1. a. $P = K[R^T | 0]$

$P' = K[R^T | -R^T {}^w t_{p'}]$

$$\Rightarrow P = \begin{bmatrix} 100 & 0 & 320 & 0 \\ 0 & 100 & 240 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad P' = \begin{bmatrix} 100 & 0 & 320 & -100 \\ 0 & 100 & 240 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$x_1 = PX_1 = [320, 240]^T$, $x_1' = P'X_1 = [220, 240]^T$

$x_2 = PX_2 = [420, 240]^T$, $x_2' = P'X_2 = [320, 240]^T$.

$x_3 = PX_3 = [370, 240]^T$, $x_3' = P'X_3 = [320, 240]^T$.

let $x = [x_1 \ x_2 \ x_3]$, $x' = [x_1' \ x_2' \ x_3']$

adding noise: $x_n = x + N$, $x_n' = x' + N'$

$x_i$ and $x_i'$ with noise:

$x_1 = [319.8932, 241.5326]^T$, $x_1' = [221.1006, 241.5442]^T$.

$x_2 = [420.3714, 239.7744]^T$, $x_2' = [318.5084, 239.2517]^T$.

$x_3 = [368.9109, 240.0326]^T$, $x_3' = [322.3505, 239.3844]^T$.

2. a. let $p^{iT}$ be the rows of $P$, $p'^{iT}$ be the rows of $P'$.

$$A_1 = \begin{bmatrix} x_{1x}p^{3T} - p^{1T} \\ x_{1y}p^{3T} - p^{2T} \\ x'_{1x}p'^{3T} - p'^{1T} \\ x'_{1y}p'^{3T} - p'^{2T} \end{bmatrix} = \begin{bmatrix} -100 & 0 & -0.1068 & 0 \\ 0 & -100 & 1.5326 & 0 \\ -100 & 0 & -98.8994 & 100 \\ 0 & -100 & 1.5442 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} x_{2x}p^{3T} - p^{1T} \\ x_{2y}p^{3T} - p^{2T} \\ x'_{2x}p'^{3T} - p'^{1T} \\ x'_{2y}p'^{3T} - p'^{2T} \end{bmatrix} = \begin{bmatrix} -100 & 0 & 100.3714 & 0 \\ 0 & -100 & -0.2256 & 0 \\ -100 & 0 & -1.4916 & 100 \\ 0 & -100 & -0.7423 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} x_{3x}p^{3T} - p^{1T} \\ x_{3y}p^{3T} - p^{2T} \\ x'_{3x}p'^{3T} - p'^{1T} \\ x'_{3y}p'^{3T} - p'^{2T} \end{bmatrix} = \begin{bmatrix} -100 & 0 & 48.9109 & 0 \\ 0 & -100 & 0.0326 & 0 \\ -100 & 0 & 2.3505 & 100 \\ 0 & -100 & -0.6156 & 0 \end{bmatrix}$$

b. $[U_i, S_i, V_i] = \text{svd}(A_i)$

reconstructed $X_i$ is the last column of $V_i$

After projective reconstruction:

$X_{R1} = [-0.0011, 0.0156, 1.0122]^T$

$X_{R2} = [0.9854, -0.0048, 0.9817]^T$

$X_{R3} = [1.0505, -0.0063, 2.1477]^T$

3. a. choose three points as the three infinity points projection

$a_1 = [50, 50]^T$, $a_2 = [152, 150]^T$, $a_3 = [250, 248]^T$.

construct $A_i = \begin{bmatrix} a_{ix} P^{3T} - P^{1T} \\ a_{iy} P^{3T} - P^{2T} \\ a_{ix} P'^{3T} - P'^{1T} \\ a_{iy} P'^{3T} - P'^{2T} \end{bmatrix}$

$[U_i, S_i, V_i] = \text{svd}(A_i)$

then infinity point i is the last column of $V_i$

$P_{\infty 1} = 10^{16} \times [2.0007, 1.4079, -0.7410]^T$.

