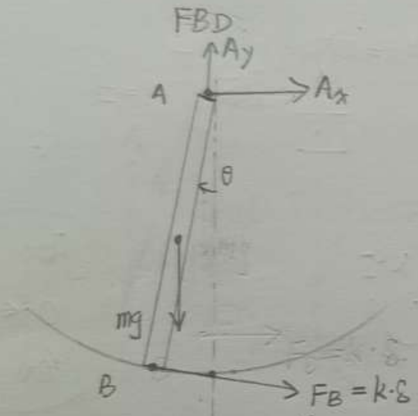
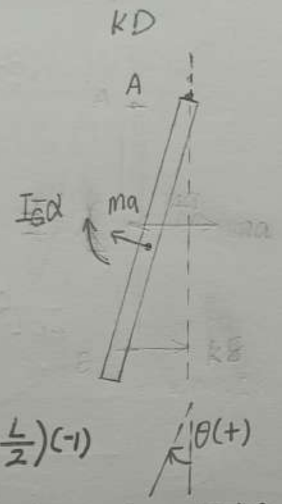
Homework

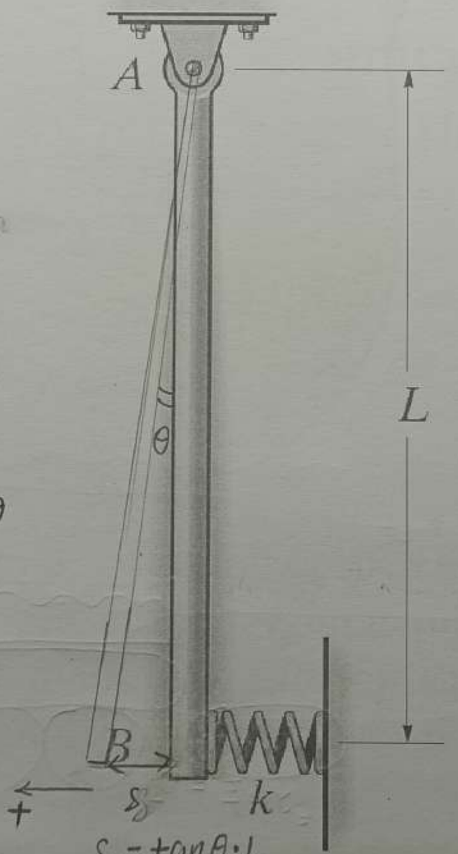
The uniform rod of mass m is supported by a pin at A and a spring at B. If B is given a small sideward displacement and released, determine the natural period of vibration.

$\tau_n = \frac{2\pi}{\omega_n}$

解:







$\Sigma M_A = mg\frac{L}{2}\sin\theta + F_B \cdot L = (I_G\alpha + ma\cdot\frac{L}{2})(-1)$

$F_B = k\cdot\delta = k\cdot\tan\theta L = kL\cdot\theta$, $\theta \approx \tan\theta \approx \sin\theta$

$\alpha = \ddot{\theta}$

$a = r\cdot\alpha = \frac{L}{2}\ddot{\theta}$

$I_G = \frac{1}{12}mL^2$

$\Rightarrow \frac{1}{3}mL^2\cdot\ddot{\theta} + (kL^2 + mg\frac{L}{2})\theta = 0$

