Assignment 3

1. DLT

a. $A_i\vec{h} = 0$ can be written as

$$\begin{bmatrix} \vec{0}^T & -w_i'\vec{x}_i^T & y_i'\vec{x}_i^T \\ w_i'\vec{x}_i^T & \vec{0}^T & -x_i'\vec{x}_i^T \end{bmatrix} \begin{pmatrix} \vec{h}^1 \\ \vec{h}^2 \\ \vec{h}^3 \end{pmatrix} = \vec{0}.$$

$\vec{x}_1 = (0,0,1)^T$, $\vec{x}_1' = (0.037, 0.086, 1)^T$.

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0.086 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -0.037 \end{bmatrix}$$

$\vec{x}_2 = (1,0,1)^T$, $\vec{x}_2' = (0.49, -0.107, 1)^T$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & -1 & -0.107 & 0 & -0.107 \\ 1 & 0 & 1 & 0 & 0 & 0 & -0.49 & 0 & -0.49 \end{bmatrix}$$

$\vec{x}_3 = (1,1,1)^T$, $\vec{x}_3' = (0.291, 0.401, 1)^T$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 & -1 & -1 & -1 & 0.401 & 0.401 & 0.401 \\ 1 & 1 & 1 & 0 & 0 & 0 & -0.291 & -0.291 & -0.291 \end{bmatrix}$$

$\vec{x}_4 = (0,1,1)^T$, $\vec{x}_4' = (-0.054, 0.548, 1)$

$$A_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0.548 & 0.548 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0.054 & 0.054 \end{bmatrix}$$

$\vec{x}_5 = (0.5, 0.5, 1)^T$, $\vec{x}_5' = (0.238, 0.174, 1)^T$

$$A_5 = \begin{bmatrix} 0 & 0 & 0 & -0.5 & -0.5 & -1 & 0.087 & 0.087 & 0.5 \\ 0.5 & 0.5 & 1 & 0 & 0 & 0 & -0.119 & -0.119 & -0.5 \end{bmatrix}$$

$$A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0.086 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -0.037 \\ 0 & 0 & 0 & -1 & 0 & -1 & -0.107 & 0 & -0.107 \\ 1 & 0 & 1 & 0 & 0 & 0 & -0.49 & 0 & -0.49 \\ 0 & 0 & 0 & -1 & -1 & -1 & 0.401 & 0.401 & 0.401 \\ 1 & 1 & 1 & 0 & 0 & 0 & -0.291 & -0.291 & -0.291 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0.548 & 0.548 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0.054 & 0.054 \\ 0 & 0 & 0 & -0.5 & -0.5 & -1 & 0.087 & 0.087 & 0.174 \\ 0.5 & 0.5 & 1 & 0 & 0 & 0 & -0.119 & -0.119 & -0.238 \end{bmatrix}$$

b. $A = UDV^T$

$\vec{h}$ is the last column of V.

$\vec{h} = [0.3030, -0.1164, 0.0598, -0.1552, 0.2828, 0.0627, -0.0820, -0.0141, 0.8424]^T$

```
clc;clear;
A = [0, 0, 0, 0, 0, -1, 0, 0, 0.086;
     0, 0, 1, 0, 0, 0, 0, 0, -0.037;
     0, 0, 0, -1, 0, -1, -0.107, 0, -0.107;
     1, 0, 1, 0, 0, 0, -0.49, 0, -0.49;
     0, 0, 0, -1, -1, -1, 0.401, 0.401, 0.401;
     1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
     0, 0, 0, 0, -1, -1, 0, 0.548, 0.548;
     0, 1, 1, 0, 0, 0, 0, 0.054, 0.054;
     0, 0, 0, -0.5, -0.5, -1, 0.087, 0.087, 0.174;
     0.5, 0.5, 1, 0, 0, 0, -0.119, -0.119, -0.238];
[U, D, V] = svd(A);
disp(V);
h = V(:,9);
disp(h);
```

