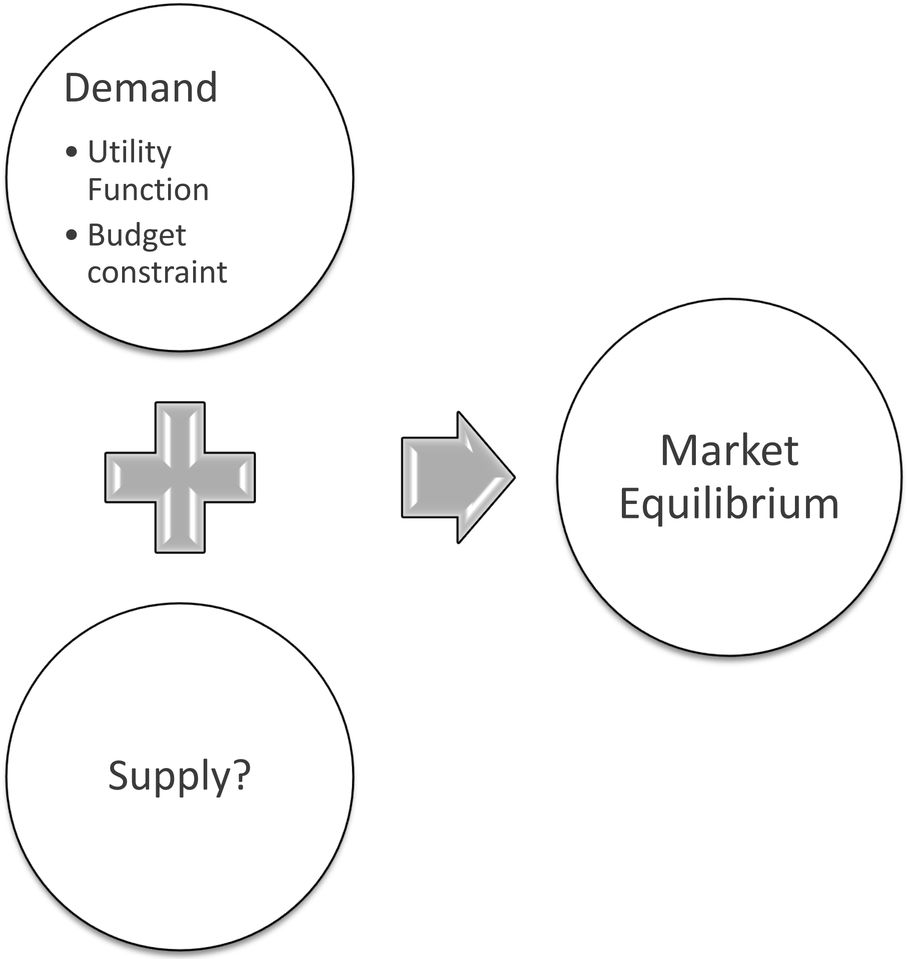
LECTURE 7
PRODUCTION

The Big Picture



Where are we?

- Production function with one variable
  - Marginal and average products
- Production function with two variables
  - Isoquants – representing the production function graphically
  - Marginal rate of technical substitution
  - Uneconomic region of production
- Returns to scale
  - Three types of returns to scale
- Technological progress
  - Three types of technological progress

New things highlighted
The rest can find analogy in consumer theory.

Part 1

Production Function with One Input

What is production?

- Firms turn inputs to outputs outputs can be tangible (goods) or intangible (service)
- *Factors of production* (inputs)
  - Labor
  - Equipment
  - Raw material
  - Land
- Production technology tells us how firms turn inputs into outputs

Given this amount of inputs, how many units of output can I produce?

Production Function

to represent production process

- Suppose the firm needs two inputs, labor (*L*) and capital (*K*), to produce outputs

workers/employees

everything else

- Definition 7.1 *Production function* tells us the *maximum* quantity (*Q*) of output the firm can produce given the amount of *L* and *K*

usually use F to represent production function

$$Q = F(L, K)$$

Production Function with One Input

- Short run in production
  - At least one input is fixed

cannot be adjusted

Within 1 year period, 1 input - size of shop not adjustable, so the firm is in the short run of the production process within this 1 year.

Beyond 1 year horizon, 2 or 3 years from now, then nothing is fixed, size of shop can be adjusted by signing new least after 1 year. That is firm in the long run, all inputs are variable.

- Long run in production
  - All inputs are variable
- Suppose capital is fixed in the short run

assumption

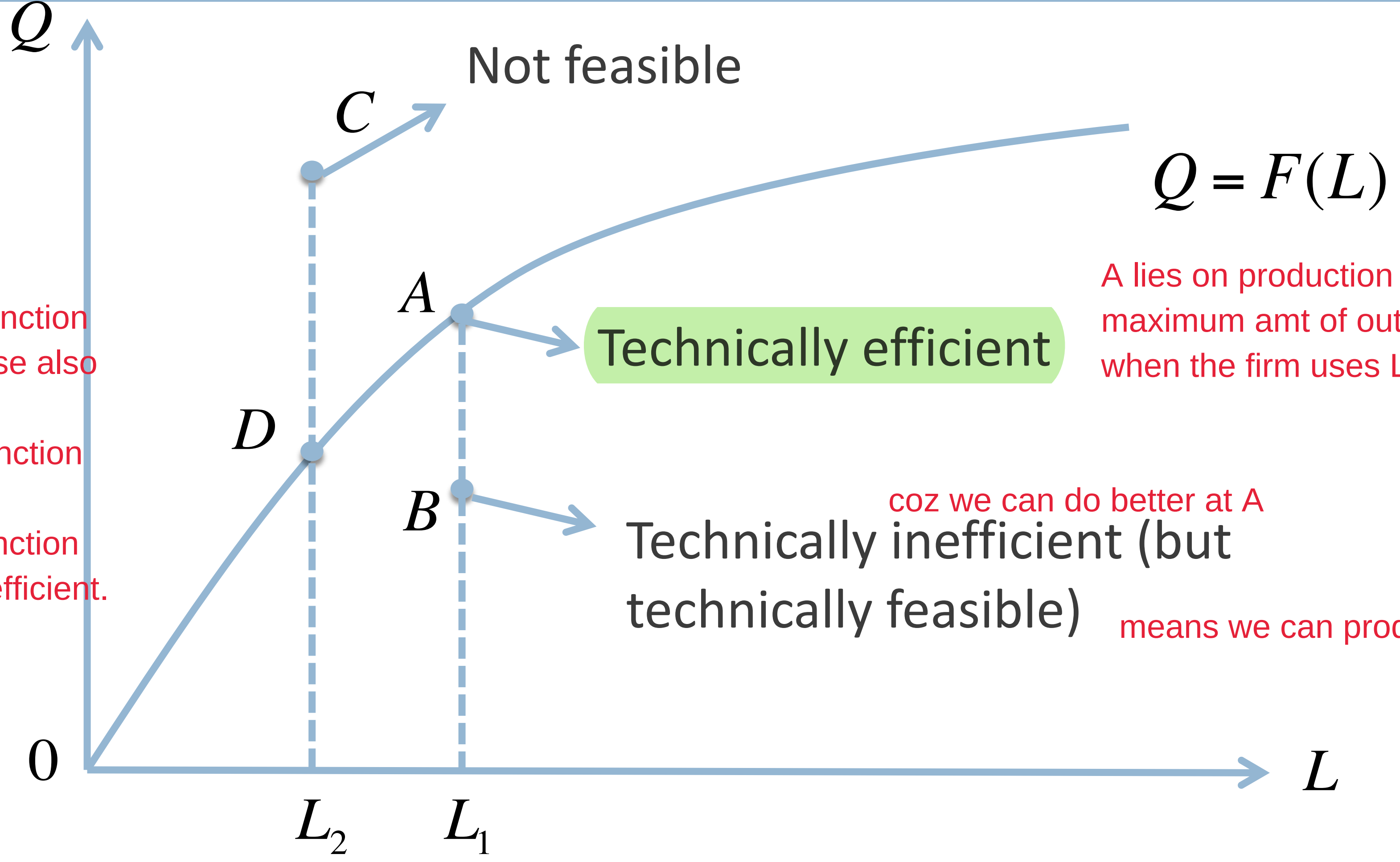
- Firm can only adjust labor
- The production function is

Effectively the production function in the short run is a function of 1 variable only, coz u cannot change how much capital u are using.

$$Q = F(L)$$

Technically Efficient and Technically Feasible

Any point on the production function is technically efficient (of course also technically feasible).
All points above production function are not feasible.
All points below production function are technically feasible but inefficient.



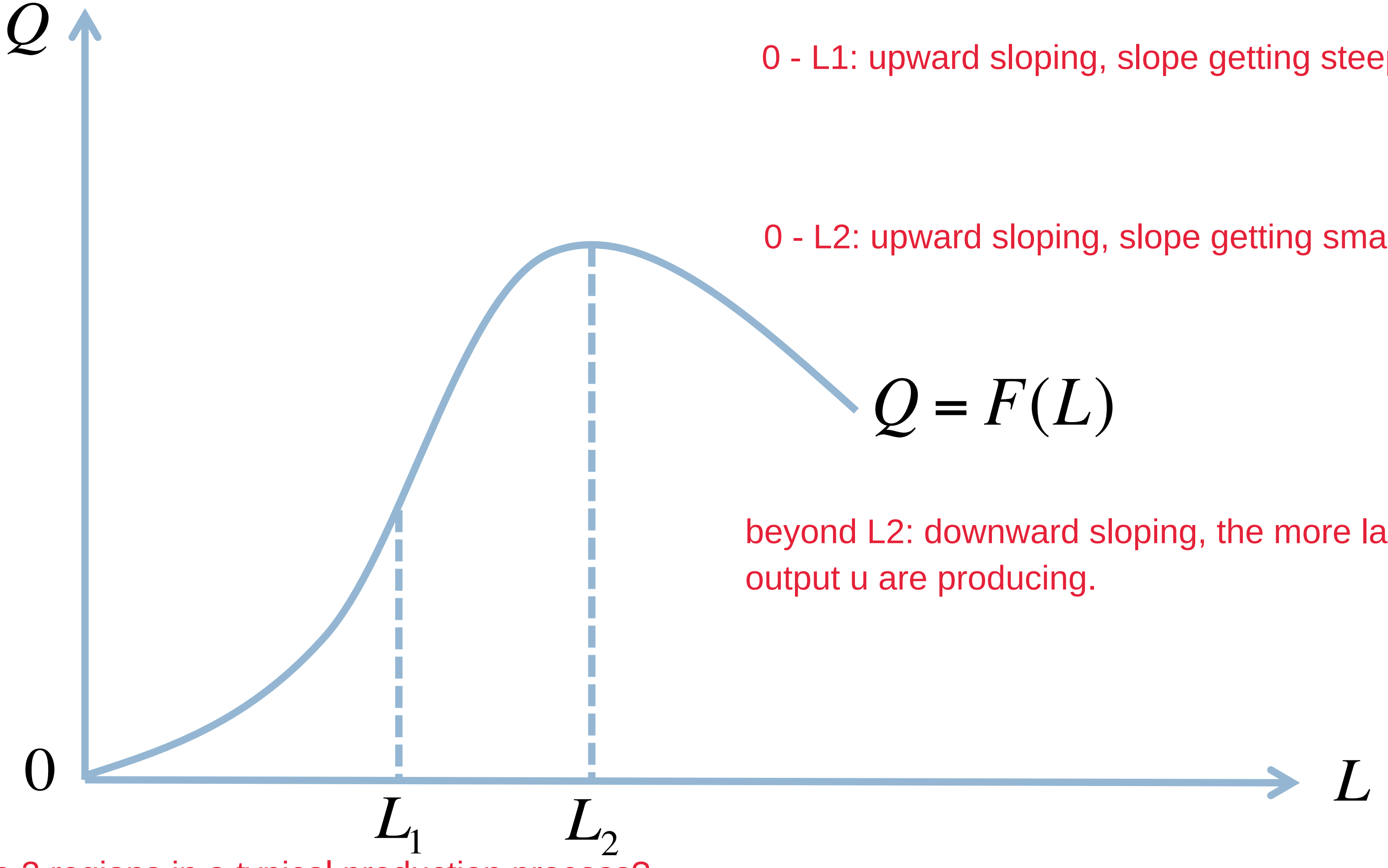
A lies on production function, tells us the maximum amt of output the firm can produce when the firm uses L1 units of labor.

coz we can do better at A

means we can produce at B if we want

A Typical Production Function

We are talking about production function in the short run for now coz we are fixing capital and only change the amt of labour we are using.



