

Assignment 3

1. DLT

a. $A\vec{h} = \vec{v}$ can be written as

$$\begin{bmatrix} \vec{0}^T & -w_i' \vec{x}_i^T & y_i' \vec{x}_i^T \\ w_i' \vec{x}_i^T & \vec{0}^T & -\vec{x}_i' \vec{x}_i^T \end{bmatrix} \begin{pmatrix} \vec{h}_1 \\ \vec{h}_2 \\ \vec{h}_3 \end{pmatrix} = \vec{v}.$$

$$\vec{x}_1 = (0, 0, 1)^T, \vec{x}_1' = (0.037, 0.086, 1)^T$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0.086 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -0.037 \end{bmatrix}$$

$$\vec{x}_2 = (1, 0, 1)^T, \vec{x}_2' = (0.49, -0.107, 1)^T$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & -1 & -0.107 & 0 & -0.107 \\ 1 & 0 & 1 & 0 & 0 & 0 & -0.49 & 0 & -0.49 \end{bmatrix}$$

$$\vec{x}_3 = (1, 1, 1)^T, \vec{x}_3' = (0.291, 0.401, 1)^T$$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 & -1 & -1 & -1 & 0.401 & 0.401 & 0.401 \\ 1 & 1 & 1 & 0 & 0 & 0 & -0.291 & -0.291 & -0.291 \end{bmatrix}$$

$$\vec{x}_4 = (0, 1, 1)^T, \vec{x}_4' = (-0.054, 0.548, 1)^T$$

$$A_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0.548 & 0.548 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0.054 & 0.054 \end{bmatrix}$$

$$\vec{x}_5 = (0.5, 0.5, 1)^T, \vec{x}_5' = (0.228, 0.174, 1)^T$$

$$A_5 = \begin{bmatrix} 0 & 0 & 0 & -0.5 & -0.5 & -1 & 0.087 & 0.087 & 0.5 \\ 0.5 & 0.5 & 1 & 0 & 0 & 0 & -0.119 & -0.119 & -0.5 \end{bmatrix}$$

$$A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0.086 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -0.037 \\ 0 & 0 & 0 & -1 & 0 & -1 & -0.107 & 0 & -0.107 \\ 1 & 0 & 1 & 0 & 0 & 0 & -0.49 & 0 & -0.49 \\ 0 & 0 & 0 & -1 & -1 & -1 & 0.401 & 0.401 & 0.401 \\ 1 & 1 & 1 & 0 & 0 & 0 & -0.291 & -0.291 & -0.291 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0.548 & 0.548 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0.054 & 0.054 \\ 0 & 0 & 0 & -0.5 & -0.5 & -1 & 0.087 & 0.087 & 0.174 \\ 0.5 & 0.5 & 1 & 0 & 0 & 0 & -0.119 & -0.119 & -0.228 \end{bmatrix}$$

$$b. A = UDV^T$$

\vec{h} is the last column of V .

$$\vec{h} = [0.3030, -0.1164, 0.0598, -0.1552, 0.2828, 0.0627, -0.0820, -0.0141, 0.8424]^T$$

```

1 clc;clear;
2 A = [0, 0, 0, 0, 0, -1, 0, 0, 0.086;
3     0, 0, 1, 0, 0, 0, 0, 0, -0.037;
4     0, 0, 0, -1, 0, -1, -0.107, 0, -0.107;
5     1, 0, 1, 0, 0, 0, -0.49, 0, -0.49;
6     0, 0, 0, -1, -1, -1, 0.401, 0.401, 0.401;
7     1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
8     0, 0, 0, 0, -1, -1, 0, 0.548, 0.548;
9     0, 1, 1, 0, 0, 0, 0, 0.054, 0.054;
10    0, 0, 0, -0.5, -0.5, -1, 0.087, 0.087, 0.174;
11    0.5, 0.5, 1, 0, 0, 0, -0.119, -0.119, -0.238];
12 [U, D, V] = svd(A);
13 disp(V);
14 h = V(:,9);
15 disp(h);

