LECTURE 6
FIRST WELFARE THEOREM
WALRAS' LAW
MIDTERM REVIEW

Where are we?

- Edgeworth box
- Pareto efficiency
- Competitive equilibrium
- First (and second) welfare theorems
  - What is the relationship between Pareto efficiency and competitive equilibrium?
- Walras' Law
  - A result derived from budget lines and optimal baskets
- Midterm Review

Part 1

First Welfare Theorem

General Competitive Equilibrium

- A pair of prices (P_1, P_2) constitutes a (general) *competitive equilibrium* if at the prices
  - Each consumer maximizes his/her utility given the budget constraint

$$x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$$

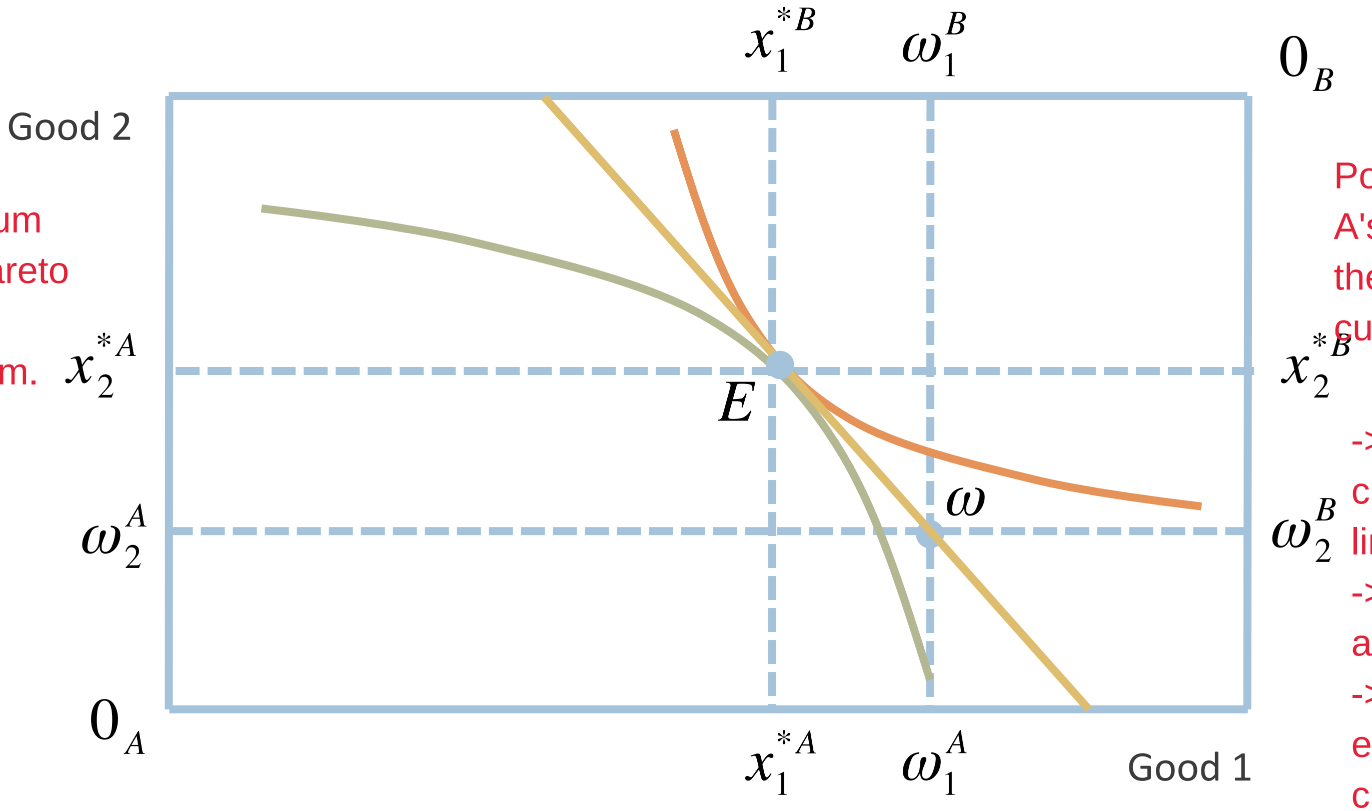
denotes the optimal consumption for each consumer given the equilibrium prices

  - Markets for both goods clear

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$
$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$$

Competitive Equilibrium in Graph

General conclusion:
As long as u have an equilibrium allocation, that allocation is Pareto efficient.
-> result of first welfare theorum.



Point E is the point of tangency between A's budget line and indiff curve. It is also the tangency point between B's indiff curve and budget line.

-> At point E, the 2 consumers' indiff curves are tangent to the same budget line.
-> without budget line, the 2 indiff curves are tangent to each other.
-> E (equilibrium allocation) is pareto efficient. It is a point on the contract curve.

At point E, the two consumers' indifference curves are tangent to each other

First Welfare Theorem

- Definition 6.1 The *First Fundamental Theorem of Welfare Economics* states that a competitive equilibrium allocation is Pareto efficient
  - Suppose the equilibrium prices are (P_1, P_2) and the allocation

$$x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$$

is the allocation given the equilibrium prices

  - Then the allocation

$$x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$$

is Pareto efficient

when u look for equilibrium allocation in the edgeworth box, u don't need to consider every possible point in the box, u only need to consider points that are on the contract curve. Because the 1st welfare theorem tells u that the equilibrium allocation has to be pareto efficient. (cannot be a point that is not on the contract curve.)
If a point is not on the contract curve, by definition, it means it's not pareto efficient.

