EN.530.663: Robot Motion Planning
Homework 6 Solution

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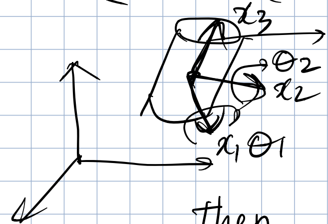
1. A cylindrical rod in $\mathbb{R}^3$

→ first of all, it is a rigid body in $\mathbb{R}^3$

so DOF = 6 SE(3).

But "If the rod is rotated about its central axis, it is assumed that the rod's position and orientation are not changed"

(→ due to the symmetry.)



Hence one rotational effect is ignored.

then rotation about the x_1- and x_2-axis,

i.e., θ_1, θ_2 → range?

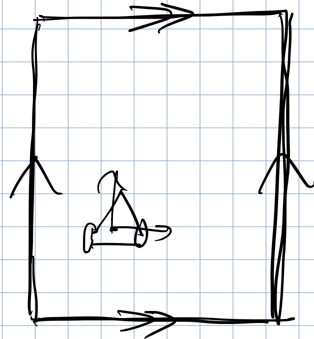
⇒ similar to positions on the unit circle

(imagine $\hat{x_3}$ as the position)

⇒ corresponding $C = \mathbb{S}^2$

$\therefore$ $C = \mathbb{S}^2 \times \mathbb{R}^3$

2.



$\leftarrow$ game screen.

$\wedge$: side-identification

$\gg$: top-bottom identification.

(No twist)

first translation : subset of $\mathbb{R}^2$

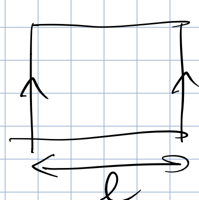
But due to identifications

we have $\begin{cases} [0, 1]/\sim & \text{for sides} \\ [0, 1]/\sim & \text{for top-bottom.} \end{cases}$

rotation $\Rightarrow$ S^1

$$\therefore \ C = [0,1]/\sim \times [0,1]/\sim \times S^1$$

Note



$$[0, l]/\sim \ \cong [0, 1]/\sim$$

L

P3 $C_{obs} = O \ominus A(0)$.

O = obstacle space   Let $b_1, b_2 \in O$

A = robot   Let $a_1, a_2 \in A$

Let $c_1, c_2 \in C_{obs}$.

$\therefore c_1 = b_1 - a_1$ and $c_2 = b_2 - a_1$

Since O and A are convex, we can write for for a point $b_3 \in O$ and $a_3 \in A$ that

$b_3 = \lambda b_1 + (1-\lambda) b_2$ and $a_3 = \lambda a_1 + (1-\lambda) a_2$

where $\lambda \in [0, 1]$.

$$\begin{aligned} b_3 - a_3 &= \lambda b_1 + (1-\lambda) b_2 - \lambda a_1 - (1-\lambda) a_2 \\ &= \lambda (b_1 - a_1) + (1-\lambda)(b_2 - a_2) \\ &= \lambda c_1 + (1-\lambda) c_2 = c_3 \in C_{obs}. \end{aligned}$$

$\therefore$ C_{obs} space is also convex.

Scanned by CamScanner

P4

$\theta_1 = -\frac{\pi}{2}$, axis : $\left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$

$h_1 = \cos\left(-\frac{\pi}{4}\right) + \frac{1}{\sqrt{3}}\sin\left(-\frac{\pi}{4}\right) + \frac{1}{\sqrt{3}}\sin\left(-\frac{\pi}{4}\right) + \frac{1}{\sqrt{3}}\sin\left(-\frac{\pi}{4}\right)$

$= \boxed{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}i - \frac{1}{\sqrt{6}}j - \frac{1}{\sqrt{6}}k}$

$\theta_2 = \pi$, axis : $[0, 1, 0]$

$h_2 = \cos\left(\frac{\pi}{2}\right) + 1 \cdot \sin\left(\frac{\pi}{2}\right)j = \boxed{j}$

$h = h_2 \cdot h_1$ (rotation in world frame)

$= (j)\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}i - \frac{1}{\sqrt{6}}j - \frac{1}{\sqrt{6}}k\right)$

$= \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}}i + \frac{1}{\sqrt{2}}j + \frac{1}{\sqrt{6}}k$

$= \frac{1}{\sqrt{6}} + \frac{\sqrt{5}}{\sqrt{6}}\left(-\frac{1}{\sqrt{5}}i + \frac{\sqrt{3}}{\sqrt{5}}j + \frac{1}{\sqrt{5}}k\right)$.

