LECTURE 3

CONSUMER CHOICE
REVEALED PREFERENCE
INDIVIDUAL DEMAND

Where are we?

- Preference
- Budget constraint
- Consumer's optimal choice
  - The tangency case
  - Other cases where the optimal basket is not a point of tangency
- Revealed preference tell us how we use the consumer choice model to infer consumer preference
  - What if we observe choice but not preference?
- Demand function
  - How does the optimal choice change with prices and income?

Part 1

Consumer Choice

Optimal Choice

- Consumer's optimal choice
  - On the budget line
  - On the highest indifference curve
- The optimal choice is the point of tangency
  - Tangency condition + budget line
  - Or the Lagrangian method
- Optimal basket is not always a point of tangency

What is the optimal basket?

- Suppose the consumer has utility function

$$U(F,C) = FC + 10F$$

its indiff curve is downward sloping and convex

- Price of food is 1, price of clothing is 2, consumer's income is 10
- The utility maximization problem is

$$\max_{F,C} \quad FC + 10F$$

$$s.t. \quad F + 2C = 10$$

What is the optimal basket? Cont'

- The tangency condition is

$$\frac{C+10}{F}=\frac{1}{2}$$

MRS = price ratio

- The budget line is

$$F+2C=10$$

- The solution is *F*=15, *C*=-2.5
- Is it the optimal basket?

Rewriting the Utility Maximization Problem

- In fact, there should be two more constraints to any utility maximization problem
  - The consumption of each good cannot be negative
- The true utility maximization problem is

$$\max_{F,C} \quad FC + 10F$$

$$\begin{aligned} & F + 2C = 10 \\ s.t. \quad & F \geq 0 \\ & C \geq 0 \end{aligned}$$

Solving the Problem

- How to solve this problem?
- Assuming the two constraints are satisfied, we just need to solve

$$\max_{F,C} \quad FC + 10F$$

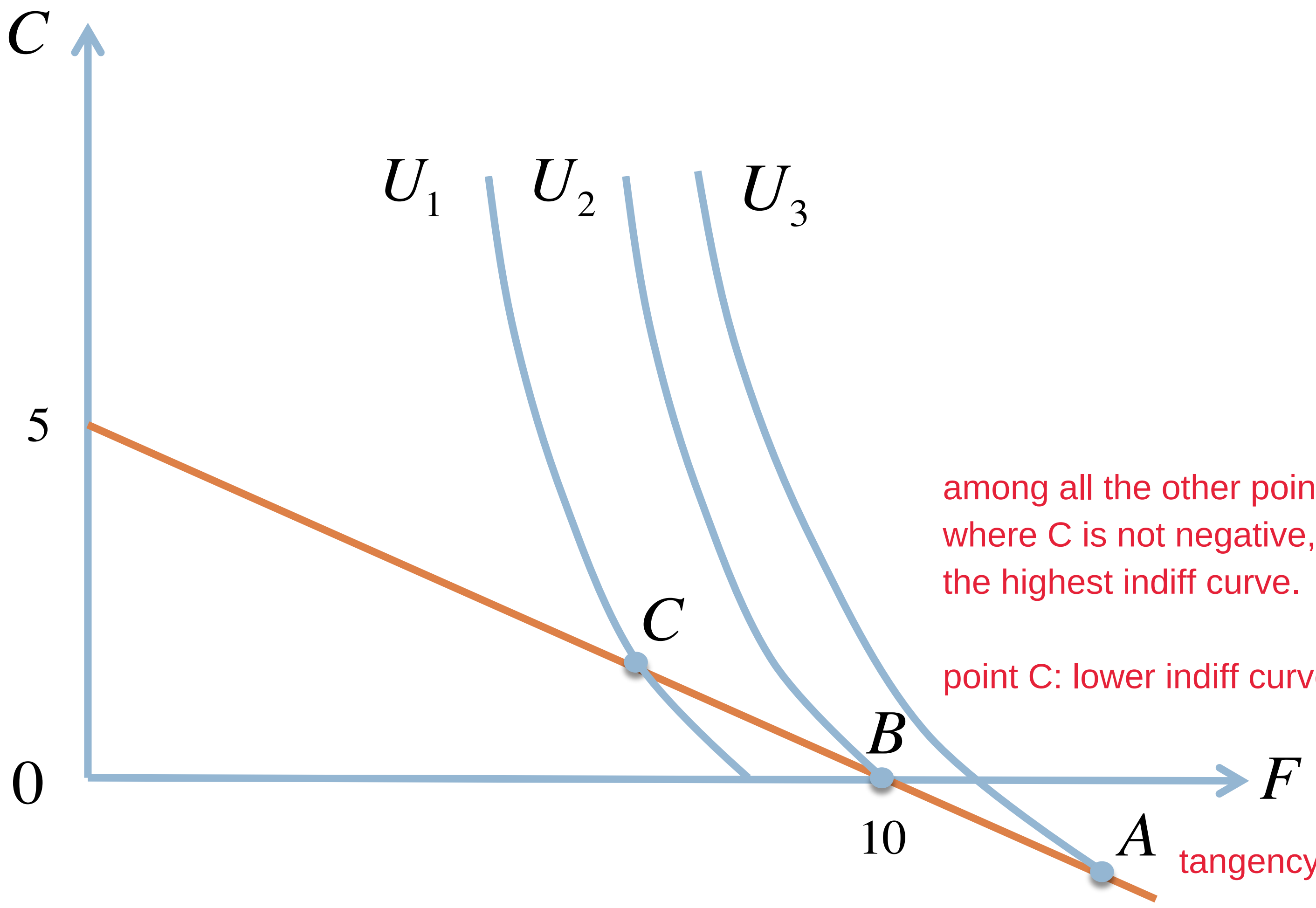
$$s.t. \quad F + 2C = 10$$

- Check if the solution indeed satisfies F>=0 and C>=0
  - If yes, we are done
- The solution F=15, C=-2.5 violates C>=0
  - This means our assumption is wrong

Solving the Problem Cont'

- The consumer wants -2.5 units of clothing
  - As C=-2.5 is not possible, C=0 is the best/closest we can get
- Thus the solution is F=10, C=0
- In this case the constraint C>=0 *binds* meaning: the inequality constraint holds with equality
  - That is, it holds with equality, C=0
- When there are inequality constraints, the constraints may or may not bind
  - In this example, the constraint C>=0 binds while the constraint F>=0 does not bind

The Scenario in Graph



among all the other points along the budget line, where C is not negative, point B is the one that's on the highest indiff curve.

point C: lower indiff curve

tangency point: F=15, C=-2.5

Corner Solution

Your optimal basket may be a corner solution, maybe an interior solution. Depends on preference & budget line.

- At optimal basket, it is *not* always true that both (all) goods are consumed
- Definition 3.1 *Corner solution* is an optimal basket at which the consumption of at least one good is 0
  - Optimal basket either on the horizontal or vertical axis
- Definition 3.2 An optimal basket in which both goods are consumed is an *interior solution*

why interior? if you are buying both goods, then your optimal basket is not going to lie on any of the axes. -> interior of the graph

- At corner solutions
  - Indifference curve may not be tangent to the budget line

At point B, indiff curve steeper than budget line.

