Brief Report

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1 Model

Latent PARMA:

$$\begin{aligned}
Z_{nT+\nu} &= \phi(\nu) Z_{nT+\nu-1} + \theta(\nu)\epsilon_{nT+\nu} \\
\epsilon_{nT+\nu} &\sim WN(0,1) \\
\phi(\nu) &= A_0 + A_1 \cos\left(\frac{2\pi(\nu - A_2)}{T}\right)
\end{aligned}$$

with zero mean and unit variance. Note that:

$$\begin{aligned}
\theta^2(\nu) &= 1 - \phi^2(\nu) \\
&= 1 - \left[A_0 + A_1 \cos\left(\frac{2\pi(\nu - A_2)}{T}\right)\right]^2
\end{aligned}$$

Define marginal binomial distribution with parameter p as function of period ν:

$$p(\nu) = B_0 + B_1 \cos\left(\frac{2\pi(\nu - B_2)}{T}\right)$$

Based on the count time paper, we have:

$$\begin{aligned}
C_m(\nu) &= \mathbb{P}\left[X_t \le m \mid p(\nu)\right] \\
G_\nu(z) &= \sum_{n=0}^{7} n \mathbf{1}_{[\Phi^{-1}(C_{m-1}(\nu)),\Phi^{-1}(C_m(\nu))]}(z)
\end{aligned}$$

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2 SIS Particle filtering

Use quasi-newton method to search vector $(A_0, A_1, A_2, B_0, B_1, B_2)$ that maximize likelihood function that

$$\mathcal{L}_T(\mathbf{A}, \mathbf{B}) = \mathbb{P}(X_0 = x_0) \prod_{s=1}^{T} \mathbb{E}_{\mathcal{H}_{s-1}} \left[w_s(\hat{Z}_s) \right]$$

above likelihood can be approximated by

$$\hat{\mathcal{L}}_T(\mathbf{A}, \mathbf{B}) = \mathbb{P}(X_0 = x_0) \frac{1}{N} \sum_{i=1}^{N} w_T^i$$

for our particular case,

$$\mathbb{P}(X_0 = x_0 | \nu = 1) = \Phi^{-1}(C_{x_0}(\nu)) - \Phi^{-1}(C_{x_0-1}(\nu))$$

due to the fact that the marginal density of latent PARMA have zero mean and unit variance.

Algorithm:

1. select initial point for $(A_0, A_1, A_2, B_0, B_1, B_2)$, I use $(0.5, .3, 0, .5, .3, 0)$ in my code.

repeat the following step for N time, i is the number of iteration.

2. sample $Z_0^i = (\mathcal{N}(0,1) | \Phi^{-1}(C_{x_0-1}(\nu_0)) \leq \mathcal{N} < \Phi^{-1}(C_{x_0}))$ and set $w_0^i = 1$

recursively in $t = 1, \cdots, T$,

3. compute $\hat{Z}_t^i = \phi_{\mathbf{B}} Z_{t-1}^i$,
4. sample $\epsilon_{\nu_t}^i = \left(\mathcal{N}(0,1) | G_{\nu_t} \left(\hat{Z}_t^i + \theta_{\mathbf{A}}(\nu_t) \mathcal{N}(0,1) \right) = x_t \right)$
5. update $Z_t^i = \hat{Z}_t^i + \theta_{\mathbf{A}} \epsilon_{\nu_t}^i$
6. update $w_t^i = w_{t-1}^i w_t(\hat{Z}_t^i)$
7. repeat steps 3 to 6 until we have x_T^i
8. repeat steps 2 to 7 until we have $x_T^1, \cdots, x_T^N$
9. calculate $\hat{\mathcal{L}}_T(\mathbf{A}, \mathbf{B})$ based on $x_T^1, \cdots, x_T^N$
10. search $(A_0, A_1, A_2, B_0, B_1, B_2)$ by quasi-newton method and repeat steps 2 to 9 until MLE is maximized.