```

$V =$

$$\begin{bmatrix} -0.2888 & 0.3380 & 0.3243 & 0.4954 & -0.0999 & 0.4885 & -0.2080 & 0.2609 & 0.3030 \\ -0.2595 & 0.3333 & -0.3681 & -0.5362 & -0.4041 & 0.4365 & 0.1678 & 0.0535 & -0.1164 \\ -0.4682 & 0.5865 & -0.1810 & -0.0107 & 0.3414 & -0.5252 & -0.0751 & -0.0479 & 0.0598 \\ -0.3014 & -0.2869 & -0.4375 & 0.4532 & -0.5508 & -0.2678 & -0.1650 & 0.0606 & -0.1552 \\ -0.3380 & -0.2906 & 0.4171 & -0.3963 & -0.2543 & 0.3280 & 0.0887 & 0.3790 & 0.3828 \\ -0.5429 & -0.5049 & -0.2257 & -0.0037 & 0.5172 & 0.3338 & 0.1019 & -0.0779 & 0.0627 \\ 0.1666 & -0.0668 & -0.2341 & -0.2381 & 0.2100 & 0.0435 & -0.7189 & 0.5355 & -0.0820 \\ 0.1880 & 0.0668 & -0.2570 & 0.2186 & 0.1685 & -0.0484 & 0.5996 & 0.6753 & -0.0141 \\ 0.2591 & -0.0057 & -0.4351 & -0.0072 & -0.0453 & 0.0002 & -0.0349 & -0.1749 & 0.8424 \end{bmatrix}$$

$$c. \vec{h} = \begin{pmatrix} \vec{h}^1 \\ \vec{h}^2 \\ \vec{h}^3 \end{pmatrix}, H = \begin{pmatrix} \vec{h}^{1T} \\ \vec{h}^{2T} \\ \vec{h}^{3T} \end{pmatrix} = \begin{bmatrix} 0.3030 & -0.1164 & 0.0598 \\ -0.1552 & 0.2828 & 0.0627 \\ -0.0820 & -0.0141 & 0.8424 \end{bmatrix}$$

d.

```

1 clc;clear;
2 A = [0, 0, 0, 0, 0, -1, 0, 0, 0.086;
3     0, 0, 1, 0, 0, 0, 0, 0, -0.037;
4     0, 0, 0, -1, 0, -1, -0.107, 0, -0.107;
5     1, 0, 1, 0, 0, 0, -0.49, 0, -0.49;
6     0, 0, 0, -1, -1, 0.401, 0.401, 0.401;
7     1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
8     0, 0, 0, 0, -1, -1, 0, 0.548, 0.548;
9     0, 1, 1, 0, 0, 0, 0, 0.054, 0.054;
10    0, 0, 0, -0.5, -0.5, -1, 0.087, 0.087, 0.174;
11    0.5, 0.5, 1, 0, 0, 0, -0.119, -0.119, -0.238];
12 [U, D, V] = svd(A);
13 % disp(V);
14 h = V(:,9);           norm ||Ah|| = 0.0710
15 % disp(h);
16 loss = norm(A*h);
17 disp(loss);

```

命令行窗口

0.0710

2. a) reprojection error function:

$$e = \sum_i d(x_i, \hat{x}_i)^2 + d(x_i^!, \hat{x}_i^!)^2 \text{ subject to } \hat{x}_i^! = \hat{H} \hat{x}_i \forall i$$

x_i is the true coordinate $\Rightarrow d(x_i, \hat{x}_i)^2 = 0 \forall i$

$$e = \sum_i d(x_i, \hat{x}_i)^2 + d(x_i^!, \hat{x}_i^!)^2 = 0.0080$$

```
1 clc;clear;
2 H = [0.3030, -0.1164, 0.0598;
3     -0.1552, 0.3828, 0.0627;
4     -0.0820, -0.0141, 0.8424];
5 H_hat = [0.455800, -0.137164, 0.064297;
6     -0.187786, 0.577227, 0.065843;
7     0.025337, 0.135953, 1.045104];
8 x1 = [0, 0, 1]';
9 x1p = [0.037, 0.086, 1]';
10 x2 = [1, 0, 1]';
11 x2p = [0.49, -0.107, 1]';
12 x3 = [1, 1, 1]';
13 x3p = [0.291, 0.401, 1]';
14 x4 = [0, 1, 1]';
15 x4p = [-0.054, 0.548, 1]';
16 x5 = [0.5, 0.5, 1]';
17 x5p = [0.238, 0.174, 1]';
18 x6 = [2, 1, 1]';
19 x6p = [0.951, -0.959, 1]';
20
21 X = [x1 x2 x3 x4 x5];
22 Xp = [x1p x2p x3p x4p x5p];
23 Xp_hat = H*X;
24 e = 0;
25 for i=1:length(X)
26     Xp_hat(:,i) = [Xp_hat(1,i)/Xp_hat(3,i);
27                     Xp_hat(2,i)/Xp_hat(3,i);
28                     1];
29     e = e + ((Xp_hat(1,i) - Xp(1,i))^2 + (Xp_hat(2,i) - Xp(2,i))^2);
30 end
31 disp(Xp_hat);
32 disp(e)
```

