LECTURE 2
PREFERENCE
BUDGET CONSTRAINT
CONSUMER CHOICE

Makeup Tutorials

- W1-W6 on 27 Jan are canceled due to CNY
- Makeup Tutorials
  - 28 Jan (Tue): 12 pm to 1 pm, AS2-0510
  - 28 Jan (Tue): 1 pm to 2 pm, AS2-0510
  - 29 Jan (Wed): 11 am to 12 pm, AS2-0510
- If you are from W1-W6, try to attend one of the above makeups if possible
  - If not, try to go to any other tutorial that fits your schedule

Where are we?

- Preference
  - Indifference curves
    - Do not cross
    - Downward sloping if “more is better” is satisfied for both goods
    - Slope of indifference curve
  - Utility functions
  - Special preferences
- Budget constraint
- Consumer choice
  - Which basket will the consumer choose to buy?

Part 1

Preference

Marginal Rate of Substitution

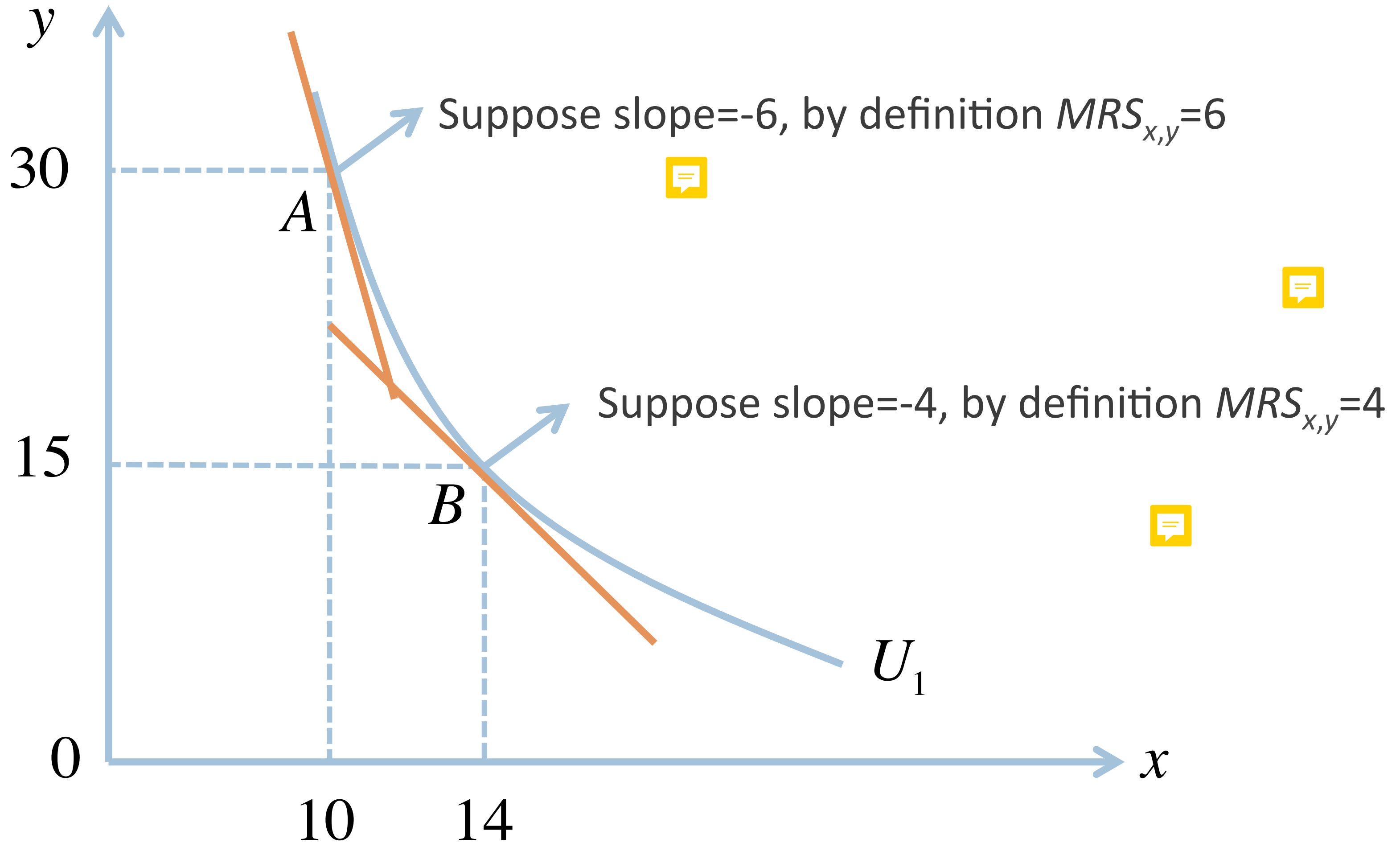
- Definition 2.1 *Marginal rate of substitution* of *x* for *y* is the rate at which the consumer is willing to give up *y* to get more of *x*, maintaining the same level of satisfaction

$$MRS_{x,y} = -\left.\frac{dy}{dx}\right|_{Same\ U} = -\left.\frac{\Delta y}{\Delta x}\right|_{Same\ U}$$

where Δx is extremely small

- $MRS_{x,y}$ is the negative of the slope of the indifference curve (with *x* on the horizontal axis and *y* on the vertical axis)

MRS and the Shape of Indifference Curve



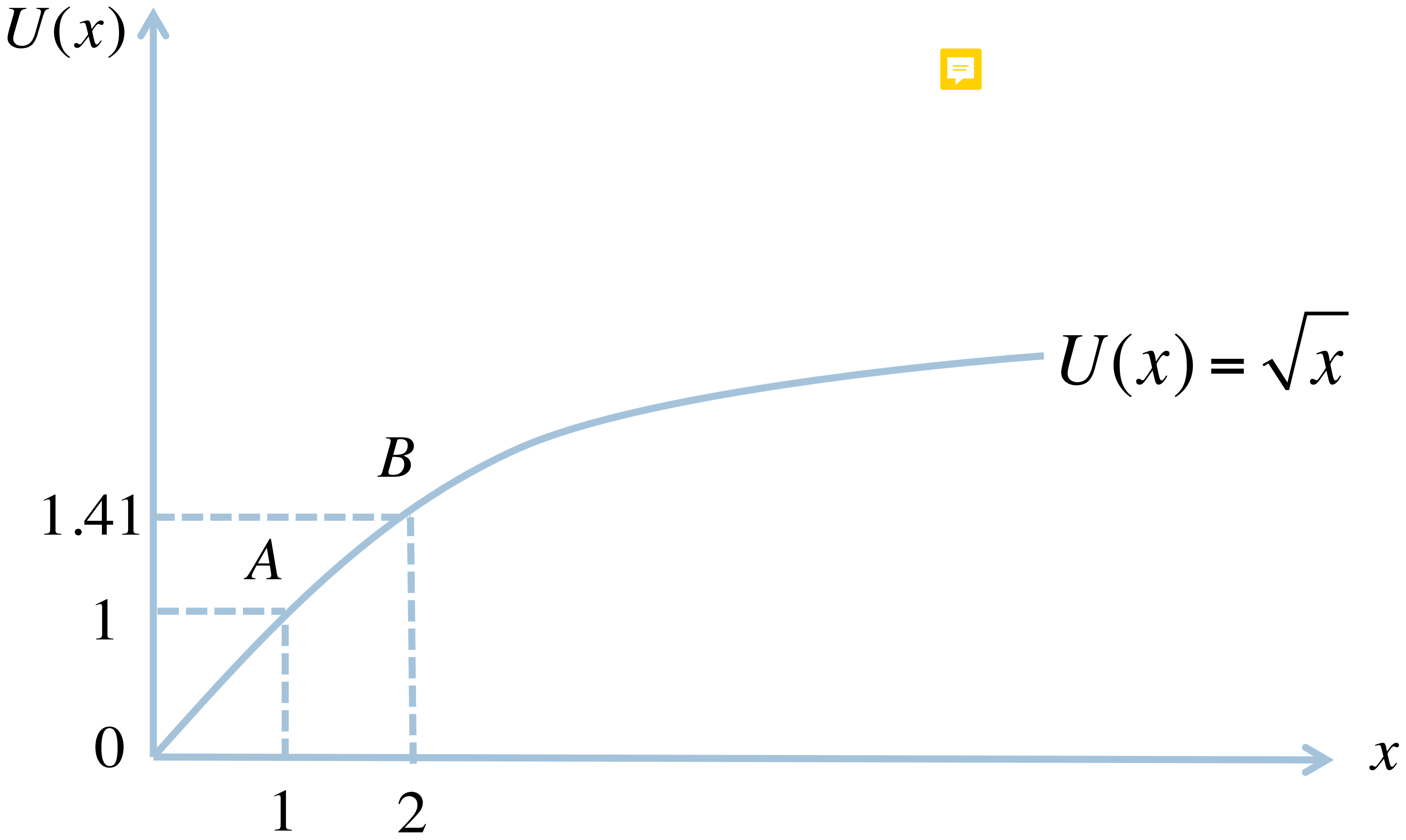
Diminishing Marginal Rate of Substitution

