

$\Sigma_x$ .  $X \sim \exp(\lambda)$   $f_X(x) = \lambda e^{-\lambda x}, x > 0$

$Y = \frac{1}{X+1} \rightarrow \text{decreasing}$

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P\left(\frac{1}{X+1} \leq y\right) = P\left(X \geq \frac{1}{y} - 1\right) \\ &= 1 - P\left(X \leq \frac{1}{y} - 1\right) = 1 - \int_0^{\frac{1}{y}-1} \lambda e^{-\lambda x} dx \\ &= 1 + e^{-\lambda x} \Big|_0^{\frac{1}{y}-1} = e^{-\lambda(\frac{1}{y}-1)} \end{aligned}$$

$$f_Y(y) = \frac{\lambda}{y^2} e^{-\lambda(\frac{1}{y}-1)}$$

$$\begin{aligned} \text{Formal} &= -f_X\left(\frac{1}{y}-1\right) \cdot \frac{d}{dy}\left(\frac{1}{y}-1\right) \\ &= -\lambda e^{-\lambda(\frac{1}{y}-1)} \cdot \left(-\frac{1}{y^2}\right) \\ &= \frac{\lambda}{y^2} e^{-\lambda(\frac{1}{y}-1)} \end{aligned}$$



