LECTURE 9
COST IN THE LONG RUN

cost-minimization problem in the long run

SHORT-RUN COST VS. LONG-RUN COST

Where are we?

- Production function
  - How firms turn L and K into Q
- Optimal choice of L and K in the short run
  - Cost curves in the short run
- Optimal choice of L and K in the long run
  - To produce a certain amount of output Q_0, how much L and K should the firm use?
  - How much does it cost to produce Q_0?
  - Cost curve: cost as a function of Q cost curve in the long run
- Short-run cost vs. long-run cost

Part 1

Long-Run Cost Minimizing Input Choice

optimal choice of inputs in the long run

How much labor and capital should the firm use?

- Recall
  - price of labor is *w* per unit
  - price of capital is *r* per unit
  - In the long run, both *L* and *K* are variable
- Assume the firm maximizes profit
- For any output level Q_0, the firm chooses *L* and *K* to *minimize* the total cost of production

constrained minimization problem, so can use Lagrange method. But how to solve this graphically?

$$\min_{L,K} \quad wL + rK$$

3 parameters: w, r, Q0 (given)
2 unknown variables: L, K

similar to the maximization problem in consumer theory

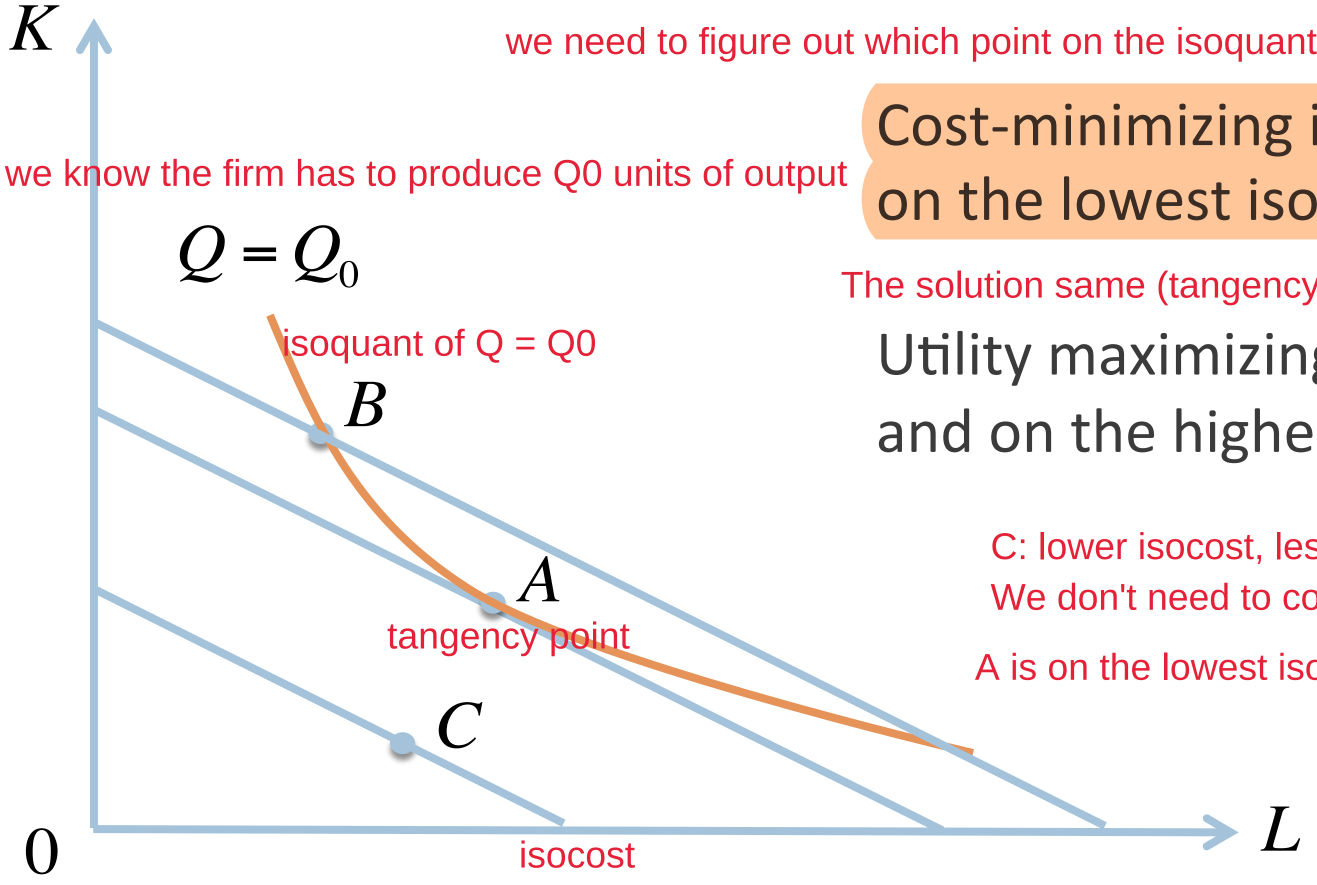
$$s.t. \quad F(L,K) = Q_0$$

cannot figure out the value of L, K just from the production function anymore

Which combination is cost-minimizing?

we need to figure out which point on the isoquant is the cheapest

we know the firm has to produce Q0 units of output



Cost-minimizing input: on the isoquant and on the lowest isocost

isoquant is fixed, many isocosts

The solution same (tangency point), but problems diff.

Utility maximizing basket: on the budget line and on the highest indifference curve

budget line is fixed, many indiff curves.

C: lower isocost, less costly than A. But it doesn't lie on the isoquant.
We don't need to consider any point that is not on the isoquant.

A is on the lowest isocost. -> cost-minimizing choice of input for this firm.

2 points on the same isocost will cost the firm same amt of money.
The higher the isocost, the higher the total cost for the firm.

Cost-Minimizing Input Choice

- The cost minimizing input combination
  - must be on the isoquant
  - must be on the lowest isocost
- On the isoquant

$$F(L,K) = Q_0$$

- Tangency condition

slope of isoquant = slope of isocost

MRTS: -ve of the slope of isoquant
w/r: -ve of the slope of isocost

$$MRTS_{L,K} = \frac{w}{r}$$

- Equivalently

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{w}{r} \Rightarrow \frac{MP_L}{w} = \frac{MP_K}{r}$$

The tangency condition and equal marginal principle are essentially the same as that in consumer theory.

Note: Depending on production function, (similar to consumer theory), optimal choice of inputs may not be the tangency point, it could be a corner solution. E.g. for perfect substitutes, linear production function, (tutorial).

At the cost-minimizing input choice, we need the per dollar marginal product of the 2 inputs (labor and capital) to be the same.

If they are not the same, e.g. per dollar MP of labor is higher than that of capital, means if u spend $1 on labor, u get more output compared to if u spend $1 on capital. So the firm should use more labor and less capital to minimize cost.

Example: Solving for the Cost-Minimizing Choice of Inputs

- Suppose the production function is

$$Q = KL$$

Cogg-Douglas production function

- Input prices are w=1 and r=2
- What is the cost-minimizing choice of inputs if the firm wants to produce 8 units?

As the production function is Cogg-Douglas, no need to worry about corner solutions. We know the optimal choice of inputs will just be a tangency point.

- To minimize cost, the firm chooses K and L such that

$$\frac{K}{L} = \frac{1}{2}$$

Example: Solving for the Cost-Minimizing Choice of Inputs Cont'

