CS540-1, HW1 Sep 21

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Question1: Warm Up Math Questions [30]

a)

$f(x)= \ln(4 + sin^2x) + e^{3x}cosx$

$\frac{dy}{dx} = \frac{2sinxcosx}{4+sin^2x} + 3e^{3x}cosx - e^{3x}sinx$

b)

Let S be the totally possibilities of the sum when we roll two 6-sided dice. And A be the number of possibilities of the sum when the two number less than 11.

Then ¬A is the number of possibility of the sum when the two number is equal or larger than 11, which is 11 or 12.

$|S| = 6 * 6 = 36;$

¬A = {(5, 6), (6, 5), (6, 6);

$|\neg A| = 3;$

So, $|A| = \frac{36-3}{36} = \frac{33}{36} = \frac{11}{12}$.

c) Using L'Hopital's rule:

$$\lim_{x\to 0}\left(\frac{(e^x-1)(sinx)}{x\ln(x+1)}\right) = \lim_{x\to 0}\left(\frac{(sinx)}{x}\right)\lim_{x\to 0}\left(\frac{(e^x-1)}{\ln(x+1)}\right)$$

$$= \lim_{x\to 0}\left(\frac{\frac{dy}{dx}(sinx)}{\frac{dy}{dx}[x]}\right)\lim_{x\to 0}\left(\frac{\frac{dy}{dx}(e^x-1)}{\frac{dy}{dx}\ln(x+1)}\right) = \lim_{x\to 0}\left(\frac{\cos x}{1}\right)\lim_{x\to 0}\left(\frac{(e^x)}{\frac{1}{x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{1}{1}\right)\lim_{x\to 0}\left(\frac{1}{1}\right) = 1$$

Question2: Hierarchical Clustering [20]

The following table shows the distance between some cities within the state of Wisconsin. The distances are given in miles.

City	Distance(Miles)				
	MA	MI	AP	CH	GC
MA	0	78.7	105.8	216.1	223.8
MI	78.7	0	107.2	283.1	291.5
AP	105.8	107.2	0	222.4	239.3
CH	216.1	283.1	222.4	0	46.0
GC	223.8	291.5	239.3	46.0	0

Use Hierarchical Clustering with single-linkage to cluster the cities by hand, until all cities are in the same cluster.

1. For each iteration, show the cluster membership.
2. For each iteration, show all pairwise distances between clusters.

Solution:

Input distance matrix.
The nearest pair of cities is CH to GC, at distance 46.0. These are merged into a single cluster called “CH/GC”.
Then we compute the distance from this new compound object to all other objects. In single link clustering the rule is that the distance from the compound object to another object is equal to the shortest distance from any member of the cluster to any others. So, the distance from “CH/GC” to MA is chosen to be 216.1, which is the distance for CH to MA, and so on.

After merging CH with GC, we obtain the following matrix:

City	Distance(Miles)			
	MA	MI	AP	CH/GC
MA	0	78.7	105.8	216.1
MI	78.7	0	107.2	283.1
AP	105.8	107.2	0	222.4
CH/GC	216.1	283.1	222.4	0

min d(i,j) = d(MA, MI) = 78.7 => merge MA and MI into a new cluster called MA/MI.

City	Distance(Miles)		
	MA/MI	AP	CH/GC
MA/MI	0	105.8	216.1
AP	105.8	0	222.4
CH/GC	216.1	222.4	0

min d(i,j) = d(MA/MI, AP) = 105.8 => merge MA/MI and AP into a new cluster called MA/MI/AP.

City	Distance(Miles)	
	MA/MI/AP	CH/GC
MA/MI/AP	0	216.1
CH/GC	216.1	0

Finally, we merge the last two clusters at level 216.1.