EN530.678 Nonlinear Control and Planning in Robotics
Practice Problems

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1. Consider the system

$$\begin{aligned}\dot{x}_1 &= -2x_1 + x_3 x_2,\\ \dot{x}_2 &= -x_1 + x_2 - x_2 x_3 + u,\\ \dot{x}_3 &= -x_3 + x_1 x_2^2 + x_1,\end{aligned}$$

   with output $y = x_2$. What is the system relative degree and over what region is it valid? Transform the system into a normal form, i.e. by finding the transformation $(z, \eta) = \Phi(x)$ and determine whether it is minimum phase.

2. Consider the system

$$\begin{aligned}\dot{x}_1 &= x_1 + \frac{x_2}{1 + x_1^2}\\ \dot{x}_2 &= x_2 + u.\end{aligned}$$

   Using backstepping, design a globally stabilizing control law and prove its stability properties.

3. Is the unicycle with linear drift (corresponding to e.g. an airplane flying through a linearly varying wind field) with dynamics given by (for some costant $a > 0$):

$$\begin{aligned}\dot{x}_1 &= \cos x_3 u_1,\\ \dot{x}_2 &= a x_2 + \sin x_3 u_1,\\ \dot{x}_3 &= u_2,\end{aligned}$$

   small-time locally controllable (STLC)? Use the definition of good and bad brackets to support your claim.

4. Consider a car-like robot subject to external disturbances, e.g. due to slipping or sliding, which results in the equations of motion

$$\begin{aligned}\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} &= \begin{pmatrix}\cos x_3 u_1\\ \sin x_3 u_1\end{pmatrix} + \delta(u_1)\\ \dot{x}_3 &= u_1 u_2,\end{aligned}$$

   where $\delta(u_1) \in \mathbb{R}^2$ represents the uncertainty, for which it is only known that

$$\|\delta(u_1)\| \leq k_0 |u_1|,$$

for a given $0 \leq k_0 < 1$. Design a backstepping controller which can stabilize the system output $y = (x_1, x_2)$ to the origin $(0,0)$ despite the uncertainties.

Hint: Employ Lyapunov redesign at each backstepping phase, i.e. in the first stage we attempt to achieve

$$\begin{pmatrix} \cos x_3 u_1 \\ \sin x_3 u_1 \end{pmatrix} \to -ky + v(y, u_1), \tag{1}$$

where v is a disturbance attenuation term.