Understanding Corner Solution

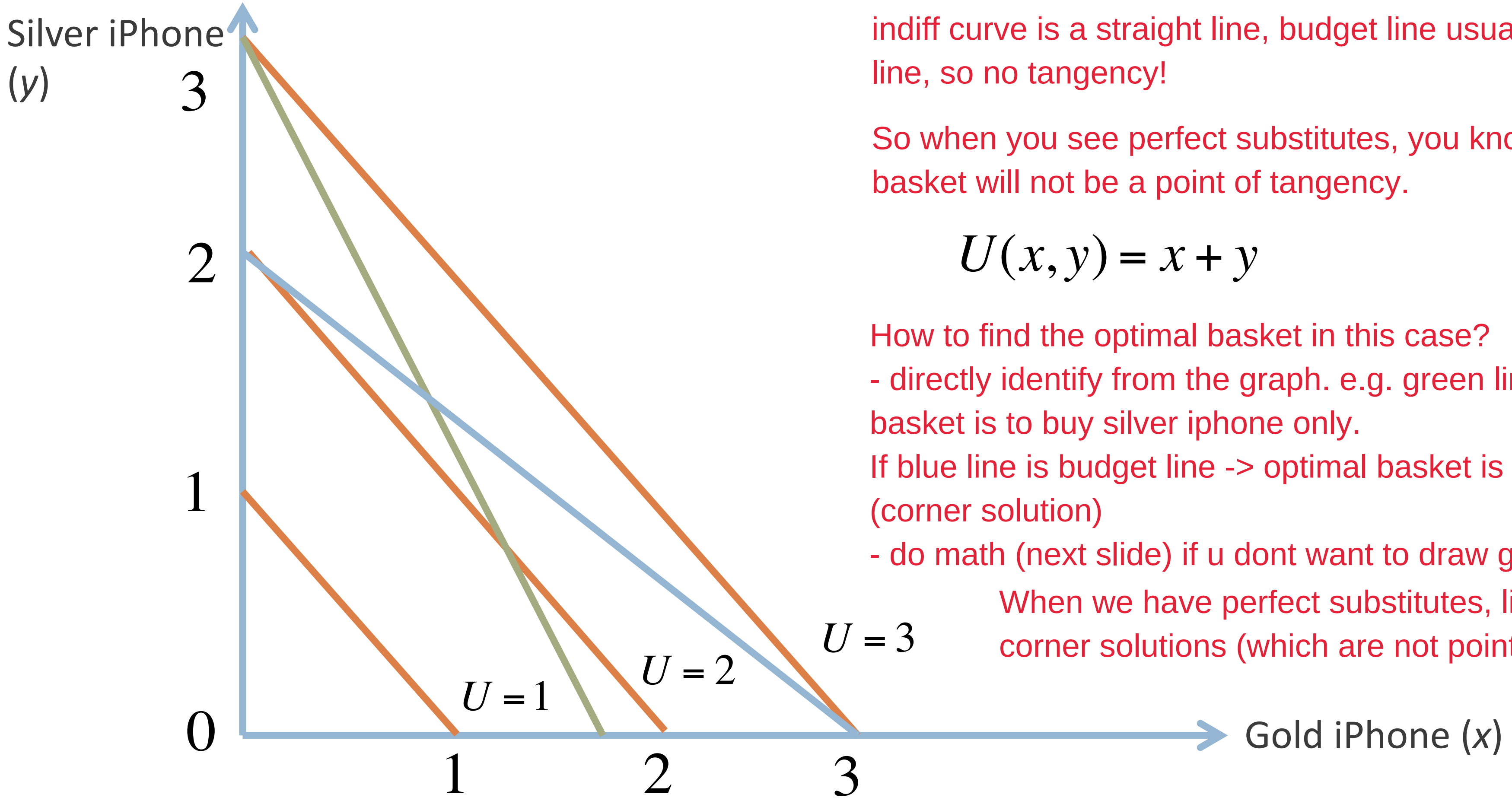
- At point B, consumer spends all the money on food
- At point B   indiff curve steeper than budget line

$$MRS_{F,C} > \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{MU_C} > \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{P_F} > \frac{MU_C}{P_C}$$

per dollar MU for food > that of clothing

- If possible, consumer wants to buy more *F* and less *C* to increase utility   coz $1 spent on food gives higher utility than $1 spent on clothing
- But consumption of *C* is already 0

Corner Solution for Perfect Substitutes: the Graph



indiff curve is a straight line, budget line usually also a straight line, so no tangency!

So when you see perfect substitutes, you know your optimal basket will not be a point of tangency.

$$U(x, y) = x + y$$

How to find the optimal basket in this case?
- directly identify from the graph. e.g. green line is budget line -> optimal basket is to buy silver iphone only.
If blue line is budget line -> optimal basket is to buy 3 gold iphones. (corner solution)
- do math (next slide) if u dont want to draw graph

When we have perfect substitutes, likely we gonna deal with corner solutions (which are not points of tangency).

Corner Solution for Perfect Substitutes: the Math

- From the utility function we know

$$MU_x = MU_y = 1$$

- Suppose P_x=1 and P_y=2

$$\frac{MU_x}{P_x} = 1 > \frac{MU_y}{P_y} = \frac{1}{2}$$

the consumer should buy more x and less y to increase utility, based on the per dollar MU

should only buy x to max utility. why? comparing these 2 per dollar MU, it's not just that MUx/Px is bigger, the 2 numbers are actually constants! Whenever you spend $1 on x, you always get utility 1. Whenever you spend $1 on y, you always get utility 0.5. So it doesn't matter how many x and y you already bought. When you think about how to spend your next dollar, you should always spend on x

  - Consumer only buys *x*

- Suppose P_x=2 and P_y=1

$$\frac{MU_x}{P_x} = \frac{1}{2} < \frac{MU_y}{P_y} = 1$$

In general this is not true. We expect the 2 MU to be functions of x or y, so per dollar MU should also be functions of x and y. This is not the case for perfect substitutes, the 2 per dollar MU never changes.

  - Consumer only buys *y*

Question: what if Px = Py?
Per dollar MU same. so? (tutorial)

Part 2

Revealed Preference

What is revealed preference?

- What we have been doing so far
  - Given preference (indifference curves/utility functions) preference is the most difficult to observe in real life among all.
  - Given budget constraint
  - We can find consumer's optimal choice
- Can we go the other way round?
  - Given budget constraint
  - Given consumer's optimal choice
  - Can we get any information on preference?
- *Revealed preference* is the analysis that enable us to infer preference based on observed prices and choices and income

Strictly Preferred vs. Weakly Preferred

- A is *strictly preferred* to B

$$A \succ B$$

A gives higher utility than B.

