Mathematical Formulation of 2E-VRP with Mobile Microhub

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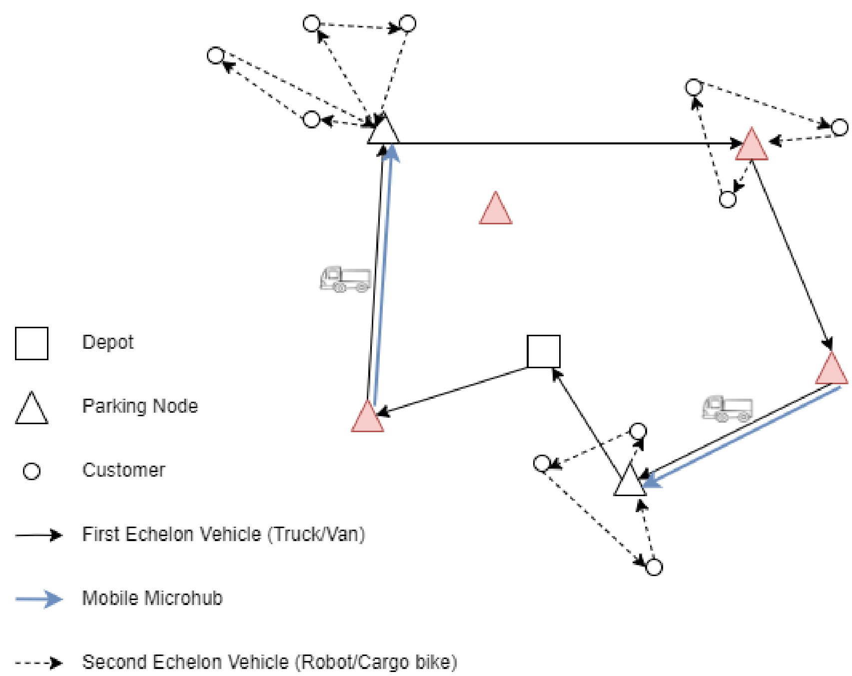
1 Problem Description

In this two-echelon urban delivery with mobile microhub problem model, the distribution network is structured around two main transportation echelons.
At the start of each planning horizon, the First Echelon Vehicle(FEV), typically a van or a truck, departs from the central depot carrying parcels destined for various urban locations. Its primary role is to transport these parcels to designated parking nodes, where the Mobile Microhub(MM) parks. The MM in our model does not possess autonomous mobility. Instead, it functions similarly to a trailer in the truck and trailer setup, relying on the FEV for relocation. It serves as a temporary storage and distribution hub at the parking nodes. Upon reaching a parking node, if customers are close, the FEV need only replenish the MM with parcels for further distribution. However, if the next set of customers is located further away, the FEV is required to tow the MM to a new parking node closer to these customers, where it will replenish the MM.
At current stage we study the case of robot as the Second Echelon Vehicle(SEV), which are stored within the MM, to carry out the final delivery of parcels to customers. In future studies, we plan to integrate cargo bikes as an additional mode of delivery within this second echelon, to provide greater flexibility. Currently, we do not account for the charging needs of our robotic delivery agents within the model. Instead, we limit their operational range based on maximum distance thresholds to manage their battery life.

2 Problem Definition

Let $G = (N, A)$ be a directed graph with the set of nodes $N = \{0\} \cup N_p \cup N_c \cup \{0'\}$ where N_p represents the parking nodes, N_c represents the customers nodes and 0, 0' represents the depot. Considering the congestion and restriction in city center area, the set of arcs $A = A_1 \cup A_2$ can be partitioned into two disjoint sets of FE arcs could be travelled by the FEV and MM and the SE arcs could be travelled only by SEV.

With each arcs $(i,j) \in A$, a non negative travel cost and travel time are associated, as the two types of vehicles travel at different speed, we denote c_{ij1}, c_{ij1} as the travel cost of FEV and SEV, tt_{ij1}, tt_{ij1} as the travel time respectively. And note that we assume the travel speed of FEV with MM is the same as SEV only because of the congestion situation of city area.
The FEV, MM and SEV have capacity Q_0, Q_1 and Q_2. In this scenario, there is only one depot and one FEV, a set of MM K_1 park in p_p as the initial place, and a set of robot K_2, each robot departs for one tour with maximum travel distance e. A non-negative service time is required at parking node as η_1 and at customer node as η_2. Each customer $i \in N_c$ has its demand d_i, which must be fulfilled by a single delivery of an SEV. Also each customer has its time window $[a_i, b_i]$ which is within the total length of planning horizon ζ.



