Axial Response of a Helical Spring in Discrete Elastic Rod

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Abstract— **This report represents the simulation of the axial response of a helical spring facing different values of characteristic force, and diameter configurations.**

I. INTRODUCTION

To Study the axial response of a helical spring, this homework would be divided into 3 parts where single point load, linear stiffness developed by various characteristic forces and the relationship between linear stiffness and different coil diameters would be studied. Results and findings would be discussed, and relevant plots would be provided.

II. SIMULATIONS AND RESULTS

A. Part 1 – Single Point Load

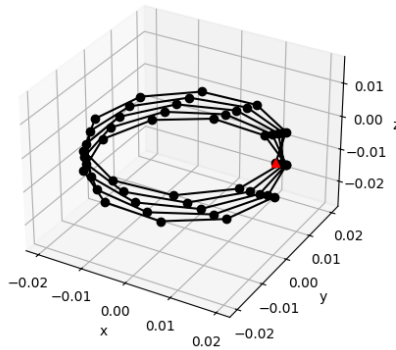Below are five snapshots taken during the simulation of twenty seconds.
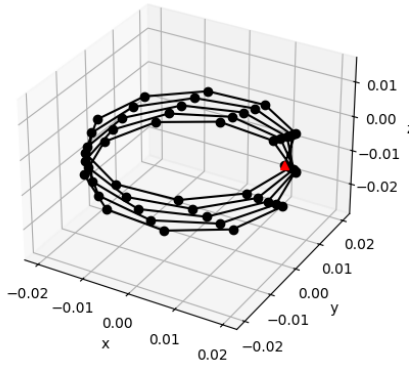
t=0.00

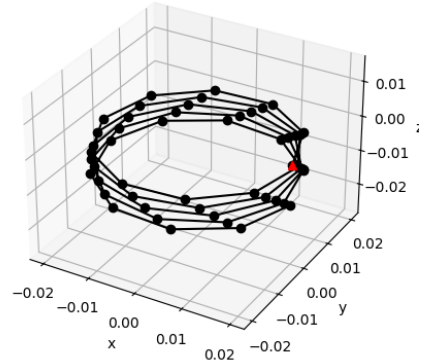*Figure 1 Helix at 0s*

t=3.00

Figure 2 Helix at 3s

t=9.00
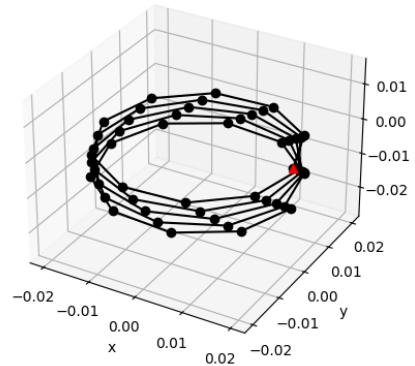

Figure 3 Helix at 9s

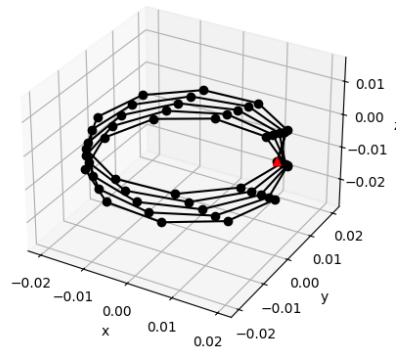t=12.00

Figure 4 Helix at 12s
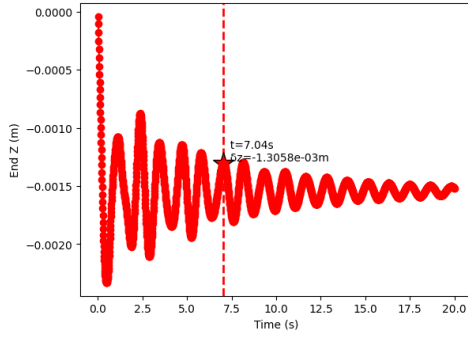
t=18.00

Figure 5 Helix at 18s

Figure 6 End Node Displacement vs Time with Steady State

Under the simple rule that the value of the end node displacement changes by less than one percent over a one-second interval, we have discovered the steady state of the spring at time 7.04s with a displacement of -1.305824e-3 m.