- Definition 3.3 A is *weakly preferred* to B if
  - Either

$$A \succ B$$

  - Or

$$A \approx B$$

indifferent, A & B give the same utility

  - We use the notation

$$A \geq B$$

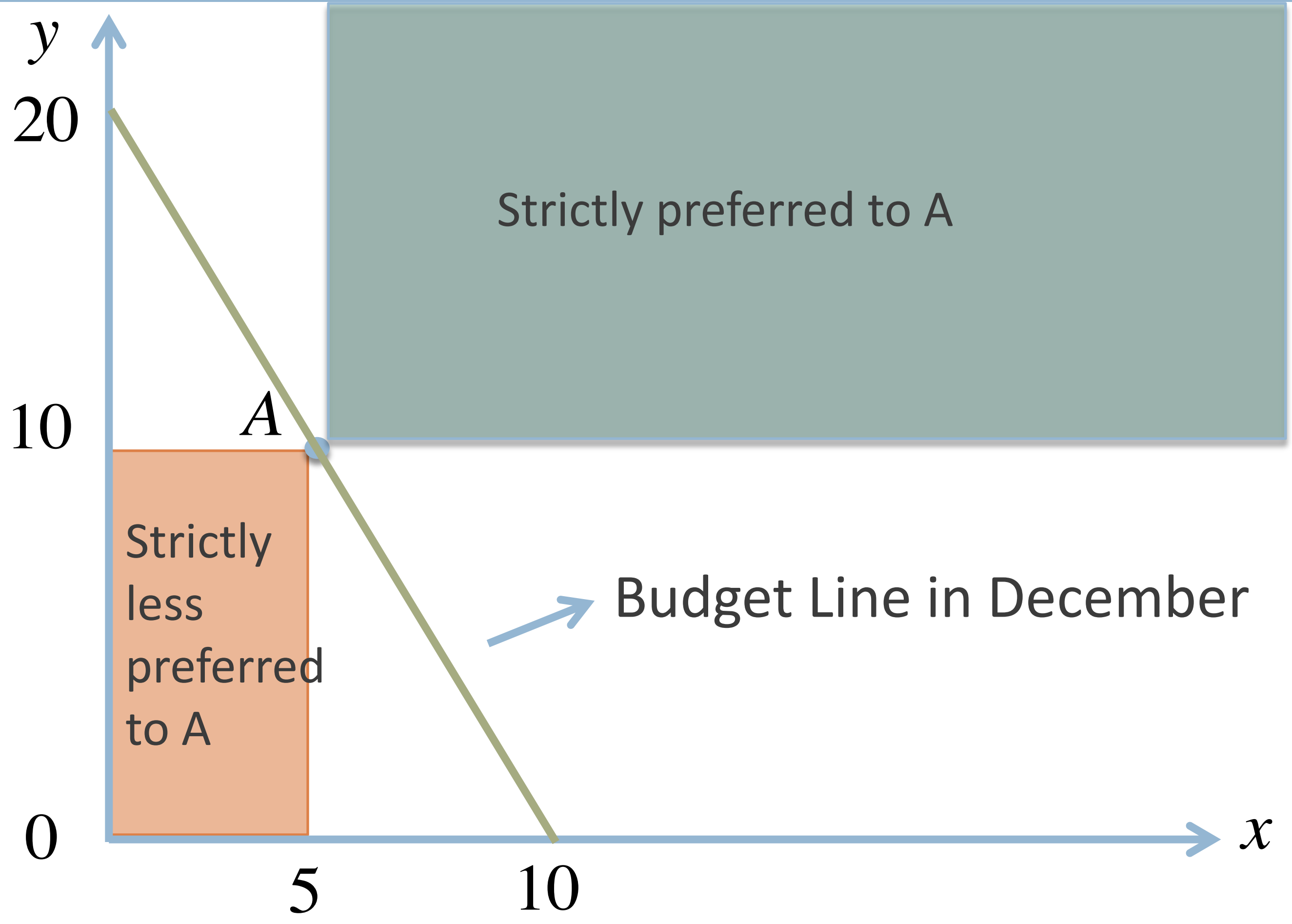
If we already know A is strictly preferred to B, can we tell A is weakly preferred to B? yes. Not the other way round.

From Choice to Preference

- Suppose we observe the budget constraint of a consumer
- We also know the optimal basket chosen given the budget constraint
- But we do not know his preference
  - We know his preference satisfies the three assumptions
  - We also know his preference does not change with prices or income
- Our goal
  - To infer preference – how he ranks different baskets

stable (reasonable coz preference is consumer's taste, which is relatively stable)

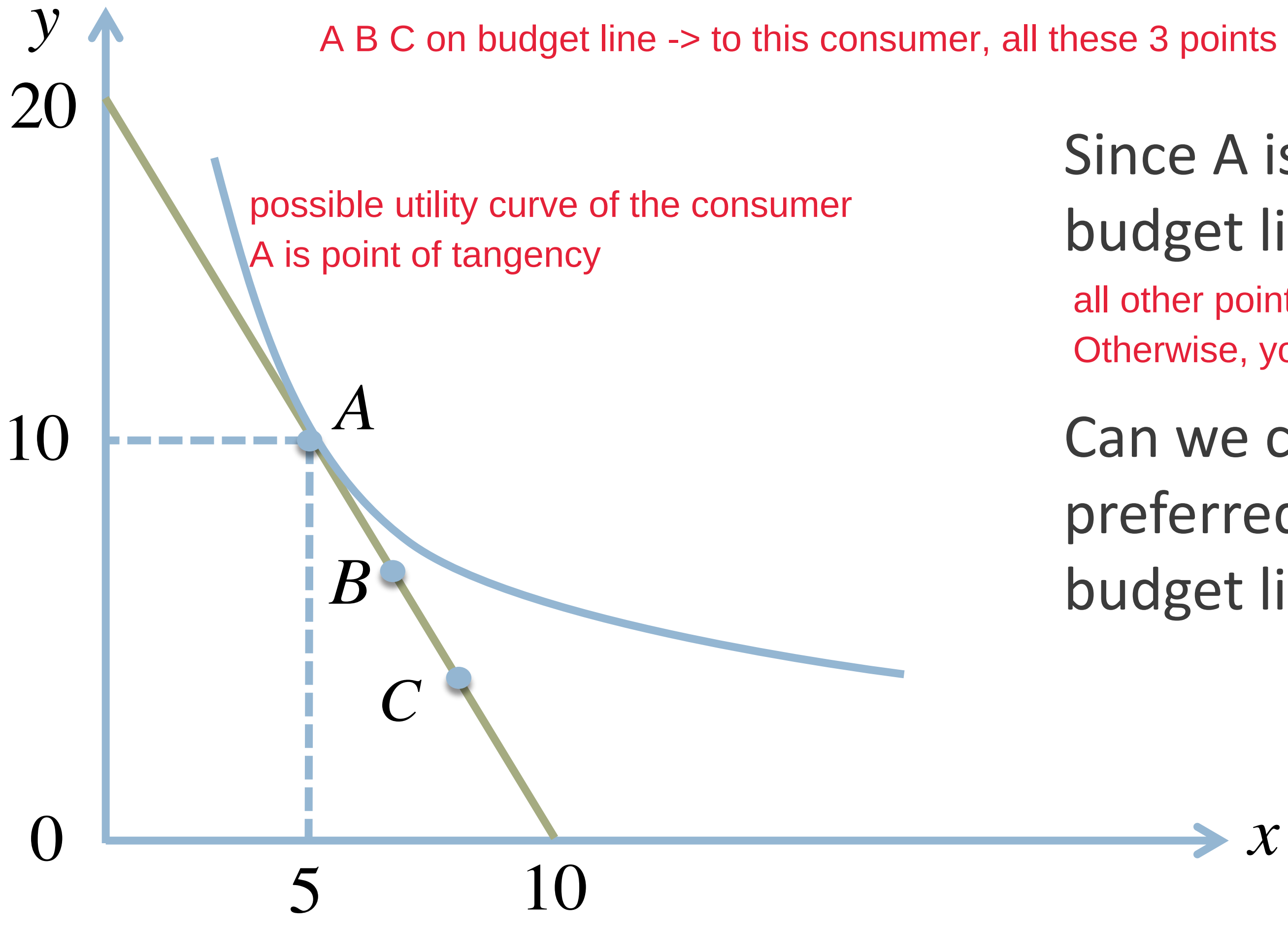
What we already know from “more is better”



Suppose A is the optimal choice in December

We know the budget line and the optimal choice, but we don't know the indiff curve of the consumer. But we assume consumer preference satisfies 'more is better', immediately we know A is strictly preferred to any points in the southwest of A. All the points to the northeast of A are strictly preferred to A.