- *Diminishing marginal rate of substitution* means $MRS_{x,y}$ decreases as the consumer gets more *x* and less *y* along the same indifference curve
  - Holding satisfaction level fixed, as the consumer gets more of *x*, the willingness to give up *y* and get additional *x* reduces
- If diminishing marginal rate of substitution holds
  - And if the three assumptions hold
  - Then indifference curves are convex to the origin

Utility Function

- *Utility* is a numeric value indicating the consumer's level of satisfaction
- Definition 2.2 *Utility function* assigns a level of utility to each consumption basket so that if $A \succ B$, $U(A) > U(B)$
  - Utility function represents preference
  - Higher utility = higher the level of satisfaction

Utility Function with One Good: An Example



Marginal Utility

- Definition 2.3 *Marginal utility* is the rate at which utility changes as the level of consumption of a good changes

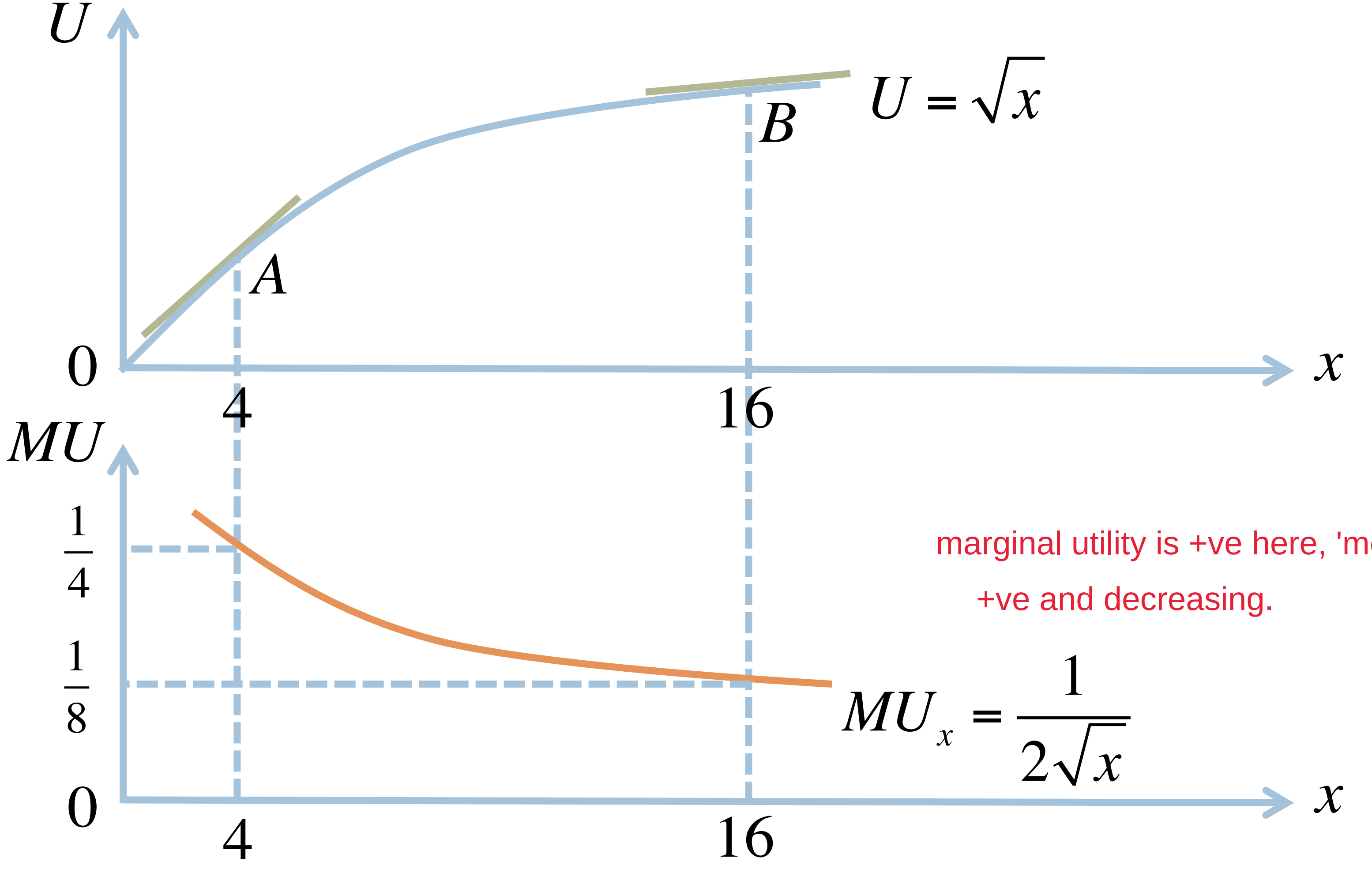
$$MU_x = \frac{dU}{dx} = \frac{\Delta U}{\Delta x}$$

where Δx is extremely small

- *MU* is the slope of the utility function
- What does the sign of marginal utility tell us?

Whether more is better is satisfied.
If marginal utility is +ve, yes more is better.
If -ve, more is not better.

Marginal Utility in Graph



marginal utility is +ve here, 'more is better' holds.

+ve and decreasing.

Principle of Diminishing Marginal Utility

call this principle, means this is quite common. Does not mean this has to be satisfied always, depends on consumer preference. (similar to diminishing MRS)

- *Principle of diminishing marginal utility*
  - Marginal utility decreases as consumption level rises
  - Utility increases slower as consumption level rises
  - Utility function becomes flatter as consumption level rises

Marginal Utility with Two Goods

2 goods, 2 MUs.
5 goods, 5 MUs.

- More often, we will deal with utility functions with two goods $U(x, y)$
- Marginal utility of x rate of change in utility with consumption of x, holding consumption of y fixed.

$$MU_x = \frac{\partial U}{\partial x}$$

partial derivative

- Marginal utility of y

$$MU_y = \frac{\partial U}{\partial y}$$

- Diminishing marginal utilities
  - MU_x decreases with x, holding y constant
  - MU_y decreases with y, holding x constant

Utility Function with Two Goods: An Example

- Suppose the utility function of the consumer is

$$U(x,y) = \sqrt{xy}$$

just from utility function, can tell 'more is better' is satisfied as utility increases with both x and y.

- "More is better" is satisfied for both goods

$$MU_x = \frac{\sqrt{y}}{2\sqrt{x}} > 0, \quad MU_y = \frac{\sqrt{x}}{2\sqrt{y}} > 0$$

here we can also do it mathematically.

