Supported Beam Simulation with Discrete Elastic Rod Model

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***Abstract*— This report describes my homework2 simulation of a supported aluminum beam under the influence of a point load using implicit Euler time integration. This beam is treated as a mass-spring network with 50 nodes, with both stretching and bending energies. The simulation results are compared with Euler-Bernoulli beam theory predictions. It later reveals that the two approaches diverge significantly as loads increasing.**

I. Introduction

Classical Euler-Bernoulli beam theory has been one of the most fundamental tools for beam deflection and deformation problems. However, this simulation provides an alternative solution when non-linearity becomes significant in the system.

In Homework2, a supported circular-tubed aluminum beam is simulated. It has length l=1m, outer radius r_out = 0.013, inner radius r_in = 0.011m, Young's modulus E=70GPa and a density of rho=2700kg/m^3. A point load P is applied at d=0.75m from the leftmost node.

This simulation involves both stretching and bending energies. With implicit discrete time-step integration and boundary conditions, it tries to provide an accurate modeling of system deformation and keep track of the falling distance as a function of time.

II. Methodology

The beam is broken down into N=50 nodes connected by elastic springs. The system's state is described by a position vector q (2N) which contains the x and y coordinates of all nodes.

Elastic Energy: The two components of total elastic energy are stretching and bending energies:

$$E_{total} = \sum_{k=1}^{N-1} E_k^s + \sum_{k=2}^{N-1} E_k^b$$

Where the stretching energy of edge k is:

$$E_k^s = \frac{1}{2} EA\Delta L \left(\frac{||\mathbf{x}_{k+1} - \mathbf{x}_k||}{\Delta L} - 1 \right)^2$$

And the bending energy of edge k is:

$$E_k^b = \frac{1}{2} EI\Delta L(\kappa_k - \bar{\kappa}_k)^2$$

where kappa_k is the discrete curvature.

Mass Distribution: provided in the homework, the mass per node is as follows:

$$m = \frac{\pi(R^2 - r^2)l\rho}{N-1}$$

III. Results

A. *Steady Value of y_max*

Figure 1 shows the maximum vertical displacement as a function of time with a point load of 2000N. As requested in the assignment, the simulation lasts for 1 s with a time step of 0.01s.

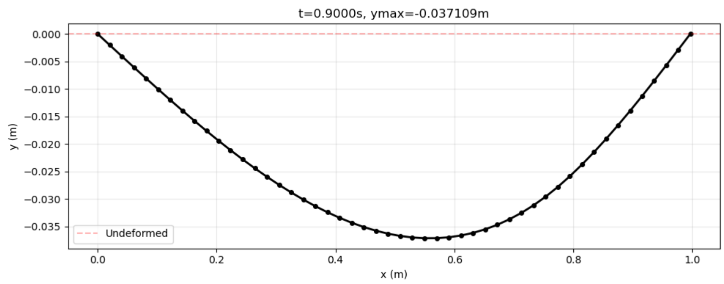

Figure 1. Final state of

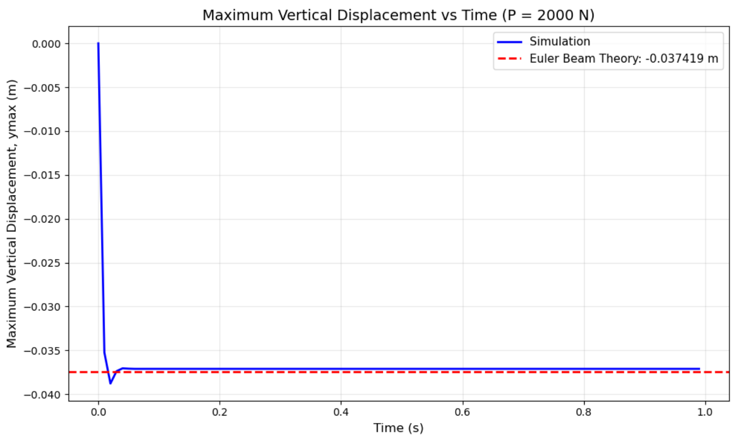

Figure 2 Comparison of simulation and Euler beam theory at P=2000N

The simulation shows that the system reaches steady state with a final displacement of -0.037419m, achieving a relative error of 0.83% given the theoretical final max displacement of -0.037109m.

B. *Large Deformation*

To determine the advantage of the simulation of the beam theory, loads ranging from 20N to 20,000N are applied to compare the results.

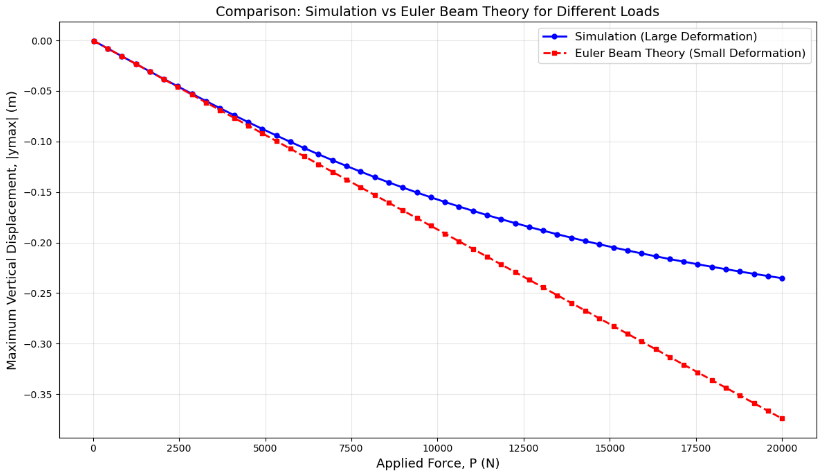

Figure 3 Comparison of simulation and Euler beam Theory for 20N<=P<=20000N

Running 50 Simulations from P=20N to P=20000N, the solutions from both methods yield a relative error < 5% until P ~= 5500N. And the relative error hits 37.13% at P = 20000N.

Euler beam theory assumes small deformation while the simulation can accurately capture the nonlinearity in the system. Therefore, the simulation provides a general solution when it comes to unusual boundary conditions and large deformation due to large loads as well as a full history and trace back of the system states as time varies.

IV. Conclusion

In this homework, an implicit Euler simulation is implemented to simulate a supported aluminum beam modeled as a discrete elastic rod. It has confirmed the Euler beam theory prediction for small loads and provides a better solution for analyzing beam-like systems undergoing large deformations.

Reference

[1] Professor M. Khalid Jawed, "Main_Falling_Beam_N_nodes (Colab notebook)," *Google Colab*, 2025. [Online]. Available: https://colab.research.google.com/drive/1O-YT-XCm3KsTVpRGLrfuAk4S2qzCUnD1?usp=sharing&authuser=1Accessed: Oct. 26, 2025.