Assignment 5

1. a) $P = K[{}^cR_w | {}^ct_w]$

$= K[{}^wR_c^T \mid -{}^wR_c^T\,{}^wt_c]$

$$= \begin{bmatrix} 100 & 0 & 320 \\ 0 & 100 & 240 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -100 & 0 & 320 & 420 \\ 0 & -100 & 240 & 340 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

b) ${}^c\tilde{x}_1 = P\,{}^wX_1 = [2020, 1540, 6]^T$ $\quad$ ${}^cx_1 = [336.67, 256.67]^T$

${}^c\tilde{x}_2 = P\,{}^wX_2 = [2560, 2020, 8]^T$ $\quad$ ${}^cx_2 = [320, 252.5]^T$

${}^c\tilde{x}_3 = P\,{}^wX_3 = [2880, 2160, 9]^T$ $\quad$ ${}^cx_3 = [320, 240]^T$

${}^c\tilde{x}_4 = P\,{}^wX_4 = [3580, 1460, 9]^T$ $\Rightarrow$ ${}^cx_4 = [397.78, 162.22]^T$

${}^c\tilde{x}_5 = P\,{}^wX_5 = [3420, 2340, 11]^T$ $\quad$ ${}^cx_5 = [310.91, 212.73]^T$

${}^c\tilde{x}_6 = P\,{}^wX_6 = [3280, 1460, 9]^T$ $\quad$ ${}^cx_6 = [364.44, 162.22]^T$

2. for each pair of corresponding points $x_i \leftrightarrow X_i$, we have

$$A_i \begin{bmatrix} \vec{p}^1 \\ \vec{p}^2 \\ \vec{p}^3 \end{bmatrix} = 0, \text{ where}$$

$$A_i = \begin{bmatrix} \vec{0}^T & -w_i \vec{X_i}^T & y_i \vec{X_i}^T \\ w_i \vec{X_i}^T & \vec{0}^T & -x_i \vec{X_i}^T \end{bmatrix}$$ and $\vec{p}^{iT}$ is the i-th row of P.

and we have $A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix}$

Then obtain the SVD of A. $A = UDV^T$, then $\vec{p} = \begin{bmatrix} \vec{p}^1 \\ \vec{p}^2 \\ \vec{p}^3 \end{bmatrix}$ is the last column of V.

then we can calculate

$$P = \begin{bmatrix} \vec{p}^{1T} \\ \vec{p}^{2T} \\ \vec{p}^{3T} \end{bmatrix} = \begin{bmatrix} 0.1456 & 0 & -0.4658 & -0.6113 \\ 0 & 0.1456 & -0.3493 & -0.4949 \\ 0 & 0 & -0.0015 & -0.0015 \end{bmatrix}$$

Because P is a homogeneous matrix, we can normalize by setting $P(3,4) = 1$.
then we get

$$P = \begin{bmatrix} -100 & 0 & 320 & 420 \\ 0 & -100 & 240 & 340 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

3. the first three column of P is $P(:,1{:}3) = K\,{}^{c}R_w = K\,{}^{w}R_c^{T}$,
where K is upper triangular with diagonal elements positive
suppose $B = P(:,1{:}3)$, we need to do an RQ decomposition of B.

$B = RQ$

First compute $\tilde{B} = MB$, where $M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

$B = K\,{}^{c}R_w = RQ$

then compute $\tilde{Q}\tilde{R} = \tilde{B}^T$ using QR decomposition

compute $Q = M\tilde{Q}^T$

compute $R = M\tilde{R}^T M$

$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $R = \begin{bmatrix} -100 & 0 & 320 \\ 0 & 100 & 240 \\ 0 & 0 & 1 \end{bmatrix}$

However, we need to make sure that the diagonal elements of R is positive.

Let $R' = RD$, $Q' = D^{-1}Q$

then $B = R'Q' = (RD)(D^{-1}Q)$ is also a solution.

Set $D_{ii} = \text{sign}(R_{ii})$, which can make the diagonal elements of (RD) positive

$D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

then $K = RD = \begin{bmatrix} 100 & 0 & 320 \\ 0 & 100 & 240 \\ 0 & 0 & 1 \end{bmatrix}$

${}^{c}R_w = D^{-1}Q = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

${}^{w}R_c = {}^{c}R_w^{T} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The fourth column of P is $P(:,4) = K\,{}^{w}t_c = K(-{}^{w}R_c^{T}\,{}^{w}t_c)$

Let $C = P(:,4)$

then ${}^{w}t_c = -{}^{w}R_c K^{-1} C$

$= [1, 1, -1]^T$.