- The firm must produce 8 units of output

$$KL = 8$$

- Solving the two equations we get

$$L = 4, \quad K = 2$$

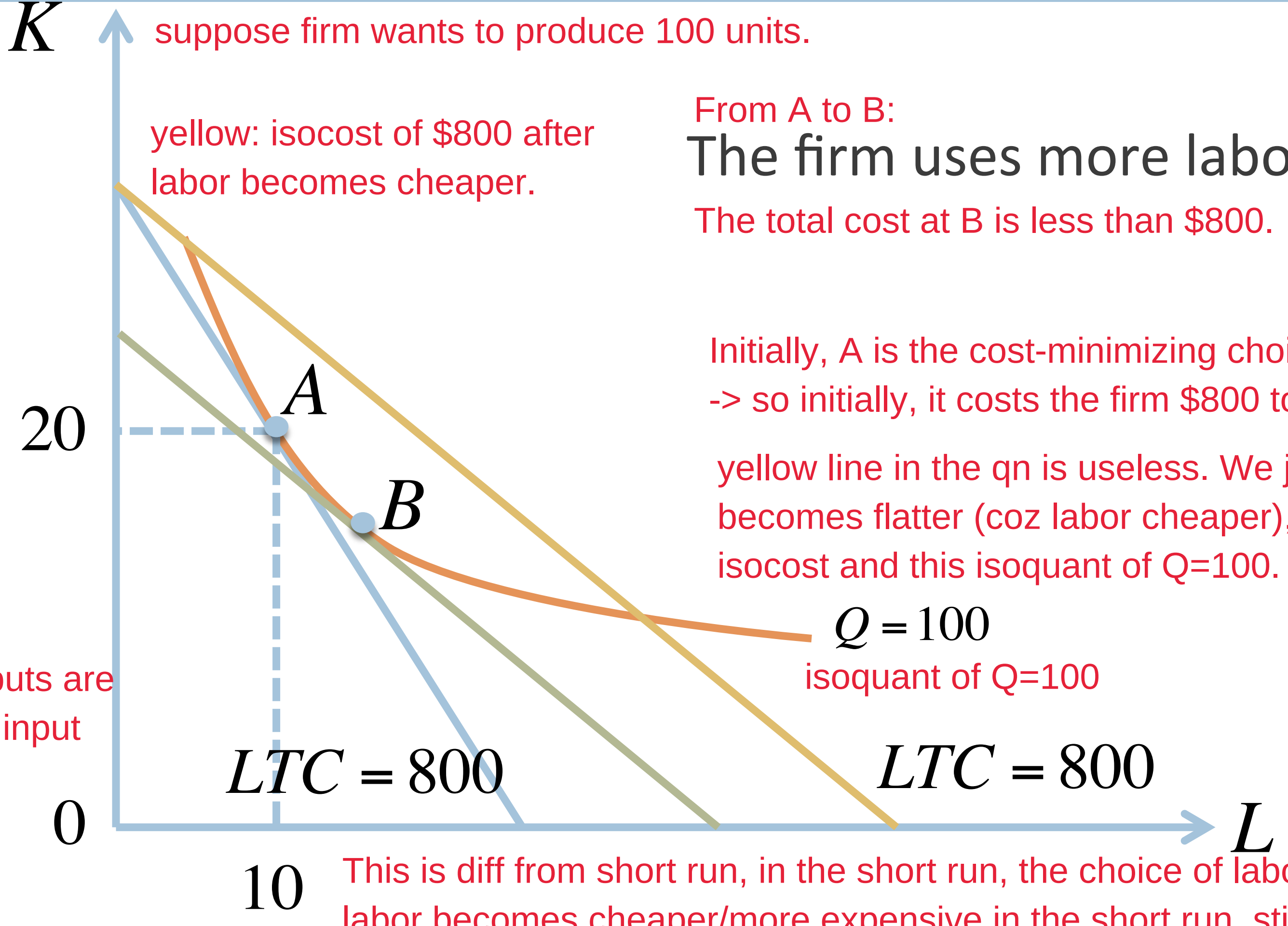
This cost-minimizing choice of input depends on parameters of the model - how many units u want to produce, prices of 2 inputs.

Comparative Statics: Changes in Input Prices and Output Level

- When input prices change
  - How does the cost-minimizing choice of L and K change?
- When output level changes
  - How does the cost-minimizing choice of L and K change?
- The above analysis is called *comparative statics*

Meaning: as long as we have a model, and we can solve the model to get the solution of the model, comparative statics means that we want to see how the solution of the model (or the solution to any minimization/minimization problem) change with the parameters of the model.

Suppose price of labor drops



suppose firm wants to produce 100 units.

yellow: isocost of $800 after labor becomes cheaper.

From A to B:

The firm uses more labor and less capital

make sense coz labor is cheaper now.

The total cost at B is less than $800.

Initially, A is the cost-minimizing choice of inputs. A lies on the isocost of 800.
-> so initially, it costs the firm $800 to produce 100 units of output.

yellow line in the qn is useless. We just need to figure out, given that isocost becomes flatter (coz labor cheaper), what is the new tangency point between isocost and this isoquant of Q=100.

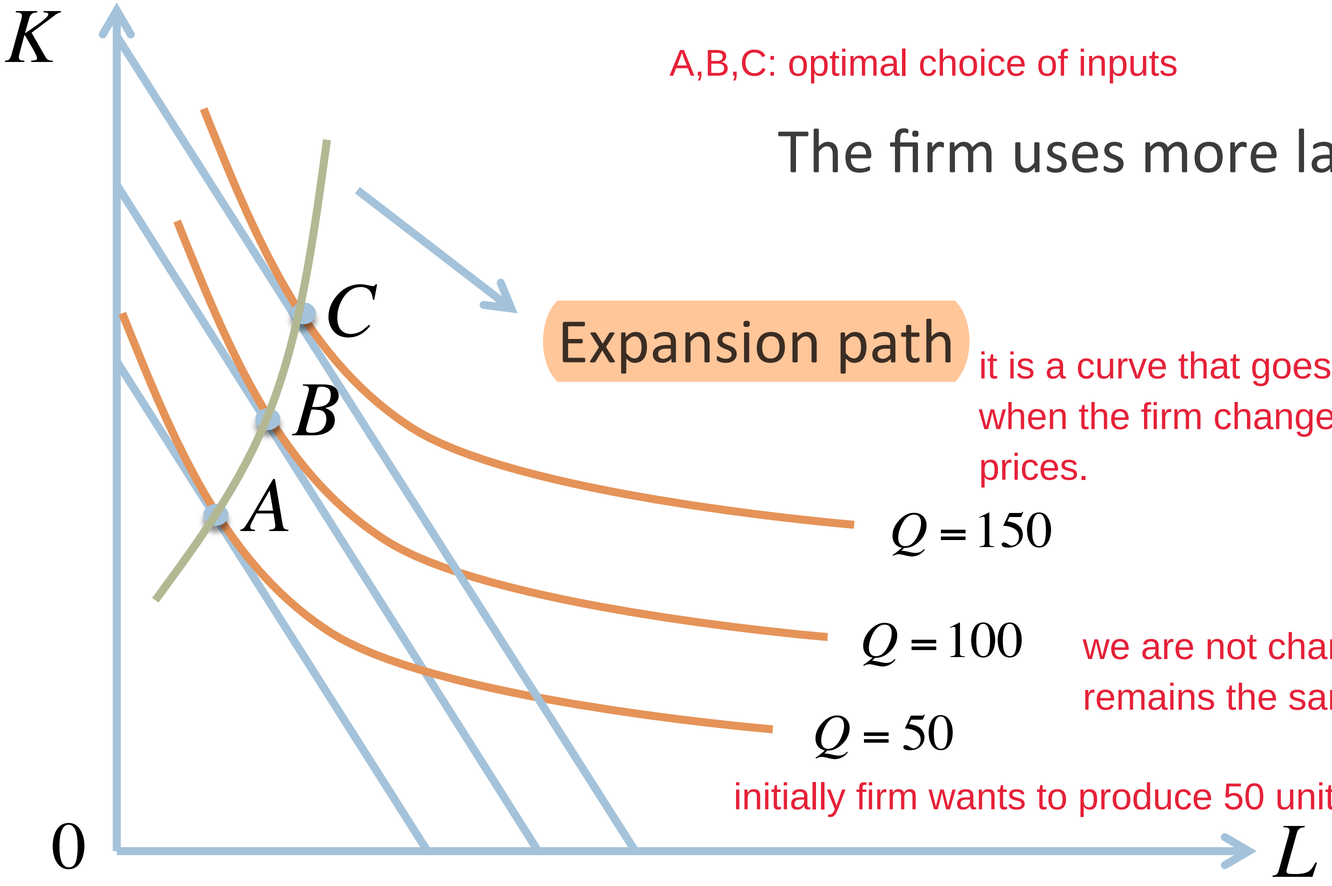
isoquant of Q=100

B is the new tangency point.

In the long run, when both inputs are variable, we expect choice of input to change with input prices.

This is diff from short run, in the short run, the choice of labor is independent of input prices. Even if labor becomes cheaper/more expensive in the short run, still gonna use the same amount of labor. It is because capital is fixed in the short run.

Suppose Q increases



A,B,C: optimal choice of inputs

The firm uses more labor and more capital

it is a curve that goes through cost-minimizing choice of inputs when the firm changes output level without changing input prices.

we are not changing input prices, so slope of isocost remains the same.

initially firm wants to produce 50 units.

Both labor & capital are normal inputs here. Expansion path sloping upwards.

Normal vs. Inferior Input

What is the shape of expansion path if labor is inferior while capital is normal?

- **Definition 9.1** ***Normal input***

  when firm produces more, it uses more of this input.

  - The cost-minimizing quantity of the input increases when output increases
  - Holding input prices fixed

- **Definition 9.2** ***Inferior input***
  - The cost-minimizing quantity of the input decreases when output increases
  - Holding input prices fixed

e.g. firm produces fishballs. At first produce in hawker center, small-scale, Q not high. When Q not high, no need to produce a lot every day, u realize that the cost-minimizing way of producing fishball is by hand. (so hire a few workers to produce fishball by hand, not worthwhile to buy a machine if your Q is small)
If now u wanna expand production, a few outlets. Q is much higher than before. Now if produce many fishballs by hand, need to hire a lot more workers, not easy and quality control. Now the cheapest way is to produce by machines.
-> As Q increases, switch to machines, capital increases, quantity of labor decreases.

