1 Calculation of the f_i functions

In the two-jet region the fixed-order thrust distribution is enhanced by large infrared logarithms which spoil the convergence of the perturbative series. The convergence can be restored by resumming the logarithms to all orders in the coupling constant α_s.

According to general theorems [1],[3],[4], the cross section **??** has a power series expansion in $\alpha_s(Q^2)$ of the form:

$$\frac{\sigma(\tau, Q^2)}{\sigma_t} = C\big(\alpha_s\big(Q^2\big)\big)\Sigma\big(\tau, \alpha_s\big(Q^2\big)\big) + F\big(\tau, \alpha_s\big(Q^2\big)\big) \tag{1}$$

where σ_t is the total hadronic cross section and

$$C(\alpha_s) = 1 + \sum_{n=1}^{\infty} C_n \alpha_s^n \tag{2a}$$

$$\Sigma(\tau, \alpha_s) = \exp\left[\sum_{n=1}^{\infty} \alpha_s^n \sum_{m=1}^{2n} G_{nm} \ln^m \tau\right] \tag{2b}$$

$$F(\tau, \alpha_s) = \sum_{n=1}^{\infty} \alpha_s^n F_n(\tau) \tag{2c}$$

Here C_n and G_{nm} are constants, while $F_n(\tau)$ are perturbatively computable functions that vanish at small τ. Thus at small τ (large thrust) it becomes morst important to resum the series of large logarithms in $\Sigma(\tau, \alpha_s)$. These are normally classified as *leading* logarithms when $n < m \leq 2n$, *next-to-leading* when $m = n$ and *subdominant* logarithms when $m < n$.

In the article by Catani, Turnock, Webber and Trentadue [2], it was observed that for a final state configuration corresponding to a large value of thrust , **??** can be approximated by

$$\tau = 1 - T \approx \frac{k_1^2 + k_2^2}{Q^2} \tag{3}$$

where k_1^2 and k_2^2 are the invariant masses squared of two back-to-back jets and Q^2 is the energy of the center of mass. Thus the key to the evaluation of the thrust distributions is its relation to the jet mass distribution $J(Q^2, k^2)$ which denotes the probability of jet invariant mass-squared k^2 at scale Q^2, then the thrust fraction

$$R(\tau, \alpha_s(Q^2)) = \frac{\sigma(\tau, Q^2)}{\sigma_t} = \frac{1}{\sigma_t}\int_0^1 \frac{\mathrm{d}\sigma(\tau', Q^2)}{\mathrm{d}\tau'}\Theta(\tau - \tau')\mathrm{d}\tau' \tag{4}$$

takes the form of a convolution of two jet mass distributions $J(Q^2, k_1^2)$ and $J(Q^2, k_2^2)$

$$R(\tau, \alpha_s(Q^2)) \underset{\tau \ll 1}{=} \int_0^\infty J(Q^2, k_1^2) J(Q^2, k_2^2) \Theta\left(\tau - \frac{k_1^2 + k_2^2}{Q^2}\right) \mathrm{d}k_1^2 \mathrm{d}k_2^2 \tag{5}$$

Introducing the Laplace transform of the jet mass distribution

$$\tilde{J}_\nu(Q^2) = \int_0^\infty J(Q^2, k^2) e^{-\nu k^2} \mathrm{d}k^2 \tag{6}$$

and using the integral representation of the Heaviside step function

$$\Theta(\tau - \frac{k^2}{Q^2}) = \frac{1}{2\pi i}\int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{e^{\nu\tau}}{\nu} e^{-\nu \frac{k^2}{Q^2}} \mathrm{d}\nu \tag{7}$$

we find that

$$\Sigma(\tau, \alpha_s(Q^2)) = \int_{\epsilon - i\infty}^{\epsilon + i\infty} \tilde{J}(Q^2, \nu_1) \tilde{J}(Q^2, \nu_2) e^{\nu_1 \tau} e^{\nu_2 \tau} \mathrm{d}\nu_1 \mathrm{d}\nu_2 \tag{8}$$

where ϵ is a real positive constant to the right of all singularities of the integrand $\tilde{J}_\nu(Q^2)$ in the complex ν plane.

An integral representation for the Laplace transform $\tilde{J}_\nu(Q^2)$ is given by

$$\ln \tilde{J}_\nu^q(Q^2) = \int_0^1 \frac{\mathrm{d}u}{u}\left(e^{-u\nu Q^2} - 1\right)\left[\int_{u^2 Q^2}^{uQ^2} \frac{1}{q^2} A\big(\alpha_s(q^2)\big)\mathrm{d}q^2 + \frac{1}{2} B\big(\alpha_s(uQ^2)\big)\right] \tag{9}$$

where

$$A(\alpha_s) = \sum_{n=1}^{\infty} \frac{A_n}{\pi^n} \alpha_s^n \qquad\qquad B(\alpha_s) = \sum_{n=1}^{\infty} \frac{B_n}{\pi^n} \alpha_s^n$$