Computational Physics Laboratory: Tree Level Gluon Amplitudes in Mathematica

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FACOLTÀ DI SCIENZE E TECNOLOGIE

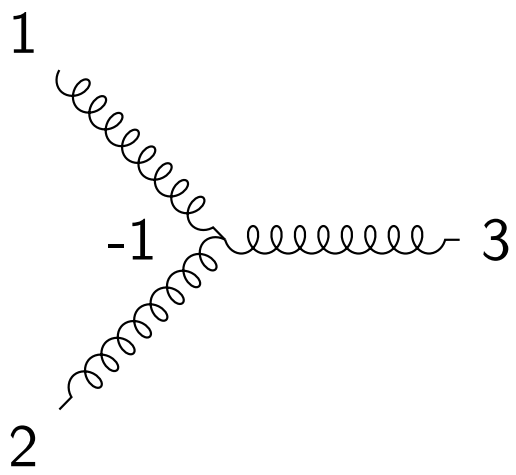
Tree Level Gluon Scattering Amplitudes

- How: Well-established theoretical framework using Feynman diagrams.
- In Practice:
  - Rapidly increasing number of diagrams as the number of external legs increases.
  - Proliferating of algebraic expressions.
  - Lorentz indices and color indices complicate the calculations.
- Number of tree-level diagrams for $n+1$ gluons grows more than factorially:

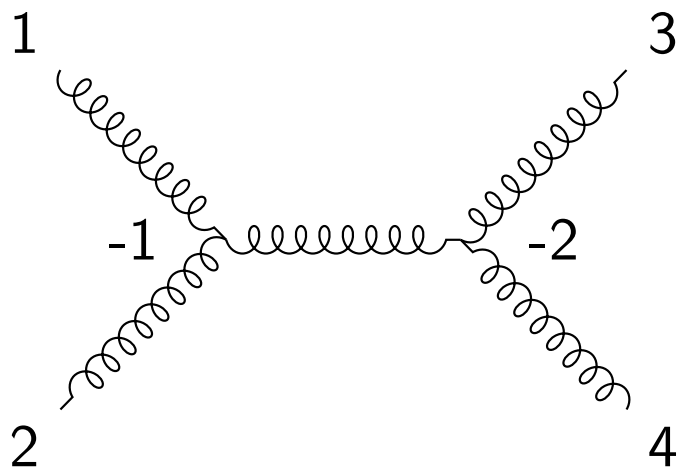
$$\mathcal{O}\left(\left(\frac{9\sqrt{3}+12}{11}\right)^{n}\frac{n!}{n^{3/2}}\right)$$

Feynman Diagrams in Mathematica

Represent each diagram as a list of pairs and use Mathematica's symbolic capabilities to manipulate these lists.



{{1,-1},{2,-1},{3,-1}}



{{1,-1},{2,-1},{-1,-2},{3,-2},{4,-2}}

Number of Diagrams

n	Diagrams	only 3-point	with 4-point
4	4	3	1
5	25	15	10
6	220	105	115
7	2485	945	1540
8	34300	10395	23905
9	559405	135135	424270
10	10525900	2027025	8498875

- Number of diagrams with only 3-point vertices: $(2n-5)!!$
- Diagrams with only 3-point vertices brings unique color structures.

Memory and Time Complexity

$$\mathcal{M} = \sum \text{diagrams}$$

n	Diagrams	Diagrams (s)	Diagrams (MB)	Amplitude (s)	Amplitude (MB)
4	4	0.0011	0.01	0.004	0.096
5	25	0.0100	0.12	0.051	2.87
6	220	0.1153	1.51	6.503	104.52
7	2485	1.5998	22.99	1112.738	4430.25
8	34300	27.5766	404.70	TBD	TBD

Ward Identities

Contracted with	Contraction Time (s)	Memory (MB)	Simplification Time (s)
$n = 4$ Gluons			
4p 0ϵ	0.0023	0.00002	0.00008
3p 1ϵ	0.0038	0.00350	0.00008
2p 2ϵ	0.0069	0.02103	0.00016
1p 3ϵ	0.0099	0.04923	0.00020
$n = 5$ Gluons			
5p 0ϵ	0.093	0.077	0.00034
4p 1ϵ	0.134	0.162	0.00062
3p 2ϵ	0.203	0.515	0.00160
2p 3ϵ	0.281	0.952	0.00327
1p 4ϵ	0.376	1.561	0.07672
$n = 6$ Gluons			
6p 0ϵ	4.04	2.67	0.23
5p 1ϵ	5.08	4.52	0.79
4p 2ϵ	7.03	11.48	9.23
3p 3ϵ	9.64	22.11	30.04
2p 4ϵ	12.59	37.03	151.34
1p 5ϵ	15.70	54.53	183.86

Modulus Squared

Operation Stage	Feynman Diagrams	Non-Independent Basis	Independent Basis
$n = 4$ Gluons			
Total Terms	10	6	3
Single Term (MB)	0.028	0.025	0.050
Times Polarization (MB)	0.945	0.693	1.512
Times Conjugate (MB)	15.449	4.186	32.592
Contract (MB)	1.364	0.358	2.220
Mandelstam (MB)	0.389	0.116	0.622
Simplify (MB)	0.005	0.003	0.006
All Contributions (MB)	0.026	0.023	0.018
Time Taken (s)	3.318	1.106	3.184
$n = 5$ Gluons			
Total Terms	325	120	21
Single Term (MB)	0.134	0.133	0.633
Times Polarization (MB)	10.455	8.970	56.295
Times Conjugate (MB)	4893.640	3695.500	TBD
Contract (MB)	6.325	1.553	TBD
Mandelstam (MB)	7.760	1.681	TBD
Simplify (MB)	0.072	0.059	TBD
All Contributions (MB)	TBD	6.867	TBD
Time Taken (min)	TBD	$\sim$50	TBD