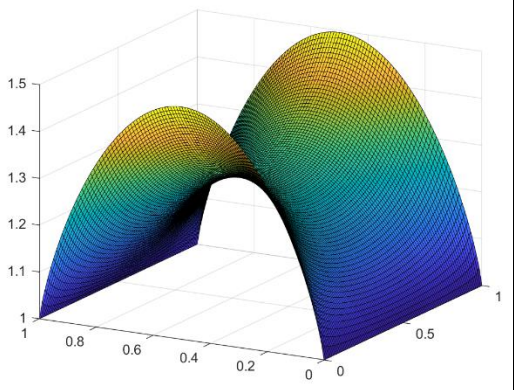
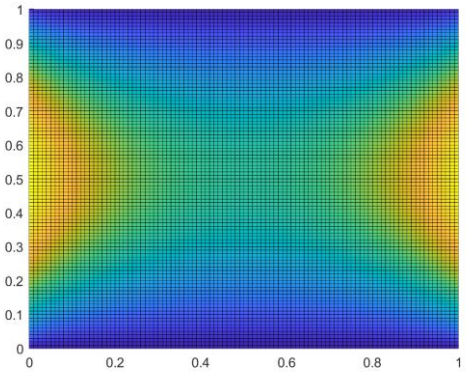
FEM Code Project Report

1. Report the errors and convergence rates measured in L2 and H1 norms (refer to Homework 5)

The temperature distribution with D and N boundaries.



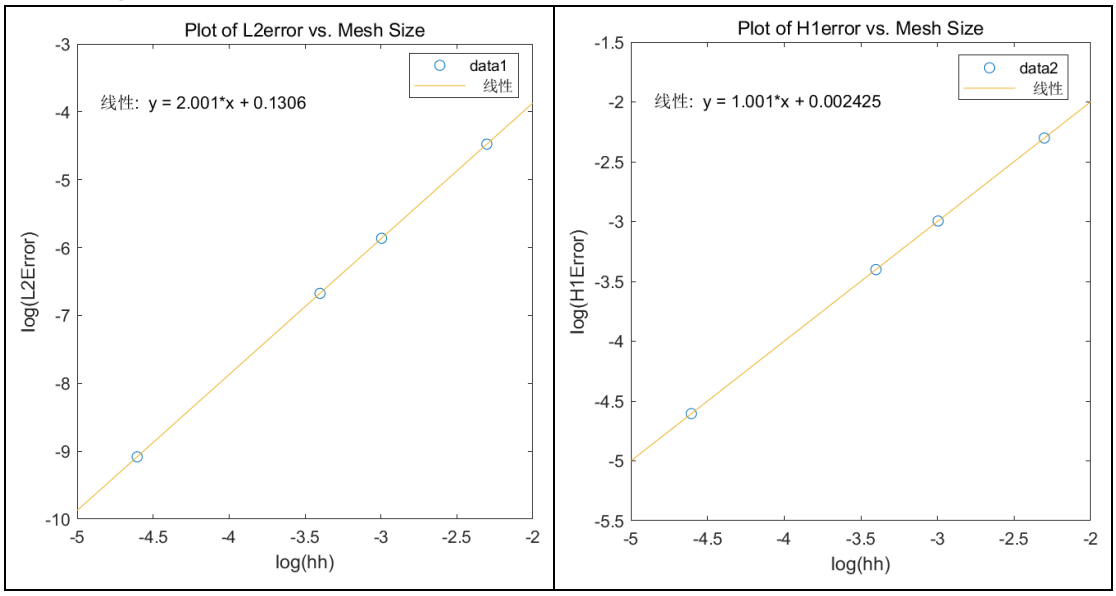
Overview



Top View

Change the hh value from 1/10 to 1/100, and the results as follows:
L2error will shrink by a factor of 100, while H1error by a factor of 10.
The slopes of the log(Error) vs log(hh) line should be 2 and 1 respectively.