A vs. Other Points on the Budget Line



A B C on budget line -> to this consumer, all these 3 points are equally expensive (cost consumer the same amt of money).

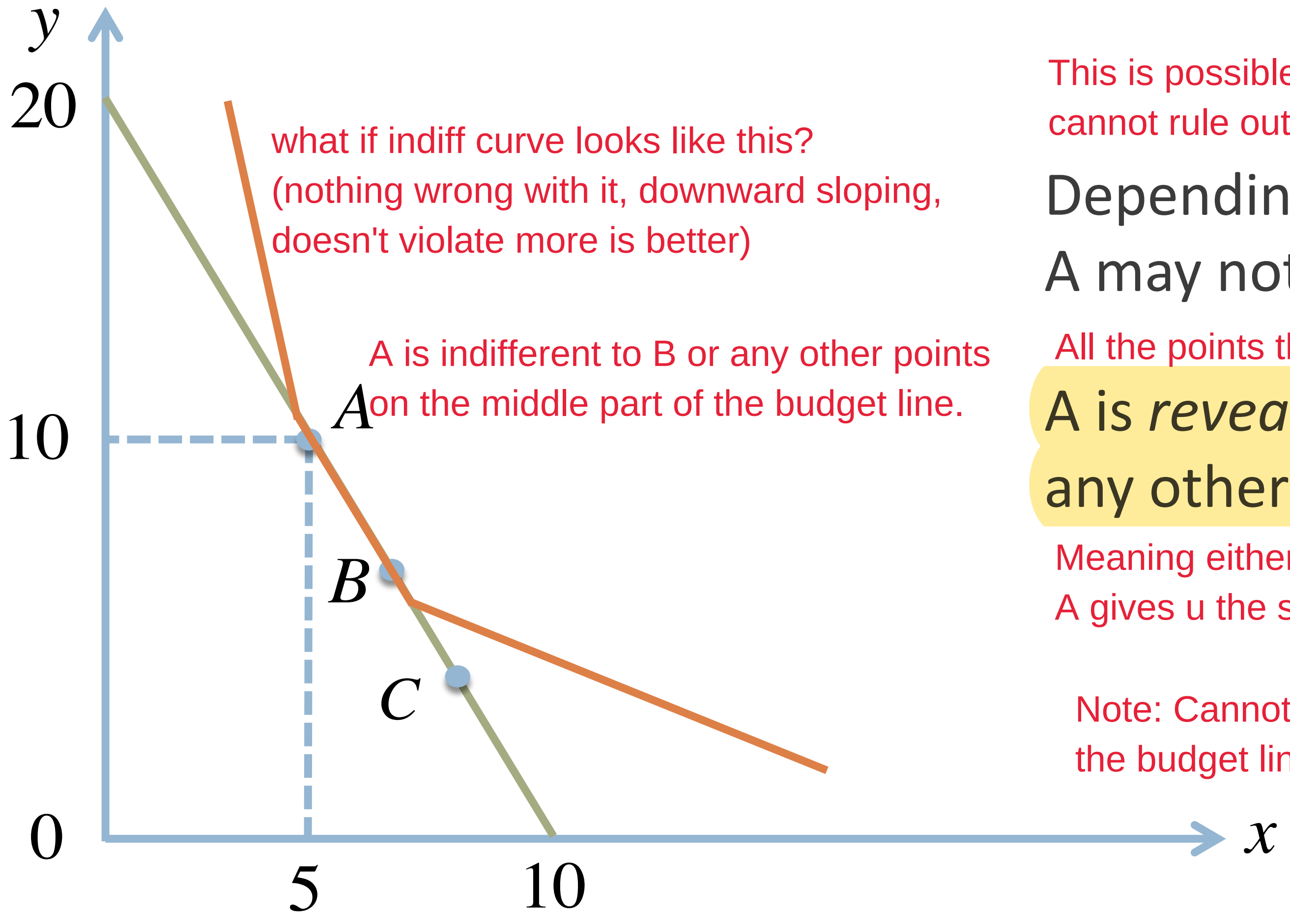
Since A is optimal, no other point on the budget line is strictly preferred to A

all other points on the budget line (besides A) do not give higher utility than A. Otherwise, you will choose other points.

Can we conclude that A is strictly preferred to any other basket on the budget line?

Yes if this is the case (A is point of tangency). But remember you don't know how indiff curve looks like. next slide.

A vs. Other Points on the Budget Line Cont'



This is possible because we don't know how indiff curve looks like, so we cannot rule out this scenario.