- Diminishing marginal utility is satisfied for both goods

For good x, look at MU x: when x increases, MU x decreases, holding y constant.
Same for good y: consume more y, MU y decreases, holding x constant.
This is by observing. If u want to be sure, can take 2nd order derivative.

From Utility Function to Indifference Curves

- Indifference curves can be drawn from the utility function
- (2,2) indifferent to (1,4) and (4,1)

we can fix a utility level, then look for a few points (baskets) that give this utility level, then connect them.

all these 3 points lie on the same indiff curve

$$U(2,2) = U(1,4) = U(4,1) = \sqrt{4} = 2$$

- (3,3) preferred to (2,2)

$$U(3,3) = 3 > U(2,2) = 2$$

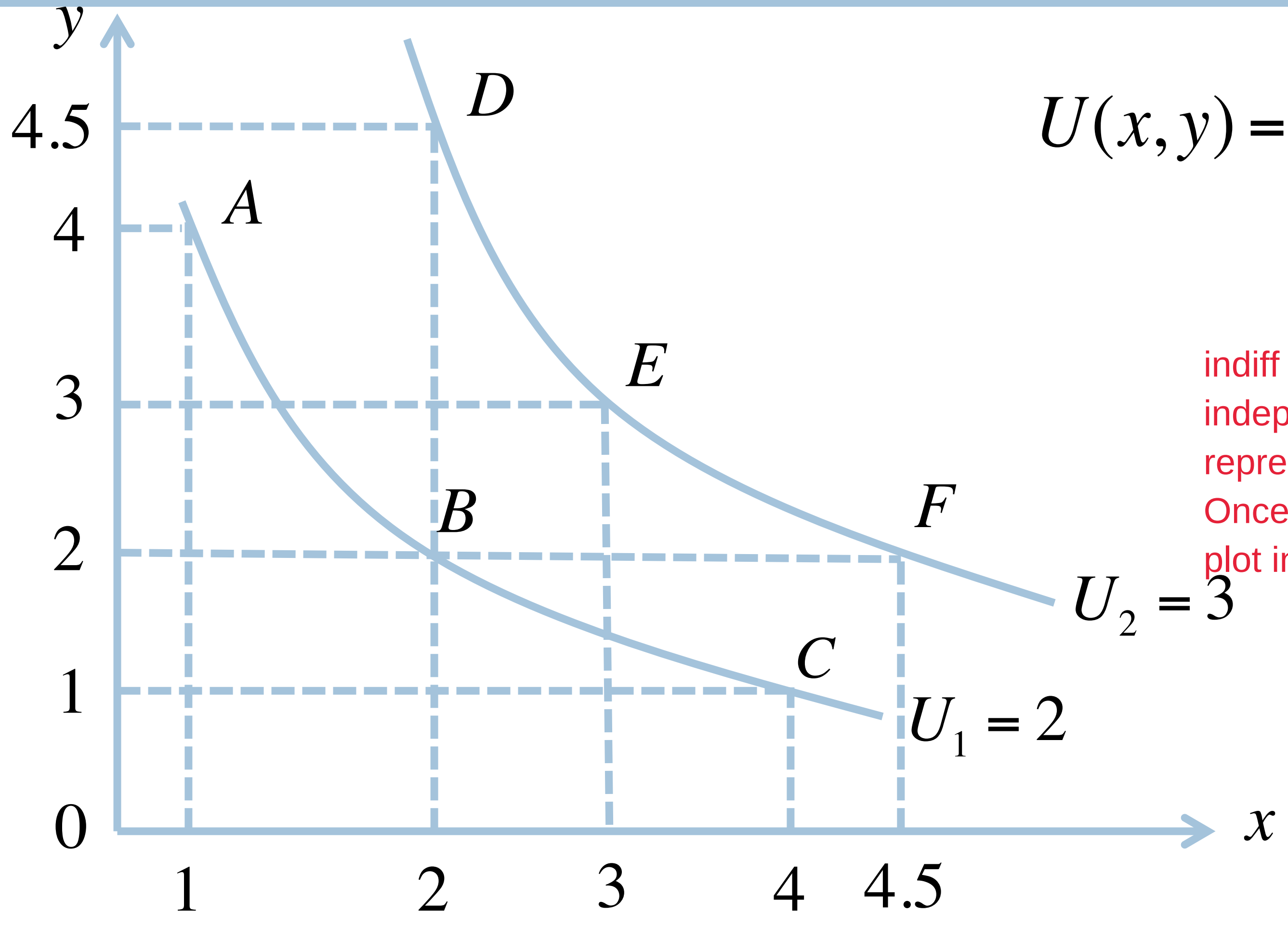
- (3,3) indifferent to (2,4.5) and (4.5,2)

same indiff curve

$$U(3,3) = U(2,4.5) = U(4.5,2) = 3$$

Plotting Indifference Curves

So instead of drawing out a 3-D utility function graph (x,y,U), we can now draw indiff curves which are 2D. These will represent this utility function.

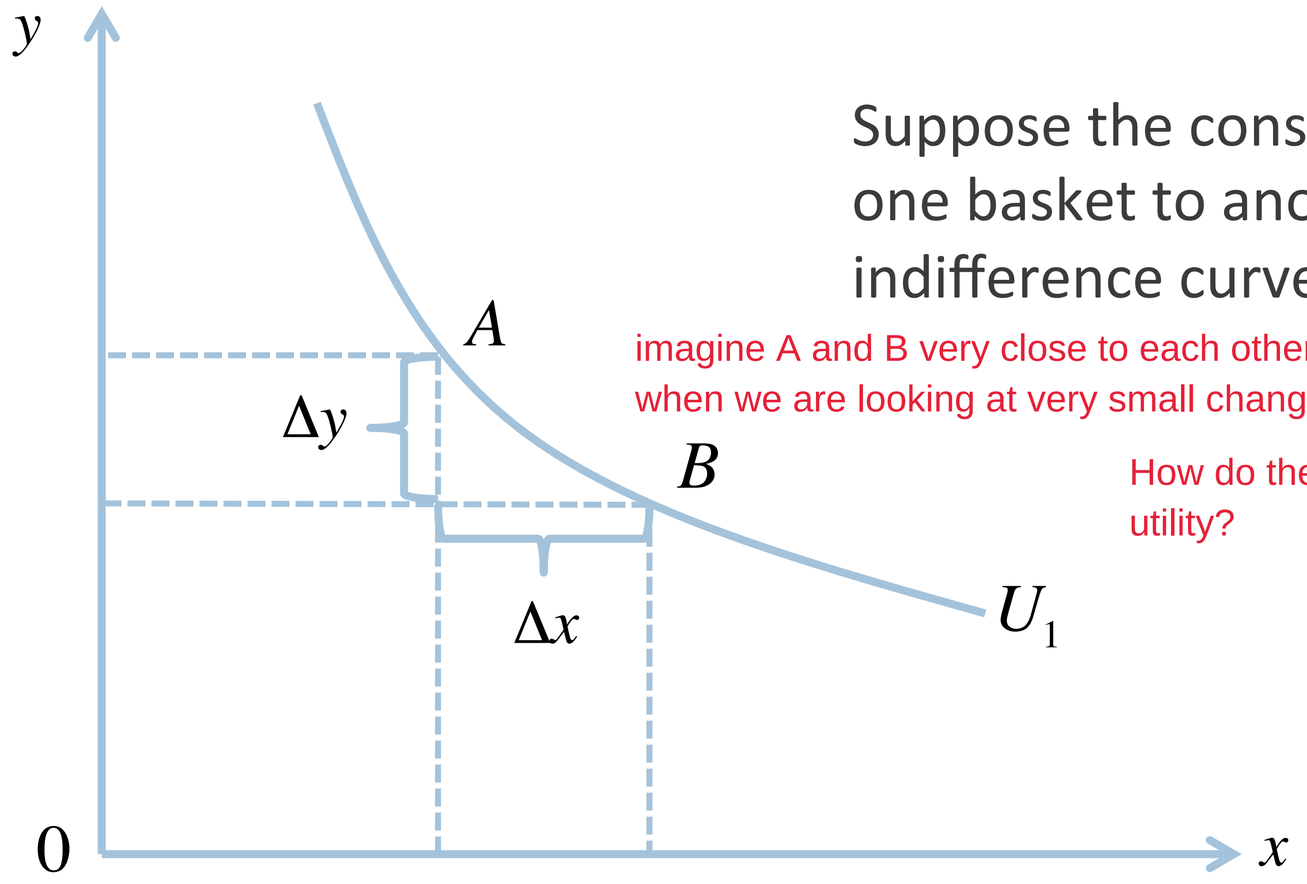


$$U(x,y) = \sqrt{xy}$$

indiff curve and utility function are not 2 independent things. They are just 2 ways of representing consumer preference.
Once u have utility function, u can use it to plot indiff curves.