Contents


```
clc;
clear;
```

Question 1

```
disp('Question 1')
K = [100,0,320;
     0,100,240;
     0,0,1];
Rwc = [-1,0,0;
       0,-1,0;
       0,0,1];
twc = [1,1,-1]';
% disp(-Rwc'*twc)
P = K*[Rwc' -Rwc'*twc];
disp('P:')
disp(P)

X1 = [0,0,5,1]';
X2 = [1,0,7,1]';
X3 = [1,1,8,1]';
X4 = [-6,8,8,1]';
X5 = [2,4,10,1]';
X6 = [-3,8,8,1]';
X = [X1 X2 X3 X4 X5 X6];

x = [];
for i = 1:6
    xi_h = P*X(:,i);
    xi = [xi_h(1)/xi_h(3) xi_h(2)/xi_h(3)]';
    x = [x xi];
end
disp('x:')
disp(x)
```

```
Question 1
P:
  -100      0    320    420
     0   -100    240    340
     0      0      1      1

x:
  336.6667   320.0000   320.0000   397.7778   310.9091   364.4444
  256.6667   252.5000   240.0000   162.2222   212.7273   162.2222
```

Question 2

```
disp('Question 2')
A = [];
for i = 1:6
    Ai = [[0,0,0,0],-X(:,i)',x(2,i)*X(:,i)';
```

```
        X(:,i)',[0,0,0,0],-x(1,i)*X(:,i)'];
    A = [A;Ai];
end
disp('A:')
disp(A)
[U, S, V] = svd(A);
p = V(:,end);
disp('p:')
disp(p);

P_DLT_h = [p(1:4)';p(5:8)';p(9:12)'];
P_DLT = P_DLT_h/P_DLT_h(3,4);
disp('P:')
disp(P_DLT)
```

```
Question 2
A:
   1.0e+03 *

  列 1 至 7

         0         0         0         0         0         0   -0.0050
         0         0    0.0050    0.0010         0         0         0
         0         0         0         0   -0.0010         0   -0.0070
    0.0010         0    0.0070    0.0010         0         0         0
         0         0         0         0   -0.0010   -0.0010   -0.0080
    0.0010    0.0010    0.0080    0.0010         0         0         0
         0         0         0         0    0.0060   -0.0080   -0.0080
   -0.0060    0.0080    0.0080    0.0010         0         0         0
         0         0         0         0   -0.0020   -0.0040   -0.0100
    0.0020    0.0040    0.0100    0.0010         0         0         0
         0         0         0         0    0.0030   -0.0080   -0.0080
   -0.0030    0.0080    0.0080    0.0010         0         0         0

  列 8 至 12

   -0.0010         0         0    1.2833    0.2567
         0         0         0   -1.6833   -0.3367
   -0.0010    0.2525         0    1.7675    0.2525
         0   -0.3200         0   -2.2400   -0.3200
   -0.0010    0.2400    0.2400    1.9200    0.2400
         0   -0.3200   -0.3200   -2.5600   -0.3200
   -0.0010   -0.9733    1.2978    1.2978    0.1622
         0    2.3867   -3.1822   -3.1822   -0.3978
   -0.0010    0.4255    0.8509    2.1273    0.2127
         0   -0.6218   -1.2436   -3.1091   -0.3109
   -0.0010   -0.4867    1.2978    1.2978    0.1622
         0    1.0933   -2.9156   -2.9156   -0.3644

p:
    0.1456
    0.0000
   -0.4658
   -0.6113
   -0.0000
    0.1456
   -0.3493
   -0.4949
    0.0000
    0.0000
   -0.0015
   -0.0015

P:
```

```
 -100.0000    -0.0000   320.0000   420.0000
    0.0000  -100.0000   240.0000   340.0000
   -0.0000    -0.0000     1.0000     1.0000
```

Question 3

```
disp('Question 3')
B = P_DLT(:,1:3);
M = [0,0,1;0,1,0;1,0,0];
B_f = M*B;
[Q_f, R_f] = qr(B_f');
Q = M*Q_f';
R = M*R_f'*M;
D = eye(3);
for i = 1:3
    D(i,i) = sign(R(i,i));
end

K_3 = R*D;
R_cw_3 = D\Q;
R_wc_3 = R_cw_3';

C = P_DLT(:,4);
t_wc_3 = -R_cw_3*inv(K_3)*C;


disp('Q:')
disp(Q)
disp('R:')
disp(R)
disp('D:')
disp(D)
disp('K:')
disp(K_3)
disp('R_wc:')
disp(R_wc_3)
disp('t_wc:')
disp(t_wc_3)
```

```
Question 3
Q:
    1.0000     0.0000     0.0000
    0.0000    -1.0000    -0.0000
   -0.0000    -0.0000     1.0000

R:
 -100.0000    -0.0000   320.0000
         0   100.0000   240.0000
         0          0     1.0000

D:
    -1     0     0
     0     1     0
     0     0     1

K:
  100.0000    -0.0000   320.0000
         0   100.0000   240.0000
         0          0     1.0000

R_wc:
```

```
   -1.0000    0.0000   -0.0000
   -0.0000   -1.0000   -0.0000
   -0.0000   -0.0000    1.0000

t_wc:
    1.0000
    1.0000
   -1.0000
```