$$V = \begin{bmatrix} -0.2888 & 0.3380 & 0.3243 & 0.4954 & -0.0999 & 0.4885 & -0.2080 & 0.2609 & 0.3030 \\ -0.2595 & 0.3333 & -0.3681 & -0.5362 & -0.4041 & 0.4365 & 0.1678 & 0.0535 & -0.1164 \\ -0.4682 & 0.5865 & -0.1810 & -0.0107 & 0.3414 & -0.5252 & -0.0751 & -0.0479 & 0.0598 \\ -0.3014 & -0.2869 & -0.4375 & 0.4532 & -0.5508 & -0.2678 & -0.1650 & 0.0606 & -0.1552 \\ -0.3380 & -0.2906 & 0.4171 & -0.3963 & -0.2543 & -0.3280 & 0.0887 & 0.3790 & 0.3828 \\ -0.5429 & -0.5049 & -0.2257 & -0.0037 & 0.5172 & 0.3338 & 0.1019 & -0.0779 & 0.0627 \\ 0.1666 & -0.0668 & -0.2341 & -0.2381 & 0.2100 & 0.0435 & -0.7189 & 0.5355 & -0.0820 \\ 0.1880 & 0.0668 & -0.2570 & 0.2186 & 0.1685 & -0.0484 & 0.5996 & 0.6753 & -0.0141 \\ 0.2591 & -0.0057 & -0.4351 & -0.0072 & -0.0453 & 0.0002 & -0.0349 & -0.1749 & 0.8424 \end{bmatrix}$$

c. $\vec{h} = \begin{pmatrix} \vec{h}^1 \\ \vec{h}^2 \\ \vec{h}^3 \end{pmatrix}$, $H = \begin{pmatrix} \vec{h}^{1T} \\ \vec{h}^{2T} \\ \vec{h}^{3T} \end{pmatrix} = \begin{bmatrix} 0.3030 & -0.1164 & 0.0598 \\ -0.1552 & 0.2828 & 0.0627 \\ -0.0820 & -0.0141 & 0.8424 \end{bmatrix}$

d.

```
clc;clear;
A = [0, 0, 0, 0, 0, -1, 0, 0, 0.086;
     0, 0, 1, 0, 0, 0, 0, 0, -0.037;
     0, 0, 0, -1, 0, -1, -0.107, 0, -0.107;
     1, 0, 1, 0, 0, 0, -0.49, 0, -0.49;
     0, 0, 0, -1, -1, -1, 0.401, 0.401, 0.401;
     1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
     0, 0, 0, 0, -1, -1, 0, 0.548, 0.548;
     0, 1, 1, 0, 0, 0, 0, 0.054, 0.054;
     0, 0, 0, -0.5, -0.5, -1, 0.087, 0.087, 0.174;
     0.5, 0.5, 1, 0, 0, 0, -0.119, -0.119, -0.238];
[U, D, V] = svd(A);
% disp(V);
h = V(:,9);
% disp(h);
loss = norm(A*h);
disp(loss);
```

norm $\|Ah\| = 0.0710$

命令行窗口

```
0.0710
```

2. a) reprojection error function:

$e = \sum_i d(x_i, \hat{x}_i)^2 + d(x_i', \hat{x}_i')^2$ subject to $\hat{x}_i' = \hat{H}\hat{x}_i \ \forall i$.

x_i is the true coordinate $\Rightarrow d(x_i, \hat{x}_i)^2 = 0 \ \forall i$

$e = \sum_i d(x_i, \hat{x}_i)^2 + d(x_i', \hat{x}_i')^2 = 0.0080$

```
clc;clear;
H = [0.3030, -0.1164, 0.0598;
     -0.1552, 0.3828, 0.0627;
     -0.0820, -0.0141, 0.8424];
H_hat = [0.455800, -0.137164, 0.064297;
         -0.187786, 0.577227, 0.065843;
         0.025337, 0.135953, 1.045104];
x1 = [0, 0, 1]';
x1p = [0.037, 0.086, 1]';
x2 = [1, 0, 1]';
x2p = [0.49, -0.107, 1]';
x3 = [1, 1, 1]';
x3p = [0.291, 0.401, 1]';
x4 = [0, 1, 1]';
x4p = [-0.054, 0.548, 1]';
x5 = [0.5, 0.5, 1]';
x5p = [0.238, 0.174, 1]';
x6 = [2, 1, 1]';
x6p = [0.951, -0.959, 1]';

X = [x1 x2 x3 x4 x5];
Xp = [x1p x2p x3p x4p x5p];
Xp_hat = H*X;
e = 0;
for i=1:length(X)
    Xp_hat(:,i) = [Xp_hat(1,i)/Xp_hat(3,i);
                   Xp_hat(2,i)/Xp_hat(3,i);
                   1];
    e = e  + ((Xp_hat(1,i) - Xp(1,i))^2 + (Xp_hat(2,i) - Xp(2,i))^2);
end
disp(Xp_hat);
disp(e)
```

