Brief Simulation Report

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1 Model

In this study, data are simulated from the following six model:

- 1. Binomial + AR(1)

$$p(\nu) = A_1 + A_2 \cos\left(2\pi(\nu - A_3)/52\right) \tag{1}$$

$$\phi = C_1 \tag{2}$$

- 2. Binomial + PAR(1)

$$p(\nu) = A_1 + A_2 \cos\left(2\pi(\nu - A_3)/52\right) \tag{3}$$

$$\phi(\nu) = C_1 + C_2 \cos\left(2\pi(\nu - C_3)/52\right) \tag{4}$$

- 3. Two States Markov Chain + AR(1)

$$\alpha(\nu) = A_1 + A_2 \cos\left(2\pi(\nu - A_3)/52\right) \tag{5}$$

$$\beta(\nu) = B_1 + B_2 \cos\left(2\pi(\nu - B_3)/52\right) \tag{6}$$

$$\phi = C_1 \tag{7}$$

  where $\alpha(\nu)$ and $\beta(\nu)$ are probability of transition from Sunny day to Sunny day and Rainy day to Rainy day, respectively;

- 4. Two States Markov Chain + PAR(1)

$$\alpha(\nu) = A_1 + A_2 \cos\left(2\pi(\nu - A_3)/52\right) \tag{8}$$

$$\beta(\nu) = B_1 + B_2 \cos\left(2\pi(\nu - B_3)/52\right) \tag{9}$$

$$\phi(\nu) = C_1 + C_2 \cos\left(2\pi(\nu - C_3)/52\right) \tag{10}$$

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- 5. Truncated Overdisperse Possion + AR(1)

$$\begin{aligned}\mu(\nu) &= A_1 + A_2 \cos\left(2\pi(\nu - A_3)/52\right) &(11)\\ \sigma(\nu) &= B_1 + B_2 \cos\left(2\pi(\nu - B_3)/52\right) &(12)\\ \phi &= C_1 &(13)\end{aligned}$$

where $\mu(\nu)$ and $\sigma_2(\nu)$ are mean and variance of overdisperse possion (not the truncated overdispersed possion distribution).

- 6. Truncated Overdispersed Possion + PAR(1)

$$\begin{aligned}\mu(\nu) &= A_1 + A_2 \cos\left(2\pi(\nu - A_3)/52\right) &(14)\\ \sigma(\nu) &= B_1 + B_2 \cos\left(2\pi(\nu - B_3)/52\right) &(15)\\ \phi(\nu) &= C_1 + C_2 \cos\left(2\pi(\nu - C_3)/52\right) &(16)\end{aligned}$$

2 Simulation Design

For each model, I generate 100 samples path, where each sample path consists of 104 data points. Then I calculate $\{\hat{\boldsymbol{\theta}}_{MLE,1}, \cdots, \hat{\boldsymbol{\theta}}_{MLE,100}\}$ by particles filtering approximation and then standard error can be estimated as well.

After that I randomly select one sample path and estimated its standard error by $\mathcal{J}^{-1}[\hat{\boldsymbol{\theta}}]_{jj}$.

Table 1: Model 1

	A_1	A_2	A_3	C_1
$\boldsymbol{\theta}$	0.5000	0.2000	20.0000	0.3000
$E[\hat{\boldsymbol{\theta}}]$	0.4949	0.2020	19.8321	0.2685
$\hat{SE}$	0.0204	0.0342	1.3847	0.1002
$\mathcal{J}^{-1}[\hat{\boldsymbol{\theta}}]_{jj}$	0.0248	0.0336	1.3914	0.0948

Table 2: Model 2

	A_1	A_2	A_3	C_1	C_2	C_3
$\boldsymbol{\theta}$	0.5000	0.2000	20.0000	0.3000	0.2000	10.0000
$E[\hat{\boldsymbol{\theta}}]$	0.4932	0.2022	20.3561	0.2496	0.2510	10.7372
$\hat{SE}$	0.0237	0.0345	1.3647	0.1178	0.1840	5.7592
$\mathcal{J}^{-1}[\hat{\boldsymbol{\theta}}]_{jj}$	0.0229	0.0291	1.2729	0.1000	0.1326	3.6574

Table 3: Model 3

	A_1	A_2	A_3	B_1	B_2	B_3	C_1
$\boldsymbol{\theta}$	0.5000	0.2000	20.0000	0.4000	0.1000	10.0000	0.3000
$E[\hat{\boldsymbol{\theta}}]$	0.4829	0.2077	20.4162	0.3815	0.1153	10.5323	0.2491
$\hat{SE}$	0.0483	0.0534	2.4721	0.0472	0.0678	5.0916	0.1213
$\mathcal{J}^{-1}[\hat{\boldsymbol{\theta}}]_{jj}$	0.0369	0.0536	1.9092	0.0506	0.0656	14.2532	0.1072

Table 4: Model 4

	A_1	A_2	A_3	B_1	B_2	B_3	C_1	C_2	C_3
$\boldsymbol{\theta}$	0.5000	0.2000	20.0000	0.4000	0.1000	10.0000	0.3000	0.1500	10.0000
$E[\hat{\boldsymbol{\theta}}]$	0.4885	0.2029	19.8599	0.3791	0.0988	9.4870	0.2640	0.1589	9.6350
$\hat{SE}$	0.0476	0.0581	2.9632	0.0449	0.1054	5.9227	0.1014	0.2080	5.7340
$\mathcal{J}^{-1}[\hat{\boldsymbol{\theta}}]_{jj}$	0.0455	0.0661	1.5924	0.0527	0.0766	5.3076	0.1069	0.1965	6.5152

Table 5: Model 5

	A_1	A_2	A_3	B_1	B_2	B_3	C_1
$\boldsymbol{\theta}$	3.0000	1.0000	10.0000	4.5000	1.5000	15.0000	0.2000
$E[\hat{\boldsymbol{\theta}}]$	3.3589	1.3222	10.8851	8.3380	6.5750	15.8437	0.1649
$\hat{SE}$	0.8897	0.7704	5.1992	10.1073	10.3963	9.7856	0.1050
$\mathcal{J}^{-1}[\hat{\boldsymbol{\theta}}]_{jj}$	0.2004	0.2449	2.7176	0.7182	1.0130	6.4951	0.0987

Table 6: Model 6

	A_1	A_2	A_3	B_1	B_2	B_3	C_1	C_2	C_3
$\boldsymbol{\theta}$	3.0000	1.0000	10.0000	4.5000	1.5000	15.0000	0.2000	0.1000	10.0000
$E[\hat{\boldsymbol{\theta}}]$	3.3735	1.3249	10.7093	9.3454	7.5416	17.0245	0.1699	0.1182	9.7886
$\hat{SE}$	0.9600	0.8685	5.1035	18.0052	18.2827	9.8846	0.0984	0.2108	6.7623
$\mathcal{J}^{-1}[\hat{\boldsymbol{\theta}}]_{jj}$	0.2647	0.3269	2.8315	1.0550	1.4260	6.7273	0.1012	0.1477	18.3851