MRS and *MU*

MRS: slope of indiff curve
relationship between utility function and MRS



Suppose the consumer moves from one basket to another along the same indifference curve

imagine A and B very close to each other because derivative is only valid when we are looking at very small changes.

How do the change in x and change in y affect utility?

MRS and *MU* Cont'

impt, use it all the time

- The total change in utility is

total differentiation

1st part: how much utility changes due to change in x.
2nd part: change in utility due to change in y.

$$\Delta U = MU_x(\Delta x) + MU_y(\Delta y)$$

- The total change in utility must be 0

This gives us a way to calculate MRS easily from utility function.
If u have utility function,u can differentiate it to get 2 marginal utilities, the ratio of the 2 MUs is the MRS.

$$0 = MU_x(\Delta x) + MU_y(\Delta y)$$

- Thus

$$\frac{MU_x}{MU_y} = -\frac{\Delta y}{\Delta x} = MRS_{x,y}$$

MRS of x for y

  - The rate at which the consumer is willing to substitute between the two goods holding utility constant is equal to the ratio of the marginal utilities of the two goods

e.g. MRS x,y = 2, means u are willing to give up 2 units of y to get 1 x, utility unchanged. -> MU x is twice of MU y. That's why u want to trade 2 y for 1 x, because utility u get from 1 x is twice as high as that from 1 y. Getting 1 x is essentially the same as getting 2y.

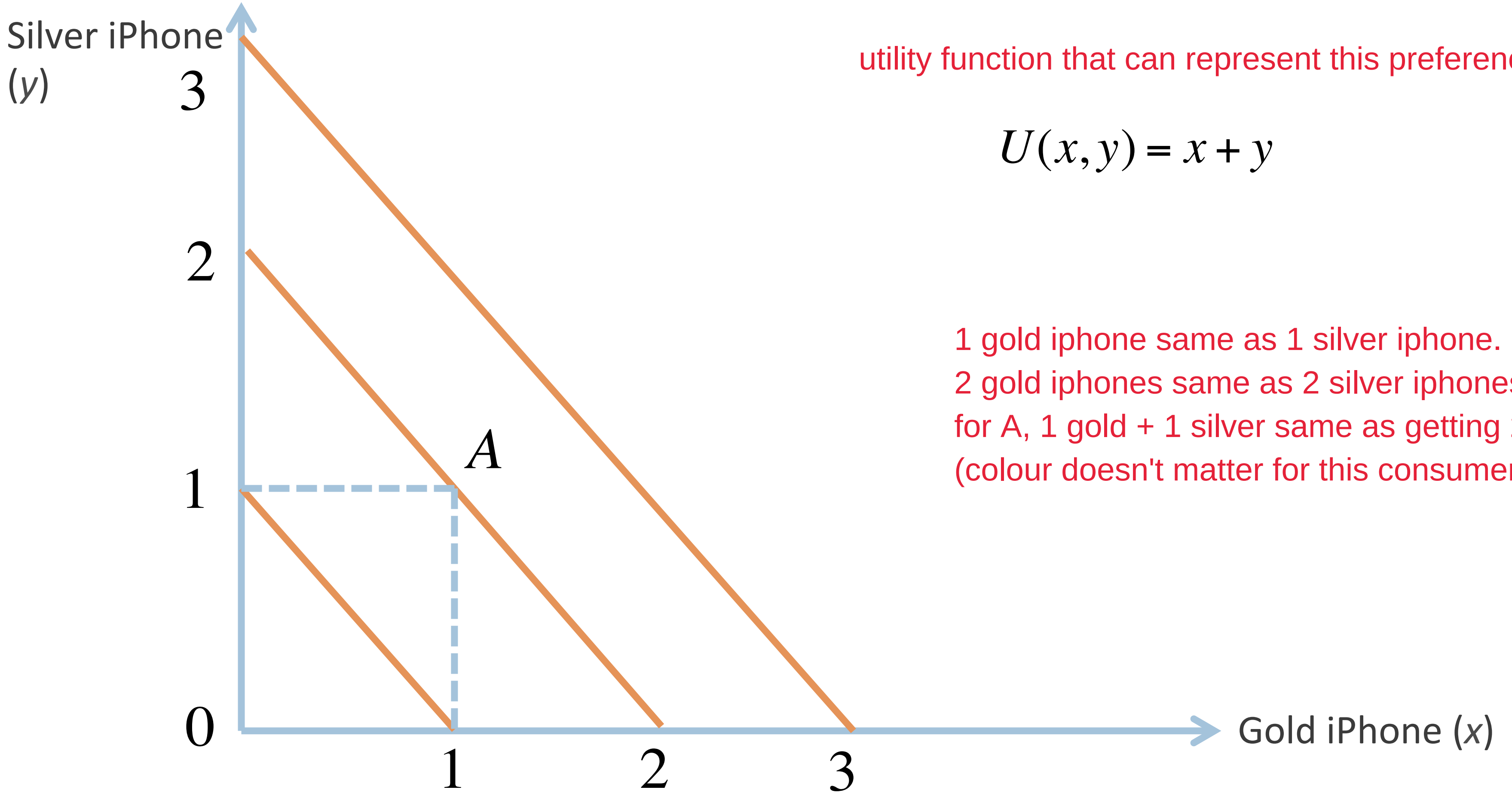
Special Preferences

- Definition 2.4 Perfect substitutes
  - Two goods are *perfect substitutes* if *MRS* is constant
  - Indifference curves are linear
- Definition 2.5 Perfect complements
  - Two goods are *perfect complements* if *MRS* is 0 or infinity
  - Indifference curves are L-shaped

Example of Perfect Substitutes

- Suppose the consumer has the following preference
  - "Gold iPhones and silver iPhones are equivalent"
  - 1 gold iPhone always brings the same utility as 1 silver iPhone
- To the consumer, gold iPhone and silver iPhone are perfect substitutes

Example of Perfect Substitutes: iPhones



utility function that can represent this preference:

$$U(x,y) = x + y$$

1 gold iphone same as 1 silver iphone.
2 gold iphones same as 2 silver iphones.
for A, 1 gold + 1 silver same as getting 2 gold or 2 silver.
(colour doesn't matter for this consumer)

Example of Perfect Substitutes: iPhones Cont'

- We know from the utility function for iPhones
  - Marginal utility of gold iPhone: $MU_x = 1$
  - Marginal utility of silver iPhone: $MU_y = 1$
  - $MRS_{x,y} = 1/1 = 1$
    - Verify from graph, slope of indifference curve is -1

all indiff curves slope is -1.

so we have verified that this utility function corresponds to the set of indiff curves we have. (another way of verifying: substitute points on the curve into the utility function to check whether it holds.)

- For perfect substitutes
  - Utility functions are linear

Perfect substitutes: linear indiff curves & linear utility function

HOWEVER, the ratio of the 2 goods might not be 1. In this example, 1 gold iphone is as good as 1 silver iphone, but it also can be 2 gold iphones as good as 1 silver. As long as the indiff curve is linear, as long as u have a constant MRS, that will be called perfect substitutes. Slope doesn't have to be 1.

