Information Provision and Pricing for Pool and Regular Services in Ride-sharing

Ying-Ju Chen
School of Business and Management and School of Engineering, Hong Kong University of Science and Technology, Hong Kong
imchen@ust.hk

Long He
NUS Business School, National University of Singapore, Singapore
longhe@nus.edu.sg

Ling-Chieh Kung
Department of Information Management, National Taiwan University, Taiwan
lckung@ntu.edu.tw

Problem definition: We study the information provision policy and pricing in a ride-sharing platform that offers both pool (i.e., shared) and regular services. We aim to evaluate the performances of the two information provision policies: (1) the *opaque* policy where the platform does not reveal request types to drivers or consequently not allow drivers to refuse any request; and (2) the *transparent* policy where the platform notifies the drivers about request types and allow drivers to freely refuse requests.

Academic/Practical relevance: Our study is motivated by the ride-sharing platforms offering both pool and regular services. Our analysis on the demand and supply management via information provision and pricing is novel to the literature. The findings in this paper also support the industry practices on emphasizing acceptance rate and imposing a high penalty of refusing a request.

Methodology: Using a queueing framework, we develop an analytical model to explicitly capture driver participation and job picking behavior, as well as rider request of service. Furthermore, we characterize the optimal prices for both services to maximize ridership under the opaque policy and discuss pairing efficiency for pool service, measured by pairing probability and detour. Using a set of customer tracking data from a ride-sharing platform, we calibrate the proposed model and evaluate the impacts of several factors.

Results: We first show that the opaque policy brings higher ridership to the platform in equilibrium, by eliminating drivers' job picking behavior. Also, we find that improving the pairing probability will make pool service more preferable to the platform and drivers, only when the detour is short. From the real data, we demonstrate that the opaque policy in information provision can bring 2.59% higher ridership compared to the transparent policy in the *status quo*. The opaque policy is also more robust than the transparent policy—the service level under the opaque policy is less sensitive to driver's reserved earning rate.

Managerial implications: We show the advantage of the opaque information provision policy when the platform provides both the pool and regular services. Moreover, while a low driver's reserved earning and high travel demand are favorable, our results also encourage algorithmic improvements in pairing efficiency to increase pairing probability and reduce detour in paired trips, especially under the transparent policy.

Key words: ride-sharing, shared service, carpool, information provision, pricing

1. Introduction

Modern information technology has enabled many innovative business models. Instead of providing services or products directly by the firms themselves, multi-sided platforms emerge in all kinds of industries, including finance (e.g., Lending Club), retailing (e.g., eBay, Taobao), mobile applications (e.g., iTunes Store, Google Play), grocery delivery (e.g., Instacart), healthcare (e.g., PatientsLikeMe), among others. In the recent years, several such platforms emerge from the so-called sharing economy are offering affordable service (and products) to customers and broadening sales channels to the providers. A major industry that is revolutionized by multi-sided platforms is transportation. Ride-sharing platforms, e.g., Uber worldwide, Didi in China and Grab in Southeast Asia, make city travel easier for passengers by mobilizing car owners as part-time drivers. With user-friendly mobile applications, efficient assignment algorithms, and IT-enabled dynamic pricing, these ride-sharing platforms improve the demand-supply matching efficiency and are getting popular around the world. As of June 2016, the valuations of Uber and Didi are around $62.5 billion and $28 billion respectively, making them top-ranked start-ups in the world (Mozur and de la Merced 2016).

To extend their market shares and fulfill their greener promises, several major ride-sharing platforms offer various services, especially the carpool (or shared) service at a discounted rate. Different from the regular services where the drivers typically take only one rider, the carpool service is regarded as the actual sharing of rides. By paying a reduced fee for the carpool service, a rider may be paired with another rider whose traveling route is similar. If such pairing is successful, both riders have to share the space until one is dropped off. When riders choose the carpool service and end up riding alone, they still pay at the discounted rate. While the regular service, as the initial product of ride-sharing, aims to match riders with drivers, the carpool service starts to match riders to share the rides. The carpool service has been marketed by various platforms such as UberPOOL, Lyft Line, Didi ExpressPool and GrabShare. In 2015, it was reported that one-third of Lyft rides in San Francisco are shared (Bloomberg 2015).

The benefits of running a carpool service (or "pool service" thereafter) include improving the environment and easing traffic congestions. In a crowded city, e.g., Beijing, with many rides happening around the same time, naturally, some rides will have close origins and destinations. By offering a pool service, a ride-sharing platform may serve more passengers with the same number of drivers. This leaves room for passengers to pay less, drivers to earn more, and a platform to economically better off. In fact, offering both regular and pool service is also a way for a platform to price discriminate. Passengers who are in a hurry or do not want to share a ride with strangers may choose the regular service, and those who are more price sensitive may choose the pool service.

To match the demand with supply, the platform receives trip information from the riders and dispatch relevant information to the drivers, after some algorithmic calculations. To improve the matching efficiency, the platform needs to manage the information flow wisely, especially when both pool and regular services are offered. A question to address is whether to inform drivers about the exact type of services along with pickup information that a rider requires. In the case where drivers know the requested service type, they may decide whether to accept requests based on types whenever allowed. However, not every driver welcomes the pool service. According to the interviews in CNNMoney (2018), UberPool means more work, but not necessarily more pay to the drivers. Consequently, some drivers tend to turn down the pool requests and only accept the regular requests. To eliminate discrimination, an alternative is not to inform drivers about the requested service types. It is analogous to the case where drivers are assigned to requests and cannot refuse any request. Specifically, we call the first case assignment by the transparent policy and the second case by the opaque policy.

In this study, we consider a platform's information provision and pricing problem when offering both pool and regular services. Under the *transparent policy*, the platform notifies a driver about the request type, i.e., regular or pool service, and the driver then freely decides whether to accept. Alternatively, the platform that adopts the *opaque policy* does not reveal the request types to drivers (or equivalently does not allow drivers to refuse any request). Using a queueing model, we analyze the performance of both information provision policies in driver supply as well as total ridership. We find that given any pricing plan, the opaque policy is more efficient in inducing drivers to serve passengers, by eliminating drivers' job picking behavior, which negatively affects the efficiency in the transparent policy. In other words, strategically making drivers less informed can benefit the whole system. We then analytically characterize the optimal prices for both regular and pool service under the opaque policy. Finally, using a set of customer tracking data from a ride-sharing platform, we calibrate the rider's mode choice model and discuss the impacts of various factors, including driver's participation, rider demand size, as well as pairing efficiency for pool service, from numerical experiments.

The rest of this paper is organized as follow. In Section 2, we review the related literature on sharing economy, shared mobility systems (e.g., car sharing and ride-sharing), and opaque selling. We then develop a queueing model in Section 3 to describe our problem. In Sections 4, we present the equilibrium under respective information provision policy and show that the opaque policy brings higher ridership. Focusing on the opaque policy in Section 5, we characterize the optimal prices for both regular and pool service. In Section 6, we numerically examine several questions in a real-world case study. We then conclude the paper in Section 7. All detailed proofs are provided in the appendix.

2. Literature Review

Our work contributes to the growing stream of studies on the sharing economy. Specifically, our research concerns ride-sharing; it addresses how to serve more riders with self-scheduled drivers in a ride-sharing platform via information provision and pricing for both pool and regular services. A number of papers have been published on decisions to purchase or rent in the peer-to-peer product sharing. For example, Jiang and Tian (2016) describe a customer's decision to purchase or rental in a two-period analytical model and discuss a firm's choice of retail price and product quality. Fraiberger and Sundararajan (2017) study the car purchase decisions of individuals in each period given matching frictions using dynamic programming. In a single-period analytical model, Benjaafar et al. (2018) examine the impact of sharing on ownership and usage under factors such as rental price, cost of ownership and moral hazard cost. While these papers investigate the product sharing context, our work exclusively concerns ride-sharing, where two ride services consume the common supply.

Our paper is closely related to research in the context of on-demand platforms, of which the ride-sharing platform is an example. Cachon et al. (2017) study several contracts in prices and wages and find that both providers and customers are generally better off with surge pricing. Hu and Zhou (2017) find that the optimal flat-commission contract can achieve satisfactory performance when the supply curves are concave in the wage. Taking a queueing perspective on the platform, Taylor (2018) discusses how delay sensitivity and agent independence impact the optimal per-service price and wage, with uncertainties in customers' valuation and agent's opportunity costs. With endogenous demand and supply in a queueing model, Bai et al. (2018) characterize the optimal price, wage, and payout ratio in equilibrium. Cohen and Zhang (2018) examine the business strategy of cooperating with competing platforms via profit sharing. Although our paper also studies a type of on-demand platform, it is different from the above papers in the following aspects. First, on the demand side, our model considers the rider's mode choice when facing two types of ride-sharing services, whereas the above papers primarily consider a single type of service. Second, on the supply side, we examine the role of information provision in driver arrivals to the platform, while the above papers investigate the impact of cash flows, e.g., price, wage, and commission. Lastly, on the matching of demand and supply, we deploy a queueing model, also seen in Banerjee et al. (2015), Bellos et al. (2017) and He et al. (2017), where the arrivals are the drivers and the server is the market with ride requests. It is different from the queueing models presented in Taylor (2018) and Bai et al. (2018), where the customers are the arrivals with waiting costs and the server is the supply whose capacity increases with the number of providers.