$P_{\infty 2} = 10^{15} \times [-4.6119, -2.4707, 2.7452]^T$

$P_{\infty 3} = 10^{15} \times [4.8818, -0.5579, -6.9739]^T$

$$\begin{bmatrix} P_{\infty 1}^T \\ P_{\infty 2}^T \\ P_{\infty 3}^T \end{bmatrix} \pi_\infty = 0 \quad \Rightarrow \quad \pi_\infty = [0, 0, 0, 1]^T$$

b. $H_A = \begin{bmatrix} I & 0 \\ \pi_\infty & \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$X_{A1} = H_A X_{R1} = [-0.0011, 0.0156, 1.0122]^T$

$X_{A2} = H_A X_{R2} = [0.9854, -0.0048, 0.9817]^T$

$X_{A3} = H_A X_{R3} = [1.0505, -0.0063, 2.1477]^T$

$P_A = P H_A^{-1} = \begin{bmatrix} 100 & 0 & 320 & 0 \\ 0 & 100 & 240 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$P_A' = P' H_A^{-1} = \begin{bmatrix} 100 & 0 & 320 & -100 \\ 0 & 100 & 240 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

4. a. $W^{-1} = KK^T$

$$\Rightarrow W = (KK^T)^{-1} = \begin{bmatrix} 0.0001 & 0 & -0.0320 \\ 0 & 0.0001 & -0.0240 \\ -0.0320 & -0.0240 & 17 \end{bmatrix}$$

b. $M = KR$.

$$AA^T = (M^T W M)^{-1}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_M = \begin{bmatrix} A^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

$X_{M1} = H_M X_{A1} = [-0.0011, 0.0156, 1.0122]^T$.

$X_{M2} = H_M X_{A2} = [0.9854, -0.0048, 0.9817]^T$.

$X_{M3} = H_M X_{A3} = [1.0505, -0.0063, 2.1477]^T$.

$$P_M = P_A H_M^{-1} = \begin{bmatrix} 100 & 0 & 320 & 0 \\ 0 & 100 & 240 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$P_M' = P_A' H_M^{-1} = \begin{bmatrix} 100 & 0 & 320 & -100 \\ 0 & 100 & 240 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

## Contents



```
clc;clear;
```

## Question 1

```
disp("Question 1")
K = [100 0 320;
     0 100 240;
     0 0 1];
R = eye(3);
twp = [0 0 0]';
twpp = [1 0 0]';
P = K*[R' -R'*twp];
Pp = K*[R' -R'*twpp];
disp('P:')
disp(P)
disp("P':")
disp(Pp)
X1 = [0 0 1 1]';
X2 = [1 0 1 1]';
X3 = [1 0 2 1]';
X = [X1 X2 X3];
x = P * X;
xp = Pp * X;
x_inhomo = [];
xp_inhomo = [];
for i=1:3
    x_inhomo(:,end+1) = [x(1,i)/x(3,i) x(2,i)/x(3,i)]';
    xp_inhomo(:,end+1) = [xp(1,i)/xp(3,i) xp(2,i)/xp(3,i)]';
end
disp('x1:')
disp(x_inhomo(:,1))
disp("x1':")
disp(xp_inhomo(:,1))
disp('x2:')
disp(x_inhomo(:,2))
disp("x2':")
disp(xp_inhomo(:,2))
disp('x3:')
disp(x_inhomo(:,3))
disp("x3':")
disp(xp_inhomo(:,3))

disp("xi and xi' with noise:")
N = [-0.1068 0.3714 -1.0891
     1.5326 -0.2256 0.0326];
Np = [1.1006 -1.4916 2.3505;
      1.5442 -0.7423 -0.6156];
x_inhomo = x_inhomo + N;
xp_inhomo = xp_inhomo + Np;
disp('x1:')
disp(x_inhomo(:,1))
disp("x1':")
disp(xp_inhomo(:,1))
```

```
disp('x2:')
disp(x_inhomo(:,2))
disp("x2':")
disp(xp_inhomo(:,2))
disp('x3:')
disp(x_inhomo(:,3))
disp("x3':")
disp(xp_inhomo(:,3))
```

```
Question 1
P:
   100     0   320     0
     0   100   240     0
     0     0     1     0

P':
   100     0   320  -100
     0   100   240     0
     0     0     1     0

x1:
   320
   240

x1':
   220
   240

x2:
   420
   240

x2':
   320
   240

x3:
   370
   240

x3':
   320
   240

xi and xi' with noise:
x1:
  319.8932
  241.5326

x1':
  221.1006
  241.5442

x2:
  420.3714
  239.7744

x2':
  318.5084
  239.2577

x3:
  368.9109
  240.0326
```

```
x3':
  322.3505
  239.3844
```

