LECTURE 5
MARKET DEMAND
EXCHANGE ECONOMY

Where are we?

- Consumer theory
  - Optimal choice
  - Individual demand
  - Consumer welfare
  - Market demand in the market, more than 1 consumer. How to derive demand curve for the entire market?
- Exchange economy
  - Edgeworth box
    - How to represent the economy graphically?
  - Pareto efficiency
    - What is the “best” allocation?
  - Competitive equilibrium

Part 1

Market Demand

Individual Demand Curve and Market Demand Curve

- Market/Aggregate demand curve is the horizontal summation of all individual demand curves add up quantities for all individual consumers in the market at any given price.
- Suppose there are 200 high-valuation consumers
  - Each high-valuation consumer's demand curve is $Q=6-2P$
- Suppose there are 100 low-valuation consumers
  - Each low-valuation consumer's demand curve is $Q=5-5P$
- What is the market demand curve?

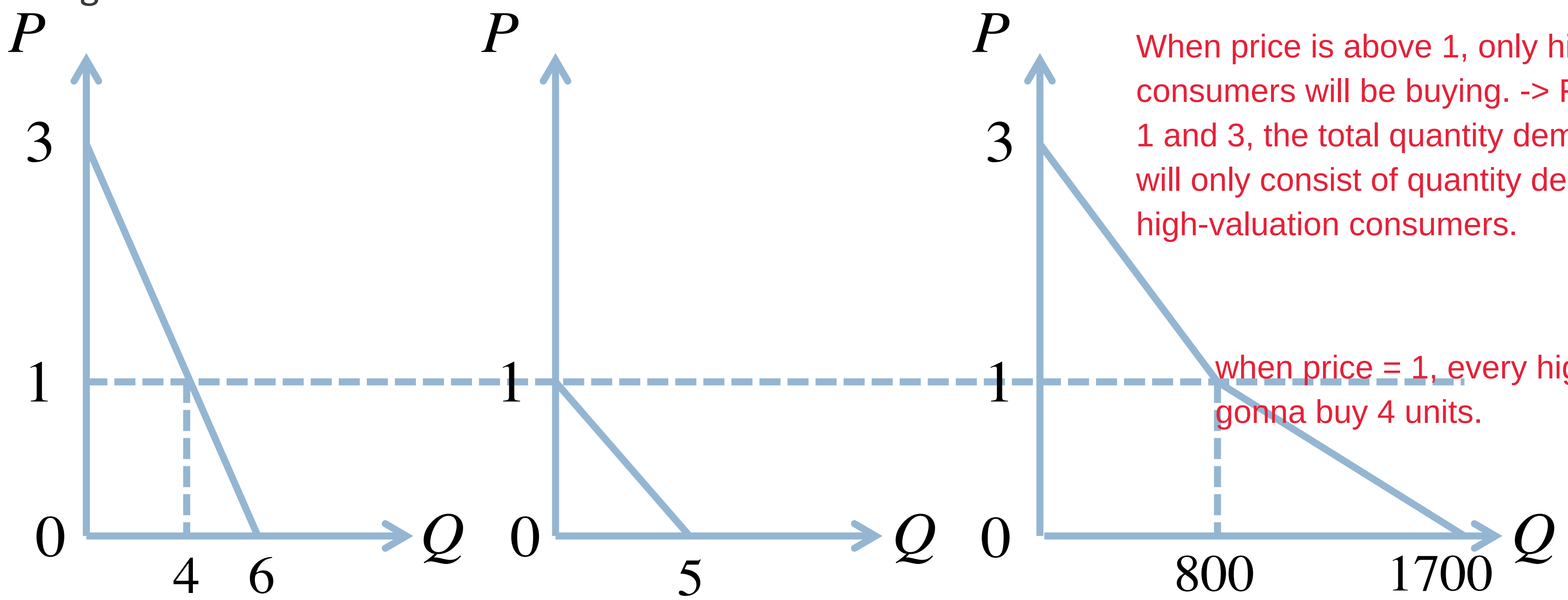
Market Demand Curve in Graph

Market demand curve tells us the total quantity demanded in this market at any price.

Q = 6 - 2P
High-valuation consumer

Q = 5 - 5P
Low-valuation consumer

Market



When price is above 1, only high-valuation consumers will be buying. -> For any price between 1 and 3, the total quantity demanded in the market will only consist of quantity demanded from the 200 high-valuation consumers.

when price = 1, every high-valuation consumer gonna buy 4 units.

From the graph, high-valuation consumers on average willing to pay more for the good compared to low-valuation consumers. (explanation: for high-valuation consumers, they start to buy the good as long as price is 3 or below. For low-valuation, consumers only start to buy the good when the price is 1 or below)

When price < 1, diff slope, flatter coz we are adding up more consumers.
1700 = 6x200 + 5x100

Equation of Market Demand Curve

- When $P>1$
  - Only high-valuation consumers will buy
  - Market demand curve: $Q=200(6-2P)$
- When $P<=1$
  - Both types of consumers will buy
  - Market demand curve: $Q=200(6-2P)+100(5-5P)$
- Market demand curve is

$$Q = \begin{cases} 1700 - 900P & if \quad P \leq 1 \\ 1200 - 400P & if \quad P > 1 \end{cases}$$

Summary: Consumer Choice

How consumers rank diff baskets

baskets consumers can afford

Preference
- What I like

+

Budget Constraint
- What I can afford

=

Optimal Choice
- What I buy

Summary: Individual Demand Curve

Optimal Choice
- What I buy at a particular price

+

Price Changes

=

Individual Demand
- What I buy as a function of price

Optimal choice of a consumer tells us how much of each good a consumer will buy at a given price and income level.

When prices change, optimal choice will change

How much the consumption of food depends on price of food.

Summary: Market Demand Curve

Exchange economy: an extension of consumer theory

In standard consumer theory model, only look at 1 consumer. How does 1 individual consumer make decisions.
But in exchange economy mode, look at 2 consumers. Allow the 2 consumers to trade with each other.

Part 2

The Edgeworth Box

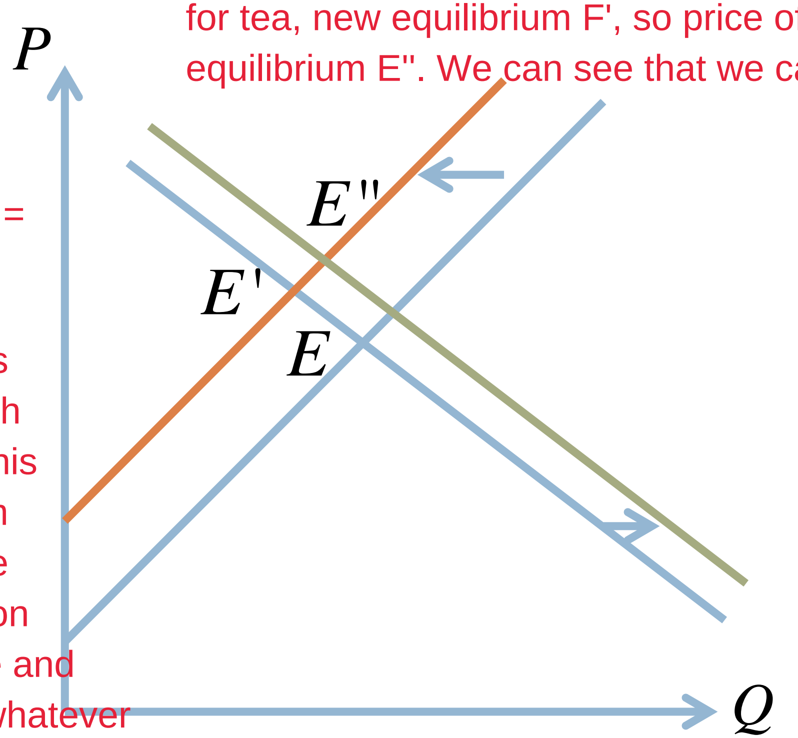
Example: Market for Coffee and Market for Tea

Assume competitive market

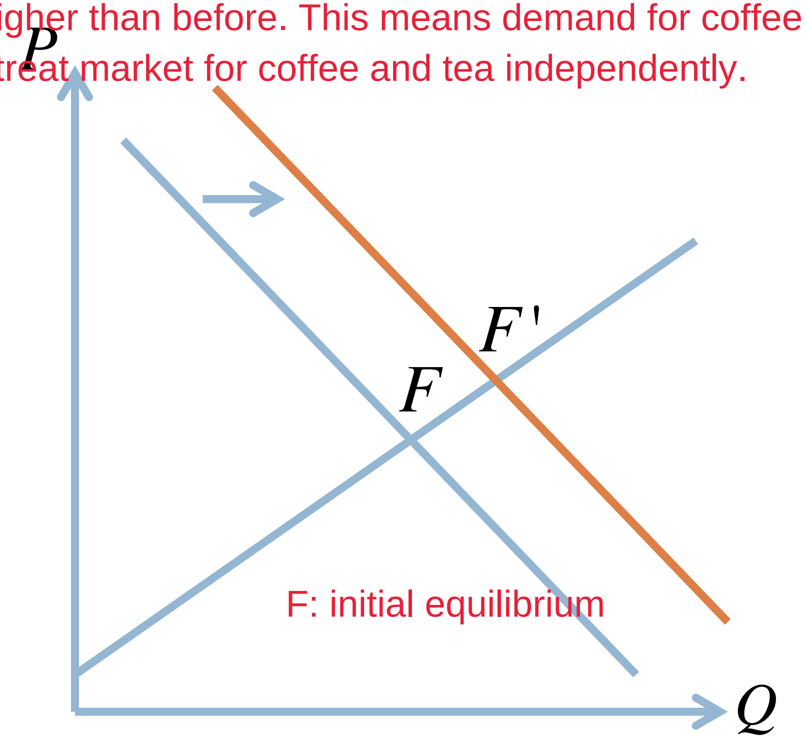
Suppose there is supply shock, supply curve for coffee shifts left, new equilibrium point E', price higher. Coffee and tea are substitutes: when coffee more ex, consumer wants to drink more tea, increase demand for tea, new equilibrium F', so price of tea higher than before. This means demand for coffee will increase, new equilibrium E''. We can see that we cannot treat market for coffee and tea independently.

At E, quantity demanded = quantity supplied

We are only looking at this market in isolation, not with other markets together. This is called partial equilibrium analysis. By doing this, we are assuming no interaction between market for coffee and market for other goods. (whatever happens in coffee market doesn't affect other markets, but in reality not true)



Market for Coffee



Market for Tea

Partial vs. General Equilibrium

- Partial Equilibrium Analysis
  - Finding the equilibrium price and quantity in a single market
  - Holding prices in all other markets fixed   we are ignoring potential interactions with other markets.
- General Equilibrium Analysis
  - Finding the equilibrium prices and quantities in more than one markets simultaneously   treat it like a system

We are not going to do a full model of general equilibrium, we will do a much simpler model of general equilibrium. This can be done with the knowledge of consumer theory only.

