《Fundamentals of Electric Circuits》 homework 6

7.7 Assuming that the switch in Fig. 7.87 has been in position A for a long time and is moved to position B at $t = 0$. Then at $t = 1$ second, the switch moves from B to C. Find $v_C(t)$ for $t \geqslant 0$. (10')

解: $t<0$, $V_c = 12V$

$t=0$, $V_c(0) = 12V$

$\tau_1 = R_B C = 10^3$

$V(t) = V_0 e^{-t/\tau} = 12e^{\frac{-t}{10^3}}$ V

$V(1) = 12e^{-10^{-3}} = 11.988V$

$\tau_2 = R_C C = 2$, $V_c(t) = 11.988 \cdot e^{-(t-1)/2}$

$$V_c(t) = \begin{cases} 12 & t=0 \\ 12e^{\frac{-t}{1000}} & 0<t<1 \\ 11.988 & t=1 \\ 11.988e^{\frac{1-t}{2}} & 1<t \end{cases}$$

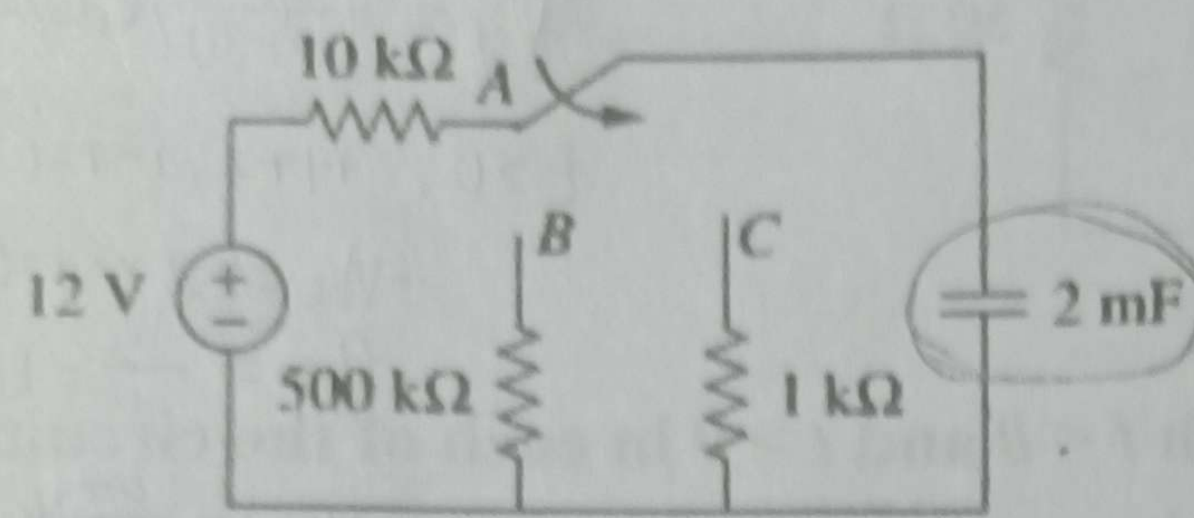


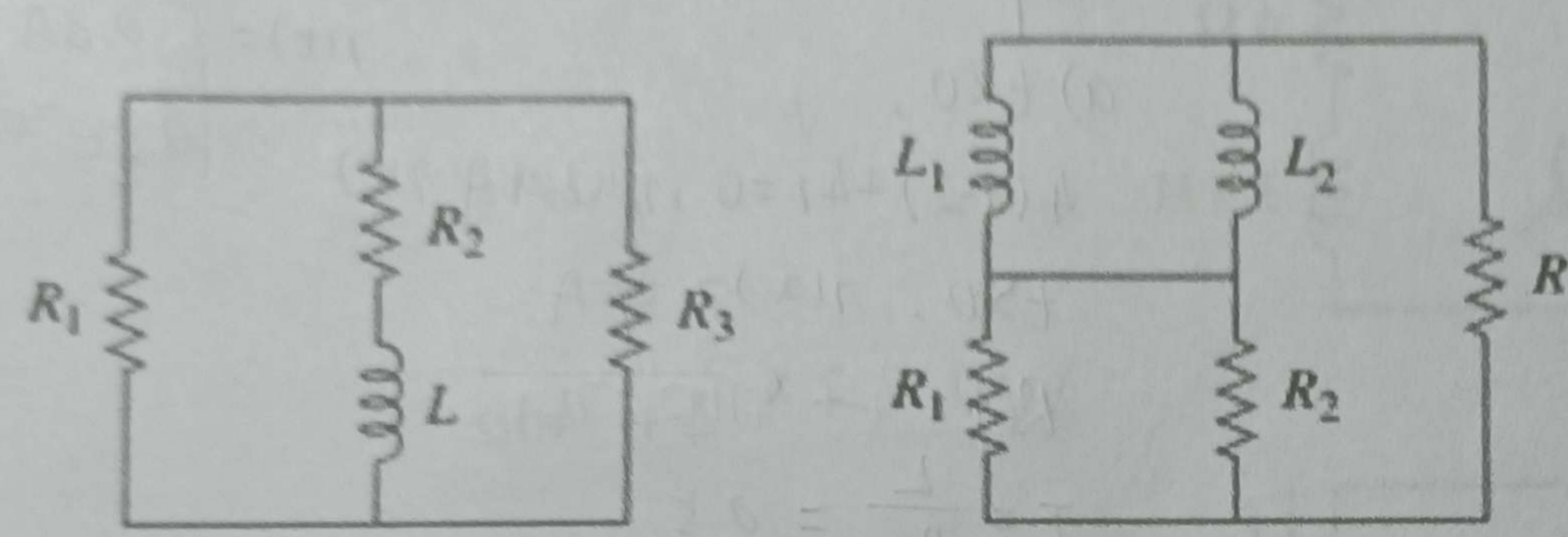
Figure 7.87

7.16 Determine the time constant for each of the circuits in Fig. 7.96. (10')

解:

a) $\tau = \frac{L}{R_{eq}} = \frac{L}{R_1 \| R_3 + R_2} = \frac{L}{\frac{R_1R_3}{R_1+R_3} + R_2}$

b) $\tau = \frac{L_{eq}}{R_{eq}} = \frac{L_1 \| L_2}{R_1 \| R_2 + R_3} = \frac{\frac{L_1L_2}{L_1+L_2}}{\frac{R_1R_2}{R_1+R_2} + R_3}$



(a) (b)

