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1.

let $V(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$ which is p.d.

$\dot{V}(x) = x_1 \cdot \dot{x}_1 + x_2 \cdot \dot{x}_2$

$= -x_1^4 + 2x_1x_2^3 - 2x_1x_2^3$

$= -x_1^4$ which is n.d.

-3

not negative definite

Hence, the origin is an asymptotically stable solution of the system

Can use LaSalle to finish proof

2. a)

let $\varphi = x_1$, $\ell = x_2$, $\theta = x_3$ then:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ -\frac{m(x_2+1)^2}{I+m(x_2+1)^2} \end{bmatrix}}_{g_1} u_1 + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{g_2} u_2$$

$$[g_1, g_2] = \begin{bmatrix} 0 \\ 0 \\ \frac{2m(x_2+1)}{(I+m(x_2+1)^2)^2} \end{bmatrix} = g_3$$ which is independent to $g_1, g_2$

so $\dim \bar{\Delta} = 3$, so the system is LA.

Since the system is driftless, LA $\Leftrightarrow$ controllability $\Leftrightarrow$ STLC

so the system is STLC

and hence nonholonomic.

2 b)

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 \\ 0 \\ 3x_1^2 + x_2^2 \end{bmatrix}}_{g_1} u_1 + \underbrace{\begin{bmatrix} 0 \\ 1 \\ -2x_1x_2 \end{bmatrix}}_{g_2} u_2$$

$$[g_1, g_2] = \begin{pmatrix} 0 \\ 0 \\ 2x_2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0 \cdot g_1 + 0 \cdot g_2$$ so the system is holonomic

$$g_1 \times g_2 = \begin{pmatrix} -3x_1^2 - x_2^2 \\ -2x_1x_2 \\ -1 \end{pmatrix}$$ which is the gradient of the manifold.

so the manifold: $x_1^3 + x_1x_2^2 + x_3 = C$

if it starts at the origin, $C = 0$

$x_f(1, 1, -2)$ is on the manifold.

so it can reach point $x_f(1, 1, -2)$

3 a

let $e_y = x_1$ $e_\psi = x_2$ $V = x_3$ $k = x_4$

then. $\dot{x}_1 = x_3 \cdot \sin x_2$

$\dot{x}_2 = x_3 \cdot \left(x_4 - \frac{k_r}{1-k_r x_1} \cos x_2\right)$

$\dot{x}_3 = u_1$

$\dot{x}_4 = u_2$

$y = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$\dot{y} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix}$

$x_2 = \arcsin\left(\frac{\dot{y}_1}{y_2}\right)$ when $y_2 \neq 0$

$u_1 = \dot{y}_2$

$\dot{x}_2 = \frac{1}{\sqrt{1-\left(\frac{\dot{y}_1}{y_2}\right)^2}} \cdot \frac{\ddot{y}_1 y_2 - \dot{y}_2 \dot{y}_1}{y_2^2}$ similarly, we can get $\ddot{x}_2$

$x_4 = \frac{\dot{x}_2}{x_3} + \frac{k_r}{1-k_r x_1} \cos x_2$ ✓

$u_2 = \dot{x}_4 = \frac{\ddot{x}_2 \cdot x_3 - \dot{x}_2 \cdot \dot{x}_3}{x_3^2} + \frac{\dot{k}_r (1-k_r x_1) - k_r \cdot (-\dot{k}_r x_1 - k_r \dot{x}_1)}{(1-k_r x_1)^2} \cdot \cos x_2 - \frac{k_r}{1-k_r x_1} \cdot \sin x_2 \cdot \dot{x}_2$

So it is differential flatness

b. use $\lambda(t) = [\lambda^7\ \lambda^6\ \lambda^5\ \lambda^4\ \lambda^3\ \lambda^2\ \lambda^1\ 1]^T$

$Y = [y_0, y_0^{(1)}, y_0^{(2)}, y_0^{(3)}, y_T, y_T^{(1)}, y_T^{(2)}, y_T^{(3)}]$

$A = Y\Lambda^{-1} \Rightarrow$ get $u_{1d}$ $u_{2d}$ $y_d$ $d y_d$ $d_2 y_d$ $d_3 y_d$

then we can have $q_d$ $d q_d$ $d_2 q_d$

so $\ddot{q}$ ~~$= d_2 x_d$~~ 3 3

$= d_2 x_d - k_d(\dot{q} - x_d) - k_q(q - d x_d)$

state $X = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$ $q = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

$k_q \cdot k_q$

-0.5: basis are generally a function of a latent variable such as time (t), on which Y is dependent

-2.5: Y = A*Lambda
The boundary conditions are used to solved for A. A is then plugged back into Y = A*Lambda to get a generalised equation for Y, Y is then used to find x and u

4.

let $u = -kw - (b \times b_0) - 3w_i^2 b \times (Ib)$ $(k>0)$

$V(x) = \frac{1}{2} w^T I w + 1 - b^T b_0$ since $\frac{1}{2} w^T I w$ is p.d

$b^T b_0 > -1$ then $1 - b^T b_0 > 0$

so $V(x)$ is p.d.

$\dot{V}(x) = w^T I \dot{w} - \dot{b}^T b_0$

$w^T I \dot{w} = w^T((Iw) \times w) + w^T u + w^T 3w_i^2 b \times (Ib)$

$= w^T(u + 3w_i^2 b \times (Ib))$

$= -w^T k w - w^T(b \times b_0)$

$\dot{b}^T b_0 = b_0^T \cdot \dot{b}$

$= b_0^T (b \times w)$

$= w^T (b_0 \times b)$

$= -w^T (b \times b_0)$

so $\dot{V}(x) = -w^T k w$ which is n.d.

Hence, the system is locally asymptotically stable