Depending on the consumer's preference, A may not be the only optimal basket

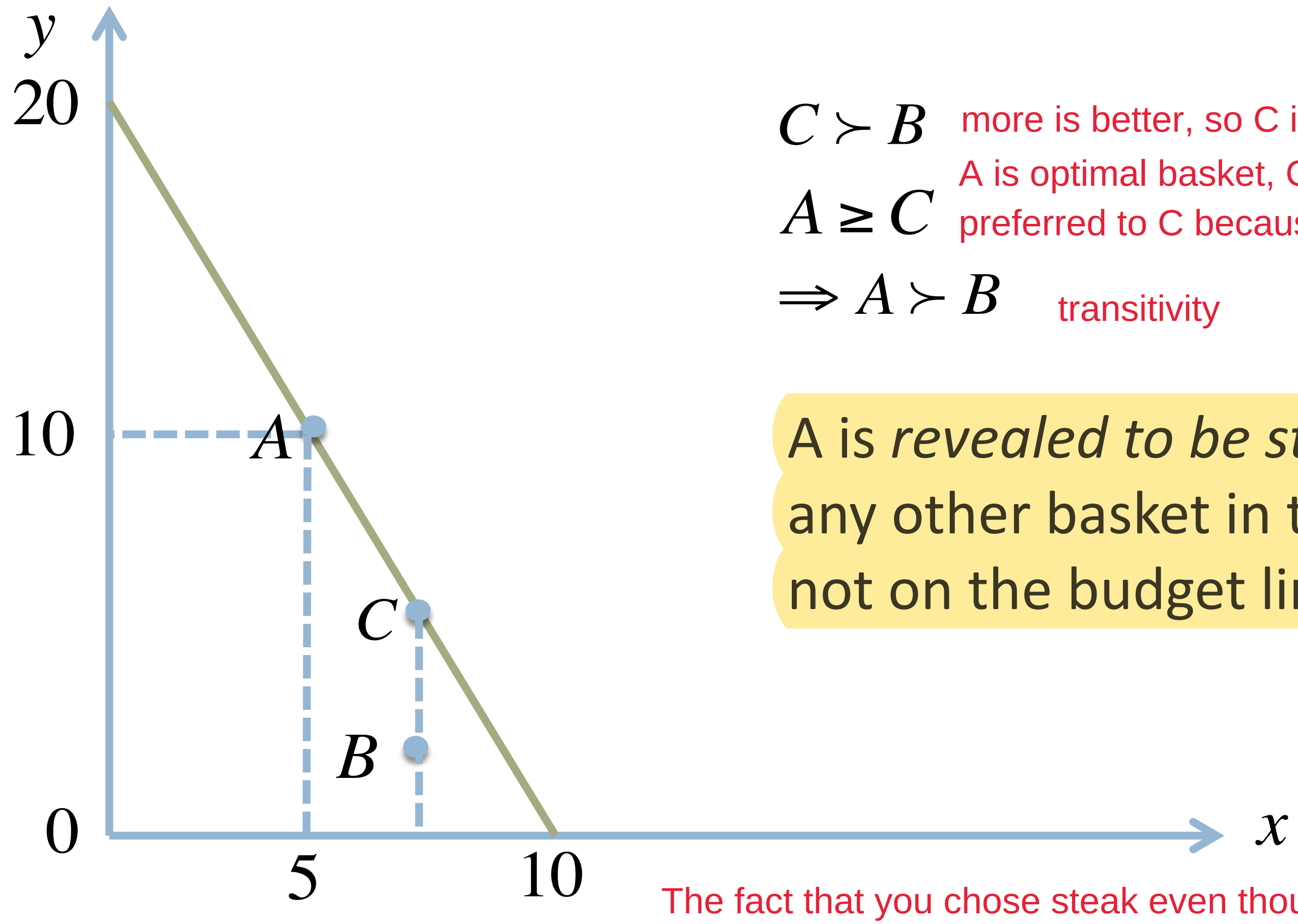
All the points that lie on that middle part of the line are optimal baskets.

A is *revealed to be weakly preferred* to any other basket on the budget line

Meaning either A gives higher utility than another point on the budget line, OR A gives u the same utility.

Note: Cannot conclude A is definitely strictly preferred to any other points on the budget line since there might be diff scenarios (this slide and last slide)

A vs. Other Points below the Budget Line



$C \succ B$ more is better, so C is strictly preferred to B.

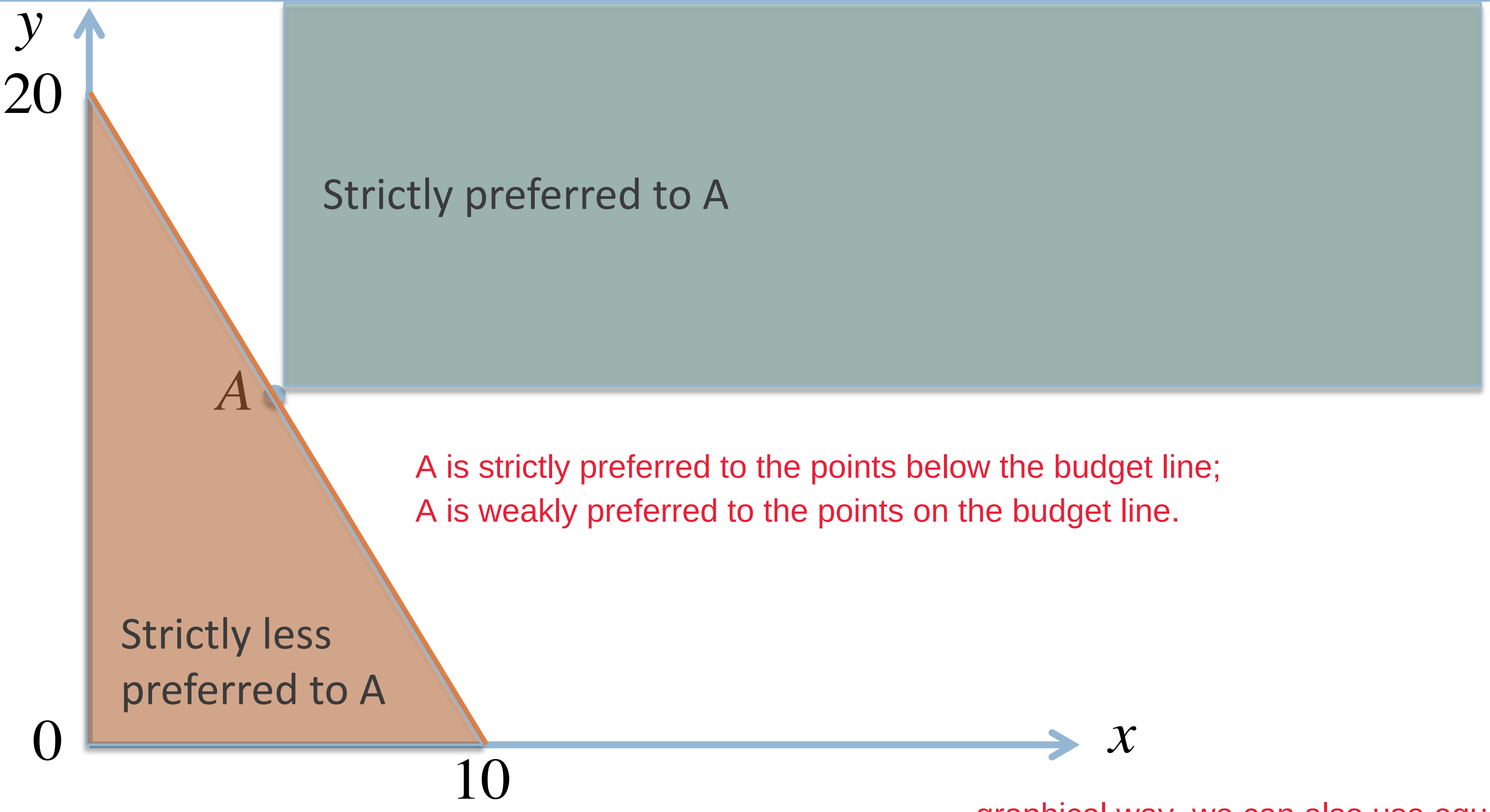
$A \geq C$ A is optimal basket, C is a point on budget line. So A is weakly preferred to C because consumer chose A.

$\Rightarrow A \succ B$ transitivity

A is *revealed to be strictly preferred* to any other basket in the budget set (but not on the budget line) below the budget line

The fact that you chose steak even though steak is more expensive than fish means that u get a higher utility with steak. -> steak is strictly preferred to fish.

How Optimal Choice "Reveals" Preference



A is strictly preferred to the points below the budget line;
A is weakly preferred to the points on the budget line.

graphical way, we can also use equations to say the same thing. See next.