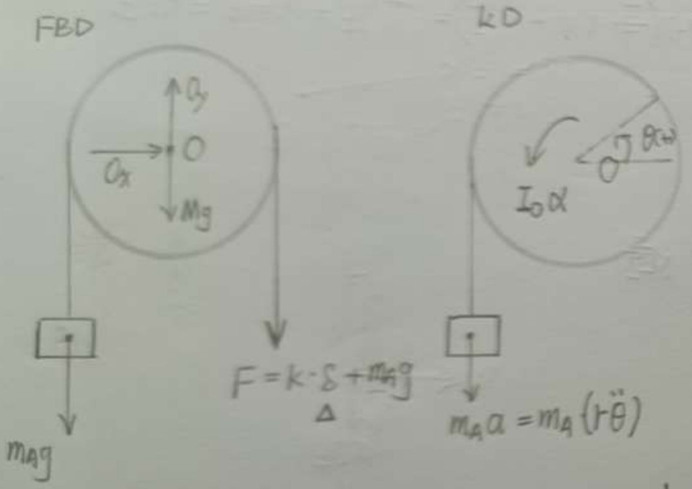
$\omega_n = \sqrt{\frac{kL^2 + mg\cdot\frac{L}{2}}{\frac{1}{3}mL^2}} = \sqrt{\frac{6kL+3mg}{2mL}}$, $\tau_n = \frac{2\pi}{\omega_n} = 2\pi\cdot\sqrt{\frac{2mL}{6kL+3mg}}$ $\boxed{\text{ANS}}$

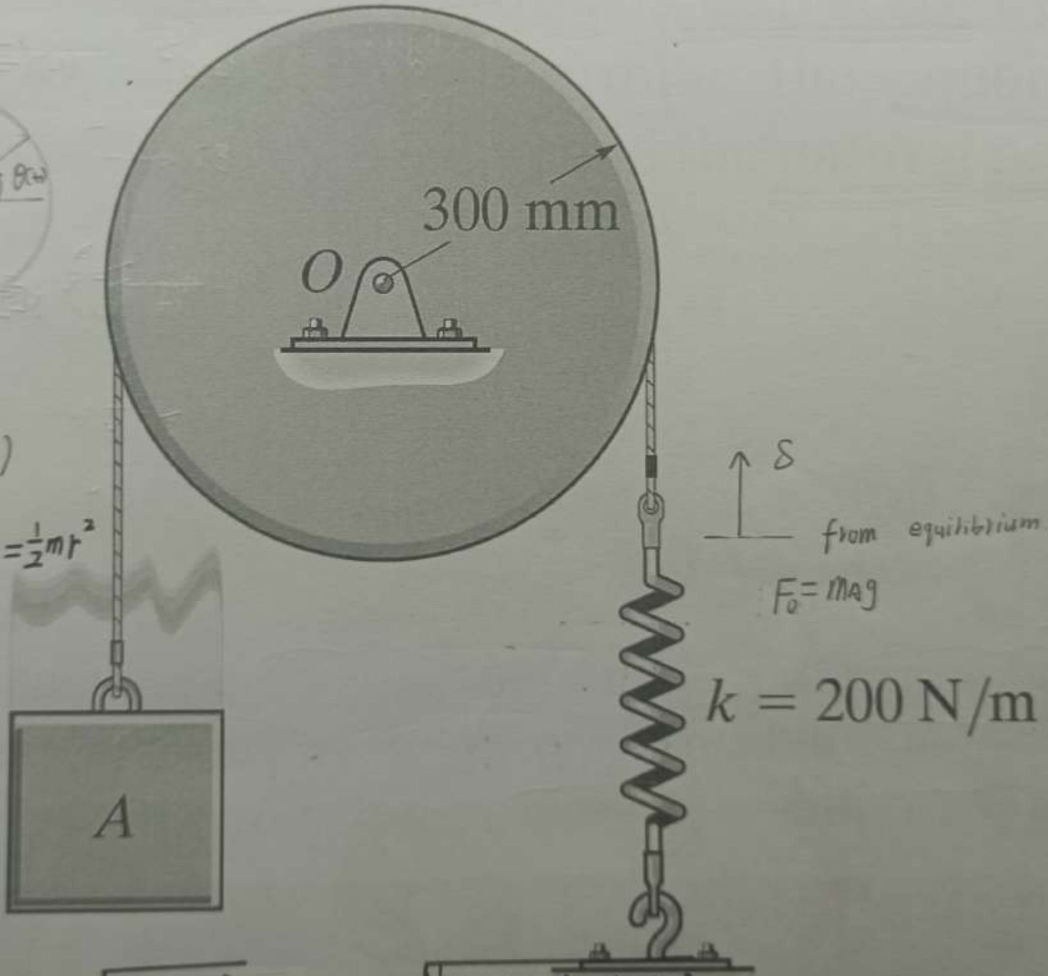
Homework

$I_G = \frac{1}{2} m R^2$

The 20-kg disk is pinned at its mass center O and supports the 4-kg block A. If the belt which passes over the disk is not allowed to slip at its contacting surface, determine the natural period of vibration of the system. τ_n , w_n