Plot the figure as follows:



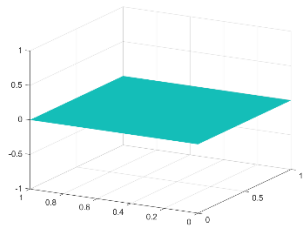
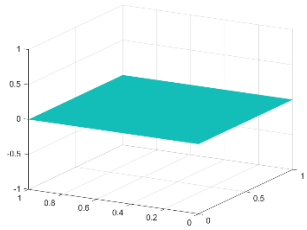
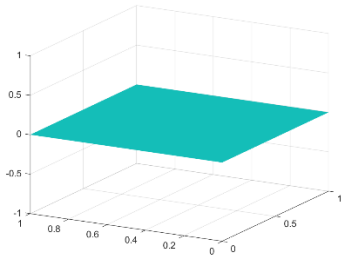
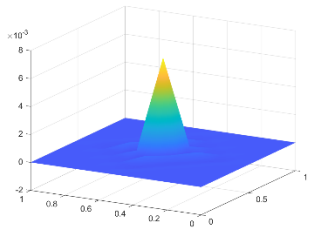
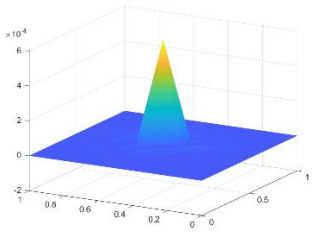
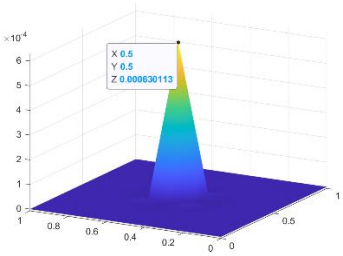
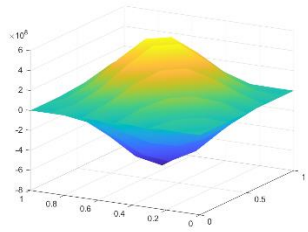
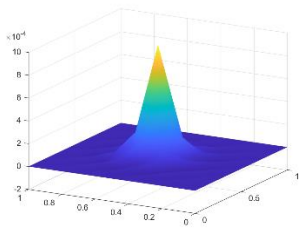
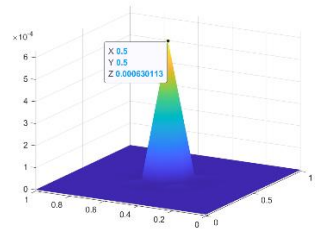
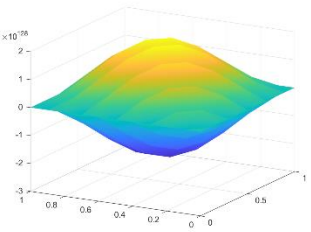
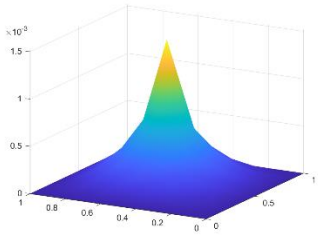
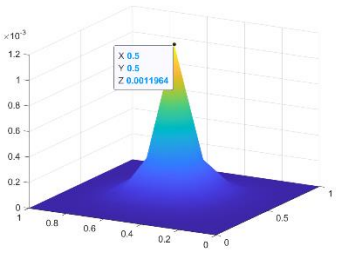
2. Determine the value of λ_{eq}^{h} in Matlab and verify this stability condition by your code.

Solution:

Use "lambda = eigs(inv(M) * K, 1)" to obtain λ_{eq}^{h} in the code attached in the Q2 file, which equals 2.2320e+03 in the given conditions(f and g).

