CS540-1, HW4 Nov 07 Jianyi Liu

Question 1: Probability [20]

The following two tables provide the full joint distribution for three boolean variables X, Y and Z. Here, X indicates X =T and ¬X indicates X =F. The same is also true for the other variables.

Z

	Y	¬Y
X	0.08	0.10
¬X	0.09	0.25

¬Z

	Y	¬Y
X	0.04	0.18
¬X	0.09	0.17

For all of these following questions, to receive full credit, please show your work.

a) [4] What is the value of P (X)?

The probability P(X=T) is computed by summing probabilities of all (x, y, z) for which X=T.

P(X) = P(X, ∑Y, ∑Z) = P(X, Y, Z) + P(X, Y, ¬Z) + P(X, ¬Y, Z) + P(X, ¬Y, ¬Z)
= 0.08 + 0.10 + 0.04 + 0.18 = 0.40.

b) [4] What is the value of P (¬X|Y)?

P(¬X|Y) = $\frac{P(\neg X, Y)}{P(Y)}$

P(¬X, Y) = P(¬X, Y, ∑Z) = P(¬X, Y, Z) + P(¬X, Y, ¬Z)
= 0.09 + 0.09 = 0.18

P(Y) = P(∑X, Y, ∑Z) = P(X, Y, Z) + P(X, Y, ¬Z) + P(¬X, Y, Z) + P(¬X, Y, ¬Z)
= 0.08 + 0.04 + 0.09 + 0.09 = 0.3

P(¬X|Y) = $\frac{P(\neg X, Y)}{P(Y)} = \frac{0.18}{0.3}$ = 0.6

c) [4] What is the value of P (¬Y|X, ¬Z)?

P (¬Y|X, ¬Z) = $\frac{P(\neg Y, X, \neg Z)}{P(X, \neg Z)}$

P (¬Y, X, ¬Z) = 0.18

P (X, ¬Z) = P (X, ¬Z, ∑Y) = P (X, ¬Z, Y) + P (X, ¬Z, ¬Y) = 0.04 + 0.18 = 0.22

P (¬Y|X, ¬Z) = $\frac{P(\neg Y, X, \neg Z)}{P(X, \neg Z)} = \frac{0.18}{0.22}$ = 0.81818

d) [4] Verify whether X and Z are conditionally independent given Y.

Two events A, and B, are independent if: P(A,B|C) = P(A|C)P(B|C)
P(A|B) = P(A)
P(B|A) = P(B)

Our goal is to verify P(X, Z|Y) = P(X|Y)P(Z|Y)

LHS: P(X, Z|Y) = $\frac{P(X,Z,Y)}{P(Y)} = \frac{0.08}{0.3} = \frac{4}{15}$

RHS: $P(X|Y)P(Z|Y) = \frac{P(X,Y)}{P(Y)}\frac{P(Z,Y)}{P(Y)} = \frac{P(X,Y,\sum Z)}{P(Y)}\frac{P(Z,Y,\sum X)}{P(Y)} = \frac{(0.08+0.04)}{0.3} * \frac{(0.08+0.09)}{0.3} = \frac{17}{75}$

LHS≠RHS
X and Z are not conditionally independent given Y.

e) [4] Verify whether X and Y are conditionally independent given Z.

Our goal is to verify P(X, Y|Z) = P(X|Z)P(Y|Z)

LHS: P(X, Y|Z) = $\frac{P(X,Y,Z)}{P(Z)} = \frac{0.08}{0.52} = \frac{2}{13}$

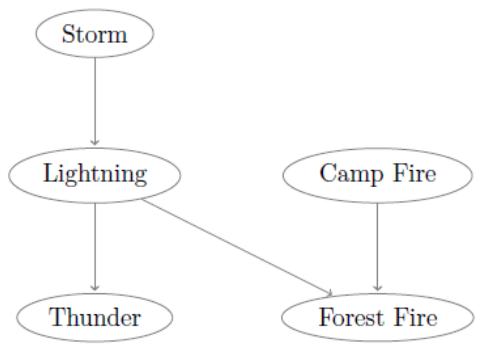
P(Z) = P(∑X, ∑Y, Z) = P(X, Y, Z) + P(X, ¬Y, Z) + P(¬X, Y, Z) + P(¬X, ¬Y, Z)
= 0.08 + 0.1 + 0.09 + 0.25 = 0.52