Does it hold for all possible utility functions? or is there any assumption behind the theorem?
We do have a very weak assumption on consumer preference. (called locally nonsatiated, not in EC2101.)
As long as preference satisfies this condition, 1st welfare theorem holds.

we can prove this result without knowing what utility function (preference) the consumer has.

Proof of First Welfare Theorem

- Suppose at the equilibrium prices P_1 and P_2, the equilibrium allocation is $x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$ we want to show this allocation is pareto efficient.
- Proof by contradiction: suppose this allocation is not Pareto efficient
- Then there must exist another feasible allocation

$$y_1^A, y_2^A, y_1^B, y_2^B$$

  - where at least one consumer is better off
  - and no one is worse off
  - compared to the equilibrium allocation

Proof of First Welfare Theorem Cont'

assume consumer A is the one that is better off -> y basket gives A higher utility than x basket.

u can assume the other way round.

- Suppose consumer A strictly prefers $(y^A{}_1, y^A{}_2)$ to $(x^{*A}{}_1, x^{*A}{}_2)$ while consumer B weakly prefers $(y^B{}_1, y^B{}_2)$ to $(x^{*B}{}_1, x^{*B}{}_2)$

consumer B can be better off or indiff.
basket y either gives B higher or equal utility than basket x*.

- By definition, the equilibrium allocation is the utility-maximizing basket for each consumer given the budget constraint, thus by revealed preference,

Given P1 P2(budget line), optimal basket for A is x*1A, x*2A.
From revealed preference, if x* is optimal basket and if u have another basket that is strictly preferred to the optimal basket, then this y basket must not be affordable to consumer A.

RHS: consumer's income
w: endowment

$$P_1 y_1^A + P_2 y_2^A > P_1 \omega_1^A + P_2 \omega_2^A \qquad (1)$$

If consumer A can afford basket y given the budget line, and y gives A higher utility than x*, then x* won't be the optimal basket for A.

P1y1B + P2y2B >= P1w1B + P2w2B (2)

For B, y basket cannot be cheaper than x* basket.
-> cannot lie below B's budget line (can lie above or on B's budget line). Or else x* is not optimal, consumer will choose y coz it's cheaper and gives higher or same utility as x*.

As x* is equilibrium allocation, optimal basket for A, so this basket lies on the budget line -> cost of this basket = consumer income (RHS)

But consumer A and B's budget line are the same line in edgeworth box!
-> contradiction! These 2 eqns cannot hold at the same time.

Proof of First Welfare Theorem Cont'

This is a general result as we are not relying on any assumption for consumer preference. We don't require diminishing MRS, we don't need it to be a tangency point.

- Add up (1) and (2), we have

$$P_1(y_1^A + y_1^B) + P_2(y_2^A + y_2^B) > P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B) \quad (3)$$

- Allocation $y_1^A, y_2^A, y_1^B, y_2^B$ must also be feasible

$$y_1^A + y_1^B = \omega_1^A + \omega_1^B$$

$$y_2^A + y_2^B = \omega_2^A + \omega_2^B$$

Eqns 1 and 2 contradicts the fact that y is a feasible allocation. As long as y is a feasible allocation, it cannot lie above A's budget line, and at the same time lie on or above B's budget line.

- Substituting into (3), we have a contradiction

$$P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B) > P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B)$$

Hence, equilibrium allocation x* has to be Pareto efficient.

The Invisible Hand

- Each consumer maximizes his/her own utility

consumer doesn't care abt society, only interested in maximising his own utility.

- No central planner

nobody tells us ' look that's pareto efficient allocation, let's move our way to it!

- Yet competitive market leads to a Pareto efficient allocation

if we just let the market allocation the resources.

The invisible hand is the competitive market. As long as we have a competitive market, and we let the market allocate the resources, we will reach the equilibrium that is pareto efficient.

Implication of First Welfare Theorem

- How should we allocate limited resources in the economy?
  - E.g., land, masks, hand sanitizers
- What is a "good" way to allocate resources?
  - There are many ways to define "good"
  - Let's suppose "good" means Pareto efficient

How can we allocate resources in an efficient way?

- Is there a mechanism we can rely on to allocate resources efficiently?

no matter what resources we are trying to allocate and doesn't matter how many ppl we have in the economy, as long as we use the mechanism, always can achieve pareto efficiency.

  - Yes, FWT tells us that we just need to create a competitive market and the market will allocate resources efficiently

As long as we have a competitive market, we are going to reach equilibrium in the market, and the equilibrium will be pareto efficient.

Comments on First Welfare Theorem

limitations of FWT

- It only holds in competitive markets
  - Not true if consumers or firms have price setting power
    e.g. monopoly / oligopoly market, then the equilibrium might not be efficient.
  - Not true if there is externality
  - Not true if there is asymmetric information

FWT doesn't tell u that markets allocate resources efficiently, only tells u that competitive markets allocate resources efficiently.

- Efficiency does not mean equity
  - A Pareto efficient allocation may or may not be an equitable allocation
    fair
  - E.g., one consumer has everything and the other consumer has nothing can be Pareto efficient
    in society, this allocation is undesirable.

e.g. tutorial question 3: as long as more is better holds for both goods for both consumers, the allocation where 1 consumer has everything and the other has nothing is pareto efficient.

Pareto Efficiency vs. Competitive Equilibrium

- Pareto Efficiency
  - An allocation where it is impossible to make someone better off without making someone else worse off
  - Does not depend on prices
  - Does not depend on endowment

You don't need to know prices and endowments to know whether an allocation is pareto efficient or not.

exactly which point in the edgeworth box is endowment allocation.