An Exchange Economy

- There are two consumers in the economy, A and B,
- There are two goods in the economy, 1 and 2
- Consumer A's consumption basket is denoted by

$$(x_1^A, x_2^A)$$

X1A: how much good 1 a consumer A is consuming

- Consumer B's consumption basket is denoted by

$$(x_1^B, x_2^B)$$

- An *allocation* is a pair of consumption baskets

$$(x_1^A, x_2^A, x_1^B, x_2^B)$$

Allocation tells us how many units of each good each consumer is consuming.

Endowment Allocation

no firms, only 2 consumers in this economy, so means no production.

- Each consumer has some amount of each good to start with
  - There is no money/income
  - They can trade with each other

good to good exchange

- The allocation the consumers start with is the *endowment allocation* denoted by

how many units of each good every consumer starts with at the beginning.

omega is given, not up to consumers to choose what their endowment is. But the x allocation is up to consumers to choose, that's how much consumer will consume.

$$(\omega_1^A, \omega_2^A, \omega_1^B, \omega_2^B)$$

not w, it's omega

- E.g. consumer A's endowment is (6, 4) and consumer B's endowment is (2, 2)

the total quantity of each good is fixed in this exchange economy.

  - The total amount of good 1 is 8 and the total amount of good 2 is 6

Feasible Allocation

- Definition 5.1 An allocation is *feasible* if

feasible/possible/attainable allocation in this economy

This is a general equilibrium model. Coz we are looking at markets for 2 goods at the same time.

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$

$$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

the total consumption = total endowment

  - The total amount of each good consumed equals to the total amount available
- Using the example from the previous slide
  - The total amount of good 1 is 8 and the total amount of good 2 is 6
  - (3, 1) and (5, 5) is feasible
  - (4, 4) and (2, 6) is not feasible

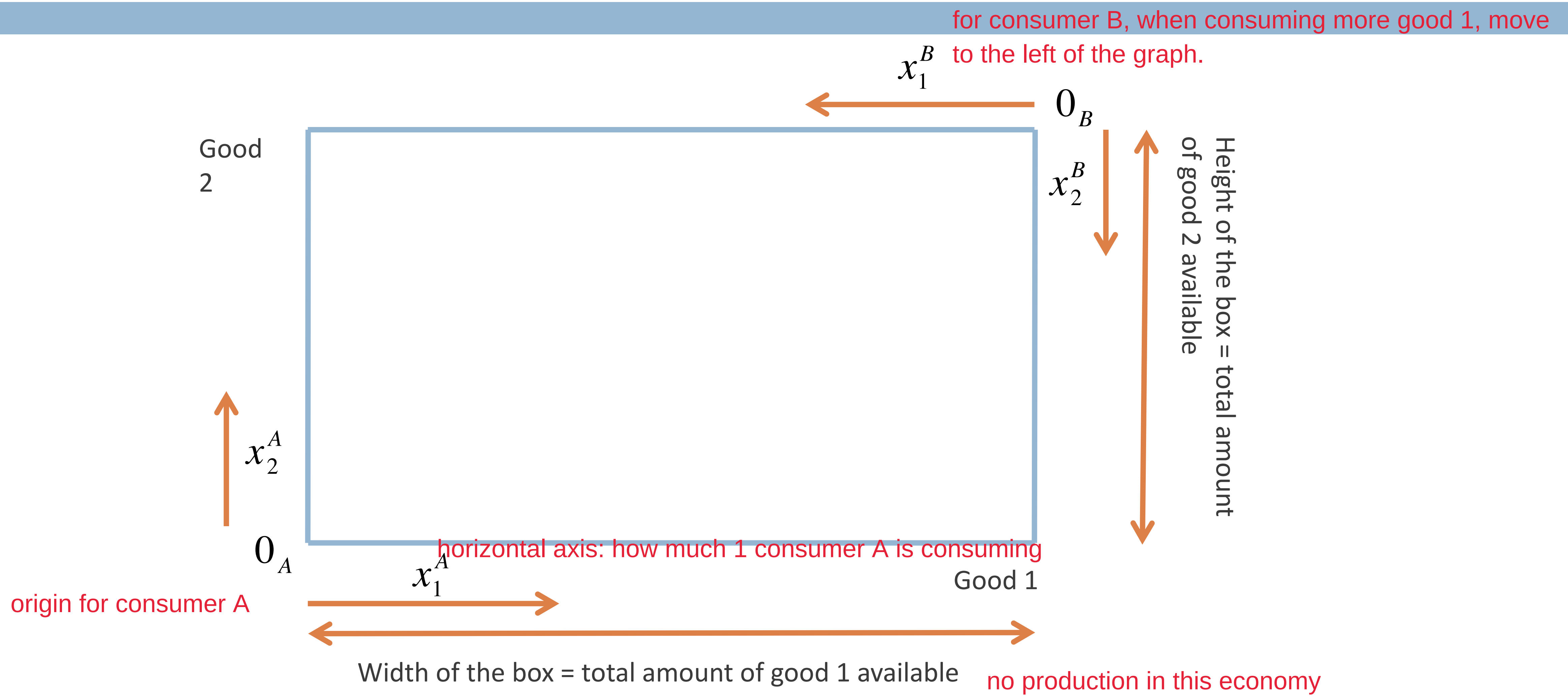
We only need to consider feasible allocations.

Question to answer: Given my endowment, at the end of day, what is the basket I will be consuming, Which allocation will I move to in this economy?
What is considered as equilibrium in this economy?

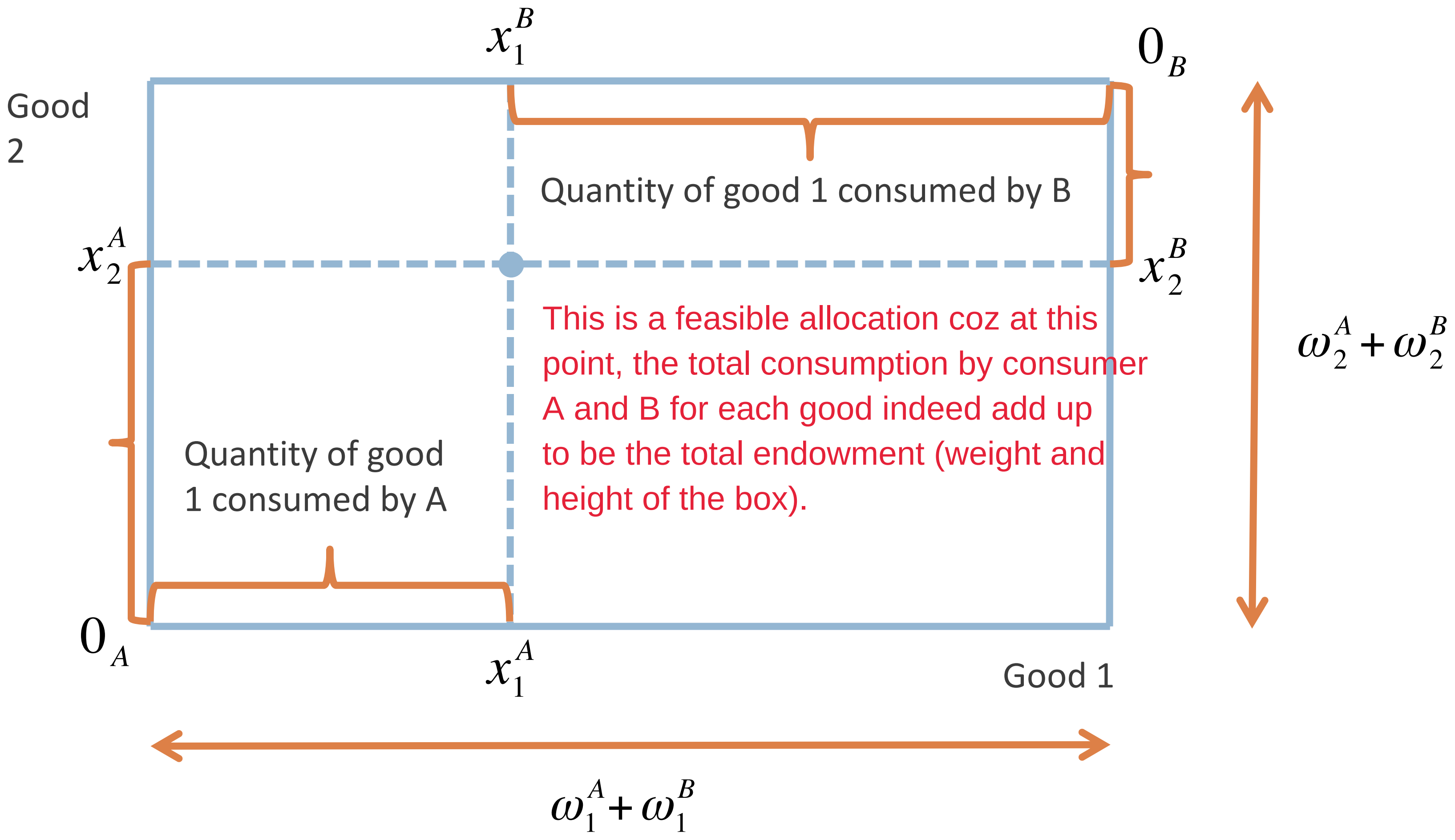
Edgeworth Box

- An *Edgeworth box* is used to graphically show all feasible allocations of the two goods between the two consumers
- Every point in the box, including those on the boundaries, represents a feasible allocation