Is it possible for labor to be a 'Giffen' input? That is, when labor becomes cheaper, to produce the same quantity of output, the firm uses less labor?
-Giffen good: price and consumption goes in the same direction.

NO! To still produce the same Q, there is 'substitution effect', but no 'income effect'. Impossible to have Giffen input as the reason for having it due to -ve and very big income effect.

diff bet cost-minimization and utility-maximization problem

Input Demand Function

- As the input prices or the output level change, firm's cost-minimizing choice of labor and capital may also change
- Definition 9.3 The *demand function of an input* is the cost-minimizing choice of input as a function of *w*, *r*, and *Q*
  - Demand function of labor
  - Demand function of capital

2 inputs here, 2 demand functions

Example: Deriving Input Demand Functions

- Suppose the production function is

$$Q = KL$$

- Input prices are w and r
- To minimize cost, the firm chooses K and L such that

$$\frac{K}{L} = \frac{w}{r}$$

it's just that we dont have numbers for w,r here now.

- This gives us

$$K = \frac{w}{r}L, \quad L = \frac{r}{w}K$$

Example: Deriving Input Demand Functions Cont'

- Substituting

$$Q = KL = (\frac{wL}{r})L = \frac{w}{r}L^2$$

- The demand function of labor is

$$L(w,r,Q) = \sqrt{\frac{rQ}{w}}$$

The diff between input demand functions and cost-minimizing input choices is really whether u treat the 3 parameters as variables or fix them at specific values.

Same idea as going from optimal basket to demand function for a good.

- The demand function of capital is

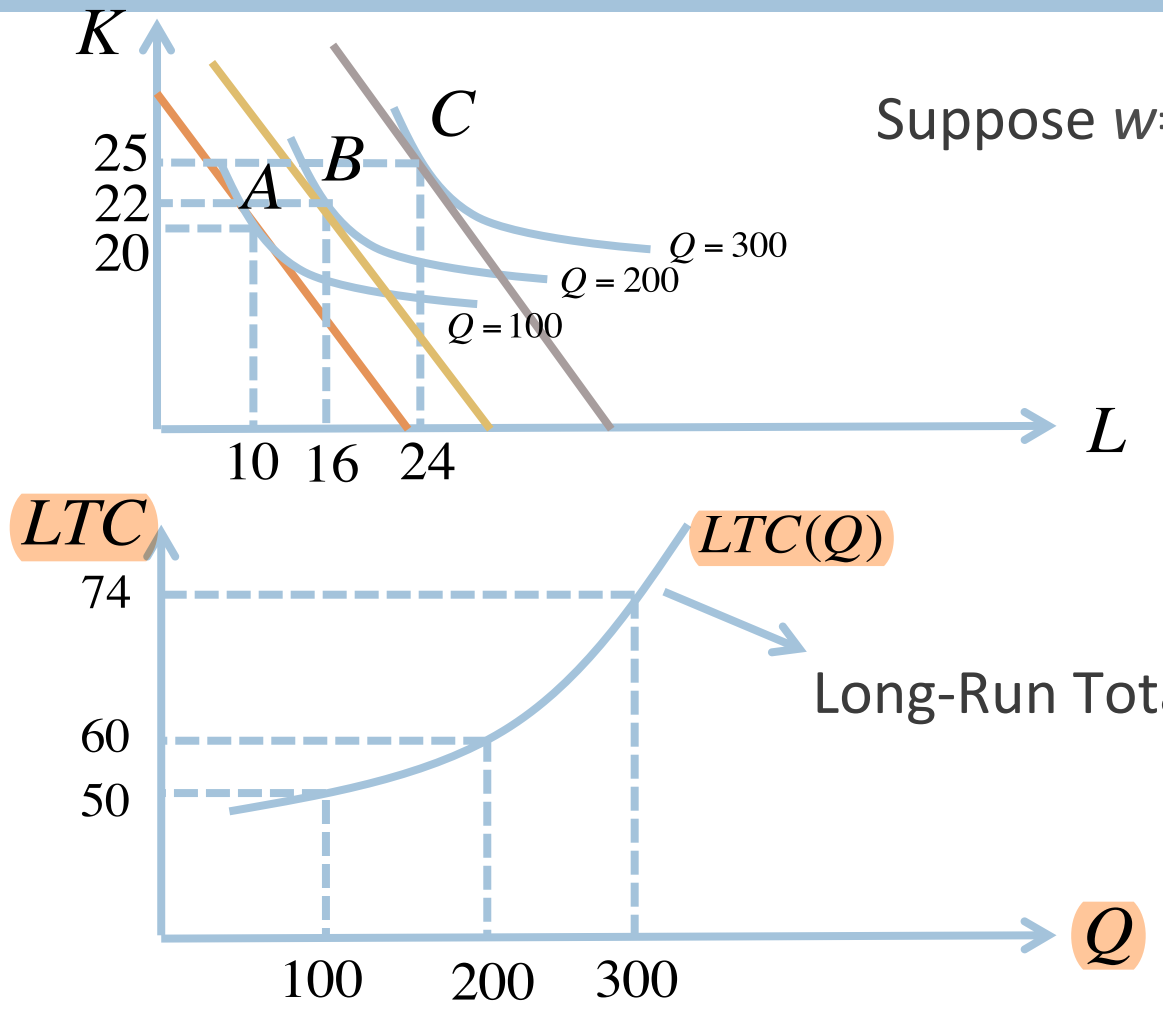
$$K(w,r,Q) = \sqrt{\frac{wQ}{r}}$$

Based on both demand functions, labor and capital are normal inputs. Because holding w and r constant, just increase Q, the firm will use more labor and capital. Similarly, when labor becomes more expensive, gonna use less labor and more capital.

Part 2

Long-Run Cost Curves

Long-Run Total Cost Curve in Graph



Suppose w=1 and r=2

Long-Run Total Cost Curve

how the total cost of production in the LR changes with quantity Q, holding input prices constant.

Long-run Total Cost Curve/Function

- Definition 9.4 *Long-run total cost curve* is total cost in the long run as a function of *Q*
  - Holding *w* and *r* constant
- Every point on the long-run total cost curve represents the firm's *minimized* total cost for a given level of output, holding input prices fixed
- No fixed cost in the long run

  both L & K are variable, everything is variable cost. By definition, variable cost=0 when the firm doesn't produce anything.
  - *LTC*=0 when *Q*=0

    So LTC curve always starts from the origin.
- Definition 9.5 *Long-run total cost function* is total cost in the long run as a function of *Q*, *w*, and *r*

Example: Deriving Long-Run Total Cost Function

- Suppose the production function is

$$Q = KL$$

- Input prices are *w* and *r*
- We have already derived the cost-minimizing choice of labor and capital

$$L(w,r,Q) = \sqrt{\frac{rQ}{w}}$$

$$K(w,r,Q) = \sqrt{\frac{wQ}{r}}$$

Example: Deriving Long-Run Total Cost Function Cont'

- The long-run total cost function is

$$LTC(Q,w,r) = wL + rK = w\sqrt{\frac{rQ}{w}} + r\sqrt{\frac{wQ}{r}}$$

- Simplifying, we get

$$LTC(Q,w,r) = 2\sqrt{wrQ}$$

Based on the equation, if w/r/Q increases, long-run total cost increases.