Therefore, the upper bound of the time step size is: $2/\lambda_{eq}^{h} = 8.9605e-04$

The following figures demonstrate the effect of change of dt:

dt = 0.01	dt = $8.9605e-04$	dt = $2e-04$ (< $2/\lambda_{eq}^{h}$)
		
		
		
		

The correct results can be obtained only when the dt < $2/\lambda_{eq}^{h}$, otherwise the results may be ridiculous such as the first column in the above table.

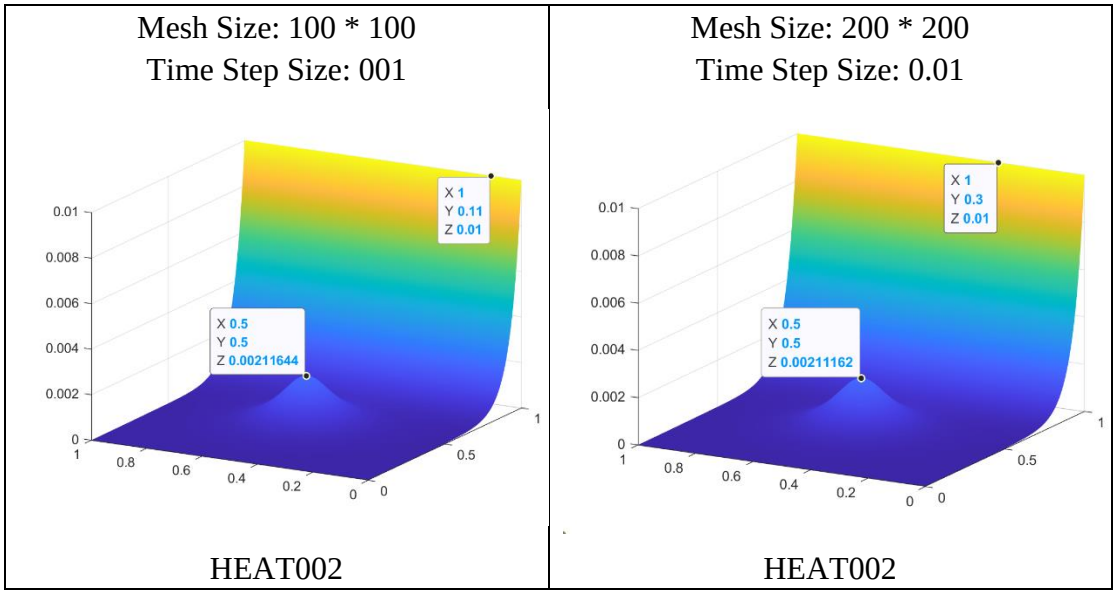
4. Investigate the temperature distribution over the time period (0,T) with T=10 using your code. Use two meshes and two different time step sizes to make sure your results are mesh-independent. Report your investigation including your numerical settings and the results.

Solution

Choose the parameters as follows:

Mesh Size	10*10	50*50	100*100	200*200
Time Step Size	0.01	0.01	0.01	0.01
	0.001	0.001	0.001	0.001

(the result is almost the same from 100*100 to 200*200 mesh size)



Mesh-Independent

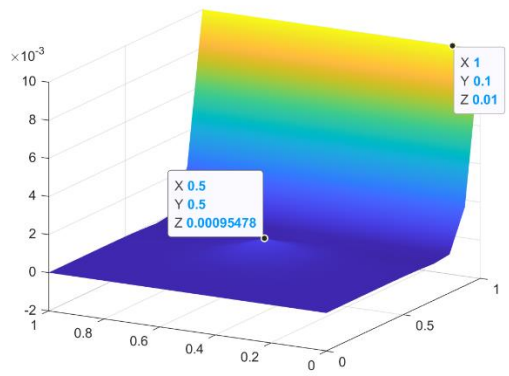
The corresponding code is attached in the Q4 file.

The corresponding results of different mesh and time step sizes are attached at the end of this report.

