第 7 周习题
常微分方程 B

March 31, 2022

1. Find the general solution for each of the following differential equations.

   (1) $y'' + 2y = 0$

   (2) $2y'' + 2y' + y = 0$

   (3) $9y'' + 6y' + y = 0$

   (4) $y'' + 8y' + 16y = 0$

2. Find the solution for each of the following initial value problem.

   (1) $y'' + 10y' + 25y = 0, \quad y(0) = 2, \quad y'(0) = -1$

   (2) $y'' + 2y' + 2y = 0, \quad y(\pi/4) = 2, \quad y'(\pi/4) = -2$

3. In each of the following exercises, use the method of reduction of order to find a second solution of the given differential equation.

   (1) $t^2y'' + 2ty' - 2y = 0, \ \ t > 0; \quad y_1(t) = t$

   (2) $t^2y'' + 3ty' + y = 0, \ t > 0; \quad y_1(t) = t^{-1}$

4. Consider the following initial value problem:

$$y'' - y' + \frac{y}{4} = 0, \quad y(0) = 2, \quad y'(0) = b.$$

   Find the solution as a function of b, and then determine the critical value of b that separates solutions that remain positive for all $t > 0$ from those that eventually become negative.

5. Consider the differential equation $ay'' + by' + cy = 0$ with constant coefficients.

   (1) If a, b, and c are all positive, show that all solutions of the equation approach zero as $t \to +\infty$.

   (2) If $a > 0$ and $c > 0$, but $b = 0$, show that all solutions are bounded as $t \to +\infty$.

(3) If $a > 0$ and $b > 0$, but $c = 0$, show that all solutions approach a constant that depends on the initial conditions as $t \to +\infty$. Determine the constant for the initial conditions $y(0) = y_0$, $y'(0) = y_0'$.

6. Find the general solution for each of the following equations.

(1) $y'' - 2y' - 3y = 3e^{2t}$

(2) $y'' - 2y' + y = te^t + 4$

(3) $y'' + 4y = 2\sin 2t$

(4) $y'' + 4y' + 4y = e^{-2t} + \sin 2t$