

Learning Notes of Acoustics

Jiaju Zou

Abstract

This document is written mostly based on the learning notes of *Sound and Sources of Sound*, a very suitable textbook for new learners to have a basic understanding of acoustics and aeroacoustics.

Classical acoustics theory will be discussed in this document, including the basic characteristics of sound, sound wave equations, sound propagation in pipes, sound wave at interfaces, Ray theory and sound generated near surfaces of discontinuity. Some aeroacoustic theories will also be discussed with a lower concentration, but we will never miss the important theory in this area, for example, the famous Lighthill Acoustic Analogy and the FW-H Equation.

Finally, some content in this document is simplified to focus on the important key points (these points are detailed and we promise you can understand them). Welcome to discuss with the author if you have any different understanding about the theories in this document since the author is still learning in this area.

1 Characteristics of Sound

1.1 Introduction

Sound

1. Elastic medium (gas, water, solid) is the necessary condition of sound propagation, therefore sound cannot propagate in the vacuum.
2. Sound wave propagation speed c_0 : in air ≈ 340 m/s , in solid ≈ 1500 m/s.

Jiaju Zou
Bachelor degree, SUSTech, Shenzhen China
Master student, BUAA, Beijing China
e-mail: zoujj2020@mail.sustech.edu.cn

3. Molecules in the medium (fluid particles) vibrate at the speed of around 0.1 m/s.
4. Longitudinal wave motion (the direction of particle vibration is the same as the direction of the wave propagation).

Mean State

First, let's have a look at the following parameters:

- ambient pressure P_0
- perturbation pressure $p'(\mathbf{x}, t)$
- ambient density ρ_0
- perturbation density $\rho'(\mathbf{x}, t)$

Then the total unsteady pressure and density can be written as:

$$\begin{aligned} P_0 + p'(\mathbf{x}, t) \\ \rho_0 + \rho'(\mathbf{x}, t) \end{aligned}$$

Now consider the perturbations can be negligible, the acoustic parameters satisfy the following condition:

$$\begin{aligned} p'(\mathbf{x}, t) &\ll P_0 \\ \rho'(\mathbf{x}, t) &\ll \rho_0 \\ v &\ll c_0 \\ s &\ll \lambda \end{aligned}$$

where v and s are the speed and displacement of particle vibration separately, λ is the wavelength of the propagating sound wave.

Mathematically, we can obtain the following results:

1. The product of perturbation quantities can be negligible.
2. The response of the acoustic field is linear.
3. Sound can be regarded as linear motion.

Here we list some common perturbation pressures in our daily life, which can help you have an intuitive understanding of the quantity of $p'(\mathbf{x}, t)$.

- We human just feel the sound at 1000Hz: 2×10^{-5} Pa
- The wind blowing the leaves: 2×10^{-4} Pa
- The talk in room with distance of 1m: $0.05 \sim 0.1$ Pa

Sound Pressure Level (SPL)

$$\text{SPL} = 20 \log_{10} \left(\frac{\sqrt{p'^2}}{2 \times 10^{-5} \text{Pa}} \right) \text{dB}$$

Remarks:

- p' is the pressure perturbation.
- $\sqrt{p'^2}$ is the root mean square of pressure perturbation, which can also be noted as p_{rms} .

- As you can notice, the bottom part of the fraction is the value we human can just feel the sound at 1000Hz.
- If you do some acoustic measurement, the data you obtained through microphone is time domain signal, you have to do Fourier Transform (FT) to get the frequency domain result. (Actually, author is still trying to figure out this step, and there are some problems author can not explain very clearly... You can refer to section 1.2 to learn more about this step. The textbook *Signals and Systems* is strongly suggested, which is the course the author missed as an undergraduate...)
- If the time domain signal p' is simple harmonic, then you can also get p_{rms} through the following equation:

$$p_{rms} = p_e = \sqrt{\frac{1}{T} \int_0^T p'^2 dt}$$

where p_e is the effective pressure.

Example 1.1

Consider two sound sources, amplitudes A and B, angular frequency ω_1 and ω_2 , phase difference σ , SPL 85dB and 80dB. Let's have a look at the SPL at a given point in this sound field.

