AMS 553.761: Nonlinear Optimization I
Midterm, Fall 2020

- **There are 4 questions on this test.**
- **You have to upload your answers on Blackboard by 4:30 pm on Tuesday, October 13, 2020. If something goes wrong with the upload you may email it to me before 4:30pm on Tuesday, October 13, 2020. Please use the email option only if the upload does not work. No answers will be accepted after this deadline.**

  Please hand in ONE submission - multiple submissions will not be tolerated.
- **You are not allowed to discuss any problem with any other human being, except the instructor.**
- **You can use a computer only as a word processor; in particular, you cannot consult the internet in regards to this midterm. You CAN use the slides from class and books from the library.**
- **You CAN cite any result we have mentioned in class or from the HWs without proof. If you cite a result (e.g., from a book) that was NOT mentioned in class, you should include a complete proof of this fact.**
- **The level of rigor expected is the same as the HW solutions. Make sure you justify all your answers.**

1. **(25 pts)** Give complete proofs of Lemmas 2.2, 2.3 and 2.6 from the lecture notes on "Conjugate Gradient".
2. **(25 pts)** Prove that the bisection method for Wolfe linesearch, i.e., Algorithm 13 from slide 76 in the "Line Search Methods" slides on the course webpage, terminates with a steplength satisfying the weak Wolfe conditions. Assume that the function f is bounded from below and has a gradient that is Lipschitz continuous with a global Lipschitz constant γ.
3. **(20 pts)** Prove Theorem 4.8 from slide 103 in the "Line Search Methods" slides on the course webpage, that establishes a global convergence rate of $O\big((\frac{1}{\epsilon})^2\big)$ for a modified or quasi Newton method with *Wolfe linesearch*. You may use Theorem 3.4 on slide 71 (due to Zoutendijk) without proof. [Recall that the condition number of B_k is given by $\frac{\lambda_{max}(B_k)}{\lambda_{min}(B_k)}$ which is the ratio of the largest eigenvalue of B_k to the smallest eigenvalue of B_k.]
4. **(10 pts)** Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function, not necessarily differentiable. Show that any local minimum is also a global minimum.