Setting up an Edgeworth Box

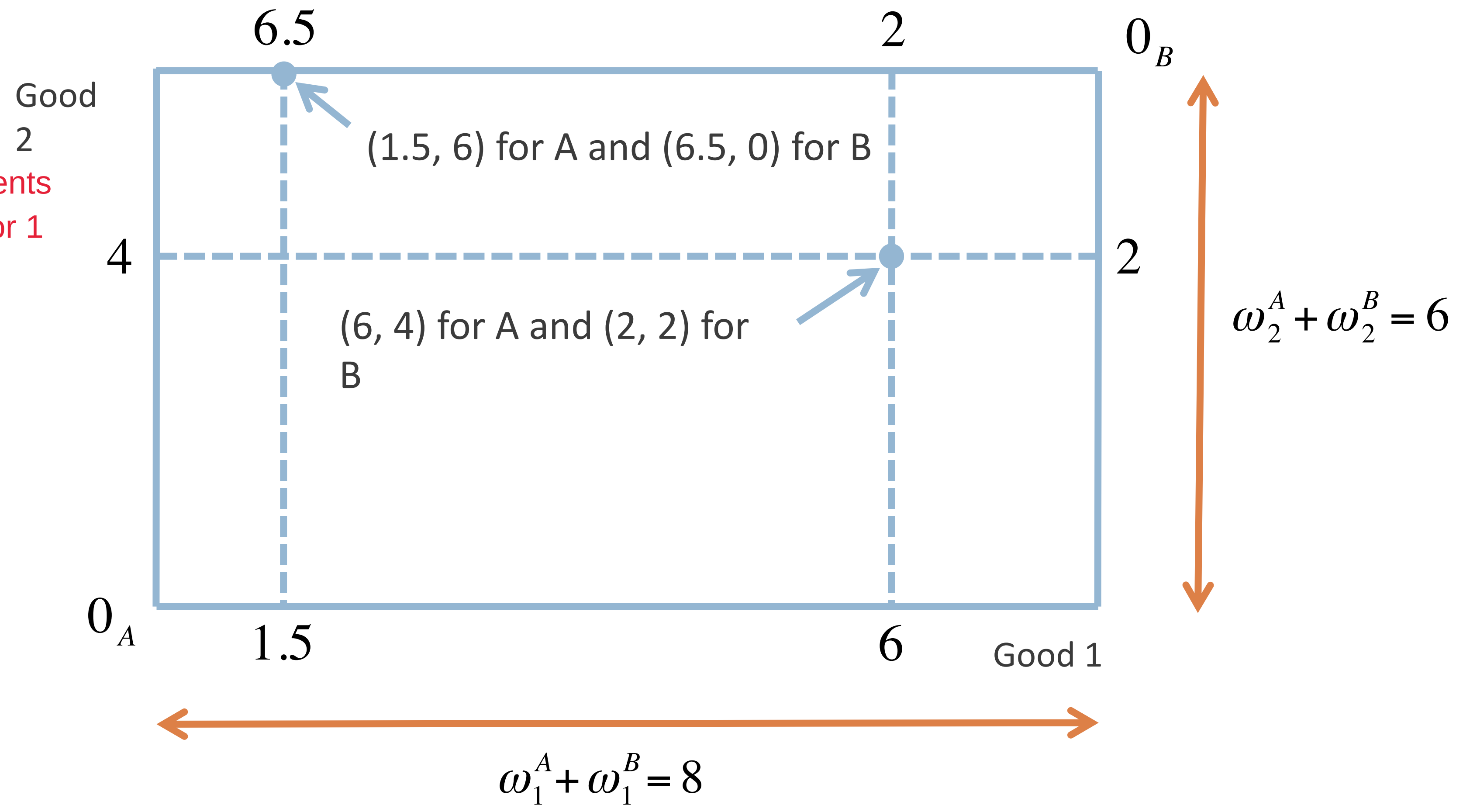


Representing a Feasible Allocation in the Edgeworth Box



Edgeworth Box: An Example

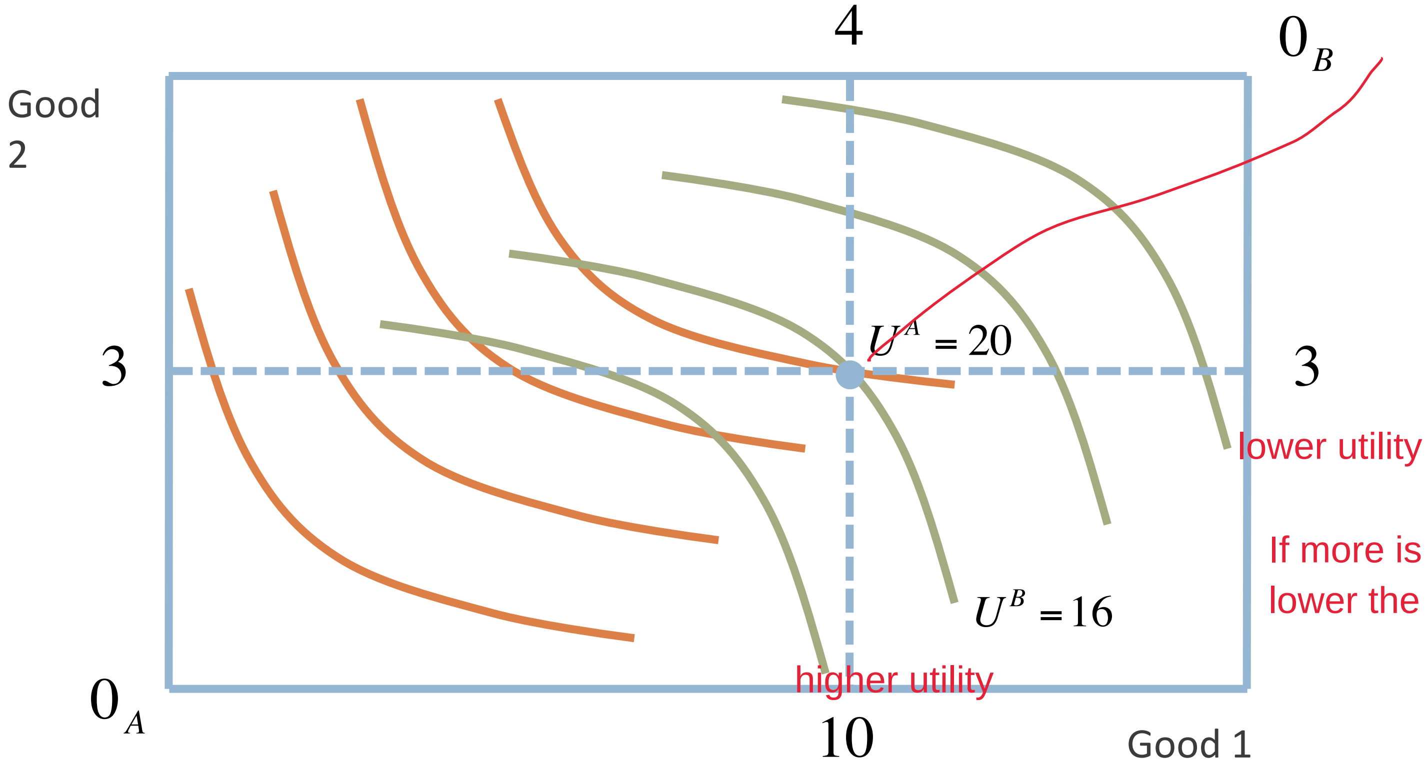
A point on the border represents consumption of 1 good = 0 for 1 consumer.



Adding Preferences to the Box

Add indiff curves to the box

If both A and B have standard convex indiff curves



For consumer A, at this point, he's consuming 10 units of good 1 and 3 units of good 2, get utility of 20. For consumer B, this point represents 4 units of good 1 and 3 units of good 2, get utility of 16.

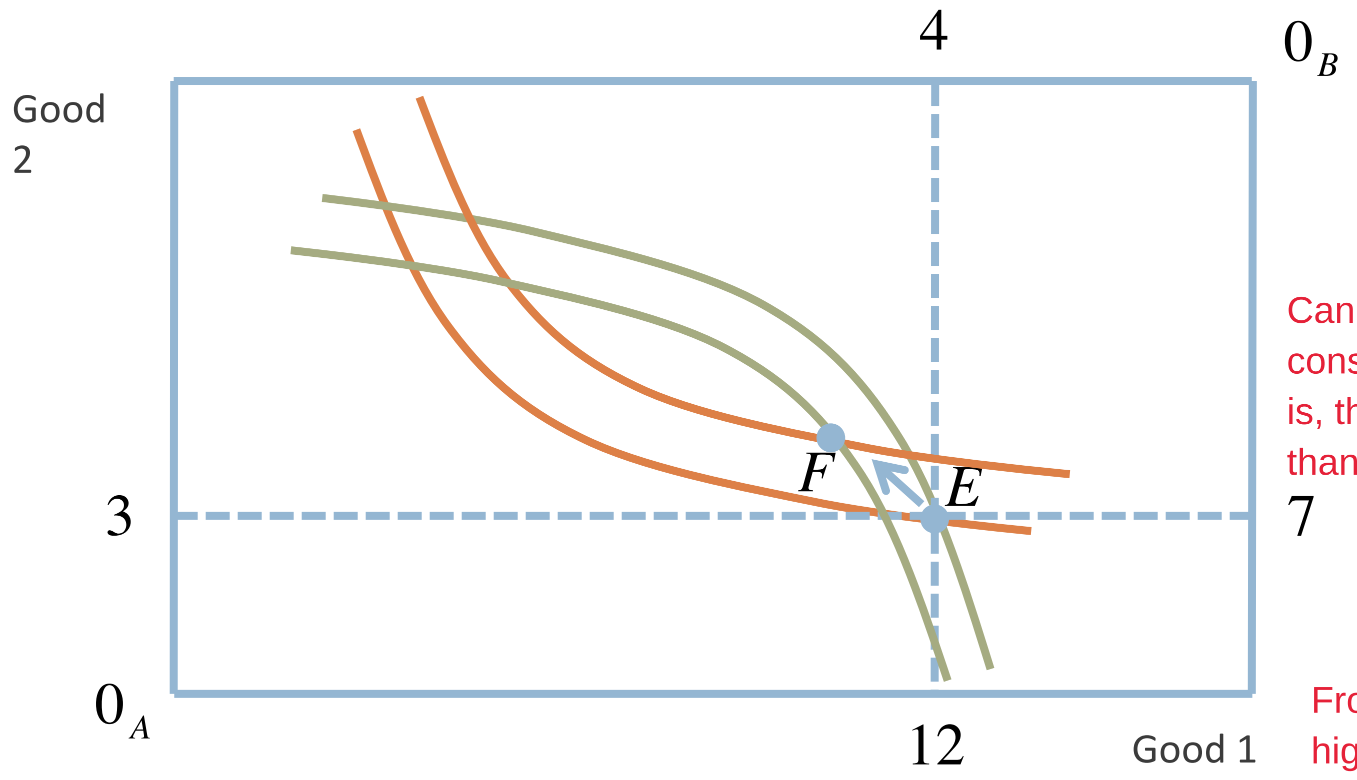
If more is better holds for consumer B, the lower the indiff curve is, the higher utility.

Part 3

Pareto Efficiency

Every point in the Edgeworth box is an allocation. Is there a way for us to rank those allocations?

Is there an allocation where both consumers are better off than at E?



orange: indiff curve for A
green: indiff curve for B

Can we find an allocation where both consumers have higher utility than E? If there is, then there is an allocation that's better than E.