Average Cost and Marginal Cost

- Definition 9.6 *Long-run average cost (LAC)*
  - Total cost per unit of output

$$LAC(Q) = \frac{LTC(Q)}{Q}$$

- Definition 9.7 *Long-run marginal cost (LMC)*

slope of LTC curve

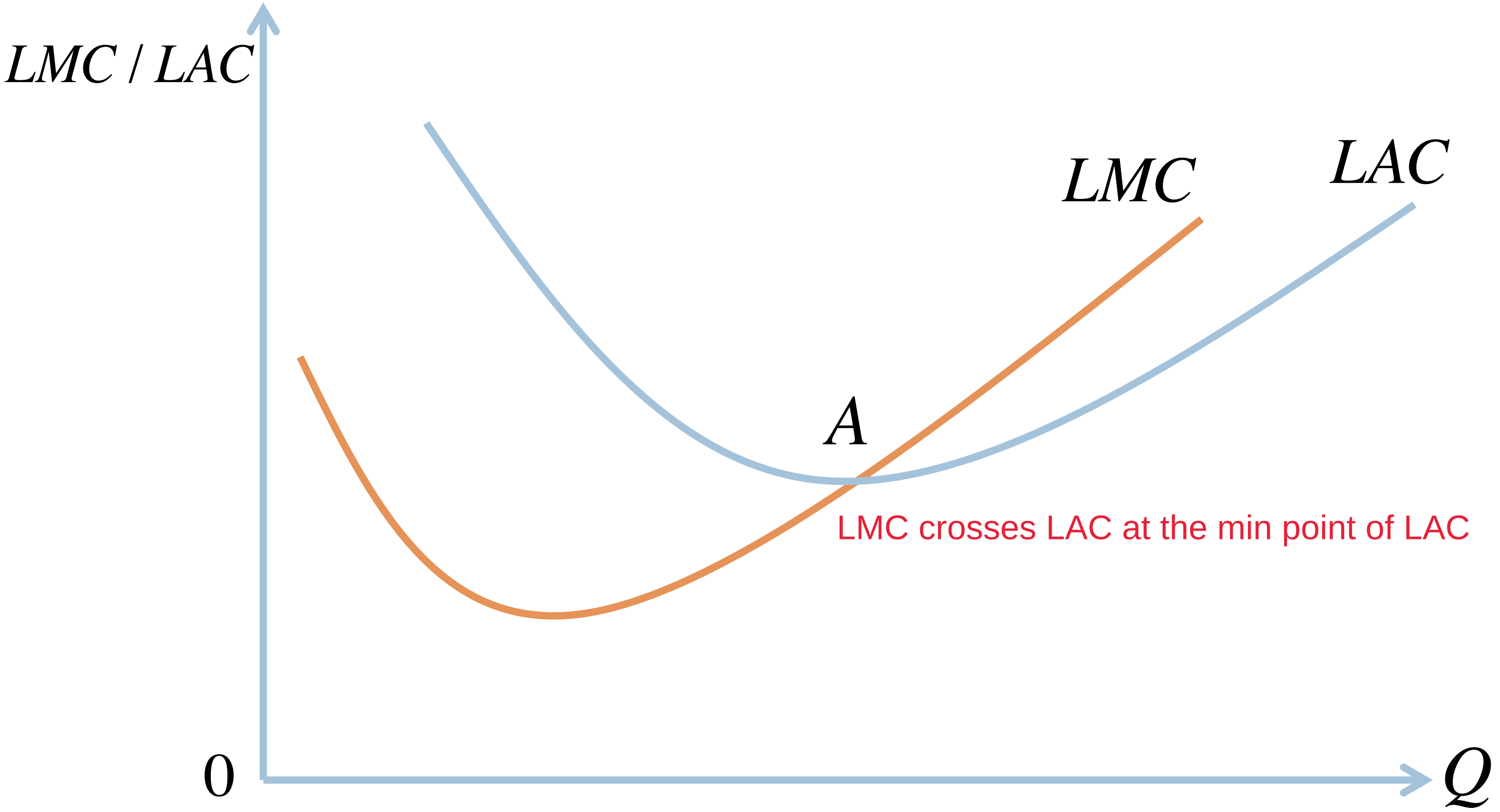
$$LMC(Q) = \frac{dLTC(Q)}{dQ} = \frac{\Delta LTC(Q)}{\Delta Q}$$

where ΔQ is extremely small

in the long run, if u produce 1 extra very small unit, how much will the long-run total cost change?

Relationship between *LMC* and *LAC*

just like in the short run



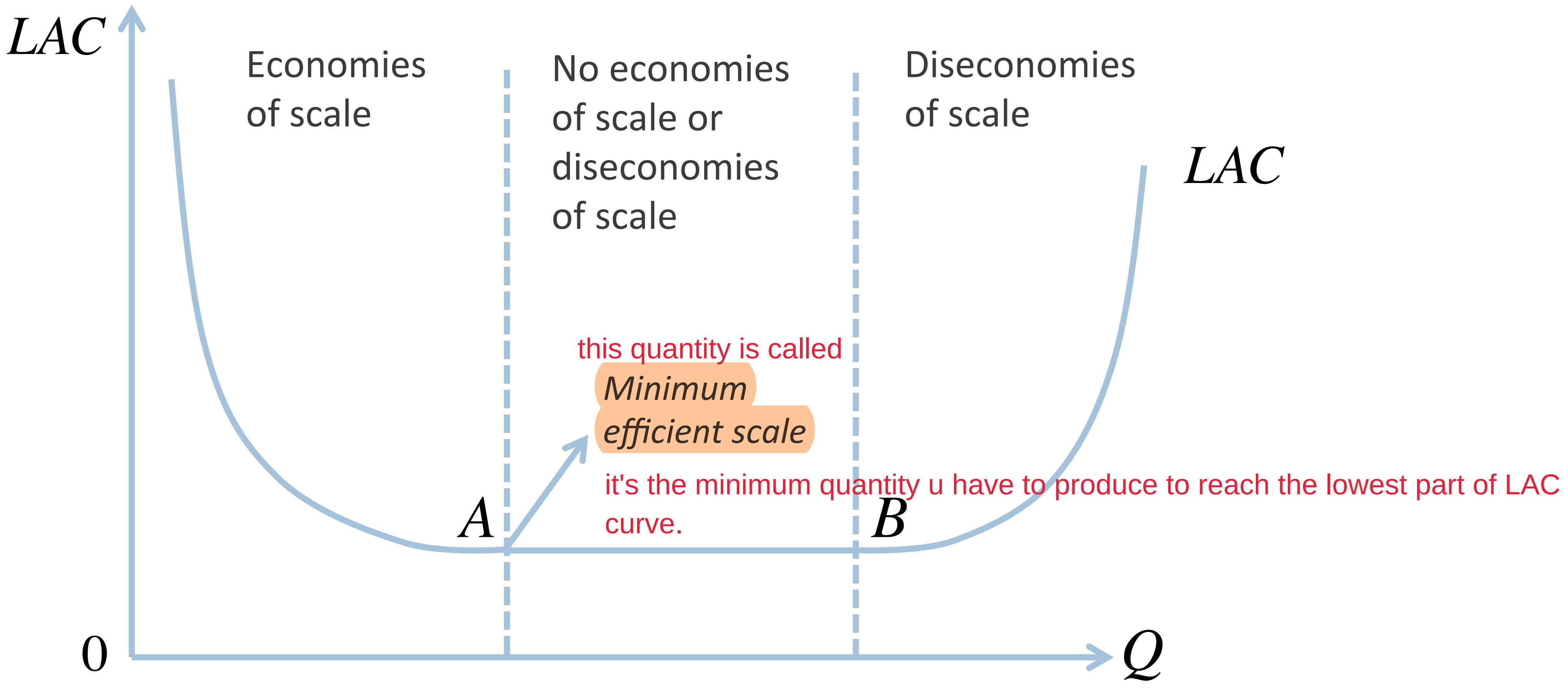
LMC crosses LAC at the min point of LAC

Economies of Scale

based on shape of LAC

- Definition 9.8 *Economies of scale*
  - If *LAC* is decreasing in *Q*
- Definition 9.9 *Diseconomies of scale*
  - If *LAC* is increasing in *Q*

Economies of Scale in Graph



Source of Economies of Scale

e.g. produce cookies, use automated production line, one production line can produce 5000 cookies per day. But yours is a small firm, only need to produce 2000 per day. But no choice, cannot find smaller machine, have to buy this production line. When ur target is 5000 now, can still use the same production line, average cost smaller.

- Indivisible input
  - The size of some input cannot be scaled down
  - The cost of the input gets spread out as quantity of output increases

this usually happens at the beginning of production process when u go from a small quantity to a reasonable quantity.

- Returns to specialization
  - More workers can lead to better specialization
  - Specialization improves productivity
  - Example
    - When L=2, K=1, Q=2, suppose w=r=1, LTC(2)=3, LAC(2)=1.5
    - When L=3, K=1, Q=4 because of better specialization of labor
    - LTC(4)=4, LAC(4)=1

likely to happen at the beginning of production process too.

Source of Diseconomies of Scale

- Managerial diseconomies of scale
  - An $a\%$ increase in Q requires a more than $a\%$ increase in the firm's spending on managers

Output Q doubled, no. of workers doubled too, but no. of managers goes from 1 to 3, more than doubled (actly it's tripled).

As u expand production scale, u may need to hire an increasing number of managers, costly. Esp a problem for a large organization whose production scale is alr quite big, when increasing production scale even further, need to add layers of managers.

