1\. a) if $x_2 = k_1 x_1$, $V_0 = \frac{1}{2}x_1^2$

then $\dot{V}_0 = -(k_1x_1+\frac{3}{2}x_1^2+\frac{1}{2}x_1^3)x_1 = -x_1^2(k_1+\frac{3}{2}x_1+\frac{1}{2}x_1^2)$

we want $\dot{V}_0$ to be n.d

so $\frac{1}{2}x_1^2+\frac{3}{2}x_1+k_1$ should be always greater than 0

so $\frac{9}{4}-2k_1<0$ so $k_1>\frac{9}{8}$, $\phi(x_1)=k_1x_1$

let $V = V_0 + \frac{1}{2}(x_2-k_1x_1)^2$ which is p.d

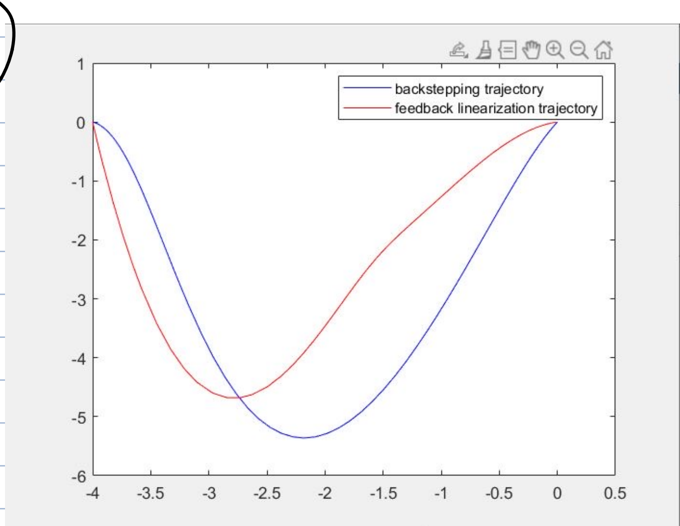
then $\dot{V} = -(k_1x_1+\frac{3}{2}x_1^2+\frac{1}{2}x_1^3)x_1 - x_1\cdot(x_2-k_1x_1)+(x_2-k_1x_1)\left(u+k_1(x_2+\frac{3}{2}x_1^2+\frac{1}{2}x_1^3)\right)$

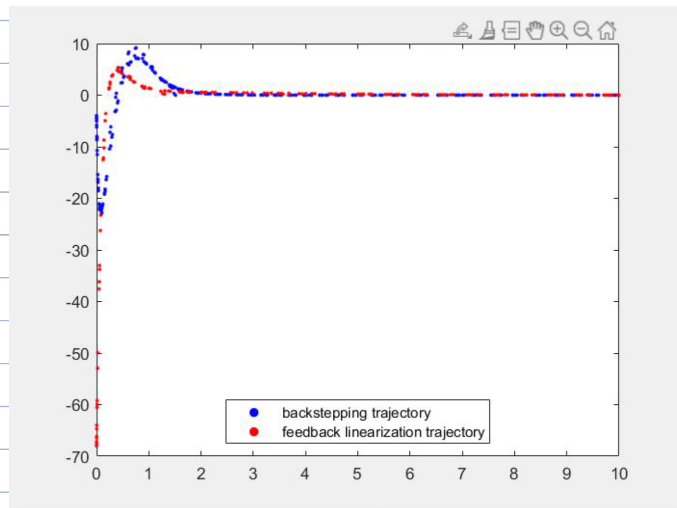
we need $\dot{V}$ to be n.d.

so $u = -k_1(x_2+\frac{3}{2}x_1^2+\frac{1}{2}x_1^3)+x_1-k_2(x_2-k_1x_1)$ $\quad k_1>\frac{9}{8}$

$k_2>0$

b)





↑ control signal .

c) if $u = k_1x_1+k_2x_2$

then $\dot{x}_1 = -x_2-\frac{3}{2}x_1^2-\frac{1}{2}x_1^3$

$\dot{x}_2 = u = k_1x_1+k_2x_2$

let $V=\frac{1}{2}x_1^2+\frac{1}{2}x_2^2$ $\quad \dot{V} = x_1\cdot\dot{x}_1+x_2\cdot\dot{x}_2 = -(x_1x_2+\frac{3}{2}x_1^3+\frac{1}{2}x_1^4)+k_1x_1x_2+k_2x_2^2$