## Question 2

```
disp('Question 2')
A1 = [x_inhomo(1,1)*P(3,:)-P(1,:);
      x_inhomo(2,1)*P(3,:)-P(2,:);
      xp_inhomo(1,1)*Pp(3,:)-Pp(1,:);
      xp_inhomo(2,1)*Pp(3,:)-Pp(2,:)];
A2 = [x_inhomo(1,2)*P(3,:)-P(1,:);
      x_inhomo(2,2)*P(3,:)-P(2,:);
      xp_inhomo(1,2)*Pp(3,:)-Pp(1,:);
      xp_inhomo(2,2)*Pp(3,:)-Pp(2,:)];
A3 = [x_inhomo(1,3)*P(3,:)-P(1,:);
      x_inhomo(2,3)*P(3,:)-P(2,:);
      xp_inhomo(1,3)*Pp(3,:)-Pp(1,:);
      xp_inhomo(2,3)*Pp(3,:)-Pp(2,:)];
disp('A1:')
disp(A1)
disp('A2:')
disp(A2)
disp('A3:')
disp(A3)

[U1,S1,V1] = svd(A1);
X1_rec = V1(:,end);
X1_rec_inhomo = [X1_rec(1)/X1_rec(4) X1_rec(2)/X1_rec(4) X1_rec(3)/X1_rec(4)]';
disp('Reconstructed X1:')
disp(X1_rec_inhomo)

[U2,S2,V2] = svd(A2);
X2_rec = V2(:,end);
X2_rec_inhomo = [X2_rec(1)/X2_rec(4) X2_rec(2)/X2_rec(4) X2_rec(3)/X2_rec(4)]';
disp('Reconstructed X2:')
disp(X2_rec_inhomo)

[U3,S3,V3] = svd(A3);
X3_rec = V3(:,end);
X3_rec_inhomo = [X3_rec(1)/X3_rec(4) X3_rec(2)/X3_rec(4) X3_rec(3)/X3_rec(4)]';
disp('Reconstructed X3:')
disp(X3_rec_inhomo)
```

```
Question 2
A1:
 -100.0000         0   -0.1068         0
         0 -100.0000    1.5326         0
 -100.0000         0  -98.8994  100.0000
         0 -100.0000    1.5442         0

A2:
 -100.0000         0  100.3714         0
         0 -100.0000   -0.2256         0
 -100.0000         0   -1.4916  100.0000
         0 -100.0000   -0.7423         0

A3:
 -100.0000         0   48.9109         0
         0 -100.0000    0.0326         0
```

```
 -100.0000         0    2.3505  100.0000
         0 -100.0000   -0.6156         0

Reconstructed X1:
   -0.0011
    0.0156
    1.0122

Reconstructed X2:
    0.9854
   -0.0048
    0.9817

Reconstructed X3:
    1.0505
   -0.0063
    2.1477
```