命令行窗口

0.0710	0.4771	0.3302	-0.0683	0.1927
0.0744	-0.1216	0.3890	0.5378	0.2222
1.0000	1.0000	1.0000	1.0000	1.0000

0.0080

fx >>

b) The reprojection error of \hat{f}_1 is 0.0074

```
1 clc;clear;
2 H = [0.3030, -0.1164, 0.0598;
3      -0.1552, 0.3828, 0.0627;
4      -0.0820, -0.0141, 0.8424];
5 H_hat = [0.455800, -0.137164, 0.064297;
6      -0.187786, 0.577227, 0.065843;
7      0.025337, 0.135953, 1.045104];
8 x1 = [0, 0, 1]';
9 x1p = [0.037, 0.086, 1]';
10 x2 = [1, 0, 1]';
11 x2p = [0.49, -0.107, 1]';
12 x3 = [1, 1, 1]';
13 x3p = [0.291, 0.401, 1]';
14 x4 = [0, 1, 1]';
15 x4p = [-0.054, 0.548, 1]';
16 x5 = [0.5, 0.5, 1]';
17 x5p = [0.238, 0.174, 1]';
18 x6 = [2, 1, 1]';
19 x6p = [0.951, -0.959, 1]';

20 X = [x1 x2 x3 x4 x5];
21 Xp = [x1p x2p x3p x4p x5p];
22 Xp_hat = H_hat*X;
23 e = 0;
24 for i=1:length(X)
25     Xp_hat(:,i) = [Xp_hat(1,i)/Xp_hat(3,i);
26                      Xp_hat(2,i)/Xp_hat(3,i);
27                      1];
28     e = e + ((Xp_hat(1,i) - Xp(1,i))^2 + (Xp_hat(2,i) - Xp(2,i))^2);
29 end
30 disp(Xp_hat);
31 disp(e)
```

命令行窗口

0.0615	0.4859	0.3174	-0.0617	0.1986
0.0630	-0.1139	0.3774	0.5445	0.2315
1.0000	1.0000	1.0000	1.0000	1.0000

0.0074

fx >>

$$3. a) \vec{x}_b = (2, 1, 1)^T, \vec{x}'_b = (0.951, -0.359, 1)^T$$

$$H_1 = \begin{bmatrix} 0.3413 & -0.0813 & 0.0279 \\ -0.1454 & 0.4773 & 0.0648 \\ 0 & 0.2358 & 0.7533 \end{bmatrix} \quad S_1 = \{1, 2, 3, 4\}$$

$$H_2 = \begin{bmatrix} 0 & -0.1614 & 0.1614 \\ 0 & -0.1180 & 0.1180 \\ 0 & -0.6782 & 0.6782 \end{bmatrix} \quad S_2 = \{3, 4, 5, 6\}$$

```

1 clc;clear;
2 A1 = [0, 0, 0, 0, 0, -1, 0, 0, 0.086;
3     0, 0, 1, 0, 0, 0, 0, 0, -0.037;
4     0, 0, 0, -1, 0, -1, -0.107, 0, -0.107;
5     1, 0, 1, 0, 0, 0, -0.49, 0, -0.49;
6     0, 0, 0, -1, -1, 0.401, 0.401, 0.401;
7     1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
8     0, 0, 0, 0, -1, -1, 0, 0.548, 0.548;
9     0, 1, 1, 0, 0, 0, 0, 0.054, 0.054];
10    [U1, D1, V1] = svd(A1);
11    h1 = V1(:, 9);
12    % disp(h1);
13    h1 = h1';
14    H1 = [h1(1:3); h1(4:6); h1(7:9)];
15    disp(H1)

16
17 A2 = [0, 0, 0, -1, -1, 0.401, 0.401, 0.401;
18     1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
19     0, 0, 0, 0, -1, -1, 0, 0.548, 0.548;
20     0, 1, 1, 0, 0, 0, 0, 0.054, 0.054;
21     0, 0, 0, -0.5, -0.5, -1, 0.087, 0.087, 0.174;
22     0.5, 0.5, 1, 0, 0, 0, -0.119, -0.119, -0.238;
23     0, 0, 0, -2, -1, -1, -1.918, -0.959, -0.959;
24     2, 1, 1, 0, 0, 0, -1.902, -0.951, -0.951];
25    [U2, D2, V2] = svd(A2);
26    h2 = V2(:, 9);
27    % disp(h2);
28    h2 = h2';
29    H2 = [h2(1:3); h2(4:6); h2(7:9)];
30    disp(H2)