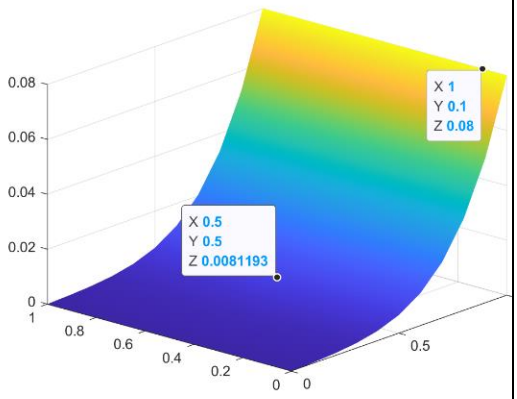
Investigation:

1) The g boundary temperature rises with time in the first 1 second, matching with the conditions given in this question.
2) The heat source effect can be neglected after some time, which means the temperature distribution is dominated by the g boundary.
3) The small top of the heat source can be observed at the first several time periods. This phenomenon may not be observed if we choose a larger time step size.
4) The temperature distribution will reach a stable state with the development of time, resulting in a plane with a slope in the end.

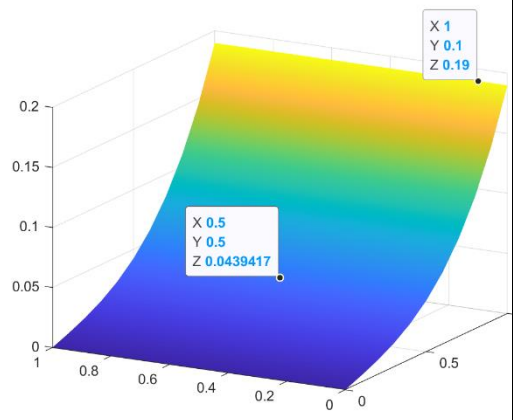
Mesh Size: 10 * 10
Time Step Size: 0.01



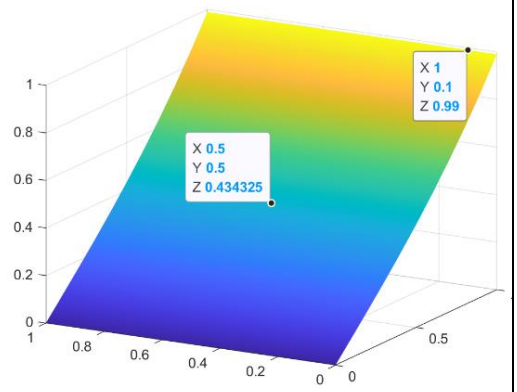
HETA002



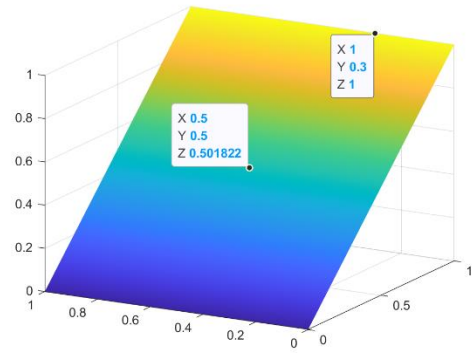
HEAT009



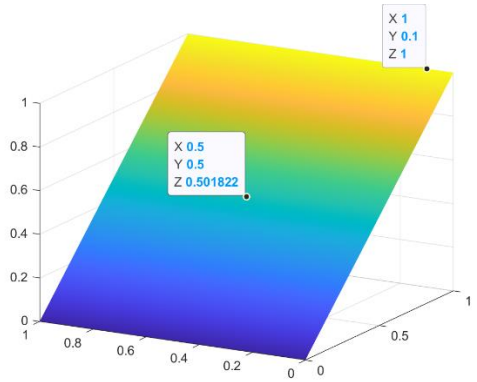
HEAT020



HEAT100

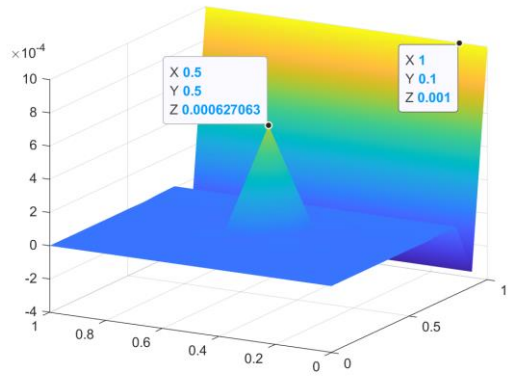


HEAT500

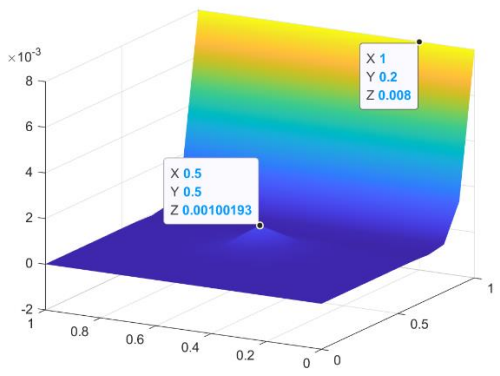


HEAT1000

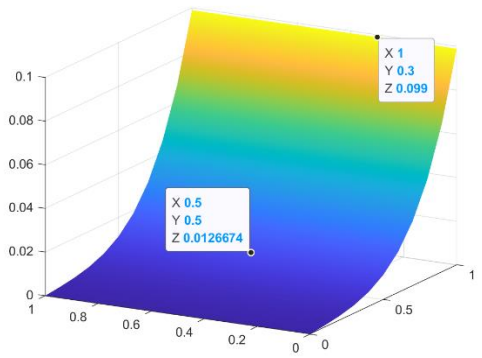
Mesh Size: 10 * 10
Time Step Size: 0.001



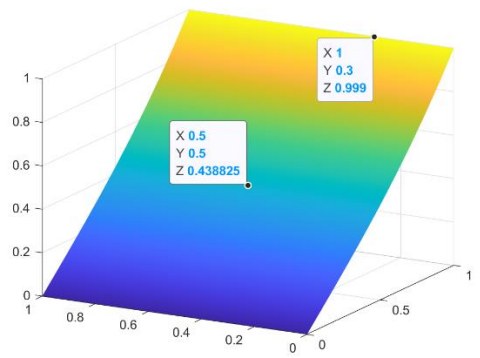
HEAT002
(Seems bug at the negative value)



