Assignment 7

1. a. $P = K[^wR_p^T \mid -^wR_p^T \, ^wt_p]$

   $P' = K[^wR_{p'}^T \mid -^wR_{p'}^T \, ^wt_{p'}]$

   $\Rightarrow P = \begin{bmatrix} 100 & 0 & 320 & -100 \\ 0 & 100 & 240 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $P' = \begin{bmatrix} 320 & 0 & -100 & 100 \\ 240 & 100 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

   b. $x = PX$

   $x' = P'X$

   $\Rightarrow x = \begin{bmatrix} 320 \\ 240 \\ 1 \end{bmatrix}$, $x' = \begin{bmatrix} 320 \\ 240 \\ 1 \end{bmatrix}$

   c. $PC = 0 \Rightarrow C$ is the null space of P.

   $e' = P'C$

   $\Rightarrow e' = \begin{bmatrix} 420 \\ 240 \\ 1 \end{bmatrix}$

2. a. $PP^+ = I \Rightarrow P^+ = pinv(P)$

$F = [e']_\times P'P^+$, and since F is homogeneous

$$\Rightarrow F = \begin{bmatrix} 0 & 0 & -0.0016 \\ 0 & 0 & -0.0014 \\ -0.0016 & -0.0027 & 1 \end{bmatrix}$$

b. Validate F with x and x' :

$$x'^T F x = 0.$$

Calculated result : $-6.5919e{-}17$

c. Validate F with e' :

$$F^T e' = 0$$

Calculated result : $1e{-}19 \times \begin{bmatrix} -0.7793 \\ 0 \\ 0 \end{bmatrix}$

3. a. $P^* = \begin{bmatrix} P \\ 0\,0\,0\,1 \end{bmatrix} = \begin{bmatrix} 100 & 0 & 320 & -100 \\ 0 & 100 & 240 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$H = P^{*-1}$

$= \begin{bmatrix} 0.01 & 0 & -3.2 & 1 \\ 0 & 0.01 & -2.4 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

We can validate by $PH = P_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

b. $P_c' = P'H$

$= \begin{bmatrix} 3.2 & 0 & -1124 & 420 \\ 2.4 & 1 & -1008 & 240 \\ 0 & 0 & -3.2 & 1 \end{bmatrix}$

c. Denote the projective equivalent point of X as X_p.

$X_p = H^{-1}X$

then

$x = P_c X_p$

$x' = P_c' X_p$

$\Rightarrow x = \begin{bmatrix} 320 \\ 240 \\ 1 \end{bmatrix}$, $x' = \begin{bmatrix} 320 \\ 240 \\ 1 \end{bmatrix}$ we can see that x and x' are equal.

d. $e' = P_c'(H^{-1}C)$

$\Rightarrow e' = \begin{bmatrix} 420 \\ 240 \\ 1 \end{bmatrix}$

4. a. $F^T e' = 0 \Rightarrow e'$ is the null space of F^T.

$$e' = \begin{bmatrix} 420 \\ 240 \\ 1 \end{bmatrix}$$

$$P_F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad , \quad P_{F'} = [[e']_\times F \mid e']$$

$$= \begin{bmatrix} -0.3750 & -0.6563 & 240.0014 & 420 \\ 0.6562 & 1.1484 & -420.0016 & 240 \\ 0.0027 & -0.0016 & -0.2266 & 1 \end{bmatrix}$$

b. F calculate by P_C and $P_{C'}$:

$$F = [e']_\times P_{C'} P_C^+ \quad , \quad \text{where } P_C P_C^+ = I$$

$$= \begin{bmatrix} 0 & 0 & -0.0016 \\ 0 & 0 & -0.0014 \\ -0.0016 & -0.0027 & 1 \end{bmatrix}$$

F calculated by P_F and $P_{F'}$:

$$F = [e']_\times P_{F'} P_F^+ \quad , \quad \text{where } P_F P_F^+ = I$$

$$= \begin{bmatrix} 0 & 0 & -0.0016 \\ 0 & 0 & -0.0014 \\ -0.0016 & -0.0027 & 1 \end{bmatrix}$$

The F calculated in Question 1:

$$F = \begin{bmatrix} 0 & 0 & -0.0016 \\ 0 & 0 & -0.0014 \\ -0.0016 & -0.0027 & 1 \end{bmatrix}$$

We can see that these three Fs are equal.