```

命令行窗口

$$H_1 = \begin{bmatrix} 0.3413 & -0.0813 & 0.0279 \\ -0.1454 & 0.4773 & 0.0648 \\ 0.0000 & 0.2358 & 0.7533 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0.0000 & -0.1614 & 0.1614 \\ -0.0000 & -0.1180 & 0.1180 \\ -0.0000 & -0.6782 & 0.6782 \end{bmatrix}$$

$$b) \quad S_1 = \{1, 2, 3, 4\}$$

$$S_2 = \{3, 5\}$$

```
1clc;clear;
2x1 = [0, 0, 1]';
3x1p = [0.037, 0.086, 1]';
4x2 = [1, 0, 1]';
5x2p = [0.49, -0.107, 1]';
6x3 = [1, 1, 1]';
7x3p = [0.291, 0.401, 1]';
8x4 = [0, 1, 1]';
9x4p = [-0.054, 0.548, 1]';
10x5 = [0.5, 0.5, 1]';
11x5p = [0.238, 0.174, 1]';
12x6 = [2, 1, 1]';
13x6p = [0.951, -0.959, 1]';
14A1 = [0, 0, 0, 0, 0, -1, 0, 0, 0.086;
15          0, 0, 1, 0, 0, 0, 0, 0, -0.037;
16          0, 0, 0, -1, 0, -1, -0.107, 0, -0.107;
17          1, 0, 1, 0, 0, 0, -0.49, 0, -0.49;
18          0, 0, 0, -1, -1, -1, 0.401, 0.401, 0.401;
19          1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
20          0, 0, 0, 0, -1, -1, 0, 0.548, 0.548;
21          0, 1, 1, 0, 0, 0, 0, 0.054, 0.054];
22[U1, D1, V1] = svd(A1);
23h1 = V1(:,9);
24% disp(h1);
25h1 = h1';
26H1 = [h1(1:3);h1(4:6);h1(7:9)];
27disp(H1)
28
29A2 = [0, 0, 0, -1, -1, -1, 0.401, 0.401, 0.401;
30          1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
31          0, 0, 0, -1, -1, 0, 0.548, 0.548;
32          0, 1, 1, 0, 0, 0, 0.054, 0.054;
33          0, 0, 0, -0.5, -0.5, -1, 0.087, 0.087, 0.174;
34          0.5, 0.5, 1, 0, 0, 0, -0.119, -0.119, -0.238;
35          0, 0, 0, -2, -1, -1, -1.918, -0.959, -0.959;
36          2, 1, 1, 0, 0, 0, -1.902, -0.951, -0.951];
37[U2, D2, V2] = svd(A2);
38h2 = V2(:,9);
39% disp(h2);
40h2 = h2';
41H2 = [h2(1:3);h2(4:6);h2(7:9)];
42disp(H2)
43
44X_total = [x1 x2 x3 x4 x5 x6];
45Xp_total = [x1p x2p x3p x4p x5p x6p];
46
47Xp_total_hat1 = H1*X_total;
48e1 = [];
49for i=1:length(X_total)
50    Xp_total_hat1(:,i) = [Xp_total_hat1(1,i)/Xp_total_hat1(3,i);
51                           Xp_total_hat1(2,i)/Xp_total_hat1(3,i);
52                           1];
53    e1 = [e1 sqrt((Xp_total_hat1(1,i) - Xp_total(1,i))^2 + (Xp_total_hat1(2,i) - Xp_total(2,i))^2)];
54end
55disp(Xp_total_hat1);
56disp(e1)
```

```
58 Xp_hat2 = H2*X_total;
59 e2 = [];
60 for i=1:length(X_total)
61     Xp_hat2(:,i) = [Xp_hat2(1,i)/Xp_hat2(3,i);
62                     Xp_hat2(2,i)/Xp_hat2(3,i);
63                     1];
64     e2 = [e2 sqrt((Xp_hat2(1,i) - Xp_total(1,i))^2 + (Xp_hat2(2,i) - Xp_total(2,i))^2)];
65 end
66 % disp(Xp_hat2);
67 disp(e2);
```

命令行窗口

distances in S1: 0.0000 0.0000 0.0000 0.0000 0.1071 1.2532

distances in S2: 0.2194 0.3774 0.0654 0.1600 0.0000 1.1803

fx >>

The distances marked in blue is the inlier points (threshold t=0.1).