Heat Transfer HW5

Basic information as follows:

编辑器 - D:\Study\Junior year\Heat Transfer\HW5\Heat_Transfer_HW5.m

Heat_Transfer_HW5.m

```
clc
clear all

%basic infomation
L=0.05; %length L=0.05m
b=0.01; %base thickness b=1cm
w=1; %width w=1m
h=15; %heat transfer coefficient h=15W/(m^2*K)
k=180; %thermal conductivity k=180W/(m*K)
T_infinity=25; %T_∞=25℃
T_base=200; %T_base=200℃
sin_theta=0.0995; %angle sin theta
cos_theta=0.9950;
```

Energy balance euqtion:

$$\left[1-\left(m-\frac{1}{2}\right)\frac{\Delta x}{L}\right](T_{m-1}-T_m)+\left[1-\left(m+\frac{1}{2}\right)\frac{\Delta x}{L}\right](T_{m+1}-T_m)+\frac{h\Delta x^2}{\lambda L\sin\theta}(T_\infty-T_m)=0$$

Rearrange:

$$\left[1-\left(m-\frac{1}{2}\right)\frac{\Delta x}{L}\right]T_{m-1}+\left[-2+2\frac{m\cdot\Delta x}{L}-\frac{h(\Delta x)^2}{\lambda L\sin\theta}\right]T_m+\left[1-\left(m+\frac{1}{2}\right)\frac{\Delta x}{L}\right]T_{m+1}=-\frac{h(\Delta x)^2}{\lambda L\sin\theta}T_\infty$$

以此来构建矩阵方程，**A·T=C**

若 M=101，则 **A** 为 100*100 的方阵

矩阵 **A** 的构建：

For m=1

$$\left(1-\tfrac{1}{2}\times\tfrac{\Delta x}{L}\right)T_{\text{base}}+\left[-2+2\tfrac{\Delta x}{L}-\tfrac{h(\Delta x)^2}{\lambda L\sin\theta}\right]T_1+\left[1-\left(1+\tfrac{1}{2}\right)\tfrac{\Delta x}{L}\right]T_2=-\frac{h(\Delta x)^2}{\lambda L\sin\theta}T_\infty$$

Rearrange:

$$\boxed{\left[-2+2\frac{\Delta x}{L}-\frac{h(\Delta x)^2}{\lambda L\sin\theta}\right]T_1+\left[1-\left(1+\frac{1}{2}\right)\frac{\Delta x}{L}\right]T_2}=-\frac{h(\Delta x)^2}{\lambda L\sin\theta}T_\infty-T_{\text{base}}\left(1-\frac{1}{2}\times\frac{\Delta x}{L}\right)$$

```
%row1
A(1,1)=-2+2*1*det_x/L-h*det_x^2/(k*L*sin_theta)
A(1,2)=1-(1+1/2)*det_x/L;
```

From m=2 to 99:

$$\boxed{\left[1-\left(m-\frac{1}{2}\right)\frac{\Delta x}{L}\right]T_{m-1}+\left[-2+2\frac{m\cdot\Delta x}{L}-\frac{h(\Delta x)^2}{\lambda L\sin\theta}\right]T_m+\left[1-\left(m+\frac{1}{2}\right)\frac{\Delta x}{L}\right]T_{m+1}}=-\frac{h(\Delta x)^2}{\lambda L\sin\theta}T_\infty$$

```
%row2 to row 99 (98*98)
for m=2:M-2
    A(m,m-1)=1-(m-1/2)*det_x/L;
    A(m,m)=-2+2*m*det_x/L-h*det_x^2/(k*L*sin_theta)
    A(m,m+1)=1-(m+1/2)*det_x/L;
end
```

For m=100, consider the boundary condition equation:

$$\lambda A_{\text{left}}\frac{T_{99}-T_{100}}{\Delta x}+hA_{\text{conv}}(T_\infty-T_{100})=0$$

$$\lambda\cdot 2w\frac{\Delta x}{2}\tan\theta\cdot\frac{T_{99}-T_{100}}{\Delta x}+h\cdot 2w\frac{\Delta x}{2\cos\theta}\cdot(T_\infty-T_{100})=0$$

$$\boxed{\lambda T_{99}+\left(-\lambda-\frac{h\Delta x}{\sin\theta}\right)T_{100}}=\frac{-h\Delta x}{\sin\theta}T_\infty$$

```
%row100 is based on the Boundary Condition
A(M-1,M-2)=k;
A(M-1,M-1)=-k-h*det_x/sin_theta;
```

矩阵 **C** 的构建

For m =1,

$$\left[-2+2\frac{\Delta x}{L}-\frac{h(\Delta x)^2}{\lambda L\sin\theta}\right]T_1+\left[1-\left(1+\frac{1}{2}\right)\frac{\Delta x}{L}\right]T_2=\boxed{-\frac{h(\Delta x)^2}{\lambda L\sin\theta}T_\infty-T_{\text{base}}\left(1-\frac{1}{2}\times\frac{\Delta x}{L}\right)}$$

```
%row 1
C(1,1)=-h*det_x^2/(k*L*sin_theta)*T_infinity-T_base*(1-(1-1/2)*det_x/L);
```