Figure 7.96

7.19 In the circuit of Fig. 7.99, find $i(t)$ for $t > 1$ if $i(0) = 6$ A. (10')

解: $i(0) = 6A$

$+0.5\times40 + 10\times1 - V_{oc} = 0$, $V_{oc} = 30V$

$R_{eq} = \frac{V_{oc}}{1} = 30\Omega$

$\tau = \frac{L}{R_{eq}} = \frac{6}{30} = 0.2$

$I(t) = i(0) \cdot e^{-t/\tau} = 6e^{-5t}$ A

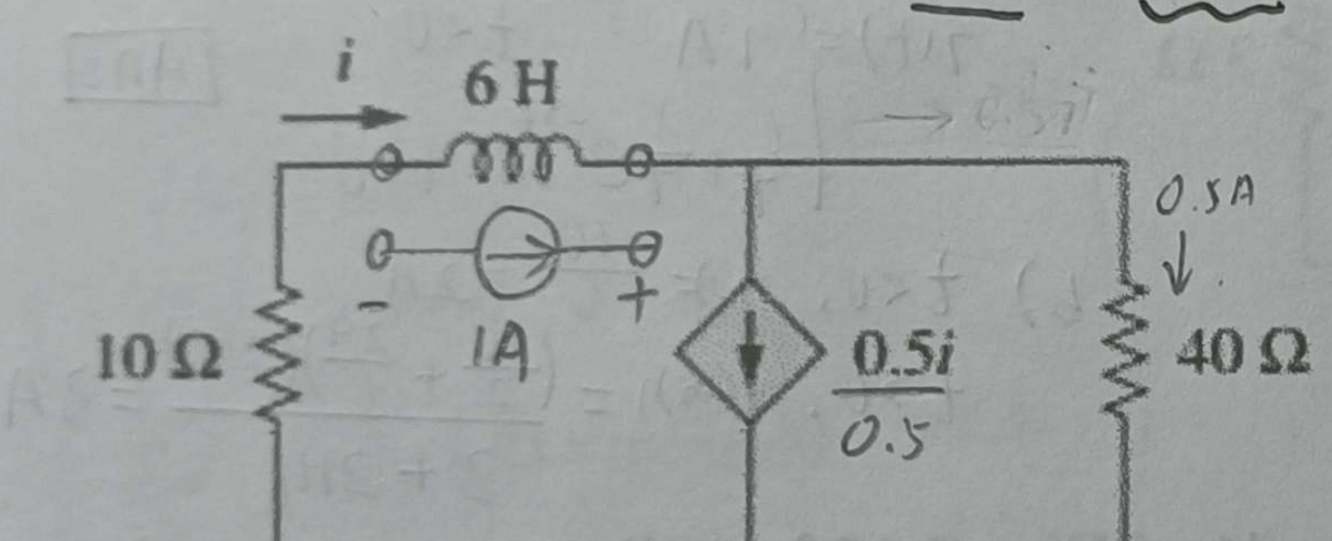


Figure 7.99

7.29 Sketch the following functions:

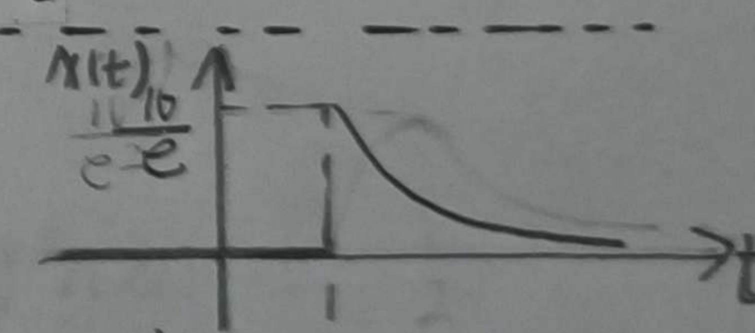
(a) $x(t) = 10e^{-t}u(t-1)$

(b) $y(t) = 10e^{-(t-1)}u(t)$

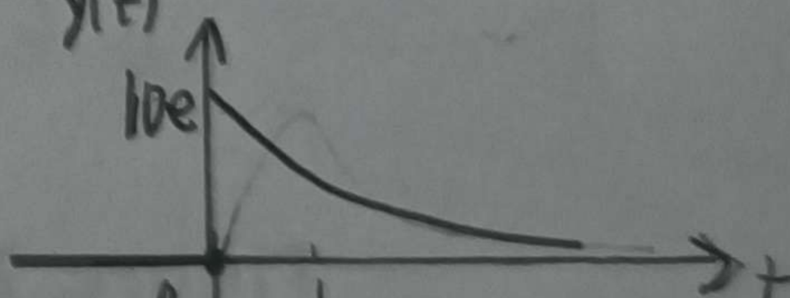
(c) $z(t) = \cos 4t\delta(t-1)$

(10')

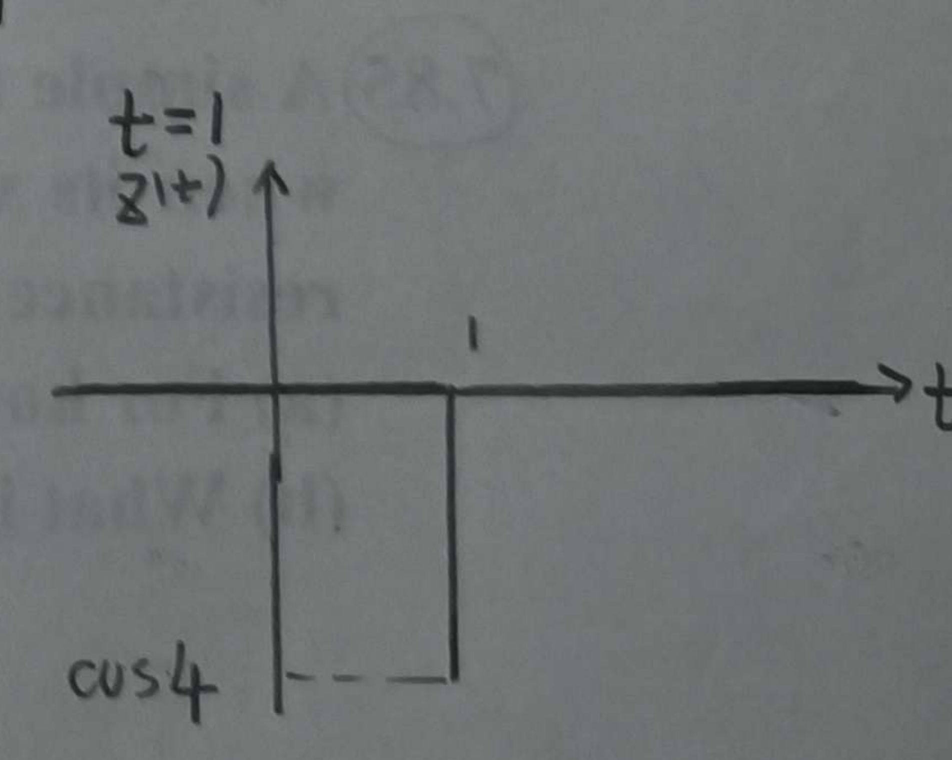
解: a) $x(t) = \begin{cases} 0 & t<1 \\ 10e^{-t} & 1\leq t \end{cases}$



b) $y(t) = \begin{cases} 0 & t<0 \\ 10e^{-(t-1)} & 0\leq t \end{cases}$



c) $z(t) = \begin{cases} 0 & t\neq1 \\ \cos4 \cdot & t=1 \end{cases}$



7.37 A circuit is described by

$$4\frac{dv}{dt} + v = 10$$

(a) What is the time constant of the circuit?