- Competitive Equilibrium
  - A pair of prices such that
    - Markets clear
    - Everyone maximizes utility given budget constraint
  - Depend on endowment
    - Endowment allocation (and prices) determine budget constraints

need to know what prices we need for consumers to consume at equilibrium

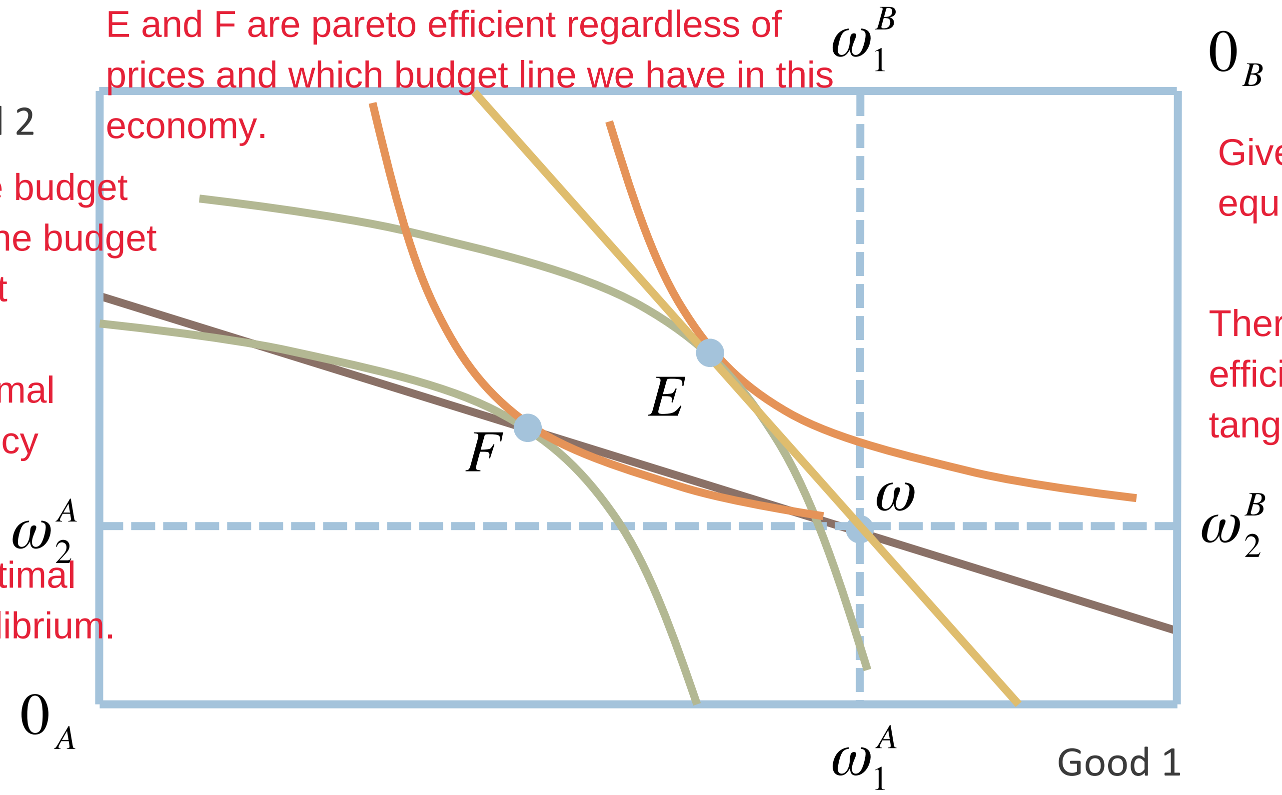
If we move the endowment from 1 point to another in the box, that gonna change the budget line of consumers, and therefore change the optimal basket.

E and F are Always Pareto Efficient Regardless of Prices

We don't need to know where the budget line is, what prices are in this economy for the 2 goods.
E and F are pareto efficient regardless of prices and which budget line we have in this economy.

Suppose F is equilibrium, F must lie on the budget line, so the budget line must be the brown line (coz the budget line has to go through endowment allocation).
With this budget line, F is not optimal basket for A coz F is not a tangency point. Similar for consumer B.

Given this budget line, F is not optimal basket for A or B, so F is not equilibrium.



Given endowment allocation w, E is equilibrium, also pareto efficient by FWT.

There are many other points that are pareto efficient in the edgeworth box. Every tangency point is pareto efficient, such as F.

Question: Is F also an equilibrium allocation?