0 - L1: upward sloping, slope getting steeper.

0 - L2: upward sloping, slope getting smaller ard = 0 when L = L2.

beyond L2: downward sloping, the more labor u use, the lower total output u are producing.

Why do we expect to see these 3 regions in a typical production process?

Marginal Product

slope of production function

- Definition 7.2 *Marginal product of labor* measures the rate at which output level changes as quantity of labor changes

$$MP_L = \frac{dQ}{dL} = \frac{\Delta Q}{\Delta L}$$

as I use a little bit more of labor, how much more output can I get.

where ΔL is extremely small

- In graph, it is the slope of the production function

Law of Diminishing Marginal Returns

as u add labor, the additional output u can get from each additional labor increases.

- Definition 7.3 *Increasing marginal returns*

  the firm is experiencing increasing marginal returns if MPL increases with L.

  - MP_L increases as L increases

    the more labor u use, the higher the marginal product of labor. e.g. When u use 1 unit of labor, can get 2 units of output. When using 2 units of labor, get 6 units of output.

- Definition 7.4 *Diminishing marginal returns*
  - MP_L decreases as L increases

    e.g. 1st unit of labor gives 4 units of output, but 2nd unit of labor only gives 2 units of output.

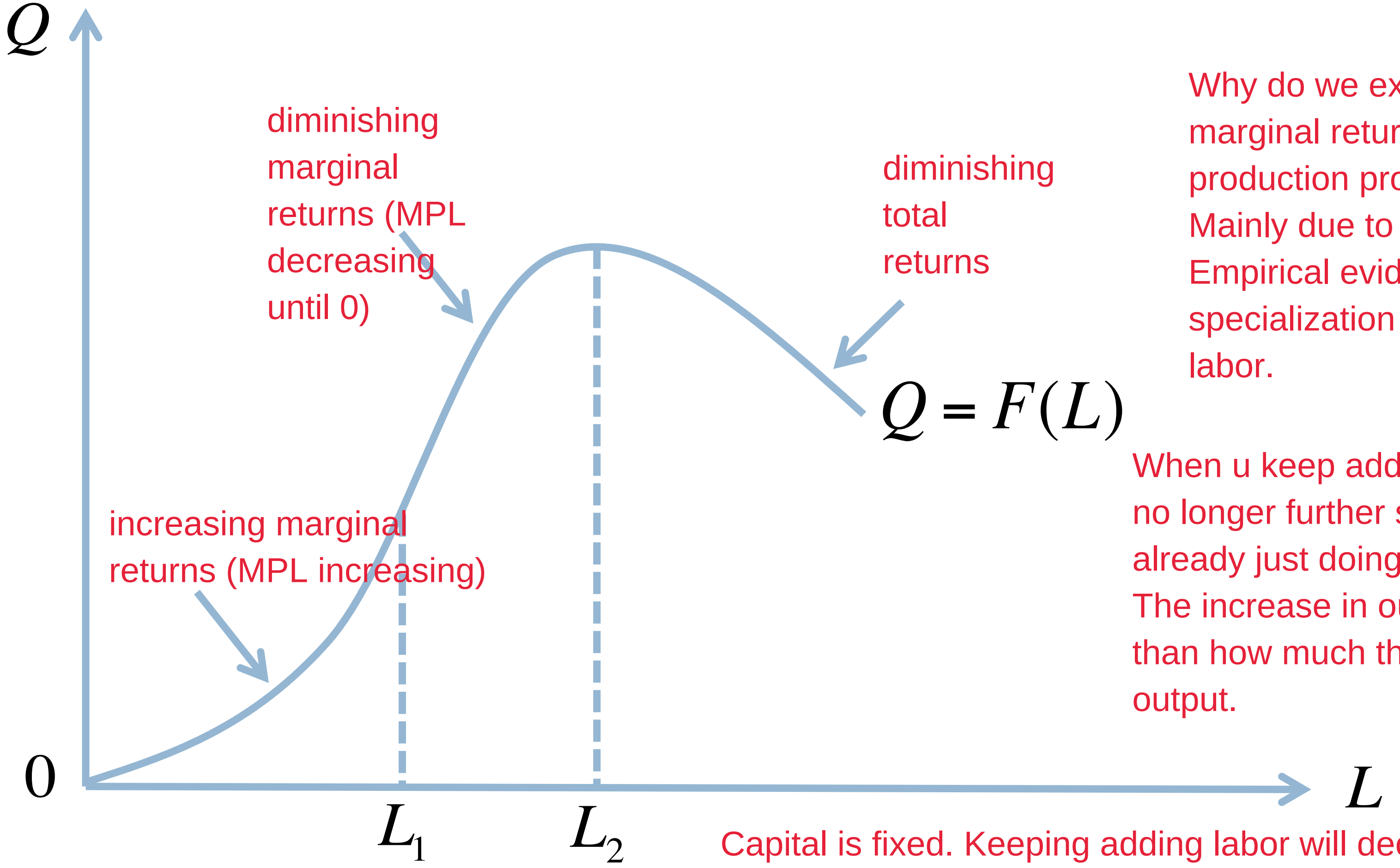
- *Law of diminishing marginal returns*
  - Suppose capital is fixed, marginal product of labor will eventually decline as the quantity of labor increases
- Definition 7.5 *Diminishing total returns*
  - Q decreases as L increases

    this is when production function is downward sloping.

  - MP_L is negative

A Typical Production Function

Typical production function, doesn't mean any production function has to look like this. Because firms use diff production technologies

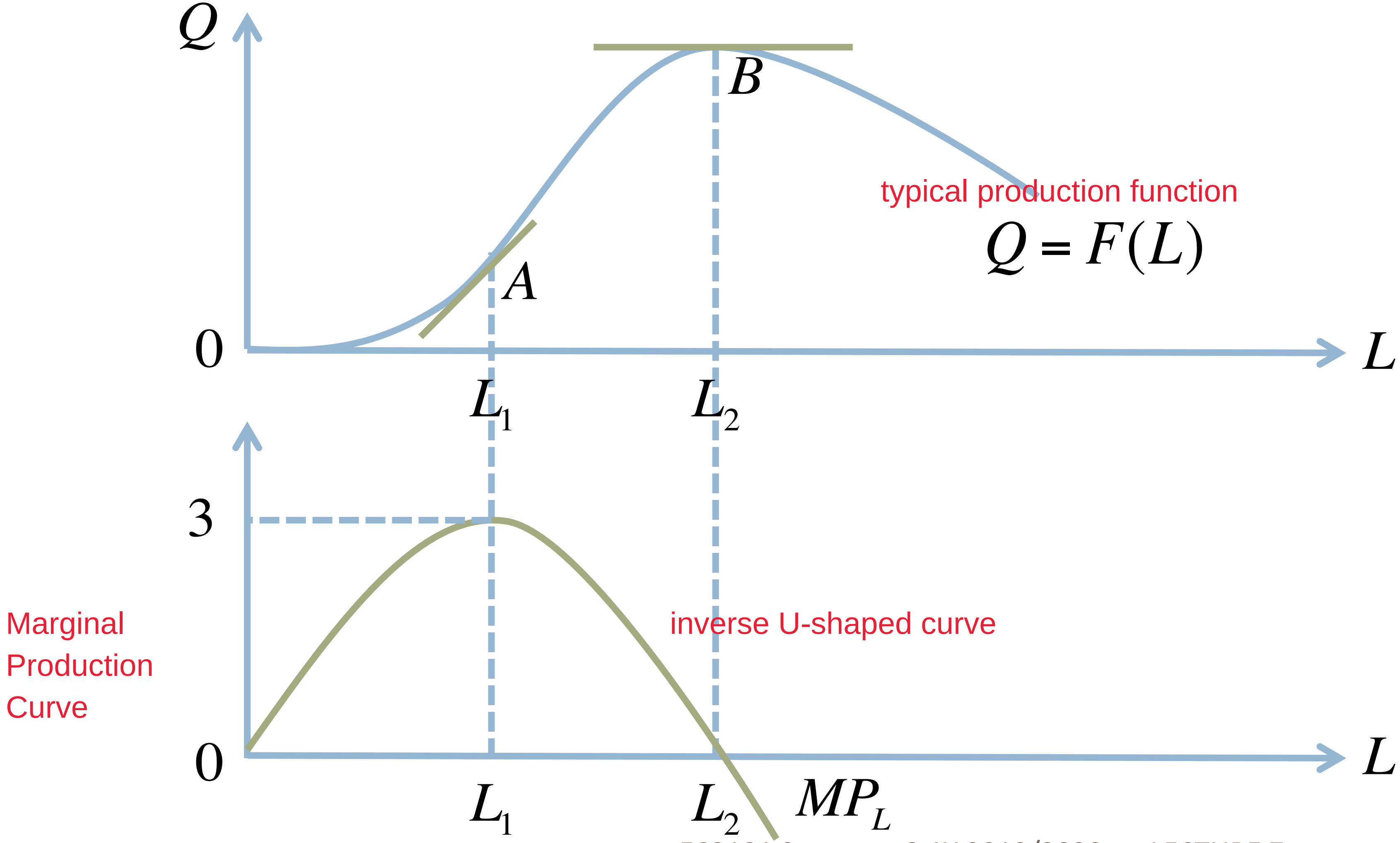


Why do we expect to see increasing marginal returns at the beginning of production process?
Mainly due to specialization of labor. Empirical evidence tells us that specialization increases productivity of labor.

When u keep adding labor, at some point, can no longer further specialize (every worker already just doing 1 thing). Not very helpful. The increase in output of the 5th worker is less than how much the 4th worker increases your output.

Capital is fixed. Keeping adding labor will decrease the output at some point coz not enough space for them to work.