(b) What is $v(\infty)$, the final value of v?

(c) If $v(0) = 2$, find $v(t)$ for $t \geq 0$. (10')

解: a) $\frac{dv}{dt} = \frac{10-v}{4}$

$\int \frac{dv}{10-v} = \int \frac{1}{4}dt$

$v = 10 - Ae^{-\frac{1}{4}t}$

$\tau = 4$

b) $t \to \infty$

$v = \lim_{t\to\infty} (10 - A\cdot e^{-\frac{1}{4}t}) = 10$

c) $v(0) = 2 \Rightarrow 2 = 10 - A$, $A = 8$

$v(t) = 10 - 8e^{-\frac{t}{4}}$

7.43 Consider the circuit in Fig. 7.110. Find $i(t)$ for $t < 0$ and $t > 0$. (10')

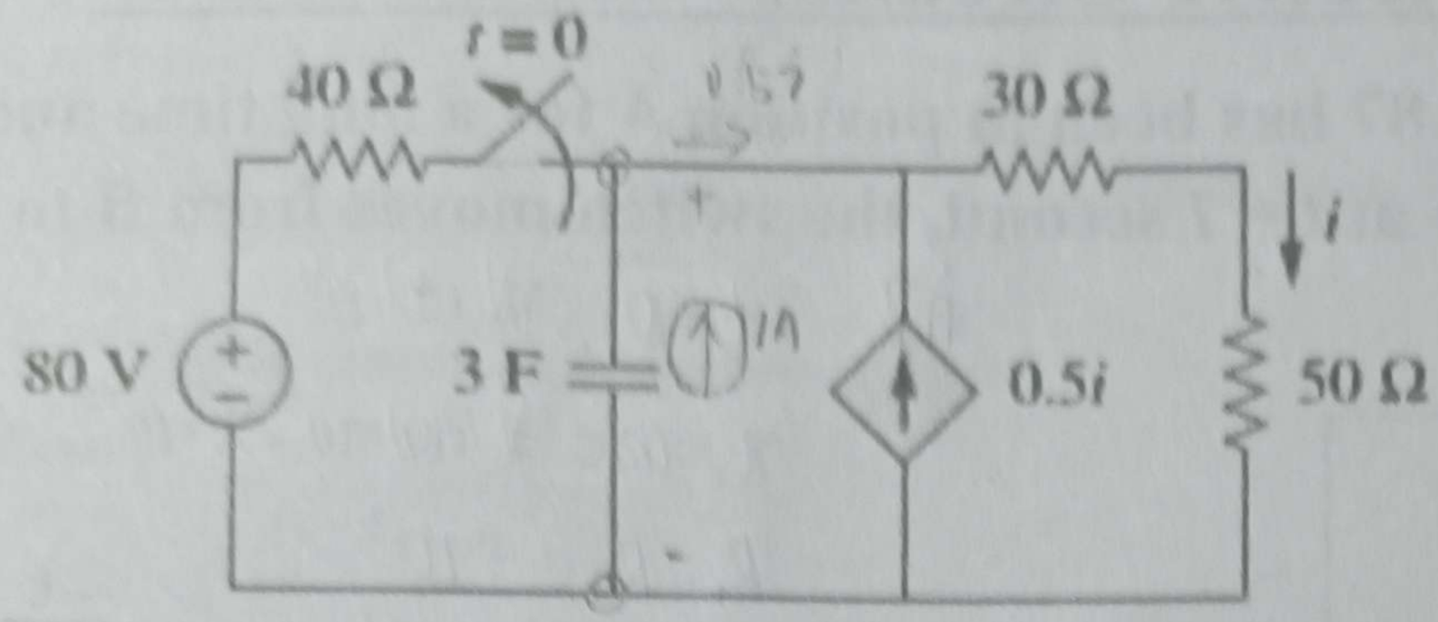


Figure 7.110

解: $t<0$,

$-80 + 40\times0.5i + 80i = 0$

$i = 0.8A$

$V_C = (30+50)\times0.8 = 64\ V$, $V_C(0) = 64\ V$

$t>0$, $+1+0.5i-i=0$, $i=2A$

$-V_{oc} + 2\times80 = 0$, $V_{oc} = 160\ V$

$R_{eq} = \frac{V_{oc}}{1} = 160\ \Omega$, $\tau = R_{eq}\cdot C = 480$

$V(t) = 64\ e^{-t/480}$

$0.5i(t)\cdot R_{eq} = V(t)$

$i(t) = \frac{2\times64}{160} e^{-t/480} = 0.8e^{\frac{-t}{480}}\ A$

$\therefore i(t) = \begin{cases} 0.8A & t<0 \\ 0.8e^{-\frac{t}{480}} & t>0 \end{cases}$