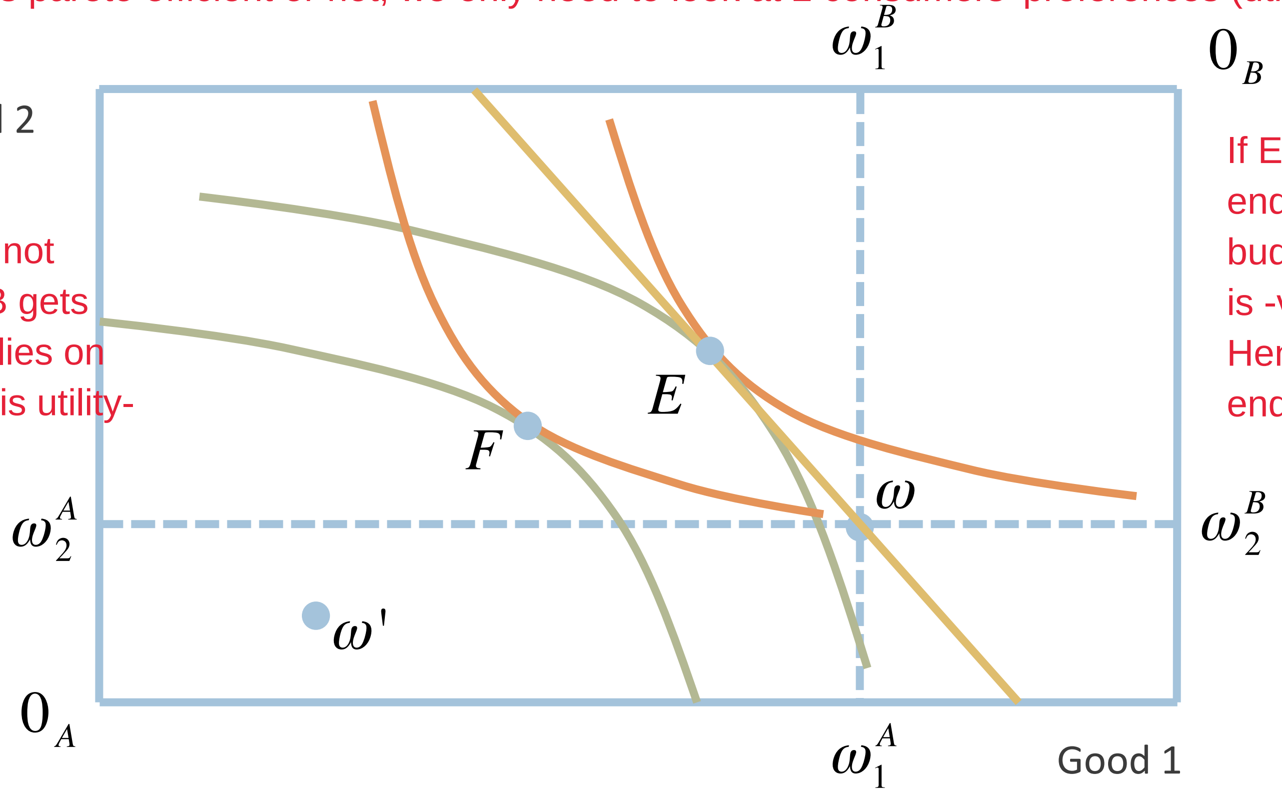
A Pareto efficient allocation may not be an equilibrium allocation

Whether a point is pareto efficient or not is a separate question from whether we can achieve this point in equilibrium.

E and F are Always Pareto Efficient Regardless of Endowment

! When we think abt whether E is pareto efficient or not, we only need to look at 2 consumers' preferences (utility functions)

Can also think from consumer B's perspective. Endowment at w', will not consume at E or F coz consumer B gets higher utility at w' than E/F (coz w' lies on higher indiff curve). No way E or F is utility-maximizing for B (optimal basket)



If E is equilibrium, budget line goes through endowment w' and E. Upward sloping budget line, means price of one of the goods is -ve, doesn't make sense.
Hence E or F cannot be equilibrium if endowment is at w'.

E or F is not the equilibrium allocation is the endowment is at point *w'*

When endowment change from w to w', E is no longer equilibrium. -> so equilibrium allocation depends on where the endowment allocation is.

For pareto efficiency, endowment is irrelevant.

Second Welfare Theorem

Not tested.

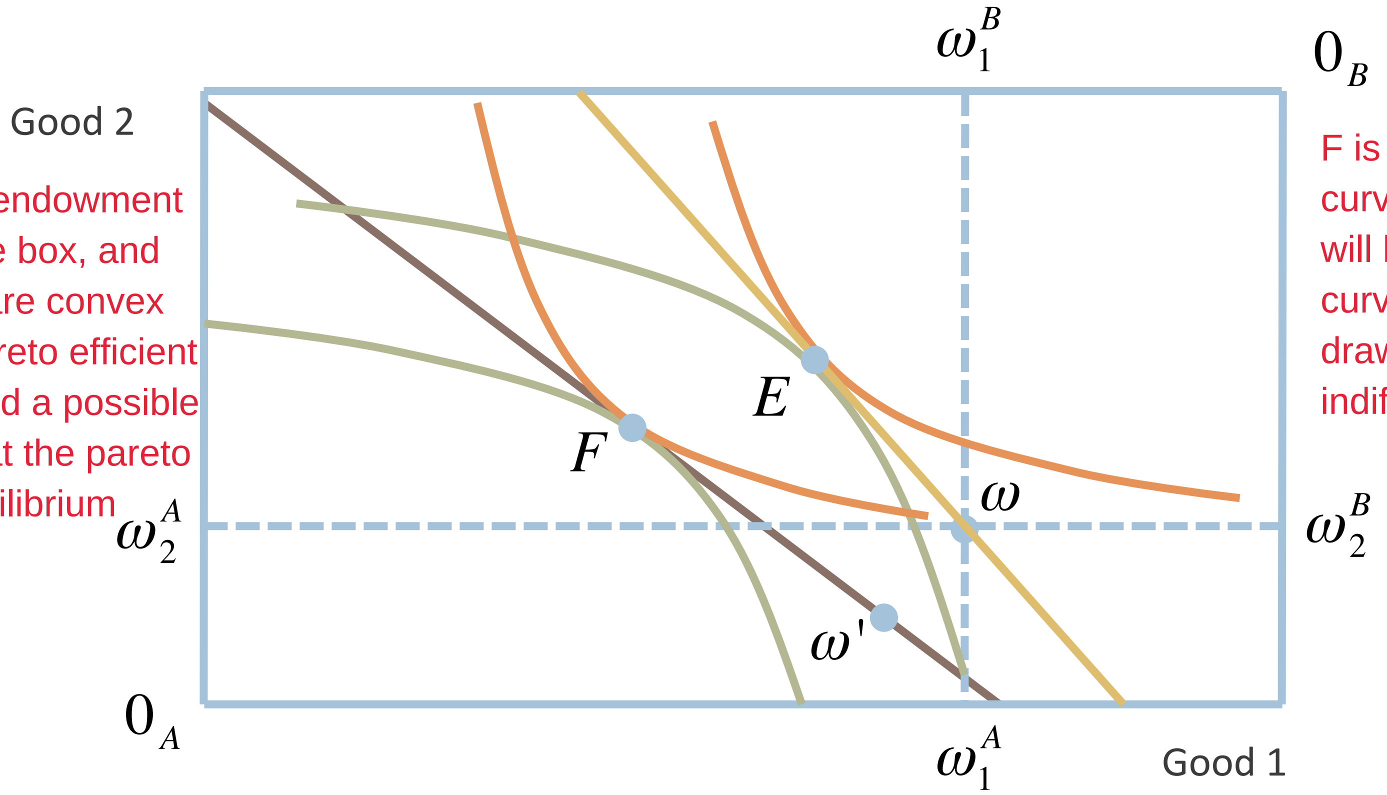
- First welfare theorem
  - Competitive equilibrium allocation is Pareto efficient
- How about the reverse?
  - We know that not every Pareto efficient allocation can be achieved in equilibrium given a particular endowment allocation
- The *Second Fundamental Theorem of Welfare Economics* states that any Pareto efficient allocation can be achieved in a competitive equilibrium through redistribution of endowments

