EN530.603 Applied Optimal Control
Homework #1

September 8, 2021

Due: September 15, 2021

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1. Find the stationary points (i.e. that satisfy $\nabla L = 0$) of the following and determine whether they are maxima, minima, or saddle points. If there are multiple local minima, specify the global minimum.

   (a) $L(x) = (2 - x_1)^2 + 3(4x_2 - x_1^2)^2$

   (b) $L(u) = (u - 3)(u + 6)(u - 2)$

   (c) $L(u) = (u_1^2 + 2u_1 - 6)(3u_2^2 + 2u_2 + 1)$

   (d) $L(x) = 3x_1^4 - 28x_1^3 + 84x_1^2 - 96x_1 + 64 + 27x_2^2$

2. Consider the classical non-linear regression problem, i.e. finding a parameter $\theta \in \mathbb{R}^n$ that minimizes the average squared error between a model $y = f(x, \theta)$ for data points (x_i, y_i) for $i = 1, ..., m$. Derive the necessary and sufficient conditions for the optimal θ^*, i.e. which minimizes the total error

$$\frac{1}{2}\|F(\theta) - Y\|^2,$$

   where $F(\theta) = (f(x_1, \theta), \cdots, f(x_m, \theta)) \in \mathbb{R}^m$ and $Y = (y_1, ..., y_m) \in \mathbb{R}^m$, and propose a numerical method for iteratively finding the optimum. You can assume that $F(\theta)$ is differentiable and that you have access to its derivatives. What assumptions on the model and the data points are necessary to find a unique minimizer?

3. Implementation: you are free to use parts of the code provided at the course homepage.

   (a) Write a MATLAB function which implements gradient descent to optimize the cost-function given by

$$L(x) = (1 - x_1)^2 + 200(x_2 - x_1^2)^2.$$

   You can use either a constant or a variable stepsize. What is the effect of the step-size choice? Use a starting point at $x = (0, 0)$.

   (b) Write a MATLAB function which implements Newton's method to optimize the function above. Comment on the rate of convergence of Newton's method compared to gradient descent.

   (c) Optimize the cost-function

$$L(x) = x_1 e^{-x_1^2 - \frac{1}{2}x_2^2} + x_1^2/10 + x_2^2/10$$

   using gradient descent and Newton's method using the starting point $x = (1.5, 3.0)$. Compare the performance of the two algorithms.

Note: upload your code using the provided link on the class website; in addition attach a printout of the code and generated plots to your homework solutions.