Source 1: $p'_1 = A \cos(\omega_1 t)$

Source 2: $p'_2 = B \cos(\omega_2 t + \sigma)$

Total: $p' = A \cos(\omega_1 t) + B \cos(\omega_2 t + \sigma)$

(We can add the two equation directly since the linear sound wave assumption)

Square:

$$p'^2 = A^2 \cos^2(\omega_1 t) + B^2 \cos^2(\omega_2 t + \sigma) + AB [\cos(\omega_1 t + \omega_2 t + \sigma) + \cos(\omega_1 t - \omega_2 t - \sigma)]$$

Time Average:

$$\overline{p'^2} = \frac{1}{T} \int_0^T p'^2 dt = \frac{1}{2} A^2 + \frac{1}{2} B^2 + AB [\overline{\dots}]$$

1. If $\omega_1 \neq \omega_2$, $AB [\overline{\dots}] = 0$.

2. If $\omega_1 = \omega_2$, $AB [\overline{\dots}] = AB \cos(\sigma)$

You can also assume more conditions to make this problem more detailed, such as identical sound waves, only a single wave etc, and we promise you can find something interesting and useful.

1.2 Sound Spectra

Spectral Analysis \Rightarrow Total Composition of Sound

Fourier transform (FT) is the key point.

As mentioned before, if you do some acoustic measurement, the data you obtained through the microphone is a time-domain signal, you have to do Fourier Transform to get the frequency domain result.

In this section, we will discuss this process in detail based on the author's understanding and knowledge.

It's warmly welcomed to share your ideas and methods with us through the e-mail on the first page :)

Now we list two equations as follows:

$$F(\omega) = \int_{-\infty}^{+\infty} s(t) e^{i\omega t} dt$$

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{i\omega t} d\omega$$

$F(\omega)$ is harmonic element,

$s(t)$ is signal function.

Example 1.2

Suppose one harmonic signal: $s(t) = I e^{i\omega_0 t}$

Its harmonic element: $F(\omega) = I \int_{-\infty}^{+\infty} e^{i(\omega_0 - \omega)t} dt = I \cdot 2\pi \delta(\omega_0 - \omega)$

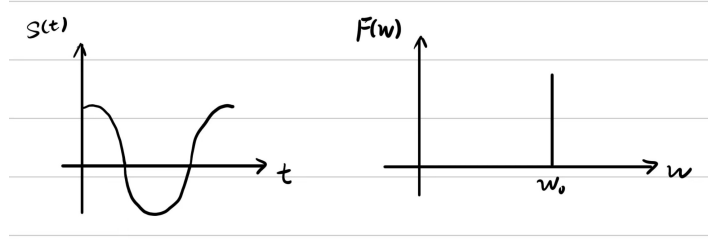


Fig. 1 Harmonic signal and its element

In this example, we used the property of Delta-Function:

$$\int_{-\infty}^{+\infty} g(t) \delta(t - t_0) dt = g(t_0) \text{ for } \forall g(t)$$

Do FT to the impulse signal $I\delta(t - t_0)$ and use the above property, we can get:

$$F(\omega) = I e^{-i\omega t_0}$$

Then, the following result can be derived:

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{i\omega t} d\omega$$

$$\delta(t - t_0) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega(t-t_0)} d\omega$$

2 Sound Wave Equations

2.1 One-Dimensional Wave Equation

In the section we will discuss the following situation of sound pressure perturbation:

$$p' = p'(x, t)$$

Note that there is **no bold** in the body of x , which means x is only a scalar, and only one direction(for example, the plane wave).

We can obtain the following equation by combining the continuity equation of mass and momentum(ignore the viscous effect in sound waves):

$$\frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x^2} = 0$$

Then we have to consider the equations of thermodynamics. The speed of heat transfer is much lower than the speed of the volume change in the process of sound propagation, therefore we can regard this process as adiabatic, the pressure is only the function of density:

$$P = P(\rho)$$

Expansion and linearization:

$$\begin{aligned} P &= P_0 + (\rho - \rho_0) \frac{dP}{d\rho}(\rho_0) + \dots \\ &\Downarrow \\ P - P_0 &= (\rho - \rho_0) \frac{dP}{d\rho}(\rho_0) \\ &\Downarrow \\ p' &= \rho' \frac{dP}{d\rho}(\rho_0) \end{aligned}$$

Introduce a new constant:

$$\begin{aligned} c^2 &= \frac{dP}{d\rho}(\rho_0) = \frac{p'}{\rho'} \\ &\text{sound speed } c \\ p' &\text{ travels with constant speed } c \end{aligned}$$

Finally we can get:

$$\frac{1}{c^2} \frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x^2} = 0$$

1-D wave equation

General solution:

$$\begin{aligned} p'(x, t) &= f(x - ct) + g(x + ct) \\ f(x - ct) &\text{ propagating to the right}(t \uparrow, x \uparrow) \\ g(x + ct) &\text{ propagating to the left}(t \uparrow, x \downarrow) \end{aligned}$$

Harmonic Waves:

$$p'(x, t) = Ae^{i\omega(t-x/c)} + Be^{i\omega(t+x/c)}$$

Only real part is meaningful.