B. Part II - Force Sweep and Linear Fit

To find the linear stiffness, 10 different values of point load ranging from 0.01F_char to 10F_char have been used to determine the spring's property, where F_char = EI/L^2. A total of 10 forces were chosen using logarithmic (numpy.logspace). A complete simulation is run starting from the undeformation and ends when a steady state is reached according to the criterion described in Part 1.
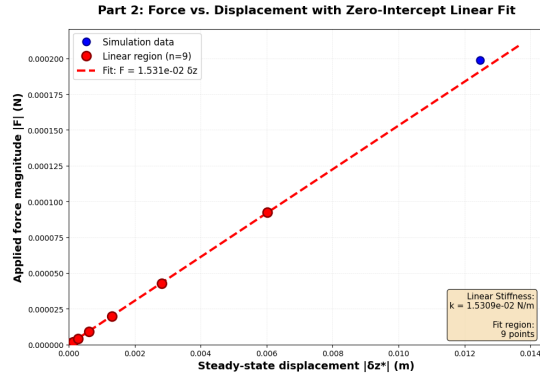

Figure 7 Applied Force vs Steady State Displacement

To stay in the small displacement region, the first 9 simulation values are used instead of the total 10. A linear stiffness of 1.531e-2 N/m is discovered and fit well along the first 9 simulation data.

C. Part III – Diameter Sweep and Textbook Trend

To examine the textbook relationship between coil diameter and the linear stiffness, 10 different diameter values ranging from D = 0.01m to D = 0.05m.

$$k_{text} = \frac{G\,d^4}{8\,N\,D^3}$$

Where G is the shear modulus, d is the wire diameter, N is the total number of turns and D is the coil diameter. This formula only works well in small deflection regime where the spring behaves like a torsion-dominated structure.

For each different value of the coil diameter D, a new helical geometry is constructed, where force sweeps (Part 2) are conducted on. The linear stiffness extracted using linear fit would be compared with the calculated value using the textbook formula. To remain inside the linear regime, only forces under 2*F_char are used. All simulations used the same criterion as in Part I.
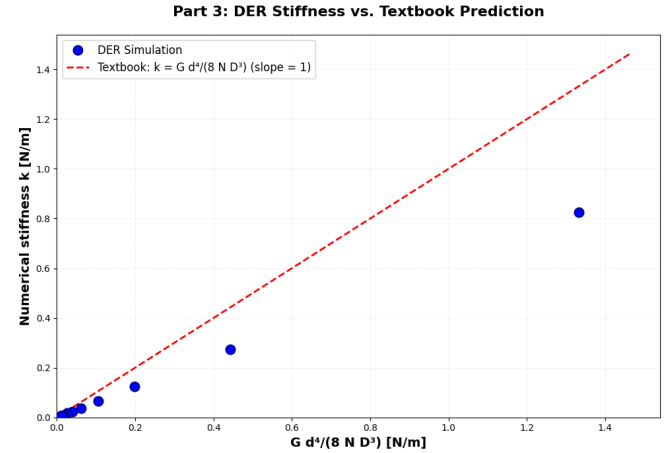

*Figure 8 Horizontal Axis: Gd^4/(8N*D3) Vertical Axis: Linear Stiffness*

III. Discussion

Even though the quantitative behaviors do not highly align between the textbook and simulation as the diameter reaches a high value, there is a qualitative agreement that as the helix diameter increases, the stiffness k decreases, and the spring becomes more flexible. The result demonstrates that the DER simulation also takes bending, twisting and stretching coupling into account. It shows that as the coil diameter increases; the axial stiffness decreases by roughly D^-3

IV. Reference

[1] M. K. Jawed. Lecture_13_Helix_Simulation.ipynb Retrieved from https://colab.research.google.com/drive/1eomYCyVkcGLNnAmxSG163sRim-9Efp6B?usp=sharing#scrollTo=k2O9AQOK3aRA