From Production Function to *MP*



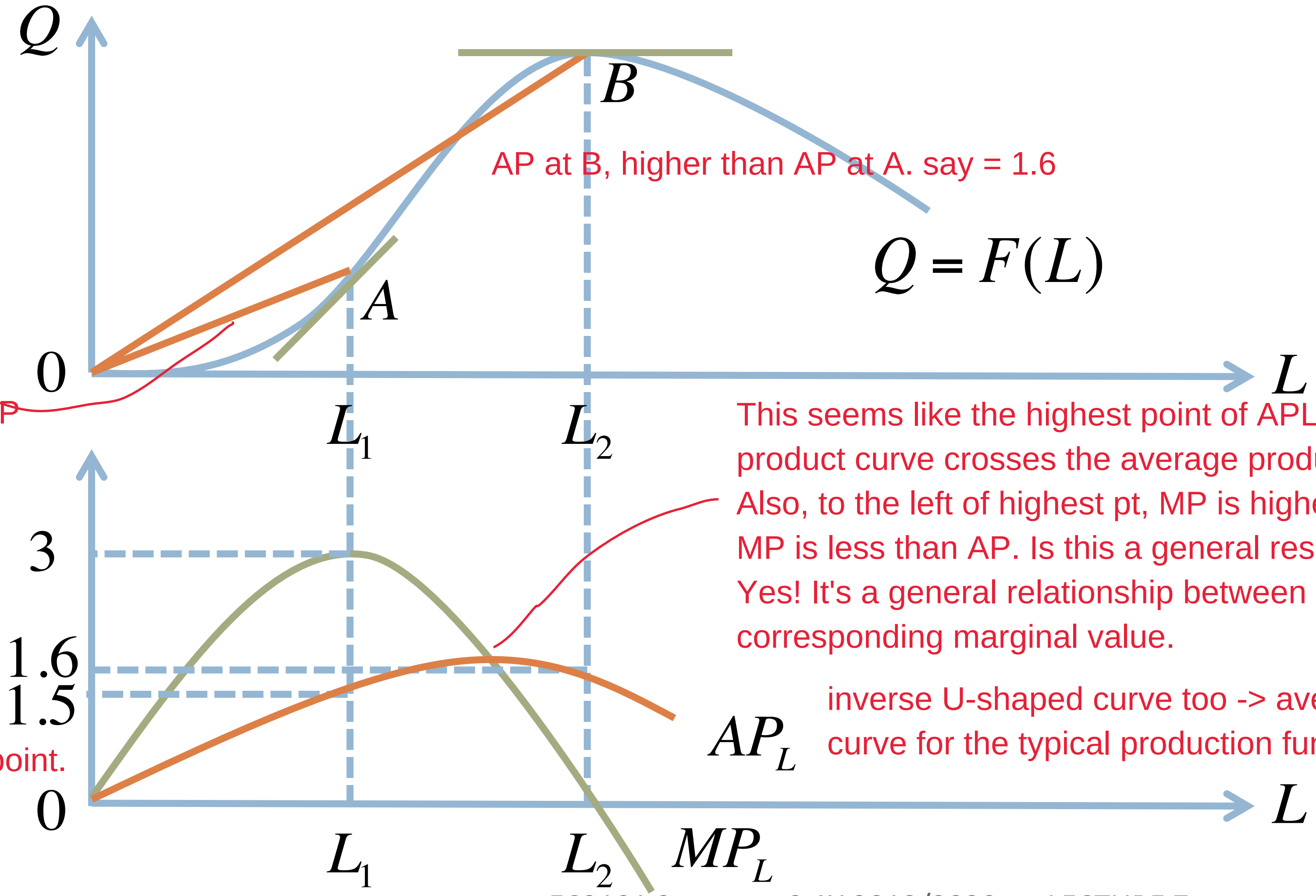
Average Product

- Definition 7.6 *Average product of labor* measures the output per unit of labor

$$AP_L = \frac{Q}{L}$$

- The slope of the ray connecting the origin and the point (*L*,*F*(*L*))
  straight line

From Production Function to *AP*



AP at B, higher than AP at A. say = 1.6

slope of this line is the average product at A

It's +ve, but less than MP of A.

This seems like the highest point of APL, and is where the marginal product curve crosses the average product curve.
Also, to the left of highest pt, MP is higher than AP. To the right of it, MP is less than AP. Is this a general result?
Yes! It's a general relationship between any average value and its corresponding marginal value.

inverse U-shaped curve too -> average product curve for the typical production function

MP cuts AP at its maximum point.

say AP at point A = 1.5

Average Value and Marginal Value

- Suppose you bought 5 apples and it cost you $5 in total
- You paid an average price of $1 per apple
- Suppose you bought 1 additional apple and the average price you paid became $0.9 per apple
- Did the 6th apple cost you more than $1 or less than $1? less than $1

MP crosses *AP* at its highest point

intuitive explanation

- When AP_L rises as L increases
  - As quantity of labor increases, average product of labor goes up
  - Output generated by an extra unit of labor is pulling up the average
  - $MP_L > AP_L$
- When AP_L falls as L increases
  - As quantity of labor increases, average product of labor goes down
  - Output generated by an extra unit of labor is pulling down the average
  - $MP_L < AP_L$

When AP neither increases nor decreases, the highest point of the AP curve, then it has to be that MP = AP, means they cross in the graph. Because as long as they are not the same, AP will either increase or decrease.

MP and *AP*: A Mathematical Explanation

- Since

AP is a function of L coz Q is a function of L.

$$AP(L) = \frac{Q(L)}{L}$$

- We have

$$\frac{dAP(L)}{dL} = \frac{d(\frac{Q(L)}{L})}{dL} = \frac{MP(L)L - Q(L)}{L^2} = \frac{MP(L) - AP(L)}{L}$$

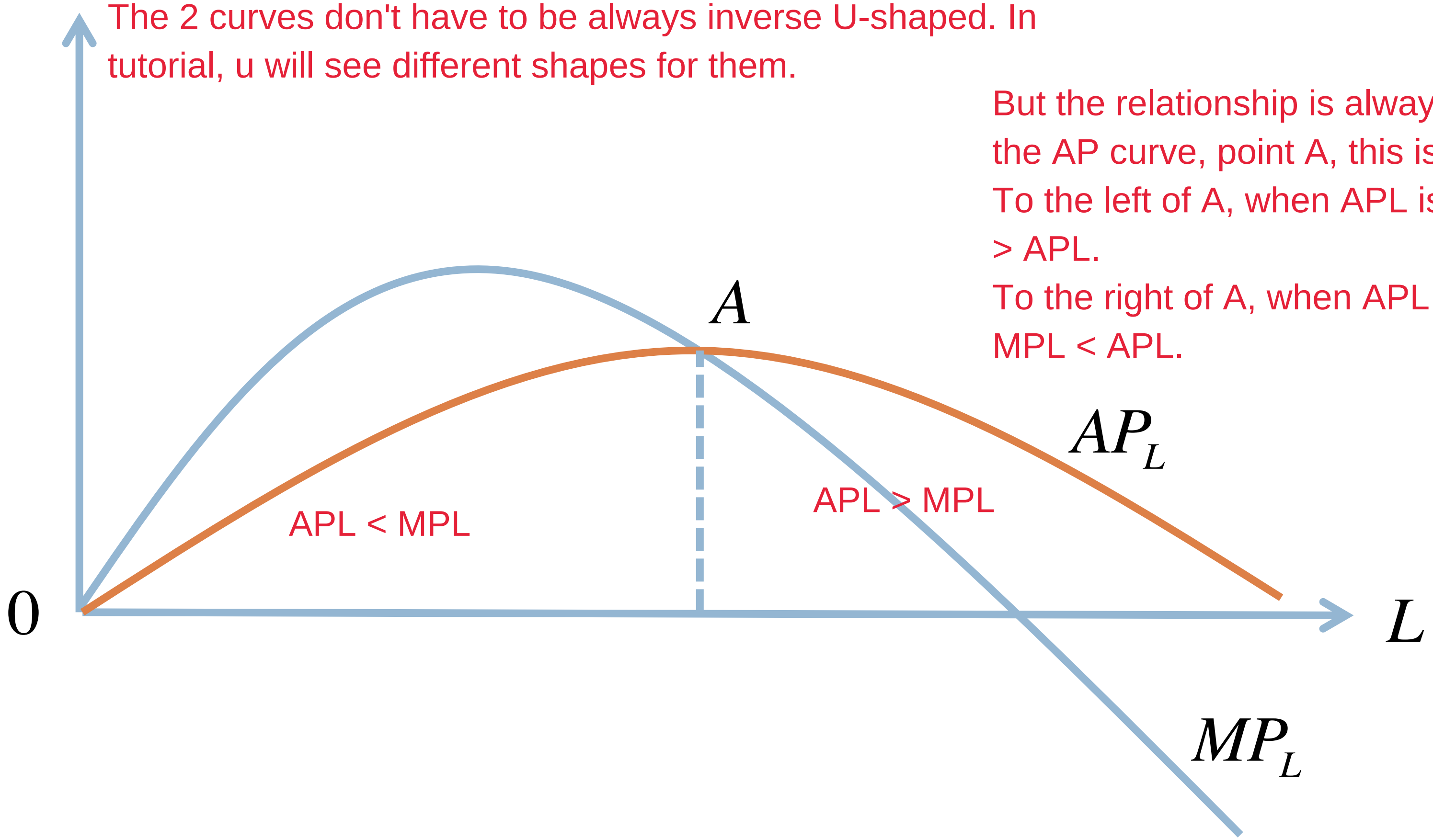
The sign of the derivative will be determined by the AP and MP.

- If as *L* increases *AP* increases, then

$$\frac{dAP(L)}{dL} > 0 \Rightarrow \frac{MP(L) - AP(L)}{L} > 0 \Rightarrow MP(L) > AP(L)$$

MP and *AP* in Graph

The 2 curves don't have to be always inverse U-shaped. In tutorial, u will see different shapes for them.



But the relationship is always true. At the highest point of the AP curve, point A, this is where APL = MPL.
To the left of A, when APL is increasing, this is where MPL > APL.
To the right of A, when APL is decreasing, this is where MPL < APL.

Analogy to Consumer Theory

- Production function
  - Utility function
- Marginal product
  - Marginal utility
- Diminishing marginal returns
  - Diminishing marginal utility

Part 2

Production Function with Two Inputs

Production Function with Two Inputs

- Suppose the firm can adjust both labor and capital in the long run
- Production function is

$$Q = F(L, K)$$

- Marginal products

Marginal Product of Labor is now defined as the partial derivative of Q wrt L. It tells us holding capital fixed, the rate at which output changes with labor.
If u don't change capital, if just increase labor by a little bit, how much more or less output will u produce?

This is like having 2 marginal utilities in the utility function where u have 2 variables (goods).

$$MP_L = \frac{\partial Q}{\partial L}$$

$$MP_K = \frac{\partial Q}{\partial K}$$

marginal product of capital

How do we represent this production function in graph, we don't want 3D graph?