7.54 Obtain the inductor current for both $t < 0$ and $t > 0$ in each of the circuits in Fig. 7.120. (15')

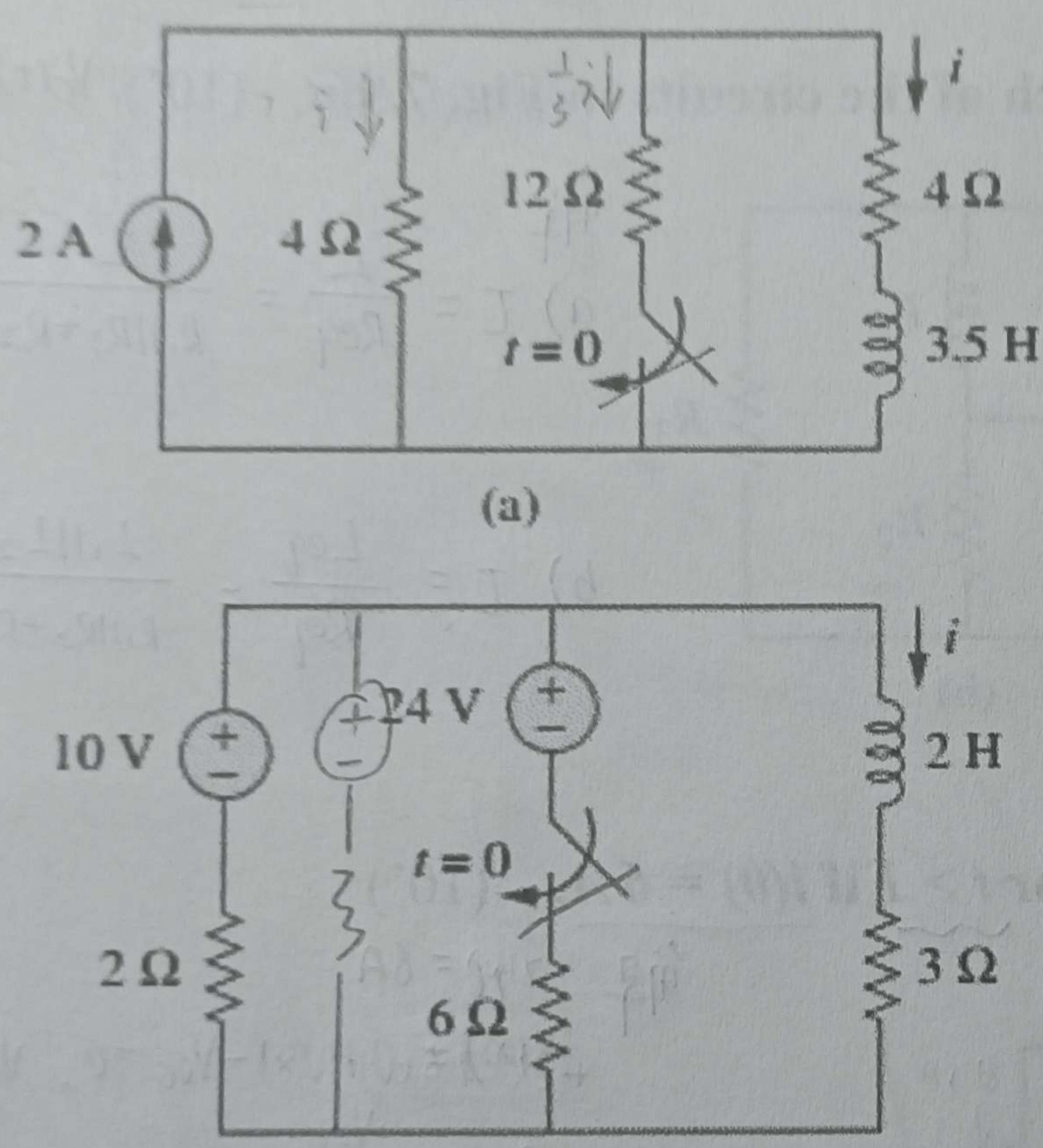


Figure 7.120

解:

a) $t<0$, $4(i-2)+4i=0$, $i(0)=1A$

$t>0$, $i(\infty) = \frac{6}{7}A$

$R_{eq} = 4 + 4\|12 = 7\Omega$

$\tau = \frac{L}{R_{eq}} = 0.5$

$i(t) = i(\infty) + [i(0)-i(\infty)]e^{-t/\tau}$

$= \frac{6}{7} + \frac{1}{7}e^{-2t}$

$\therefore i(t) = \begin{cases} 1A & t<0 \\ \frac{6}{7}+\frac{1}{7}e^{-2t} & t>0 \end{cases}$ $\boxed{Ans}$

b) $t<0$, $i(0) = \frac{10}{5} = 2A$

$t>0$, $i(\infty) = \frac{(\frac{10}{2}+\frac{24}{6})(2\|6)}{3+2\|6} = 3A$

$R_{eq} = 3+2\|6 = 4.5\Omega$, $\tau = \frac{L}{R_{eq}} = \frac{2}{4.5} = \frac{4}{9}$

$i(t) = 3 + (-1)\cdot e^{-\frac{9t}{4}}$

$\therefore i(t) = \begin{cases} 2A & t<0 \\ 3-e^{-\frac{9t}{4}}\ A & t>0 \end{cases}$ $\boxed{Ans}$