$\therefore \theta = 2\arccos\left(\frac{1}{\sqrt{6}}\right)$ or $\theta = 2\operatorname{atan2}(\sqrt{5})$

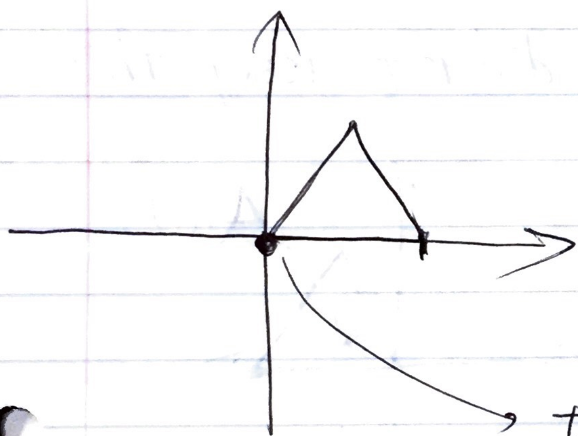
axis : $\left[\frac{-1}{\sqrt{5}}, \frac{\sqrt{3}}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right]$.

Exercise 11

A and O : $(0,0)$, $(2,0)$ and $(1,\sqrt{3})$

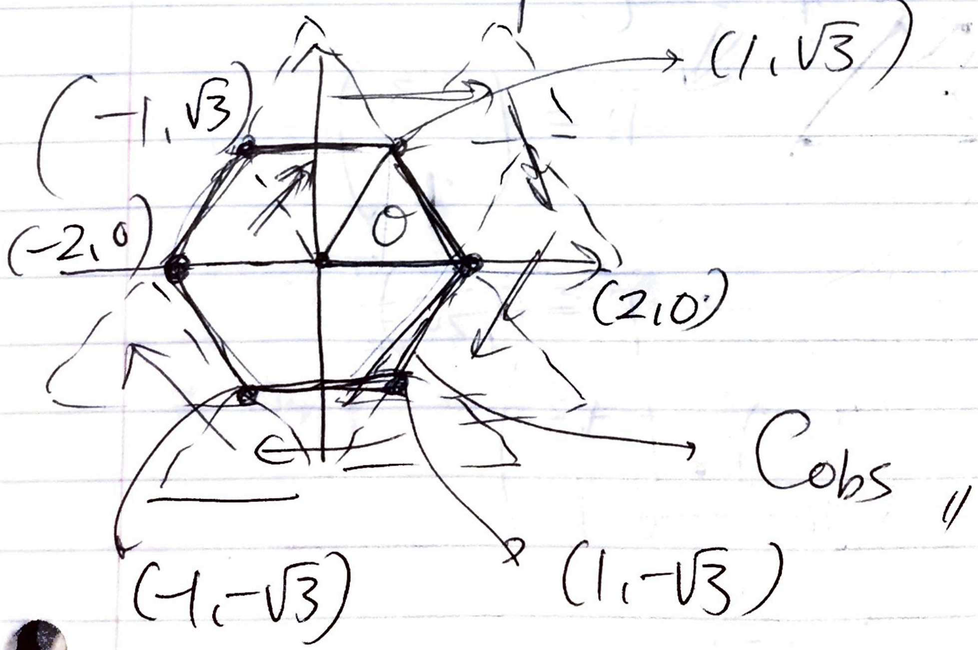
for A, this corresponds to $A(0)$

with $x_t = y_t = \theta = 0$.



this is $(0,0) \Rightarrow$ representative point of A.

Consider sweep



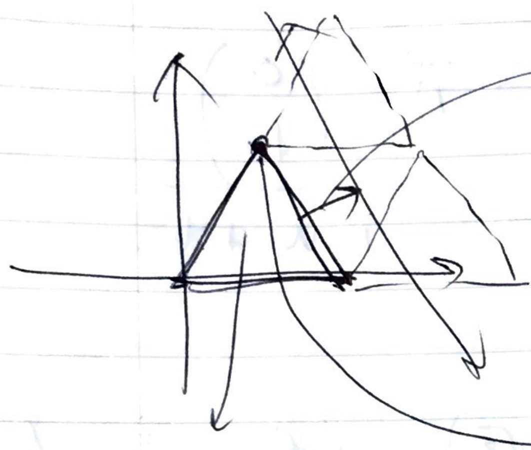
How to compute half-plane primitive?