Isoquants

- We can describe production function using isoquants

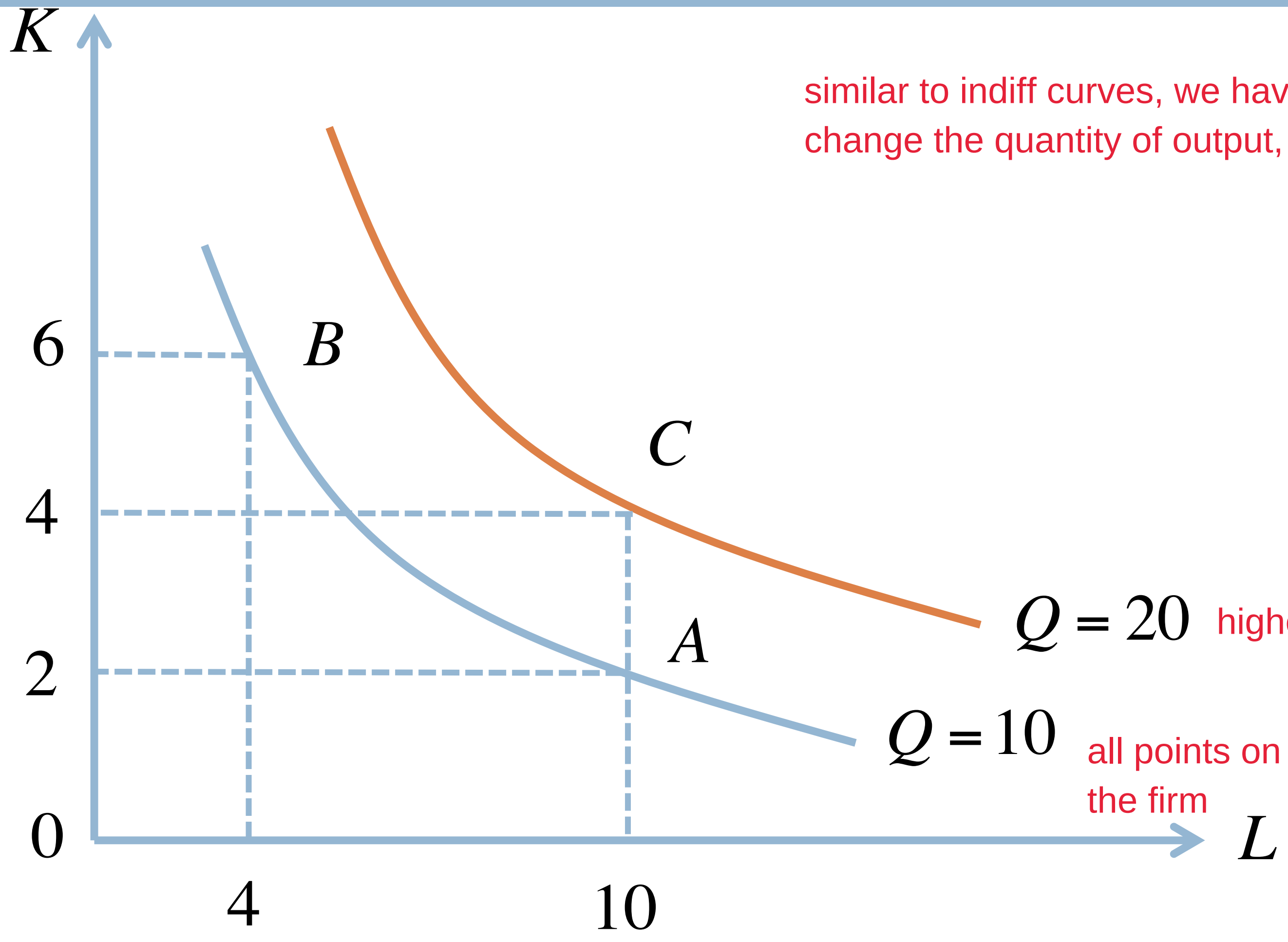
iso: same
quant: quantity

- Definition 7.7 An *isoquant* is a curve that connects all combinations of labor and capital that generate the same level of output

same quantity curve

If u have 2 points that give the firm the same level of output, then these 2 points will be lying on the same isoquant.

Isoquants in Graph



similar to indiff curves, we have many isoquants as well. As long as u change the quantity of output, you are going to get a new isoquant.

higher isoquant

all points on this curve generate 10 units of output for the firm

Marginal Rate of Technical Substitution

similar to MRS, marginal rate of substitution

- Definition 7.8 *Marginal rate of technical substitution* of labor for capital is the rate at which the firm can reduce the quantity of capital for more labor, holding the output level fixed

If I use a little bit more labor, how much capital can I save to still produce the same quantity as before.

$$MRTS_{L,K} = -\left.\frac{dK}{dL}\right|_{Same \quad Q} = -\left.\frac{\Delta K}{\Delta L}\right|_{Same \quad Q}$$

where ΔL is extremely small

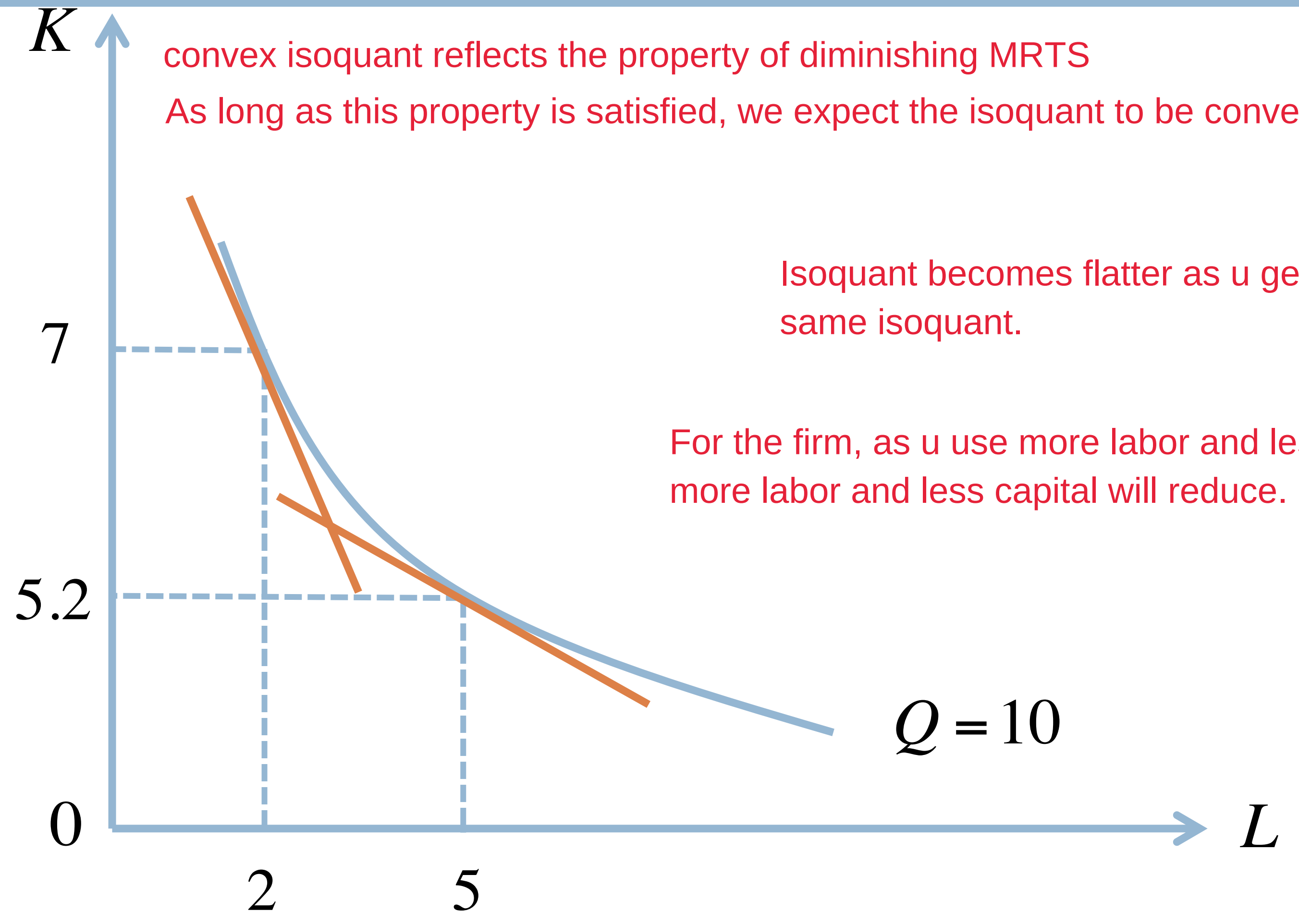
- *MRTS* is the negative of the slope of the isoquant

Diminishing Marginal Rate of Technical Substitution

similar to diminishing MRS property in consumer theory, we are not saying any production function has to have diminishing MRTS property.

convex isoquant reflects the property of diminishing MRTS

As long as this property is satisfied, we expect the isoquant to be convex to the origin.



Isoquant becomes flatter as u get more labor and less capital along the same isoquant.

For the firm, as u use more labor and less capital, your willingness to use even more labor and less capital will reduce.

MRTS and *MP*

MU and MRS in consumer theory

- Suppose the firm changes the quantity of labor and capital, but keeps the output level fixed

moving from 1 point to another point that's very close to it, u are changing both labor and capital

- The total change in output is

delta Q = MPL (delta L) + MPK (delta K)

The 1st term: how much Q changes due to change in L
The 2nd term: how much Q changes due to change in K

- The total change in output must be 0

coz we are moving along the same isoquant

0 = MPL (delta L) + MPK (delta K)