Recent developments in the sharing economy encourage research in various innovative business models, especially shared mobility systems. Bellos et al. (2017) discuss the manufacturer's strategy in offering car sharing. He et al. (2017) optimize the service region for free-float electric vehicle sharing systems where the fleet operations, e.g., repositioning and recharging, are modeled using a queueing network. Furthermore, Qi et al. (2018) explore the opportunities of utilizing ride-sharing in last-mile delivery and evaluate its environmental impact. Besides the strategic designs of shared mobility systems, operational decisions have also been extensively studied. Kung and Zhong (2017) consider crowdsourcing delivery platforms, e.g., Instacart, and compares the performance of membership-based pricing, transaction-based pricing, and cross subsidization. Using a set of operational data from a bike sharing system, Kabra et al. (2016) measure impacts of station accessibility and bike availability on ridership. To solve the detailed matching problem with heterogeneous supply and demand, Hu and Zhou (2016) propose a priority rule in the optimal matching policy. In a product rental network, Benjaafar et al. (2017) characterize the structural properties of optimal repositioning policy. In the context of vehicle sharing, He et al. (2018) study the fleet repositioning problem and develop a computationally efficient optimization framework. While firms have full control over supply in vehicle sharing, ride-sharing platforms usually use price and wage as tools to align the demand and supply in a network of locations. For instance, Bimpikis et al. (2016) develop a tractable model to account for spatial price discrimination in a ride-sharing network and highlight the impact of demand pattern on profits and consumer surplus. Our paper complements this part of existing literature by exploring the use of information provision. Furthermore, in our model, riders face two types of services, i.e., the pool and regular services, that consume the common supply, i.e., the drivers. By exploiting the nature of pool service, we simplify the pricing optimization problem under the opaque information provision policy.

Finally, our work connects to the literature on probabilistic or opaque selling. Under the opaque information provision policy, drivers do not see the job types of either pool or regular services, which can be regarded as the probabilistic jobs from the drivers' perspective. In the setting with a monopolist offering two products, Fay and Xie (2008) show that offering probabilistic goods can solve the mismatch between capacity and demand and enhance efficiency, assuming customers are rational. When there is limited capacity, Jerath et al. (2010) compares last-minute selling and opaque selling in a two-period model, where customer behaviors follow rational expectations. Instead of the rational expectation framework, Huang and Yu (2014) explains the benefits of opaque selling when the customers are bounded rational using the anecdotal reasoning framework. In a queueing model for service systems, Huang and Chen (2015) extends the anecdotal reasoning framework to model customer's inference of waiting time. Different from the aforementioned papers, our model considers a two-sided platform where the drivers form rational expectation on the idle time which depends on both the intensities of driver participation and rider request. In particular, we find more drivers join the platform as they anticipate a lower expected idle time when the ride requests are aggregated under the opaque policy.

3. The Model

We consider a ride-sharing platform that provides both pool and regular services. In the pool service, a rider, who is willing to share a ride by paying a reduced fee, may be paired with another rider heading in the same direction. The regular service, on the other hand, does not pick up additional riders. To focus on the discussion of information provision policy, we study the drivers (i.e., supply) and riders (i.e., demand) in a single region where the platform sets prices c_p and c_r for pool and regular services respectively. For the ease of presentation, we consider c_p and c_r as the prices for an average trip. Once informed about the prices, the drivers (riders) decide whether to join (use) the platform and which type of service request to accept (request).

Figure 1 A Queueing Model for Ride-sharing Operations in a Region

We model the operations of ride-sharing as a queue in Figure 1. Different from the settings in typical models for call centers, we consider the *drivers* (i.e., cars) as the Poisson arrivals in the queue, instead of the riders. Similar perspective is also seen in Bellos et al. (2017) and He et al. (2017) for car sharing systems and Banerjee et al. (2015) for ride-sharing platforms. After a car arrives in the region, it becomes idle, waiting for the next ride request. We assume that the riders' service requests also occur following a Poisson process with rates depending on prices. Therefore, the region itself works as a server in the queueing model. Furthermore, when a departure happens (i.e., pickup of a rider), all drivers waiting in the region has equal probability to be the departure. In fact, this assumption is not restrictive, as later we are only interested in the average idle time of a driver in the region, which is given by Little's Law and does not depend on the priority discipline in the job assignment. When there is no driver available in the region, a ride request is lost, since the riders usually have other travel options in cities.

In the remaining of this section, we refer to *requests* as the orders received from the riders, and *jobs* as the equivalent demand for drivers, under both pool and regular services. As noted in the transportation literature, e.g., Ben-Akiva and Lerman (1985) on travel demand modeling, the key attributes that influence riders' travel mode choices include travel time, cost and comfort. Presumably, the regular service has the higher level of comfort compared to both the pool service and the public transport, where the riders need to share the space with others. Thus, the riders choose travel modes, i.e., regular, pool or public transport, based on the above listed factors. Given the prices (c_p, c_r), we denote a rider's probabilities of requesting the regular and pool service as

$q_r = q_r(c_p, c_r)$ and $q_p = q_p(c_p, c_r)$ respectively. Therefore, the probability of not using any ride-sharing services is $1 - q_r - q_p$.

Let μ be the total travel request rate in the region. The request rates for regular and pool service are therefore given by $\mu_r = q_r\mu$ and $\mu_p = q_p\mu$. Upon receiving the information about ride requests, the platform converts them into jobs sent to the drivers. A key difference between the regular and pool service is in the resource utilization. That is, each car can take *only one* request for regular service but *up to two* requests for pool service. Therefore, the cars needed for regular service equals the regular requests while those for pool service may be less than the pool requests. In this paper, we assume a car can take up to two pool requests for the clarity of exposition. The same analysis follows when more than two pool requests can be accommodated by one car. The following part discusses the details of conversion between ride requests and jobs.

The platform deploys its pairing algorithm for pool requests. We use the notion of pairing probability and detour to measure the efficiency of the pairing algorithm. Let η be the pairing probability that a pool request will be successfully paired with another and Δt be the detour (in time) for an additional pickup or dropoff. Consequently, there are $(1-\eta)\mu_p$ pool requests unpaired and thus $(1-\eta)\mu_p$ jobs for pool service with single riders are created. Meanwhile, $\eta\mu_p$ pool requests are paired and they only need $\frac{\eta}{2}\mu_p$ cars. Therefore, the total number of jobs created is

$$\hat{\mu} = (1-\eta/2)\mu_p + \mu_r, \tag{1}$$

where $(1-\eta/2)\mu_p$ and μ_r jobs are for pool and regular services respectively.

Among all jobs created, let ρ be the proportion of pool service jobs. We can explicitly write ρ as below:

$$\rho = \frac{(1-\eta/2)\mu_p}{(1-\eta/2)\mu_p + \mu_r} = \frac{(1-\eta/2)q_p}{(1-\eta/2)q_p + q_r}. \tag{2}$$

Furthermore, for each driver in pool service, its probability of having paired requests is thus

$$\psi = \frac{\eta/2}{1-\eta/2}. \tag{3}$$

In particular, when $\eta = 1$, ψ is also 1, as all pool requests are successfully paired. We provide a summary of the notation in Appendix A.

4. Information Provision

Having described the requests from riders, we now present the formation of driver supply with respect to the prices (c_p, c_r) as well as information provision policy. When deciding whether to join the ride-sharing platform, drivers need to evaluate the expected earnings together with their job picking strategies: regular service only, pool service only, or both, depending on the information provision policy. In this paper, we consider the following two information provision policies: 1. the opaque policy where the driver is not informed about the request types; and 2. the transparent policy where the driver sees the request types. Under either policy, drivers join the platform if their expected earning rates are no less than the (expected) reserved earning rate R_0, which represents the expected earning from other job opportunities. Similar to the setting in Cachon et al. (2017) where R_0 is interpreted as the driver's opportunity cost, all drivers are assumed to share the same value of R_0. Such simplification facilitates our discussion on driver's participation in the platform.

4.1. The Opaque Policy

Under the opaque policy, the drivers are not informed about the request types and thus treat all jobs as homogeneous. We can then model the system as a single M/M/1 queue shown in Figure 2. The drivers arrive following a Poisson process with rate λ_d and service rate equals the total number of jobs per unit time $\hat{\mu}$ given in Equation (1). With prices (c_p, c_r) known by the drivers, we characterize the driver behavior and derive the arrival rate λ_d as follows.

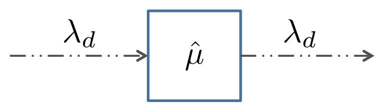

Figure 2 M/M/1 Queue for the Opaque Policy

Driver's expected earning rate depends on the price and travel time of trips, as well as the idle time between trips. While we assume all drivers are equally likely to receive a job regardless their arrival sequence in Section 3, we provide the average idle time in Lemma 1 from Little's law and it does not depend on the priority discipline. All detailed proofs are provided in Appendix B.