Part 3

Short-Run Cost Vs. Long-Run Cost

in-class exercise webcast 8:00

Short-Run Expansion Path

Cost-minimizing inputs in SR:

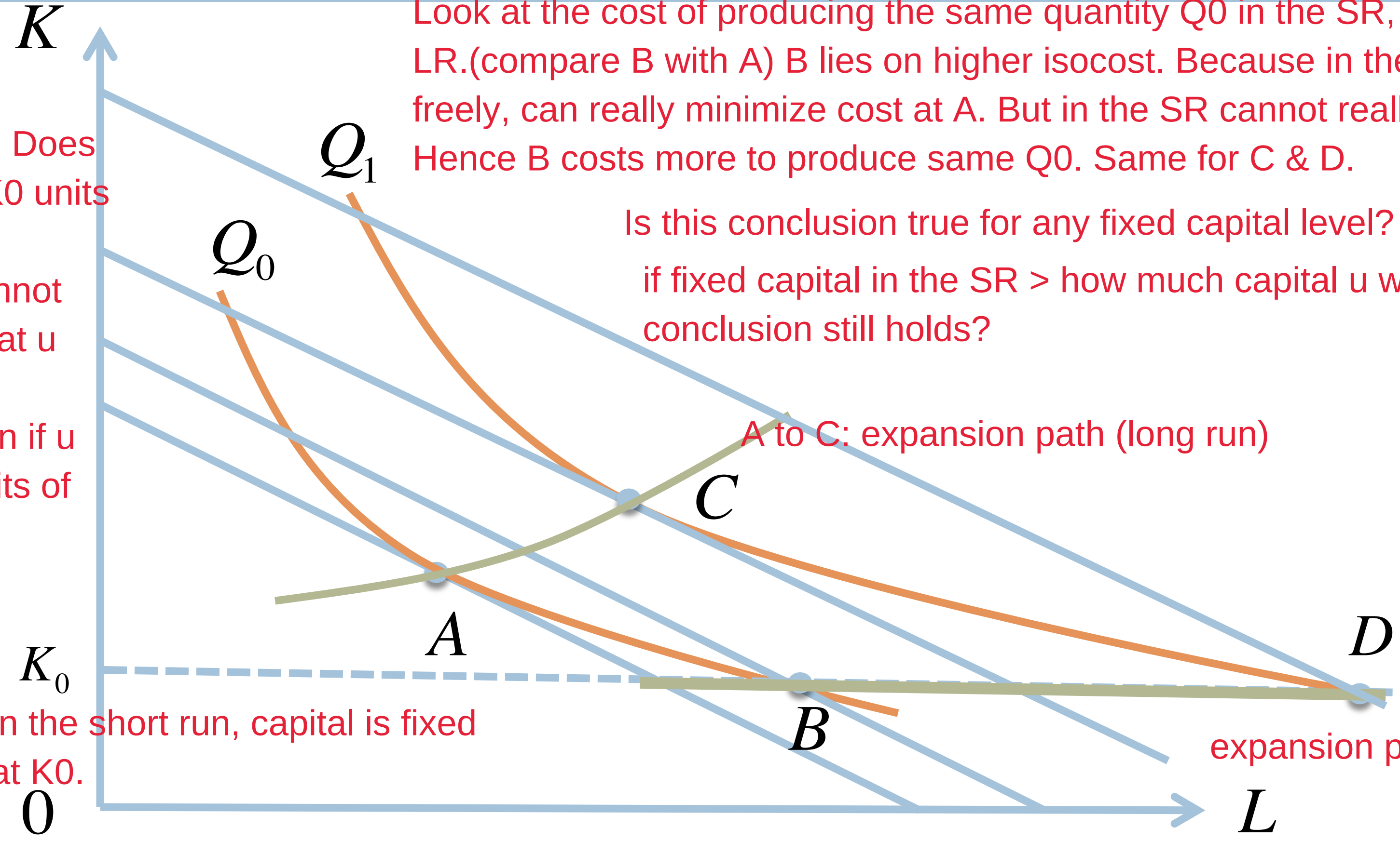
In the shirt run, capital is fixed at K0. Does it mean that I cannot use less than K0 units of capital? Depends. Capital may be indivisible, meaning if fixed at K0 cannot use less than K0. But it's possible that u can use less than K0 capital.

Capital fixed at K0 in SR means even if u use <K0 capital, u still pay for K0 units of capital in SR.

Look at the cost of producing the same quantity Q0 in the SR, higher than producing in the LR.(compare B with A) B lies on higher isocost. Because in the LR, can choose L & K freely, can really minimize cost at A. But in the SR cannot really minimize cost coz K fixed. Hence B costs more to produce same Q0. Same for C & D.

Is this conclusion true for any fixed capital level?

if fixed capital in the SR > how much capital u would need in the LR, conclusion still holds?



A to C: expansion path (long run)

in the short run, capital is fixed at K0.

a horizontal line expansion path in the short run

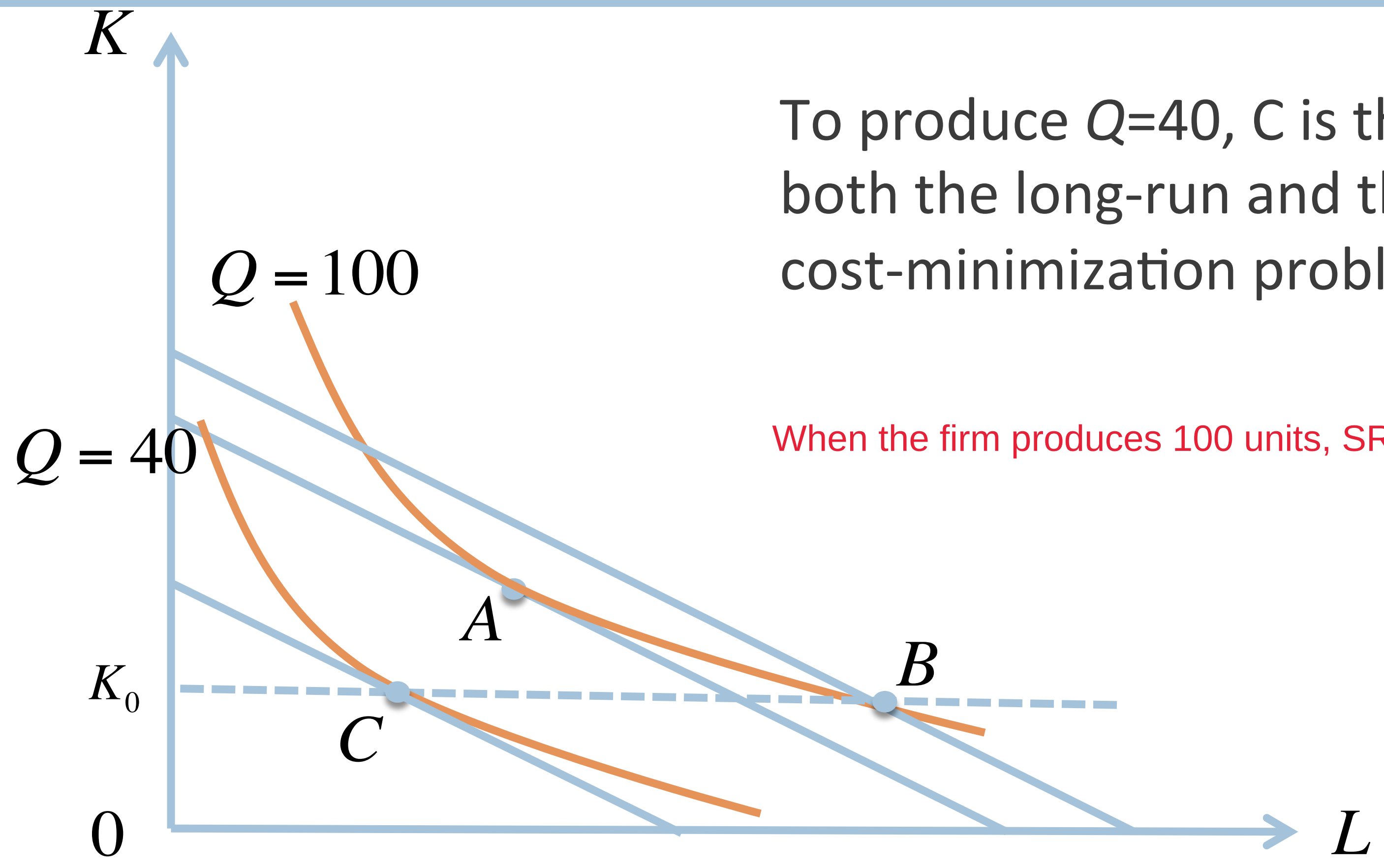
In the SR, no matter what is the fixed capital level, more than or less than the optimal capital level in the LR, the cost-minimizing choice for the firm is always to produce by using all the fixed capital.

B: to produce Q0 in the short run, can only produce at pt B as capital is fixed.

Is *STC*=*LTC* possible?

In the previous eg, we expect the firm to pay a higher cost in the SR than in LR.
Coz in SR, capital fixed, cannot really minimize cost.

Possible for the firm to incur same cost in the SR and LR?