Example of Perfect Complements

- Suppose the consumer has the following preference
  - "For every right shoe, I need exactly one left shoe"
  - 2 right shoes and 1 left shoe brings the same utility as 1 right shoe and 1 left shoe
- To the consumer, right shoes and left shoes are perfect complements

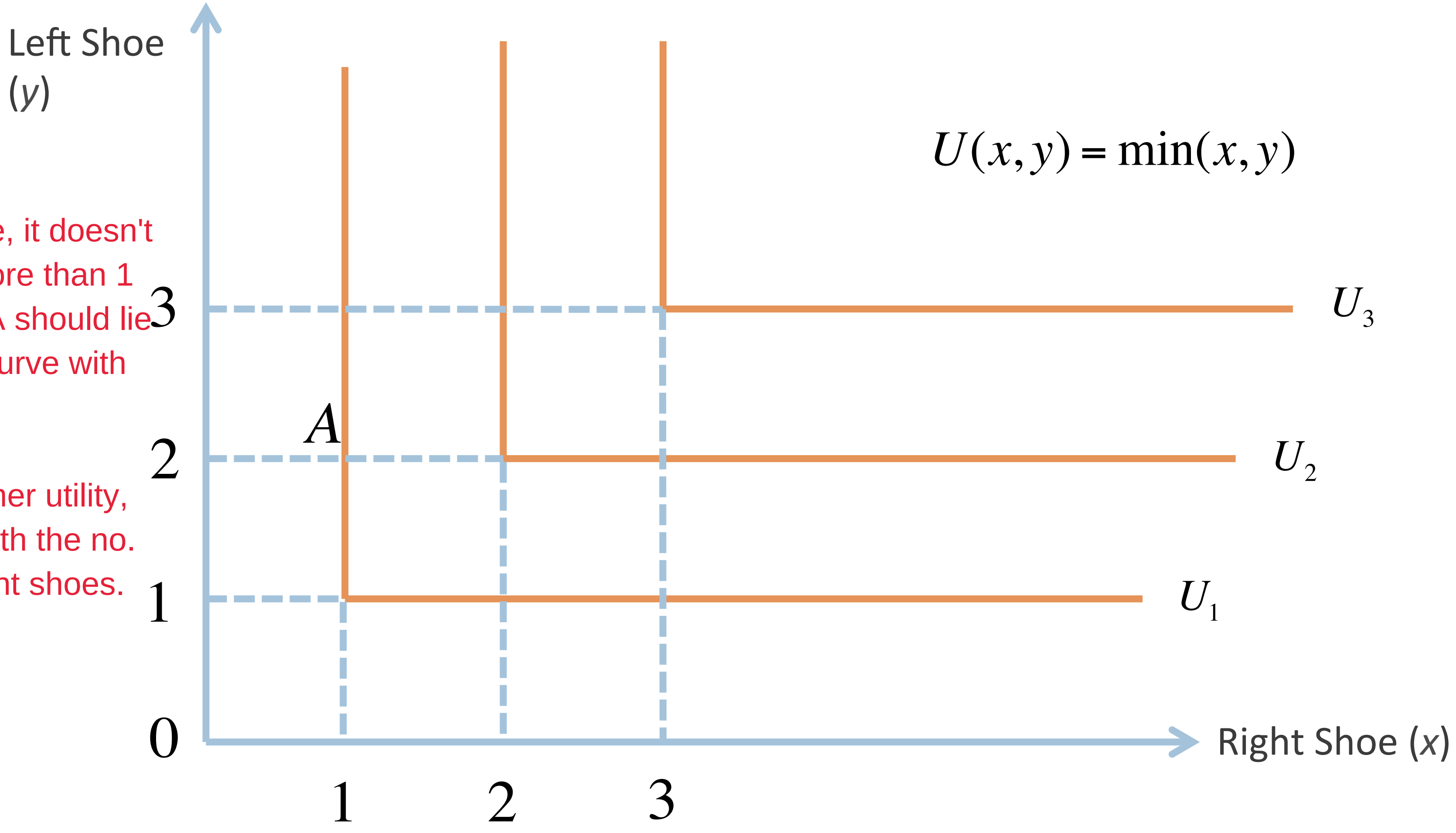
For this consumer and these 2 goods, need to consume these 2 goods in a fixed proportion. 1 left shoe & 1 right shoe.

Example of Perfect Complements: Shoes

indifference curve

if i have 1 right shoe, it doesn't help if u give me more than 1 left shoe. So point A should lie on the same indiff curve with point (1,1).

To increase consumer utility, need to increase both the no. of left and no. of right shoes.



Example of Perfect Complements: Shoes Cont’

- How does the “min” function work?
  - min (x,y) = the smaller of x and y
  - E.g. if $x=1$ and $y=2$, $U(x,y) = \min(1,2)=1$
  - E.g. if $x=1$ and $y=1$, $U(x,y) = \min(1,1)=1$

This also shows us why this utility function corresponds to indiff curves.
utility = 1 even if u increase y from 1 to 2.

- $MRS_{x,y}$ is
  - Infinity in the vertical part
  - 0 in the horizontal part

Important note: this utility function is not differentiable. To be differentiable, the function has to be smooth!
So we cannot just differentiate it and get the MU for this utility function.
So how do we determine MRS in this case? just look at graph, slope of indiff curve is either 0 or -infinity.

- For perfect complements
  - Utility functions are the “min” functions

The ratio of the 2 goods doesn't need to be 1.
Can be 1 left shoe tgt with 2 right shoes. As long as u have L-shaped indiff curves, they are called perfect complements.

Part 2

Budget Constraint

Budget Constraint

- Suppose consumer chooses *F* units of food and *C* units of clothing
- The price of food is P_F
- The price of clothing is P_C
- Consumer has limited income *I*
- *Budget constraint* is

$$P_F F + P_C C \leq I$$

Budget Set vs. Budget Line

- Budget set
  - The set of all baskets the consumer can afford, that is, all baskets that satisfy the budget constraint
- Budget line subset of budget set
  - The set of all baskets consumer can afford by spending all income

$$P_F F + P_C C = I$$

  - Rearranging the budget line

$$C = \frac{I}{P_C} - \frac{P_F}{P_C} F$$

Example of Budget Constraint

- Suppose
  - The price of food is 20
  - The price of clothing is 40
  - Consumer's income is 600
- Budget constraint (and budget set) is

