PRE-LECTURE VIDEO

COBB-DOUGLAS UTILITY FUNCTION

can think of this as the 3rd special case for preference (first 2: perfect substitute, perfect complement)

Definition

- A utility function of the following form is called a Cobb-Douglas utility function

a power function with +ve power

$$U(x,y) = Ax^{\alpha}y^{\beta}, A > 0, \alpha > 0, \beta > 0$$

- Examples of Cobb-Douglas utility function

$$U(x,y) = xy$$

$$U(x,y) = \frac{1}{3}x^2y^3$$

$$U(x,y) = \sqrt{xy}$$

$$U(x,y) = 4x^{\frac{1}{3}}y^5$$

Marginal Utilities of Cobb-Douglas Utility Functions

- Partially differentiating the utility function

$$MU_x = A\alpha x^{\alpha-1} y^{\beta}$$

$$MU_y = A\beta x^{\alpha} y^{\beta-1}$$

- Both marginal utilities are always positive
- “More is better” satisfied for both goods
- Indifference curves are downward sloping

as long as 'more is better' holds, indiff curve is downward sloping.

Marginal Rate of Substitution of Cobb-Douglas Utility Functions

- The marginal rate of substitution is

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{A\alpha x^{\alpha-1}y^{\beta}}{A\beta x^{\alpha}y^{\beta-1}} = \frac{\alpha y}{\beta x}$$

- As the consumer gets more *x* and less *y* along the same indifference curve
  - *$MRS_{x,y}$* diminishes
- Indifference curves are convex

