# PID in State Space Form

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A PID controller normally looks like the following:

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \dot{e}(t) \tag{1}$$

We want to try rewriting this in state space form.

$$\dot{x} = ax + bu \tag{2}$$

$$y = cx \tag{3}$$

$$e \equiv y - s = cx - s \tag{4}$$

$$K_g \equiv (1 + k_d cb)^{-1} \tag{5}$$

$$y_p \equiv e = \dot{z} = cx - s \tag{6}$$

$$y_i \equiv z \tag{7}$$

$$y_d \equiv K_g cax \tag{8}$$

$$u = -k_p y_p - k_i y_i - k_d y_d \tag{9}$$

State space:

$$u = -KY \tag{10}$$

$$K \equiv \begin{bmatrix} 0 & k_p & k_i & k_d \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u \\ s \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} y \\ y_p \\ y_i \\ y_d \end{bmatrix} = \begin{bmatrix} c & 0 \\ c & 0 \\ 0 & 1 \\ K_g ca & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ s \end{bmatrix} \tag{13}$$

Closed loop, after plugging in our control law for $u$:

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} a - b(k_p c + k_d K_g ca) & -bk_i \\ c & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} bk_p \\ -1 \end{bmatrix} s \tag{14}$$

$$y = \begin{bmatrix} c & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} \tag{15}$$