LEMMA 1. *The average idle time of a driver is* $\frac{1}{\hat{\mu}-\lambda_d}$.

Once a driver takes a job, there is ρ probability that it is for pool service. We denote the expected travel time as t_r for a trip with a single rider, i.e., a regular trip or an unpaired pool trip. We then consider a simple fixed commission payout structure to the drivers. That is, suppose ω is the given payout ratio, then the driver earns ωc_r and ωc_p for a regular trip and an unpaired pool trip respectively, and $2\omega c_p$ for a paired pool trip. For the ease of notation, it is equivalent to consider the drivers earn c_r and c_p, and rescale the reserved earning rate to $\hat{R}_0 = \frac{R_0}{\omega}$, as if the payout ratio was 1, in the following discussion.

When a job is for a paired pool trip, the driver earns $2c_p$ by serving two pool requests which has value c_p each. However, it takes additional $2\Delta t$ time to make additional pickup and dropoff. Recall that ψ in Equation (3) is the probability of serving paired requests in a pool trip. For a pool trip, its expected travel time is therefore $t_p = t_r + 2\psi\Delta t$ and its expected earning rate is

$c_p(1-\psi)+2c_p\psi = c_p(1+\psi)$. For an opaque job received by the driver, as there is ρ probability to be a pool service, the expected travel time is $t_r(1-\rho)+t_p\rho$ and the expected earning rate is $c_r(1-\rho)+c_p(1+\psi)\rho$, where ρ is given by Equation (2).

When the expected earning rate from joining the platform is higher than $\hat{R}_0$, a driver joins the platform and accepts the opaque jobs. We formally state the driver participation in Proposition 1.

PROPOSITION 1. *Under the opaque policy, a driver joins the platform if and only if*

$$\frac{c_r(1-\rho)+c_p(1+\psi)\rho}{\frac{1}{\hat{\mu}-\lambda_d}+t_r(1-\rho)+t_p\rho} \geq \hat{R}_0, \tag{4}$$

where $\frac{1}{\hat{\mu}-\lambda_d}+t_r(1-\rho)+t_p\rho$ *is the average time per job cycle, including idle time and travel time.*

On the one hand, we see that when there are more jobs, i.e., $\hat{\mu}$ is high, the platform is more attractive to the drivers. On the other hand, the drivers will stop joining the platform when there are too many drivers that lead to long idle time. In equilibrium, the equality holds in (4) and it results in the following expression of λ_d, assuming abundant driver supply:

$$\lambda_d = \hat{\mu} - \frac{\hat{R}_0}{\rho\left(c_p(1+\psi)-\hat{R}_0 t_p\right)+(1-\rho)(c_r-\hat{R}_0 t_r)}, \tag{5}$$

where the condition $\rho\left(c_p(1+\psi)-\hat{R}_0 t_p\right)+(1-\rho)(c_r-\hat{R}_0 t_r)>0$ holds.

For each opaque job, the expected number of requests served is $(1-\rho)+\rho(1+\psi)=1+\rho\psi$, where $(1+\psi)$ is the expected number of requests in a pool job. With ψ and ρ from Equations (3) and (2), we note that $1+\rho\psi = \frac{q_p+q_r}{(1-\eta/2)q_p+q_r}$ and $1+\psi=\frac{1}{1-\eta/2}$.

The expected *ridership*, i.e., number of requests served per unit time, is equal to the jobs completed λ_d multiplied by $1+\rho\psi$ requests in each job. Therefore, the platform's ridership under the opaque policy is:

$$\begin{aligned}\Pi_o(c_p,c_r) =& \lambda_d(1+\rho\psi)\\ =& \mu\left(q_p+q_r\right) - \frac{\hat{R}_0(q_p+q_r)}{q_p\left(c_p-(1-\eta/2)\hat{R}_0 t_p\right)+q_r(c_r-\hat{R}_0 t_r)}.\end{aligned}$$

4.2. The Transparent Policy

Under the transparent policy, the drivers are notified about the job type, e.g. pool or regular. They are also allowed to decide their job picking strategies on which jobs to take. The supply of drivers can therefore be split into three streams with arrival rates denoted as λ_r, λ_p and λ_b for regular service only, pool service only and both.

Figure 3 illustrates the corresponding queueing model for the transparent policy. The driver arrivals for pool or regular jobs only join the first and second queue respectively. The flexible drivers

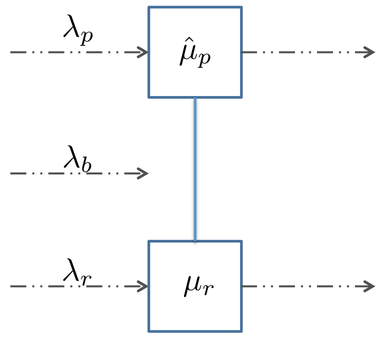

Figure 3 Queues for the Transparent Policy

for both services can be served by any available server. As discussed in Section 3, service rate in the first server node is $\hat{\mu}_p = (1-\eta/2)\mu_p$ which equals the request rate for pool service. Similarly, the service rate in the second server node is the request rate for regular service μ_r.

The flexible drivers that take both types of jobs can be implicitly tagged by the jobs they eventually take when we look backward. With probability ρ, a flexible driver receives a pool job. Effectively, ρ proportion of flexible drivers compete for jobs with the "pool only" drivers. In this case, the conditional expected idle time is $\frac{1}{\hat{\mu}_p-\lambda_p-\rho\lambda_b}$ in the first queue for pool jobs, as the corresponding total driver arrival rate is $\lambda_p+\rho\lambda_b$. Similarly, there is $1-\rho$ probability that a flexible driver is tagged by a regular job with conditional expected idle time $\frac{1}{\mu_r-\lambda_r-(1-\rho)\lambda_b}$ in the queue for regular jobs. Hence, the expected total idle time for a flexible driver is given by $\frac{\rho}{\hat{\mu}_p-\lambda_p-\rho\lambda_b}+\frac{1-\rho}{\mu_r-\lambda_r-(1-\rho)\lambda_b}$.

A rational driver chooses the job picking strategy that maximizes his expected earning rate. The expected earning rate for each job picking strategy, i.e., joining each queue, is provided below:

$$R_p = \frac{c_p(1+\psi)}{\frac{1}{\hat{\mu}_p-\lambda_p-\rho\lambda_b}+t_p} \tag{6}$$

$$R_r = \frac{c_r}{\frac{1}{\mu_r-\lambda_r-(1-\rho)\lambda_b}+t_r} \tag{7}$$

$$R_b = \frac{c_p\rho(1+\psi)+c_r(1-\rho)}{\frac{\rho}{\hat{\mu}_p-\lambda_p-\rho\lambda_b}+\frac{1-\rho}{\mu_r-\lambda_r-(1-\rho)\lambda_b}+t_p\rho+t_r(1-\rho)} \tag{8}$$

We now characterize the driver supplies for pool and regular services in equilibrium.

PROPOSITION 2. *Under the transparent policy, in equilibrium, the total driver arrival rates for pool and regular services are* $\lambda_p+\rho\lambda_b = \hat{\mu}_p - \frac{\hat{R}_0}{c_p(1+\psi)-\hat{R}_0 t_p}$ *and* $\lambda_r+(1-\rho)\lambda_b = \mu_r - \frac{\hat{R}_0}{c_r-\hat{R}_0 t_r}$ *respectively.*

Recall that a pool job takes $(1+\psi)$ pool requests on average and each regular job serves one regular request. The platform's total ridership is therefore $\Pi_t = (\lambda_p+\rho\lambda_b)(1+\psi)+(\lambda_r+(1-\rho)\lambda_b)$.

4.3. Comparison of Policies

We then compare the total driver supplies and ridership between the opaque and transparent policies. Proposition 3 below shows that the total driver arrivals under the opaque policy are larger than that under the transparent policy. As the ride requests are aggregated under the opaque policy, the drivers anticipate a lower expected idle time and thus is more likely to join the platform.

PROPOSITION 3. *Given any pair of c_p and c_r, the total driver arrival rate under the transparent policy is strictly less than that under the opaque policy, i.e., $\lambda_p + \lambda_b + \lambda_r < \lambda_d$.*

In ride-sharing business, providing responsive service is crucial for the platform to grow its market share. Often, the ridership or equivalently number of transactions served as a key performance indicator (KPI) when investors evaluate ride-sharing platforms (see Bloomberg 2017, Financial Times 2017). Thus, we use the ridership, defined as the expected number of requests completed per unit time (e.g., monthly rides), as the objective to be maximized. In fact, it is also straightforward to translate the ridership to service level by normalizing the ridership with the total ride requests.

We can easily extend the result in Proposition 3 to compare the ridership under both policies in Theorem 1.

