1. a) if $x_2 = k_1 x_1$, $V_0 = \frac{1}{2}x_1^2$

then $\dot{V}_0 = -(k_1x_1 + \frac{3}{2}x_1^2 + \frac{1}{2}x_1^3)x_1 = -x_1^2(k_1 + \frac{3}{2}x_1 + \frac{1}{2}x_1^2)$

we want $\dot{V}_0$ to be n.d

so $\frac{1}{2}x_1^2 + \frac{3}{2}x_1 + k_1$ should be always greater than 0

so $\frac{9}{4} - 2k_1 < 0$ so $k_1 > \frac{9}{8}$, $\phi(x_1) = k_1 x_1$

let $V = V_0 + \frac{1}{2}(x_2 - k_1x_1)^2$ which is p.d

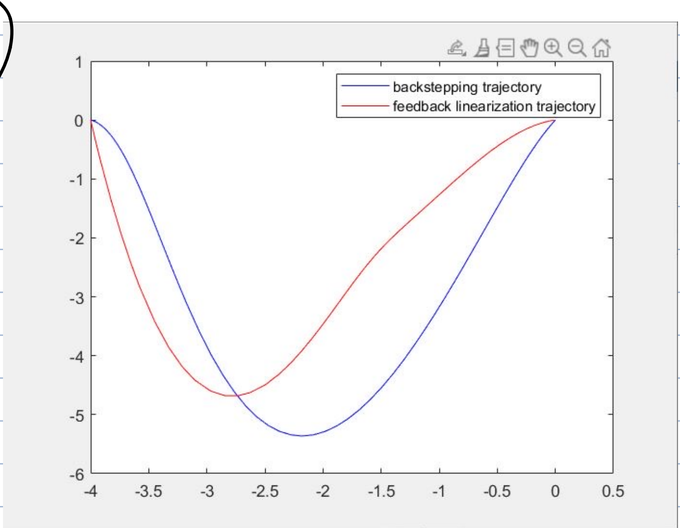
then $\dot{V} = -(k_1x_1 + \frac{3}{2}x_1^2 + \frac{1}{2}x_1^3)x_1 - x_1 \cdot (x_2 - k_1x_1) + (x_2 - k_1x_1)\left(u + k_1(x_2 + \frac{3}{2}x_1^2 + \frac{1}{2}x_1^3)\right)$

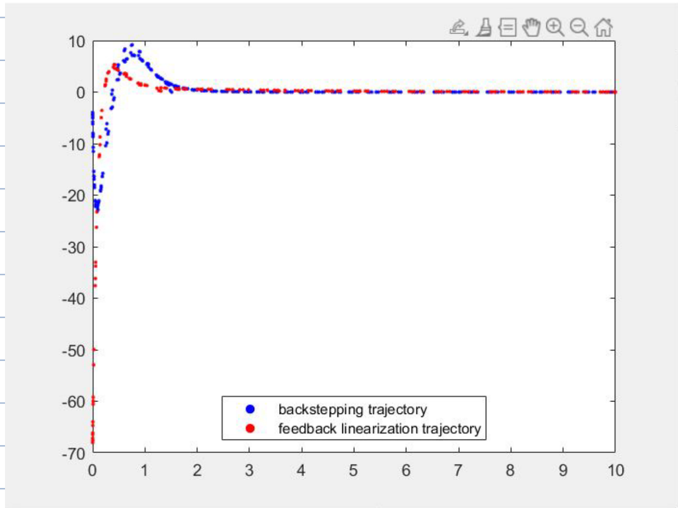
we need $\dot{V}$ to be n.d.

so $u = -k_1(x_2 + \frac{3}{2}x_1^2 + \frac{1}{2}x_1^3) + x_1 - k_2(x_2 - k_1x_1)$ $\quad k_1 > \frac{9}{8}$

$k_2 > 0$

b)





control signal.

c) if $u = k_1x_1 + k_2x_2$

then $\dot{x}_1 = -x_2 - \frac{3}{2}x_1^2 - \frac{1}{2}x_1^3$

$\dot{x}_2 = u = k_1x_1 + k_2x_2$

let $V = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$ $\quad \dot{V} = x_1 \cdot \dot{x}_1 + x_2 \cdot \dot{x}_2 = -(x_1x_2 + \frac{3}{2}x_1^3 + \frac{1}{2}x_1^4) + k_1x_1x_2 + k_2x_2^2$