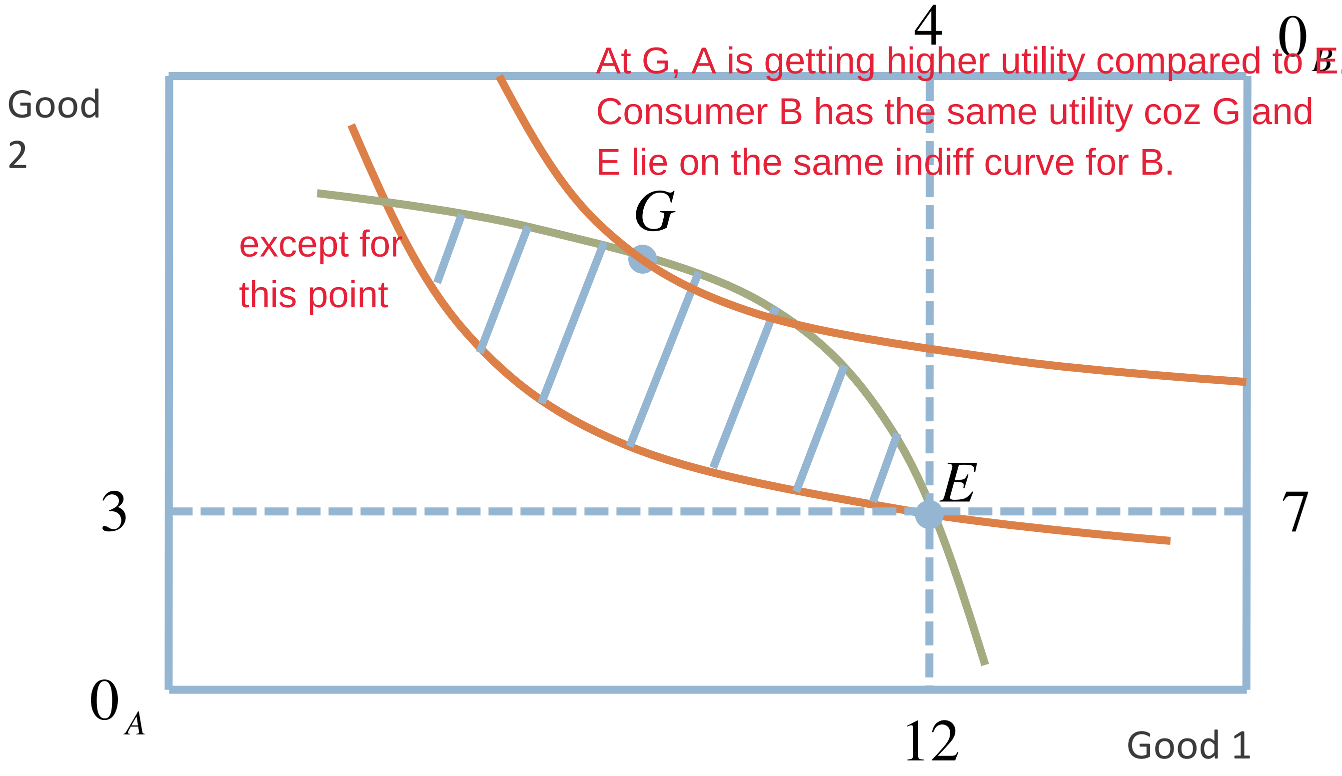
From E to F, both consumers are on a higher indiff curve.

At point F, both consumers get higher utility compared to point E

Point F is not the only point that is better than E.

Both Consumers are at Least as Well off as at E in the Shaded Region

G is better than E.



At G, A is getting higher utility compared to E.
Consumer B has the same utility coz G and E lie on the same indiff curve for B.

except for this point

Any pts in the shaded area, compared with E, we gonna come to the conclusion that at least 1 consumer will get higher utility. No consumer is going to get lower utility.

At any allocation in the shaded region, each consumer's utility is either higher than or the same as the utility at E

Pareto Improvement

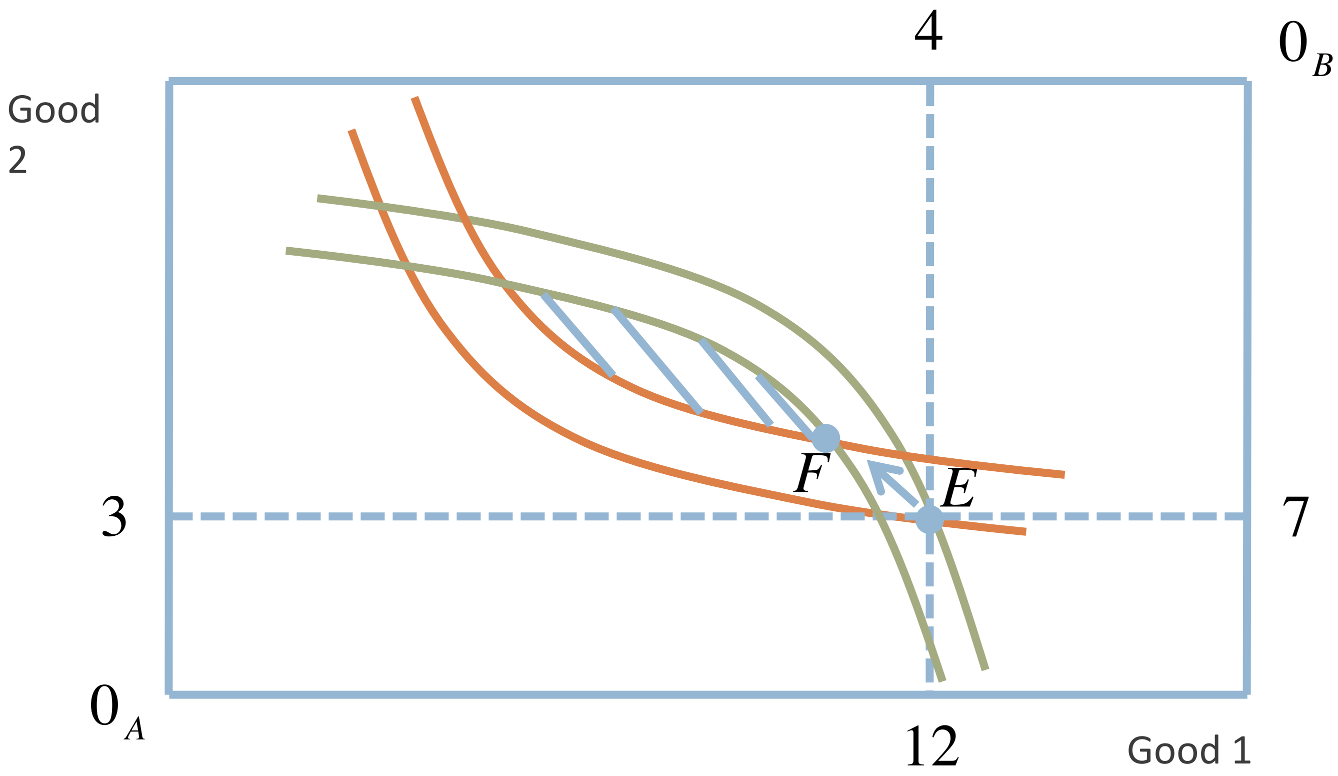
How we decide a better allocation.

- Definition 5.2 From some allocation X to some other allocation Y is a *Pareto improvement* if from X to Y, at least one consumer is better off and no one else is worse off

At least 1 consumer gets higher utility. The other consumer either gets higher utility or the same utility.

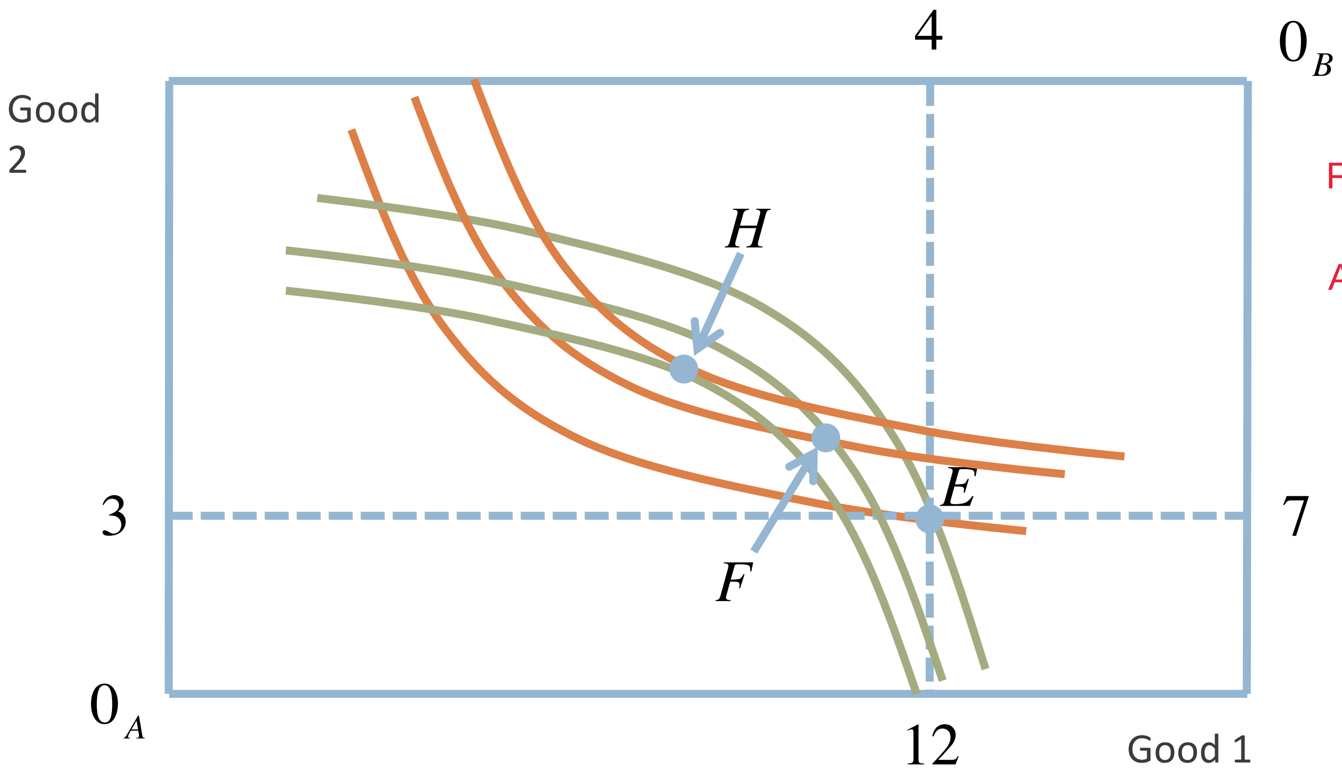
- On slide 22
  - E to F is a Pareto improvement
- On slide 23
  - E to G is a Pareto improvement

At F, can we make a Pareto improvement?