- Thus

$$\frac{MP_L}{MP_K} = -\frac{\Delta K}{\Delta L} = MRTS_{L,K}$$

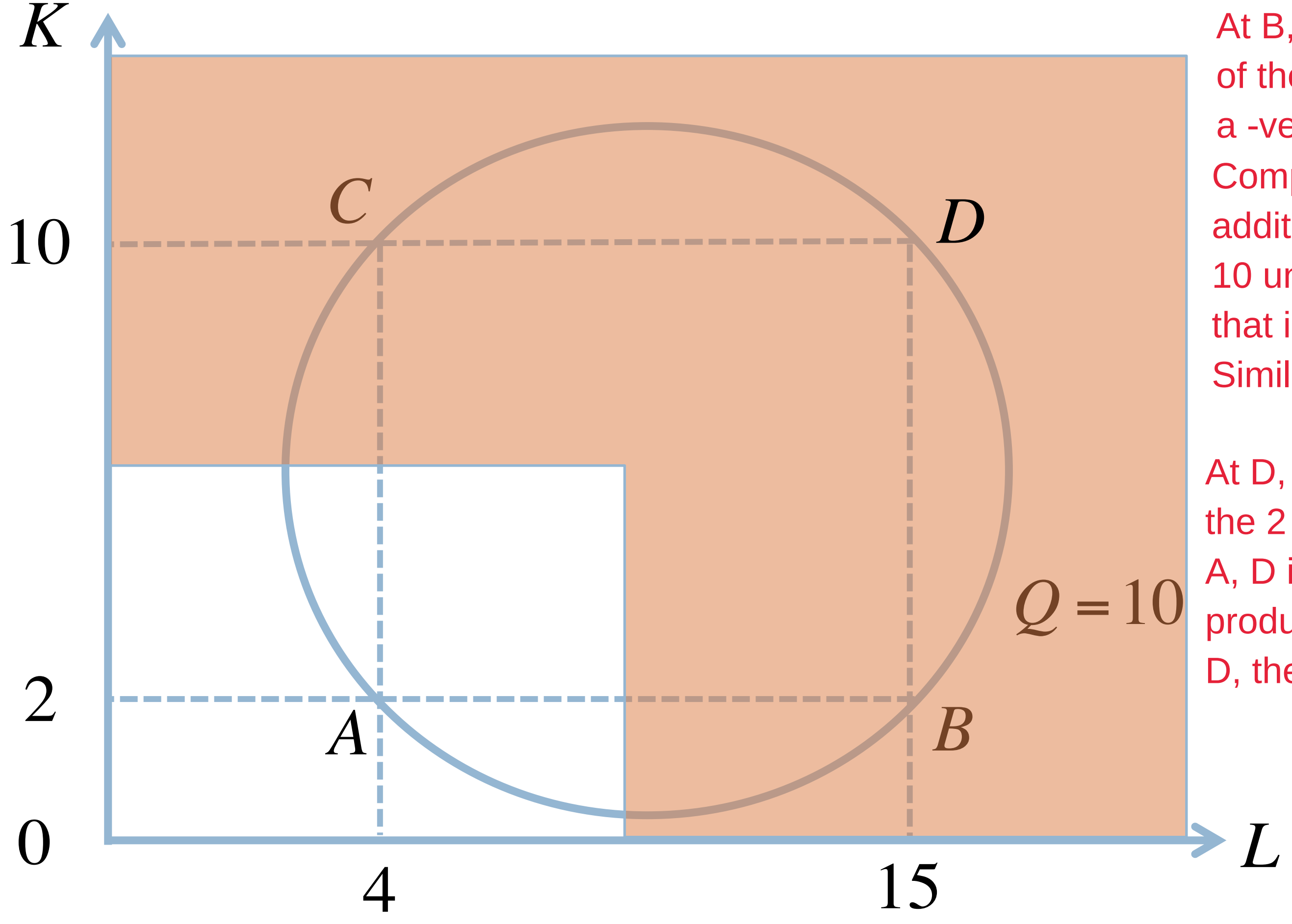
The firm's willingness to substitute between labor and capital while holding output level fixed is defined by the relative marginal product of the 2 inputs.

Analogy to Consumer Theory

- Isoquant
  - Indifference curve
- Marginal rate of technical substitution
  - Marginal rate of substitution
- Diminishing marginal rate of technical substitution
  - Diminishing marginal rate of substitution

Uneconomic Region of Production

Isoquant here is a circle. Possible as shape of isoquant is determined by production function, and production function can be any function.



At B, slope of isoquant is +ve, MRTS at B is -ve. MRTS = ratio of the 2 MP, MPL / MPK. Hence one of the marginal product is a -ve number. Same for point C.

Comparing B to A, same capital but much more labor. But the additional labor doesn't give the firm more output, still producing 10 units. So at B, likely the marginal product of labor is the one that is -ve.

Similarly at C, MP of capital is the one that is negative.

At D, downward sloping isoquant, MRTS is +ve, means at D the 2 marginal products have the same sign. Compare D with A, D is using more labor and capital but still the firm doesn't produce more output. Both MPL and MPK are -ve. Not just for D, the whole downward sloping region ard D.

Qn: Think of an equation of production function that represents this isoquant.

Need the 2 MP to be +ve at A, MPL -ve at B, MPK -ve at C, both MPL and MPK -ve at D (or the parts ard D).

not ecomonic for the firm to produce at that region

Marginal Product and Uneconomic Region of Production

- Definition 7.9 In the *uneconomic region of production*
  - At least one marginal product is negative

one is -ve or both -ve.

firms want to maximize profits, minimize cost of production. profit = total revenue - total cost

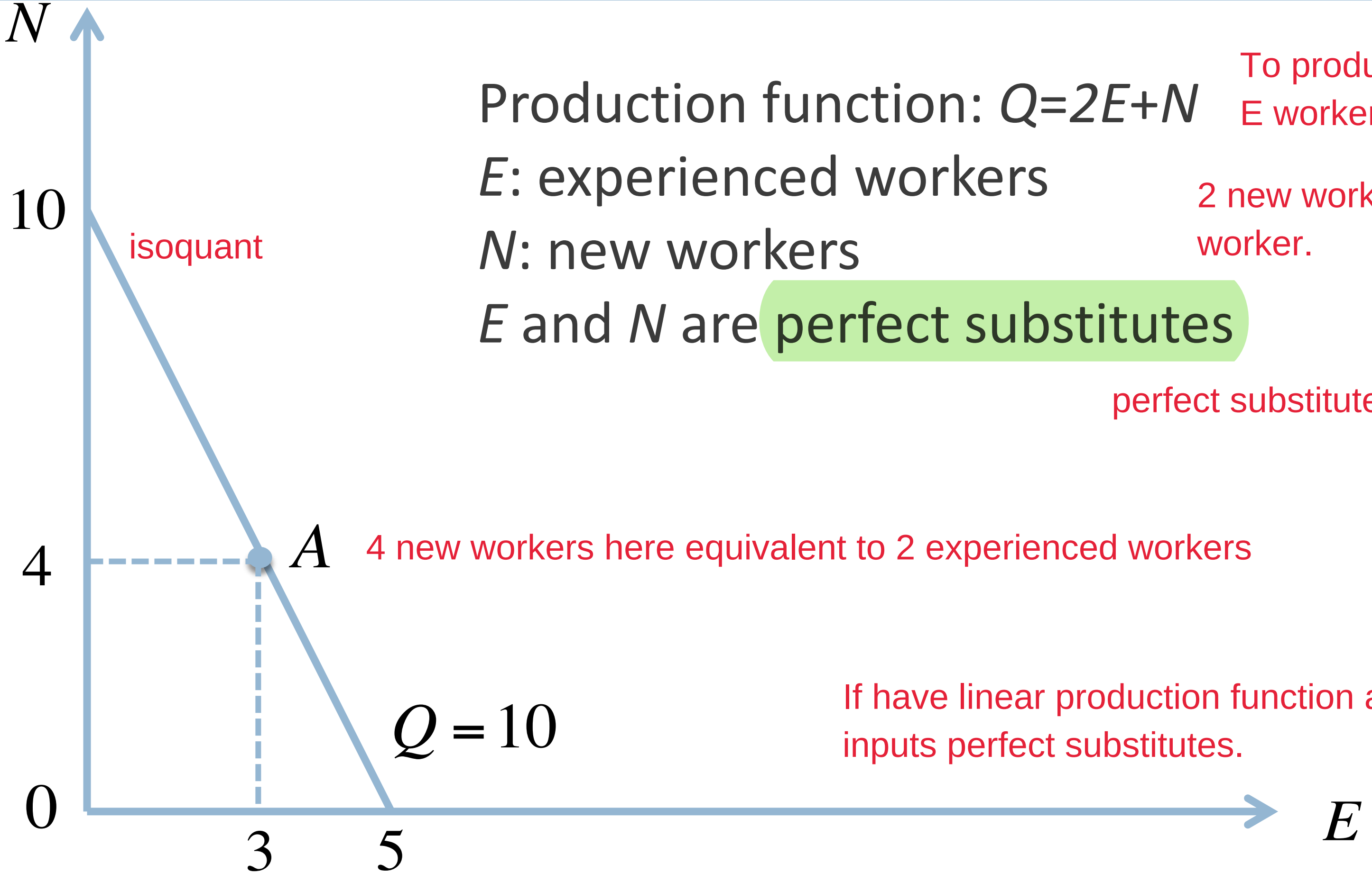
- Cost-minimizing firms never produce in the uneconomic region of production
  - E.g., if the firm produces at point B, it uses 15 labor and 2 capital
  - The firm can produce the same quantity at point A with 4 labor and 2 capital

A is cheaper than B and C.

Point A may not be the best, may not be the cost-minimizing choice of inputs, but we know any point in this uneconomic region of production are not going to be cost-minimizing. Because for any point in this region, we can always find a point that gives the firm same output with less input (cheaper for the firm).

This region may exist, depending on the equation of production function. However, it doesn't matter coz as long as the firm wants to minimize cost, the firm will never produce in the uneconomic region.

Linear Production Function



Production function: $Q=2E+N$

E: experienced workers

N: new workers

E and N are perfect substitutes

To produce 10 units of output, either use 5 E workers or 10 N workers.

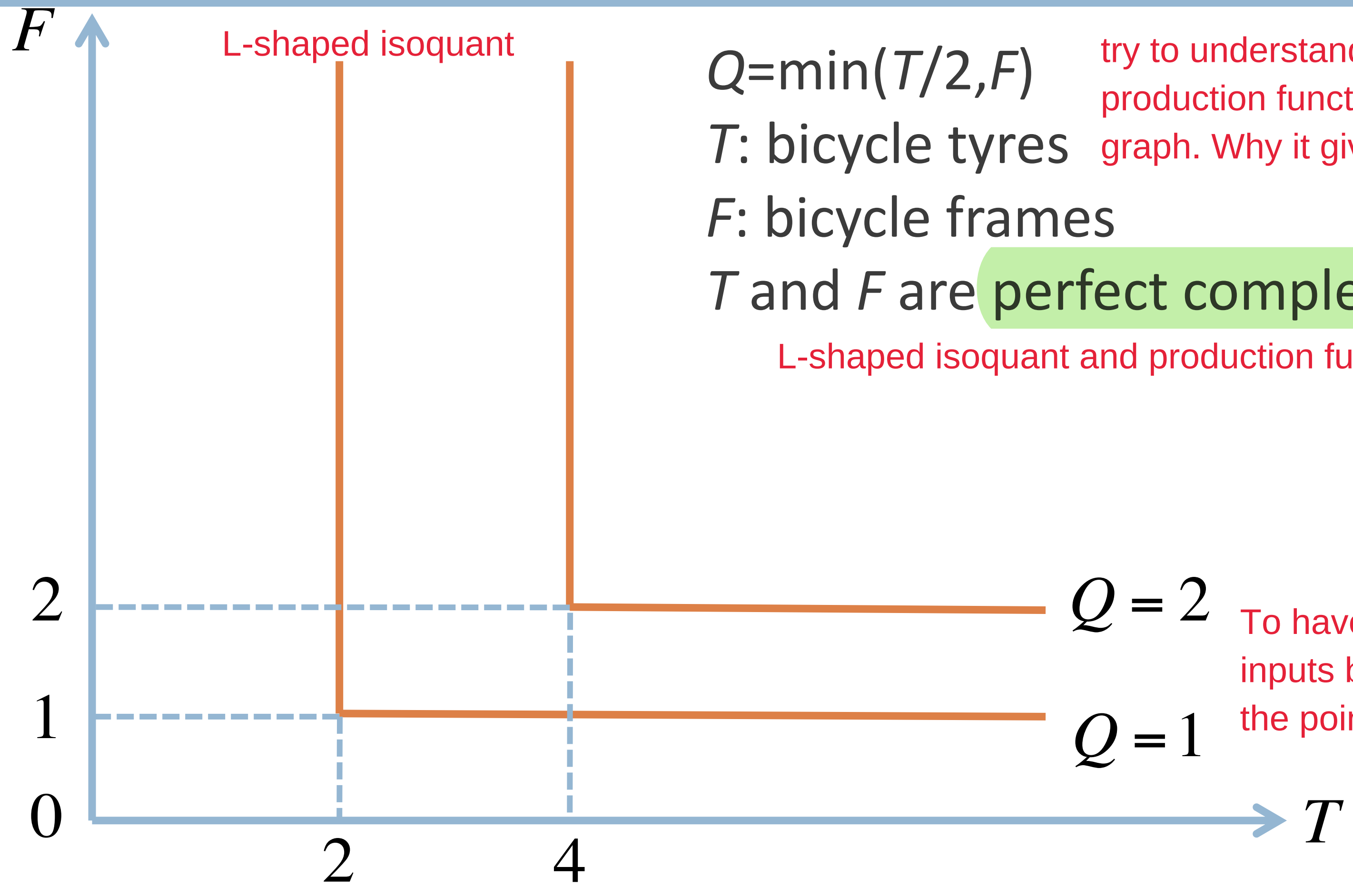
2 new workers are as good as 1 experienced worker.

perfect substitutes: MRTS constant

4 new workers here equivalent to 2 experienced workers

If have linear production function and linear isoquant, we call the 2 inputs perfect substitutes.

