rank}(j | \sigma(c)) \in \{1, 2, \dots, |\mathcal{J}|\}$  is the rank of the employer  $j$  in the ranked list  $\sigma(c)$ , and  $v$  is an examination function.

## 3 Deferred Acceptance Algorithm Approach

The deferred acceptance algorithm for the recommendation system I propose here includes two parameters:  $n$  and  $m$ . Here  $n$  is the number of firms recommended to a single candidate.  $m$ , on the other hand, is the

number of candidates a firm can be recommended to. These two parameters are chosen by the recommendation system.

The recommendation system then works in the following way:

Step 1: each job candidate  $c$  proposes to the top  $n$  jobs according to the preference score  $p_{c,j}$ . Each firm  $j$  ranks the job candidates proposing to it according to the preference score  $p_{j,c}$  and reject those ranks  $m + 1$  or lower.

Step  $k$ : each job candidate  $c$  rejected by  $l \leq n$  firms in Step  $k - 1$  proposes to  $l$  firms that are ranked the highest among the firms that he/she has not proposed to yet according to  $p_{c,j}$ . Each firm  $j$  ranks the job candidates proposing to it and job candidates who are not rejected in  $k - 1$  according to the preference score  $p_{j,c}$  and reject those ranks  $m + 1$  or lower.

## 4 Experiment

See <https://github.com/Jiahao76/Recommendation-System-based-on-DA-algorithm> for the implementation of this recommendation system. I follow the same procedure as [1] to generate data.

## References

- [1] Y. Tomita, R. Togashi, Y. Hashizume, and N. Ohsaka, “Fast and examination-agnostic reciprocal recommendation in matching markets,” 2023.
- [2] A. Saini, “Privatejobmatch: A privacy-oriented deferred multi-match recommender system for stable employment,” 2019.