Deformation of a Clamped Thin Beam Using a Plate Model

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***Abstract*— This report represents the work of Homework 5, which is the simulation of a beam subjected to gravity using discrete plate model.**

I. Introduction

In this set up, the beam is clamped at one end and free at the other, representing a classic cantilever beam problem. The goal of this homework is to compare the steady tip displacement predicted by the Euler-Bernoulli beam theory and the discrete plate model.

II. Procedure for Paper Submission

A. Configuration

A thin rectangular beam has the following properties:

Length L = 0.1m

Width w = 0.01m

Thickness h = 0.002m

Young's modulus Y = 1 * 10 ^ 7 Pa

Density rho = 1000 kg/m^3

The beam is discretized into a plate mesh with 20 nodes in total, with 10 nodes in the direction of x axis and 2 nodes in the y axis.

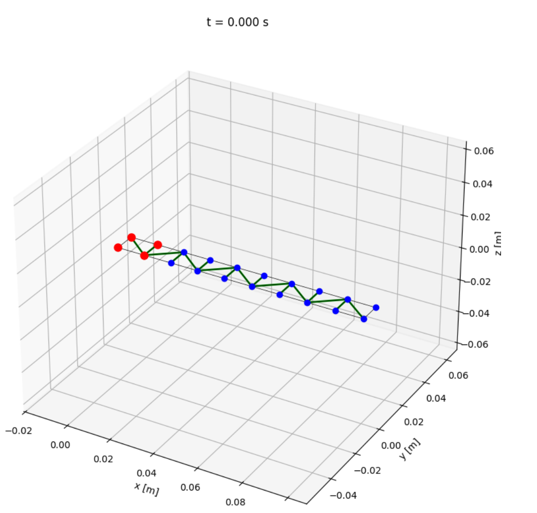

Figure 1 initial state at time 0

B. Discrete Plate Model

The plate is a node-spring network containing stretching and bending springs. Edges connecting adjacent nodes that resists stretching or compressing are stretching springs. Shared edges (the green ones representing the diagonal and shared vertical edges) that resist bending are bending springs. The mesh contains 20 nodes, with spacing of dx = 0.0125m and dy = 0.01m. There are totally 37 edges and 17 hinges. The leftmost 4 nodes are fixed, and all other modes are free to move. All nodes start at rest with 0 velocity.

III. Math

The total energy of the system consists of stretching energy:

$$E_s = \sum_k \frac{1}{2} k_s \epsilon_k^2$$

And bending energy:

$$E_b = \sum_j \frac{1}{2} k_b (\theta_j - \bar{\theta}_j)^2$$

The equation of motion then is solved by implicit Euler Integration with the Newton-Raphson method:

$$\mathbf{M}\frac{\mathbf{u}_{new} - \mathbf{u}_{old}}{\Delta t} = \mathbf{F}(\mathbf{q}_{new})$$

$$\mathbf{q}_{new} = \mathbf{q}_{old} + \Delta t \times \mathbf{u}_{new}$$

At each time step, the method solves

$$\mathbf{f} = \frac{\mathbf{M}}{\Delta t}\left(\frac{\mathbf{q}_{new} - \mathbf{q}_{old}}{\Delta t} - \mathbf{u}_{old}\right) - \mathbf{F} = \mathbf{0}$$

$$\mathbf{J} = \frac{\mathbf{M}}{\Delta t^2} - \frac{\partial \mathbf{F}}{\partial \mathbf{q}}$$

For Comparison, the final stable tip displacement would be compared with the analytical solution provided by Euler-Bernoulli Beam Theory:

$$\delta_{EB} = -\frac{qL^4}{8YI}$$

where q is the distributed load and I is the second moment of area.

IV. Results

The simulation was run for 20 seconds to allow the beam to reach a steady state, and the tip displacement was tracked at the centerline.

The Euler-Bernoulli predicts a final displacement of -36.75mm while the simulation converges at a final displacement of -28.85mm, with a normalized difference of 21.49%. For improvements, hinge setup and stiffness parameters would be investigate to mitigate the difference between the simulation result and the theory prediction.

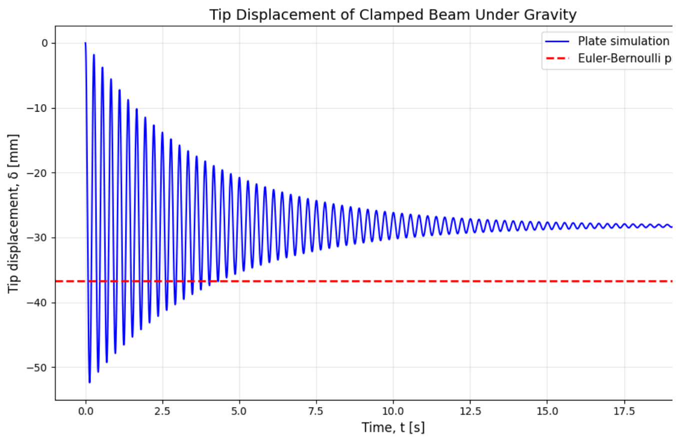

Figure 2 Displacement vs time t and the theoretical prediction

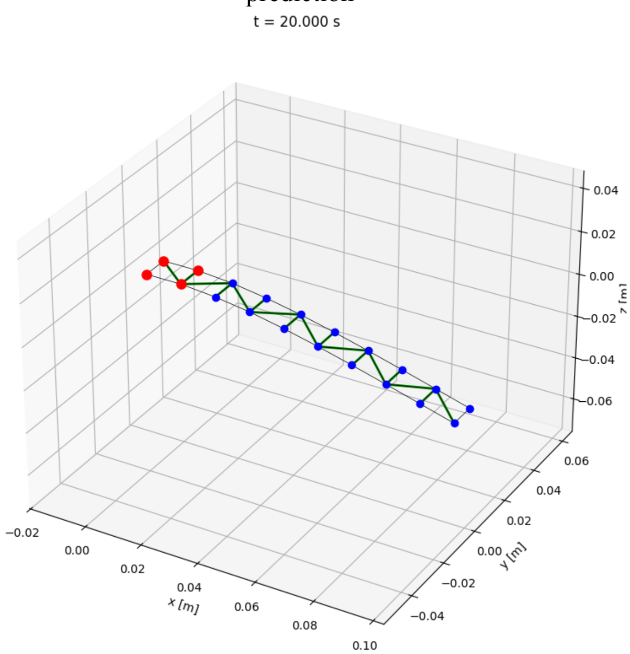

Figure 3 Final State at time = 20s

REFERENCES

[1] M. K. Jawed. Lecture_14_Main_Discrete_Shell_Simple.ipynb Retrieved from https://colab.research.google.com/drive/1IUVx3ND3KXZSEXGRjM4_VtTLRDWOJxAA?usp=sharing#scrollTo=0wtfqRWzqqyZ