7.57 Find $i_1(t)$ and $i_2(t)$ for $t > 0$ in the circuit of Fig. 7.123.(10')

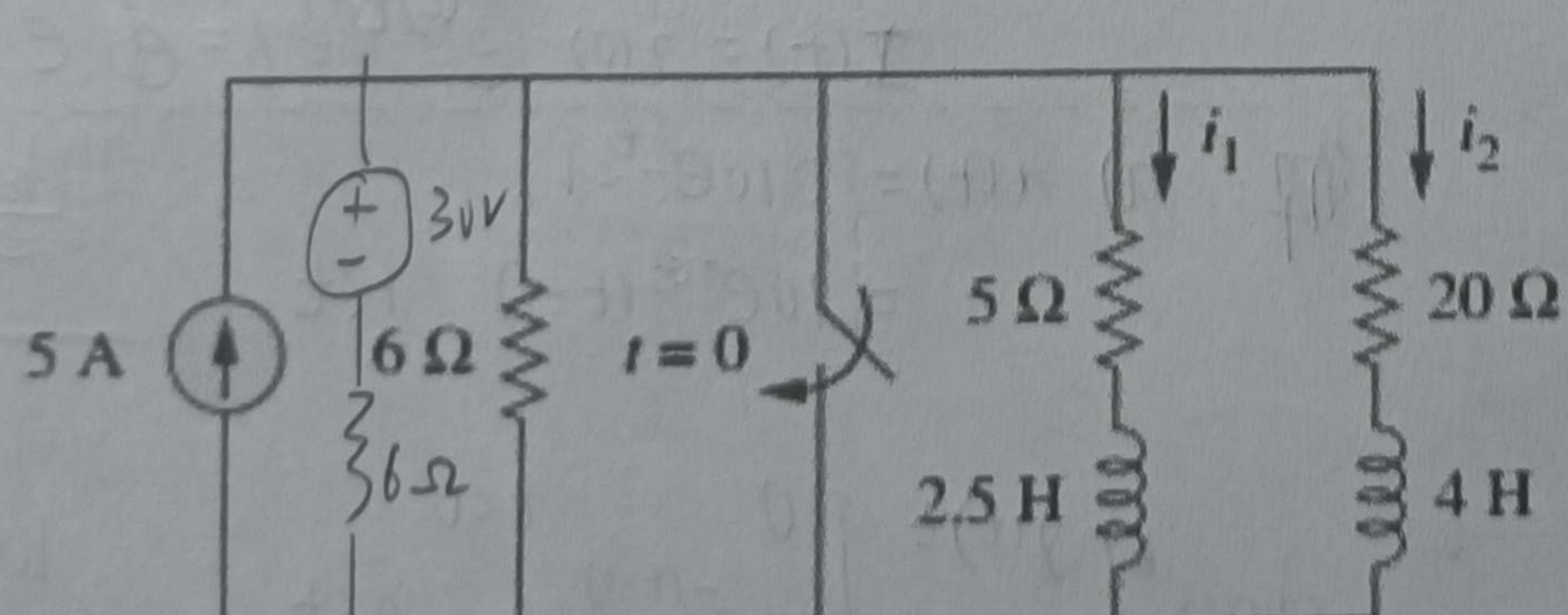


Figure 7.123

解: $t<0$, $i_1(0) = \frac{30}{6+5\|20}\times\frac{5\|20}{5} = 2.4A$

$i_2(0) = \frac{30}{6+5\|20}\times\frac{5\|20}{20} = 0.6A$

$t>0$, $i_1(\infty) = 0 = i_2(\infty)$

$\tau_1 = \frac{2.5}{5} = 0.5$, $\tau_2 = \frac{4}{20} = 0.2$

$i_1(t) = i_1(\infty)+[i_1(0)-i_1(\infty)]e^{-t/\tau_1}$

$= 2.4e^{-2t}$ $(t>0)$

$i_2(t) = i_2(\infty)+[i_2(0)-i_2(\infty)]e^{-t/\tau_2}$