THEOREM 1. *The platform's ridership under the transparent policy is strictly less than that under the opaque policy, i.e., $\Pi_t = (\lambda_p + \rho\lambda_b)(1+\psi) + (\lambda_r + (1-\rho)\lambda_b) < \Pi_o = \lambda_d(1+\rho\psi)$.*

Therefore, the opaque policy is more responsive in fulfilling requests than the transparent policy. This result supports the industry practices on emphasizing acceptance rate and imposing a high penalty of refusing request (CNNMoney 2018), in order to maintain desired ridership of the platform. With transparent information, drivers are able to pick jobs so that platform's matching efficiency is undermined. When the platform implements the transparent policy, it is better to market pool and regular services as different products and manage the two streams of drivers for each product separately. Since the opaque policy has higher ridership, we will focus on its optimal pricing in the next section.

5. Optimal Pricing in the Opaque Policy

In this section, we explicitly model the rider's mode choice following the multinomial logit (MNL) model. Let α_r (α_p) be the intrinsic utility, e.g., comfort levels, from the regular (pool) service and β be the price sensitivity. We normalize the utility from other travel modes (e.g., public transport) to zero. The probabilities of requesting the regular and pool service $q_r(c_p, c_r)$ and $q_p(c_p, c_r)$ are expressed as:

$$q_r(c_p, c_r) = \frac{e^{\alpha_r - \beta c_r}}{1 + \sum_{i\in\{r,p\}} e^{\alpha_i - \beta c_i}} \quad \text{and} \quad q_p(c_p, c_r) = \frac{e^{\alpha_p - \beta c_p}}{1 + \sum_{i\in\{r,p\}} e^{\alpha_i - \beta c_i}} \tag{9}$$

Recall the formulation of $\Pi_o(c_p, c_r)$ in Section 4.1, we maximize the ridership in the following optimization model:

$$\begin{aligned}
\max_{c_r, c_p, q_r, q_p \geq 0} \quad & \mu\,(q_p + q_r) - \frac{\hat{R}_0(q_p+q_r)}{q_p\left(c_p - (1-\eta/2)\hat{R}_0 t_p\right) + q_r(c_r - \hat{R}_0 t_r)} \\
\text{s.t.} \quad & c_p = \frac{1}{\beta}[\alpha_p + \log(1-q_r-q_p) - \log q_p] \\
& c_r = \frac{1}{\beta}[\alpha_r + \log(1-q_r-q_p) - \log q_r] \\
& q_p\left(c_p - (1-\eta/2)\hat{R}_0 t_p\right) + q_r(c_r - \hat{R}_0 t_r) \geq 0
\end{aligned} \tag{10}$$

The first and second constraints are obtained by reorganizing Equation (9) to write c_p and c_r as functions of q_r and q_p. The last constraint is the feasibility condition specified in Equation (5).

Let $\kappa_r = \alpha_r - \beta(\hat{R}_0 t_r)$ and $\kappa_p = \alpha_p - \beta(1-\eta/2)\hat{R}_0 t_p$. To avoid trivial solutions, we assume $\kappa_r \geq 0$, i.e., $\alpha_r \geq \beta(\hat{R}_0 t_r)$, such that the rider's utility from regular service is positive under the lowest possible price $\hat{R}_0 t_r$. By defining $\theta = \frac{q_p}{q_p+q_r} \in [0,1]$ as the relative proportion of pool requests among all requests received by the platform and $\hat{q} = q_p + q_r \in [0,1]$ as the total market share, we reformulate problem (10) with decision variables θ and $\hat{q}$ as below:

$$\begin{aligned}
\max_{0 \leq \theta, \hat{q} \leq 1} \quad & \mu\hat{q} - \frac{\beta\hat{R}_0}{\kappa_r + (\kappa_p - \kappa_r)\theta + \log(1-\hat{q}) - \log\hat{q} - \theta\log\theta - (1-\theta)\log(1-\theta)} \\
\text{s.t.} \quad & \kappa_r + (\kappa_p - \kappa_r)\theta + \log(1-\hat{q}) - \log\hat{q} - \theta\log\theta - (1-\theta)\log(1-\theta) \geq 0
\end{aligned} \tag{11}$$

In Proposition 4, we are able to provide a closed-form solution to the optimal θ^* in problem (11).

Proposition 4. *The optimal θ^* in problem* (11) *is $\theta^* = \frac{e^{\kappa_p}}{e^{\kappa_r}+e^{\kappa_p}}$.*

We note that θ^* only depends on $(\kappa_p - \kappa_r)$ which is the difference between the expected utilities of pool and regular services, when the platform sets $c_p = (1-\eta/2)\hat{R}_0 t_p$ and $c_r = R_o t_r$. The result follows from the choice probabilities in (9). As θ^* is increasing in $(\kappa_p - \kappa_r)$, we analyze the properties of θ^* in Corollary 1 through the monotonicity of $(\kappa_p - \kappa_r)$.

Corollary 1. *The optimal $\theta^* = \frac{q_p^*}{q_p^*+q_r^*}$ is*

1. *increasing in α_p and decreasing in α_r*
2. *increasing in $\hat{R}_0$ and η if $\Delta t \leq \frac{t_r}{2}$; and decreasing in $\hat{R}_0$ and η if $\Delta t > \frac{t_r}{2}$.*

The first result in Corollary 1 is straightforward that the relative proportion of pool requests becomes higher when the intrinsic utility in pool service improves. The second result, however, suggests that improving pairing efficiency η reduces the optimal proportion of pool requests, when the additional pickup or dropoff time exceeds 50% of the regular trip. Thus, it is better for the platform to discourage pool requests, when the detour to pick up additional rider is long. In the similar case, pool service becomes less attractive to the drivers when their reserved earnings are higher and it therefore makes the platform favor the regular service more. When it is convenient to pick up additional rider, e.g., $\Delta t \le \frac{t_r}{2}$, the pool service is more profitable when the platform has higher pairing efficiency η. In that case, the pool service also becomes relatively more attractive to the drivers compared to the regular service, when their reserved earnings $\hat{R}_0$ increase. Note that θ^* represents the relative proportion of pool requests compared with the total requests received by the platform. To characterize the market share, we further examine $\hat{q} = q_p + q_r$.

With the optimal θ^* provided in Proposition 4, problem (11) can be simplified as:

$$\max_{0 \le \hat{q} \le \frac{e^{\kappa_r}+e^{\kappa_p}}{1+e^{\kappa_r}+e^{\kappa_p}}} \quad H(\hat{q}) = \mu\hat{q} - \frac{\beta\hat{R}_0}{\log(e^{\kappa_r}+e^{\kappa_p}) + \log(1-\hat{q}) - \log\hat{q}} \tag{12}$$

where $H(0) = 0$ and $H\left(\frac{e^{\kappa_r}+e^{\kappa_p}}{1+e^{\kappa_r}+e^{\kappa_p}}\right) = -\infty$.

We first characterize the behavior of the objective function $H(x)$ in Lemma 2.

LEMMA 2. *There exist* $x_1 \in [0, \frac{1}{2}]$ *and* $x_2 \in [\frac{1}{2}, 1]$, *such that*

1. $H(x)$ *is convex in* x, *if* $x \in [0, x_1]$.
2. $H(x)$ *is concave in* x, *if* $x \in \left(x_1, \frac{e^{\kappa_r}+e^{\kappa_p}}{1+e^{\kappa_r}+e^{\kappa_p}}\right]$.

where x_1 *and* x_2 *are the roots to equation* $(2x-1)\left[\log(e^{\kappa_r}+e^{\kappa_p}) + \log(1-x) - \log x\right] + 2 = 0$.

The results in Lemma 2 can be visualized by the example in Figure 4, where the parameters are selected only for illustration purpose.

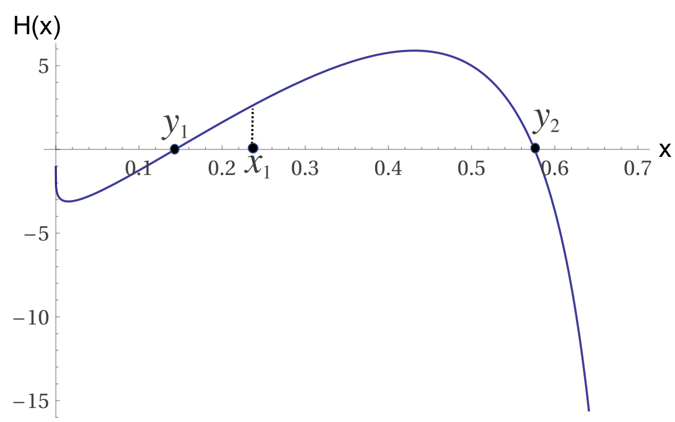

Figure 4 **An Example of** $H(x)$ **with** $\mu = 0.0177$, $\beta = 0.007$, $\hat{R}_0 = 0.556$, $\kappa_r = 0.221$ **and** $\kappa_p = 0.093$.

As we aim to maximize the ridership, we are interested in the range of $\hat{q}$ such that $H(\hat{q}) \ge 0$. Define $F(x) = x\left[\log(e^{\kappa_r}+e^{\kappa_p}) + \log(1-x) - \log x\right]$. One can check that $F(0) = 0$, $F\left(\frac{e^{\kappa_r}+e^{\kappa_p}}{1+e^{\kappa_r}+e^{\kappa_p}}\right) =$

0 and $F(x)$ is concave, i.e., $F''(x) \leq 0$. Then there exist at most two roots $y_1 > 0$ and $y_2 > 0$ to $F(x) = \frac{\beta \hat{R}_0}{\mu}$. Equivalently, y_1 and y_2 are also the roots to $H(x) = 0$ as shown in Figure 4. Suppose $y_2 \geq y_1$. For any $x \in [y_1, y_2]$, we have $F(x) \geq \frac{\beta \hat{R}_0}{\mu}$ and hence $H(x) \geq 0$. Thus, we only need to focus on the interval $\hat{q} \in [y_1, y_2]$. The above observations help us to derive the optimal $\hat{q}^*$ provided below.