redistribution means we move the endowment from 1 point in the box to some other point in the box.

e.g. A has 20 apples and 10 bananas, if we take 2 apples and 1 banana from A to give it to B -> redistribution of endowments.

Second Welfare Theorem in Graph

SWT: If we can change our endowment from 1 point to another in the box, and as long as the indiff curves are convex and smooth, then for any pareto efficient allocation, we can always find a possible endowment allocation so that the pareto efficient allocation is the equilibrium allocation.



F is pareto efficient, as long as the indiff curves are smooth and convex, this means F will be a tangency point between the 2 indiff curves. As long as this is true, we can always draw a budget line that is tangent to both indiff curves at point F.

Now we just move endowment from w to w'. All we need is that w' lies on the brown budget line.

F will be an equilibrium allocation if the endowment is *w'*

Part 2

Walras' Law

Gross Demand at Any Given Prices

- Let P_1, P_2 be any pair of prices

Given any prices, as long as u are an utility-maximizing consumer, u will just consume your optimal basket.

  - May or may not be the equilibrium prices
- Let $(x^A{}_1, x^A{}_2)$ be A's gross demand and $(x^B{}_1, x^B{}_2)$ be B's gross demand given P_1, P_2

x1A: gross demand of consumer A for good 1, given prices P1, P2.

gross demand:
  - The utility-maximizing quantity of each good for each consumer at the given prices

gross demand is just the optimal basket.

- Since P_1, P_2 may not be the equilibrium prices, it is possible that

we don't need the markets for the 2 goods to clear.

$$x_1^A + x_1^B \neq \omega_1^A + \omega_1^B$$

$$x_2^A + x_2^B \neq \omega_2^A + \omega_2^B$$

Net Demand

- **Definition 6.2** The *net demand* of a consumer for a good is the difference between the gross demand for that good and his/her endowment for that good
- A's net demand for good 1 is

$$x_1^A - \omega_1^A$$

- A's net demand for good 2 is

$$x_2^A - \omega_2^A$$

e.g. A has 10 units of good 1 to start with (endowment). If A wants to buy 12 units of good 1 given the prices, then net demand for good 1 is 2.
-> tells us relative to endowment (how many of these goods u already have), how much more/less do u want to consume. Net demand can be a -ve number.

Aggregate Net Demand

- Definition 6.3 The *aggregate net demand* for a good is the sum of the net demand for that good for the two consumers

how much good 1 and 2 consumers want to consume in total

total endowment of good 1

$$x_1^A + x_1^B - \omega_1^A - \omega_1^B, \quad x_2^A + x_2^B - \omega_2^A - \omega_2^B$$

- When the aggregate net demand for a good is positive
  - There is excess demand for that good
- When the aggregate net demand for a good is negative
  - There is excess supply for that good

When the aggregate net demand for a good = 0?
total consumption of good 1 = total endowment of good 1 -> market clears

=> The market clear condition is equivalent to saying that the aggregate net demand of the good is 0.

Value of Net Demand

optimal basket

P1, P2 give A's budget line

- $(x^A{}_1, x^A{}_2)$ lies on consumer A’s budget line

$$P_1 x_1^A + P_2 x_2^A = P_1\omega_1^A + P_2\omega_2^A$$

cost of optimal basket = consumer's income

- Rearranging

net demand for good 2

$$P_1(x_1^A - \omega_1^A) + P_2(x_2^A - \omega_2^A) = 0$$

We got this just from rearranging the budget line.

net demand for good 1

monetary

- The total value of consumer A’s net demand for the two goods is 0

this is just another way to interpret the budget line

  - The value of A’s net demand for good 1 is

monetary value

$$P_1(x_1^A - \omega_1^A)$$

  - The value of A’s net demand for good 2 is

$$P_2(x_2^A - \omega_2^A)$$

Walras' Law

- Similarly, the total value of consumer B's net demand for the two goods is 0

monetary

$$P_1(x_1^B-\omega_1^B)+P_2(x_2^B-\omega_2^B)=0$$

- Adding up the equation for A and B

the 2 eqns are just budget lines of the 2 consumers.

1st term: total value of aggregate net demand for good 1

$$P_1(x_1^A+x_1^B-\omega_1^A-\omega_1^B)+P_2(x_2^A+x_2^B-\omega_2^A-\omega_2^B)=0$$

now we can express the budget lines of the 2 consumers using the aggregate net demand

- Definition 6.4 The equation above is the *Walras' Law*
  - The total value of the aggregate net demand for the two goods is 0

Implications of Walras' Law

We cannot have a scenario where 1 market is in equilibrium but the other is not.