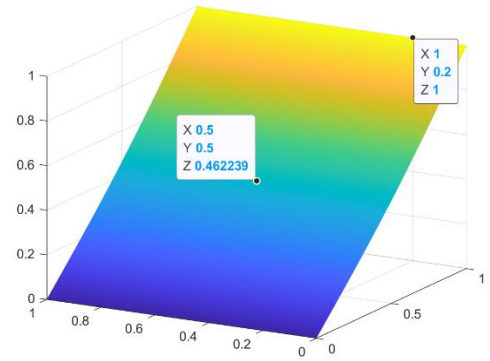
HEAT009



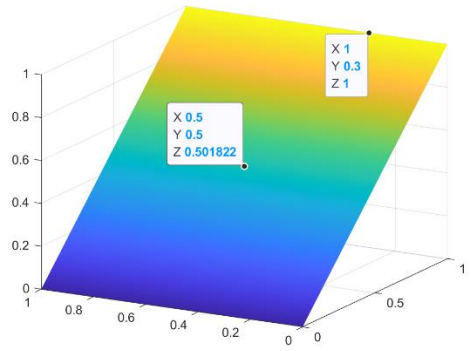
HEAT100



HEAT1000

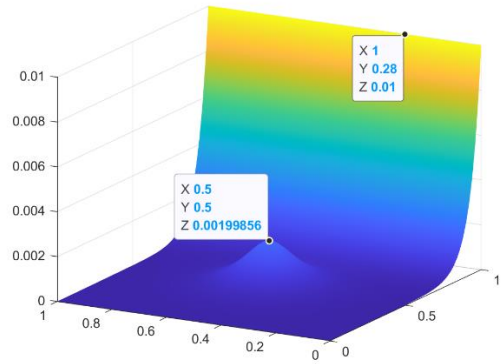


HEAT1050

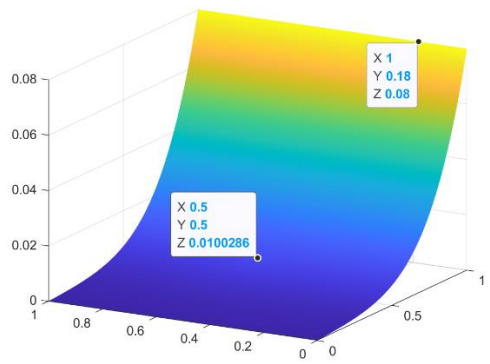


HEAT10000

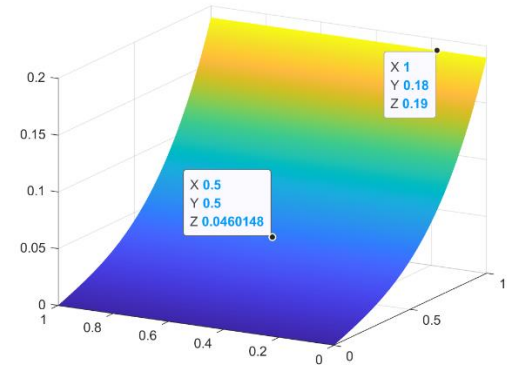
Mesh Size: 50 * 50
Time Step Size: 0.01



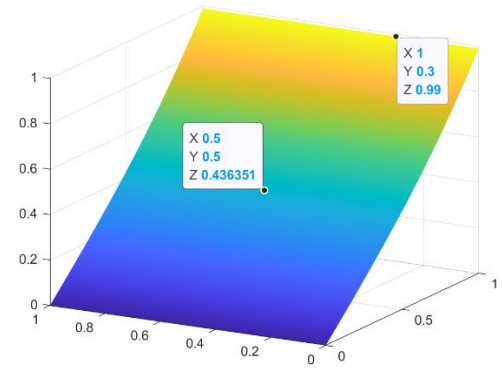
HEAT002



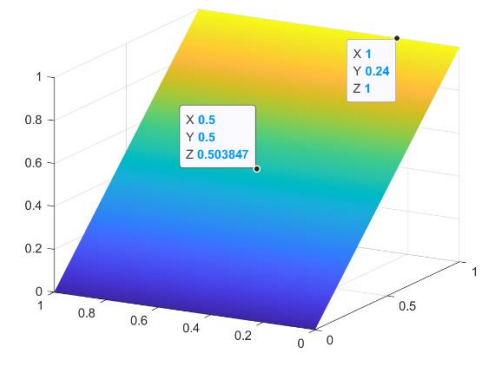
HEAT009