3 Mathematical Formulation

We propose two mathematical formulations in this paper(draft). The difference is if the robot need to return to exactly the microhub where it dispatched from. In formulation F1, every robot is linked to one microhub, so the variable describe the route of robot contains an index of microhub. In formulation F2 since we assume that all the robots belong to the same logistics company and are the same model, and all the microhub possess the same number of robots, so it is possible to get rid of the microhub index.

3.1 Formulation F_1

The model uses six variables. For the first echelon, binary routing variable x_{ij} denotes whether the arc $(i, j) \in A_1$ is traversed by FEV. Binary routing variable y_{ij} denotes if the arc $(i, j) \in A_1$ is traversed by MM(being towed by FEV). Binary variable z_{ijp} denote if arc $(i, j) \in A_2$ is traversed from robot dispatched from MM in parking node $p \in N_p$. The amount of freight transported from depot to MM is denoted by $w_p \in \mathbb{R}^+$. The freight flow of arc $(i, j) \in A_2$ is denoted by $f_{ij} \in \mathbb{R}^+$. And finally variable t_i denotes the arrival time of any node besides the depot $i \in N_p \cup N_c$.

$$\text{minimize} \quad \sum_{(i,j)\in A_1} c_{ij1}x_{ij} + \sum_{(i,j)\in A_1} c_{ij1}y_{ij} + \sum_{r\in K_2}\sum_{(i,j)\in A_2} c_{ij2}z_{ij}^r$$

subject to

Constraints for FEV

Flow conservation for FEV at each parking node

$$\sum_{(i,j)\in A_1} x_{ij} = \sum_{(j,i)\in A_1} x_{ji}, \quad i \in N_p \tag{1}$$

$$\sum_{(i,j)\in A_1} x_{ij} \le 1, \quad i \in N_p \tag{2}$$

$$\sum_{(j,i)\in A_1} y_{ji} + \sum_{(j,i)\in A_1} y_{ij} \le 1, \quad i \in N_p \tag{3}$$

$$y_{ij} \le x_{ij}, (i,j) \in A_1 \tag{4}$$

Flow conservation for FEV at depot

$$\sum_{(0,j)\in A_1} x_{0j} = \sum_{(j,0')\in A_1} x_{j0'} = 1 \tag{5}$$

$$\sum_{(0,j)\in A_1} y_{0j} = \sum_{(j,0')\in A_1} y_{j0'} = 0 \tag{6}$$

The capacity constraints for the FEV

$$\sum_{p\in N_p} w_p \le Q_0 \tag{7}$$

Constraints for Mobile Microhub

Constraints for initial parking place

$$\sum_{(i,j)\in A_1} y_{ij} \le p_i, \quad i \in N_p \tag{8}$$

$$\sum_{(i,j)\in A_1} y_{ij} \le 1 - p_j, \quad j \in N_p \tag{9}$$

$$w_p \le Q_1(1 - \sum_{(p,j)\in A_1} y_{pj}), \quad p \in N_p \tag{10}$$

Link the 1st and 2nd echelon and impose that the total flow from the depot to MM equals to total demand served from MM

$$w_p = \sum_{(p,j)\in A_2} f_{pj}, \quad p \in N_p \tag{11}$$

Capacity constraints for MM (The freight flow from depot to MM can only be positive if parking node is used)

$$w_p \le Q_1 \sum_{(i,p)\in A_1} x_{ip}, \quad p \in N_p \tag{12}$$

$$w_p \le Q_1 (\sum_{(i,p)\in A_1} y_{ip} + p_p), \quad p \in N_p \tag{13}$$

Constraints for SEV(Multiple Robots, each one dispatched once)

Flow conservation at parking nodes and customer nodes

$$\sum_{(i,j)\in A_2} z_{ij}^r = \sum_{(j,i)\in A_2} z_{ji}^r, \quad i \in N_p \cup N_c, r \in K_2 \tag{14}$$

$$\sum_{r\in K_2} \sum_{(p,j)\in A_2} z_{pj}^r \le |K_2|, \quad p \in N_p \tag{15}$$

$$\sum_{(p.j)\in A_2} z_{pj}^r \le 1, \quad p \in N_p, r \in K_2 \tag{16}$$

$$\sum_{r\in K_2} \sum_{(i,j)\in A_2} z_{ij}^r = 1, \quad i \in N_c \tag{17}$$

The customer demands are met

$$\sum_{(j,i)\in A_2} f_{ji} - \sum_{(i,j)\in A_2} f_{ij} = d_i, \quad i \in N_c \tag{18}$$