To produce Q=40, C is the solution to both the long-run and the short-run cost-minimization problem

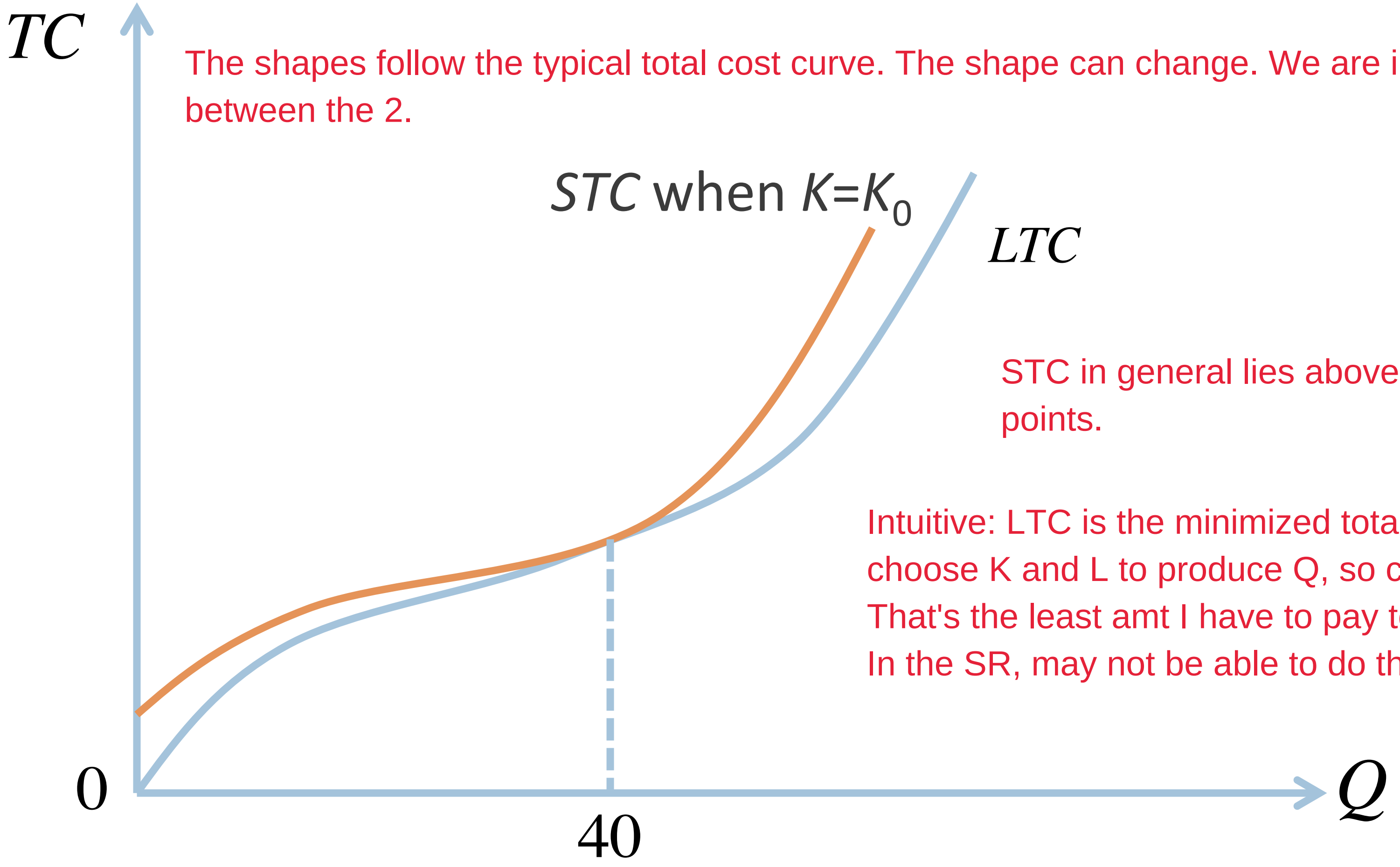
When the firm produces 100 units, SR cost is higher than LR cost.

STC cannot be lower than *LTC*

general conclusion

can be the same

The shapes follow the typical total cost curve. The shape can change. We are interested in the relationship between the 2.

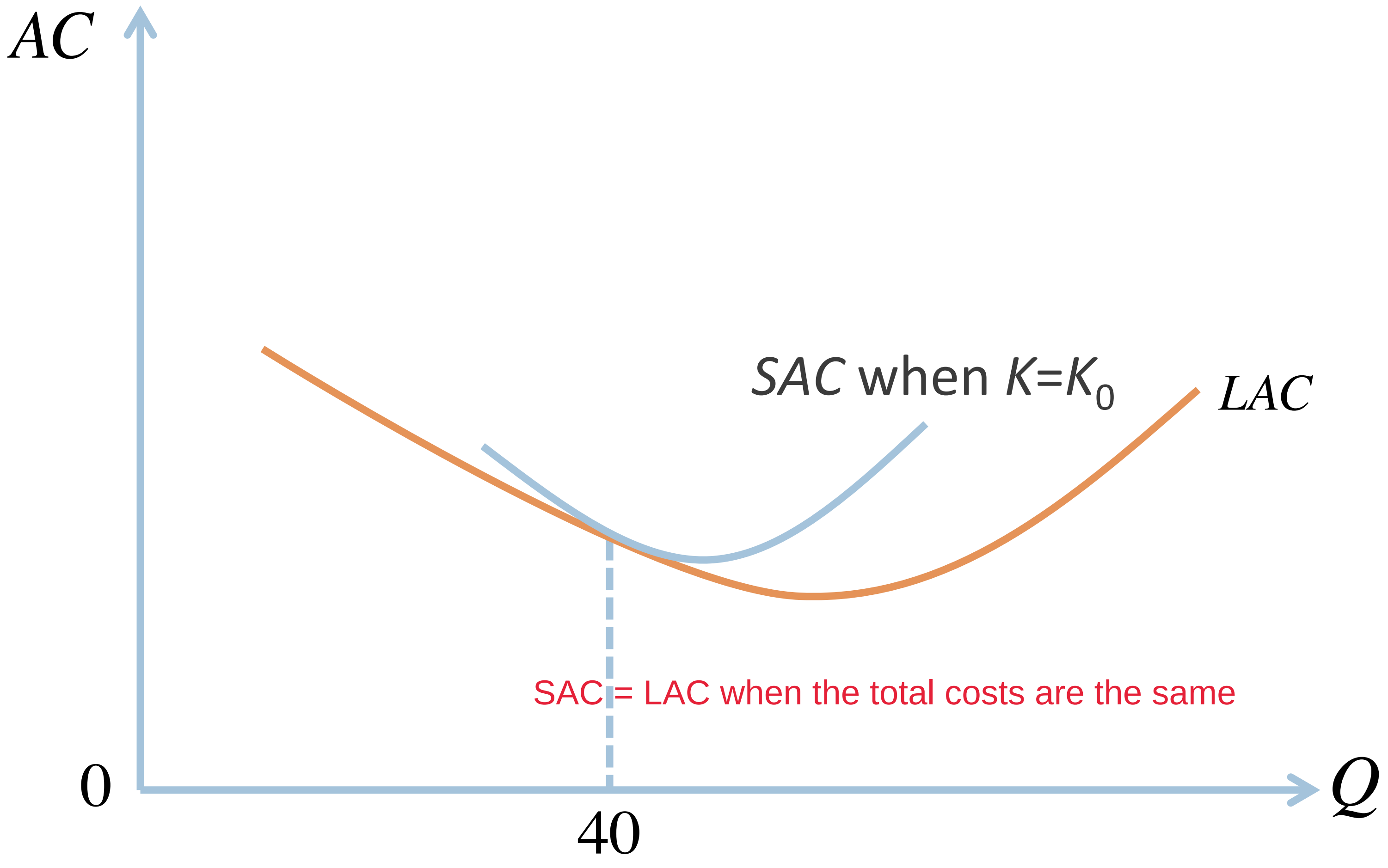


STC in general lies above LTC, can be the same at certain points.

Intuitive: LTC is the minimized total cost of production. Can freely choose K and L to produce Q, so can minimize cost of production. That's the least amt I have to pay to produce Q.
In the SR, may not be able to do that, may cost u more/same.