解:





$\circlearrowleft M_O = m_A g\cdot r - F\cdot r = I_O\alpha + m_A\cdot \underset{r\ddot{\theta}}{\underline{a}}\cdot r$, $I_O = \frac{1}{2} m r^2$

$\delta = \theta\cdot r$

$\alpha = \ddot{\theta}$

$a = \alpha\cdot r = r\ddot{\theta}$

$\Rightarrow m_A g r - (\theta r k + m_A g) r = I_O\ddot{\theta} + m_A r^2\ddot{\theta}$

$\Rightarrow (I_O + m_A r^2)\ddot{\theta} + r^2 k\theta = 0$

$w_n = \sqrt{\dfrac{r^2 k}{I_O + m_A r^2}}$, $\tau_n = \dfrac{2\pi}{w_n} = 2\pi\cdot\sqrt{\dfrac{I_O + m_A r^2}{r^2 k}} = 2\pi\cdot\sqrt{\dfrac{\frac{1}{2}\times 20\times 0.3^2 + 4\times 0.3^2}{0.3\times 0.3\times 200}} = 1.66\,s$ $\boxed{\text{ANS}}$

Homework

W

The disk has a weight of 30 lb and rolls without slipping on the horizontal surface as it oscillates about its equilibrium position. If the disk is displaced, by rolling it counterclockwise 0.2 rad, determine the equation which describes its oscillatory motion and the natural period when it is released.

解：Apply energy method

$V = \frac{1}{2} k x^2 = \frac{1}{2} k (r\theta)^2$

$T = \frac{1}{2} \bar{I} w^2 + \frac{1}{2} m v^2$

$= \frac{1}{2} \bar{I} \dot{\theta}^2 + \frac{1}{2} m (r\dot{\theta})^2 = (\frac{1}{2} \bar{I} + \frac{1}{2} m r^2) \dot{\theta}^2$

$T + V = \text{Constant}$

$\frac{d(T+V)}{dt} = 0$

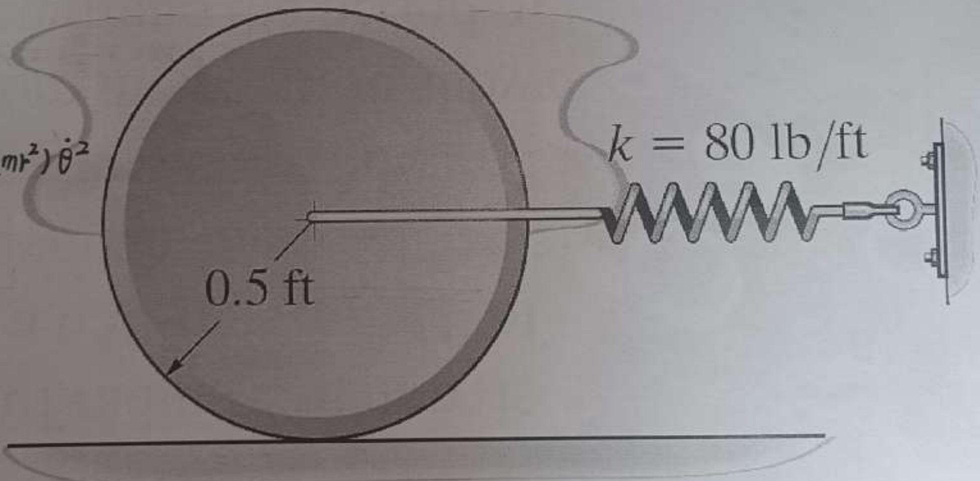
$\frac{1}{2} k r^2 \cdot 2 \cdot \theta \cdot \dot{\theta} + (\frac{1}{2} \bar{I} + \frac{1}{2} m r^2) \cdot 2 \cdot \dot{\theta} \cdot \ddot{\theta} = 0$

