Homework 5

Due: 2023 Nov. 27

1. Report your github account and the link to your repository. Please make your repository **private** and add me and the course TA to your repository.

2. Work on 1D Matlab code developed in class. Your code development shall be submitted to your github repository.

a. Read the code and draw a detailed flowchart of it.

b. Calculate the following two relative errors of the solution using your code,

$$e_{L2} := \frac{\left(\int_0^1 (u^h - u)^2 dx\right)^{1/2}}{\left(\int_0^1 u^2 dx\right)^{1/2}},$$

$$e_{H1} := \frac{\left(\int_0^1 (u^h_{,x} - u_{,x})^2 dx\right)^{1/2}}{\left(\int_0^1 u^2_{,x} dx\right)^{1/2}},$$

with 2, 4, 6, 8, 10, 12, 14, and 16 elements and uniform mesh size. Plot the relative errors against the mesh size in the log-log plot (i.e., plot log(error) against log(h)). Determine the slope of the curves.

c. Enable the code for higher-order elements using quadratic and cubic elements. Modify the code. Repeat the calculation of the errors using the two higher-order elements.

d. (bonus) Matlab also provides iterative methods for solving linear systems. Learn the usage of the command gmres in Matlab. You may set restart=maxit=10000. Set the value of tol to be 10^{-2}, 10^{-4}, and 10^{-6} and compare the solutions against those obtained by the direct method (i.e. LU-factorization). Give your comments. You may need to consult the following page for references.
https://www.mathworks.com/help/matlab/ref/gmres.html

e. The equivalence between (G) and (M) is contingent upon the accuracy of the quadrature. The effect of numerical quadrature is discussed in Hughes book p. 191. Run the code with the element of degree 3 (i.e. cubic element). Experiment the code with 1,2,3,4,5,6 quadrature points, respectively. Report your observation and make comments.