## Question 3

```
disp('Question 3:')
x_new = [50 152 250;
         50 150 248];
A1_3 = [x_new(1,1)*P(3,:)-P(1,:);
       x_new(2,1)*P(3,:)-P(2,:);
       x_new(1,1)*Pp(3,:)-Pp(1,:);
       x_new(2,1)*Pp(3,:)-Pp(2,:)];
A2_3 = [x_new(1,2)*P(3,:)-P(1,:);
       x_new(2,2)*P(3,:)-P(2,:);
       x_new(1,2)*Pp(3,:)-Pp(1,:);
       x_new(2,2)*Pp(3,:)-Pp(2,:)];
A3_3 = [x_new(1,3)*P(3,:)-P(1,:);
       x_new(2,3)*P(3,:)-P(2,:);
       x_new(1,3)*Pp(3,:)-Pp(1,:);
       x_new(2,3)*Pp(3,:)-Pp(2,:)];
% disp('A1 new:')
% disp(A1_3)
% disp('A2 new:')
% disp(A2_3)
% disp('A3 new:')
% disp(A3_3)

[U1_3,S1_3,V1_3] = svd(A1_3);
X1_new = V1_3(:,end);
X1_new = [X1_new(1)/X1_new(4) X1_new(2)/X1_new(4) X1_new(3)/X1_new(4)]';
disp('Infinity point 1:')
disp(X1_new)

[U2_3,S2_3,V2_3] = svd(A2_3);
X2_new = V2_3(:,end);
X2_new = [X2_new(1)/X2_new(4) X2_new(2)/X2_new(4) X2_new(3)/X2_new(4)]';
disp('Infinity point 2:')
disp(X2_new)

[U3_3,S3_3,V3_3] = svd(A3_3);
X3_new = V3_3(:,end);
X3_new = [X3_new(1)/X3_new(4) X3_new(2)/X3_new(4) X3_new(3)/X3_new(4)]';
disp('Infinity point 3:')
disp(X3_new)

X_new = [X1_new' 1; X2_new' 1; X3_new' 1];
[Unew,Snew,Vnew] = svd(X_new);
pi_infinity = Vnew(:,end);
```

```
% pi_infinity = null(X_new);
disp('Plane at infinity:')
disp(pi_infinity)
% disp(X_new*pi_infinity)

HA = [eye(3) [0 0 0]';
     pi_infinity'];
disp('HA:')
disp(HA)
XA1 = HA*X1_rec;
XA1_inhomo = [XA1(1)/XA1(4) XA1(2)/XA1(4) XA1(3)/XA1(4)]';
XA2 = HA*X2_rec;
XA2_inhomo = [XA2(1)/XA2(4) XA2(2)/XA2(4) XA2(3)/XA2(4)]';
XA3 = HA*X3_rec;
XA3_inhomo = [XA3(1)/XA3(4) XA3(2)/XA3(4) XA3(3)/XA3(4)]';
PA = P/HA;
PAp = Pp/HA;
disp('XA1:')
disp(XA1_inhomo)
disp('XA2:')
disp(XA2_inhomo)
disp('XA3:')
disp(XA3_inhomo)
disp('PA:')
disp(PA)
disp("PA':")
disp(PAp)
```

```
Question 3:
Infinity point 1:
   1.0e+16 *

    2.0007
    1.4079
   -0.7410

Infinity point 2:
   1.0e+15 *

   -4.6119
   -2.4707
    2.7452

Infinity point 3:
   1.0e+15 *

    4.8818
   -0.5579
   -6.9739

Plane at infinity:
   -0.0000
    0.0000
   -0.0000
    1.0000

HA:
    1.0000         0         0         0
         0    1.0000         0         0
         0         0    1.0000         0
   -0.0000    0.0000   -0.0000    1.0000

XA1:
   -0.0011
```

```
    0.0156
    1.0122

XA2:
    0.9854
   -0.0048
    0.9817

XA3:
    1.0505
   -0.0063
    2.1477

PA:
   100     0   320     0
     0   100   240     0
     0     0     1     0

PA':
  100.0000    0.0000  320.0000 -100.0000
         0  100.0000  240.0000         0
         0         0    1.0000         0
```

## Question 4

```
disp('Question 4:')
w = inv(K*K');
disp('w:')
disp(w)
M = K*R;
AAt = inv(M'*w*M);
A_4 = chol(AAt);
% disp(A_4)
HM = [inv(A_4) [0 0 0]';
      0 0 0 1];
XM1 = HM * XA1;
XM1_inhomo = [XM1(1)/XM1(4) XM1(2)/XM1(4) XM1(3)/XM1(4)]';
XM2 = HM * XA2;
XM2_inhomo = [XM2(1)/XM2(4) XM2(2)/XM2(4) XM2(3)/XM2(4)]';
XM3 = HM * XA3;
XM3_inhomo = [XM3(1)/XM3(4) XM3(2)/XM3(4) XM3(3)/XM3(4)]';
PM = PA/HM;
PMp = PAp/HM;
disp('XM1:')
disp(XM1_inhomo)
disp('XM2:')
disp(XM2_inhomo)
disp('XM3:')
disp(XM3_inhomo)
disp('PM:')
disp(PM)
disp("PM':")
disp(PMp)
```

```
Question 4:
w:
    0.0001   -0.0000   -0.0320
   -0.0000    0.0001   -0.0240
   -0.0320   -0.0240   17.0000

XM1:
```

```
   -0.0011
    0.0156
    1.0122

XM2:
    0.9854
   -0.0048
    0.9817

XM3:
    1.0505
   -0.0063
    2.1477

PM:
  100.0000    0.0000  320.0000         0
         0  100.0000  240.0000         0
         0         0    1.0000         0

PM':
  100.0000    0.0000  320.0000 -100.0000
         0  100.0000  240.0000         0
         0         0    1.0000         0
```

---

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