$= 0.6e^{-5t}$ $(t>0)$

7.85 A simple relaxation oscillator circuit is shown in Fig. 7.145. The neon lamp fires when its voltage reaches *75 V* and turns off when its voltage drops to *30 V*. Its resistance is *120Ω* when on and infinitely high when off.

(a) For how long is the lamp on each time the capacitor discharges?

(b) What is the time interval between light flashes? (15')

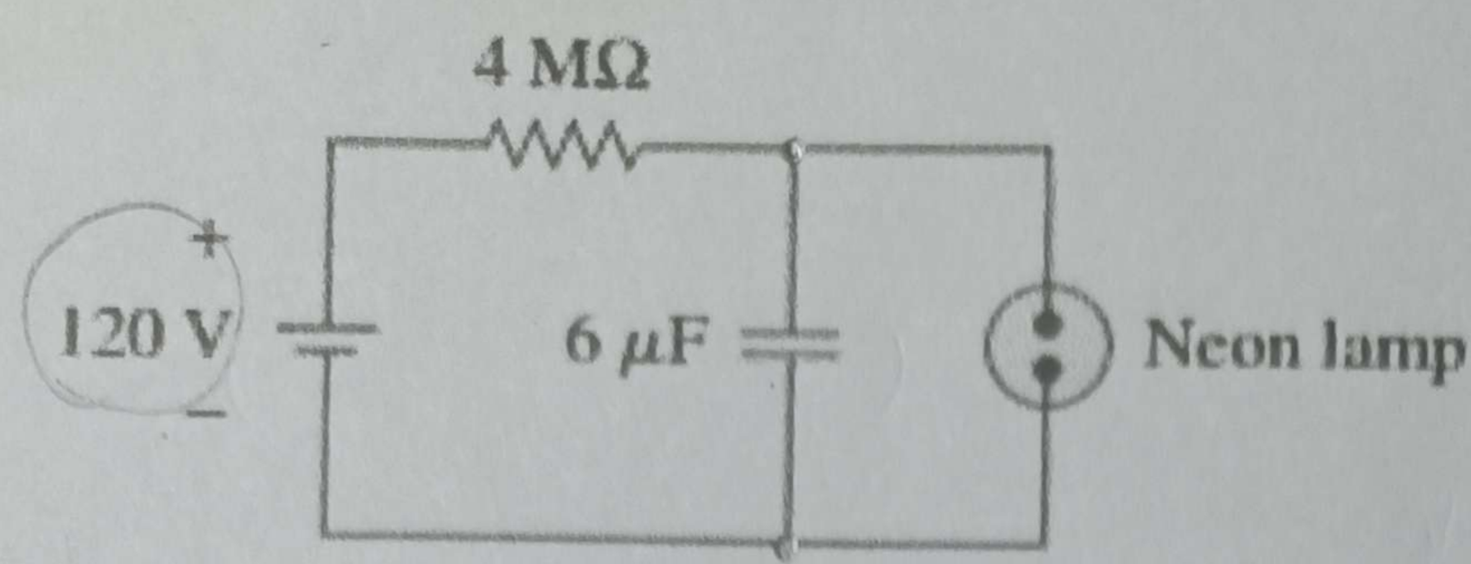
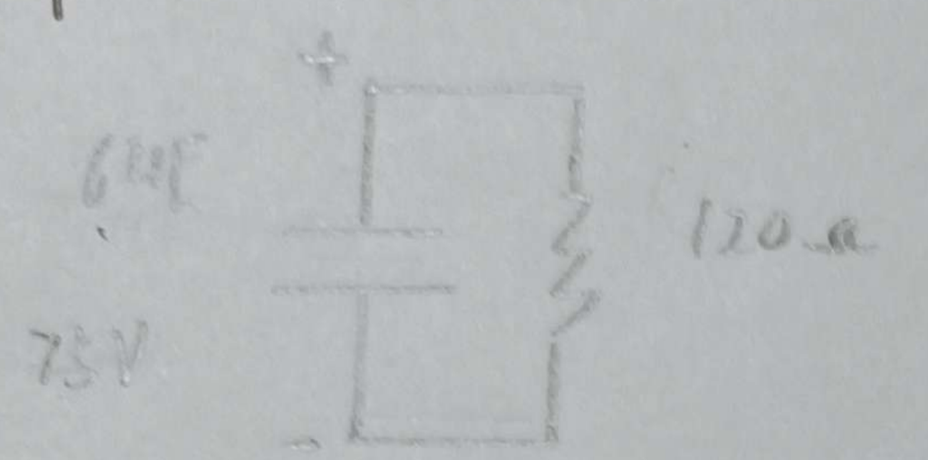