Fixed Proportions Production Function



$Q=\min(T/2,F)$

try to understand why this function is the correct production function for the isoquant we see in the graph. Why it gives us kink point at (2,1) and (4,2)

T: bicycle tyres

F: bicycle frames

T and *F* are perfect complements

L-shaped isoquant and production function is min function

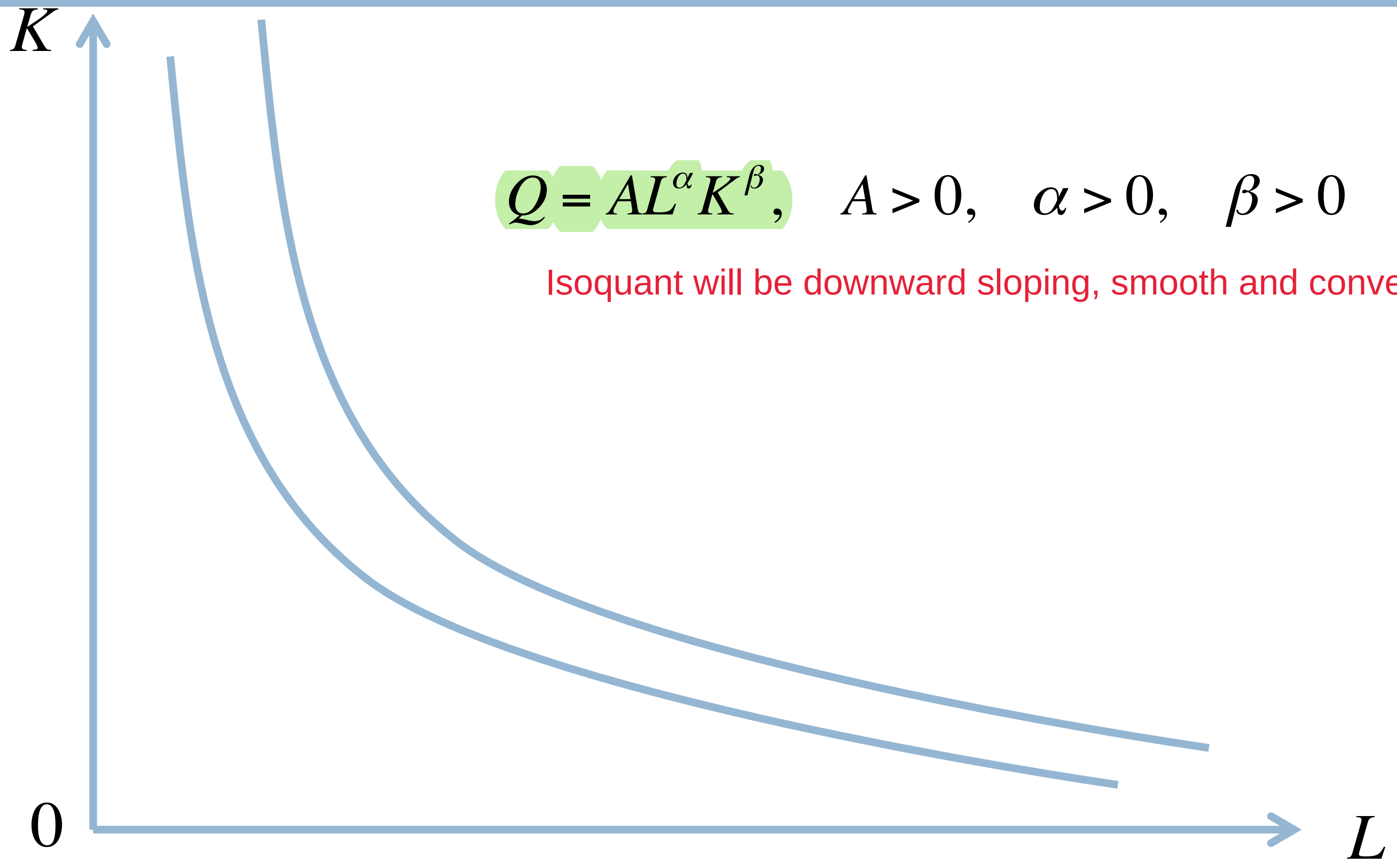
To have 2 bicycles, need to increase both inputs by the same proportion to move to the point (4,2)

Cobb-Douglas Production Function

just like Cobb-Douglas utility function

$$Q = AL^{\alpha}K^{\beta}, \quad A > 0, \quad \alpha > 0, \quad \beta > 0$$

Isoquant will be downward sloping, smooth and convex and doesn't touch axes.



Part 3

Returns to Scale and Technological Progress

Returns to Scale

increase scale of production

- How much more Q can the firm produce when using more L and K?
- *Returns to scale* measures the rate at which output increases when all inputs increase proportionally

increase inputs by the same proportion

  - E.g., how much will output increase if both labor and capital increase by 25%?
  - E.g., how much will output increase if both labor and capital increase by 100%?

Interpreting Returns to Scale

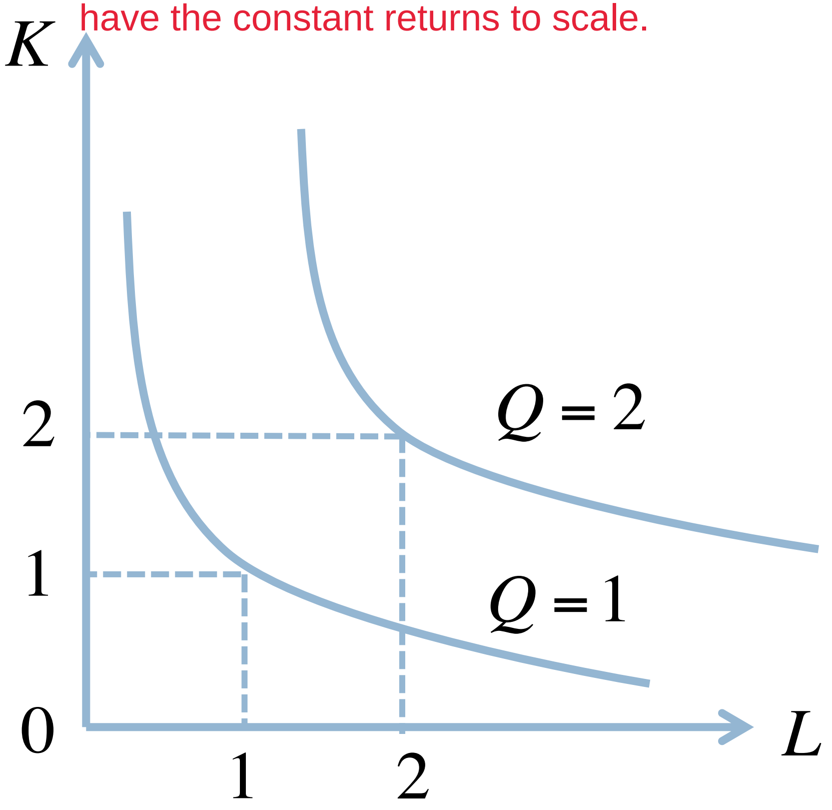
- Suppose when L increases to aL and K increases to aK (a>1)
- Output increases to bQ from Q to bQ
- Definition 7.10 *Increasing returns to scale*
  - If b>a
- Definition 7.11 *Constant returns to scale*
  - If b=a
- Definition 7.12 *Decreasing returns to scale*
  - If b<a

Depending on what production function a firm has, it may fall into increasing/constant/ decreasing return to scale.
So we need to know production function to decide what kind of return to scale the firm is experiencing. (tutorial)

when increase inputs by a factor of a, your output will go up by the same factor.

Returns to Scale and Isoquants

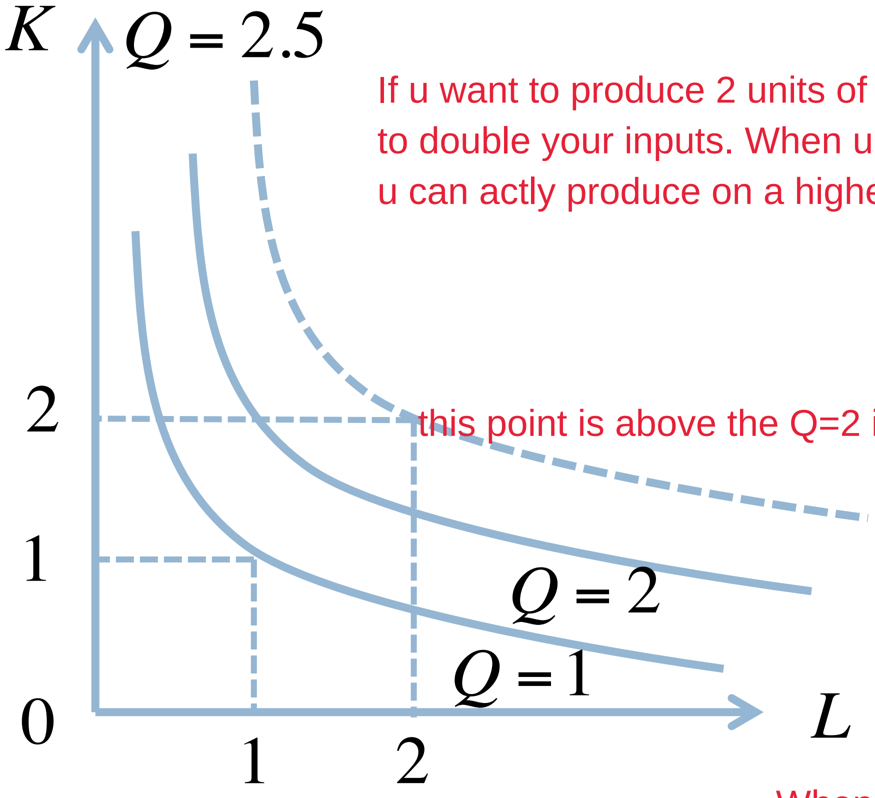
When u have increasing returns to scale, your isoquants are closer to each other compared to the case where u have the constant returns to scale.