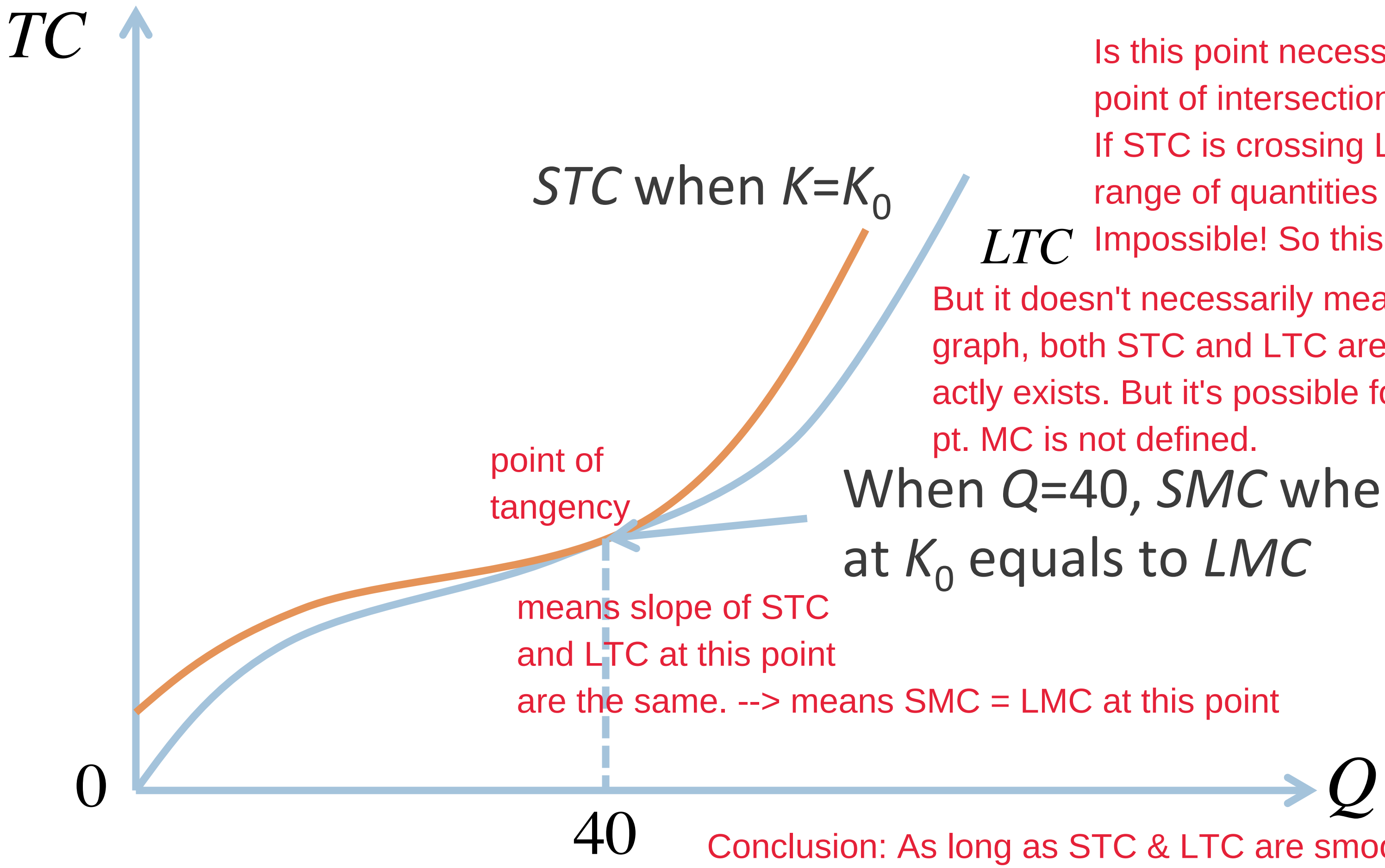
SAC cannot be lower than *LAC*

If STC cannot be lower than LTC, it follows that SAC cannot be lower than LAC.



How about marginal cost?

MC is the slope of TC curve



Is this point necessarily a tangency point? Can it be a point of intersection?
If STC is crossing LTC, it means that there exists a range of quantities where STC lies below LTC.
Impossible! So this point cannot be an intersection pt.
But it doesn't necessarily mean it's a tangency point. In this graph, both STC and LTC are smooth (differentiable). So MC actly exists. But it's possible for LTC/STC to have a kink at this pt. MC is not defined.

When Q=40, *SMC* when K is fixed at K_0 equals to *LMC*

means slope of STC and LTC at this point are the same. --> means SMC = LMC at this point

Conclusion: As long as STC & LTC are smooth (Or as long as MC is defined), then the point where STC=LTC will be a tangency point and SMC=LMC.

When does Long-Run Cost=Short-Run Cost?

- Suppose in the short run capital is fixed at K_0
- Suppose when the firm produces Q_0, K_0 is the cost-minimizing capital choice in the long run
- When $Q=Q_0$
  - The choice of inputs in the long-run and in the short-run are the same
  - $STC=LTC$
  - $SAC=LAC$
  - $SMC=LMC$ as long as MC is defined

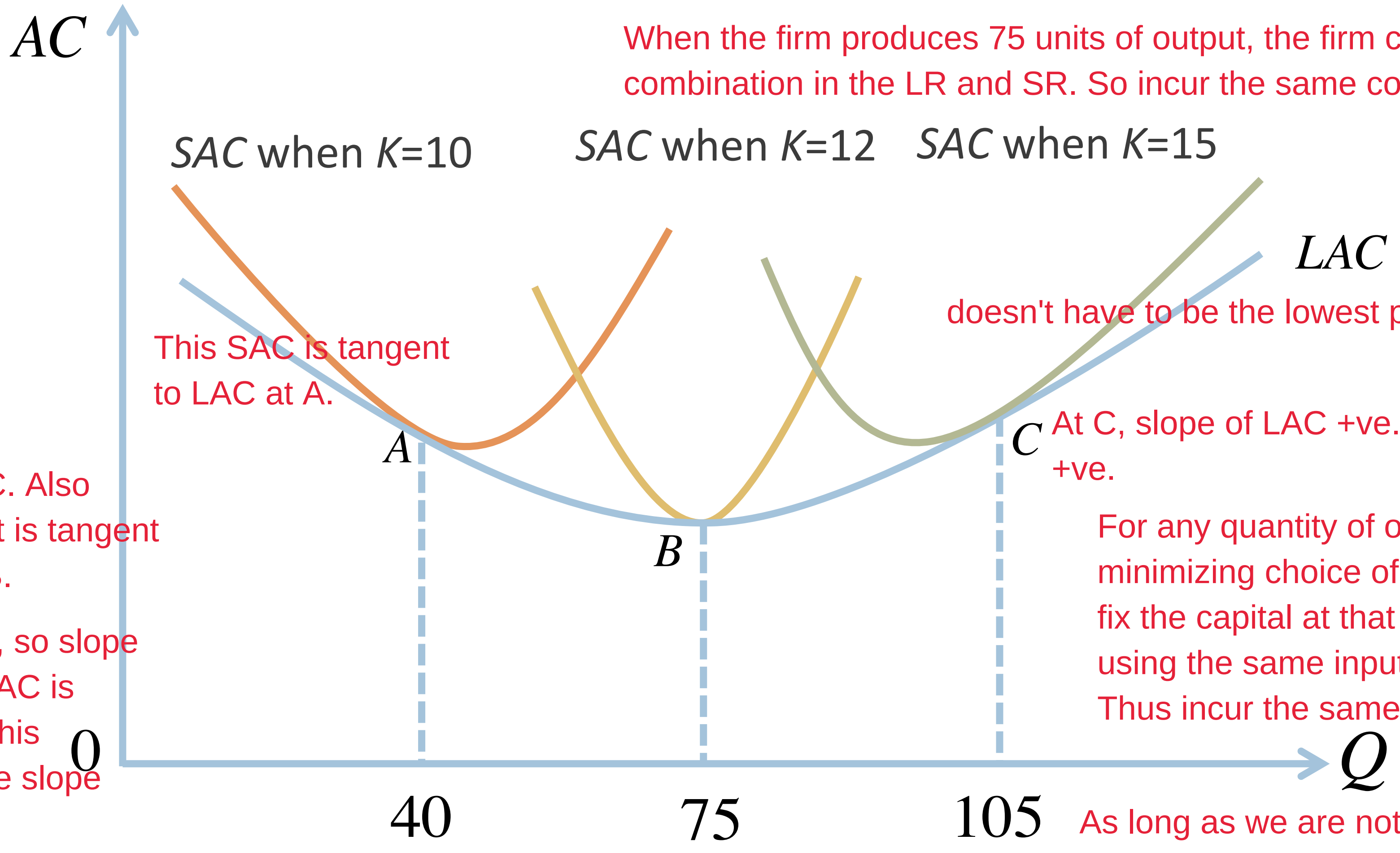
Depending on fixed level K0, there exists many short-run cost curves. Now we want to compare more than 1 short-run curves to the long-run cost curve.

Long-run Average Cost Curve vs. Short-run Average Cost Curves

- Suppose if the firm produces 40 units
  - Its optimal choice of capital in the long run is 10
- Suppose if the firm produces 75 units
  - Its optimal choice of capital in the long run is 12
- Suppose if the firm produces 105 units
  - Its optimal choice of capital in the long run is 15

LAC is the lower envelope of *SAC*

When the firm produces 75 units of output, the firm chooses the same input combination in the LR and SR. So incur the same cost in the LR and SR.



doesn't have to be the lowest pt of SAC.

This SAC is tangent to LAC at A.

At C, slope of LAC +ve. So slope of this SAC at C also +ve.

Point B: min point of LAC. Also the min point of SAC that is tangent to the LAC at this point B.

Why? B is min pt of LAC, so slope of LAC at B = 0. Since SAC is tangent to LAC at B, so this SAC must have the same slope as LAC at B, 0.

For any quantity of output, we can figure out the cost-minimizing choice of capital in the LR. As long as we fix the capital at that level in the SR, the firm will be using the same input combination in the SR and LR. Thus incur the same TC and AC in the SR and LR.

As long as we are not at the min pt of LAC, tangency pt between SAC and LAC is not the min pt of SAC.

