

1.1

The number of training data < dimensionality of ω

1.2

We know

$$RSS = \sum_n (y - (\omega^T x_n + b))^2$$

Then take the derivative

$$\begin{aligned} \frac{\partial RSS}{\partial b} &= \sum_n 2(\omega^T x_n + b - y_n) = 0 \\ &= \sum_n 2(\omega^T x_n) + \sum_n 2(b - y_n) = 0 \end{aligned}$$

From the question we know:

$$\begin{aligned} \frac{1}{N} \sum_n x_{nd} &= 0 \\ 1_N X &= 0_N \\ 1_N X \omega &= \sum_n (\omega^T x_n) = 0 \end{aligned}$$

plug in:

$$\begin{aligned} \sum_n 2(b^* - y_n) &= 0 \\ b^* &= \frac{1}{N} \sum_n y_n \end{aligned}$$

2.1

The error function:

$$\min \epsilon(b) = - \sum_n [y_n \log \sigma(b) + (1 - y_n) \log(1 - \sigma(b))]$$

Take derivatives:

$$\begin{aligned} \frac{\partial \epsilon}{\partial b} &= \sum_n \left[\frac{y_n \sigma'(b)}{\sigma(b)} - (1 - y_n) \frac{\sigma'(b)}{1 - \sigma(b)} \right] = 0 \\ \sum_n \left[\frac{y_n}{\sigma(b)} - \frac{1 - y_n}{1 - \sigma(b)} \right] &= 0 \end{aligned}$$

$$\sum_n [y_n - \sigma(b)] = 0$$

$$b = \sigma^{-1}\left(\frac{\sum_n y_n}{N}\right) = \log\left(\frac{\frac{\sum_n y_n}{N}}{1 - \frac{\sum_n y_n}{N}}\right)$$

The optimal classifier will be assign the sample to $y=1$ in probability $\frac{\sum_n y_n}{N}$, $y=0$ in probability $1 - \frac{\sum_n y_n}{N}$. The probability of assigning a sample to $y=1$ is $\frac{\sum_n y_n}{N}$.