RHS: $P(X|Z)P(Y|Z) = \frac{P(X,Z)}{P(Z)}\frac{P(Y,Z)}{P(Z)} = \frac{P(X,Z,\sum Y)}{P(Z)}\frac{P(Y,Z,\sum X)}{P(Z)} = \frac{(0.08+0.1)}{0.52} * \frac{(0.08+0.09)}{0.52} = \frac{153}{1352}$

LHS≠RHS
X and Y are not conditionally independent given Z.

Question 2: Directed Graphical Model [30]

Consider the following Bayes Net over five boolean-valued random variables.



P(Storm=T)	P(Storm=F)
0.2	0.8

P(Camp Fire=T)	P(Camp Fire=F)
0.6	0.4

Storm	P(Lightning=T | Storm)	P(Lightning=F | Storm)
T	0.7	0.3
F	0.05	0.95

Lightning	P(Thunder=T | Lightning)	P(Thunder=F | Lightning)
T	0.94	0.06
F	0.25	0.75

Lightning	Camp Fire	P(Forest Fire=T | Lightning, Camp)	P(Forest Fire=F | Lightning, Camp)
T	T	0.6	0.4
T	F	0.5	0.5
F	T	0.15	0.85
F	F	0.01	0.99

For all of these following questions, to receive full credit, please show your work.

a) [6] What is the probability of thunder?

$\Pr(T) = \Pr(T, \sum L) = \Pr(T, L) + \Pr(T, \neg L) = \Pr(T|L)\Pr(L) + \Pr(T|\neg L)\Pr(\neg L)$

$\Pr(L) = \Pr(L, \sum S) = \Pr(L, S) + \Pr(L, \neg S) = \Pr(L|S)\Pr(S) + \Pr(L|\neg S)\Pr(\neg S)$

$= 0.7 * 0.2 + 0.05 * 0.8 = 0.18$

$\Pr(\neg \mathrm{L}) = \Pr(\neg L, \sum S) = \Pr(\neg L, S) + \Pr(\neg L, \neg S) = \Pr(\neg L|S)\Pr(S) +$
$\Pr(\neg L|\neg S)\Pr(\neg S)$
$= 0.3 * 0.2 + 0.95 * 0.8 = 0.82$
$\Pr(\mathrm{T}) = 0.94 * 0.18 + 0.25 * 0.82 = 0.3742$

b) [6] What is the probability of thunder given a forest fire?