- In the two-good exchange economy, if one market is in equilibrium, the other market must also be in equilibrium
- Suppose the market for good 1 clears

suppose market for good 1 is in equilibrium
-> means market for good 1 must clear (aggregate net demand for good 1 = 0)

$$x_1^A + x_1^B - \omega_1^A - \omega_1^B = 0$$

- By the Walras' law

$$P_1(x_1^A + x_1^B - \omega_1^A - \omega_1^B) + P_2(x_2^A + x_2^B - \omega_2^A - \omega_2^B) = 0$$

- Market for good 2 clears as well

$$x_2^A + x_2^B - \omega_2^A - \omega_2^B = 0$$

Implications of Walras' Law Cont'

Walras' Law doesn't mean the markets have to be in equilibrium.

- In the two-good exchange economy, an excess supply in one market implies an excess demand in the other market

You cannot have excess supply or excess demand in both markets.

- Suppose there is excess supply of good 1

$$x_1^A + x_1^B - \omega_1^A - \omega_1^B < 0$$

- By the Walras' law

$$P_1(x_1^A + x_1^B - \omega_1^A - \omega_1^B) + P_2(x_2^A + x_2^B - \omega_2^A - \omega_2^B) = 0$$

- There will be excess demand of good 2

The Walras' Law is a simple result which we get just by adding up the budget lines of the 2 consumers and reinterpret the budget lines using the aggregate net demand.

Walras' Law vs. Competitive Equilibrium

no relationship between the 2.

- Walras' law holds for ANY prices
  - Not just the equilibrium prices

Walras' Law always hold because we only use budget lines, we don't need the markets, economy to be in equilibrium.

- At the equilibrium prices, the aggregate net demand for each good is 0

Definition: If the 2 markets are in equilibrium, then the 2 markets will clear. -> aggregate net demand for both goods = 0

$$P_1\underbrace{(x_1^A + x_1^B - \omega_1^A - \omega_1^B)}_{=0} + P_2\underbrace{(x_2^A + x_2^B - \omega_2^A - \omega_2^B)}_{=0} = 0$$

- At non-equilibrium prices, the aggregate net demand is not 0

$$P_1\underbrace{(x_1^A + x_1^B - \omega_1^A - \omega_1^B)}_{\neq 0} + P_2\underbrace{(x_2^A + x_2^B - \omega_2^A - \omega_2^B)}_{\neq 0} = 0$$

Part 3

Midterm Review

Basic Information

- 3 Mar, 6:30 pm to 7:45 pm
  - Split into multiple venues: MPSH + faculty venues
- Format
  - MCQ + structured questions
- Coverage
  - Lecture 1 – Lecture 5
- Bring
  - Student ID
  - Non-programmable calculator
- Do not write in pencil
- Memorize your tutorial number

Logistics

- Topics in the textbook that are not covered in lectures or tutorials are not required
- There will be no lecture or tutorial in week 7
- Midterm practice problems will be uploaded on LumiNUS
- My consultation hours
  - 25 Feb – 27 Feb: Zoom consultation
    - L1: 2 pm to 3:30 pm
    - L2: 3:30 pm to 5 pm
  - 28 Feb and 2 Mar: walk-in consultation
    - 2:30 pm to 5 pm

Our Topics

- Consumer Theory
  - Optimal choice
  - Revealed preference
  - Demand curve/demand function
  - Substitution and income effects
  - Consumer welfare
- Exchange
  - Pareto efficiency
  - Competitive equilibrium

Common Utility Functions

- Cobb Douglas
- Perfect substitutes
- Perfect complements
- Quasi linear

Feasible Allocation

- Recall an allocation is feasible if

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

the feasibility conditions are the same as the markets clear conditions here.

- An alternative definition says that an allocation is feasible if

$$x_1^A + x_1^B \le \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B \le \omega_2^A + \omega_2^B$$

If define feasibility conditions this way, then not the same as market clear conditions.

  - The total amount of each good consumed does not exceed the total amount available

Does it matter?

- Given the alternative definition, when you solve for the contract curve, you will have
- Tangency condition

$$MRS^{A}_{1,2} = MRS^{B}_{1,2} \quad (1)$$

- The allocation must be feasible

$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B \quad (2)$$

$$x_2^A + x_2^B \leq \omega_2^A + \omega_2^B \quad (3)$$

So when solving for contract curve, even with this feasibility definition, we can still use the = sign here (due to the reason below).

