EN.601.461/661

Computer Vision

Assignment #7

Due date: November 5th 11:59PM

5 marks (10% per day late submission)

Instructions: Answer your questions on paper or an electronic/printout document. Use a new sheet for each question (i.e. don't answer two questions on the same sheet). Scan/save your calculations as a pdf document or take pictures of your solution and submit it on Gradescope. You are encouraged to use a calculator or other software to help with math but you **must** include all the steps/printouts of the calculations in your submission. No need to submit code.

1. 3D Projection of a 3D point (1pt).
    a. You have two cameras P and P' in a world frame W. Both cameras have the same calibration matrix
    $$K = \begin{bmatrix} 100 & 0 & 320 \\ 0 & 100 & 240 \\ 0 & 0 & 1 \end{bmatrix}.$$
    The first camera has a rotation and translation ${}^{W}R_{P} = I$ and ${}^{W}\boldsymbol{t}_{P} = [1 \quad 0 \quad 0]^{T}$, the second camera has ${}^{W}R_{P'} = R_{y}\left(\frac{\pi}{2}\right)$ and ${}^{W}\boldsymbol{t}_{P'} = [0 \quad 0 \quad 1]^{T}$, where $R_{y}(\theta)$ indicates a rotation about the axis Y by an angle θ. Calculate the camera matrices P and P'.
    b. What are the image homogeneous coordinates $\boldsymbol{x}$ and $\boldsymbol{x}'$ of the 3D point $\boldsymbol{X} = [1 \quad 0 \quad 1]^{T}$?
    c. What are the homogeneous coordinates of the epipole $\boldsymbol{e}'$?
2. Fundamental matrix (1pt)
    a. What is the fundamental matrix F (0.5pt)?
    b. Validate F with $\boldsymbol{x}$ and $\boldsymbol{x}'$ (0.25pt).
    c. Validate F with $\boldsymbol{e}'$ (0.25pt).
3. Canonical cameras (2pt)
    a. Projective reconstruction and cameras are determined up to a projective transformation H. Given P from question 1, find the matrix H that transforms P to $P_{C} = [I \quad \mathbf{0}]$ (1pt).
    b. Apply H to P' to obtain $P'_{c} = [M \quad \boldsymbol{m}]$ (0.5pt).
    c. Using P_{C} and P'_{c}, validate that the projective equivalent point of $\boldsymbol{X}$ projects to the same $\boldsymbol{x}$ and $\boldsymbol{x}'$ (0.25pt)
    d. Use P'_{c} and the (projective equivalent) coordinates of camera center of the first camera to calculate the epipole $\boldsymbol{e}'$(0.25pt).
4. Retrieving the camera matrices (1pt).
    a. Using the fundamental matrix of question 2, retrieve the (canonical) matrices P_{F} and P'_{F} (0.5pt).
    b. You now have two pairs of canonical camera matrices: P_{C} and P'_{c} from question 3 and now P_{F} and P'_{F}. Compute the fundamental matrix from each pair and show they are equal (it should also be equal to the fundamental matrix of question 1) (0.5pt).