~~空气~~动力学HW1　12012127　邹佳驹

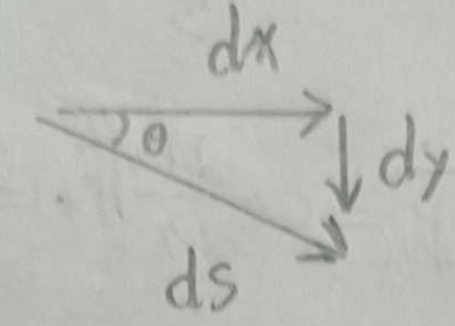
1.2

解:

①

$$N' = -\int_{LE}^{TE} (p_u\cos\theta + \tau_u \sin\theta)\,ds_u + \int_{LE}^{TE} (p_l\cos\theta - \tau_l\sin\theta)\,ds_l$$

$$C_n = \frac{N'}{q_\infty C}, \quad dx = ds\cdot\cos\theta, \quad dy = -ds\cdot\sin\theta$$



$$C_n = \frac{1}{q_\infty C}\cdot\Big[-\int_{LE}^{TE}(p_u\,dx_u + \tau_u(-1)dy_u) + \int_{LE}^{TE}(p_l\,dx_l + \tau_l\,dy_l)\Big]$$

$$= \frac{1}{C}\cdot\Big[-\int_{LE}^{TE}\frac{p_u}{q_\infty}dx_u + \frac{-\tau_u}{q_\infty}\cdot\frac{dy_u}{dx_u}\cdot dx_u + \int_{LE}^{TE}\frac{p_l}{q_\infty}dx_l + \frac{\tau_l}{q_\infty}\cdot\frac{dy_l}{dx_l}\cdot dx_l\Big]$$

$$= \frac{1}{C}\cdot\Big[-\int_0^C\Big(C_{p.u}\,dx - C_{f.u}\cdot\frac{dy_u}{dx}\cdot dx\Big) + \int_0^C C_{p.l}\,dx + C_{f.l}\cdot\frac{dy_l}{dx}\cdot dx\Big]$$

$$= \frac{1}{C}\cdot\Big[\int_0^C(C_{p.l} - C_{p.u})\,dx + \int_0^C\Big(C_{f.l}\frac{dy_l}{dx} + C_{f.u}\frac{dy_u}{dx}\Big)dx\Big]$$

②

$$A' = \int_{LE}^{TE}(-p_u\sin\theta + \tau_u\cos\theta)\,ds_u + \int_{LE}^{TE}(p_l\sin\theta + \tau_l\cos\theta)\,ds_l$$

$$C_a = \frac{A'}{q_\infty\cdot C} = \frac{1}{q_\infty\cdot C}\Big[\int_{LE}^{TE}(p_u\,dy_u + \tau_u\,dx_u) + \int_{LE}^{TE}(-p_l\,dy_l + \tau_l\,dx_l)\Big]$$

$$= \frac{1}{C}\cdot\int_0^C\Big(\frac{p_u}{q_\infty}\cdot\frac{dy_u}{dx_u}\cdot dx_u + \frac{\tau_u}{q_\infty}\cdot dx_u - \frac{p_l}{q_\infty}\cdot\frac{dy_l}{dx_l}\cdot dx_l + \frac{\tau_l}{q_\infty}dx_l\Big)$$

$$= \frac{1}{C}\Big[\int_0^C\Big(C_{p.u}\frac{dy_u}{dx} - C_{p.l}\frac{dy_l}{dx}\Big)dx + \int_0^C(C_{f.u} + C_{f.l})\,dx\Big]$$