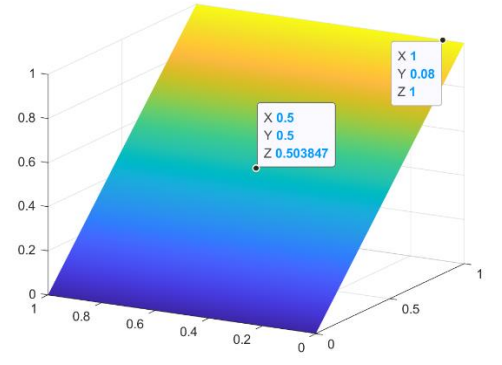
HEAT020



HEAT100

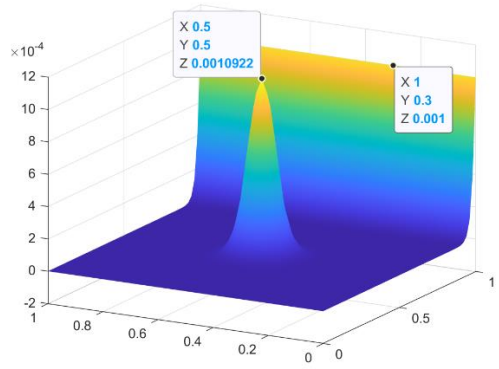


HEAT500

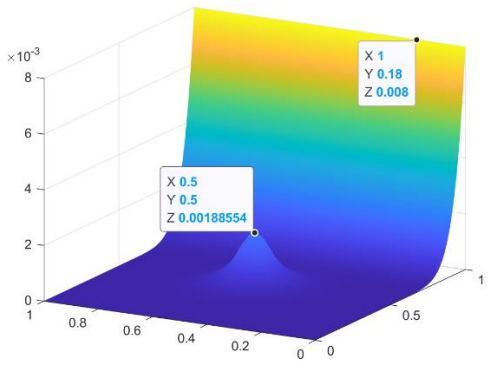


HEAT1000

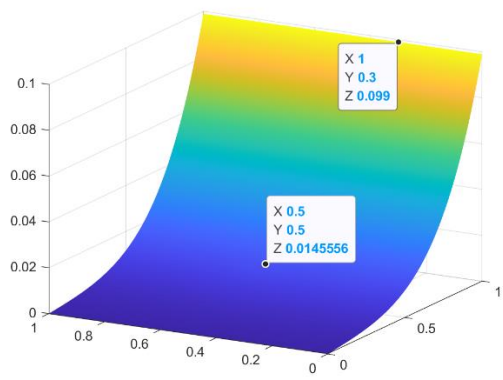
Mesh Size: 50 * 50
Time Step Size: 0.001



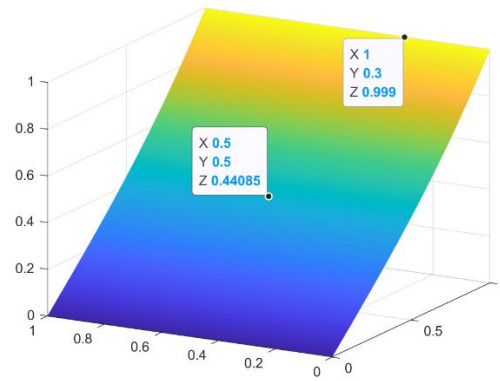
HEAT002



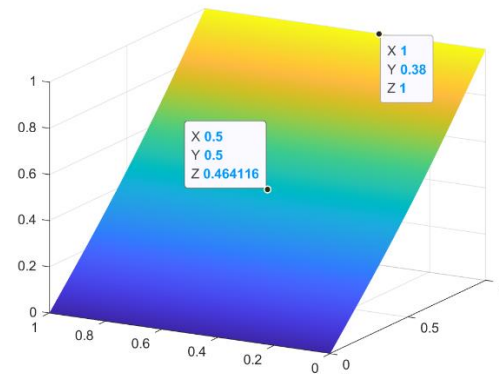
HEAT009



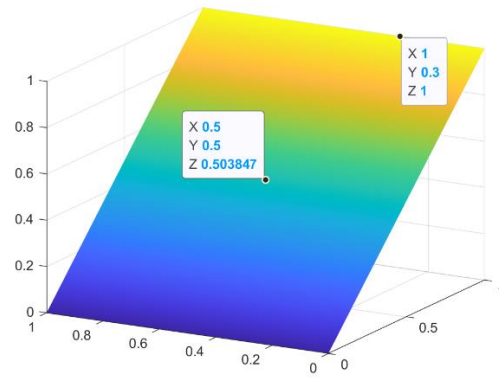
HEAT100



HEAT1000



HEAT1050



HEAT10000