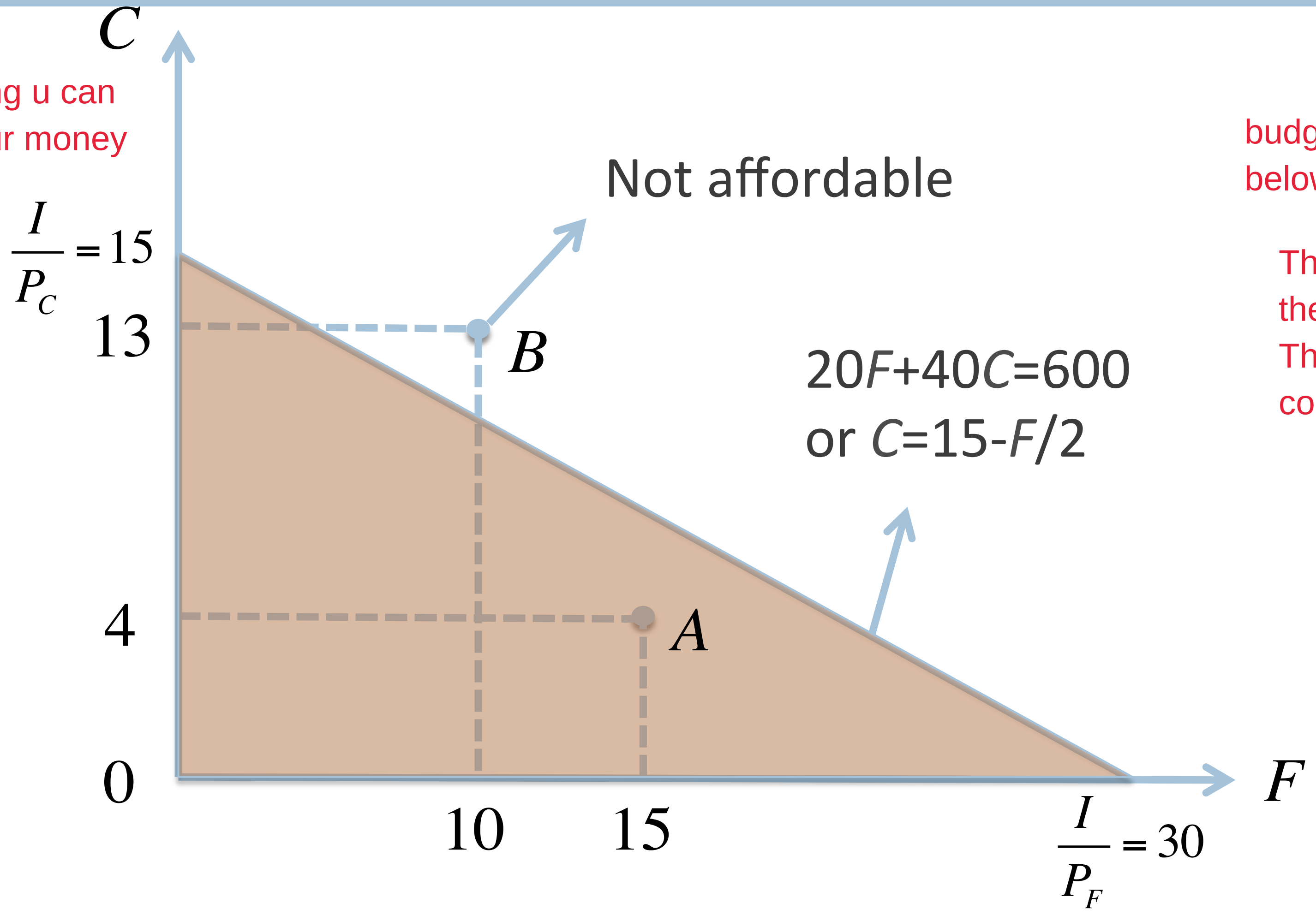
$$20F + 40C \leq 600$$

- Budget line is

$$20F + 40C = 600$$

Budget Constraint in Graph

15: how much clothing u can buy if u spend all your money on clothing.



budget set: any point that's on or below the budget line. (shaded area)

The points on the budget line cost the consumer exactly $600.
The points below the budget line cost consumer <$600.

Slope of Budget Line

- The budget line has a slope of

$$-\frac{P_F}{P_C} = -\frac{20}{40} = -\frac{1}{2}$$

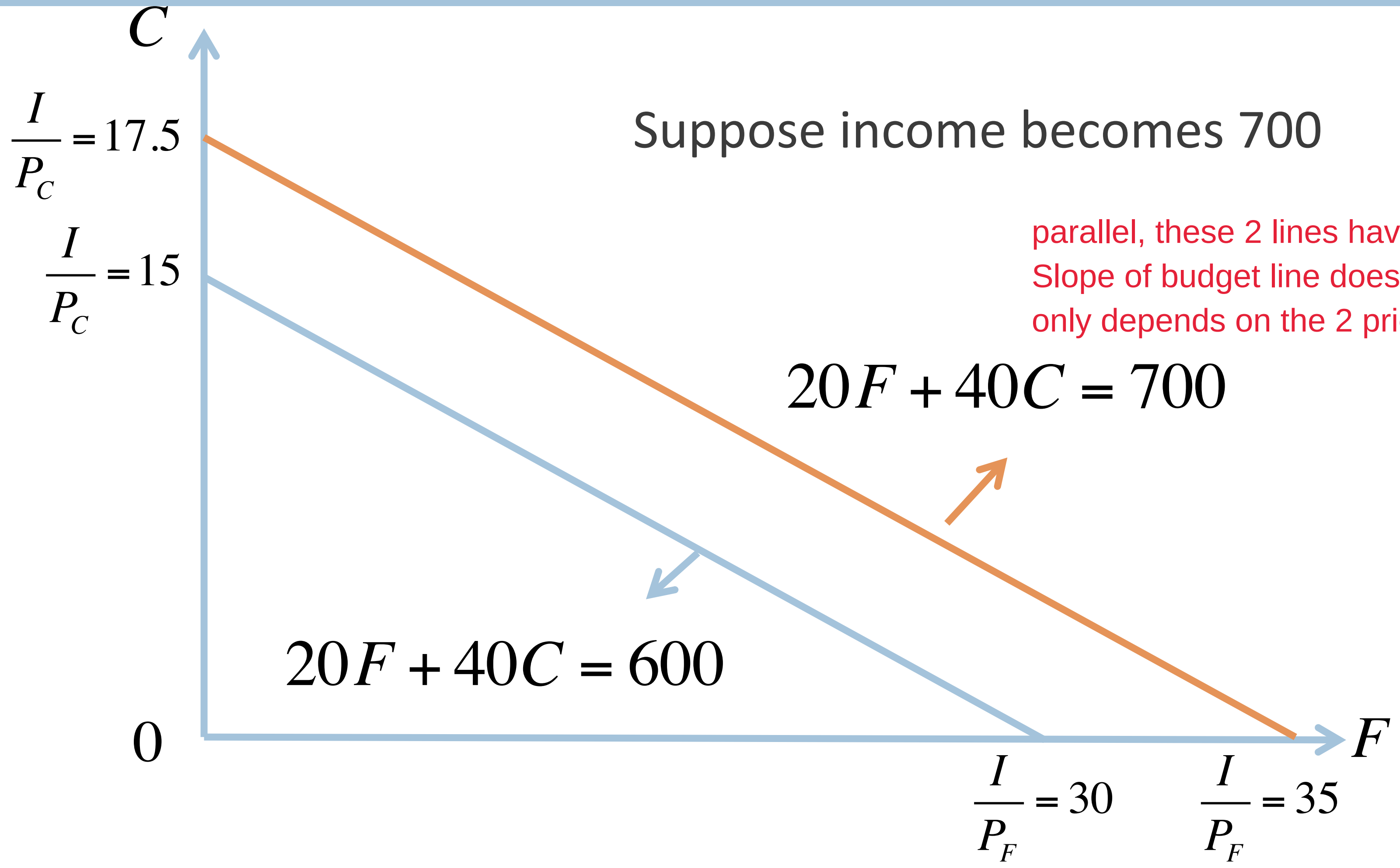
- Slope of budget line represents the rate at which two goods can be substituted in the market, that is, based on the prices
  - Because clothing is twice as expensive as food
  - To get 1 additional unit of food, the consumer *must* give up 0.5 unit of clothing

U are on the budget line, if u want to get 1 additional unit of food, need to give up some clothing because u don't have money to buy food alr. (on budget line means u used all your money alr)