⇒ Follow the textbook method (or you can calculate directly from C_{obs} that is obtained)

<u>Note</u>: Fig 4.17 ⇒ I told you that $\vec{n}$ should be reversed.

But if you look at fig 4.19. $\vec{n}$ is not defined so in some cases. So here let's use the exact way that we defined in the class. (i.e., $\vec{n}$: inward normal for $A(0)$).

Now H_1



$$\vec{n} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$$

what is $\vec{p}$

this is $\vec{p}$

O

$$\vec{p} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$$

$$\therefore f(x_t, y_t) = \vec{n} \cdot \vec{v}(x_t, y_t)$$

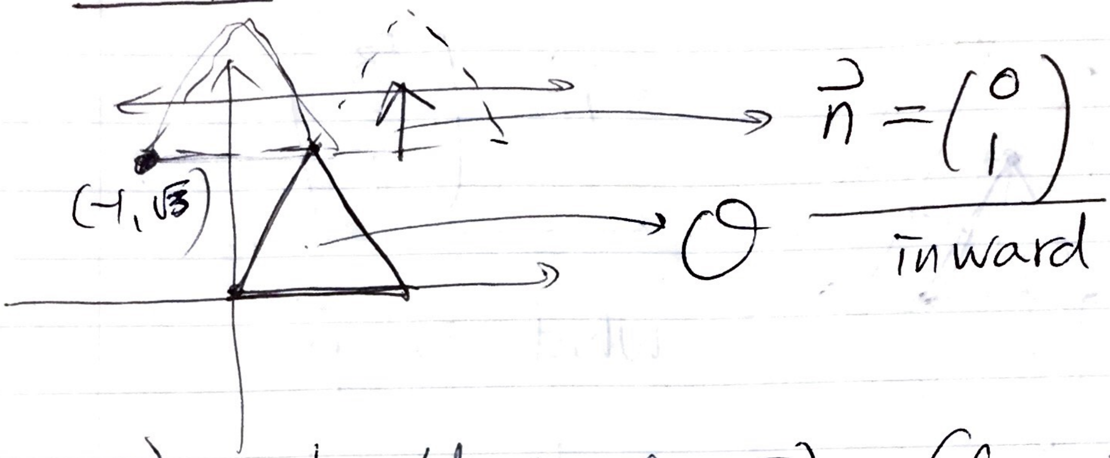
$$= \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x_t - 1 \\ y_t - \sqrt{3} \end{pmatrix}$$

$$= \sqrt{3} x_t + y_t - 2\sqrt{3} \leq 0$$

$$\therefore H_1 = \{ (x, y) \mid \sqrt{3} x + y - 2\sqrt{3} \leq 0 \}$$

type = VE

- $\underline{H_2}$ in solution



$\vec{p} \Rightarrow$ should be $\underline{(-1, \sqrt{3})}$. (from Fig 4.17).

Hence $\vec{v} = \begin{pmatrix} x_t + 1 \\ y_t - \sqrt{3} \end{pmatrix}$ //

- Then $f(x_t, y_t) = \vec{n} \cdot \vec{v}$

$= \underline{y_t - \sqrt{3} \leq 0}$

$\hookrightarrow$ This is H_2 //

i.e., $H_2 = \{(x, y) \mid y - \sqrt{3} \leq 0\}$ //

type = EV

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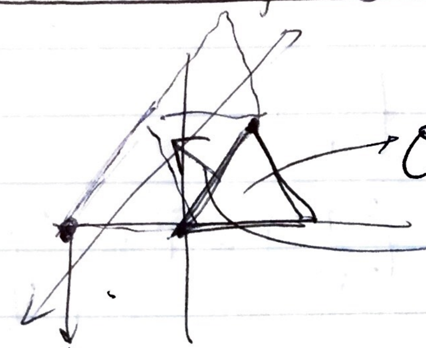
- let's keep going (briefly)

