1.1
The matrix is

$$K = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \tag{1}$$

we can know that the eigenvalue of K is $\lambda_1 = \lambda_2 = \cdots = \lambda_n = 1$. All positive.So it is SDP.

1.2
From the answer of Q1.1, we can know that K=I. From the slides we know that $\alpha = K^{-1}y = y$.

$$J(\alpha) = \frac{1}{2}\alpha^T\alpha - y^T\alpha + \frac{\lambda}{2}\alpha^T\alpha + \frac{1}{2}y^Ty$$

$$= \frac{1}{2}y^Ty - y^Ty + \frac{1}{2}y^Ty = 0$$

1.3

$$w^* = \Phi^T(K)^{-1}y = \Phi^T y$$

$$X_{new}w^* = \phi(X_{new})\Phi^T y$$

$$= [\phi(X_1)\phi(X_{new})^T\phi(X_2)\phi(X_{new})^T \cdots \phi(X_n)\phi(X_{new})^T]y$$

$$= [K_{1new}K_{2new} \cdots K_{nnew}]y = 0$$

2.1
Assume it is linear separable. Assume the classifier is $y = sign[w_1x + b]$. Then w_1 and b should match:

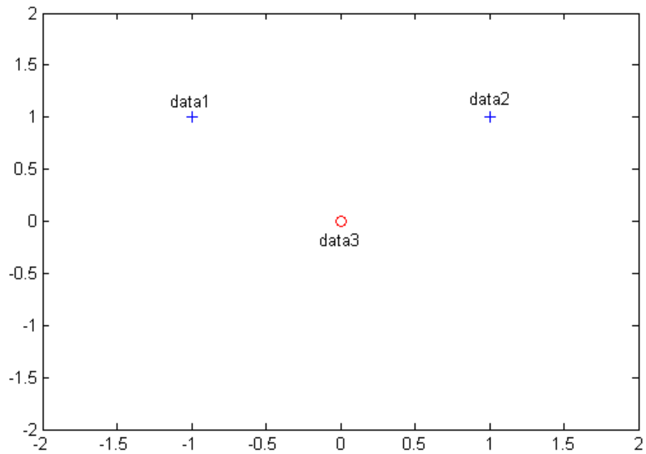
$$b > 0 \ (1)$$

$$w_1 + b < 0 \ (2)$$

$$-w_1 + b < 0 \ (3)$$

Add (2) and (3), we know b¡0,whch contradicts (1).
So it is not linear separable.

2.2



It is linear separable now. $x_2 = 0.5$ can linearly separate two classes.

2.3

$$k(x, x') = (x)(x') + (x^2)(x'^2) = xx + (xx')^2$$

$K_{11} = (-1,1)(-1,1) = 2, K_{12} = (-1,1)(1,1) = 0, K_{13} = (-1,1)(0,0) = 0, K_{22} = (1,1)(1,1) = 2, K_{23} = (1,1)(0,0) = 0, K_{33} = (0,0)(0,0)$
$K = K^T$. So

$$K = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2}$$

2.4
primer:

$$min \ \frac{w_1^2 + w_2^2 + b^2}{2}$$

$$\begin{cases} b > 0 \\ -w_1 + w_2 + b >= 1 \\ w_1 + w_2 + b >= 1 \end{cases} \tag{3}$$

dual:

$$min \ \alpha_2^2 + \alpha_3^2 - \alpha_1 - \alpha_2 - \alpha_3 \ (0)$$

$$\begin{cases} \alpha_1 >= 0 & (1) \\ \alpha_2 >= 0 & (2) \\ \alpha_3 >= 0 & (3) \\ \alpha_1 - \alpha_2 - \alpha_3 = 0 & (4) \end{cases} \qquad (4)$$