$\Pr(\mathrm{T}|\mathrm{FF}) = \frac{\Pr(\mathrm{T},\mathrm{FF})}{\Pr(\mathrm{FF})}$
$\Pr(\mathrm{T},\mathrm{FF}) = \Pr(\mathrm{T},\mathrm{FF}, \sum CF, \sum L, \sum S)$
$= \Pr(\mathrm{T},\mathrm{FF}, CF, L, S) + \Pr(\mathrm{T},\mathrm{FF}, CF, L, \neg S) +$
$\Pr(\mathrm{T},\mathrm{FF}, CF, \neg L, S) + \Pr(\mathrm{T},\mathrm{FF}, CF, \neg L, \neg S) +$
$\Pr(\mathrm{T},\mathrm{FF}, \neg CF, L, S) + \Pr(\mathrm{T},\mathrm{FF}, \neg CF, L, \neg S) +$
$\Pr(\mathrm{T},\mathrm{FF}, \neg CF, \neg L, S) + \Pr(\mathrm{T},\mathrm{FF}, \neg CF, \neg L, \neg S)$
$\Pr(\mathrm{T},\mathrm{FF}, CF, L, S) = \Pr(\mathrm{T}|L)\Pr(\mathrm{FF}|CF, L)\Pr(CF)\Pr(L|S)\Pr(S)$
$= 0.94 * 0.6 * 0.6 * 0.7 * 0.2 = 0.047376$
$\Pr(\mathrm{T},\mathrm{FF}, CF, L, \neg S) = \Pr(\mathrm{T}|L)\Pr(\mathrm{FF}|CF, L)\Pr(CF)\Pr(L|\neg S)\Pr(\neg S)$
$= 0.94 * 0.6 * 0.6 * 0.05 * 0.8 = 0.013536$
$\Pr(\mathrm{T},\mathrm{FF}, CF, \neg L, S) = \Pr(\mathrm{T}|\neg L)\Pr(\mathrm{FF}|CF, \neg L)\Pr(CF)\Pr(\neg L|S)\Pr(S)$
$= 0.25 * 0.15 * 0.6 * 0.3 * 0.2 = 0.00135$
$\Pr(\mathrm{T},\mathrm{FF}, CF, \neg L, \neg S) = \Pr(\mathrm{T}|\neg L)\Pr(\mathrm{FF}|CF, \neg L)\Pr(CF)\Pr(\neg L|\neg S)\Pr(\neg S)$
$= 0.25 * 0.15 * 0.6 * 0.95 * 0.8 = 0.0171$
$\Pr(\mathrm{T},\mathrm{FF}, \neg CF, L, S) = \Pr(\mathrm{T}|L)\Pr(\mathrm{FF}|\neg CF, L)\Pr(\neg CF)\Pr(L|S)\Pr(S)$
$= 0.94 * 0.5 * 0.4 * 0.7 * 0.2 = 0.02632$
$\Pr(\mathrm{T},\mathrm{FF}, \neg CF, L, \neg S) = \Pr(\mathrm{T}|L)\Pr(\mathrm{FF}|\neg CF, L)\Pr(\neg CF)\Pr(L|\neg S)\Pr(\neg S)$
$= 0.94 * 0.5 * 0.4 * 0.05 * 0.8 = 0.00752$
$\Pr(\mathrm{T},\mathrm{FF}, \neg CF, \neg L, S) = \Pr(\mathrm{T}|\neg L)\Pr(\mathrm{FF}|\neg CF, \neg L)\Pr(\neg CF)\Pr(\neg L|S)\Pr(S)$
$= 0.25 * 0.01 * 0.4 * 0.3 * 0.2 = 0.00006$
$\Pr(\mathrm{T},\mathrm{FF}, \neg CF, \neg L, \neg S) = \Pr(\mathrm{T}|\neg L)\Pr(\mathrm{FF}|\neg CF, \neg L)\Pr(\neg CF)\Pr(\neg L|\neg S)\Pr(\neg S)$
$= 0.25 * 0.01 * 0.4 * 0.95 * 0.8 = 0.00076$
$\Pr(\mathrm{T},\mathrm{FF}) = 0.047376 + 0.013536 + 0.00135 + 0.0171 + 0.02632 + 0.0752 + 0.00006 + 0.00076 = 0.114022$
$\Pr(\mathrm{FF}) = \Pr(\mathrm{FF}, \sum L, \sum CF)$
$= \Pr(\mathrm{FF}, L, CF) + \Pr(\mathrm{FF}, L, \neg CF) + \Pr(\mathrm{FF}, \neg L, CF) + \Pr(\mathrm{FF}, \neg L, \neg CF)$
$= \Pr(\mathrm{FF}|L, CF)\Pr(\mathrm{L})\Pr(\mathrm{CF}) + \Pr(\mathrm{FF}|L, \neg CF)\Pr(L)\Pr(\neg CF)$
$+ \Pr(\mathrm{FF}|\neg L, CF)\Pr(\neg L)\Pr(CF) + \Pr(\mathrm{FF}|\neg L, \neg CF)\Pr(\neg L)\Pr(\neg CF)$
$\mathrm{From(a)}, \Pr(\mathrm{L}) = 0.18, \Pr(\neg \mathrm{L}) = 0.82,$
$\Pr(\mathrm{FF}) = 0.6 * 0.18 * 0.6 + 0.5 * 0.18 * 0.4 + 0.15 * 0.82 * 0.6 + 0.01 * 0.82 * 0.4 = 0.17788$
$\Pr(\mathrm{T}|\mathrm{FF}) = \frac{0.114022}{0.17788} = 0.641005$

c) [6] What is the probability that there is a storm given that there is thunder?

