FEM Code Project Report

1. Report the errors and convergence rates measured in L2 and H1 norms (refer to Homework 5)

Change the hh value from 1/10 to 1/100, and the results as follows:

L2error will shrink by a factor of 100, while H1error by a factor of 10.

The slopes of the log(Error) vs log(hh) line should be 2 and 1 respectively.

Plot the figure as follows:

For the all-D boundary condition case:

|  |  |
| --- | --- |
|  |  |

2. Determine the value of in Matlab and verify this stability condition by your code.

Solution:

Use “lambda = eigs(inv(M) \* K, 1)” to obtainin the code attached in the Q2 file, which equals 2.2320e+03 in the given conditions(f and g).

Therefore, the upper bound of the time step size is: 2/

The following figures demonstrate the effect of change of dt:

|  |  |  |
| --- | --- | --- |
| dt = 0.01 | dt = | dt = 2  ( < 2/) |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

The correct results can be obtained only when the dt < 2/, otherwise the results may be ridiculous such as the first column in the above table.

4. Investigate the temperature distribution over the time period (0,T) with T=10 using your code.

Use two meshes and two different time step sizes to make sure your results are mesh-independent. Report your investigation including your numerical settings and the results.

Solution

Choose the parameters as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Mesh Size | 10\*10 | 50\*50 | 100\*100 | 200\*200 |
| Time Step Size | 0.01 | 0.01 | 0.01 | 0.01 |
| 0.001 | 0.001 | 0.001 | 0.001 |

(the result is almost the same from 100\*100 to 200\*200 mesh size)

|  |  |
| --- | --- |
| Mesh Size: 100 \* 100  Time Step Size: 001    HEAT002 | Mesh Size: 200 \* 200  Time Step Size: 0.01    HEAT002 |

Mesh-Independent

The corresponding code is attached in the Q4 file.

The corresponding results of different mesh and time step sizes are attached at the end of this report.

Investigation:

1. The g boundary temperature rises with time in the first 1 second, matching with the conditions given in this question.
2. The heat source effect can be neglected after some time, which means the temperature distribution is dominated by the g boundary.
3. The small top of the heat source can be observed at the first several time periods. This phenomenon may not be observed if we choose a larger time step size.
4. The temperature distribution will reach a stable state with the development of time, resulting in a plane with a slope in the end.

Mesh Size: 10 \* 10

Time Step Size: 0.01

|  |  |
| --- | --- |
| HETA002 | HEAT009 |
| HEAT020 | HEAT100 |
| HEAT500 | HEAT1000 |

Mesh Size: 10 \* 10

Time Step Size: 0.001

|  |  |
| --- | --- |
| HEAT002  (Seems bug at the negative value) | HEAT009 |
| HEAT100 | HEAT1000 |
| HEAT1050 | HEAT10000 |

Mesh Size: 50 \* 50

Time Step Size: 0.01

|  |  |
| --- | --- |
| HEAT002 | HEAT009 |
| HEAT020 | HEAT100 |
| HEAT500 | HEAT1000 |

Mesh Size: 50 \* 50

Time Step Size: 0.001

|  |  |
| --- | --- |
| HEAT002 | HEAT009 |
| HEAT100 | HEAT1000 |
| HEAT1050 | HEAT10000 |