Another Way to Understand Revealed Preference

- Suppose basket A=(x_A, y_A) is the optimal basket given prices P_x, P_y, and income I
  - Basket A must be on the budget line

$$P_x x_A + P_y y_A = I$$

- No other affordable basket is strictly preferred to A otherwise you won't choose A.
- Therefore, if basket B=(x_B, y_B) is strictly preferred to basket A, it must be that

$$P_x x_B + P_y y_B > P_x x_A + P_y y_A = I$$ means this basket has to lie above budget line

Another Way to Understand Revealed Preference Cont’

- Similarly, if basket C=(x_C, y_C) is indifferent to basket A, it must be that

PxXc + PyYc >= PxXa + PyYa = I

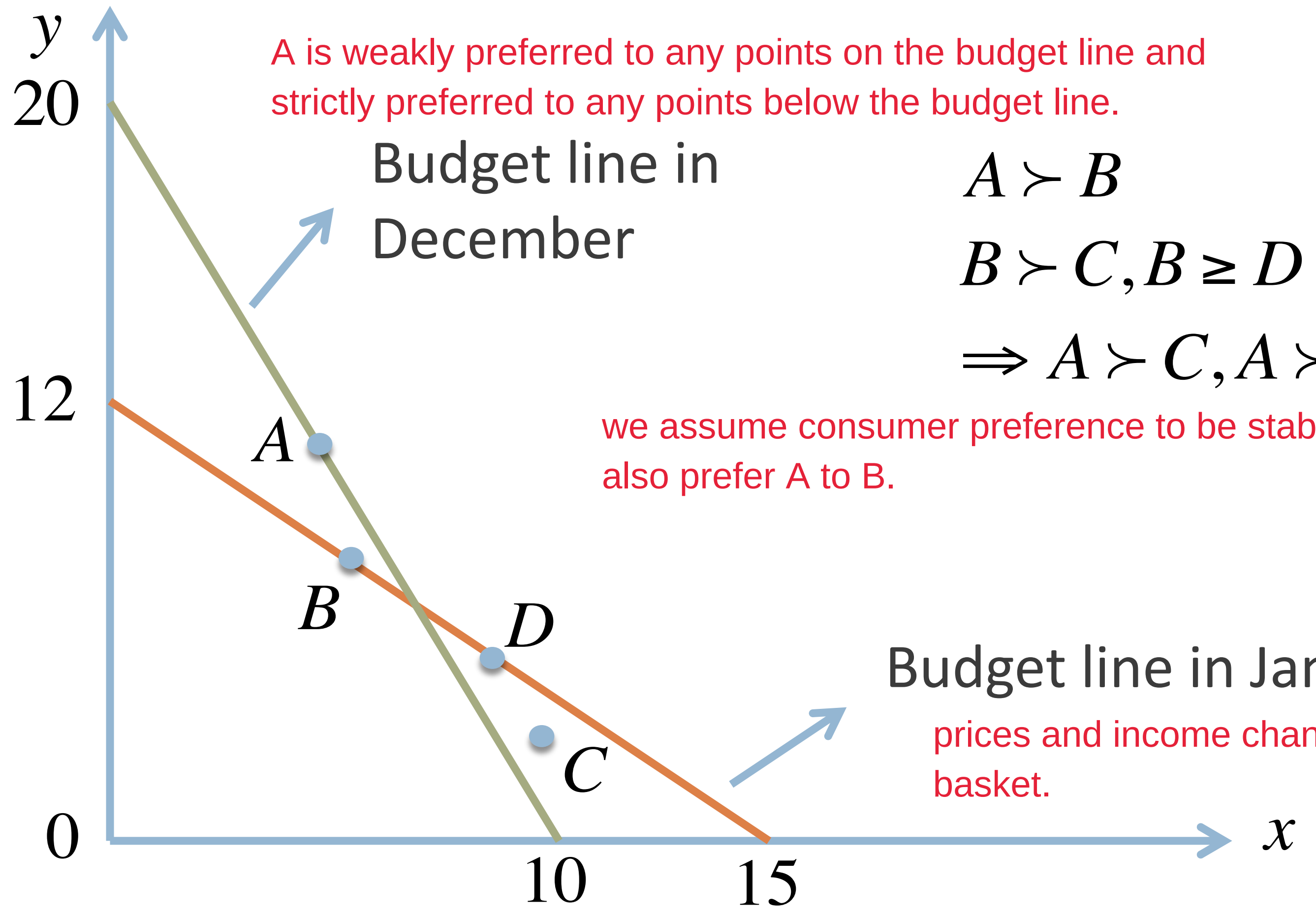
C cannot be below budget line, or else will choose C since C gives u the same utility as A and C is cheaper. So point C cannot be cheaper than A.

Meaning C can lie on the same budget line as A (both optimal), or C can lie above budget line

- To summarize
  - If A is the optimal basket given the budget constraint
  - Any basket that is strictly preferred to A cannot be affordable
  - Any basket that is indifferent to A cannot cost less than A

B is the Optimal Choice in January

A is weakly preferred to any points on the budget line and strictly preferred to any points below the budget line.



$$A \succ B$$

$$B \succ C, B \geq D$$

$$\Rightarrow A \succ C, A \succ D$$ transitivity

we assume consumer preference to be stable. -> so if in Dec u prefer A to B, in Jan u also prefer A to B.

prices and income change, now let's say B is the new optimal basket.

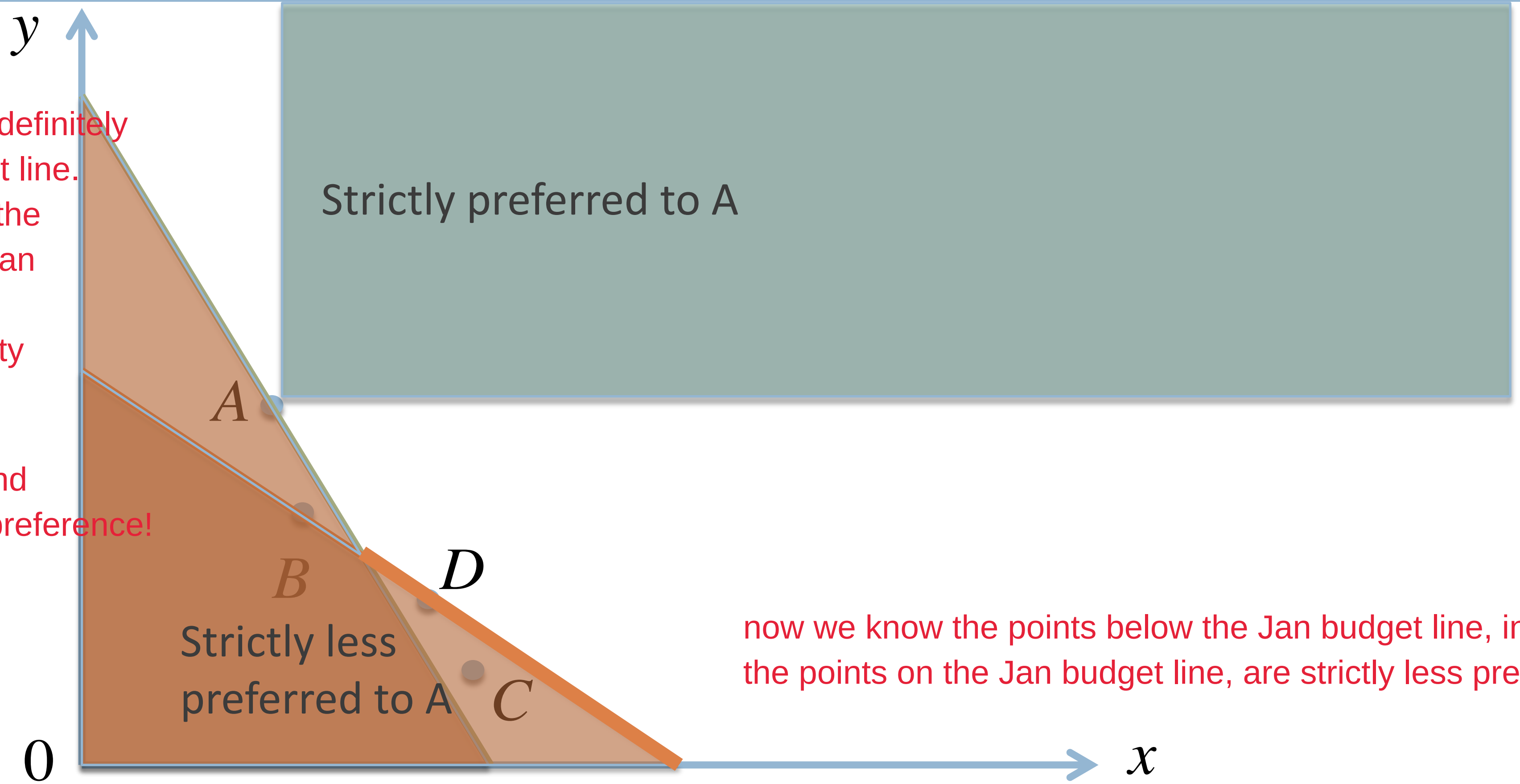
more preference is revealed now (with only Dec budget line, won't be able to compare A with C and D)