Yes, any allocation in the shaded region is a Pareto improvement of F

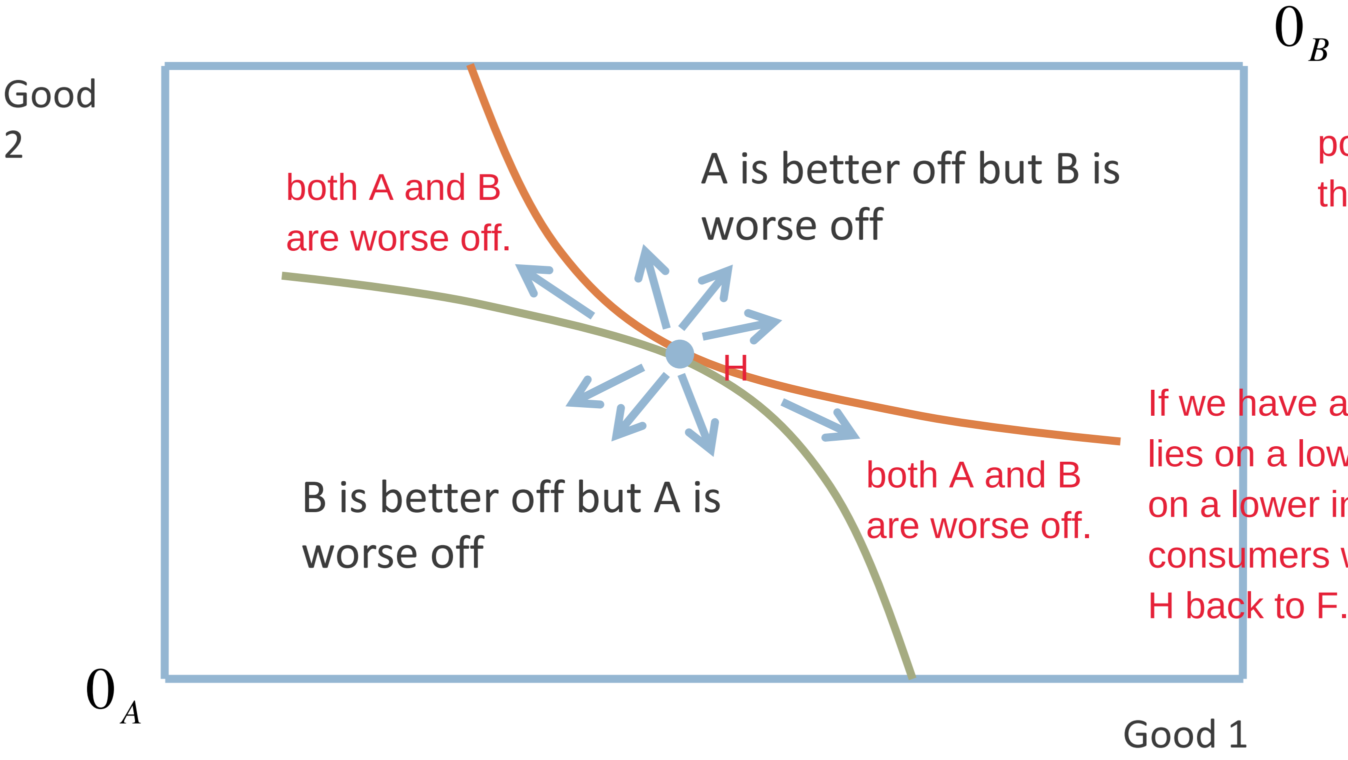
At H, can we make a Pareto improvement?



From F to H, pareto improvement.

At H, can we find a better allocation?

A Closer Look at Point H



point H is the point of tangency between the 2 consumer indiff curves.

If we have a point here, for consumer A, this point lies on a lower indiff curve. For consumer B, also lies on a lower indiff curve. So at this point, both consumers will get lower utility. It's like moving from H back to F.

At point H, we cannot make one consumer better off without making the other consumer worse off

cannot have pareto improvement anymore, cannot find an allocation that's better than this allocation H.

Pareto Efficiency

- Definition 5.3 An allocation is *Pareto efficient* if there is no way to make one consumer better off without making someone else worse off

Point H is pareto efficient.

it means that this is the best allocation, cannot find allocation that is better than this one.

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A Pareto efficient allocation can be described as an allocation where:

1. ~~There is no way to make all the people involved better off; or~~
2. there is no way to make some individual better off without making someone else worse off; or
3. ~~all of the gains from trade have been exhausted; or~~
4. ~~there are no mutually advantageous trades to be made, and so on.~~

Pareto Efficiency and Pareto Improvement

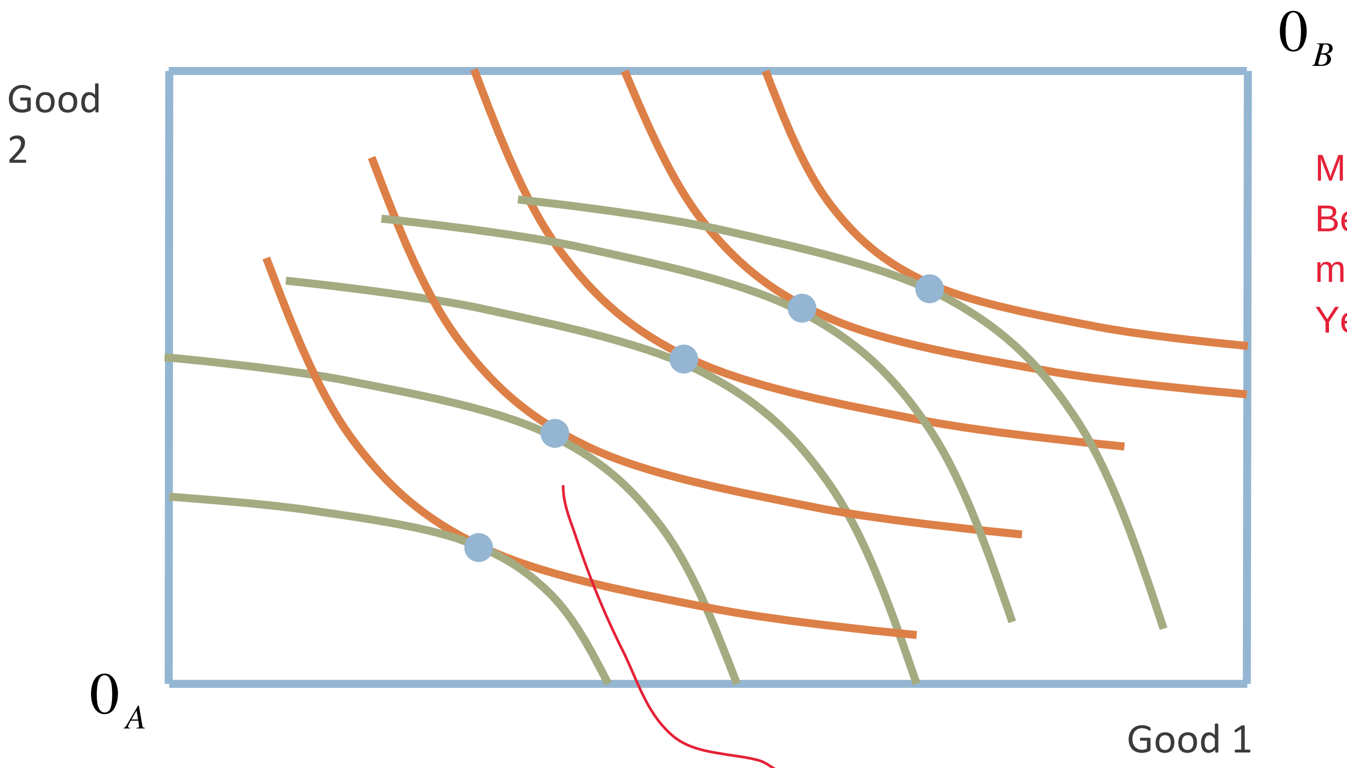
- If an allocation is Pareto efficient
  - There is no room for Pareto improvement

smooth, downward-sloping, convex

- When each consumer's indifference curves are smooth with diminishing *MRS* and when we have interior solutions, Pareto efficient allocations will be the tangency points between the two indifference curves
  - When indifference curves are not tangent to each other, it is not Pareto efficient
  - Hence there is room for Pareto improvement

if we are at an allocation that is not pareto efficient, means we can still have pareto improvement.

There is More than One Pareto Efficient Allocation

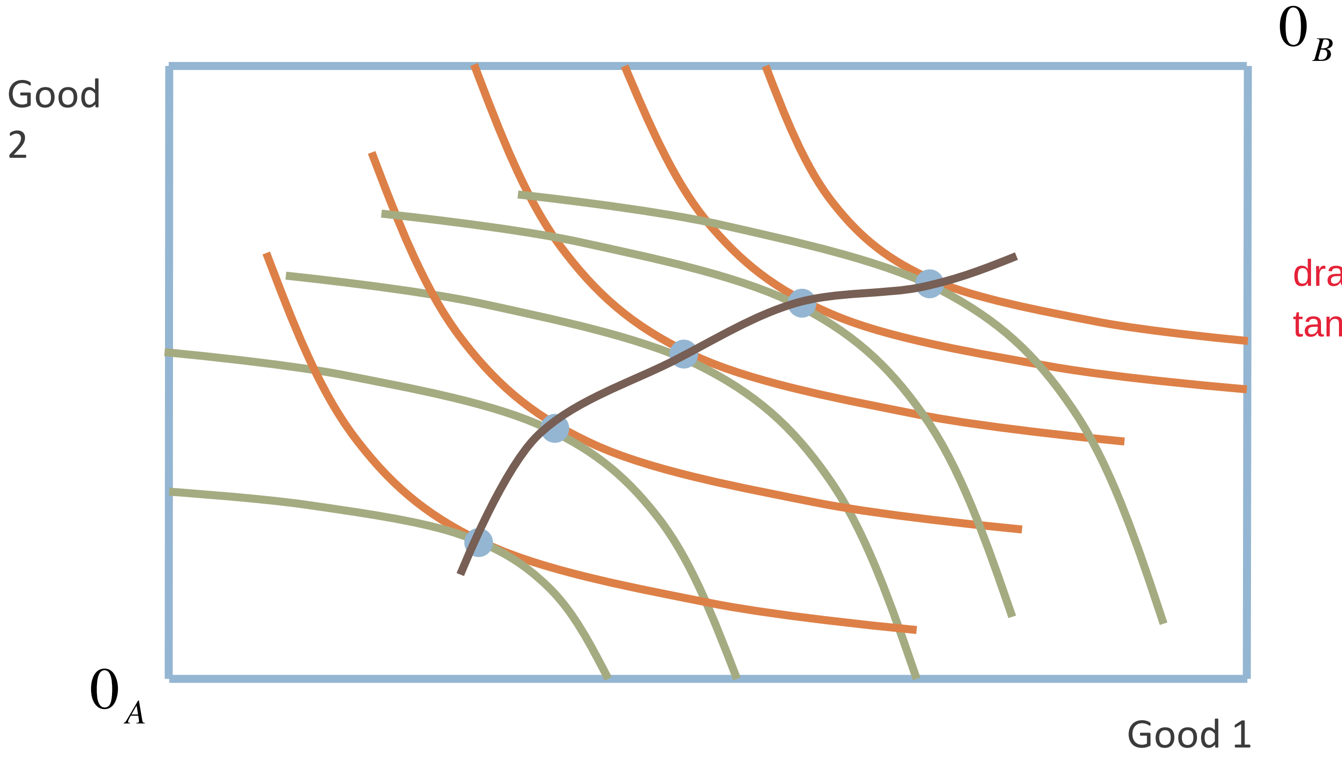


More than 1 tangency points in this box. Because every consumer has indefinitely many indiff curves. All are pareto efficient? Yes.

moving from the lower point to the higher point, pareto improvement? NO. Consumer A will be better off, but consumer B is worse off coz lower indiff curve.