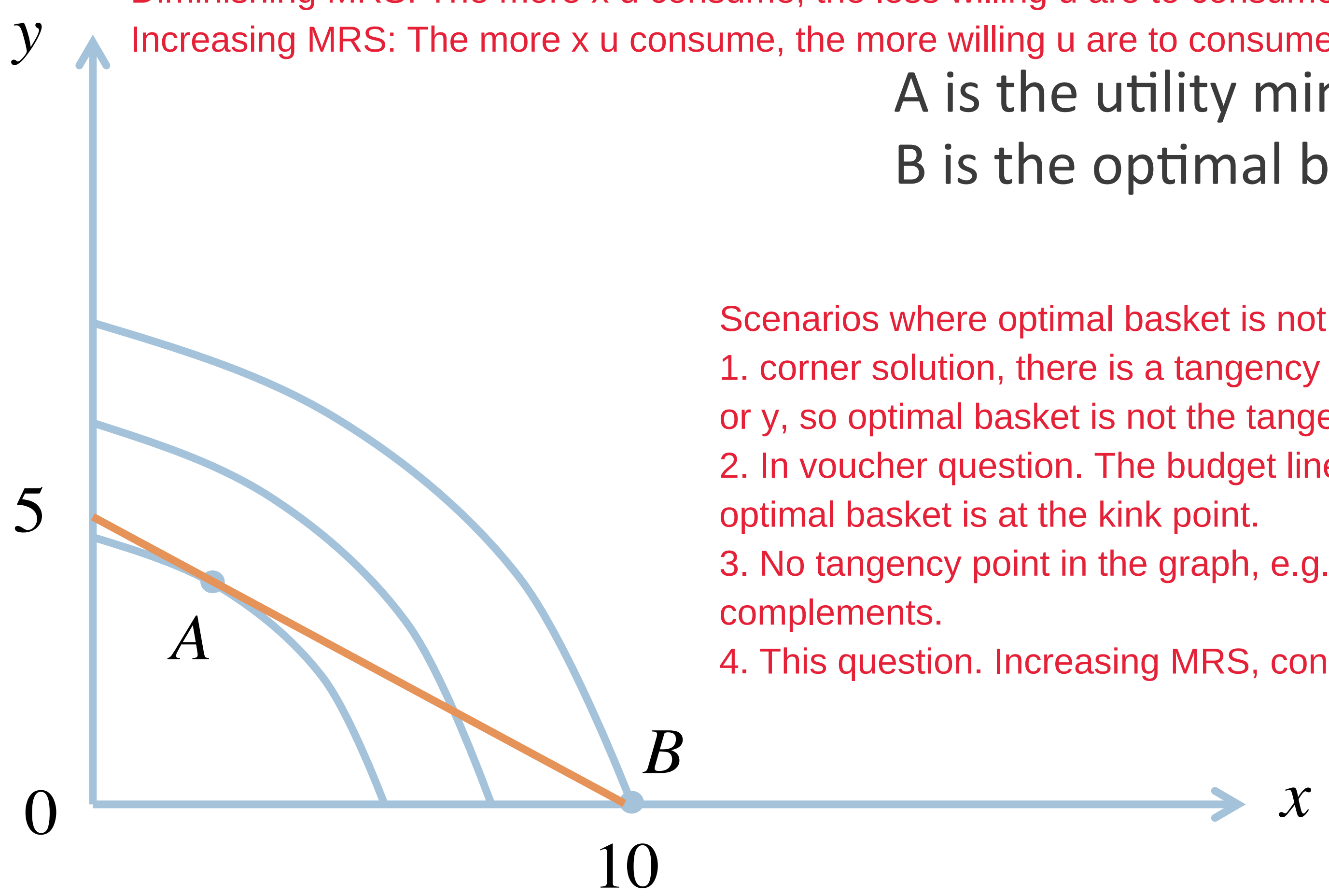
From these 2 inequalities now, we cannot express x1B in terms of x1A anymore. Then how to solve for contract curve now?

The alternative feasibility definition, 2 types of allocations that are considered as feasible: one is the allocation where the total consumption is equal to the total endowment, another one is the allocation where the total consumption is less than total endowment. Do u think the allocation where the total consumption of each good < total endowment will be pareto efficient? No as long as more is better holds.

Hence, using the alternative feasibility definition, doesn't create a problem with contract curve.

Homework 1 Question 1 d)

Diminishing MRS: The more x u consume, the less willing u are to consume even more x.
Increasing MRS: The more x u consume, the more willing u are to consumer even more x and reduce y.



A is the utility minimizing basket

B is the optimal basket corner solution

Scenarios where optimal basket is not tangency point:

1. corner solution, there is a tangency point but it involves a -ve number for x or y, so optimal basket is not the tangency point.
2. In voucher question. The budget line has a kink point, possible that the optimal basket is at the kink point.
3. No tangency point in the graph, e.g. perfect substitutes / perfect complements.
4. This question. Increasing MRS, concave indiff curve.

Homework 1 Question 2 b)

- This is about the direction of substitution effect when there is a price increase
- Given the initial price ($1), A is optimal and the consumer is indifferent between A and C, this means

$$x_c + 3y_c \geq x_A + 3y_A = 12$$

revealed preference

- Given the new price ($2), C is optimal and the consumer is indifferent between A and C, this means