$-(x_1x_2+\frac{3}{2}x_1^3+\frac{1}{2}x_1^4)+k_1x_1x_2+k_2x_2^2 = -\frac{1}{2}x_1^4+k_2x_2^2-\frac{3}{2}x_1^3+(k_1-1)x_1x_2$

so $k_2<0$, $k_1=1$, $\dot{V} = -\frac{1}{2}x_1^4-\frac{3}{2}x_1^3+k_2x_2^2 = -\frac{1}{2}x_1^3(x_1+3)+k_2x_2^2$

When $x_1<-3$ or $x_1>0$, $\dot{V}<0$, $k_2<0$ $k_1=1$, then the system is locally A.S

2 a) let $x=x_1$ $y=x_2$ $\theta=x_3$ $V=x_4$

then $\dot{x}_1 = x_4\cos x_3$ $\qquad e(t)=q(t)-q_d(t)$ $\qquad q:\begin{pmatrix}x_1\\x_2\end{pmatrix}$

$\dot{x}_2 = x_4\cdot\sin x_3$ $\qquad V_0=\frac{1}{2}e^Te$

$\dot{x}_3 = u_1' x_4$ $\quad u_1'=\frac{\tan u_1}{L}$ $\qquad \dot{V}_0 = e^T\dot{e}$ $\quad$ we hope $\dot{e}=-k_0e$ $\quad k_0>0$

$\dot{x}_4 = u_2$ $\qquad \dot{V}_0 = e^T(-k_0e+z)$ $\quad z=k_0e+\dot{e}$ $\quad \dot{z}=k_0\dot{e}+\ddot{e}$

let $V=V_0+\frac{1}{2}z^Tz$ $\qquad \dot{V} = -k_0e^Te + z^T(e+k_0\dot{e}+\ddot{q}-\ddot{q}_d)$

$$z = k_0e + \begin{pmatrix}x_4\cos x_3\\ x_4\sin x_3\end{pmatrix} - \dot{q}_d$$

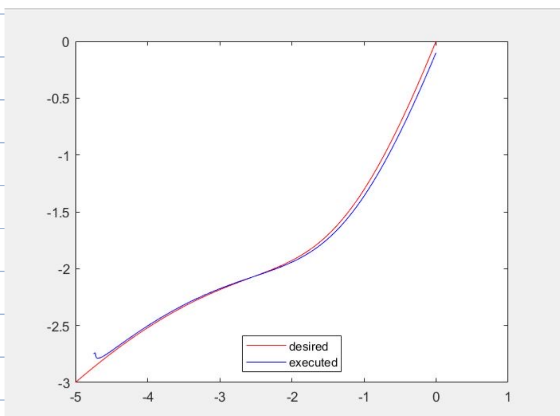
$$\dot{z} = k_0\dot{e} + \begin{pmatrix}\cos x_3 & -\sin x_3\\ \sin x_3 & \cos x_3\end{pmatrix}\begin{pmatrix}u_2\\ x_4^2\cdot u_1'\end{pmatrix} - \ddot{q}_d$$

so $$\begin{bmatrix}u_2\\u_1'\end{bmatrix} = \begin{bmatrix}-\frac{\sin x_3}{x_4^2} & \frac{\cos x_3}{x_4^2}\\ \cos x_3 & \sin x_3\end{bmatrix}\cdot\left(-e-k_0\dot{e}-\ddot{q}_d-k_1 z^T\right)$$

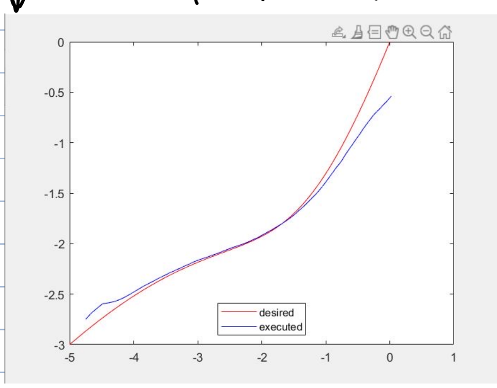
$k_0>0, k_1>0$

$u_1 = \operatorname{atan}(u_1'\cdot L)$

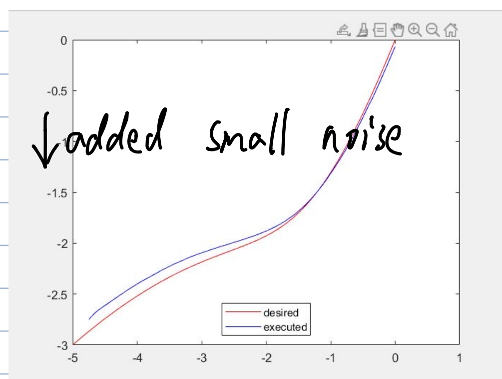
b)