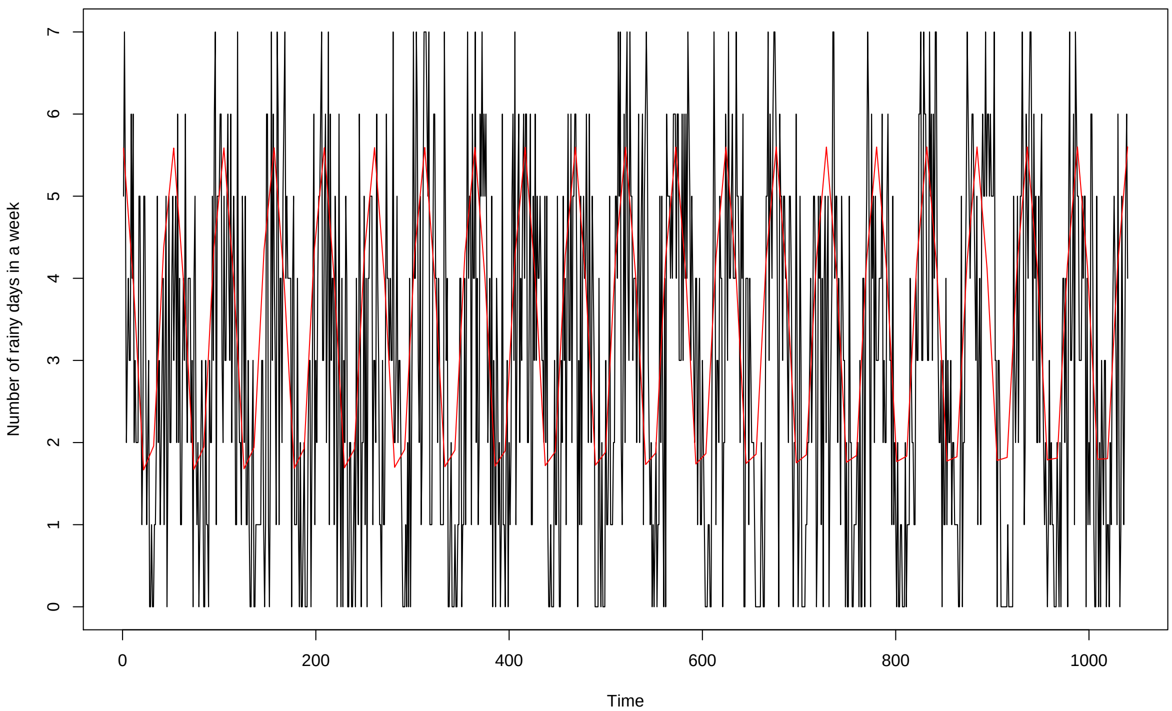


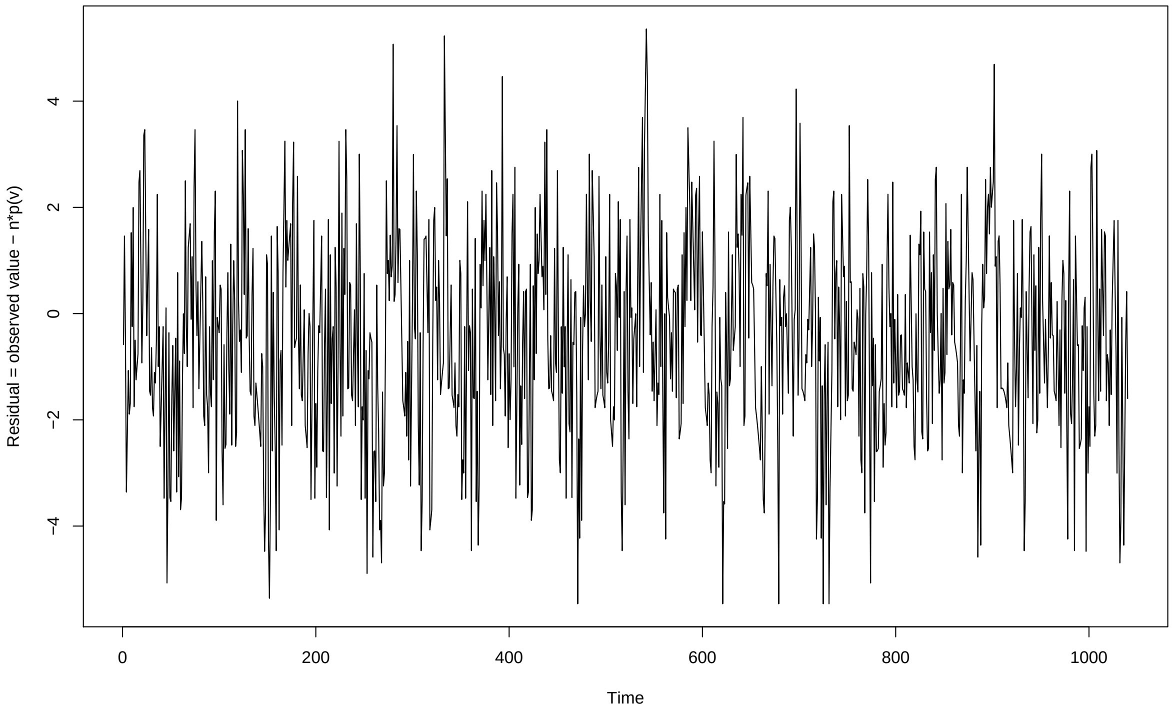
Figure 1: Black line is the count time series of observed data, Red line is fitted by $np(\nu) = 7 \times [0.418 + 0.175\cos(2\pi(\nu_t + 0.369)/52)]$, where $\nu_t = t \mod 52$.