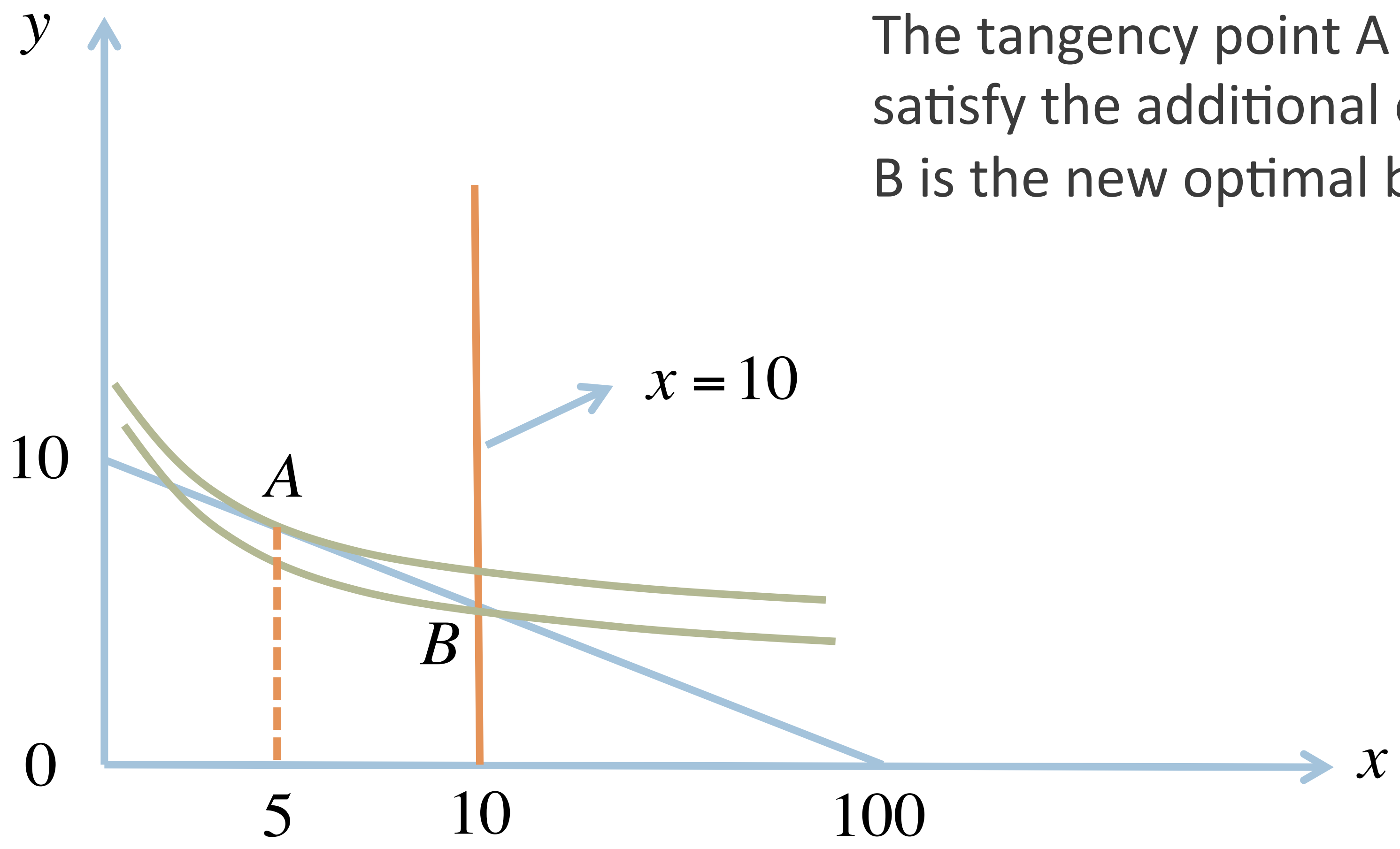
$$18 = 2x_A + 3y_A \geq 2x_c + 3y_c$$

- Thus

$$x_c \leq x_A = 6$$

Conclusion: when there is price increase, then substitution effect will be non-positive. (Xc - Xa)
In class, we have shown that in case of price drop, substitution effect will be non-negative.

Homework 1 Question 4 c)



The tangency point A does not satisfy the additional constraint
B is the new optimal basket

MCQ: Example 1

- A consumer buys two goods, rice and housing. At the initial optimal basket, the consumer buys both goods. When the price of rice decreases while the price of housing and the consumer's income remain constant, the consumption of rice increases by 4 units. If rice is an inferior good, regarding the substitution effect (*SE*) and income effect (*IE*) with respect to rice, which of the following is true?
  - A. 0<=*SE*<=4, 0<=*IE*<=4
  - B. -4<=*SE*<0, -4<=*IE*<=0
  - C. *SE*>4, *IE*<0
  - D. *SE*<0, *IE*>4

Since rice is inferior, when rice becomes cheaper, purchasing power increases, income effect should be -ve.
We know the total effect is 4, i.e. SE +IE = 4.
Hence, C.

Solution for MCQ 1

MCQ: Example 2

This questions tests on demand function

- Suppose a consumer has utility function $U(x,y)=\min(ax,y)$, where $a>0$ .Which of the following statements is true?
  - A. The consumer always buys the same amount of *x* and *y*.
  - B. The consumer's expenditure on *y* is always greater than the expenditure on *x*.
  - C. When income doubles, the consumer doubles his consumption of both *x* and *y*.
  - D. When *x* becomes more expensive, the consumer buys more *y*.

Solving for demand functions: ax = y, PxX + PyY = I
x = I / (Px + aPy)
y = aI / (Px + aPy)

Expenditure on x and y: PxX = PxI / (Px + aPy), PyY = aPyI / (Px + aPy)

Solution for MCQ 2

MCQ: Example 3

U are not given utility function, so cannot solve anything, this is a revealed preference question.

- Suppose a consumer's preference satisfies the three assumptions. The consumer has an income of \$18. When the price of *x* is \$2 and the price of *y* is \$1, the consumer's optimal choice is 6 units of *x* and 6 units of *y*. When the price of *x* becomes \$1 and the price of *y* becomes \$2, assuming income does not change, the optimal choice <u>CANNOT</u> be
  - A. 10 units of *x* and 4 units of *y*
  - B. 8 units of *x* and 5 units of *y*
  - C. 6 units of *x* and 6 units of *y*
  - D. 4 units of *x* and 7 units of *y*

Draw out 2 budget lines, point (6, 6) is the point of intersection between the 2 lines.

All the 4 baskets here lie on the new budget line.

Basket D is below initial budget line, coz 2x4 + 1x7 = 15 < 18, thus the initial optimal basket is strictly preferred to D.
Since the initial optimal basket is still affordable given new budget line, D cannot be optimal, coz at least there is 1 basket, (6, 6) that gives consumer higher utility and still affordable given new budget line.

When x becomes more expensive, will consumer buy more y?
No, depends on utility function. Can see the demand function for MCQ2.

Solution for MCQ 3