Contract Curve

not always a curve, might be a line or more complicated, depends on consumer preference.



draw a curve that goes through all these tangency points

Any point on the contract curve, by definition, is pareto efficient.

Definition 5.4 The *contract curve* is the set of all Pareto efficient allocations

Means that if u have an allocation that is pareto efficient, that allocation will definitely lie on the contract curve.

Deriving the Contract Curve Mathematically

The 2 consumers' indiff curves are tangent to each other.
MRS for consumer A between the 2 goods, 1 and 2, is the same as MRS for consumer B for the 2 goods, 1 and 2.

- Tangency condition

$$MRS^{A}_{1,2} = MRS^{B}_{1,2} \qquad (1)$$

How many variables you are going to have from this eqn? 4. MRS for consumer A usually contains 2 variables x1A (consumption of good 1 for consumer A), x2A. MRS for consumer B contains 2 variables for B, x1B and x2B. We want to end up with an eqn of 2 variables so that we can plot the contract curve in graph.

- The allocation must be feasible

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B \qquad (2)$$

$$x_2^A + x_2^B = \omega_2^A + \omega_2^B \qquad (3)$$

omegas are given, we are not solving them.

- Substituting (2) and (3) into (1), we can express the contract curve in terms of x^A_1 and x^A_2 or x^B_1 and x^B_2

From (2), we can write x1B as a function of x1A. Similarly, from (3). we can write x2B as a function of x2A.
Then we can plug in to (1) to get rid of x1B and x2B.
The equation will only contain x1A and x2A.

However, this can only be applied if we have tangency points in the box. But this is not always the case, depends on consumer preference.
Question: what if we don't have tangency point? What if we have perfect complements or perfect substitutes for example? Impossible to find tangency points in the box.
Does it mean there is no contract curve and no pareto efficient allocations?
No, still have contract curve and pareto efficient allocation. It's just that the pareto efficient allocations will no longer be points of tangency.

How do we find contract curves if we don't have points of tangency?

Part 4

General Competitive Equilibrium

Budget Set in the Exchange Economy

the defining characteristic of a competitive market is price-taking behaviour (means no individual consumer/ firm have power to decide market price)

consumer decision-making problem again

- Given an endowment allocation, which allocation will the consumers end up consuming?
  - optimal basket
  - Each consumer will choose the utility-maximizing basket given the budget constraint
  - Budget constraint determined by prices and endowments

in this economy, consumers do not have income, then what is budget constraint? they are having money in terms of having goods. We can figure out how much endowments for each consumer is worth, monetary value of endowment, can still treat it as income

- Suppose the market for each good is perfectly competitive
  - That is, consumers are price takers everyone takes price as given
- Let P_1 be the price of good 1 and P_2 be the price of good 2, the two consumers' budget constraints are

total expenditure of good 1 of consumer A + total expenditure of good 2 of consumer A

RHS: monetary value of endowments. How much money is the endowment worth to the consumer.

$$P_1 x_1^A + P_2 x_2^A \leq P_1 \omega_1^A + P_2 \omega_2^A \quad (A)$$

budget constraint for consumer A

$$P_1 x_1^B + P_2 x_2^B \leq P_1 \omega_1^B + P_2 \omega_2^B \quad (B)$$

Example: Budget Constraints

- Suppose the price of good 1 is \$2 and the price of good 2 is \$1
- Consumer A's endowment is (12, 3)
  - The endowment is worth 12*\$2+3*\$1=\$27
  - Equivalent to having \$27 of income
- Consumer B's endowment is (4, 7)
  - The endowment is worth 4*\$2+7*\$1=\$15
  - Equivalent to having \$15 of income
- The budget constraints are

$$2x_1^A + x_2^A \leq 27 \quad (A)$$

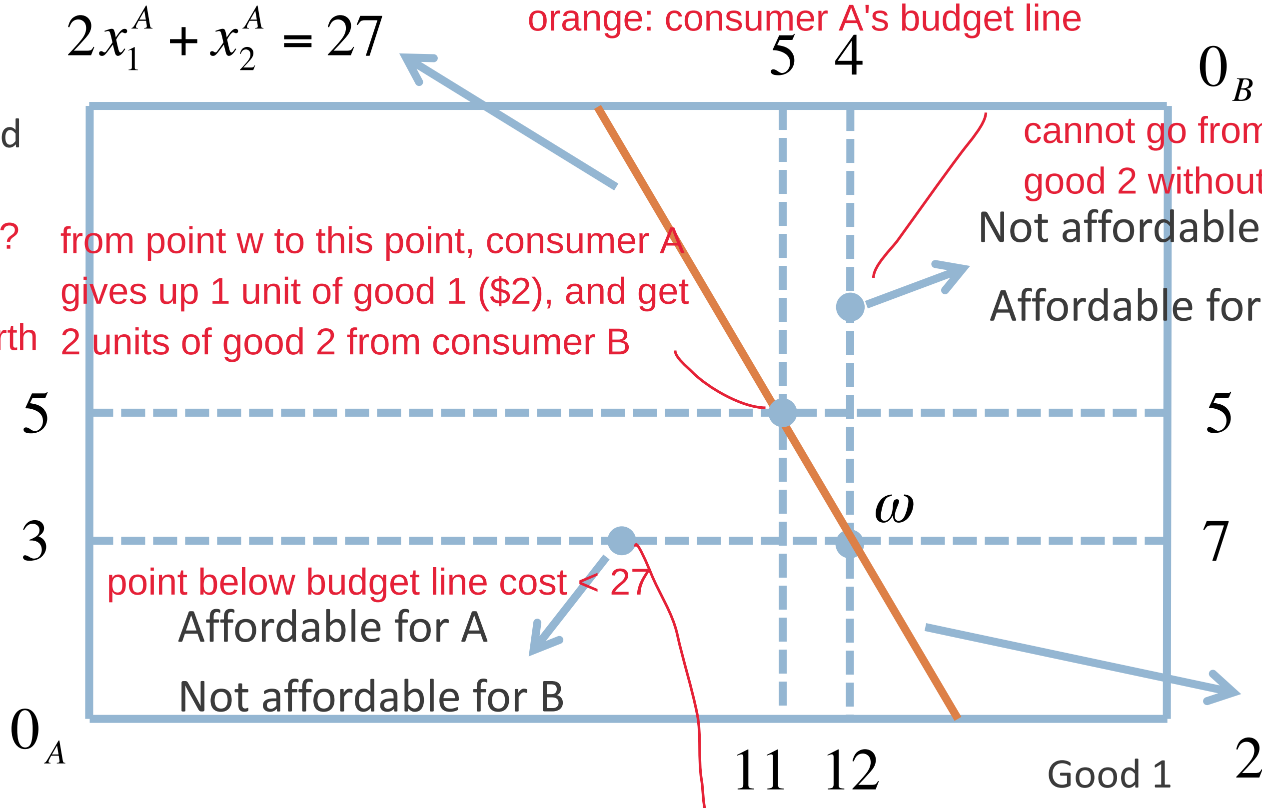
$$2x_1^B + x_2^B \leq 15 \quad (B)$$

This is an economy where we don't have money, then what do prices mean?
What we care about is not whether the price of good 1 is 2 or 4, doesn't mean anything coz we don't have money.
What we want to know is the relative price of the 2 goods.
If u want to get 1 unit of good 1, need to use 2 units of good 2 to exchange for it. So we want to know how the 2 goods should be exchanged in this economy between these 2 consumers. We want to get exchange rate, which is the relative price of the 2 goods.

Example: Budget Constraints in Graph

The allocations that are affordable for both A and B are only allocations on the budget line!

orange: consumer A's budget line



Points on the budget line meaning? just like a usual budget line. Any point on the budget line will be worth \$27 to the consumer A. All points on budget line cost the consumer the same (same monetary value).

from point w to this point, consumer A gives up 1 unit of good 1 (\$2), and get 2 units of good 2 from consumer B

cannot go from w to this point, coz cannot get more good 2 without giving up some good 1.

cost A > 27 in terms of monetary value

budget line goes through the endowment allocation.

point below budget line cost < 27

exact same line as A's budget line!

1. same prices, so same slope
2. B's budget line must go through endowment

from endowment w to this point, just give up some good 1 to B for free.