THEOREM 2. *The optimal $\hat{q}^*$ in problem* (12) *is given by the following:*

1. *If $H(x) = 0$ has no or only one non-zero root, we have $\hat{q}^* = 0$.*

2. *Otherwise, let y_1 and y_2 be the nonzero roots to equation $H(x) = 0$. We have that $\hat{q}^* \in [y_1, y_2]$ is a root to*

$$(1 - \hat{q}^*)\hat{q}^* \left[\log(e^{\kappa_r} + e^{\kappa_p}) + \log(1 - \hat{q}^*) - \log \hat{q}^*\right]^2 = \frac{\beta \hat{R}_0}{\mu}.$$

Recall that the objective function $H(x)$ can be viewed as the total request $\mu \hat{q}$ less the supply shortage $\frac{\beta \hat{R}_0}{\log(e^{\kappa_r} + e^{\kappa_p}) + \log(1-\hat{q}) - \log \hat{q}}$. The first case in Theorem 2 corresponds to the condition $H(x) \leq 0$, i.e., the supply shortage is larger than the total request, that leads to no request fulfillment, i.e., $\hat{q}^* = 0$. For instance, if the driver's reserved earning R_0 is very high, few driver would join the platform. It creates a huge shortage in supply that poses operational challenges under such market condition.

With both θ^* and $\hat{q}^*$ obtained, we get the corresponding optimal choice probabilities $q_r^* = (1 - \theta^*)\hat{q}^*$ and $q_p^* = \theta^* \hat{q}^*$. The optimal prices for regular and pool service are therefore given by $c_r^* = \frac{1}{\beta}[\alpha_r + \log(1 - q_r^* - q_p^*) - \log q_r^*]$ and $c_p^* = \frac{1}{\beta}[\alpha_p + \log(1 - q_r^* - q_p^*) - \log q_p^*]$. In the following section, we will examine a real dataset to discuss the optimal pricing for pool and regular services under various conditions.

6. Case Study

In this section, we use a set of detailed customer tracking data from a ride-sharing platform to specify the proposed model. We first explain the data preparation and then discuss the observations from the numerical experiments.

Data Preparation

The dataset is from a major ride-sharing platform offering both pool and regular services in a Chinese city. It tracks a sample of 49,897 customers from July to August in 2016. The tracking data contain every "bubble" record when a customer entered origin-destination (OD) information and checked the prices on the mobile application, as well as the subsequent request information if a ride request is made and the trip information if an associated trip is completed.

To calibrate the travel mode choices from more than 4 million "bubbles", we need to identify the travel demands of riders and their choices: regular service, pool service or leave the platform and seek for an outside option including taxi and public transport. For a single travel demand, the rider may check on the mobile application several times and may or may not eventually place a request. For each customer, we cluster the "bubbles" that have the same OD pair and appear within a short period, e.g., within an hour, and refer to the clustered record as a *session*. In each session, we identify the rider's choice of regular or pool service based on the request information when available and classify the choice to be the outside option of no request in the cluster. Our clustering criteria lead to a total 1,068,551 sessions. By checking whether exactly a single choice is made within a session, we then removed 0.87% mis-clustered sessions with multiple choices. We further remove the noisy sessions with the OD distance less than 2km or larger than 200km, as the most city driving range is between the two thresholds. The resulting dataset contains 924,595 sessions with 45.13% regular requests, 16.13% pool requests and 38.74% no request.

Furthermore, our model considers average trips in a single market. Hence, we need to standardize the different distances of all trips to an average trip distance $\bar{d} = 8.484$ km. The distance-normalized price is calculated as $c_i = \frac{\hat{c}_i}{\hat{d}_i}\bar{d}$, where $\hat{c}_i$ and $\hat{d}_i$ are the prices and distances for $i = \{\text{regular, pool}\}$ in a session. Similar distance adjustment on price is also applied to the outside option. We fit the multinomial choice model (9) and the results from the multinomial logistic regression are provided in Table 1. As noted from Table 1, for a typical trip, i.e., with distance $\bar{d} = 8.484$ km, a rider's expected utility is decreasing in price. Moreover, as we normalize the intrinsic utility from the outside option to zero, the result suggests positive intrinsic utility from the regular service with a higher comfort level. The pool service, however, has negative intrinsic utility due to possibly higher inconvenience to the regular service and developed outside option, e.g., taxi or mass transit railway. Thus, we then set the parameters as $\alpha_r = 0.534$, $\alpha_p = -0.735$ and $\beta = 0.028$ in the choice model.

Table 1 The Results of Multinomial Logistic Regression

Coefficients	Estimate	Std. Error	t-value	Pr(> |t|)
α_r	0.534	0.0044	120.44	$< 2.2\text{e}{-16}$ ***
α_p	-0.735	0.0034	-217.08	$< 2.2\text{e}{-16}$ ***
$-\beta$	-0.028	0.0003	-100.37	$< 2.2\text{e}{-16}$ ***

Signif. codes: 0 '***', 0.001 '**', 0.01 '*'

To conduct numerical experiments, we further calibrate the other parameters. We use the average number of sessions per minute as the travel demand rate $\mu = 8.86$/min. Considering both the traffic congestion and city expressway, we set the average speed $s = 500$m/min (30km/hr). Therefore, the average travel time t_r is given by $\frac{\bar{d}}{s} = 16.97$ min. Among all pool requests, there are 78% of them successfully paired, i.e., $\eta = 0.78$. To account for the pairing algorithm bias in the estimation of Δt, we first normalize the actual distance traveled by the predicted OD distance (which is unit free after the normalization) for all pool trips and measure the average difference on the normalized distance. The calculation shows that $\Delta t = 0.19 t_r$ for an additional pickup or dropoff. Finally, we use the taxi fare (adjusted with rental fees) as an approximate to the reserved earning rate for the driver as $R_0 = 0.525$ CNY/min (Xinhua News 2013) and it is equivalent to $\hat{R}_0 = 0.656$ CNY/min, using a payout ratio $\omega = 80\%$ reported in Bai et al. (2018).

Numerical Results

Before computing the optimal prices for both services, we examine the effectiveness of current prices. The average distance-normalized prices are $c_r = 21.82$ and $c_p = 13.42$ for regular and pool service respectively. The fitted MNL model predicts the market shares as 41.06% for regular service, 14.60% for pool service and 44.34% for the outside option. Our model suggests that in equilibrium, the per-minute driver arrival rate under the opaque policy is 4.363 while the total driver arrival rate under the transparent policy is 4.284 with 3.576 for regular service and 0.708 for pool service. The total ridership under the opaque policy is 4.86 trips per minute, that is 2.59% higher than the ridership under the transparent policy. In the discussion below, we refer to this case as the *status quo*.

We now optimize over the prices c_r and c_p for the opaque policy using the proposed framework. Our solution suggests the optimal prices $c_r = 14.45$ and $c_p = 11.76$ under the opaque policy, which lead to market shares 45.83% for regular service and 13.89% for pool service. The reduction on both prices increase the total market share from the *status quo*. Moreover, the solution narrows the gap, i.e., the optimal pool price is around 20% off from the optimal regular price instead of around 40% gap in the *status quo*. It enhances the market share for regular service. The opaque policy results in a total ridership of 5.09 trips per minute, which is 4.04% higher than the transparent policy and also an improvement of 4.73% from the *status quo*.

Table 2 Impacts of Reserved Earning Rate R_0

Scale of R_0	c_p	c_r	θ	$\mu(q_r + q_p)$	Π_o	γ_o	Π_t	γ_t
1	11.76	14.45	23.26%	5.29	5.09	96.26%	4.90	92.52%
1.1	12.75	15.72	23.40%	5.22	5.01	96.01%	4.80	92.02%
1.2	13.74	16.97	23.53%	5.15	4.93	95.76%	4.71	91.53%
1.3	14.73	18.23	23.67%	5.08	4.85	95.52%	4.62	91.04%
1.4	15.70	19.47	23.80%	5.00	4.77	95.27%	4.53	90.54%
1.5	16.68	20.71	23.94%	4.93	4.69	95.02%	4.44	90.05%
1.6	17.64	21.95	24.08%	4.86	4.61	94.77%	4.35	89.55%
1.7	18.61	23.19	24.22%	4.79	4.53	94.53%	4.26	89.05%
1.8	19.57	24.42	24.36%	4.72	4.45	94.27%	4.18	88.55%
1.9	20.53	25.65	24.49%	4.64	4.37	94.02%	4.09	88.04%
2	21.49	26.88	24.63%	4.57	4.29	93.76%	4.00	87.53%

In the remaining of the numerical results, we study the impacts of driver's reserved earning rate, demand size and pairing efficiency in the optimal prices under the opaque policy, ridership and service levels under both policies. That is, we optimize the prices under the opaque policy and use the optimized prices to evaluate the performance metrics under both the opaque and transparent policies. We define the service level as the proportion of requests fulfilled, i.e., $\gamma_o = \frac{\Pi_o}{\mu}$ and $\gamma_t = \frac{\Pi_t}{\mu}$ for the opaque and transparent policies respectively. In Table 2, we observe the changes in prices, ridership and service levels, as R_0 scales from 100% to 200%. When R_0 is higher, the prices for both services increase under the opaque policy. It indicates that if the prices in the *status quo* were (near) optimal, the actual R_0 in that market is likely to be $40 \sim 50\%$ higher than our estimation. Since $\Delta t < 50\% t_r$ in the study, the optimal θ increases in R_0 as indicated in Corollary 1. We also notice that the opaque policy is relatively more robust to the transparent policy, as the service level under the transparent policy decreases faster.