③

$$M'_{LE} = \int_{LE}^{TE}\big[(p_u\cos\theta + \tau_u\sin\theta)x - (p_u\sin\theta - \tau_u\cos\theta)y_u\big]ds_u + \int_{LE}^{TE}\big[(-p_l\cos\theta + \tau_l\sin\theta)x + (p_l\sin\theta + \tau_l\cos\theta)y_l\big]ds_l$$

$$C_{m_{LE}} = \frac{M'_{LE}}{q_\infty\cdot C^2}$$

$$= \frac{1}{q_\infty\cdot C^2}\Big[\int_{LE}^{TE}(p_u\,dx_u - \tau_u\,dy_u)x + (p_u\,dy_u + \tau_u\,dx_u)y_u + \int_{LE}^{TE}(-p_l\,dx_l - \tau_l\,dy_l)x + (-p_l\,dy_l + \tau_l\,dx_l)y_l\Big]$$

$$= \frac{1}{C^2}\Big[\int_0^C\Big(\frac{p_u}{q_\infty}\cdot x\,dx_u - \frac{\tau_u}{q_\infty}\cdot\frac{dy_u}{dx_u}\cdot dx_u\cdot x\Big) + \int_0^C\Big(\frac{p_u}{q_\infty}y_u\frac{dy_u}{dx_u}\cdot dx_u + \frac{\tau_u}{q_\infty}\cdot y_u\,dx_u\Big)$$
$$+ \int_0^C\Big(\frac{-p_l}{q_\infty}\cdot x\cdot dx_l - \frac{\tau_l}{q_\infty}\cdot x\cdot\frac{dy_l}{dx_l}\cdot dx_l\Big) + \int_0^C\Big(\frac{-p_l}{q_\infty}\cdot y_l\frac{dy_l}{dx_l}\cdot dx_l + \frac{\tau_l}{q_\infty}\cdot y_l\,dx_l\Big)\Big]$$

$$= \frac{1}{C^2}\Big[\int_0^C\Big(\underline{C_{p.u}\,x\,dx} - \underline{C_{f.u}\cdot\frac{dy_u}{dx}\cdot x\,dx}\Big) + \int_0^C\Big(C_{p.u}\cdot y_u\frac{dy_u}{dx}\cdot dx + C_{f.u}\cdot y_u\,dx\Big)$$
$$+ \int_0^C\Big(\underline{-C_{p.l}\cdot x\cdot dx} - \underline{C_{f.l}\cdot x\cdot\frac{dy_l}{dx}\cdot dx}\Big) \int_0^C\Big(-C_{p.l}\cdot y_l\frac{dy_l}{dx}\cdot dx + C_{f.l}\cdot y_l\,dx\Big)\Big]$$

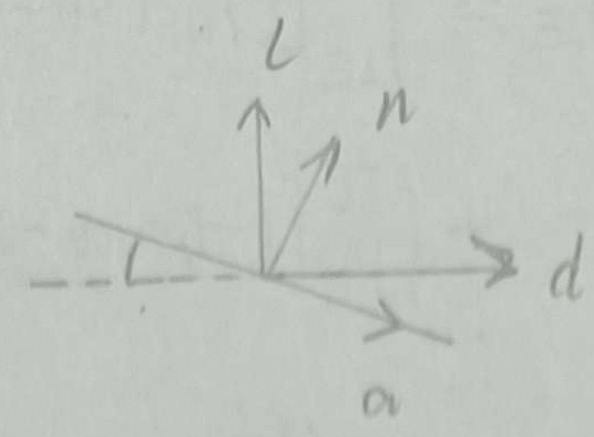
$$= \frac{1}{C^2}\cdot\Big[\int_0^C(C_{p.u} - C_{p.l})\,x\,dx - \int_0^C\Big(C_{f.u}\frac{dy_u}{dx} + C_{f.l}\frac{dy_l}{dx}\Big)x\,dx + \int_0^C\Big(C_{f.u} + C_{p.u}\frac{dy_u}{dx}\Big)y_u\,dx + \int_0^C\Big(C_{f.l} - C_{p.l}\frac{dy_l}{dx}\Big)y_l\cdot dx\Big]$$

1.5

解: $\alpha = 12^\circ$

$C_n = 1.2$, $C_a = 0.03$

Ask C_l, C_d



$$C_l = \cos\alpha \cdot C_n - \sin\alpha \cdot C_a = 1.168$$

$$C_d = \cos\alpha \cdot C_a + \sin\alpha \cdot C_n = 0.279$$

2.1

解:

Proof: p is const, scalar field.