Constant Returns to Scale

When firm doubles labor and capital, output also doubled.

If u want to produce 2 units of output, u don't have to double your inputs. When u double your inputs, u can actly produce on a higher isoquant.



this point is above the Q=2 isoquant

Increasing returns to scale

When inputs are doubled, the output is more than doubled.

Technological Progress

Changes in technology changes firm's production function.

- So far we assumed production technology is fixed
  - Production function is fixed

we have been assuming production function is fixed, not true in real life coz technology does improve over time.

- What if technology improves?

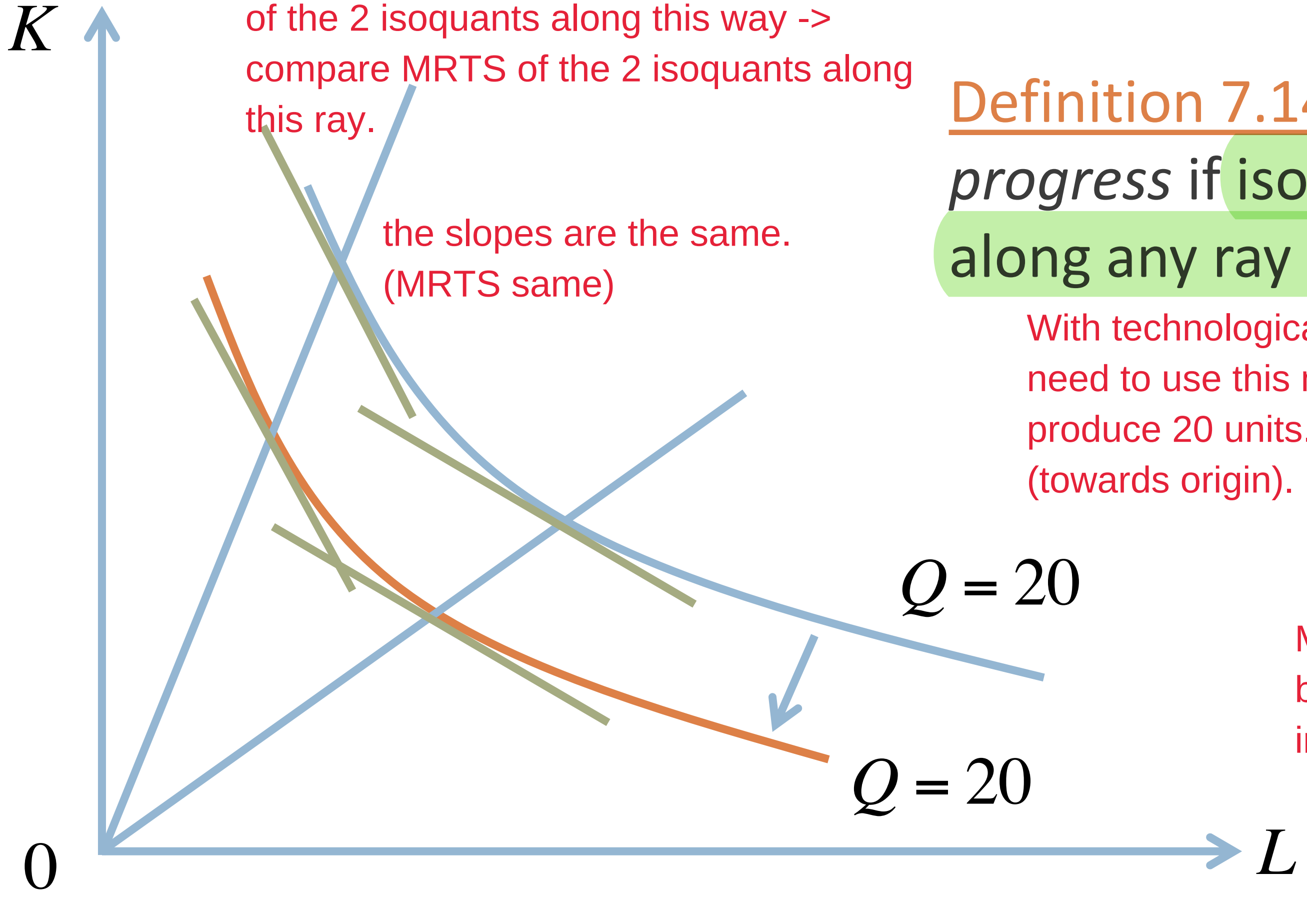
So for the same firm over time, we may be observing a different production function due to an improvement in technology.

- Definition 7.13 We have *technological progress* if for any given combination of inputs, the firm produces higher Q
  - Or, to produce any Q, the firm uses less input

Neutral Technological Progress

3 types of technological progress

draw a ray here & compare the slopes of the 2 isoquants along this way -> compare MRTS of the 2 isoquants along this ray.



the slopes are the same. (MRTS same)

Definition 7.14 We have a *neutral technological progress* if isoquant shifts inward and $MRTS_{L,K}$ along any ray from the origin remains the same

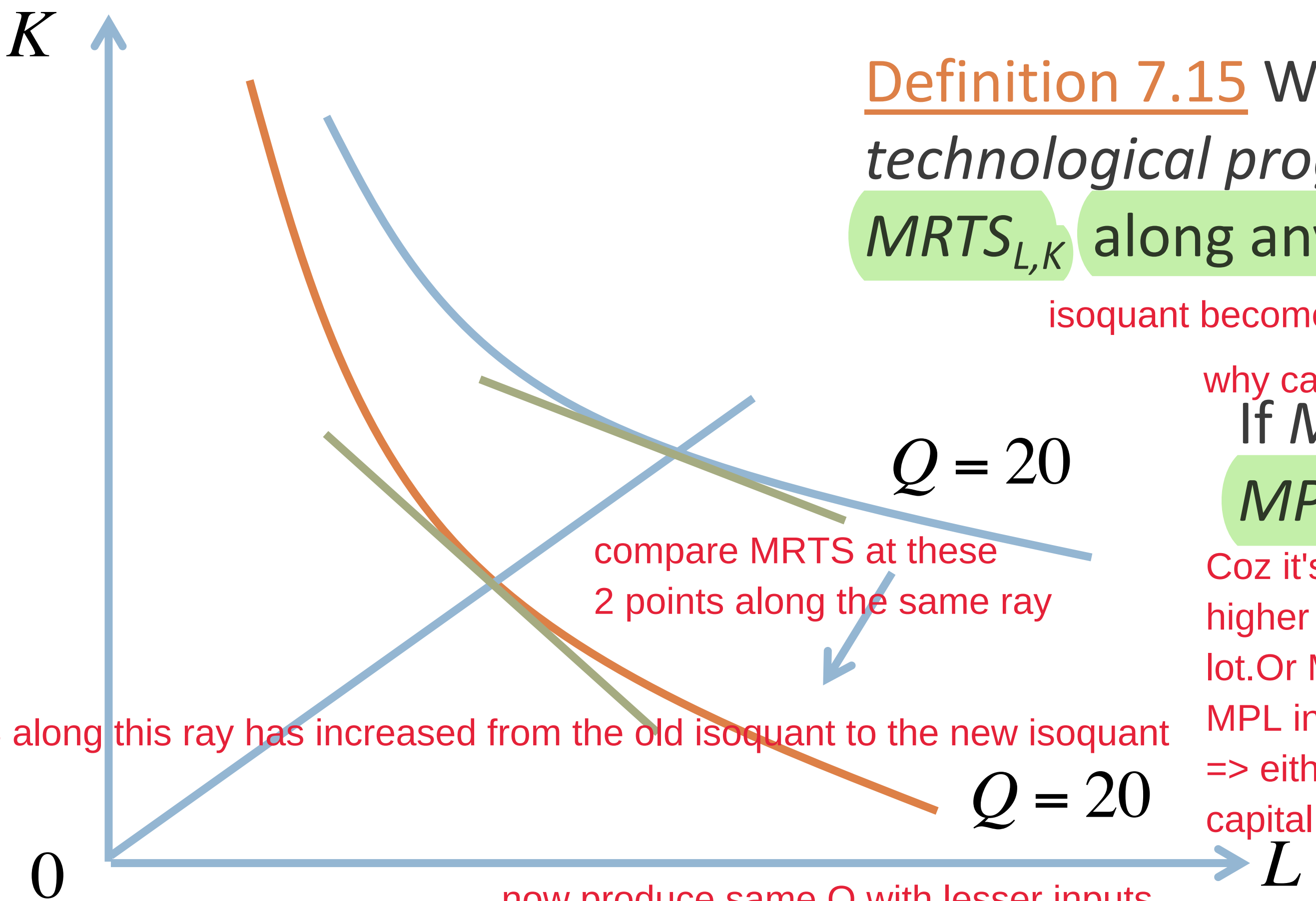
With technological progress, to produce 20 units of output, we don't need to use this many inputs anymore. Can use less inputs still produce 20 units. -> Graphically isoquant of Q=20 shifts inwards (towards origin).

MRTS doesn't change -> ratio stays same -> labor and capital become more productive by the same proportion (balanced improvement of technology)

Capital-Saving Technological Progress

Labor more productive, so save capital

The source of technological progress in this case is coz labor has become more productive, MPL increases relative to MPK.
maybe better training for employees, better specialization. etc.



compare MRTS at these 2 points along the same ray

MRTS along this ray has increased from the old isoquant to the new isoquant

now produce same Q with lesser inputs

Definition 7.15 We have a *capital-saving technological progress* if isoquant shifts inward and $MRTS_{L,K}$ along any ray from the origin increases

isoquant becomes steeper than before.

why capital saving?