The derivation of the 1-D wave equation is based on some assumptions(**inviscid, adiabatic, small perturbation**), and the result can be referred to as linear acoustics, which is still useful under lots of situations.

2.1.1 Acoustic Particle velocity

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Further on please use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.1, see also Fig. 2¹

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

Paragraph Heading

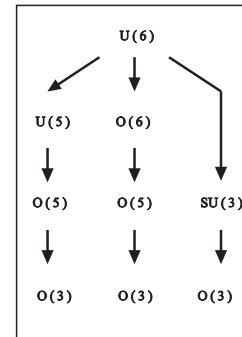
Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Further on please use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

For typesetting numbered lists we recommend to use the `enumerate` environment – it will automatically rendered in line with the preferred layout.

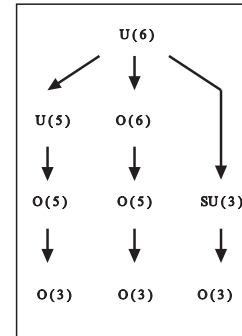
1. Livelihood and survival mobility are oftentimes outcomes of uneven socioeconomic development.

Fig. 2 If the width of the figure is less than 7.8 cm use the `sidecaption` command to flush the caption on the left side of the page. If the figure is positioned at the top of the page, align the sidecaption with the top of the figure – to achieve this you simply need to use the optional argument `[t]` with the `sidecaption` command



¹ If you copy text passages, figures, or tables from other works, you must obtain *permission* from the copyright holder (usually the original publisher). Please enclose the signed permission with the manuscript. The sources must be acknowledged either in the captions, as footnotes or in a separate section of the book.

Fig. 3 If the width of the figure is less than 7.8 cm use the `sidecaption` command to flush the caption on the left side of the page. If the figure is positioned at the top of the page, align the sidecaption with the top of the figure – to achieve this you simply need to use the optional argument `[t]` with the `sidecaption` command



- a. Livelihood and survival mobility are oftentimes coutcomes of uneven socioe-
conomic development.
 - b. Livelihood and survival mobility are oftentimes coutcomes of uneven socioe-
conomic development.
2. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeco-
nomic development.

Subparagraph Heading

In order to avoid simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2, see also Fig. 3.

For unnumbered list we recommend to use the `itemize` environment – it will automatically be rendered in line with the preferred layout.

- Livelihood and survival mobility are oftentimes coutcomes of uneven socioeco-
nomic development, cf. Table 1.
 - Livelihood and survival mobility are oftentimes coutcomes of uneven socioe-
conomic development.
 - Livelihood and survival mobility are oftentimes coutcomes of uneven socioe-
conomic development.
- Livelihood and survival mobility are oftentimes coutcomes of uneven socioeco-
nomic development.

Run-in Heading Boldface Version Use the \LaTeX automatism for all your cross-
references and citations as has already been described in Sect. 2.

Run-in Heading Boldface and Italic Version Use the \LaTeX automatism for all your
cross-references and citations as has already been described in Sect. 2.

Table 1 Please write your table caption here

Classes	Subclass	Length	Action Mechanism
Translation	mRNA ^a	22 (19–25)	Translation repression, mRNA cleavage
Translation	mRNA cleavage	21	mRNA cleavage
Translation	mRNA	21–22	mRNA cleavage
Translation	mRNA	24–26	Histone and DNA Modification

^a Table foot note (with superscript)

Run-in Heading Displayed Version

Use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.

3 Section Heading

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Further on please use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

If you want to list definitions or the like we recommend to use the `description` environment – it will automatically rendered in line with the preferred layout.

- Type 1 That addresses central themes pertaining to migration, health, and disease. In Sect. 1.1, Wilson discusses the role of human migration in infectious disease distributions and patterns.
- Type 2 That addresses central themes pertaining to migration, health, and disease. In Sect. 2.1, Wilson discusses the role of human migration in infectious disease distributions and patterns.