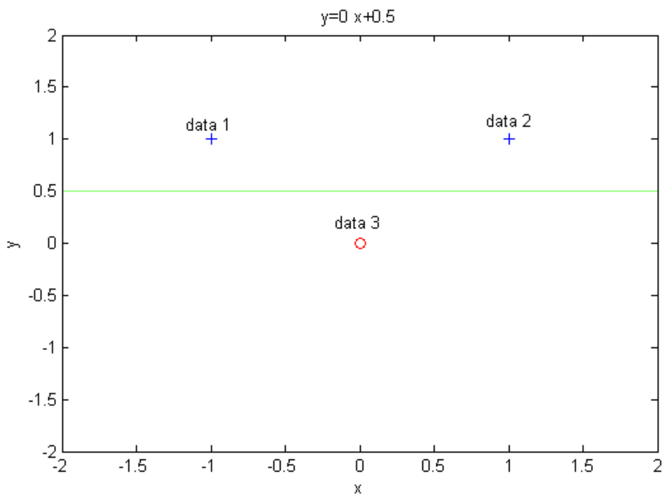
2.5

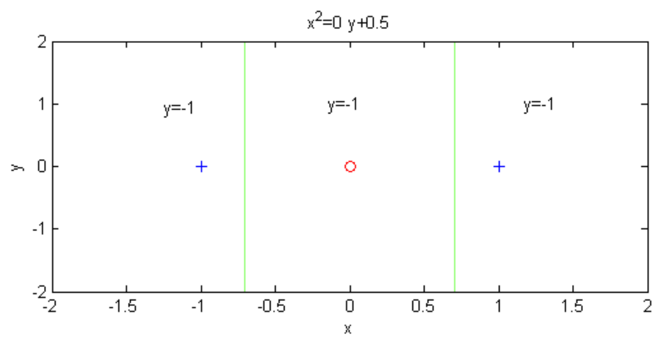
Add (4) and(0) we know we just need to minimize $\alpha_2^2 + \alpha_3^2 - 2\alpha_2 - 2\alpha_3$

So $\alpha_2 = 1, \alpha_3 = 1, \alpha_1 = 2$ Then b=1/2=0.5,$-w_1 + w_2 + b = 1/1 = 1, w_1 + w_2 + b = 1$

$w = [0, 0.5], b = 0.5$

2.6





3.1

(s,b,d)=(1,0.5,2)

$\epsilon_1 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$\beta_1 = \frac{1}{2} log \frac{1-1/2}{1/2} = 0$

3.2

$$w_2(1) = w_1(1)e^{\beta_1} = \frac{1}{4} * 1 = \frac{1}{4}$$
$$w_2(2) = w_1(2)e^{\beta_1} = \frac{1}{4} * 1 = \frac{1}{4}$$
$$w_2(3) = w_1(3)e^{-\beta_1} = \frac{1}{4} * 1 = \frac{1}{4}$$
$$w_2(4) = w_1(4)e^{-\beta_1} = \frac{1}{4} * 1 = \frac{1}{4}$$

3.3
(s,b,d)=(1,-0.5,2)
$\epsilon_1 = \frac{1}{4} = \frac{1}{4}$
$\beta_1 = \frac{1}{2} log \frac{1-1/4}{1/4} = 0.55$

3.4

$$w_2(1) = w_1(1)e^{\beta_1} = 0.43$$
$$w_2(2) = w_1(2)e^{-\beta_1} = 0.14$$
$$w_2(3) = w_1(3)e^{-\beta_1} = 0.14$$
$$w_2(4) = w_1(4)e^{-\beta_1} = 0.14$$

Nomalize:

$$w_2(1) = 0.51$$
$$w_2(2) = 0.16$$
$$w_2(3) = 0.16$$
$$w_2(4) = 0.16$$
$$f2 = (-1, 0.5, 2)$$
$$\epsilon_2 = 0.16$$

$$\beta_2 = \frac{1}{2} log \frac{1 - 0.16}{0.16} = 0.829$$

3.5

$$w_3(1) = w_2(1)e^{-\beta_2} = 0.19$$
$$w_3(2) = w_2(2)e^{-\beta_2} = 0.07$$
$$w_3(3) = w_2(3)e^{\beta_2} = 0.37$$
$$w_3(4) = w_2(4)e^{-\beta_2} = 0.07$$

Nomalize:

$$w_3(1) = 0.27$$
$$w_3(2) = 0.1$$
$$w_3(3) = 0.53$$
$$w_3(4) = 0.1$$
$$f3 = (1, 0.5, 1)$$
$$\epsilon_2 = 0.1$$

$$\beta_3 = \frac{1}{2} log \frac{1-0.1}{0.1} = 1.09$$

3.6

$$F(x) = sign[0.55h_{(1,0.5,2)} + 0.829h_{(-1,0.5,2)} + 1.09h_{(1,0.5,1)}]$$

x1 is assigned -1,x2 is assigned 1,x3 is assigned -1,x4 is assigned 1. All 4 are correctly assigned.