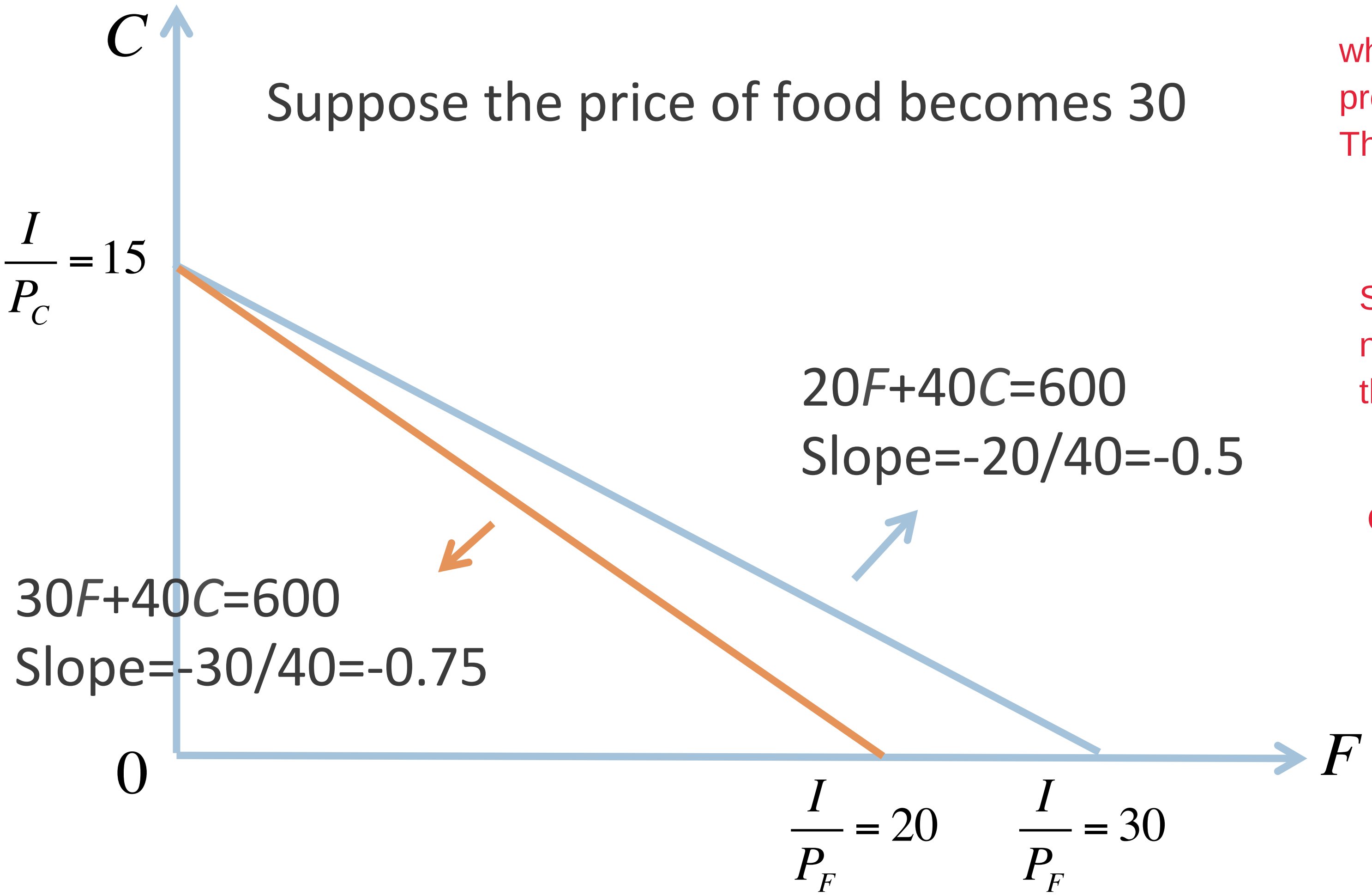
What if income increases?

Suppose income becomes 700

parallel, these 2 lines have the same slope.
Slope of budget line doesn't depend on income,
only depends on the 2 prices.



What if food becomes more expensive?



what if price of clothing also increases by the same proportion (50% increase)? from 40 to 60
The slope will remain the same as the ratio of prices same.

So even if both prices are changing, it does not necessarily mean that slope will change. It depends on the relative price of the 2 goods has changed or not.

Question: is the budget line always a straight line?

maybe buy more units the price is cheaper? etc.

horizontal intercept becomes smaller as food price increases, now can buy fewer units of food if u spend all the money on food.

Part 3

Consumer Choice

Optimal Choice

- What is the optimal basket?
  - Consumer chooses the basket that gives him/her the highest utility given the budget constraint
- Let $U(F,C)$ be the utility function of F units of food and C units of clothing, the consumer solves

we are choosing F & C to maximize utility

$$\max_{F,C} \quad U(F,C)$$

subject to

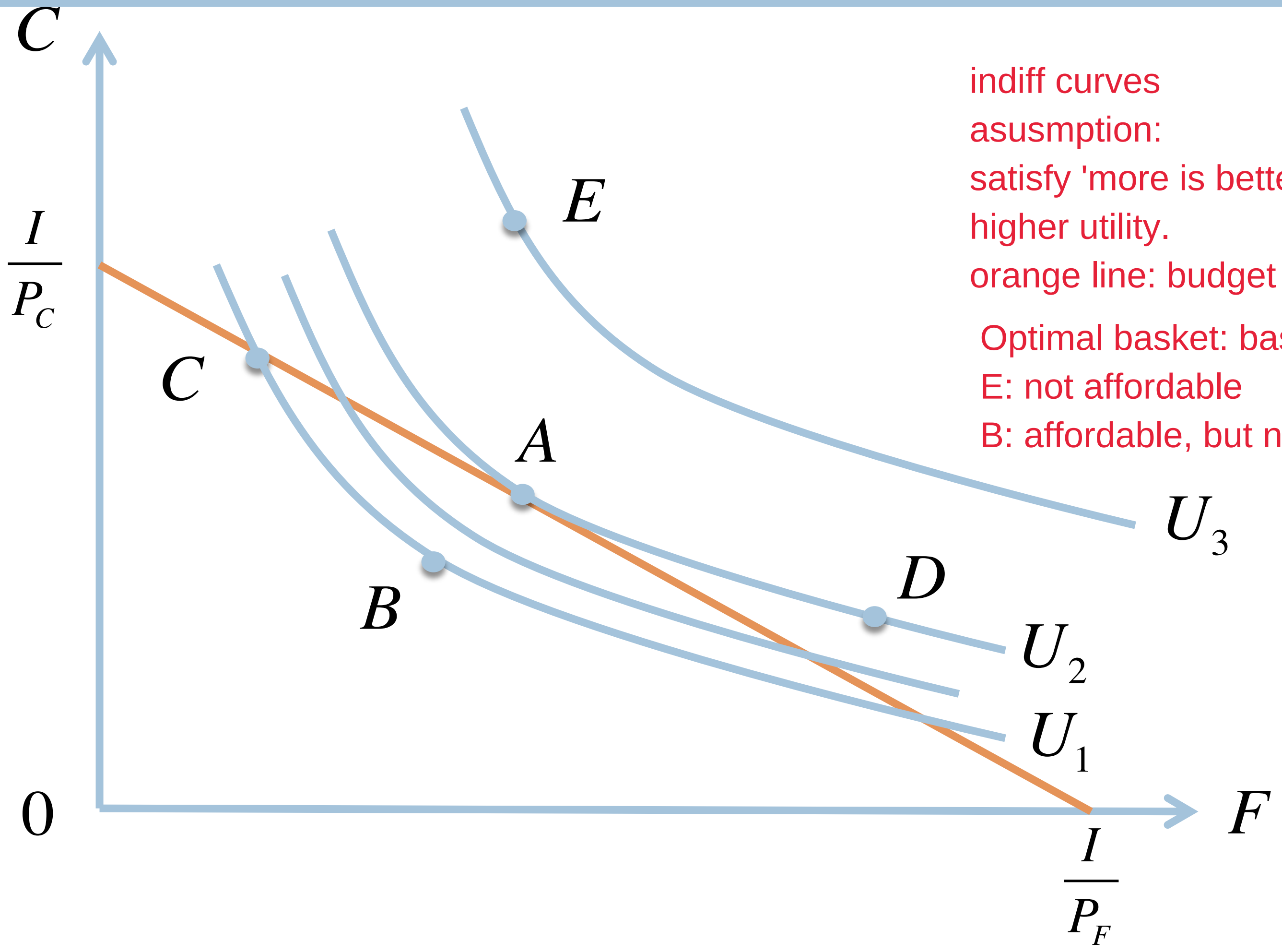
$$s.t. \quad P_F F + P_C C \leq I$$

Mathematically, the solution to this problem will give us the optimal basket, among all the baskets that satisfy the budget constraint, which one maximizes my utility, that's the one I'm going to choose.