H_3



$\underline{type} = VE$

$\mathcal{O}$

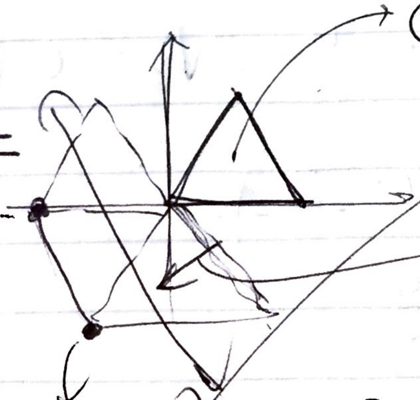
$\vec{n} = \begin{pmatrix} -\sqrt{3} \\ 1 \end{pmatrix}$

$\vec{p} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$ ($\because$ $\vec{v}$ depends on the translation $\xi = (x_t, y_t)$ of the point p)

$$\Rightarrow f(x_t, y_t) = \vec{n} \cdot \vec{v}(x_t, y_t)$$
$$= \begin{pmatrix} -\sqrt{3} \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x_t + 2 \\ y_t \end{pmatrix}$$
$$= -\sqrt{3}x_t + y_t - 2\sqrt{3} \leq 0$$

$$H_3 = \{ (x, y) \mid -\sqrt{3}x + y - 2\sqrt{3} \leq 0 \}$$ //

H_4



$\mathcal{O}$

NOTE: you can choose $\vec{p} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

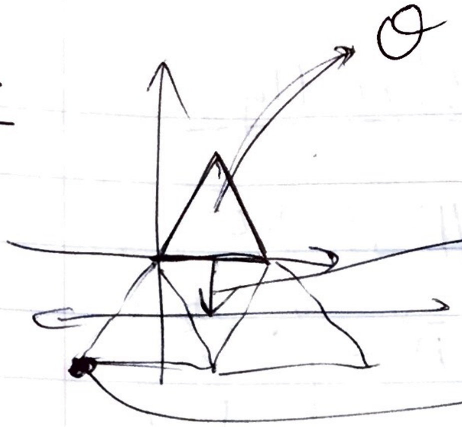
$\vec{n} = \begin{pmatrix} -\sqrt{3} \\ -1 \end{pmatrix}$

$\vec{p} = \begin{pmatrix} -1 \\ -\sqrt{3} \end{pmatrix} \rightarrow \vec{v} = (x_t + 1, \ y_t + \sqrt{3})$

$f(x_t, y_t) \leq 0$

$$\Rightarrow H_4 = \{ (x, y) \mid -\sqrt{3}x - y - 2\sqrt{3} \leq 0 \}.$$ //

H_5



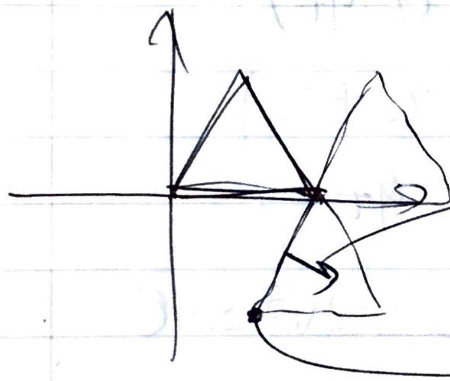
$$\vec{n} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\vec{p} = \begin{pmatrix} -1 \\ -\sqrt{3} \end{pmatrix} \rightarrow \vec{v} = \begin{pmatrix} x_t + 1 \\ y_t + \sqrt{3} \end{pmatrix}$$

$$f(x_t, y_t) = \vec{n} \cdot \vec{v} \leq 0$$

$$\Rightarrow H_5 = \{(x,y) \mid -y - \sqrt{3} \leq 0\}$$

H_6:



$$\vec{n} = \begin{pmatrix} \sqrt{3} \\ -1 \end{pmatrix}$$

$$\vec{p} = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \rightarrow \vec{v} = \begin{pmatrix} x-1 \\ y+\sqrt{3} \end{pmatrix}$$

$$\Rightarrow H_6 = \{(x,y) \mid \sqrt{3}x - y - 2\sqrt{3} \leq 0\}$$

If $\vec{p} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \rightarrow \vec{v} = \begin{pmatrix} x-2 \\ y \end{pmatrix}$

$$\Rightarrow f(x,y) = \vec{n} \cdot \vec{v} = \sqrt{3}x - 2\sqrt{3} + y$$

the same!