More Choices Observed, More Information Revealed on Preference

Revealed preference does not allow u to compare your optimal choice with an unaffordable basket unless that basket is to the northeast (more is better).

Note: A point on the budget line definitely preferred to a point below budget line. Wrong! Can only say a point on the budget line is more expensive than a point below the budget line, doesn't mean it'll give higher utility than a point below budget line.

We need both optimal choice and budget line to know consumer preference!



now we know the points below the Jan budget line, including the points on the Jan budget line, are strictly less preferred to A.

If you get even more budget lines, and more optimal choices, we can uncover more info on consumer preference.

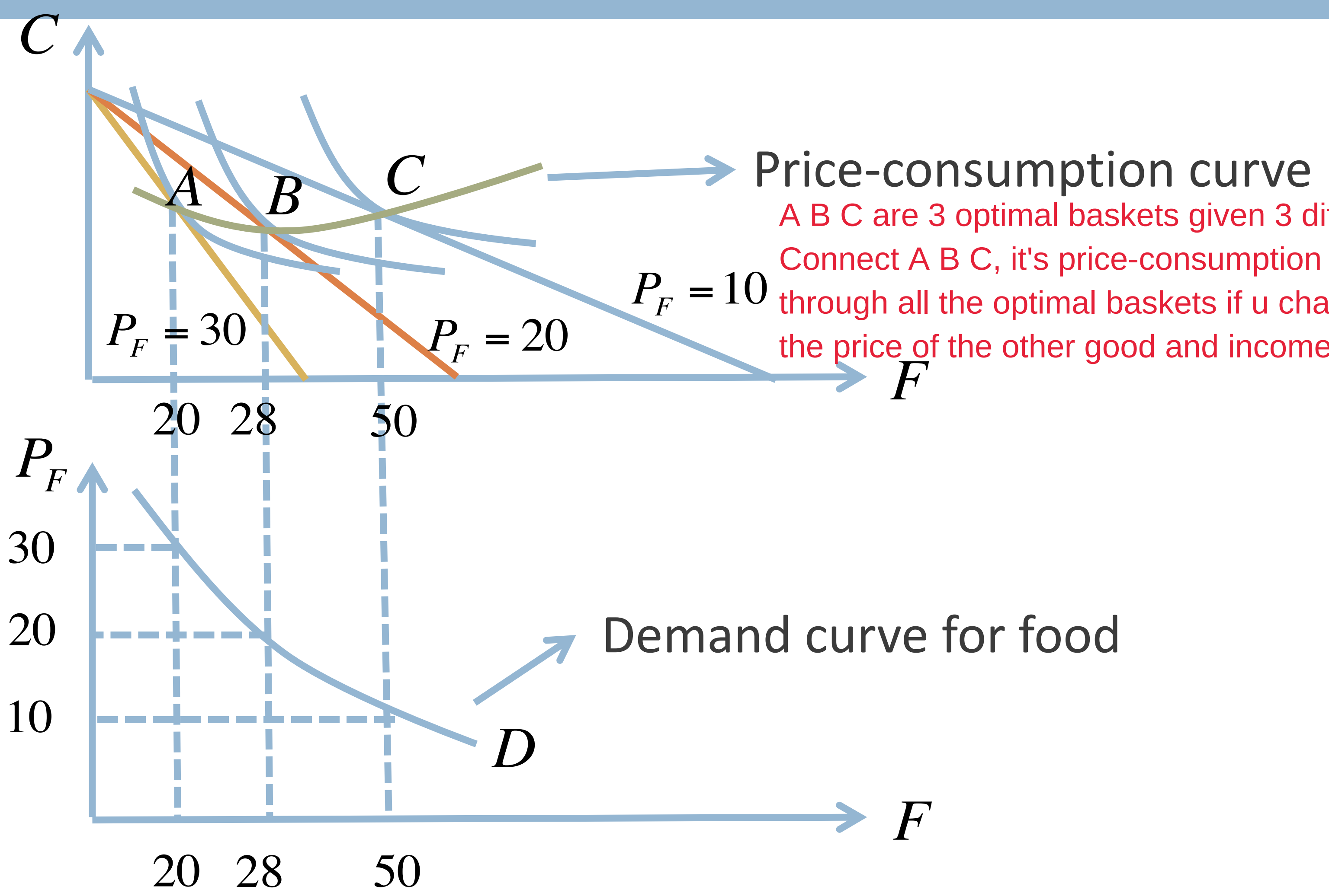
Part 3

Individual Demand

From Optimal Baskets to Individual Demand Curve

- Assume the consumer chooses food and clothing
- Suppose the price of food changes
  - The price of clothing and income are fixed
- How does the optimal basket change?
  - In particular, how does the consumption of food change?
- An individual consumer’s demand curve for food captures the relationship between the optimal consumption of food and the price of food for that consumer

Example: Demand Curve for Food in Graph



A B C are 3 optimal baskets given 3 diff budget lines.
Connect A B C, it's price-consumption curve -> a curve that goes through all the optimal baskets if u change the price of 1 good, holding the price of the other good and income constant.