↓added loud noise



↓added small noise



3. let $V_0 = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$, then $\dot{V}_0 = -x_1^2 + x_2(x_1^2 + x_2 + x_3)$

when $x_3 = -x_1^2 - 2x_2$ $\quad \dot{V}_0$ is n.d.

$= \phi(x_1, x_2)$

let $V = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}(x_3 + x_1^2 + 2x_2)^2$

then $\dot{V} = -x_1^2 - x_2^2 + (x_3 + x_1^2 + 2x_2)(\underbrace{x_1^2 + \delta + u}_{\dot{x}_3} \underbrace{- 2x_1 \cdot \dot{x}_1}_{\dot{x}_1} + \underbrace{2x_2 + 2x_3}_{2\dot{x}_2} + x_2)$

$u = -x_1^2 - 2x_1(-x_1 + x_1 x_2) - 2(x_2 + x_3) - x_2 + v$

$= x_1^2 - 2x_1^2 x_2 - 3x_2 - 2x_3 + v$

then $\dot{V} = -x_1^2 - x_2^2 - (x_3 + x_1^2 + x_2^2)^2 + \|x_3 + x_1^2 + 2x_2\| \cdot (\delta + v)$

let $v = -k_1\|x\|$ $\quad k_1 > k$

then, $\|x_3 + x_1^2 + 2x_2\| \cdot (\delta + v) \le \|x_3 + x_1^2 + 2x_2\| \cdot (k - k_1) \cdot \|x\| < 0$

so $\dot{V} \le 0$

$u = x_1^2 - 2x_1^2 x_2 - 3x_2 - 2x_3 - k_1\|x\|$ $\quad k_1 > k$

4.

1) $\ddot{q} = m^{-1}(u - c\dot{q} - g)$

nominal: $\ddot{q} = -\hat{m}^{-1}[\hat{c}\dot{q} + \hat{g}] + \hat{m}^{-1}\hat{u}$

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -k_d & -k_p \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$ ← what we want.

$$\ddot{q} = \underbrace{-\hat{m}^{-1}\hat{c}}_{-k_p}\dot{q} - \hat{m}^{-1}\hat{g} + \hat{m}^{-1}\hat{u}.$$

so $\hat{u} = \hat{g} - k \cdot q \quad k > 0$

2) $$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \underbrace{\begin{pmatrix} \dot{q} \\ -\hat{m}^{-1}[\hat{c}\dot{q} + \hat{g}] \end{pmatrix}}_{f} + \underbrace{\begin{pmatrix} 0 \\ \hat{m}^{-1} \end{pmatrix}}_{G}(\hat{u} + \delta)$$

$$\delta = \hat{m}(m^{-1} - \hat{m}^{-1})\hat{u} - \hat{m}(m^{-1}c - \hat{m}^{-1}\hat{c})\dot{q} - \hat{m}(m^{-1}g - \hat{m}^{-1}\hat{g})$$

3)

$u = \hat{g} - kq + v \qquad v < \hat{m}kq$

$\ddot{q} = -\hat{m}^{-1}\hat{c}\dot{q} - kq + \hat{m}^{-1} \cdot v \qquad v < \hat{c}\dot{q}$

so $\hat{m}^{-1}v < kq \quad \hat{m}^{-1}v < \hat{m}^{-1}\hat{c}\dot{q}$

so $v < |\hat{m}k\hat{c}| \cdot \|x\|$

let $v = \left|\frac{\hat{m}k\hat{c}}{2}\right| \cdot \|x\|$

4) $a = \hat{m}m^{-1}$

$\delta = (a - I)(\hat{g} - kq) + (a - I) \cdot r - a \cdot c \cdot \dot{q} + \hat{c}\dot{q} - ag + \hat{g}$

$\|\hat{m}m^{-1} - I\| < I$ ①

for ρ. $\rho_1 \geq \|k(\hat{m}m^{-1} - I)\| + |a(\hat{g} - g)/q|$

$\rho_2 \geq \|\hat{c} - \hat{m}m^{-1}c\|$