EN.601.461/661

Computer Vision

Assignment #5

Due date: October 22nd 11:59PM

5 marks (10% per day late submission)

Instructions: Answer your questions on paper or an electronic/printout document. Use a new sheet for each question (i.e. don't answer two questions on the same sheet). Scan/save your calculations as a pdf document or take pictures of your solution and submit it on Gradescope. You are encouraged to use a calculator or other software to help with math but you **must** include all the steps/printouts of the calculations in your submission. No need to submit code.

1. 3D points projection (1pt)

   You have the following 6 3D points with coordinates in a "world" frame

   $$ {}^{w}\boldsymbol{X}_1 = [0 \quad 0 \quad 5]^T, {}^{w}\boldsymbol{X}_2 = [1 \quad 0 \quad 7]^T, $$
   $$ {}^{w}\boldsymbol{X}_3 = [1 \quad 1 \quad 8]^T, {}^{w}\boldsymbol{X}_4 = [-6 \quad 8 \quad 8]^T $$
   $$ {}^{w}\boldsymbol{X}_5 = [2 \quad 4 \quad 10]^T, {}^{w}\boldsymbol{X}_6 = [-3 \quad 8 \quad 8]^T $$

   A camera with calibration matrix $K = \begin{bmatrix} 100 & 0 & 320 \\ 0 & 100 & 240 \\ 0 & 0 & 1 \end{bmatrix}$, is placed in the world frame. The position and orientation of the camera frame with respect to the world frame are ${}^{w}R_C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and ${}^{W}\boldsymbol{t_C} = [1 \quad 1 \quad -1]^T$.

   a) Calculate the matrix P
   b) Calculate the image coordinate (inhomogeneous) of the points $\boldsymbol{x}_i$.

2. DLT (2pts)

   Estimation of P from $\boldsymbol{x}_i \leftrightarrow \boldsymbol{X}_i$, by using DLT (use your results from question 1b). Use all 6 points (12 equations).

3. Recover ${}^{w}R_C$, ${}^{W}\boldsymbol{t_C}$ and K, from P (2pts) by using your result of 2. To recover K and ${}^{w}R_C$, you will need a "RQ" decomposition (a matrix A is decomposed in a product of two matrices A=RQ), where R is upper triangular (not to be confused with the rotation matrix) and Q is an orthogonal matrix. Most software will provide you with a "QR" decomposition (A=QR) instead. You will need to find a way around this by using QR decomposition (here are some slides that can help). Also, be aware that several RQ decompositions exist for a given matrix A, such as $A = (RD)(D^{-1}Q)$, where D can be a diagonal matrix with elements ± 1. Whatever result you obtain, you need to find a D that ensures that the diagonal elements of K are positive (as they should). At the end, you should obtain the same matrices ${}^{w}R_C$, ${}^{W}\boldsymbol{t_C}$ and K that you started with in question 1.