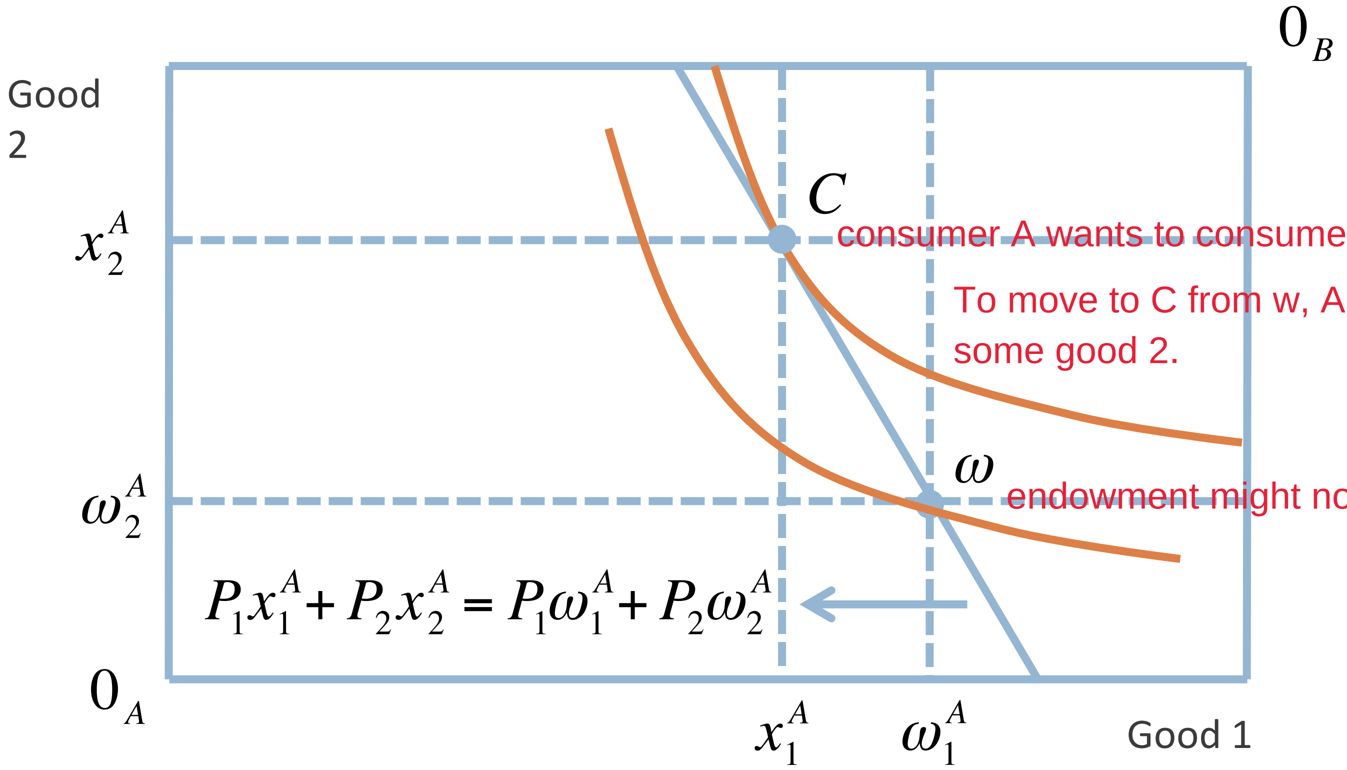
same as before, -ve of relative price of 2 goods.

Slope of budget line = $-P_1/P_2$

The endowment allocation is on the budget line

All points below budget line are affordable to consumer A.

Consumer A's Optimal Choice



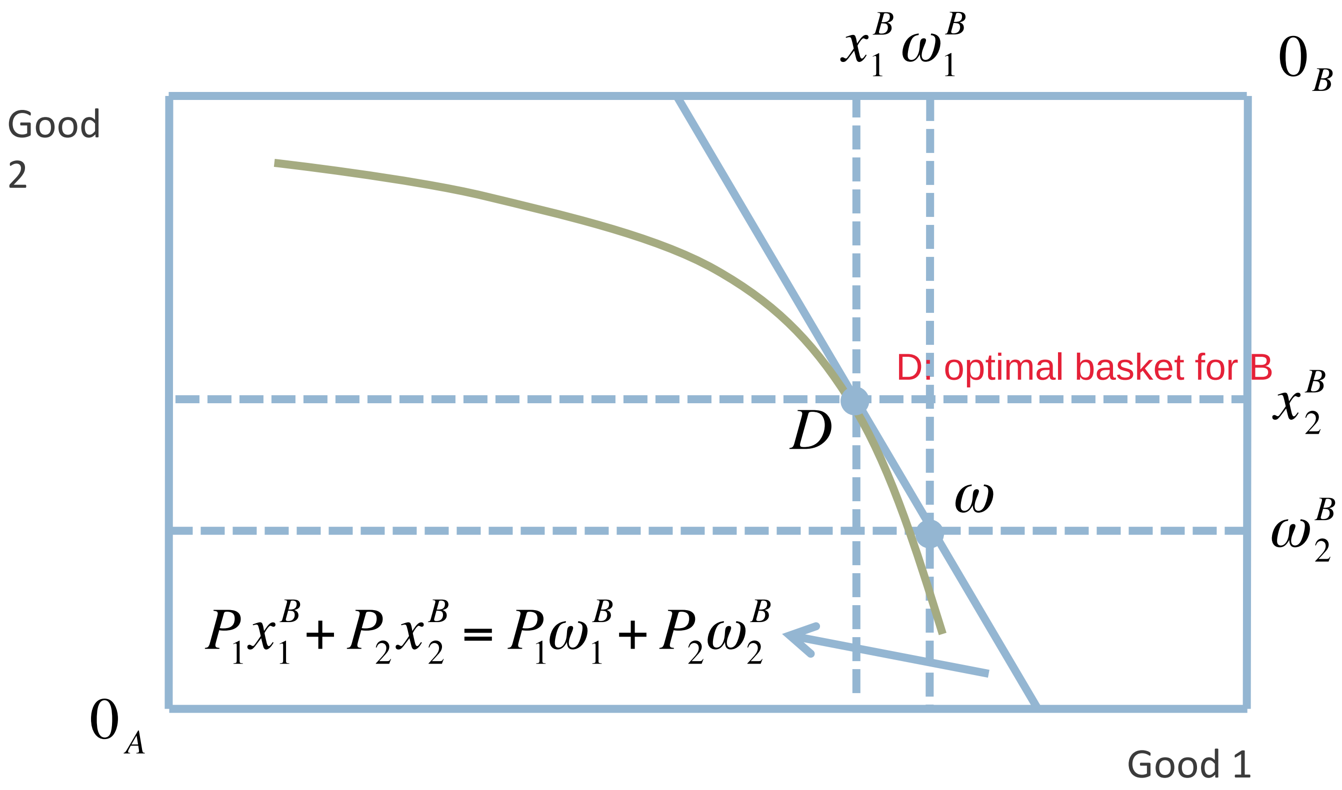
consumer A wants to consumer at C, optimal basket.

To move to C from w, A want to sell good 1 to B in exchange for some good 2.

endowment might not be optimal (utility-maximizing) basket

Given the endowment, consumer A wants to sell some good 1 in exchange for some good 2

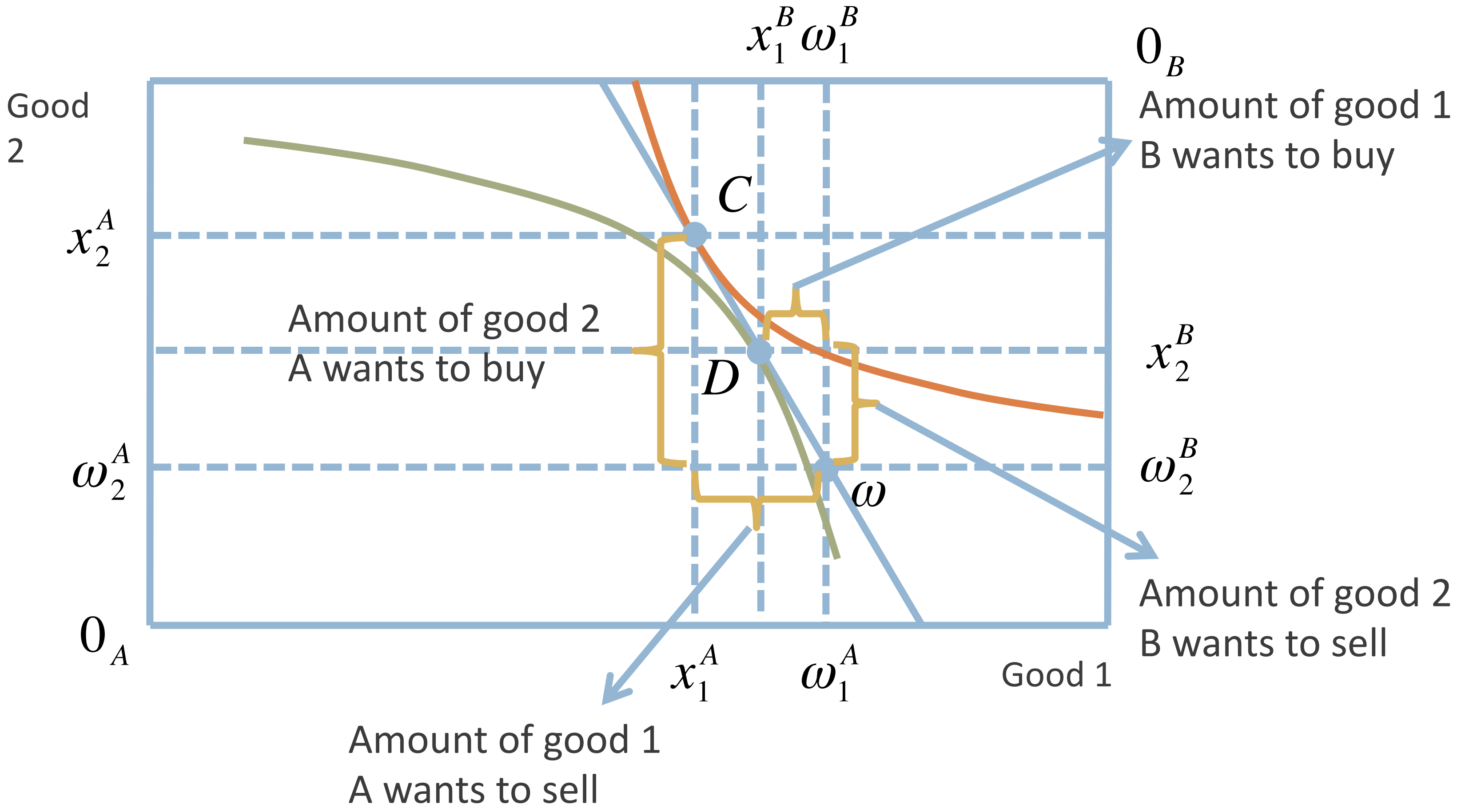
Consumer B's Optimal Choice



Given the endowment, consumer B wants to sell some good 2 in exchange for some good 1

Can the consumers complete their desired transactions?

A wants to be at C,
B wants to be at D.



Markets do not clear at the current prices

meaning: demand != supply

- There is excess supply of good 1
  - The amount B wants to buy is less than the amount A wants to sell

market not in equilibrium. If equilibrium, shouldn't have excess demand or supply.