$-(x_1x_2 + \frac{3}{2}x_1^3 + \frac{1}{2}x_1^4) + k_1x_1x_2 + k_2x_2^2 = -\frac{1}{2}x_1^4 + k_2x_2^2 - \frac{3}{2}x_1^3 + (k_1 - 1)x_1x_2$

so $k_2 < 0$, $k_1 = 1$, $\dot{V} = -\frac{1}{2}x_1^4 - \frac{3}{2}x_1^3 + k_2x_2^2 = -\frac{1}{2}x_1^3(x_1 + 3) + k_2x_2^2$

When $x_1 < -3$ or $x_1 > 0$, $\dot{V} < 0$, $k_2 < 0$ $k_1 = 1$, then the system is locally A.S

2 a) let $x = x_1$ $\quad y = x_2$ $\quad \theta = x_3$ $\quad v = x_4$

then $\dot{x}_1 = x_4 \cos x_3$ $\qquad e(t) = q(t) - q_d(t)$ $\qquad q : \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$\dot{x}_2 = x_4 \cdot \sin x_3$ $\qquad V_0 = \frac{1}{2} e^T e$

$\dot{x}_3 = u_1' x_4$ $\quad u_1' = \frac{\tan u_1}{L}$ $\qquad \dot{V}_0 = e^T \dot{e}$ $\quad$ we hope $\dot{e} = -k_0 e$ $\quad k_0 > 0$

$\dot{x}_4 = u_2$ $\qquad \dot{V}_0 = e^T(-k_0 e + z)$ $\quad z = k_0 e + \dot{e}$ $\quad \dot{z} = k_0 \dot{e} + \ddot{e}$

let $V = V_0 + \frac{1}{2} z^T z$ $\qquad \dot{V} = -k_0 e^T e + z^T(e + k_0 \dot{e} + \ddot{q} - \ddot{q}_d)$

$$z = k_0 e + \begin{pmatrix} x_4 \cos x_3 \\ x_4 \sin x_3 \end{pmatrix} - \dot{q}_d$$

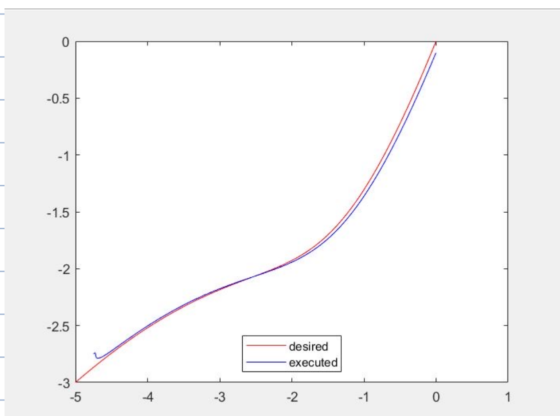
$$\dot{z} = k_0 \dot{e} + \begin{pmatrix} \cos x_3 & -\sin x_3 \\ \sin x_3 & \cos x_3 \end{pmatrix} \begin{pmatrix} u_2 \\ x_4^2 \cdot u_1' \end{pmatrix} - \ddot{q}_d$$

so
$$\begin{bmatrix} u_2 \\ u_1' \end{bmatrix} = \begin{bmatrix} -\frac{\sin x_3}{x_4^2} & \frac{\cos x_3}{x_4^2} \\ \cos x_3 & \sin x_3 \end{bmatrix} \cdot \left(-e - k_0 \dot{e} - \ddot{q}_d - k_1 z^T\right)$$

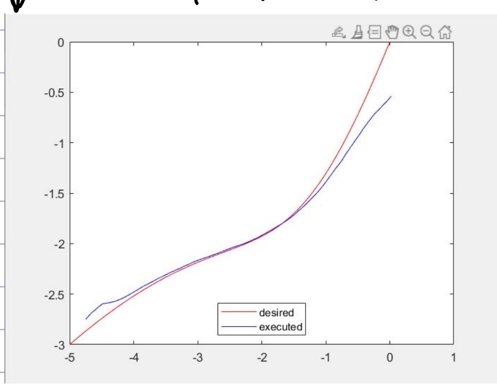
$k_0 > 0, k_1 > 0$

$u_1 = \operatorname{atan}(u_1' \cdot L)$

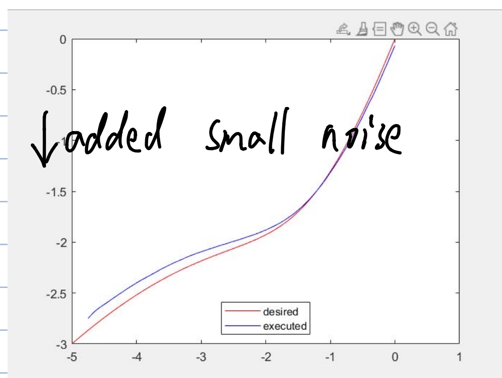
b)



