MAE 310: Computational Solid Mechanics

Lecture 22: Plane elasticity

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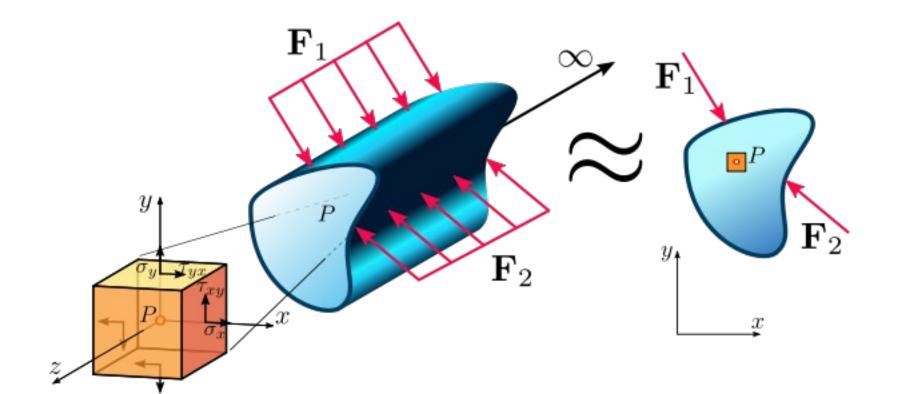
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Isotropic elasticity

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} 2\mu + \lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2\mu + \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2\mu + \lambda & 0 & 0 & 0 \\ \lambda & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{bmatrix}$$

ر σ_{11}	$\Gamma^2\mu + \lambda$	λ	λ	0	0	70	ړ $arepsilon_{11}$
σ_{22}	λ	$2\mu + \lambda$	λ	0	0	0	$ \mathcal{E}_{22} $
$ \sigma_{33} _{\cdot}$	λ	λ	$2\mu + \lambda$	0	0	0	0
$ \sigma_{23} ^{2}$	0	0	0	μ	0	0	0
$ \sigma_{13} $	0	0	0	0	μ	0	0
$\lfloor \sigma_{12} \rfloor$	0	0	0	0	0	μ	$\lfloor arepsilon_{12} floor$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} 2\mu + \lambda & \lambda & 0 \\ \lambda & 2\mu + \lambda & 0 \\ 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix} \text{ and } \sigma_{33} = \lambda(\varepsilon_{11} + \varepsilon_{22})$$



Isotropic elasticity

The stress-strain relation

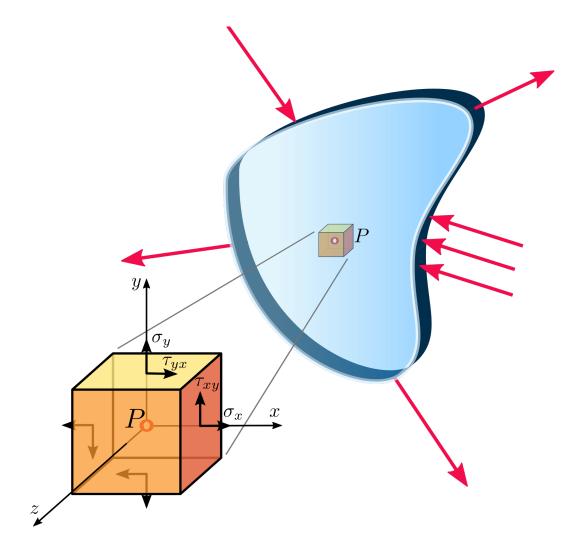
$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} 2\mu + \lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2\mu + \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2\mu + \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{bmatrix}$$

can be inverted as a matrix problem.

Plane stress elasticity

Γ ε ₁₁ -	1	г 1	-v	-v	0	0	0 7	$\lceil^{\sigma{11}} ceil$
\mathcal{E}_{22}		-v	1	-v	0	0	0	$ \sigma_{22} $
ε_{33}	1	-v	-v	1	0	0	0	0
$ \varepsilon_{23} $	$= \frac{1}{E}$	0	0	0	2(1 + v)	0	0	0
ε_{13}		0	0	0	0	2(1 + v)	0	0 0
$L\varepsilon_{12}$		0	0	0	0	0	2(1+v)	

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -v & 0 \\ -v & 1 & 0 \\ 0 & 0 & 2(1+v) \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}$$
 and
$$\varepsilon_{33} = \frac{-v}{E} (\sigma_{11} + \sigma_{22})$$



$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \frac{E}{1 - v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1 - v}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix}$$

and

$$\varepsilon_{33} = \frac{-v}{E}(\sigma_{11} + \sigma_{22})$$