命令行窗口

```
    0.0710    0.4771    0.3302   -0.0683    0.1927
    0.0744   -0.1216    0.3890    0.5378    0.2222
    1.0000    1.0000    1.0000    1.0000    1.0000


    0.0080

fx >>
```

b) The reprojection error of $\hat{H}$ is 0.0074

```
clc;clear;
H = [0.3030, -0.1164, 0.0598;
     -0.1552, 0.3828, 0.0627;
     -0.0820, -0.0141, 0.8424];
H_hat = [0.455800, -0.137164, 0.064297;
         -0.187786, 0.577227, 0.065843;
         0.025337, 0.135953, 1.045104];
x1 = [0, 0, 1]';
x1p = [0.037, 0.086, 1]';
x2 = [1, 0, 1]';
x2p = [0.49, -0.107, 1]';
x3 = [1, 1, 1]';
x3p = [0.291, 0.401, 1]';
x4 = [0, 1, 1]';
x4p = [-0.054, 0.548, 1]';
x5 = [0.5, 0.5, 1]';
x5p = [0.238, 0.174, 1]';
x6 = [2, 1, 1]';
x6p = [0.951, -0.959, 1]';

X = [x1 x2 x3 x4 x5];
Xp = [x1p x2p x3p x4p x5p];
Xp_hat = H_hat*X;
e = 0;
for i=1:length(X)
    Xp_hat(:,i) = [Xp_hat(1,i)/Xp_hat(3,i);
                   Xp_hat(2,i)/Xp_hat(3,i);
                   1];
    e = e  + ((Xp_hat(1,i) - Xp(1,i))^2 + (Xp_hat(2,i) - Xp(2,i))^2);
end
disp(Xp_hat);
disp(e)
```

命令行窗口

```
    0.0615    0.4859    0.3174   -0.0617    0.1986
    0.0630   -0.1139    0.3774    0.5445    0.2315
    1.0000    1.0000    1.0000    1.0000    1.0000

    0.0074

fx >>
```

3. a) $\vec{x_0} = (2,1,1)^T$, $\vec{x_0}' = (0.951, -0.959, 1)^T$

$$H_1 = \begin{bmatrix} 0.3413 & -0.0813 & 0.0279 \\ -0.1454 & 0.4773 & 0.0648 \\ 0 & 0.2358 & 0.7533 \end{bmatrix} \quad S_1 = \{1,2,3,4\}$$

$$H_2 = \begin{bmatrix} 0 & -0.1614 & 0.1614 \\ 0 & -0.1180 & 0.1180 \\ 0 & -0.6782 & 0.6782 \end{bmatrix} \quad S_2 = \{3,4,5,6\}$$

```
clc;clear;
A1 = [0, 0, 0, 0, 0, -1, 0, 0, 0.086;
     0, 0, 1, 0, 0, 0, 0, 0, -0.037;
     0, 0, 0, -1, 0, -1, -0.107, 0, -0.107;
     1, 0, 1, 0, 0, 0, -0.49, 0, -0.49;
     0, 0, 0, -1, -1, -1, 0.401, 0.401, 0.401;
     1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
     0, 0, 0, 0, -1, -1, 0, 0.548, 0.548;
     0, 1, 1, 0, 0, 0, 0, 0.054, 0.054];
[U1, D1, V1] = svd(A1);
h1 = V1(:,9);
% disp(h1);
h1 = h1';
H1 = [h1(1:3);h1(4:6);h1(7:9)];
disp(H1)
```

```

A2 = [0, 0, 0, -1, -1, -1, 0.401, 0.401, 0.401;
     1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
     0, 0, 0, 0, -1, -1, 0, 0.548, 0.548;
     0, 1, 1, 0, 0, 0, 0, 0.054, 0.054;
     0, 0, 0, -0.5, -0.5, -1, 0.087, 0.087, 0.174;
     0.5, 0.5, 1, 0, 0, 0, -0.119, -0.119, -0.238;
     0, 0, 0, -2, -1, -1, -1.918, -0.959, -0.959;
     2, 1, 1, 0, 0, 0, -1.902, -0.951, -0.951];
[U2, D2, V2] = svd(A2);
h2 = V2(:,9);
% disp(h2);
h2 = h2';
H2 = [h2(1:3);h2(4:6);h2(7:9)];
disp(H2)
```