Typical Indifference Curves for Cobb-Douglas Utility Functions

downward sloping & convex

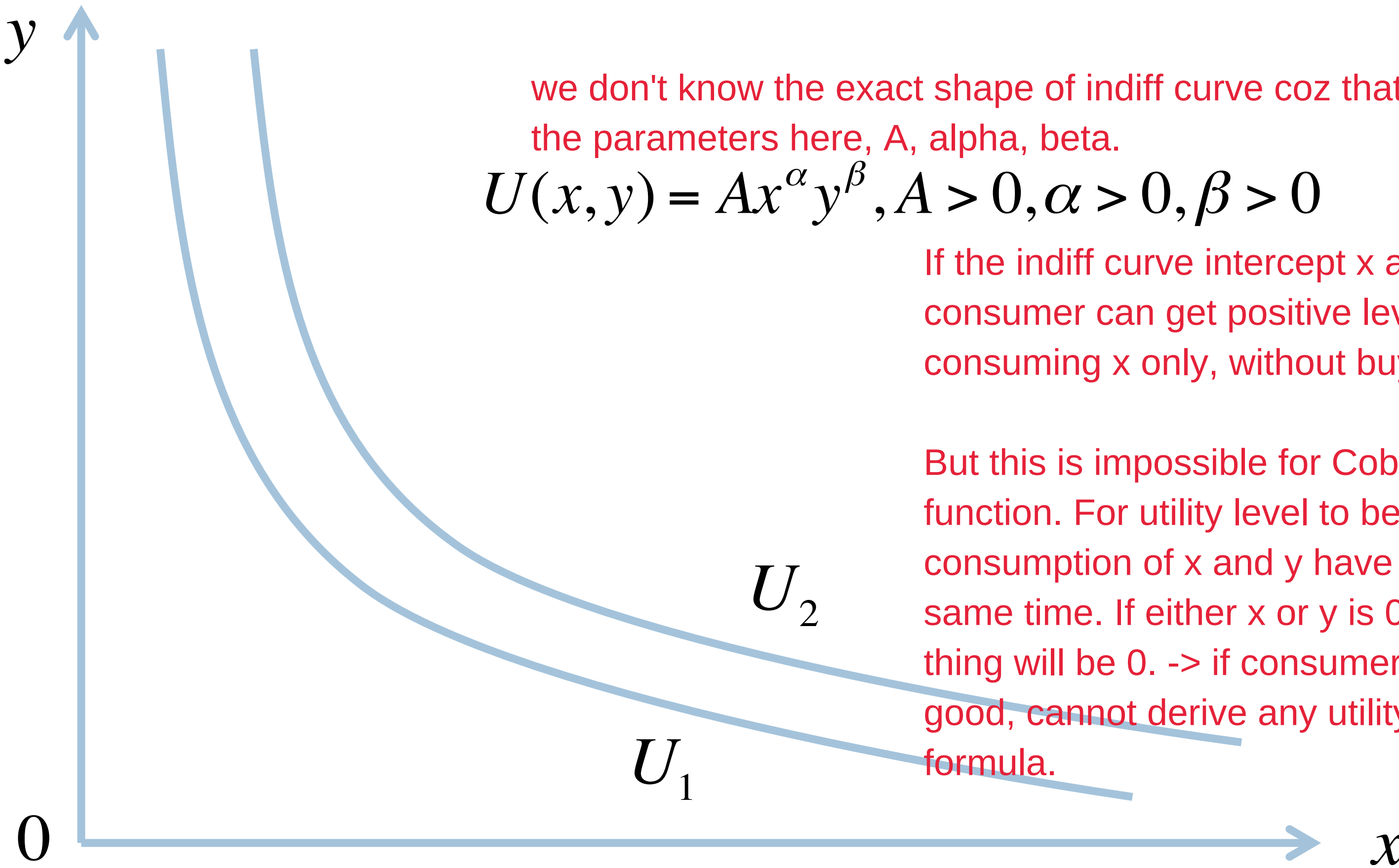
cannot intersect axes

we don't know the exact shape of indiff curve coz that depends on the parameters here, A, alpha, beta.

$$U(x,y) = Ax^{\alpha}y^{\beta}, A > 0, \alpha > 0, \beta > 0$$

If the indiff curve intercept x axis, means the consumer can get positive level of utility by consuming x only, without buying y.

But this is impossible for Cobb-Douglas utility function. For utility level to be +ve, consumption of x and y have to be +ve at the same time. If either x or y is 0, the entire thing will be 0. -> if consumer only buys 1 good, cannot derive any utility from the formula.



since we know 'more is better' holds, higher indiff curve U2 represents higher utility.

Is the principle of diminishing marginal utility satisfied?

Only satisfied when alpha < 1

- Recall the marginal utilities are

$$MU_x = A\alpha x^{\alpha-1}y^{\beta}$$

$$MU_y = A\beta x^{\alpha}y^{\beta-1}$$

- Differentiating MU_x with respect to x, we get

$$\frac{\partial MU_x}{\partial x} = A\alpha(\alpha-1)x^{\alpha-2}y^{\beta}$$

A, alpha, beta, power terms all +ve

- The derivative is negative when $\alpha < 1$
- Marginal utility for Cobb-Douglas utility functions may or may not be diminishing

If try MU y, MU will only diminish in y when beta < 1.

Examples

- Consider the utility function

$$U(x, y) = x^2 y^2$$

- The marginal utilities and the marginal rate of substitution are

$$MU_x = 2xy^2$$

MU x increasing when x increases.

$$MU_y = 2x^2 y$$

$$MRS_{x,y} = \frac{2xy^2}{2x^2 y} = \frac{y}{x}$$

MRS is diminishing when consumer buys more x and less y along the same indiff curve, even though the MU for 2 goods not diminishing.

- Marginal utilities are increasing, not diminishing

Examples Cont'

- Consider the utility function

$$U(x,y) = \sqrt{xy}$$

- The marginal utilities and the marginal rate of substitution are

$$MU_x = \frac{1}{2}\sqrt{\frac{y}{x}}$$

MU x is decreasing in x (consume more x, MU decreases)

$$MU_y = \frac{1}{2}\sqrt{\frac{x}{y}}$$

$$MRS_{x,y} = \frac{y}{x}$$

- Marginal utilities are diminishing

As long as power of x < 1, MU x is diminishing for x

Why do we study Cobb-Douglas Utility Functions?

- Convenient mathematical/economic properties
  - Simple functional form
  - "More is better" satisfied indiff curve downward sloping
  - Diminishing marginal rate of substitution indiff curve convex
  - Indifference curves do not intersect the axes
- What kind of preferences can be represented by a Cobb-Douglas utility function?

No typical example.

We will talk about demand functions, there are certain special properties about the optimal consumption for consumers who have Cobb-Douglas utility function.