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1 Introduction

In this thesis we are interested in a specific aspect of theoretical high energy physics. In particular, the topic of interest is that electron-positron (e^+e^-) collisions. As is typical in physics, the equations that govern the interactions are quite complicated and it is almost impossible to find exact solutions, so all functions of interest are perturbatively expanded, meaning they are expanded in a power series of a small parameter. When the force in question is the electromagnetic interaction, one can simply use the fine structure constant (or electromagnetic coupling constant) $\alpha_{em} \sim \frac{1}{137}$.

When discussing particles that interact with the strong interaction, it is then natural to use the strong coupling constant α_S . The function we are interested in is the transverse momentum differential cross section. We are able to calculate it to the first order, but when discussing higher order corrections, one must include radiative corrections, meaning one must consider the emission of additional soft gluons. These corrections lead to logarithmically enhanced terms such as $\alpha_S \log(M/qT)$, where M is the mass of the final system and qT is the transverse momentum. For small qT , where the bulk of events is produced, $\log(M/qT)$ can become quite large, but when $\alpha_S \log(M/qT) \sim 1$ the perturbative expansion becomes meaningless because the neglected terms can be of the same order of the included terms.

In order to obtain realistic cross sections, these logarithmically enhanced terms must then be treated differently. A promising solution is the theory of resummation: instead of stopping at a certain power of α_S as in standard perturbation theory, we calculate an all order resummation of the $\alpha_S \log(M/qT)$ terms which lead to a consistent exponential term, the Sudakov form factor. It turns out that the Sudakov form factor itself is a power expansion of α_S but now with no logarithmic enhancement. The first order calculation is called Leading Logarithmic

(LL) order, the next order calculation is called Next to Leading Logarithmic (NLL) and so on. In the literature, it has been calculated up to N3 LL. We were able to calculate the next order, N4 LL, and the main objective of this thesis is to estimate the uncertainty in this function due to the missing higher order corrections. In the first chapter we introduce the forces and particles of the Standard Model, also describing the Quantum Field Theory formalism which is at the base of all the forces described in the Standard Model. In the second chapter we then describe the process of interest and illustrate the theorems and formulas necessary to calculate the relevant cross sections. In particular we introduce the resummation formalism and the Sudakov form factor which is the main object of this work. In chapter 3, we describe the calculations that lead us to obtain the Sudakov fom factor at different logarithmic orders and finally in chapter 4 we present plots of this function and discuss its uncertainty, first using the standard methods and then exploring different methods.

2 QCD running coupling

Classically, the force between two sources is then given by $F = \frac{\alpha}{r^2}$, characterized by a universal coefficient – the coupling constant α, which quantifies the force between two static bodies of unit "charge" at distance r, *i.e.*, the electric charge for QED, the color charge for QCD, the weak isospin for the weak force, or the mass for gravity. Consequently, the coupling α is defined as being proportional to the elementary charge squared, *e.g.*, $\alpha_{em} \equiv \frac{e^2}{4\pi}$ where e is the elementary electric charge, or $\alpha_s \equiv \frac{g^2}{4\pi}$ where g is the elementary gauge field coupling in QCD. In quantum field theory (QFT), $\frac{1}{r^2}$ is the coordinate-space expression for the propagator of the force carrier (gauge boson) at leading-order in perturbation theory: in momentum space, the analogous propagator is proportional to $\frac{1}{q^2}$, where q the boson 4-momentum ($Q^2 = -q^2 > 0$).

For sources interacting weakly, the one-boson exchange representation of interactions is a good first approximation. However, when interactions become strong (with "strong" to be defined below), higher orders in perturbation theory become noticeable and the $\frac{1}{r^2}$ law no longer stands. In such cases, it makes good physics sense to fold the extra r-dependence into the coupling, which thereby becomes r, or equivalently Q^2, dependent.

The running of the coupling is due to vacuum polarization, the vacuum is not empty, but is filled with virtual particles that are constantly created and annihilated which can interact with the propagating particles, leading to a modification of the interaction strength.

While in QED, the extra r-dependence comes only from the vacuum polarization. In QCD, α_s receives contributions from the vacuum polarization and from gluon self-interactions since the gluon has a color charge.

The two couplings have opposite trends: the QED coupling increases with energy and the theory becomes strongly coupled at high energies, whereas the opposite happens for the QCD coupling as it is large at low energies and decreases with energy. This property of being weakly coupled at high energies is known as *asymptotic freedom* and it means that perturbative calculations in QCD can only be done at high energies where α_s becomes small enough that a power expansion is meaningful.