3.1 Subsection Heading

In order to avoid simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

If you want to emphasize complete paragraphs of texts we recommend to use the newly defined class option `graybox` and the newly defined environment `svgraybox`. This will produce a 15 percent screened box 'behind' your text.

If you want to emphasize complete paragraphs of texts we recommend to use the newly defined class option and environment `svgraybox`. This will produce a 15 percent screened box 'behind' your text.

3.1.1 Subsubsection Heading

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Further on please use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

Theorem 1 *Theorem text goes here.*

Definition 1 Definition text goes here.

Proof Proof text goes here. □

Paragraph Heading

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Further on please use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.

Note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

Theorem 2 *Theorem text goes here.*

Definition 2 Definition text goes here.

Proof Proof text goes here. □

Trailer Head

If you want to emphasize complete paragraphs of texts in an Trailer Head we recommend to use

```
\begin{trailer}{Trailer Head}  
...  
\end{trailer}
```

? Questions

If you want to emphasize complete paragraphs of texts in an `Questions` we recommend to use

```
\begin{question}{Questions}  
...  
\end{question}
```

> Important

If you want to emphasize complete paragraphs of texts in an **Important** we recommend to use

```
\begin{important}{Important}
...
\end{important}
```

! Attention

If you want to emphasize complete paragraphs of texts in an **Attention** we recommend to use

```
\begin{warning}{Attention}
...
\end{warning}
```

Program Code

If you want to emphasize complete paragraphs of texts in an **Program Code** we recommend to use

```
\begin{programcode}{Program Code}
\begin{verbatim}...\end{verbatim}
\end{programcode}
```

Tips

If you want to emphasize complete paragraphs of texts in an **Tips** we recommend to use

```
\begin{tips}{Tips}
...
\end{tips}
```

Overview

If you want to emphasize complete paragraphs of texts in an Overview we recommend to use

```
\begin{overview}{Overview}
...
\end{overview}
```

Background Information

If you want to emphasize complete paragraphs of texts in an Background Information we recommend to use

```
\begin{backgroundinformation}{Background Information}
...
\end{backgroundinformation}
```

Legal Text

If you want to emphasize complete paragraphs of texts in an Legal Text we recommend to use

```
\begin{legalttext}{Legal Text}
...
\end{legalttext}
```

Acknowledgements If you want to include acknowledgments of assistance and the like at the end of an individual chapter please use the `acknowledgement` environment – it will automatically be rendered in line with the preferred layout.

Appendix

When placed at the end of a chapter or contribution (as opposed to at the end of the book), the numbering of tables, figures, and equations in the appendix section continues on from that in the main text. Hence please *do not* use the `appendix` command when writing an appendix at the end of your chapter or contribution. If there is only one the appendix is designated “Appendix”, or “Appendix 1”, or “Appendix 2”, etc. if there is more than one.

$$a \times b = c \tag{1}$$

References

References may be *cited* in the text either by number (preferred) or by author/year.² If the citation in the text is numbered, the reference list should be arranged in ascending order. If the citation in the text is author/year, the reference list should be *sorted* alphabetically and if there are several works by the same author, the following order should be used:

1. all works by the author alone, ordered chronologically by year of publication
2. all works by the author with a coauthor, ordered alphabetically by coauthor
3. all works by the author with several coauthors, ordered chronologically by year of publication.

The *styling* of references³ depends on the subject of your book:

- The *two* recommended styles for references in books on *mathematical, physical, statistical and computer sciences* are depicted in [1, 2, 3, 4, 5] and [6, 7, 8, 9, 10].
- Examples of the most commonly used reference style in books on *Psychology, Social Sciences* are [11, 12, 13, 14, 15].
- Examples for references in books on *Humanities, Linguistics, Philosophy* are [16, 17, 18, 19, 20].
- Examples of the basic Springer Nature style used in publications on a wide range of subjects such as *Computer Science, Economics, Engineering, Geosciences, Life Sciences, Medicine, Biomedicine* are [21, 22, 24, 23, 25].