Demand Curve

- Definition 3.4 A consumer's *demand curve* for a good is the optimal consumption of the good as a function of its price
  - Holding all other factors fixed

    income & price of the other good
- Law of demand
  - Demand curve is downward sloping
  - Higher price, lower quantity demanded

Example: Deriving Demand Curve

- Suppose the consumer has utility function

$$U(F,C) = FC$$

- Suppose price of clothing is 2, income is 10
- What is the demand curve for food?
- The consumer solves

consumer is still maximising utility. Just that now we are maximising utility at diff prices of food. Price of food is a variable now.

$$\max_{F,C} \quad FC$$

$$s.t. \quad P_F F + 2C = 10$$

Example: Deriving Demand Curve Cont'

- Tangency condition

$$\frac{P_F}{2} = \frac{C}{F}$$

- Or

$$P_F F = 2C$$

- Budget line

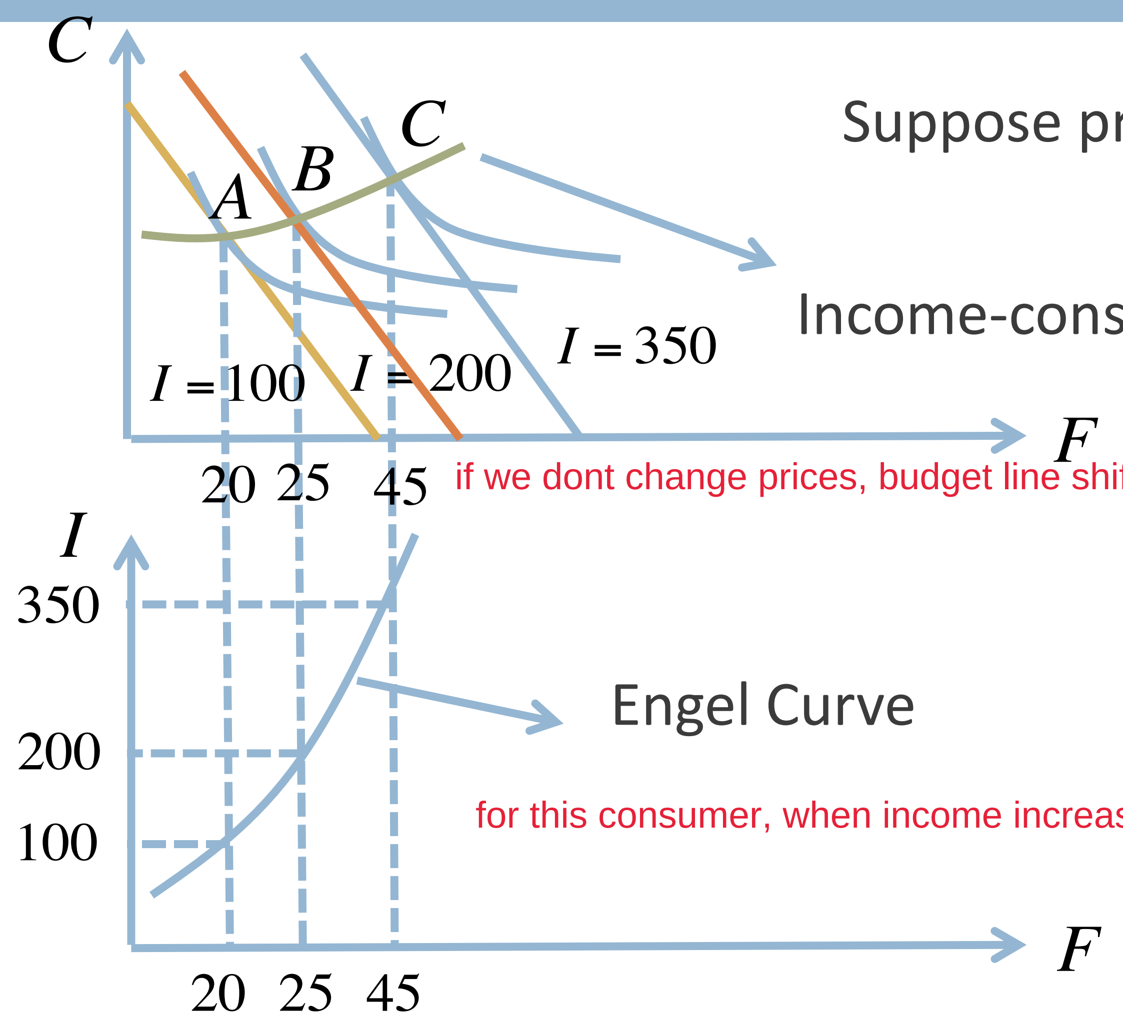
$$P_F F + 2C = 10$$

need to get rid of C

- Demand curve for food is $F = \dfrac{5}{P_F}$

What if income changes?

relationship between income and consumption



Suppose prices are fixed

Income-consumption curve

connect A B C, it's income-consumption curve, shows us how optimal basket moves as we change consumer income without changing any prices.

if we dont change prices, budget line shifts in a parallel way.

Engel Curve

for this consumer, when income increases, he buys more food.

Engel Curve

- Definition 3.5 A consumer's *Engel curve* of a good is the curve that shows the relationship between income and optimal consumption
  - Holding other factors fixed
- Definition 3.6 If the good is a *normal good*
  - Engel curve is upward sloping 5-star hotels
- Definition 3.7 If the good is an *inferior good*
  - Engel curve is downward sloping sth u buy less when u are richer

e.g. budget hotel

Demand Function

- Quantity demanded (optimal consumption) depends on
  same meaning
  utility-maximizing amount of the good u buy
  - Price of the good demand curve
  - Income engel curve
  - Prices of other goods
- Can we write down a general formula?
  - Quantity demanded as a function of all parameters (income and all prices)
- Definition 3.8 A consumer's *demand function* for a good is quantity demanded as a function of income and all prices

Demand Function for Cobb-Douglas Utility Function

- The consumer solves

$$\max_{x,y} \quad Ax^{\alpha}y^{\beta}$$

$$s.t. \quad P_x x + P_y y = I$$

now we don't have numbers, we have Px, Py and I as variables.

- The tangency condition is

MRS $$\frac{\alpha y}{\beta x} = \frac{P_x}{P_y}$$

- Tangency condition can be written as

if u are looking for demand function for x, what variables do u want to get rid of? y.

$$P_y y = \frac{\beta}{\alpha} P_x x$$

Demand Function for Cobb-Douglas Utility Function Cont'

- Plugging into the budget line

$$P_x x + \frac{\beta}{\alpha} P_x x = I$$

- Thus the demand function for *x* is

$$x = \frac{\alpha}{\alpha + \beta} \times \frac{I}{P_x}$$

x is normal good. coz when income increases, holding price constant, x will increase.

- And the demand function for *y* is

$$y = \frac{\beta}{\alpha + \beta} \times \frac{I}{P_y}$$

Properties of Cobb-Douglas Utility Function

- Demand for one good does not depend on the price of the other good

this is true for Cobb-Douglas utility function, not generally true

seen from equations.

- Consumer always spends a fixed proportion of income on each good
  - The total expenditure on *x* is

No matter how Px, Py and income changes, the proportion of income spent on each good never changes.

$$P_x x = P_x \times \frac{\alpha}{\alpha+\beta} \times \frac{I}{P_x} = \frac{\alpha I}{\alpha+\beta}$$

alpha & beta are powers in the utility function, constant number.

  - The total expenditure on *y* is

$$P_y y = P_y \times \frac{\beta}{\alpha+\beta} \times \frac{I}{P_y} = \frac{\beta I}{\alpha+\beta}$$

real-life example that can be represented by Cobb-Douglas utility function?