In the framework of perturbative QCD (*pQCD*), predictions for observables are expressed in terms of the renormalized coupling $\alpha = \alpha(\mu_R^2)$, a function of an (unphysical) renormalization scale μ_R . The coupling satisfies the following renormalization group equation (RGE):

$$\mu_R^2 \frac{d\alpha}{d\mu_R^2} = \beta(\alpha) = -\left(b_0\alpha^2 + b_1\alpha^3 + b_2\alpha^4 + \ldots\right) \tag{1}$$

where $b_0 = \frac{11C_A - 4n_f T_R}{12\pi} = \frac{33-2n_f}{12\pi}$ is the 1-loop β-function coefficient, $b_1 = \frac{17C_A^2 - n_f T_R(10C_A + 6C_F)}{24\pi^2} = \frac{153-19n_f}{24\pi^2}$ is the 2-loop coefficient, $b_2 = \frac{2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2}{128\pi^3}$ is the 3-loop coefficient. $C_A = 3$ and $C_F = \frac{4}{3}$ are the Casimir operators of the adjoint and fundamental representations of $SU(3)$, $T_R = \frac{1}{2}$ is the trace normalization, n_f is the number of active quark flavors.

It is not possible to solve **??** as it is for two reasons: only the first few b_n coefficients are known (up to b_4); the exact equation becomes more and more complicated as more terms of the series are included, making it impossible to obtain an analytic solution.

In order to solve both problems, the equation is solved in the following way: at first only b_0 is included and the obtained solution is called α_{LO} , as it will only contain a term proportional to α ; then also b_1 is included and only terms up to the second order in α are kept to obtain α_{NLO} ; this same procedure is used to obtain α_{NNLO} , α_{N^3LO} , α_{N^4LO} . There will be a complication in calculating α_{NLO} and higher orders which will be explained and resolved in the following sections.

2.1 One-loop running coupling

The one-loop running coupling α_{LO} is obtained by solving the RGE with only the first term of the β-function:

$$\mu_R^2 \frac{d\alpha}{d\mu_R^2} = -b_0 \alpha^2 \tag{2}$$

This equation can be solved by separation of variables and imposing the boundary condition $\alpha(Q^2) = \alpha_s$:

$$\int_{\alpha(Q^2)}^{\alpha(\mu_R^2)} \frac{d\alpha}{\alpha^2} = \int_{Q^2}^{\mu_R^2} -b_0 \frac{d\mu_R^2}{\mu_R^2} \tag{3}$$

and one obtains:

$$\alpha_{LO}(\mu_R^2) = \frac{\alpha_s}{1 + b_0\alpha_s \log\left(\frac{\mu_R^2}{Q^2}\right)} \tag{4}$$

In which we can confirm the decreasing with energy trend of the running coupling that we anticipated above.

It is useful to define the variable $\lambda_\mu = b_0\alpha_s \log\left(\frac{\mu_R^2}{Q^2}\right)$ so that:

$$\alpha_{LO}(\mu_R^2) = \frac{\alpha_s}{1 + \lambda_\mu} \tag{5}$$

2.2 Two-loop running coupling

In order to obtain the two-loop running coupling α_{NLO}, we need to solve the RGE with the first two terms of the β-function **??**:

$$\mu_R^2 \frac{d\alpha}{d\mu_R^2} = -b_0\alpha^2 - b_1\alpha^3 \tag{6}$$

but this equation is not solvable in a straightforward way as the one-loop equation, we have to use the perturbative approach. We can rewrite the equation as:

$$\frac{d\alpha}{d\mu_R^2} = -\frac{b_0\alpha^2}{\mu_R^2}(1 - \frac{b_1}{b_0}\alpha) \tag{7}$$

and expand the α term in the parenthesis as:

$$\alpha = \alpha_{LO} + \delta_\alpha \tag{8}$$

where α_{LO} is the one-loop running coupling and δ_α contains the higher order corrections.

A Equivalence between Laplace weight and Mellin weight in Soft-gluon resummation

In this appendix, we will show the equivalence between the Laplace weight and Mellin weight in the context of soft-gluon resummation. The Laplace weight is defined as

$$\int_0^1 \mathrm{d}z \, \frac{e^{-N(1-z)} - 1}{1 - z} \tag{9}$$

while the Mellin weight is defined as

$$\int_0^1 \mathrm{d}z \, \frac{z^N - 1}{1 - z} \tag{10}$$

we can show that in the limit $N \to \infty$, the two weights are equivalent, meaning they have the same approximation in the limite.

[**Catani:1992ua**]