  - F and C are the choice variables unknown variables
  - P_F, P_C, and I are the parameters known

We are solving for F and C, the consumer optimal consumption.

Finding the Optimal Basket from the Graph



indiff curves
asusmption:
satisfy 'more is better', so higher indiff curve means higher utility.
orange line: budget line

Optimal basket: basket A
E: not affordable
B: affordable, but not optimal

Two Conditions for Optimal Choice

- Consumer chooses the basket
  - On the budget line
  - On the highest indifference curve
- At the optimal basket
  - Consumer spends all the money

$$P_F F + P_C C = I$$

  - Budget line tangent to indifference curve

slope of budget line = slope of indiff curve

$$-\frac{P_F}{P_C} = -MRS_{F,C} \Rightarrow \frac{P_F}{P_C} = MRS_{F,C}$$

The Tangency Condition

$$MRS_{F,C} = \frac{P_F}{P_C}$$

- To maximize utility, the amount of *F* and *C* consumed should be such that
  - The rate at which the consumer is willing to substitute between the two goods holding utility constant is equal to the rate at which the two goods are exchanged in the market
  - Otherwise the consumer is not maximizing utility

The Equal Marginal Principle

= tangency condition (they are the same condition)

- Since

$$MRS_{F,C} = \frac{MU_F}{MU_C}$$

- The tangency condition can be rewritten as

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{P_F} = \frac{MU_C}{P_C}$$

LHS: per dollar marginal utility of food
RHS: per dollar marginal utility of clothing

- To maximize utility, consumer sets marginal utility per dollar of expenditure equal for both goods
  - Extra utility per dollar spent on food is the same as the extra utility per dollar spent on clothing

What if *MU* per dollar are not the same?

- The equal marginal principle requires

$$\frac{MU_F}{P_F} = \frac{MU_C}{P_C}$$

So as long as they are not equal, the consumer can actly reallocate how to spend the money to get higher utility. Either buy more clothing or more food that will give me higher utility, meaning I'm not maximizing utility, i can improve.

- Suppose

$$\frac{MU_F}{P_F} < \frac{MU_C}{P_C}$$

Thus, when the consumer is already maximising utility, these 2 per dollar marginal utility must be the same. Cannot reallocate my money to get higher utility, already doing the best.

- Assume the current basket is on the budget line so no additional money to spend
- To maximize utility, the consumer should buy more C and less F.
  - \$1 spent on *C* brings higher extra utility than \$1 spent on *F*

Why is basket C not optimal?

All these are tryng to understand why point A is the optimal. (tangency condition)

- At point C on slide 36

At C, indiff curve & budget line do not have the same slope. Indiff curve is steeper.

$$MRS_{F,C} > \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{MU_C} > \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{P_F} > \frac{MU_C}{P_C}$$

- Per dollar marginal utility of food higher than per dollar marginal utility of clothing
- Consumer's utility will increase if more money spent on food
- Consumer should buy more food

take away $1 spent on clothing to spend on food, can get a higher utility.

Indeed, from point C to point A, we are buying more food and less clothing.

Example: Finding the Optimal Basket

- Suppose the consumer's utility function is

$$U(F,C) = FC$$

- Suppose the price of food is 20, price of clothing is 40 and the consumer's income is 600
- The marginal utility is

differentiate to get 2 marginal utilities

$$MU_F = C, \quad MU_C = F$$

- The tangency condition requires

$$\frac{P_F}{P_C} = MRS_{F,C} = \frac{MU_F}{MU_C}$$

Example: Finding the Optimal Basket Cont’

- Thus

$$\frac{20}{40} = \frac{C}{F}$$

- Simplifying

$$F = 2C \quad (1)$$

- The optimal basket must also lie on the budget line

$$20F + 40C = 600 \quad (2)$$

- Solving (1) and (2) together we have *F*=15, *C*=7.5

Example: Finding the Optimal Basket Using Langrange Multiplier Method

- This is a constrained maximization problem

$$\max_{F,C} \quad FC$$

$$s.t. \quad 20F + 40C \le 600$$

as long as 'more is better' holds, a consumer will never maximize utility if the consumer doesn't spend all the money. The optimal basket will necessarily lie on the budget line.

- To maximize utility, consumer spends all the money

$$\max_{F,C} \quad FC$$

$$s.t. \quad 20F + 40C = 600$$

Example: Finding the Optimal Basket Using Langrange Multiplier Method Cont'

- Rewriting the budget constraint

$$\max_{F,C} \quad FC$$

$$s.t. \quad 600 - 20F - 40C = 0$$

- The Lagrangian function is

$$\Lambda(F,C,\lambda) = FC + \lambda(600 - 20F - 40C)$$

we need to maximize Lagrangian function

Example: Finding the Optimal Basket Using Langrange Multiplier Method Cont'

- The first-order conditions are

$$\frac{\partial \Lambda}{\partial F} = C - 20\lambda = 0$$

$$\frac{\partial \Lambda}{\partial C} = F - 40\lambda = 0$$

first 2 equations combine them and remove lambda,
that's tangency condition.

$$\frac{\partial \Lambda}{\partial \lambda} = 600 - 20F - 40C = 0$$

this is budget line actly

- Solving the three equations, we get

$$F = 15, \quad C = 7.5, \quad \lambda = 0.375$$

same answer. SO using the Langrange method
or the 2 conditions (tangency & budget line) method
is exactly the same.

What is the meaning of the multiplier?

- Recall the general form of the constrained maximization problem

$$\max_{x,y} \quad f(x,y)$$

$$s.t. \quad g(x,y) = 0$$

- In consumer theory, it is

$$\max_{x,y} \quad U(x,y)$$ utility function

$$s.t. \quad I - P_x x - P_y y = 0$$ budget line

The Multiplier is the Extra Utility of One Extra Dollar of Consumption

- The Lagrangian function is

$$\Lambda(x,y,\lambda) = U(x,y) + \lambda(I - P_x x - P_y y)$$

- The first-order conditions w.r.t. *x* and *y* are

$$\frac{\partial \Lambda}{\partial x} = MU_x - \lambda P_x = 0$$

$$\frac{\partial \Lambda}{\partial y} = MU_y - \lambda P_y = 0$$

- Rearranging the equations we have

$$\lambda = \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

Lambda: per dollar marginal utility of either x or y at the optimal basket.
The additional utility u are going to get if u spend 1 more dollar in consumption, either x or y.

Price and Willingness to Pay

- Rearranging the equation

$$\lambda = \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

- We have

$$P_x = \frac{MU_x}{\lambda}, \quad P_y = \frac{MU_y}{\lambda}$$

e.g. lambda = 1, MU x = 2.
If i spend $1, get utility of 1.
If i get 1 unit of x, get utility of 2.
So I am willing to pay $2 for 1 unit of x.

- Price of a good is equal to the extra utility from consuming one more unit of the good divided by the extra utility of one dollar spending
- Price represents consumer's willingness to pay for one more unit of the good

When the consumer is maximizing utility, price represents how much is 1 unit of good worth to the consumer, not just how much is the consumer paying.

The Sign of the Multiplier

- If you write down the Lagrangian function as

$$\Lambda(x,y,\lambda) = U(x,y) + \lambda(P_x x + P_y y - I)$$

- The first-order conditions w.r.t. *x* and *y* are

- The multiplier now becomes

sign changed, now is a -ve number.
Now lambda is not the per dollar marginal utility, it's the -ve of it.

X and Y solutions won't change.