$$\vec{F} = \oiint_S p \cdot d\vec{s} = \iiint_{\forall} \nabla p \, d\forall$$

$$= \nabla p \iiint_{\forall} d\forall$$

$$= \left(\frac{\partial p}{\partial x}\vec{i} + \frac{\partial p}{\partial y}\vec{j} + \frac{\partial p}{\partial z}\vec{k} \right) \iiint_{\forall} d\forall$$

$$= (0\vec{i} + 0\vec{j} + 0\vec{k}) \iiint_{\forall} d\forall$$

$$= \vec{0}$$

2.3

解: streamlines, $f(x,y) = C$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$v\,dx = u\,dy$$

$$a y\,dx = a x \cdot dy$$

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\frac{1}{y}dy = \frac{1}{x}dx$$

$y = Ax$, A is constant

2.5

解: $V_r = \frac{1}{r} \cdot \frac{\partial \psi}{\partial \theta} = 0$, $\psi = C_1 + \psi(r)$

$V_\theta = -\frac{\partial \psi}{\partial r} = Cr$, $\psi = -\frac{1}{2}C \cdot r^2 + C_2$

$\therefore$ streamlines: $\psi = -\frac{1}{2}C r^2 + C_2$

$$= -\frac{1}{2}C \cdot (x^2 + y^2) + C_2$$

namely, $x^2 + y^2 = C_3$

$u = \frac{cx}{x^2+y^2}$, $v = \frac{cy}{x^2+y^2}$

a) namely, we need $\nabla\cdot\vec{V}$

$$\nabla\cdot\vec{V} = \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$$

$$= \frac{2c\cdot(x^2+y^2) - c\cdot x\cdot 2x - cy\cdot 2y}{(x^2+y^2)^2}$$

$$= \frac{2c(x^2+y^2)-2c(x^2+y^2)}{(x^2+y^2)^2} = 0$$

the time rate of change of the volume of a fluid element per unit volume is 0. $\boxed{\text{ANS}}$

b) $\vec{\xi} = \nabla\times\vec{V}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{cx}{x^2+y^2} & \frac{cy}{x^2+y^2} & 0 \end{vmatrix}$$

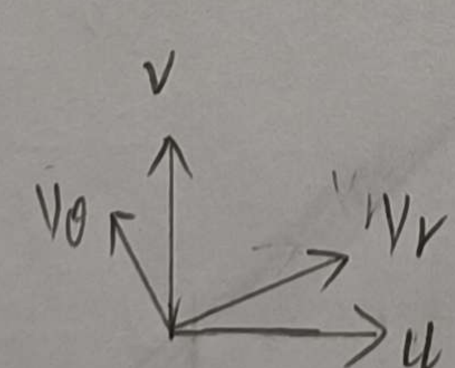
$$= -\frac{\partial}{\partial z}\frac{cy}{x^2+y^2}\vec{i} + \frac{\partial}{\partial z}\frac{cx}{x^2+y^2}\vec{j}$$

$$+\left(\frac{\partial}{\partial x}\frac{cy}{x^2+y^2} - \frac{\partial}{\partial y}\frac{cx}{x^2+y^2}\right)\vec{k}$$

$$= 0\vec{i} + 0\vec{j} + \left[\frac{-cy\cdot 2x}{(x^2+y^2)^2} + \frac{cx\cdot 2y}{(x^2+y^2)^2}\right]\vec{k}$$

$$= 0\vec{i} + 0\vec{j} + 0\vec{k} \quad \boxed{\text{ANS}}$$

Method 2，极坐标.