Figure 7.145

解: a) the lamp on



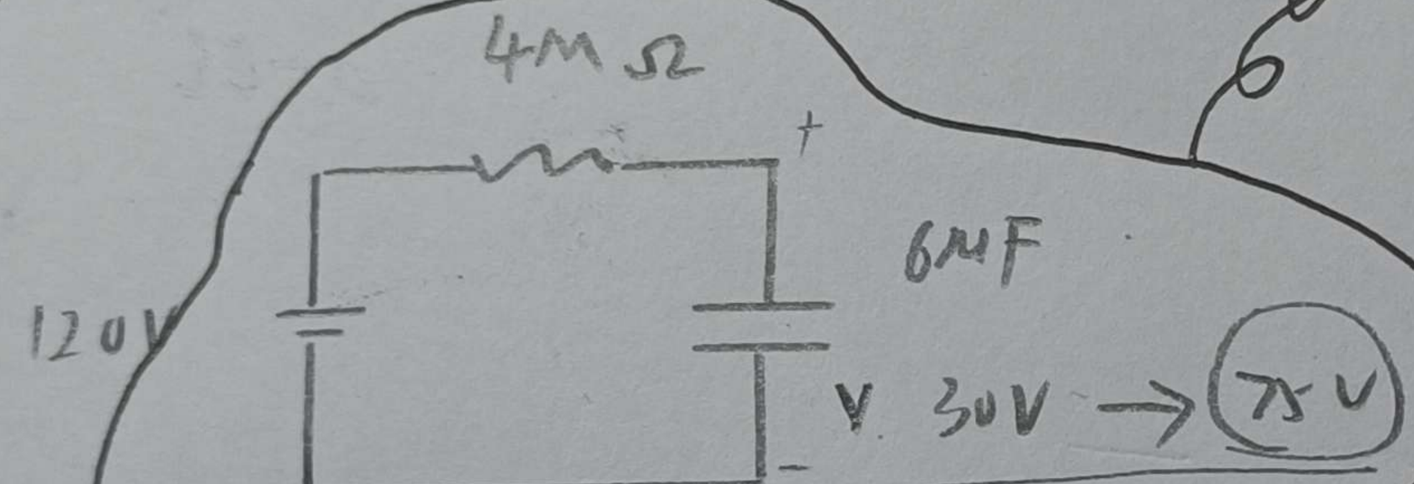
$\tau = RC = 120 \times 6 \times 10^{-6} = 7.2 \times 10^{-4}$

$V(t) = 75\, e^{\frac{-t}{7.2} \times 10^{4}}$ V

$75\, e^{\frac{-t}{7.2} \times 10^{4}} = 30$

$t = 6.597 \times 10^{-4}$ s

b)



$V(0) = 30V$

$V(\infty) = 120V$

$\tau = R \cdot C = 4 \times 10^{6} \times 6 \times 10^{-6} = 24$

$V(t) = V(\infty) + [V(0) - V(\infty)] e^{-t/\tau}$

$75 = 120 + (-45) e^{-t/24}$

$t =$

$V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)] e^{-t/\tau}$

At t_1, the lamp off

t_2 on

find $T = t_2 - t_1$

$V_c(t_1) = 30$ V, $V_c(t_2) = 75$ V

$V_c(\infty) = 120$ V

$V_c(t_1) = V_c(\infty) + [V_c(0) - V_c(\infty)] e^{-t_1/\tau_1}$

$V_c(t_2) = V_c(\infty) + [V_c(0) - V_c(\infty)] e^{-t_2/\tau_2}$

$$\frac{30-120}{75-120} = \frac{e^{-t_1/24}}{e^{-t_2/24}} = e^{\frac{1}{24}(t_2 - t_1)}$$

$$\frac{1}{24}(t_2 - t_1) = \ln \frac{90}{45}$$

$T = 24 \ln 2 = 16.64$ s