Homework 0

findingnothing

July 6, 2025

- If you want to submit this homework, please send it in PDF format to findingnothing's Wechat before **July 11th, 23:00 (GMT+8)**.

Question 1.

Suppose a random variable X follows a uniform distribution on $[0, 1]$. Consider $a \in [0, 1]$. Define a random variable $Y = |X - a|$. What is the cumulative distribution function of Y? (15)

Question 2.

There are two urns A and B. In urn A, there are $100p$ red balls and $100(1-p)$ blue balls. In urn B, there are $100q$ red balls and $100(1-q)$ blue balls. Here, $p, q \in (0, 1)$.

A man draws a ball randomly from the urns. With probability $\mu \in (0, 1)$, he draws the ball from urn A. With probability $1 - \mu$, he draws the ball from urn B. If the ball drawn is blue, this man will paint it red with probability $\pi \in (0, 1)$.

The man will show you the ball after all the above. Suppose you cannot observe which urn is chosen and what color the ball is originally.

(a) If you are shown a blue ball, what is the probability that it is originally a blue ball? (5)

(b) If you are shown a red ball, what is the probability that it is originally a red ball? (10)

Question 3.

Consider the following function $f : [0, \infty) \to \mathbb{R}$ such that

$$f(x) = ax^2 + bx + c$$

where $a, b, c \in \mathbb{R}$. Calculate $\max_{x\in[0,\infty)} f(x)$. (10)

Question 4.

Suppose $m > 0$. The series

$$1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \dots$$

converges to e^m. Using this fact, we define a function $f : \mathbb{Z}_+ \to \mathbb{R}$ such that

$$f(n) = \frac{m^n e^{-m}}{n!}$$

Let X be a random variable taking values in the nonnegative integers $\mathbb{Z}_+$ such that for any $n \in \mathbb{Z}_+$,

$$Pr(X = n) = f(n)$$

We say X follows a Poisson(m) distribution. Calculate $E[X]$. (10)