$V_r = u\cos\theta + v\sin\theta = \frac{c}{r}$

$V_\theta = v\cos\theta - u\sin\theta = 0$

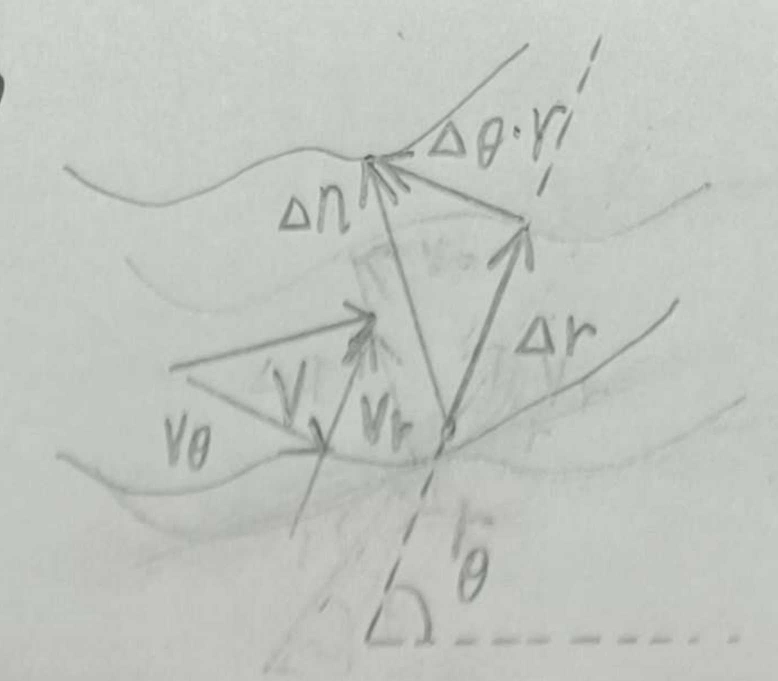
a) $\nabla\cdot\vec{V} = \frac{1}{r}\frac{\partial rV_r}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial\theta} = 0$

b) $\nabla\times\vec{V} = \begin{vmatrix} \vec{e}_r & \vec{e}_\theta & \vec{e}_z \\ \frac{1}{r}\frac{\partial r}{\partial r} & \frac{1}{r}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ \frac{c}{r} & 0 & 0 \end{vmatrix}$

$$= 0\vec{e}_r + 0\vec{e}_\theta + 0\vec{e}_z$$

2.10

解:



$$\rho V\Delta n = \rho V_r\Delta\theta\cdot r - \rho V_\theta\Delta r$$

$\Delta n \to 0$

$$d\bar{\psi} = \rho V_r\cdot r\,d\theta - \rho V_\theta\,dr$$

$$= \frac{\partial\bar{\psi}}{\partial\theta}d\theta + \frac{\partial\bar{\psi}}{\partial r}dr \quad \text{(chain rule)}$$

$$\rho V_r = \frac{1}{r}\cdot\frac{\partial\bar{\psi}}{\partial\theta}$$

$$\rho V_\theta = -\frac{\partial\bar{\psi}}{\partial r}$$

2.11

解: $u = cx$, $v = -cy$,

incompressible → ρ constant

$\frac{u}{dx} = \frac{v}{dy}$

$\frac{dy}{dx} = \frac{-y}{x}$

$\frac{1}{-y}dy = \frac{1}{x}dx$

$(-1)\ln y = \ln x + C_1$

$u = \frac{\partial\psi}{\partial y}$, $\psi = cxy + C_1(x)$

$v = -\frac{\partial\psi}{\partial x}$, $\psi = cxy + C_2$

$\underbrace{\psi = xy\cdot c + C_2 = \text{Const}, (xy = A)}$ $\boxed{\text{ANS}}$

$\nabla\phi = \vec{V}$

$u = \frac{\partial\phi}{\partial x} = cx$, $\phi = \frac{1}{2}cx^2 + C_3(y)$

$v = \frac{\partial\phi}{\partial y} = -cy$, $\underbrace{\phi = \frac{1}{2}cx^2 - \frac{1}{2}cy^2 + C_4}$ $\boxed{\text{ANS}}$

(namely , $x^2 - y^2 = C_5$)

To prove perpendicular

$\frac{d\psi}{dx} = c\cdot\frac{d(xy)}{dx} = c\cdot(y + x\cdot\frac{dy}{dx}) = 0$, $\frac{dy}{dx} = \frac{-y}{x} = k_1$

$\frac{d\phi}{dx} = c\cdot x - \frac{1}{2}\cdot c\cdot\frac{dy^2}{dx} = cx - \cancel{\frac{1}{2}}c\cdot\cancel{2}y\cdot\frac{dy}{dx} = 0$, $\frac{dy}{dx} = \frac{x}{y} = k_2$

$k_1\cdot k_2 = -1$, so the lines are perpendicular.