Table 3 Impacts of Demand Size

Demand Scale	c_p	c_r	$\mu(q_r + q_p)$	Π_o	γ_o	Π_t	γ_t
0.1	18.78	21.48	0.49	0.42	86.96%	0.36	73.91%
0.2	15.79	18.48	1.01	0.92	91.15%	0.83	82.31%
0.3	14.46	17.15	1.54	1.43	92.91%	1.32	85.82%
0.4	13.66	16.35	2.07	1.95	93.93%	1.82	87.86%
0.5	13.12	15.81	2.61	2.46	94.61%	2.32	89.22%
0.6	12.71	15.41	3.14	2.99	95.11%	2.83	90.21%
0.7	12.40	15.09	3.68	3.51	95.49%	3.35	90.98%
0.8	12.15	14.84	4.21	4.04	95.80%	3.86	91.59%
0.9	11.94	14.63	4.75	4.57	96.05%	4.38	92.09%
1	11.76	14.45	5.29	5.09	96.26%	4.90	92.52%
1.1	11.61	14.30	5.83	5.62	96.44%	5.42	92.88%
1.2	11.47	14.17	6.37	6.15	96.60%	5.94	93.20%
1.3	11.35	14.04	6.91	6.69	96.73%	6.46	93.47%
1.4	11.25	13.94	7.45	7.22	96.86%	6.98	93.72%
1.5	11.16	13.85	7.99	7.75	96.97%	7.51	93.94%
1.6	11.07	13.76	8.53	8.28	97.07%	8.03	94.14%
1.7	10.99	13.68	9.07	8.82	97.16%	8.56	94.32%
1.8	10.92	13.61	9.62	9.35	97.24%	9.09	94.48%
1.9	10.86	13.55	10.16	9.88	97.32%	9.61	94.64%
2	10.80	13.49	10.70	10.42	97.39%	10.14	94.78%

By scaling the demand from 0.1 to 2 times from the *status quo*, we summarize the results in Table 3. As the demand scales up, both prices for the regular and pool service decrease to induce more customers to use ride-sharing services and it also becomes easier to attract drivers due to less idle time. The platform also benefits from the increasing demand with improvements in the service levels under both policies, e.g., from 87.0% to 97.4% for the opaque policy and from 73.9%

Table 4 Impact of Paring Efficiency: η

η	c_p	c_r	$q_p \times \mu$	$q_r \times \mu$	γ_o^r	γ_o^p	γ_t^r	γ_t^p
0.1	14.10	14.45	1.16	4.10	96.23%	96.23%	95.16%	82.96%
0.2	13.76	14.45	1.17	4.09	96.24%	96.24%	95.16%	83.11%
0.3	13.41	14.45	1.18	4.09	96.24%	96.24%	95.15%	83.25%
0.4	13.07	14.45	1.19	4.08	96.25%	96.25%	95.15%	83.39%
0.5	12.72	14.45	1.20	4.08	96.25%	96.25%	95.14%	83.54%
0.6	12.38	14.45	1.21	4.07	96.25%	96.25%	95.14%	83.68%
0.7	12.04	14.45	1.22	4.07	96.26%	96.26%	95.13%	83.82%
0.8	11.69	14.45	1.23	4.06	96.26%	96.26%	95.13%	83.95%
0.9	11.35	14.45	1.24	4.05	96.27%	96.27%	95.12%	84.09%
1	11.00	14.45	1.25	4.05	96.27%	96.27%	95.12%	84.22%

to 94.8% for the transparent policy. Furthermore, in all cases, the opaque policy dominates the transparent policy in the ridership and subsequently service level, as discussed in Theorem 1.

We continue to examine the impact of the efficiency of pairing algorithm in two dimensions: the pairing probability η and the additional travel time for pickup (or dropoff) Δt in a paired trip. We include Δt as it measures the similarity of the paired trips, e.g., longer detour suggests less trip similarity. In Table 4, we vary the pairing probability from 0.1 to 1 and report the detailed service levels γ_o^r and γ_t^r for regular service and γ_o^p and γ_t^p for pool service. As η increases, the optimal price for pool service decreases as it becomes less costly to offer pool service via more efficient pairing. Under the opaque policy, the service levels for both services increase slightly as the pairing efficiency improves. Moreover, both service levels are equal, as the drivers can not picking jobs based on the types. Under the transparent policy, we see lower service level for pool service, as a result of the driver's job picking behavior. We also notice that the improvement in pairing probability brings the highest benefits to the pool service under the transparent policy.

Table 5 Impact of Paring Efficiency: Δt

$\Delta t/t_r$	c_p	c_r	$q_p \times \mu$	$q_r \times \mu$	γ_o^r	γ_o^p	γ_t^r	γ_t^p
0.05	10.55	14.46	1.27	4.04	96.28%	96.28%	95.11%	84.40%
0.10	10.98	14.45	1.25	4.05	96.27%	96.27%	95.11%	84.23%
0.15	11.41	14.45	1.24	4.06	96.26%	96.26%	95.12%	84.06%
0.20	11.85	14.45	1.23	4.06	96.26%	96.26%	95.13%	83.89%
0.25	12.28	14.45	1.22	4.07	96.25%	96.25%	95.14%	83.72%
0.30	12.71	14.45	1.20	4.08	96.25%	96.25%	95.14%	83.54%
0.35	13.15	14.45	1.19	4.08	96.24%	96.24%	95.15%	83.36%
0.40	13.58	14.45	1.18	4.09	96.24%	96.24%	95.16%	83.18%
0.45	14.01	14.45	1.17	4.10	96.23%	96.23%	95.16%	83.00%
0.50	14.45	14.45	1.15	4.10	96.23%	96.23%	95.17%	82.81%

To discuss trip similarity, we define the detour ratio as $\frac{\Delta t}{t_r}$ and vary it from 5% to 50% in Table 5. With higher similarity in paired trips, i.e., smaller $\frac{\Delta t}{t_r}$, the platform is able to offer lower price for pool service as the detour inconvenience of a paired trip is reduced to the drivers. Similar to the observation in Table 4, both the price and demand for regular services are relatively insensitive to the pairing efficiency. We also observe significant improvement in service level for pool service under the transparent policy. The results from Table 4 and 5 suggest that both the pairing probability and trip similarity are the key aspects of the platform's pairing algorithm in offering market competitive pool service, especially for the platform using the transparent policy.

7. Conclusion

This paper studies the information provision policy and pricing in a ride-sharing platform that offers both pool and regular services. Using a queueing model for the dynamics of riders and drivers, we solve the equilibrium under the opaque and transparent information provision policies respectively. Our analysis shows higher ridership under the opaque policy, where the optimal prices for both services are further characterized. In a real-world case study, we calibrate the proposed model using the customer tracking data from a ride-sharing platform and conduct numerical experiments to study the impacts of several factors, including driver's reserved earning rate, demand size and pairing efficiency for pool service. Besides the favorable market conditions, such as low driver's reserved earning rate and large demand size, our results encourage the algorithmic improvements in pairing efficiency to increase pairing probability and reduce detour in paired trips, especially when the platform adopts the transparent policy.

To focus on the performances of information provision policies, we simplify our model to consider a single market with a fixed commission payout scheme. Further research can potentially extend in either dimension. First, by relaxing the fixed commission setting, the platform can optimize the prices of both services and driver's wage simultaneously. One may examine the performance of fixed commission and explore simple (near) optimal payout schemes, when the platform offers two types of services. Second, an extension that expands the single market setting to a network of locations is also interesting and yet challenging. One may discuss the information provision policy on whether to reveal destination information to the drivers. By designing effective information provision policy together with spatial pricing scheme on a network, the platform needs to deal with driver's job picking behavior based on trip destination, in order to sustain desirable ridership while maintaining fairness to riders.

