

EN.530.663: Robot Motion Planning

Homework 4 Solution

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Problem 2

$$(a) \text{Rouler}(\alpha, \beta, \gamma) = R_z(\alpha) R_y(\beta) R_z(\gamma)$$

$$= \begin{bmatrix} c\alpha c\beta c\gamma - s\alpha s\gamma & -s\alpha c\gamma - c\alpha c\beta s\gamma & c\alpha s\beta \\ c\alpha s\gamma + s\alpha c\beta c\gamma & c\alpha c\gamma - s\alpha c\beta s\gamma & s\alpha s\beta \\ -s\beta c\gamma & s\beta s\gamma & c\beta \end{bmatrix}$$

$$(b) \sin(\alpha - \pi) = -\sin\alpha, \quad \cos(\alpha - \pi) = -\cos\alpha$$

$$\sin(-\beta) = -\sin\beta, \quad \cos(-\beta) = \cos\beta$$

$$\sin(\gamma - \pi) = -\sin\gamma, \quad \cos(\gamma - \pi) = -\cos\gamma$$

$$\text{Rouler}(\alpha - \pi, -\beta, \gamma - \pi)$$

$$= \begin{bmatrix} -c\gamma c\beta(-c\gamma) - (-s\alpha) - s\gamma & c\alpha c\beta(-s\gamma) + s\alpha(-c\gamma) & (-c\alpha)(-s\beta) \\ (-s\alpha)c\beta(-c\gamma) + (-s\gamma)(-c\alpha) & (-c\alpha)(-c\gamma) - (-s\alpha)c\beta(-s\gamma) & (-s\alpha)(-s\beta) \\ -(-s\beta)(-c\gamma) & (-s\beta)(-s\gamma) & c\beta \end{bmatrix}$$

$$= \text{Rouler}(\alpha, \beta, \gamma)$$

$$(c) \frac{r_{23}}{r_{13}} = \frac{s\alpha s\beta}{c\alpha s\beta} = \tan(\alpha) \Rightarrow \alpha = \text{atan2}(r_{23}, r_{13})$$

$$r_{33} = \cos\beta, \quad \sqrt{1 - r_{33}^2} = |\sin\beta|, \quad \beta = \text{atan2}(\sqrt{1 - r_{33}^2}, r_{33})$$

$$r_{32} = \sin\beta \sin\gamma, \quad r_{31} = -\sin\beta \cos\gamma, \quad \gamma = \text{atan2}(r_{32}, -r_{31})$$

Problem 3

$$H = \{(x, y) \mid x^2 + y^2 \leq 1\}$$

Transformed coordinates is

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = R(\theta) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \cos \theta - y \sin \theta \\ x \sin \theta + y \cos \theta \end{bmatrix}$$

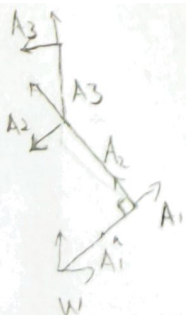
$$x'^2 + y'^2 = (x \cos \theta - y \sin \theta)^2 + (x \sin \theta + y \cos \theta)^2 = x^2 + y^2 \leq 1$$

Therefore $x^2 + y^2 \leq 1 \Leftrightarrow x'^2 + y'^2 \leq 1$

$H' = \{(x', y') \mid x'^2 + y'^2 \leq 1\}$ is equivalent to H

So the primitive is unchanged when rotation is applied.

Problem 4



(a) Consider homogeneous transformation to A_1, A_2, A_3 frame

$$R_{A1W} = \begin{bmatrix} \cos(\frac{\pi}{4}) & -\sin(\frac{\pi}{4}) & 5\sqrt{2} \\ \sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) & 5\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$a = R_{A1W} \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.5\sqrt{2} \\ 2.5\sqrt{2} \\ 1 \end{bmatrix} \quad (x_a, y_a) = (2.5\sqrt{2}, 2.5\sqrt{2})$$

$$R_{A2A1} = \begin{bmatrix} \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) & 0 \\ \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$b = R_{A_1 W} R_{A_2 A_1} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 5\sqrt{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 5\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 10\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.5\sqrt{2} \\ 10.5\sqrt{2} \\ 1 \end{bmatrix}$$

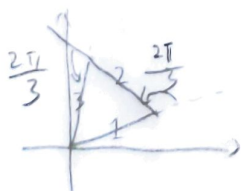
$$(x_b, y_b) = (-0.5\sqrt{2}, 10.5\sqrt{2})$$

$$R_{A_3 A_2} = \begin{bmatrix} \cos(-\frac{\pi}{4}) & -\sin(-\frac{\pi}{4}) & 5\sqrt{2} \\ \sin(-\frac{\pi}{4}) & \cos(\frac{\pi}{4}) & -5\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 5\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -5\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$c = R_{A_1 W} R_{A_2 A_1} R_{A_3 A_2} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 10+10\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 11+10\sqrt{2} \\ 1 \end{bmatrix}$$

$$(x_c, y_c) = (-1, 11+10\sqrt{2})$$

(b)

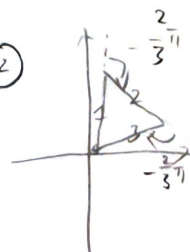


① $\theta_1 \in \mathbb{R}$

$$\theta_2 = \frac{2}{3}\pi + 2k\pi, k=0, \pm 1, \dots$$

$$\theta_3 = \frac{2}{3}\pi + 2k\pi, k=0, \pm 1, \dots$$

②



$\theta_1 \in \mathbb{R}$

$$\theta_2 = -\frac{2}{3}\pi + 2k\pi, k=0, \pm 1, \dots$$

$$\theta_3 = -\frac{2}{3}\pi + 2k\pi, k=0, \pm 1, \dots$$