$\Pr(\mathrm{S}|\mathrm{T}) = \frac{\Pr(\mathrm{S},\mathrm{T})}{\Pr(\mathrm{T})}$
$\Pr(\mathrm{S},\mathrm{T}) = \Pr(\mathrm{S},\mathrm{T}, \sum L) = \Pr(\mathrm{S},\mathrm{T}, L) + \Pr(\mathrm{S},\mathrm{T}, \neg L)$
$\Pr(\mathrm{S},\mathrm{T}, L) = \Pr(\mathrm{S})\Pr(\mathrm{T}|L)\Pr(L|S) = 0.2 * 0.94 * 0.7 = 0.1316$
$\Pr(\mathrm{S},\mathrm{T}, \neg L) = \Pr(\mathrm{S})\Pr(\mathrm{T}|\neg L)\Pr(\neg L|S) = 0.2 * 0.25 * 0.3 = 0.015$

$\Pr(\mathrm{S},\mathrm{T}) = 0.1316 + 0.015 = 01466$
From (a), $\Pr(\mathrm{T}) = 0.3742$
$\Pr(\mathrm{S}|\mathrm{T}) = \frac{0.1466}{0.3742} = 0.3918$

d) [6] What is the probability of camp fire given that there is no thunder?
$\Pr(\mathrm{CF}|\neg\mathrm{T}) = \Pr(\mathrm{CF}) = 0.6$

e) [6] What is the probability of a lightning and a forest fire?
$\Pr(\mathrm{L},\mathrm{FF}) = \Pr(\mathrm{L},\mathrm{FF},\sum\mathrm{CF},\sum\mathrm{S}) = \Pr(\mathrm{L},\mathrm{FF},\mathrm{CF},\mathrm{S}) + \Pr(\mathrm{L},\mathrm{FF},\mathrm{CF},\neg\mathrm{S}) + \Pr(\mathrm{L},\mathrm{FF},\neg\mathrm{CF},\mathrm{S}) + \Pr(\mathrm{L},\mathrm{FF},\neg\mathrm{CF},\neg\mathrm{S})$
$\Pr(\mathrm{L},\mathrm{FF},\mathrm{CF},\mathrm{S}) = \Pr(L|S)\Pr(CF)\Pr(\mathrm{FF}|CF,L)\Pr(S) = 0.7 * 0.6 * 0.6 * 0.2 = 0.0504$
$\Pr(\mathrm{L},\mathrm{FF},\mathrm{CF},\neg\mathrm{S}) = \Pr(L|\neg S)\Pr(CF)\Pr(\mathrm{FF}|CF,L)\Pr(\neg S) = 0.05 * 0.6 * 0.6 * 0.8 = 0.0144$
$\Pr(\mathrm{L},\mathrm{FF},\neg\mathrm{CF},\mathrm{S}) = \Pr(L|S)\Pr(\neg CF)\Pr(\mathrm{FF}|\neg CF,L)\Pr(S) = 0.7 * 0.4 * 0.5 * 0.2 = 0.028$
$\Pr(\mathrm{L},\mathrm{FF},\neg\mathrm{CF},\neg\mathrm{S}) = \Pr(L|\neg S)\Pr(\neg CF)\Pr(\mathrm{FF}|\neg CF,L)\Pr(\neg S) = 0.05 * 0.4 * 0.5 * 0.8 = 0.008$
$\Pr(\mathrm{L},\mathrm{FF}) = 0.1008$