If $MRTS_{L,K} = MP_L/MP_K$ increases

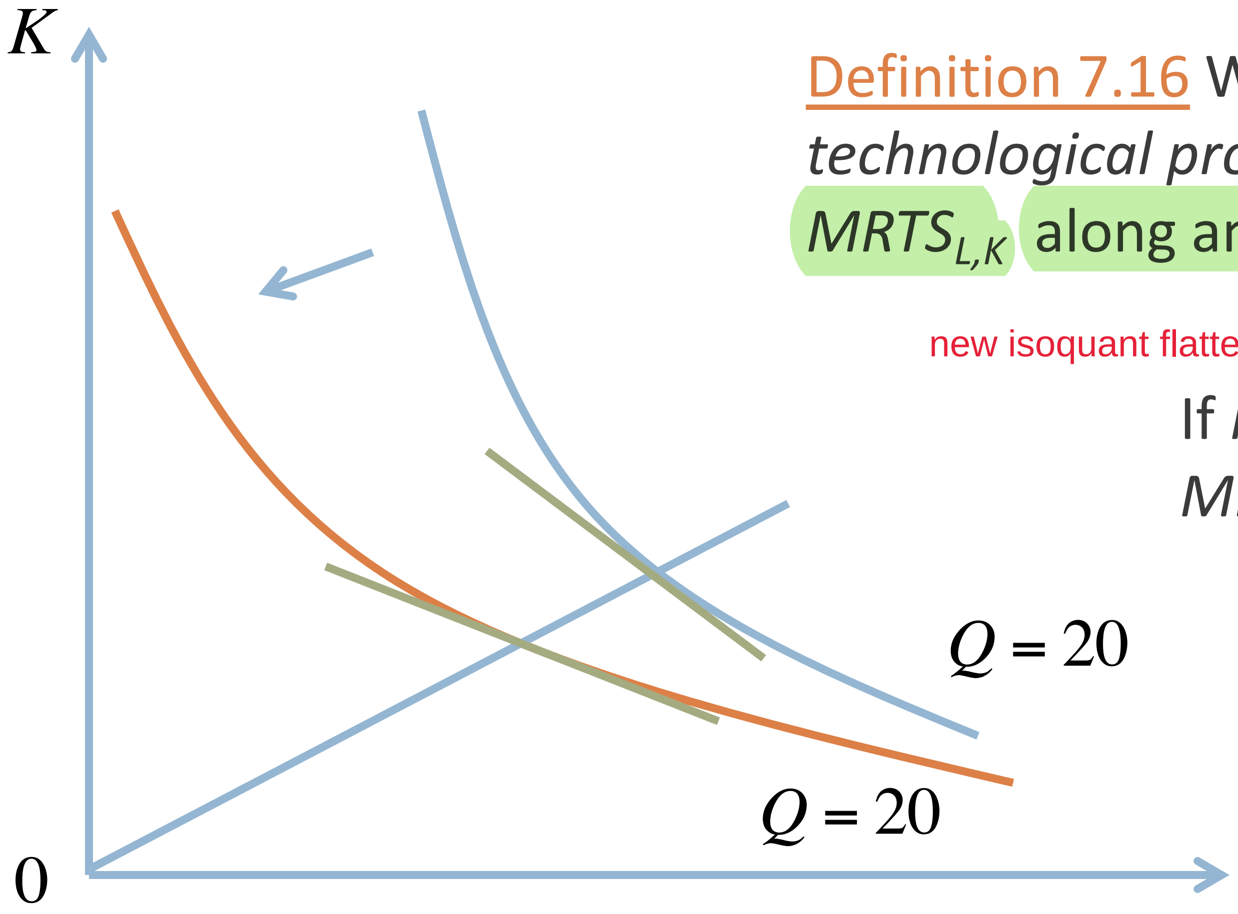
MP_L increases relative to MP_K

Coz it's technological progress, we expect MP to increase, better/ higher than before. Maybe MPk increase a bit and MPL increases a lot.Or MPk doesn't change, so productivity of capital is the same, but MPL increases, so ratio goes up.
=> either way, it's labor that becomes more productive relative to capital.

Capital saving: MPL higher than before -> If u use a little bit more labor, u can save more capital than before since 1 unit of labor is more productive than before.

Labor-Saving Technological Progress

Labor-saving, capital more productive, so can save more labor than before -> MPK increases relative to MPL, so ratio goes down -> MRTS decreases -> new isoquant flatter



Definition 7.16 We have a *labor-saving technological progress* if isoquant shifts inward and $MRTS_{L,K}$ along any ray from the origin decreases

new isoquant flatter.

If $MRTS_{L,K} = MP_L/MP_K$ decreases

MP_K increases relative to MP_L

so maybe MPL doesn't change (no change in productivity of labor), just capital becomes more productive.
Or maybe MPL increases, but MPK increases more.
Capital is driving the technological progress. (e.g. better machines)

So if u use a little bit more capital, can save more labor than before.

NOTE: we cannot just look at how MRTS has changed, we need to look at how MRTS along a ray from the origin has changed.

Does Neutral Technological Progress Mean *MRTS* does not change?

- Suppose the initial production function is

$$Q^1 = KL + K$$

- The new production function is

obviously there is tech progress, coz with same amt of inputs, now we can double the output.

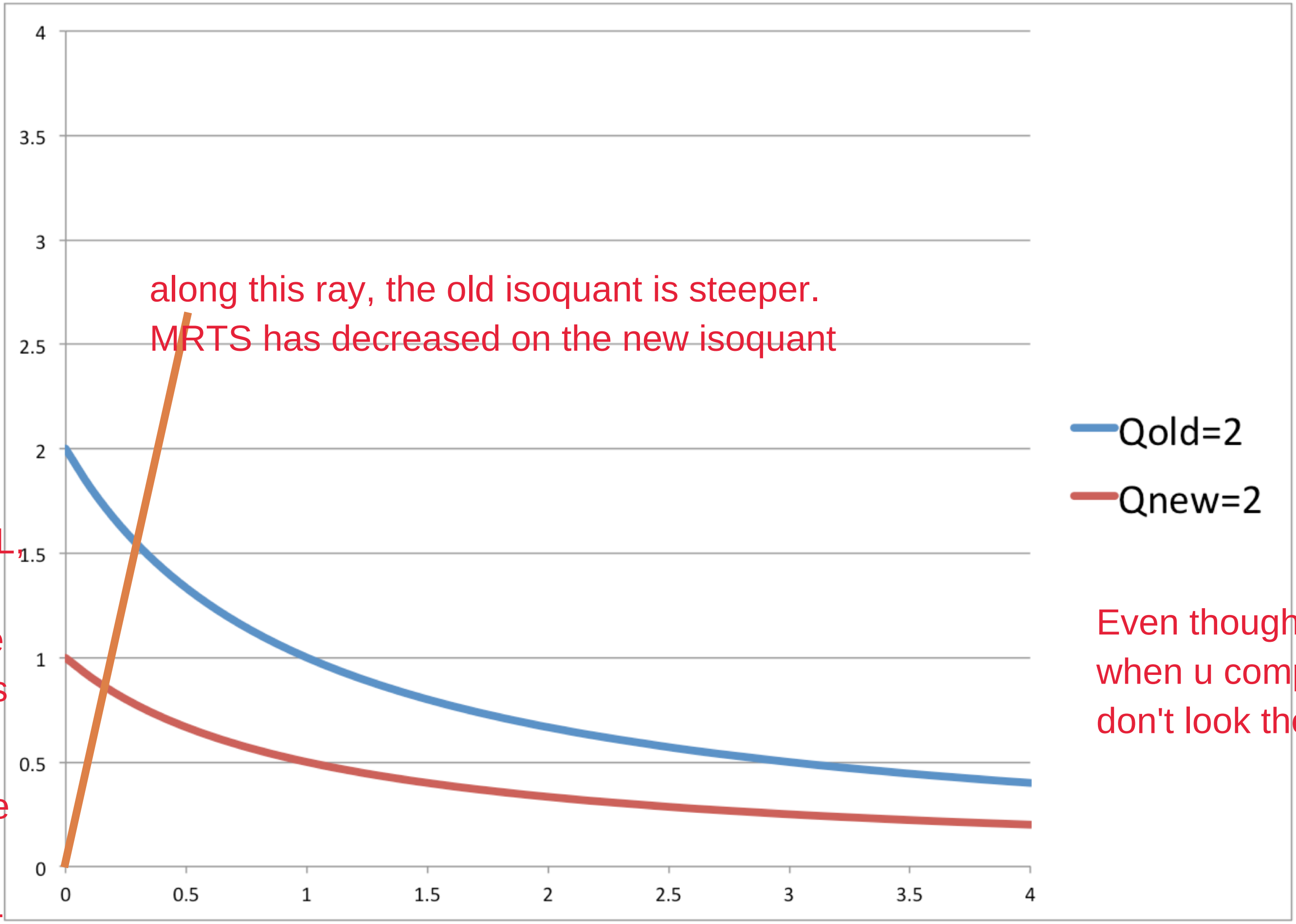
$$Q^2 = 2(KL + K)$$

- $MRTS_{L,K}$ does not change

$$MRTS^1_{L,K} = MRTS^2_{L,K} = \frac{K}{L+1}$$

- Is this neutral technological progress? since MRTS for the 2 production functions are the same?

Isoquants of Q=2 before and after



From graph, the 2 curves do not seem to have the same shape.

A ray from origin is K = number*L, e.g. K=L, K=2L...

When u compare 2 pts along the same ray from origin, these 2 pts gonna have the same K/L ratio.

What we need to do to determine the type of tech progress is to compare 2 pts with the same K/L ratio. And the 2 pts should lie on the isoquant with the same Q, given 2 production functions.

Even though the equation of MRTS hasn't changed, when u compare MRTS along a ray from origin, they don't look the same.

MRTS along a ray from the origin not the same

- With the initial production function
  - When $L=K=1$, $Q=2$
  - ($L=1$, $K=1$) is on the ray $K=L$ this is a ray from origin
  - At this point $MRTS_{L,K}=1/(1+1)=0.5$
- With the new production function
  - The point on $Q=2$ and $K=L$ is ($L=0.62$, $K=0.62$)
  - At this point $MRTS_{L,K}=0.62/(0.62+1)=0.38$
- $MRTS_{L,K}$ along the ray $K=L$ not the same!

NOTE: comparing the equation of MRTS and compare MRTS along a ray from the origin are NOT the same thing!

Hence this is not neutral tech progress coz it doesn't satisfy the definition.

Technological Progress for Cobb-Douglas Production Functions

- Suppose the initial production function is

$$Q^1 = KL$$

- The new production function is

$$Q^2 = 2KL$$

- $MRTS_{L,K}$ does not change

$$MRTS^1_{L,K} = MRTS^2_{L,K} = \frac{K}{L}$$

for cobb-douglas function, MRTS = alpha K / beta L.
For this example, alpha and beta = 1, so MRTS = K/L.

-> Cobb-douglas production function, the MRTS itself is the K/L ratio. Either K/L, or some number times K/L.

- This is indeed a neutral technological progress

When u require the 2 pts to be on the same ray from origin, means same K/L ratio, of course u gonna get same number for MRTS.

We can just Compare *MRTS* for Cobb-Douglas Production Functions

- With the initial production function
  - When $L=1$, $K=2$, $Q=2$
  - $(L=1, K=2)$ is on the ray $K=2L$
  - At this point, $MRTS_{L,K}=2$
- With the new production function
  - The point on $Q=2$ and $K=2L$ is $(L=0.71, K=1.41)$
  - At this point $MRTS_{L,K}=2$
- $MRTS_{L,K}$ along the ray $K=2L$ are the same
- Same applies to any ray $K=aL$

comparing the MRTS along a ray from origin is the same as just compare equations of MRTS.

If u have a Cobb-Douglas production function, and if u know the MRTS equation doesn't change, then u can indeed conclude that this is a neutral tech progress.

Note: Not true: if have Cobb-Douglas production function, always have neutral tech progress. Can still have all 3 types.

Just that when we have Cobb-Douglas, we don't need to compare MRTS along a ray from origin, we can just compare the MRTS. Cannot do this for other production function.