From m=2 to 99,

$$\left[1-\left(m-\frac{1}{2}\right)\frac{\Delta x}{L}\right]T_{m-1}+\left[-2+2\frac{m\cdot\Delta x}{L}-\frac{h(\Delta x)^2}{\lambda L\sin\theta}\right]T_m+\left[1-\left(m+\frac{1}{2}\right)\frac{\Delta x}{L}\right]T_{m+1}=\boxed{-\frac{h(\Delta x)^2}{\lambda L\sin\theta}T_\infty}$$

```
%row2 to row99, the energy balance equation constant term
for m=2:M-2
    C(m,1)=-h*det_x^2/(k*L*sin_theta)*T_infinity;
end
```

For m=100, the boundary condition equation:

$$\lambda A_{\text{left}}\frac{T_{99}-T_{100}}{\Delta x}+hA_{\text{conv}}(T_\infty-T_{100})=0$$

$$\lambda\cdot 2w\frac{\Delta x}{2}\tan\theta\cdot\frac{T_{99}-T_{100}}{\Delta x}+h\cdot 2w\frac{\Delta x}{2\cos\theta}\cdot(T_\infty-T_{100})=0$$

$$\lambda T_{99}+\left(-\lambda-\frac{h\Delta x}{\sin\theta}\right)T_{100}=\boxed{\frac{-h\Delta x}{\sin\theta}T_\infty}$$

```
%row 100, the boundary condition constant term
C(M-1,1)=-h*det_x/sin_theta*T_infinity;
```

Calculate the rate of heat transfer from the fin:

$$\emptyset_{fin}=\frac{hw\Delta x}{cos\theta}\left[(T_{base}-T_\infty)+2\sum_{i=1}^{M-2=99}(T_i-T_\infty)+(T_{M-1}-T_\infty)\right]$$

$$\emptyset_{fin}=\frac{hw\Delta x}{cos\theta}\left[(T_{base}+T_{M-1})+2\sum_{i=1}^{M-2}(T_i)-(2M-2)T_\infty\right]$$

```
%the rate of heat transfer from the fin
T_total=0; %node temperature sum except the base and tip nodes
for i=1:M-2
    T_total=T(i)+T_total;
end
Q_fin=h*w*det_x/cos_theta*(T_base+T(M-1)+2*T_total-(2*M-2)*T_infinity);
```

Fin efficiency:

$$\Phi_{\max} = hA_{\text{fin,total}}\left(T_0 - T_\infty\right) = h\frac{2wL}{\cos\theta}\left(T_0 - T_\infty\right)$$

$$\eta = \frac{\Phi_{fin}}{\Phi_{\max}}$$

```
Q_max=h*2*w*L*(T_base-T_infinity)/cos_theta;
efficiency=Q_fin/Q_max;
```

The solutions are as follows:

1) Nodes temperatures: please see the Excel Temperature attached.

2) $\emptyset_{fin} = 258.4451488078796W$

3) $\eta = 0.979630183100343$

They are quite close to the results on the lecture solution.

As for M=201, the process is similar, and the results are as follows:

1) Nodes temperatures: please see the Excel Temperature attached.

2) $\emptyset_{fin} = 258.4451433579625W$

3) $\eta = 0.979630162442563$

The results of these two cases are very close.

The Matlab code is attached behind.

```
clc
clear all

%basic infomation
L=0.05; %length L=0.05m
b=0.01; %base thickness b=1cm
w=1; %width w=1m
h=15; %heat transfer coefficient h=15W/(m^2*K)
k=180; %thermal conductivity k=180W/(m*K)
T_infinity=25; %T_∞=25℃
T_base=200; %T_base=200℃
sin_theta=0.0995; %angle sin theta
cos_theta=0.9950;


M=201; %node number
det_x=L/(M-1); %nodal spacing del_x

%From energy balance equation to obtain 99 equations
A=zeros(M-1);
%row1 to row M-2 are based on the energy balance equation
%row1
A(1,1)=-2+2*1*det_x/L-h*det_x^2/(k*L*sin_theta);
A(1,2)=1-(1+1/2)*det_x/L;
```

```
%row2 to row M-2 (M-3*M-3)
for m=2:M-2
    A(m,m-1)=1-(m-1/2)*det_x/L;
    A(m,m)=-2+2*m*det_x/L-h*det_x^2/(k*L*sin_theta);
    A(m,m+1)=1-(m+1/2)*det_x/L;
end
%rowM-1 is based on the Boundary Condition
A(M-1,M-2)=k;
A(M-1,M-1)=-k-h*det_x/sin_theta;

%Constant terms
C = zeros(M-1,1);
%row 1
C(1,1)=-h*det_x^2/(k*L*sin_theta)*T_infinity-T_base*(1-(1-1/2)*det_x/L);
%row2 to rowM-2, the energy balance equation constant term
for m=2:M-2
    C(m,1)=-h*det_x^2/(k*L*sin_theta)*T_infinity;
end
%row M-1, the boundary condition constant term
C(M-1,1)=-h*det_x/sin_theta*T_infinity;

%node temperatures
T=A \ C;
```

```
%the rate of heat transfer from the fin

T_total=0; %node temperature sum except the base and tip nodes

for i=1:M-2

    T_total=T(i)+T_total;

end

Q_fin=h*w*det_x/cos_theta*(T_base+T(M-1)+2*T_total-(2*M-
2)*T_infinity);


Q_max=h*2*w*L*(T_base-T_infinity)/cos_theta;

efficiency=Q_fin/Q_max;
```