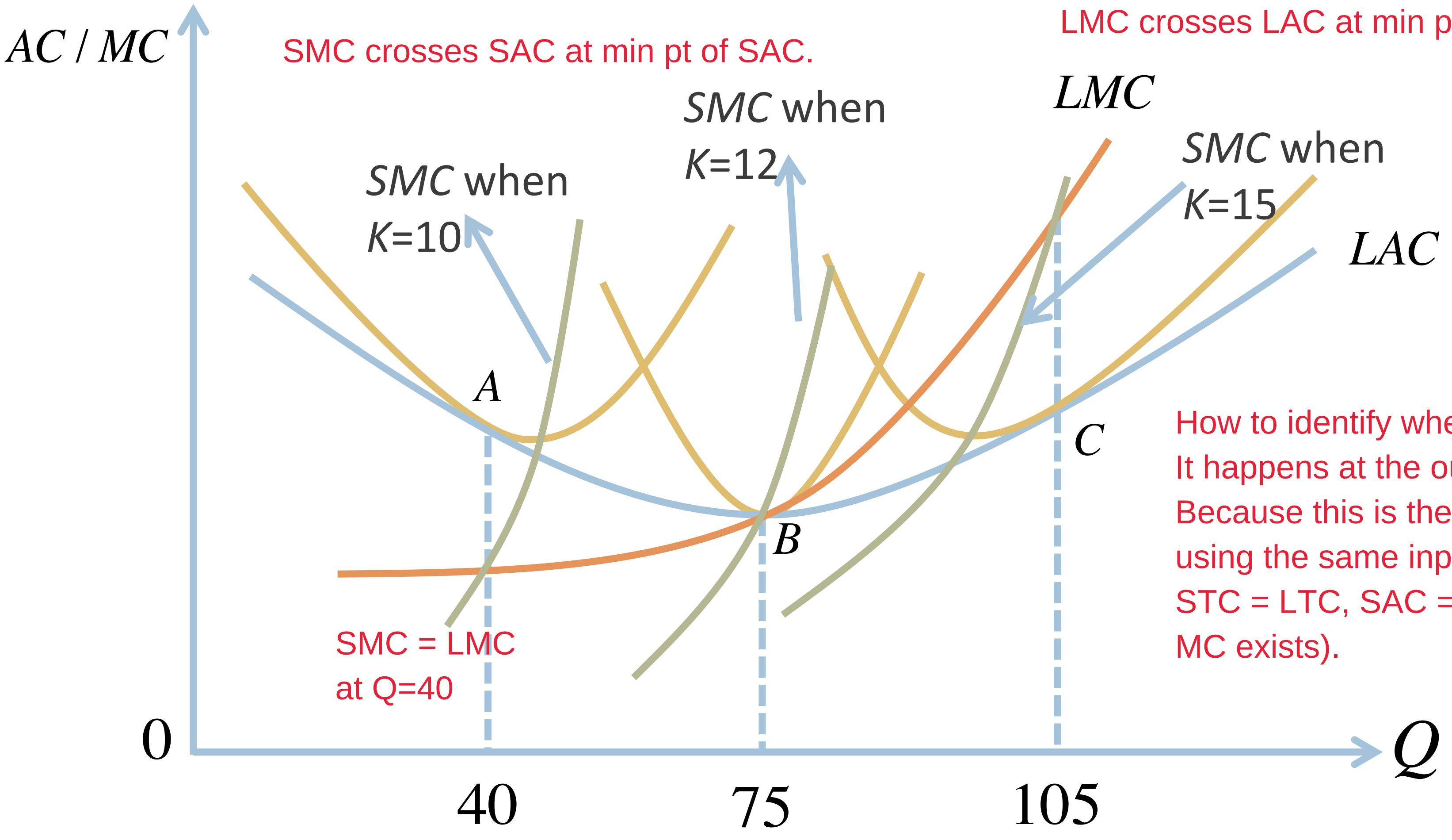
When *LAC* is at its minimum

- When the firm produces 75 units its *LAC* is the lowest across all possible output levels
- At this output level, the *SAC* when *K*=12 must also reach its minimum
  - When *LAC* is at its minimum, its slope is 0
  - At the point where the *SAC* is tangent to *LAC*, they have the same slope
  - The slope of the *SAC* at the point where it is tangent to *LAC* is also 0
  - Thus *SAC* is at its minimum

When *LAC* is not at its minimum

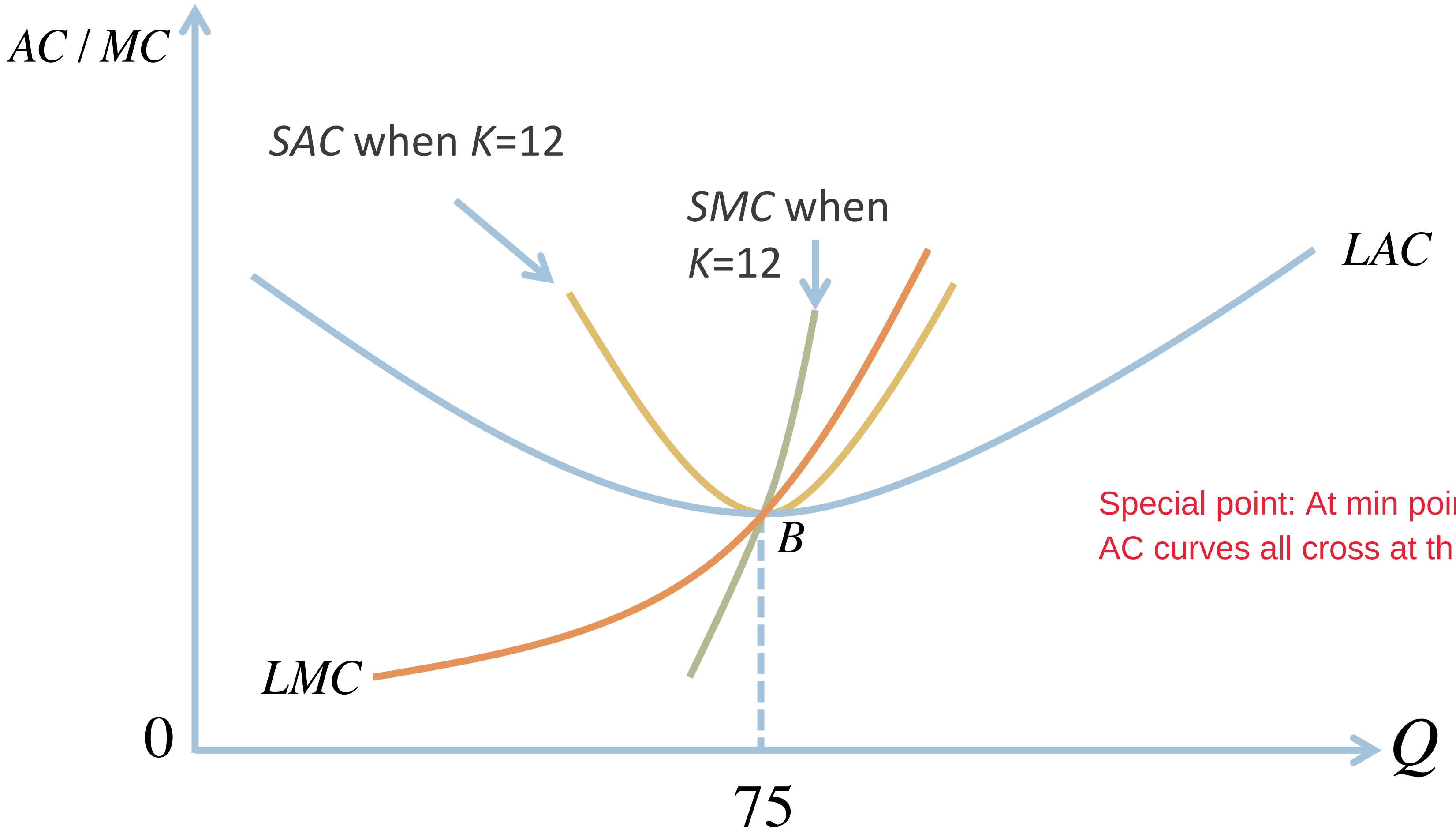
- *SAC* is not tangent to *LAC* at *SAC*'s minimum point
  - When *LAC* is not at its minimum, it is either decreasing or increasing, i.e., its slope is either negative or positive
  - At the point where *SAC* is tangent to *LAC*, they have the same slope
  - The slope of the *SAC* at the point where it is tangent to *LAC* is also either negative or positive
  - Thus *SAC* is not at its minimum

LMC vs. *SMC*



How to identify where SMC = LMC?
It happens at the output level where LAC = SAC.
Because this is the output level at which the firm is using the same input combination in the LR and SR.
STC = LTC, SAC = LAC, SMC = LMC (as long as MC exists).

The Minimum Point of *LAC*



Special point: At min point of LAC, the 2 MC curves and 2 AC curves all cross at this point B.