Appendix A: Notation

Table 6 Notation

Parameter	Definition
t_r	Expected travel time in regular service and unpaired pool trip
Δt	Expected time for additional pickup or dropoff in paired pool trip
t_p	Expected travel time in pool service
α_r, α_p	Intrinsic utility from regular/pool service
β	Price sensitivity of riders
μ	Demand size: total travel demand rate
λ	Supply size: total driver arrival rate
η	Pairing probability in pool service
$\psi(\eta)$	Probability of a driver in pool service having paired requests
$\hat{R}_0$	Driver's reserved earning rate

Variables	Definition
c_r	Price of regular service
c_p	Price of pool service
q_r	Rider's probability of using regular service
q_p	Rider's probability of using pool service
μ_r	Request rate of regular service
μ_p	Request rate of pool service
$\hat{\mu}$	Total equivalent rate of cars needed
λ_d	Driver arrival rate in the opaque policy
λ_p	Driver arrival rate for only pool service in the transparent policy
λ_r	Driver arrival rate for only regular service in the transparent policy
λ_b	Driver arrival rate for both services in the transparent policy
ρ	Proportion of pool jobs

Appendix B: Proofs

Proof of Lemma 1:

We first derive the distribution of queue length. Since the cars are all identical and interchangeable from the platform's perspective, we can temporarily assume the cars depart the queue in a first-in-first-out manner to in the derivation of the queue length. With the FIFO discipline, the total number of drivers in the queue follows the geometric distribution with probability $1-\frac{\lambda_d}{\hat{\mu}}$ with the average queue length as $L=\frac{\lambda_d}{\hat{\mu}-\lambda_d}$. By Little's law that does not require the FIFO assumption, we have the average waiting time as $W=\frac{L}{\lambda_d}=\frac{1}{\hat{\mu}-\lambda_d}$. $\square$

Proof of Proposition 2:

In order to analyze the equilibrium, we first provide the following lemma.

LEMMA 3. *For positive real numbers $A,B,C,D \in \mathbb{R}^+$, we have $\frac{A+C}{B+D} \leq \max\{\frac{A}{B}, \frac{C}{D}\} < \frac{A}{B} + \frac{C}{D}$.*

Let $\frac{A}{B}=x>0$ and $\frac{C}{D}=y>0$. Without loss of generality, we assume that $x\leq y$. We then have $\frac{A+C}{B+D}=\frac{Bx+Dy}{B+D}\leq\frac{By+Dy}{B+D}=y$. The second inequality holds, e.g., $\max\{x,y\}=y<x+y$, since $x>0$. This completes the proof of Lemma 3.

Let $A=c_p(1+\psi), B=\frac{1}{\hat{\mu}_p-\lambda_p-\rho\lambda_b}+t_p, C=c_r$ and $D=\frac{1}{\mu_r-\lambda_r-(1-\rho)\lambda_b}+t_r$, we see that $R_p=\frac{A}{B}, R_r=\frac{C}{D}$ and $R_b=\frac{\rho A+(1-\rho)C}{\rho B+(1-\rho)D}$. From Lemma 3, we note that $R_b\leq\max\{\frac{\rho A}{\rho B},\frac{(1-\rho)C}{(1-\rho)D}\}=\max\{R_p,R_r\}$ with equality holds if and only if $R_p=R_r$. Thus, we have $R_p=R_r=R_b=\hat{R}_0$ in equilibrium and the results follow by solving equations (6) to (8). Since there are multiple solutions, e.g., by setting different λ_b, the equilibrium is not unique. □

Proof of Proposition 3:

In equilibrium, we have

$$\begin{aligned}
\lambda_p+\lambda_b+\lambda_r &= \hat{\mu}_p+\mu_r-\frac{\hat{R}_0}{c_p(1+\psi)-t_p\hat{R}_0}-\frac{\hat{R}_0}{c_r-t_r\hat{R}_0}\\
&< \hat{\mu}_p+\mu_r-\frac{\rho\hat{R}_0+(1-\rho)\hat{R}_0}{\rho(c_p(1+\psi)-\hat{R}_0t_p)+(1-\rho)(c_r-\hat{R}_0t_r)}\\
&= \hat{\mu}-\frac{\hat{R}_0}{\rho\left(c_p(1+\psi)-\hat{R}_0t_p\right)+(1-\rho)(c_r-\hat{R}_0t_r)}\\
&= \lambda_d
\end{aligned}$$

where the inequality is a direct result from Lemma 3. □

Proof of Theorem 1:

The expected riders served in a pool request is $(1+\psi)$ and that by a regular trip is 1. Hence, the response rate in the transparent policy is

$$\begin{aligned}
(\lambda_p+\rho\lambda_b)(1+\psi)+(\lambda_r+(1-\rho)\lambda_b) &= \hat{\mu}_p(1+\psi)-\frac{\hat{R}_0(1+\psi)}{c_p(1+\psi)-\hat{R}_0t_p}+\mu_r-\frac{\hat{R}_0}{c_r-\hat{R}_0t_r}\\
&= \mu_p(1-\frac{\eta}{2})(1+\psi)-\frac{\hat{R}_0(1+\psi)}{c_p(1+\psi)-\hat{R}_0t_p}+\mu_r-\frac{\hat{R}_0}{c_r-\hat{R}_0t_r}\\
&= \mu(q_p+q_r)-\frac{\hat{R}_0(1+\psi)\rho}{\rho(c_p(1+\psi)-\hat{R}_0t_p)}-\frac{\hat{R}_0(1-\rho)}{(1-\rho)(c_r-\hat{R}_0t_r)}\\
&< \mu(q_p+q_r)-\frac{\hat{R}_0[\rho(1+\psi)+(1-\rho)]}{\rho(c_p(1+\psi)-\hat{R}_0t_p)+(1-\rho)(c_r-\hat{R}_0t_r)}=\lambda_d(1+\rho\psi).
\end{aligned}$$

The RHS is the response rate in the opaque policy. □

Proof of Proposition 4:

Define $g(\theta)=(\kappa_p-\kappa_r)\theta-\theta\log\theta-(1-\theta)\log(1-\theta)$. The first and second derivatives are

$$\begin{aligned}
\frac{\partial g}{\partial\theta} &= (\kappa_p-\kappa_r)-\log\theta+\log(1-\theta)\\
\frac{\partial^2 g}{\partial\theta^2} &= -\frac{1}{\theta}-\frac{1}{1-\theta}\leq 0
\end{aligned}$$

Therefore, the function $g(\theta)$ is concave in θ. By solving the first order condition, i.e., $(\kappa_p - \kappa_r) - \log\theta + \log(1-\theta) = 0$, we have $\theta^* = \frac{e^{\kappa_p}}{e^{\kappa_r}+e^{\kappa_p}}$. $\square$

Proof of Corollary 1:

Note that $t_p = t_r + 2\psi\Delta t$ and $\psi = \frac{\eta/2}{1-\eta/2}$, we rewrite $(\kappa_p - \kappa_r)$ as follows:

$$\begin{aligned}(\kappa_p - \kappa_r) &= \alpha_p - \beta(1-\eta/2)\hat{R}_0 t_p - \alpha_r + \beta\hat{R}_0 t_r \\ &= \alpha_p - \alpha_r + \beta\hat{R}_0\left(t_r - (1-\eta/2)t_p\right) \\ &= \alpha_p - \alpha_r + \beta\hat{R}_0\eta\left(\frac{t_r}{2} - \Delta t\right)\end{aligned}$$

When $\frac{t_r}{2} - \Delta t \geq 0$, it is clear that $(\kappa_p - \kappa_r)$ is increasing in $\hat{R}_0$ and η; otherwise, $(\kappa_p - \kappa_r)$ is decreasing in $\hat{R}_0$ and η. Furthermore, it is straightforward that $(\kappa_p - \kappa_r)$ is increasing in α_p and decreasing in α_r. Since θ^* is increasing in $(\kappa_p - \kappa_r)$, the above monotonicity preserves. $\square$

Proof of Lemma 2:

We first derive the second order derivative of $H(x)$:

$$H''(x) = -\beta\hat{R}_0 \frac{(2x-1)\left[\log(e^{\kappa_r}+e^{\kappa_p}) + \log(1-x) - \log x\right] + 2}{(x-1)^2x^2\left[\log(e^{\kappa_r}+e^{\kappa_p}) + \log(1-x) - \log x\right]^3}.$$

Define the function $G(x) = (2x-1)\left[\log(e^{\kappa_r}+e^{\kappa_p}) + \log(1-x) - \log x\right] + 2$. By checking the second order derivative of $G(x)$:

$$G''(x) = -\frac{1}{(1-x)^2x^2} < 0,$$

we notice that $G(x)$ is concave in $x \in [0,1]$. Furthermore, we have

$$G(0) = \lim_{x\to 0} G(x) = -\infty, G(\frac{1}{2}) = 2, \text{and } G(1) = \lim_{x\to 1} G(x) = -\infty$$

Therefore, there exist $x_1 \in [0, \frac{1}{2}]$ and $x_2 \in [\frac{1}{2}, 1]$ such that $G(x_1) = G(x_2) = 0$, as shown in Figure 5.

