1 Equivalence between Laplace weight and Mellin weight in Soft-gluon resummation

In this appendix, we will show the equivalence between the Laplace weight and Mellin weight in the context of soft-gluon resummation. The Laplace weight is defined as

$$\int_0^1 \mathrm{d}z \, \frac{e^{-N(1-z)} - 1}{1-z} \tag{1}$$

while the Mellin weight is defined as

$$\int_0^1 \mathrm{d}z \, \frac{z^N - 1}{1-z} \tag{2}$$

we can show that in the limit $N \to \infty$, the two weights are equivalent, meaning they have the same approximation in the limite.