3 Application

Data: seattle weekly rainy days record from 2000 week 1st to 2019 week 52th, containing 1040 data points.

Use the algorithm given in section 2, the estimated parameters are

$$\begin{aligned} A_0 &= 0.099, A_1 = -0.128, A_2 = -0.243 \\ B_0 &= 0.418, B_1 = 0.175, B_2 = -0.369 \end{aligned}$$



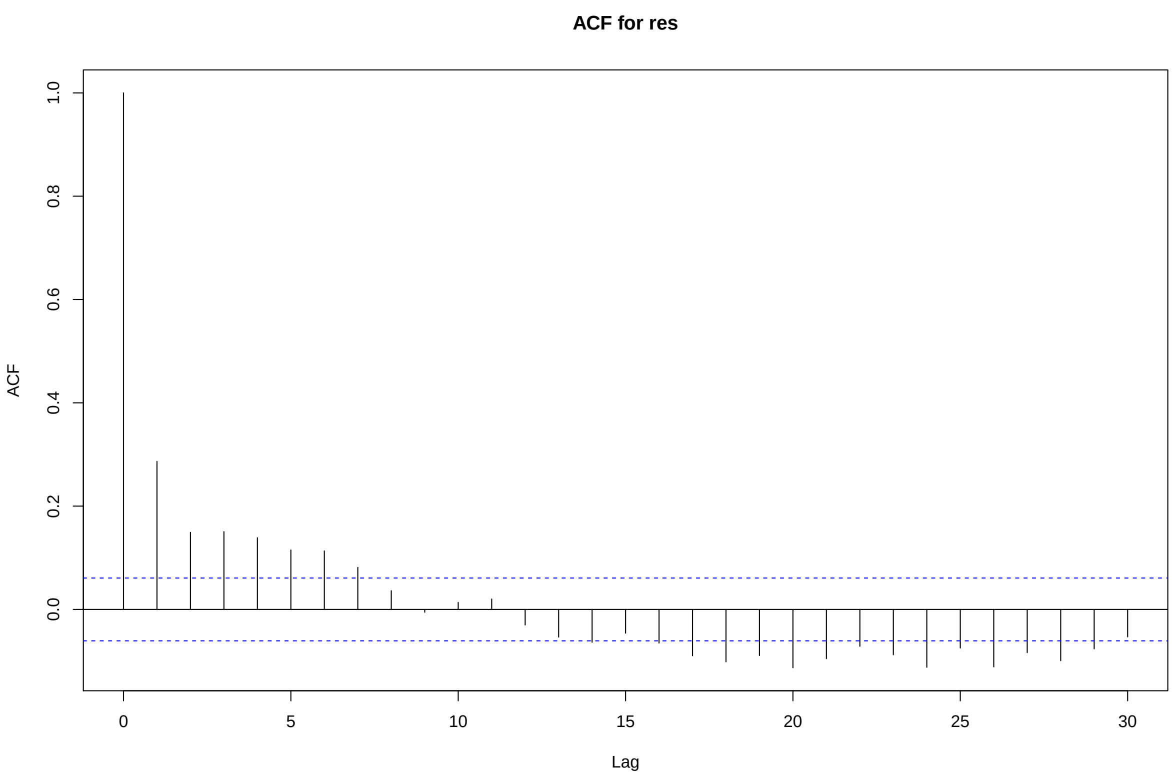


Figure 2: Above: Residual Plot: $res = observation - n * \hat{p}(\nu)$; Below: ACF for residual series in figure (2)