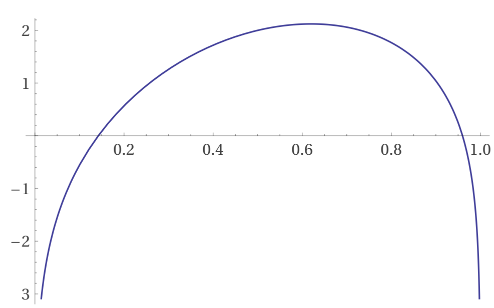

Figure 5 **An Example of** $G(x)$

Since $\kappa_r > 0$, we have $\frac{e^{\kappa_r}+e^{\kappa_p}}{1+e^{\kappa_r}+e^{\kappa_p}} > \frac{1}{1+1} = \frac{1}{2} > x_1$. Moreover, since $G(\frac{e^{\kappa_r}+e^{\kappa_p}}{1+e^{\kappa_r}+e^{\kappa_p}}) = 2 > G(x_2) = 0$, we have $\frac{e^{\kappa_r}+e^{\kappa_p}}{1+e^{\kappa_r}+e^{\kappa_p}} < x_2$. Thus, we only need to consider the intervals $[0, x_1]$ and $(x_1, \frac{e^{\kappa_r}+e^{\kappa_p}}{1+e^{\kappa_r}+e^{\kappa_p}}]$. As discussed above, $G(x) \le 0$ when $x \in [0, x_1]$ and $G(x) > 0$ when $x \in (x_1, \frac{e^{\kappa_r}+e^{\kappa_p}}{1+e^{\kappa_r}+e^{\kappa_p}}]$. Therefore, we have $H''(x) \ge 0$ and $H(x)$ is convex when $x \in [0, x_1]$; and $H''(x) < 0$ and $H(x)$ is concave when $x \in (x_1, \frac{e^{\kappa_r}+e^{\kappa_p}}{1+e^{\kappa_r}+e^{\kappa_p}}]$. $\square$

Proof of Theorem 2:

If $F(x) = \frac{\beta \hat{R}_0}{\mu}$ has no or only one root, then $H(\hat{q}) \le 0$ for $\hat{q} \in \left[0, \frac{e^{\kappa_r}+e^{\kappa_p}}{1+e^{\kappa_r}+e^{\kappa_p}}\right]$. It is optimal to simply set $\hat{q}^* = 0$ with $H(0) = 0$.

Otherwise, let y_1 and y_2 be the nonzero roots to equation $F(x) = \frac{\beta \hat{R}_0}{\mu}$. We discuss by the following cases.

1. If $x_1 < y_1$, then we have $H(x)$ is concave in $x \in [y_1, y_2]$. Moreover, since $H(y_1) = H(y_2) = 0$, there is an maximum exists in $[y_1, y_2]$ given by the first order condition $H'(x) = 0$.

2. If $x_1 \in [y_1, y_2]$, then $H(x)$ is convex in $[y_1, x_1]$ and concave in $[x_1, y_2]$. Note that $H(0) = H(y_1) = 0$ and $H(x)$ is convex in $[0, y_1]$. We have that $H'(y_1) > 0$ and $H''(y_1) > 0$. Therefore, $H(x_1) > 0$, $H'(x_1) > 0$ and $H''(x_1) = 0$, where the last equality is by definition of x_1. Since $H(x_1) > H(y_2) = 0$ and $H'(x_1) > 0$, by the concavity of $H(x)$ in $[x_1, y_2]$, we also have the maximum exists between $[x_1, y_2]$ given by the first order condition $H'(x) = 0$.

3. If $x_1 > y_2$, then $H(x)$ is strictly convex in $[y_1, y_2]$. Since $H'(y_1) > 0$, $H'(x) > 0$ for all $x \in [y_1, y_2]$. Therefore, $H(y_2) > H(y_1)$. Recall that $H(y_1) = H(y_2) = 0$ by definition. It leads to the contradiction. Thus, this case will not happen.

In summary, when there exists two distinguished roots y_1 and y_2, we can explicitly write the optimal solution $\hat{q} \in [y_1, y_2]$ satisfies the first order condition as

$$(1-\hat{q}^*)\hat{q}^* \left[\log(e^{\kappa_r} + e^{\kappa_p}) + \log(1-\hat{q}^*) - \log \hat{q}^*\right]^2 = \frac{\beta \hat{R}_0}{\mu}.$$

$\square$

References

Bai, Jiaru, Kut C So, Christopher S Tang, Xiqun Chen, Hai Wang. 2018. Coordinating supply and demand on an on-demand service platform with impatient customers. *Manufacturing & Service Operations Management* .

Banerjee, Siddhartha, Carlos Riquelme, Ramesh Johari. 2015. Pricing in ride-share platforms: A queueing-theoretic approach. *Working paper, Cornell University* .

Bellos, Ioannis, Mark Ferguson, L Beril Toktay. 2017. The car sharing economy: Interaction of business model choice and product line design. *Manufacturing & Service Operations Management* .

Ben-Akiva, Moshe E, Steven R Lerman. 1985. *Discrete choice analysis: theory and application to travel demand*, vol. 9. MIT press.

Benjaafar, Saif, Guangwen Kong, Xiang Li, Costas Courcoubetis. 2018. Peer-to-peer product sharing: Implications for ownership, usage, and social welfare in the sharing economy. *Management Science* .

Benjaafar, Saif, Xiang Li, Xiaobo Li. 2017. Inventory repositioning in on-demand product rental networks. *Working paper, University of Minnesota* .

Bimpikis, Kostas, Ozan Candogan, Saban Daniela. 2016. Spatial pricing in ride-sharing networks. *Working Paper* .

Bloomberg. 2015. The Future of Uber and Lyft: A Crowded Back Seat. `https://www.bloomberg.com/news/articles/2015-01-29/uber-lyft-bet-big-on-carpool-services`. [29-Jan-2015].

Bloomberg. 2017. Lyft Bookings and Ridership Soar, While Losses Shrink . `https://www.bloomberg.com/news/articles/2017-04-27/lyft-bookings-and-ridership-soar-while-losses-shrink`. [27-April-2017].

Cachon, Gerard P, Kaitlin M Daniels, Ruben Lobel. 2017. The role of surge pricing on a service platform with self-scheduling capacity. *Manufacturing & Service Operations Management* **19**(3) 368–384.

CNNMoney. 2018. How Uber punishes drivers who refuse to use UberPool. `https://money.cnn.com/2016/07/28/technology/uber-uberpool-timeouts/index.html`. [25-September-2018].

Cohen, Maxime C, Renyu Philip Zhang. 2018. Coopetition and profit sharing for ride-sharing platforms .

Fay, Scott, Jinhong Xie. 2008. Probabilistic goods: A creative way of selling products and services. *Marketing Science* **27**(4) 674–690.

Financial Times. 2017. Didi seeks up to $6bn to expand beyond Chinas borders. `https://www.ft.com/content/04bd4032-2a95-11e7-9ec8-168383da43b7`. [27-April-2017].

Fraiberger, Samuel P, Arun Sundararajan. 2017. Peer-to-peer rental markets in the sharing economy .

He, Long, Zhenyu Hu, Meilin Zhang. 2018. Robust repositioning for vehicle sharing. *Manufacturing & Service Operations Management, Forthcoming* .

He, Long, Ho-Yin Mak, Ying Rong, Zuo-Jun Max Shen. 2017. Service region design for urban electric vehicle sharing systems. *Manufacturing & Service Operations Management* .

Hu, Ming, Yun Zhou. 2016. Dynamic type matching. *Working Paper* .

Hu, Ming, Yun Zhou. 2017. Price, wage and fixed commission in on-demand matching .

Huang, Tingliang, Ying-Ju Chen. 2015. Service systems with experience-based anecdotal reasoning customers. *Production and Operations Management* **24**(5) 778–790.

Huang, Tingliang, Yimin Yu. 2014. Sell probabilistic goods? a behavioral explanation for opaque selling. *Marketing Science* **33**(5) 743–759.

Jerath, Kinshuk, Serguei Netessine, Senthil K Veeraraghavan. 2010. Revenue management with strategic customers: Last-minute selling and opaque selling. *Management Science* **56**(3) 430–448.

Jiang, Baojun, Lin Tian. 2016. Collaborative consumption: Strategic and economic implications of product sharing. *Management Science* **64**(3) 1171–1188.

Kabra, Ashish, Elena Belavina, Karan Girotra. 2016. Bike share systems: Accessibility and availability. *Working paper, INSEAD* .

Kung, Ling-Chieh, Guan-Yu Zhong. 2017. The optimal pricing strategy for two-sided platform delivery in the sharing economy. *Transportation Research Part E: Logistics and Transportation Review* **101** 1–12.

Mozur, Paul, Michael J. de la Merced. 2016. Uber rival's $28 billion valuation shows size of China's ride-sharing market. The New York Times, 2016/6/16, https://www.nytimes.com/.

Qi, Wei, Lefei Li, Sheng Liu, Zuo-Jun Max Shen. 2018. Shared mobility for last-mile delivery: Design, operational prescriptions, and environmental impact. *Manufacturing & Service Operations Management* .

Taylor, Terry A. 2018. On-demand service platforms. *Manufacturing & Service Operations Management* .

Xinhua News. 2013. Let taxi fare reform be fair. `http://news.xinhuanet.com/english/indepth/2013-05/24/c_132405060.htm`. [24-May-2013].