EN530.603 Applied Optimal Control
Homework #3

September 22, 2021

Due: September 29, 2021 (before class)

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1. Find the curve x^* that minimizes

$$J(x) = \int_0^1 \left(\frac{1}{2}\dot{x}^2(t) + 3x(t)\dot{x}(t) + 2x^2(t) + 3x(t) \right) dt$$

and passes through the points $x(0) = 0$ and $x(1) = 4$.

2. Find the extremals for

$$J(x) = \int_0^{t_f} [\dot{x}_1^2(t) + \dot{x}_2^2(t) + 3x_1(t)x_2(t)]dt,$$

with

a) boundary conditions:

$$x_1(0) = 0, x_2(0) = 0, t_f = 1, x_1(t_f) \text{ free }, x_2(t_f) = 1$$

b) boundary conditions:

$$x_1(0) = 0, x_2(0) = 0,$$

where t_f is free and $x(t_f)$ must lie on the surface

$$x_1(t) + 3x_2(t) + 5t = 15$$

Note: the last constraint is of the general form $\psi(x(t_f), t_f) = 0$. See last section in Lecture#4 posted on the website for details.

3. (Kirk 4-5.) Let η be a continuously differentiable function of time t that is arbitrary on the interval $[t_0, t_f]$ except at the end-points where $\eta(t_0) = \eta(t_f) = 0$. If ϵ is an arbitrary real parameter, then $x^* + \epsilon\eta$ represents a family of curves. Evaluating the functional

$$J(x) = \int_{t_0}^{t_f} g(x(t), \dot{x}(t), t)dt$$

on $x = x^* + \epsilon\eta$ makes J a function of ϵ, and if x^* is an extremal this function must have a relative extremum at $\epsilon = 0$.

Show that the Euler-Lagrange equations can be equivalently obtained from the necessary condition

$$\left.\frac{dJ(x^* + \epsilon\eta)}{d\epsilon}\right|_{\epsilon=0} = 0.$$

Note: upload your code using the File Upload link on the website. In addition attach a printout of the code and plots to your homework solutions.