命令行窗口

$$H_1 = \begin{bmatrix} 0.3413 & -0.0813 & 0.0279 \\ -0.1454 & 0.4773 & 0.0648 \\ 0.0000 & 0.2358 & 0.7533 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0.0000 & -0.1614 & 0.1614 \\ -0.0000 & -0.1180 & 0.1180 \\ -0.0000 & -0.6782 & 0.6782 \end{bmatrix}$$

b) $S_1 = \{1,2,3,4\}$

$S_2 = \{3,5\}$

```
clc;clear;
x1 = [0, 0, 1]';
x1p = [0.037, 0.086, 1]';
x2 = [1, 0, 1]';
x2p = [0.49, -0.107, 1]';
x3 = [1, 1, 1]';
x3p = [0.291, 0.401, 1]';
x4 = [0, 1, 1]';
x4p = [-0.054, 0.548, 1]';
x5 = [0.5, 0.5, 1]';
x5p = [0.238, 0.174, 1]';
x6 = [2, 1, 1]';
x6p = [0.951, -0.959, 1]';
A1 = [0, 0, 0, 0, 0, -1, 0, 0, 0.086;
     0, 0, 1, 0, 0, 0, 0, 0, -0.037;
     0, 0, 0, -1, 0, -1, -0.107, 0, -0.107;
     1, 0, 1, 0, 0, 0, -0.49, 0, -0.49;
     0, 0, 0, -1, -1, -1, 0.401, 0.401, 0.401;
     1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
     0, 0, 0, 0, -1, -1, 0, 0.548, 0.548;
     0, 1, 1, 0, 0, 0, 0, 0.054, 0.054];
[U1, D1, V1] = svd(A1);
```

```
h1 = V1(:,9);
% disp(h1);
h1 = h1';
H1 = [h1(1:3);h1(4:6);h1(7:9)];
disp(H1)

A2 = [0, 0, 0, -1, -1, -1, 0.401, 0.401, 0.401;
     1, 1, 1, 0, 0, 0, -0.291, -0.291, -0.291;
     0, 0, 0, 0, -1, -1, 0, 0.548, 0.548;
     0, 1, 1, 0, 0, 0, 0, 0.054, 0.054;
     0, 0, 0, -0.5, -0.5, -1, 0.087, 0.087, 0.174;
     0.5, 0.5, 1, 0, 0, 0, -0.119, -0.119, -0.238;
     0, 0, 0, -2, -1, -1, -1.918, -0.959, -0.959;
     2, 1, 1, 0, 0, 0, -1.902, -0.951, -0.951];
[U2, D2, V2] = svd(A2);
h2 = V2(:,9);
% disp(h2);
h2 = h2';
H2 = [h2(1:3);h2(4:6);h2(7:9)];
disp(H2)
```

```
X_total = [x1 x2 x3 x4 x5 x6];
Xp_total = [x1p x2p x3p x4p x5p x6p];

Xp_total_hat1 = H1*X_total;
e1 = [];
for i=1:length(X_total)
    Xp_total_hat1(:,i) = [Xp_total_hat1(1,i)/Xp_total_hat1(3,i);
                          Xp_total_hat1(2,i)/Xp_total_hat1(3,i);
                          1];
    e1 = [e1 sqrt((Xp_total_hat1(1,i) - Xp_total(1,i))^2 + (Xp_total_hat1(2,i) - Xp_total(2,i))^2)];
end
disp(Xp_total_hat1);
disp(e1)
```

```
58  Xp_hat2 = H2*X_total;
59  e2 = [];
60  for i=1:length(X_total)
61      Xp_hat2(:,i) = [Xp_hat2(1,i)/Xp_hat2(3,i);
62                      Xp_hat2(2,i)/Xp_hat2(3,i);
63                      1];
64      e2 = [e2 sqrt((Xp_hat2(1,i) - Xp_total(1,i))^2 + (Xp_hat2(2,i) - Xp_total(2,i))^2)];
65  end
66  % disp(Xp_hat2);
67  disp(e2);
```

命令行窗口

distances in S_1 : 0.0000 0.0000 0.0000 0.0000 0.1071 1.2532

distances in S_2 : 0.2194 0.3774 0.0654 0.1600 0.0000 1.1803

fx >>

The distances marked in blue is the inlier points (threshold $t = 0.1$).