$(\bar{I} + m r^2) \ddot{\theta} + k r^2 \theta = 0$

$\frac{3}{2} m r^2 \ddot{\theta} + k r^2 \theta = 0$

$\frac{3}{2} \times \frac{30}{32.2} \times 0.5^2 \ddot{\theta} + 80 \times 0.5^2 \theta = 0$

$0.3494 \ddot{\theta} + 20\theta = 0$ $\boxed{\text{ANS}}$



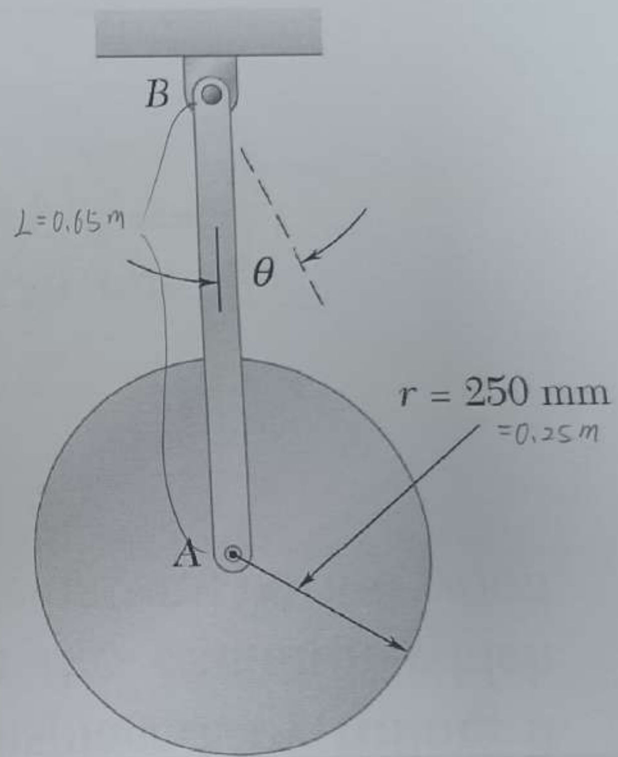
$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{20}{0.3494}}} = 0.8305$ s $\boxed{\text{ANS}}$

Homework

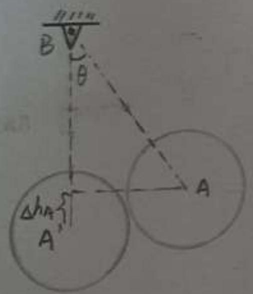
$r=0.25m$

A uniform disk of radius 250 mm is attached at A to a 650-mm rod AB of negligible mass which can rotate freely in a vertical plane about B. If the rod is displaced $2°$ from the position shown and released, determine the period of the resulting oscillation.

$L=0.65m$; $\theta = \frac{2}{180}\pi$ rad



解: Apply energy method



$$V = mg\Delta h_A = mgL(1-\cos\theta)$$

$$T = \frac{1}{2} I_B \dot{\theta}^2 \qquad I_B = \bar{I} + mL^2$$

$$T+V = \text{Const}$$

$$\Rightarrow \frac{d(T+V)}{dt} = 0$$

$$mgL(0+\sin\theta\cdot\dot{\theta}) + I_B\dot{\theta}\cdot\ddot{\theta} = 0$$

$$I_B\ddot{\theta} + mgL\sin\theta = 0$$

$$I_B\ddot{\theta} + mgL\theta = 0$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{mgL}{I_B}}} = \frac{2\pi}{\sqrt{\frac{mgL}{\frac{1}{2}mr^2+mL^2}}} = \frac{2\pi}{\sqrt{\frac{2gL}{r^2+2L^2}}} = 1.676\ s \quad \boxed{\text{ANS}}$$