EN.601.461/661

Computer Vision

Assignment #1

Due date: September 17th 11:59PM

5 marks (10% per day late submission)

Instructions: Answer your question on blank sheets of paper. Use a new sheet for each question (i.e. don't answer two questions on the same sheet. It will be easier to grade). Scan or take pictures of your solution and submit it on Gradescope.

1. Show that the points

$$\begin{bmatrix} \frac{1}{2} \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ \frac{1}{3} \\ \frac{4}{3} \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

   of the real projective plane are collinear and find an equation of the line containing them.

2. You are given a unit square with coordinates (0,0), (1,0), (1,1) and (0,1). You are also given a projective transformation

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

   that transforms the square.
   a) Calculate the transformed points according to H and draw or sketch the transformed square (draw what the square "looks" like in an image according to H).
   b) From a), calculate two vanishing points, the line at infinity and draw them in your drawing of a)
   c) From b), calculate the transformation H_p for projective rectification
   d) Apply H_p to the points in a)

3. Calculate the vanishing point from length ratios. You are given the homogeneous coordinates of three points $\boldsymbol{p}_1$, $\boldsymbol{p}_2$, and $\boldsymbol{p}_3$, on a Euclidean line (you are free to give these coordinates any value you want, including non-numeric values, i.e. a letter instead of a number). You measure the corresponding image coordinates to be $\boldsymbol{p}'_1$, $\boldsymbol{p}'_2$ and $\boldsymbol{p}'_3$ (you are also free to give these coordinates any value you want).
   a) What is the 2x2 homography H that maps $\boldsymbol{p}'_i = H\,\boldsymbol{p}_i$?
   b) What are the image coordinates $\boldsymbol{p}'$ of the point at infinity $\boldsymbol{p} = [1 \quad 0]^T$?

4. Let the points p_1, p_2, p_3 and p_4 be on a projective line. Show that the cross-ratio $\{p_1, p_2, p_3, p_4\} = \frac{\overline{p_1 p_3}}{\overline{p_2 p_3}}$ if p_4 is the point at infinity. (We will cover cross-ratio during lecture Tuesday).