Capacity constraints of SEV

$$f_{ij} \le Q_2 \sum_{r\in K_2} z_{ij}^r, \quad (i,j) \in A_2 \tag{19}$$

Time constraints

Total working time does not exceed ζ

$$\sum_{(i,j)\in A_1} tt_{ij}^1 x_{ij} + \eta_1 \sum_{(i,p)\in A_1} p_p x_{ip} \le \zeta \tag{20}$$

$$t_i + tt_{ij}^2 + \eta_2 \le \zeta + M(1 - z_{ij}^r), \quad i \in N_c, j \in N_p, r \in K_2 \tag{21}$$

Subtour elimination constraints and synchronization constraints

$$t_i + \eta_1(1 - x_{ij}) + tt_{ij}^1 x_{ij} \le t_j + M(1 - x_{ij}), \quad (i,j) \in A_1 \tag{22}$$

$$t_i + \eta_2(1 - z_{ij}^r) + tt_{ij}^2 z_{ij}^r \le t_j + M(1 - z_{ij}^r), \quad (i,j) \in A_2, r \in K_2 \tag{23}$$

Time window constraint

$$a_i \le t_i \le b_i, \quad i \in N_c \tag{24}$$

Arrival time initialization

$$t_p + tt_{pj}^2 z_{pj}^r \le t_j, \quad p \in N_p, j \in N_c, r \in K_2 \tag{25}$$

$$tt_{0j}^1 x_{0i} \le t_i, \quad i \in N_p \tag{26}$$

Max travel distance of robot (TBD)

4 Appendix: Notation table

Set	
$G = (N, A)$	Directed graph
$N = \{0\} \cup N_p \cup N_c \cup \{0'\}$	Set of nodes
$N_p = \{1 \ldots n_p\}$	Set of parking nodes
$N_c = \{n_p+1 \ldots n_p+n_c\}$	Set of customers nodes
$A = A_1 \cup A_2$	Set of arcs
A_1	The set of FEV arcs, $\{(i,j) \| i, j \in \{0\} \cup N_p \cup \{0'\}, i \neq j\}$
A_2	The set of SEV arcs, $\{(i,j) \| i, j \in N_p \cup N_c, i \neq j\} \setminus \{(i,j) \| i, j \in N_p\}$
N_c	The set of customer nodes
N_p	The set of parking nodes
K_1	The set of MM
K_2	The set of SEV from one MM
Parameter	
c_{ij1}	Travel cost of arc $(i,j) \in A_1$
c_{ij2}	Travel cost of arc $(i,j) \in A_2$
p_p	Parking node $p \in N_p$ is occupied by a MM or not
d_i	Demand of customer $i \in N_c$
$dist_{ij}$	Travel distance of robot in arc $(i,j) \in A_2$
Q_0	Capacity of FEV
Q_1	Capacity of MM
Q_2	Capacity of SEV, $Q_0 \gg Q_1 \gg Q_2$
η_1, η_2	Operation time of replenishment at parking node, or service time at customer node. $\eta_1 > \eta_2$
ζ (zeta)	Length of planning horizon
$[a_i, b_i]$	Time window of customer i $\in N_c$
e	Max travel distance of robot
$tt1_{ij}$	Time to travel arc$(i,j) \in A_1$ for truck
$tt2_{ij}$	Time to travel arc$(i,j) \in A_2$ for robot
Variable	
$x_{ij} \in \{0,1\}$	$= 1$ if arc $(i,j) \in A_1$ is traveled by a FEV
$y_{ij} \in \{0,1\}$	$= 1$ if arc $(i,j) \in A_1$ is traveled by FEV with MM
$z_{ij}^r \in \{0,1\}$	$= 1$ if arc $(i,j) \in A_2$ is used by SEV $r-th \in K_2$ dispatched from a mobile microhub
$w_p \in \mathbb{R}^+$	Amount of freight transported from depot to MM located in $p \in N_p$
$f_{ij} \in \mathbb{R}^+$	Freight flow of arc $(i,j) \in A_2$
t_i	Arrival time of SEV at customer $i \in N_c$ or the arrival time of FEV at parking node $i \in N_p$