1. Broy, M.: Software engineering — from auxiliary to key technologies. In: Broy, M., Dener, E. (eds.) *Software Pioneers*, pp. 10-13. Springer, Heidelberg (2002)
2. Dod, J.: Effective substances. In: *The Dictionary of Substances and Their Effects*. Royal Society of Chemistry (1999) Available via DIALOG.
<http://www.rsc.org/dose/title> of subordinate document. Cited 15 Jan 1999
3. Geddes, K.O., Czapor, S.R., Labahn, G.: *Algorithms for Computer Algebra*. Kluwer, Boston (1992)
4. Hamburger, C.: Quasimonotonicity, regularity and duality for nonlinear systems of partial differential equations. *Ann. Mat. Pura. Appl.* **169**, 321–354 (1995)
5. Slifka, M.K., Whitton, J.L.: Clinical implications of dysregulated cytokine production. *J. Mol. Med.* (2000) doi: 10.1007/s001090000086
6. J. Dod, in *The Dictionary of Substances and Their Effects*, Royal Society of Chemistry. (Available via DIALOG, 1999), <http://www.rsc.org/dose/title> of subordinate document. Cited 15 Jan 1999
7. H. Ibach, H. Lüth, *Solid-State Physics*, 2nd edn. (Springer, New York, 1996), pp. 45-56
8. S. Preuss, A. Demchuk Jr., M. Stuke, *Appl. Phys. A* **61**
9. M.K. Slifka, J.L. Whitton, *J. Mol. Med.*, doi: 10.1007/s001090000086
10. S.E. Smith, in *Neuromuscular Junction*, ed. by E. Zaimis. *Handbook of Experimental Pharmacology*, vol 42 (Springer, Heidelberg, 1976), p. 593

² Make sure that all references from the list are cited in the text. Those not cited should be moved to a separate *Further Reading* section or chapter.

³ Always use the standard abbreviation of a journal's name according to the *ISSN List of Title Word Abbreviations*, see <http://www.issn.org/en/node/344>

11. Calfee, R. C., & Valencia, R. R. (1991). *APA guide to preparing manuscripts for journal publication*. Washington, DC: American Psychological Association.
12. Dod, J. (1999). Effective substances. In: The dictionary of substances and their effects. Royal Society of Chemistry. Available via DIALOG.
<http://www.rsc.org/dose/Effective substances>. Cited 15 Jan 1999.
13. Harris, M., Karper, E., Stacks, G., Hoffman, D., DeNiro, R., Cruz, P., et al. (2001). Writing labs and the Hollywood connection. *J Film Writing*, 44(3), 213–245.
14. O’Neil, J. M., & Egan, J. (1992). Men’s and women’s gender role journeys: Metaphor for healing, transition, and transformation. In B. R. Wainrig (Ed.), *Gender issues across the life cycle* (pp. 107–123). New York: Springer.
15. Kreger, M., Brindis, C.D., Manuel, D.M., Sassoubre, L. (2007). Lessons learned in systems change initiatives: benchmarks and indicators. *American Journal of Community Psychology*, doi: 10.1007/s10464-007-9108-14.
16. Alber John, Daniel C. O’Connell, and Sabine Kowal. 2002. Personal perspective in TV interviews. *Pragmatics* 12:257–271
17. Cameron, Deborah. 1997. Theoretical debates in feminist linguistics: Questions of sex and gender. In *Gender and discourse*, ed. Ruth Wodak, 99–119. London: Sage Publications.
18. Cameron, Deborah. 1985. *Feminism and linguistic theory*. New York: St. Martin’s Press.
19. Dod, Jake. 1999. Effective substances. In: The dictionary of substances and their effects. Royal Society of Chemistry. Available via DIALOG.
<http://www.rsc.org/dose/title of subordinate document>. Cited 15 Jan 1999
20. Suleiman, Camelia, Daniel C. O’Connell, and Sabine Kowal. 2002. ‘If you and I, if we, in this later day, lose that sacred fire...’: Perspective in political interviews. *Journal of Psycholinguistic Research*. doi: 10.1023/A:1015592129296.
21. Brown B, Aaron M (2001) The politics of nature. In: Smith J (ed) The rise of modern genomics, 3rd edn. Wiley, New York
22. Dod J (1999) Effective Substances. In: The dictionary of substances and their effects. Royal Society of Chemistry. Available via DIALOG.
<http://www.rsc.org/dose/title of subordinate document>. Cited 15 Jan 1999
23. Slifka MK, Whitton JL (2000) Clinical implications of dysregulated cytokine production. *J Mol Med*, doi: 10.1007/s001090000086
24. Smith J, Jones M Jr, Houghton L et al (1999) Future of health insurance. *N Engl J Med* 341:325–329
25. South J, Blass B (2001) The future of modern genomics. Blackwell, London