Contents

```
clc;
clear;
```

Question 1

```
disp('Question 1')
K = [100, 0, 320;
     0, 100, 240;
     0, 0, 1];
Rwc1 = eye(3);
twc1 = [1, 0, 0]';
Rwc2 = ROTY(pi/2);
twc2 = [0, 0, 1]';
P1 = K*[Rwc1' -Rwc1'*twc1];
P2 = K*[Rwc2' -Rwc2'*twc2];
disp('P:')
disp(P1)
disp("P':")
disp(P2)
X = [1, 0, 1, 1]';
x1 = P1*X;
x1 = [x1(1)/x1(3) x1(2)/x1(3) 1]';
x2 = P2*X;
x2 = [x2(1)/x2(3) x2(2)/x2(3) 1]';
disp('x:')
disp(x1)
disp("x':")
disp(x2)
C1 = null(P1);
e2 = P2*C1;
e2 = [e2(1)/e2(3) e2(2)/e2(3) 1]';
disp("e':")
disp(e2)
```

```
Question 1
P:
   100      0    320   -100
     0    100    240      0
     0      0      1      0

P':
  320.0000         0 -100.0000  100.0000
  240.0000  100.0000    0.0000   -0.0000
    1.0000         0    0.0000   -0.0000

x:
   320
   240
     1

x':
  320.0000
```

```
    240.0000
      1.0000

e':
    420
    240
      1
```

Question 2

```
disp('Question 2')
P_plus = pinv(P1);
F = SKEW3(e2)*P2*P_plus;
F = F/F(3,3);
disp('F:')
disp(F)
a = x2'*F*x1;
disp("Validate F with x'T*F*x=0")
disp("x'T*F*x:")
disp(a)
b = F'*e2;
disp("Validate F with FT*e'=0")
disp("FT*e':")
disp(b)
```

```
Question 2
F:
      0.0000      0.0000     -0.0016
      0.0000     -0.0000     -0.0014
     -0.0016     -0.0027      1.0000

Validate F with x'T*F*x=0
x'T*F*x:
  -6.5919e-17

Validate F with FT*e'=0
FT*e':
    1.0e-19 *

   -0.7793
         0
         0
```

Question 3

```
disp('Question 3')
Pc1 = [eye(3) [0,0,0]'];
P1_star = [P1;[0 0 0 1]];
H = inv(P1_star);
disp('H:')
disp(H)
% disp('P1*H:')
% disp(P1*H)
Pc2 = P2*H;
disp("Pc':")
disp(Pc2)
X_new = H\X;
x3 = Pc1*X_new;
x3 = [x3(1)/x3(3) x3(2)/x3(3) 1]';
```

```
disp('x:')
disp(x3)
x4 = Pc2*X_new;
x4 = [x4(1)/x4(3) x4(2)/x4(3) 1]';
disp("x':")
disp(x4)
e2_new = Pc2*(H\C1);
e2_new = [e2_new(1)/e2_new(3) e2_new(2)/e2_new(3) 1]';
disp("e':")
disp(e2_new)
```

```
Question 3
H:
    0.0100         0   -3.2000    1.0000
         0    0.0100   -2.4000         0
         0         0    1.0000         0
         0         0         0    1.0000

Pc':
   1.0e+03 *

    0.0032         0   -1.1240    0.4200
    0.0024    0.0010   -1.0080    0.2400
    0.0000         0   -0.0032    0.0010

x:
   320
   240
     1

x':
   320.0000
   240.0000
     1.0000

e':
   420.0000
   240.0000
     1.0000
```

Question 4

```
disp('Question 4')
PF1 = [eye(3) [0 0 0]'];
% v = [1 2 3]';
% lamda = 5;
e2_star = null(F');
e2_star = [e2_star(1)/e2_star(3) e2_star(2)/e2_star(3) 1]';
disp("e' by null(F):")
disp(e2_star)
PF2 = [SKEW3(e2_star)*F e2_star];
disp('PF:')
disp(PF1)
disp("PF':")
disp(PF2)
Pc1_plus = pinv(Pc1);
Fc = SKEW3(e2)*Pc2*Pc1_plus;
Fc = Fc/Fc(3,3);
PF1_plus = pinv(PF1);
Ff = SKEW3(e2_star)*PF2*PF1_plus;
Ff = Ff/Ff(3,3);
```

```
disp("F calculated by (Pc,Pc'):")
disp(Fc)
disp("F calculated by (PF,PF'):")
disp(Ff)
disp("F of Question 1:")
disp(F)
```

Question 4
e' by null(F):
    420.0000
    240.0000
      1.0000

PF:
     1   0   0   0
     0   1   0   0
     0   0   1   0

PF':
    -0.3750   -0.6563   240.0014   420.0000
     0.6562   1.1484 -420.0016   240.0000
     0.0027   -0.0016   -0.2266   1.0000

F calculated by (Pc,Pc'):
         0   0.0000   -0.0016
    0.0000   0   -0.0014
   -0.0016   -0.0027   1.0000

F calculated by (PF,PF'):
   -0.0000   0.0000   -0.0016
    0.0000   0.0000   -0.0014
   -0.0016   -0.0027   1.0000

F of Question 1:
    0.0000   0.0000   -0.0016
    0.0000   -0.0000   -0.0014
   -0.0016   -0.0027   1.0000

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