- There is excess demand of good 2
  - The amount A wants to buy is more than the amount B wants to sell

shortage of good 2

- Sum of the demand for each good does not equal to the total quantity available

$$x_1^A + x_1^B < \omega_1^A + \omega_1^B$$

some units of good 1 nobody wants to consume

$$x_2^A + x_2^B > \omega_2^A + \omega_2^B$$

can see these 2 eqns from the graph on last slide

General Competitive Equilibrium

Important

- Definition 5.5 A pair of prices (P_1, P_2) constitutes a (general) *competitive equilibrium* if at these prices
  - Each consumer maximizes his/her utility given the budget constraint

$$x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$$

denotes the optimal consumption for each consumer given the equilibrium prices

  - Markets for both goods clear at the utility-maximizing basket

market-clearing conditions

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

the total demand for each good = total supply

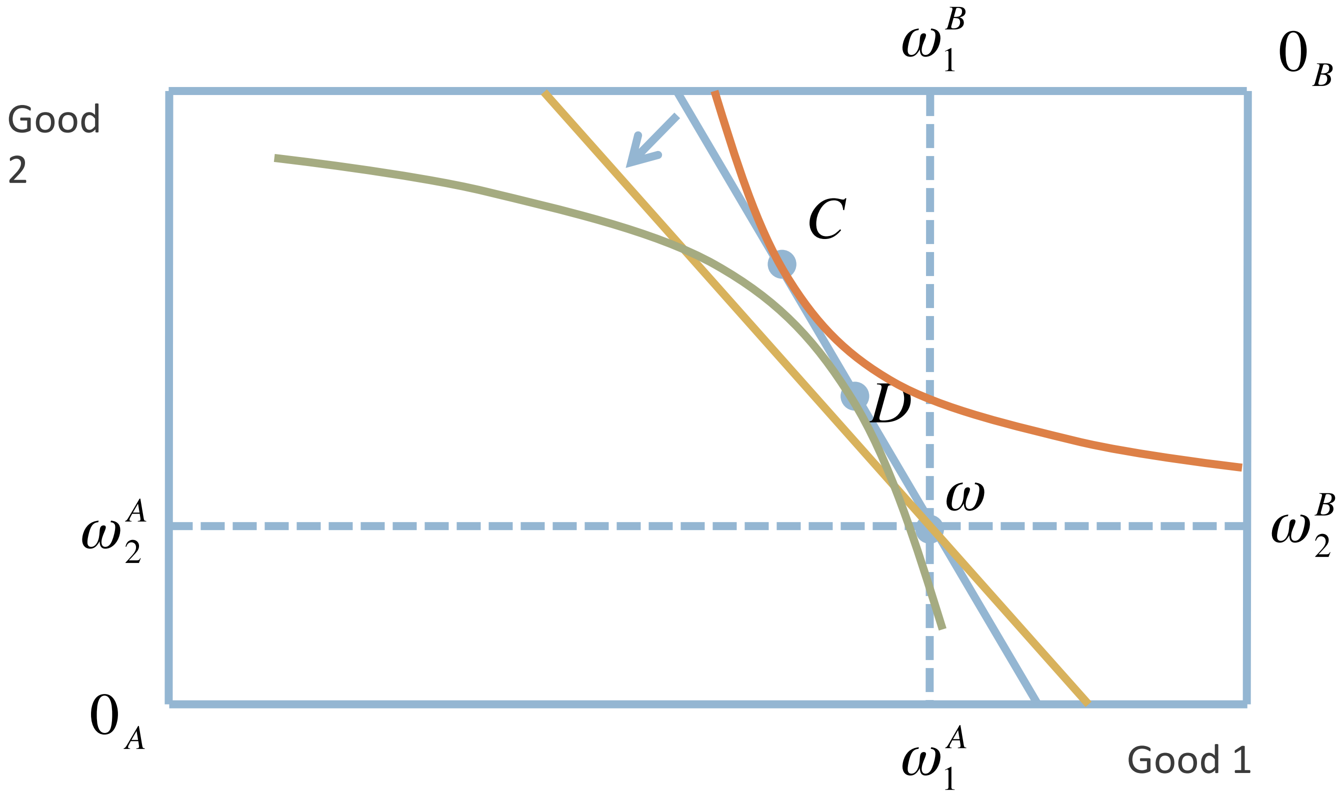
$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$$

Back to Slide 39

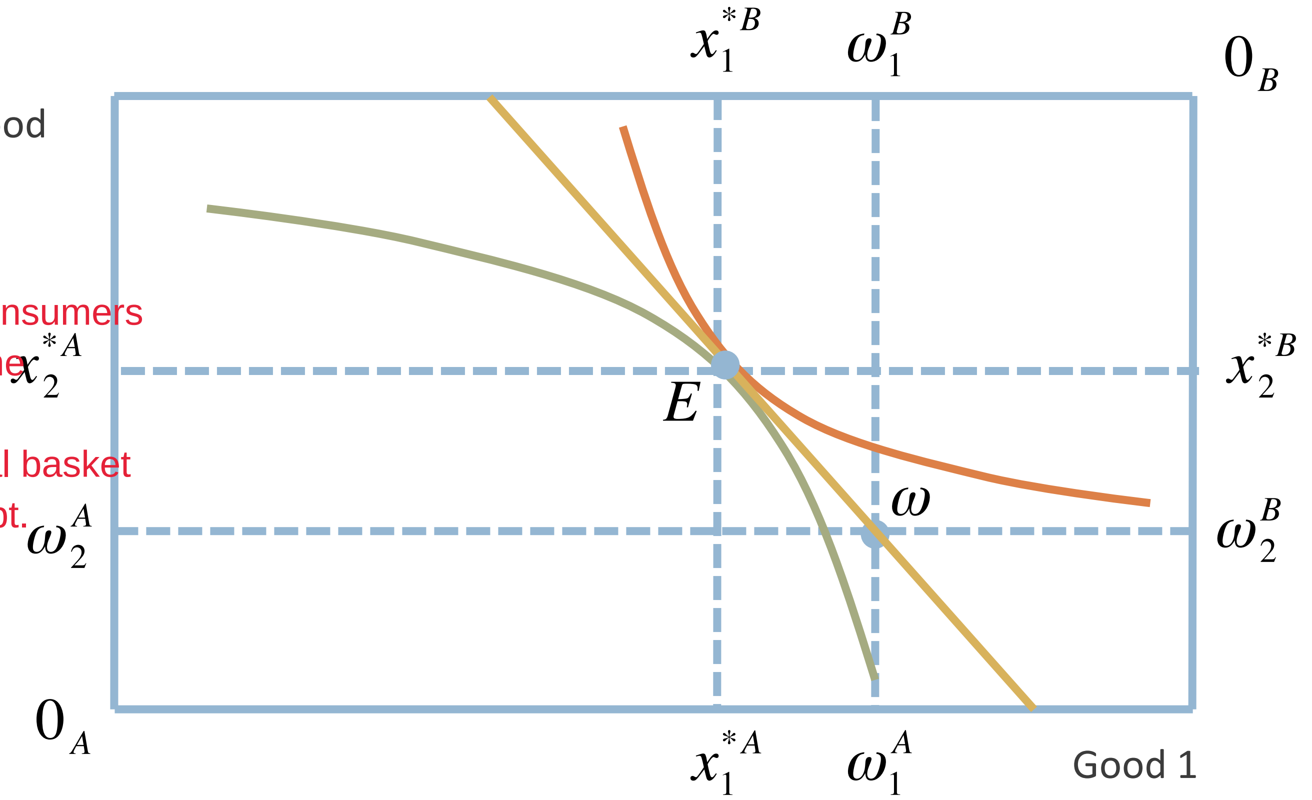
- Since there is excess supply of good 1
  - The price of good 1 will
- Since there is excess demand of good 2
  - The price of good 2 will
- Thus P_1/P_2 will decrease relative price measures the slope of budget line
- Budget line will become flatter
  - But it still goes through the endowment allocation

the endowment allocation is always exactly affordable to each consumer

Reaching an Equilibrium



Reaching an Equilibrium Cont’



When will an economy reach an equilibrium?
When optimal baskets of the 2 consumers are the same point. If not the same point, market for the 2 goods will not clear. Only clear when optimal basket of the 2 consumers is the same pt.

E is optimal basket for consumer A and B, given this budget line.

total consumption of each good = total endowment -> markets for the 2 goods clear -> satisfy definition of competitive equilibrium

At the new prices, markets for the two goods clear and each consumer maximizes utility given the budget constraint

Thus, E is equilibrium allocation and slope of budget line tells us the equilibrium price ratio

Example: Solving for Competitive Equilibrium

- Suppose consumer A's utility function is

$$U^A(x_1^A, x_2^A) = x_1^A x_2^A$$

x1A is 1 variable, not power!

- Suppose consumer B's utility function is

$$U^B(x_1^B, x_2^B) = x_1^B x_2^B$$

- Consumer A's endowment is (10, 6) and consumer B's endowment is (10, 4)
- Find the equilibrium prices P_1 and P_2

6 unknown variables: allocation (4 variables), prices (2 variables)

Example: Solving for Competitive Equilibrium Cont’

- Consumer A’s optimal choice utility function is Cogg Douglas, we know optimal basket is the tangency pt.

$$\frac{x_2^A}{x_1^A} = \frac{P_1}{P_2} \qquad (1)$$

tangency eqn, MRS = price ratio

$$P_1 x_1^A + P_2 x_2^A = 10P_1 + 6P_2 \qquad (2)$$

budget line, RHS: monetary value of endowment of consumer A

- Consumer B’s optimal choice

$$\frac{x_2^B}{x_1^B} = \frac{P_1}{P_2} \qquad (3)$$

$$P_1 x_1^B + P_2 x_2^B = 10P_1 + 4P_2 \qquad (4)$$

- Market clearing

$$x_1^A + x_1^B = 10 + 10 = 20 \qquad (5)$$

$$x_2^A + x_2^B = 6 + 4 = 10 \qquad (6)$$

Example: Solving for Competitive Equilibrium Cont'

- (1) and (3) give us

$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B} = \frac{P_1}{P_2} \qquad (7)$$

- Plugging (5) and (6) into (7)

$$\frac{x_2^A}{x_1^A} = \frac{10 - x_2^A}{20 - x_1^A} = \frac{P_1}{P_2} \qquad (8)$$

- Solving

$$\frac{x_2^A}{x_1^A} = \frac{10 - x_2^A}{20 - x_1^A}$$

- We get

$$x_1^A = 2x_2^A \qquad (9)$$

Example: Solving for Competitive Equilibrium Cont’

- Plugging (9) into (8)

$$\frac{P_1}{P_2} = 0.5 \quad (10)$$

- Plugging (9) and (10) into (2)

$$P_1 2x_2^A + 2P_1 x_2^A = 10P_1 + 12P_1 \quad \Rightarrow \quad x_2^A = 5.5$$

- The equilibrium allocation is

$$x_1^{*A} = 11, \quad x_2^{*A} = 5.5, \quad x_1^{*B} = 9, \quad x_2^{*B} = 4.5$$

cannot solve p1 and p2 separately, things get cancelled out, the only thing u get eventually is still the price ratio. Mathematically, because although we have 6 eqns, we only have 5 independent eqns

Relative Price

- In the previous example, we can only solve for the relative price P_1/P_2
- Relative price is what matters
  - In the previous example, we just need the price ratio to be P_1/P_2=0.5 in equilibrium
  - It does not matter if P_1=2, P_2=4 or P_1=3, P_2=6
- It is convenient to set one of the prices to 1 at the beginning. -> reduce 6 variables to 5
  - Such a price is called a *numeraire price*, such a good is called a *numeraire*
  - If we set good 2 as a numeraire in the example, then P_1=0.5