1 Introduction

Quantum Chromodynamics (QCD), the fundamental theory of the strong interaction, describes the behavior of quarks and gluons. Understanding high-energy collider phenomena, such as jet production, necessitates the precise calculation of scattering amplitudes involving these fundamental particles.

While the theoretical framework for computing these amplitudes using Feynman diagrams is well-established, the practical execution of these calculations quickly becomes an arduous and error-prone task due to the rapidly increasing number of diagrams, the complexity of algebraic expressions, and the intricate handling of momentum, color, and Lorentz indices.

For gluon scattering in QCD, the number of tree-level Feynman diagrams grows more than factorially with the number of external legs $n+1$, following an asymptotic growth pattern given by:

$$\mathcal{O}\left(\left(\frac{9\sqrt{3}+12}{11}\right)^n \frac{n!}{n^{3/2}}\right) \tag{1}$$

For instance, the number of tree-level diagrams for the first few $n+1$ gluons can be computed using a recurrence relation listing 1 or counted directly after generating the diagrams in Mathematica in the following section.

Listing 1: Number of tree-level diagrams for $n+1$ gluons

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a[0] = 0; a[1] = 1; a[2] = 1;
a[n_] := a[n_]:=a[n]=(12(2n-3)a[n-1]+(3n-5)(3n-7)a[n-2])/11;
Table[a[n], {n, 3, 12}];
Output: {4, 25, 220, 2485, 34300, 559405, 10525900, 224449225, 5348843500, 140880765025}
```

This increasing complexity, particularly in perturbative QCD, makes a computational approach essential. For this project, Mathematica [1] was selected as the primary computational environment due to its unparalleled symbolic manipulation capabilities, which are uniquely well-suited for the challenges of calculating Feynman amplitudes. Unlike compiled languages such as C++ or Fortran, which excel in numerical computations and low-level control, Mathematica provides a high-level, interactive environment that natively understands and operates on symbolic expressions.

This report details the computational implementation of gluon amplitude calculations in QCD using Mathematica. We will demonstrate how Mathematica's unique strengths in symbolic computation, automated algebraic simplification, and high-level programming facilitate the systematic construction of Feynman diagrams, the automated assignment of momenta, the application of Feynman rules, and ultimately, the derivation of explicit amplitude expressions and modulus squared.

The subsequent sections will walk through the design of our data structures, the algorithms for momentum assignment and constraint solving, the implementation of Feynman rules as substitution rules, color algebra handling, verification of key properties such as Ward identities, and the final evaluation of scattering amplitudes.