↓added loud noise



↓added small noise



3. let $V_0 = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$, then $\dot{V}_0 = -x_1^2 + x_2(x_1^2 + x_2 + x_3)$

when $x_3 = -x_1^2 - 2x_2$ $\quad \dot{V}_0$ is n.d.

$= \phi(x_1, x_2)$

let $V = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}(x_3 + x_1^2 + 2x_2)^2$

then $\dot{V} = -x_1^2 - x_2^2 + (x_3 + x_1^2 + 2x_2)\big(\underbrace{x_1^2 + \delta + u}_{\dot{x}_3} \underbrace{- 2x_1 \cdot \dot{x}_1}_{\dot{x}_1} + \underbrace{2x_2 + 2x_3}_{2\dot{x}_2} + x_2\big)$

$u = -x_1^2 - 2x_1(-x_1 + x_1 x_2) - 2(x_2 + x_3) - x_2 + v$

$= x_1^2 - 2x_1^2 x_2 - 3x_2 - 2x_3 + v$

then $\dot{V} = -x_1^2 - x_2^2 - (x_3 + x_1^2 + x_2^2)^2 + \|x_3 + x_1^2 + 2x_2\| \cdot (\delta + v)$

let $v = -k_1\|x\|$ $\quad k_1 > k$

then, $\|x_3 + x_1^2 + 2x_2\| \cdot (\delta + v) \leq \|x_3 + x_1^2 + 2x_2\| \cdot (k - k_1) \cdot \|x\| < 0$

so $\dot{V} \leq 0$

$u = x_1^2 - 2x_1^2 x_2 - 3x_2 - 2x_3 - k_1\|x\|$ $\quad k_1 > k$

4.

1) $\ddot{q} = m^{-1}(u - c\dot{q} - g)$

nominal: $\ddot{q} = -\hat{m}^{-1}[\hat{c}\dot{q} + \hat{g}] + \hat{m}^{-1}\hat{u}$

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -k_d & -k_p \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$ ← what we want.

$$\ddot{q} = \underbrace{-\hat{m}^{-1}\hat{c}}_{-k_p}\dot{q} - \hat{m}^{-1}\hat{g} + \hat{m}^{-1}\hat{u}.$$

so $\hat{u} = \hat{g} - k \cdot q \quad k > 0$

2) $$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \underbrace{\begin{pmatrix} \dot{q} \\ -\hat{m}^{-1}[\hat{c}\dot{q} + \hat{g}] \end{pmatrix}}_{f} + \underbrace{\begin{pmatrix} 0 \\ \hat{m}^{-1} \end{pmatrix}}_{G}(\hat{u} + \delta)$$

$$\delta = \hat{m}(m^{-1} - \hat{m}^{-1})\hat{u} - \hat{m}(m^{-1}c - \hat{m}^{-1}\hat{c})\dot{q} - \hat{m}(m^{-1}g - \hat{m}^{-1}\hat{g})$$

3)

$u = \hat{g} - kq + v \qquad v < \hat{m}kq$

$\ddot{q} = -\hat{m}^{-1}\hat{c}\dot{q} - kq + \hat{m}^{-1} \cdot v \qquad V < \hat{c}\dot{q}$

so $\hat{m}^{-1}v < kq \quad \hat{m}^{-1}V < \hat{m}^{-1}\hat{c}\dot{q}$

so $v < |\hat{m}k\hat{c}| \cdot \|x\|$

let $v = \left|\frac{\hat{m}k\hat{c}}{2}\right| \cdot \|x\|$

4) $a = \hat{m}m^{-1}$

$\delta = (a - I)(\hat{g} - kq) + (a - I) \cdot r - a \cdot c \cdot \dot{q} + \hat{c}\dot{q} - ag + \hat{g}$

$\|\hat{m}m^{-1} - I\| < I$ ①

for ρ. $\rho_1 \geq \|k(\hat{m}m^{-1} - I)\| + |a(\hat{g} - g)/q|$

$\rho_2 \geq \|\hat{c} - \hat{m}m^{-1}c\|$