力学与航空航天工程系
DEPARTMENT OF MECHANICS AND AEROSPACE ENGINEERING

MECHANICS OF MATERIALS

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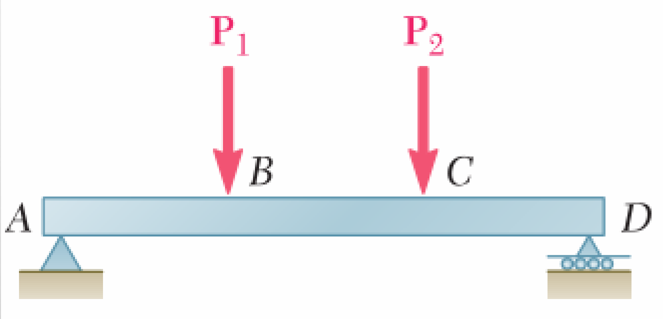
Lesson 6： Analysis and Design of Beams for Bending

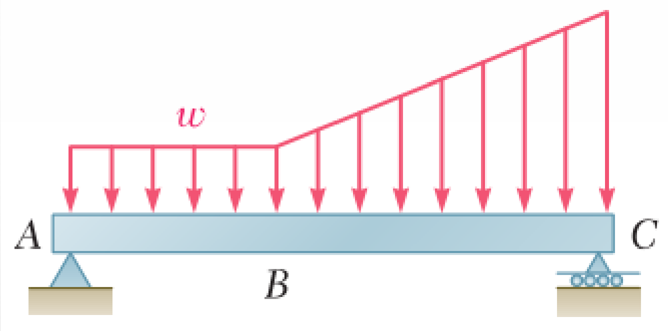
- Load, Shear, and Bending Moment
- Singularity Functions to Determine Shear and Bending Moment in a Beam

§ 6.1 Introduction

- Members that are slender and support loadings that are applied perpendicular to their longitudinal axis are called **beams**.



Concentrated loads

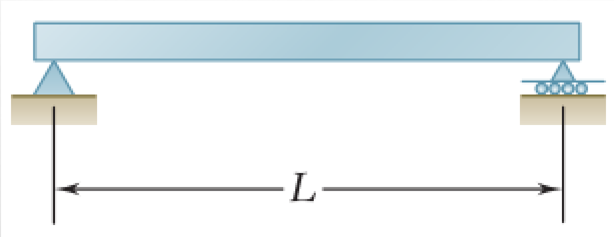


Distributed loads

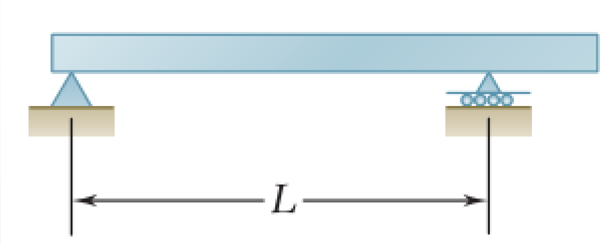
§ 6.1 Introduction

- Members that are slender and support loadings that are applied perpendicular to their longitudinal axis are called **beams**.

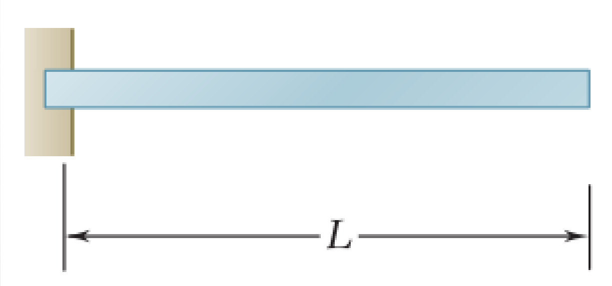
Determinate



Simply supported beam

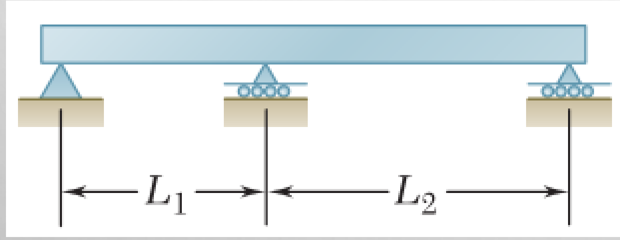


Overhanging beam

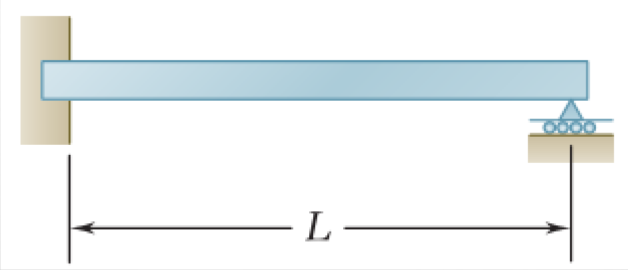


Cantilever beam

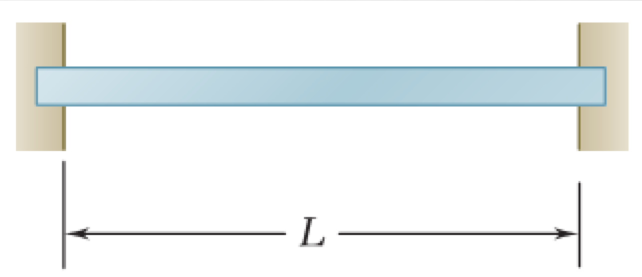
Indeterminate



Continuous beam



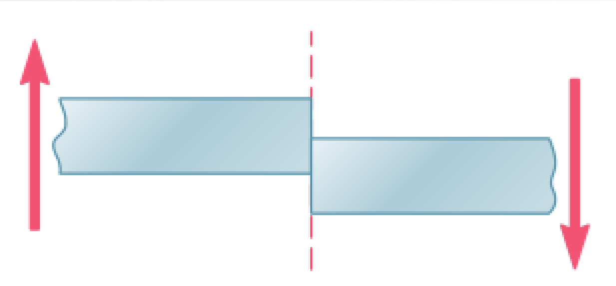
Beam fixed at one end and simply supported at the other end



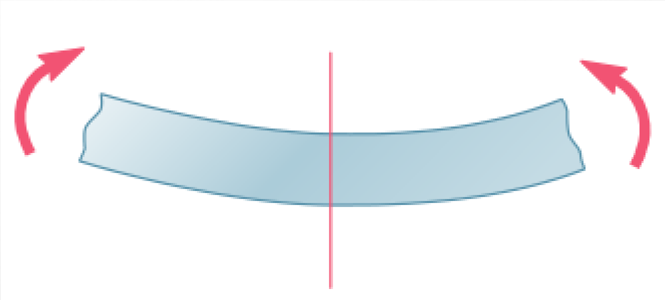
Fixed beam

§ 6.2 Beam sign convention

- The shear *V* and the bending moment *M* at a given point of a beam are said to be positive when…



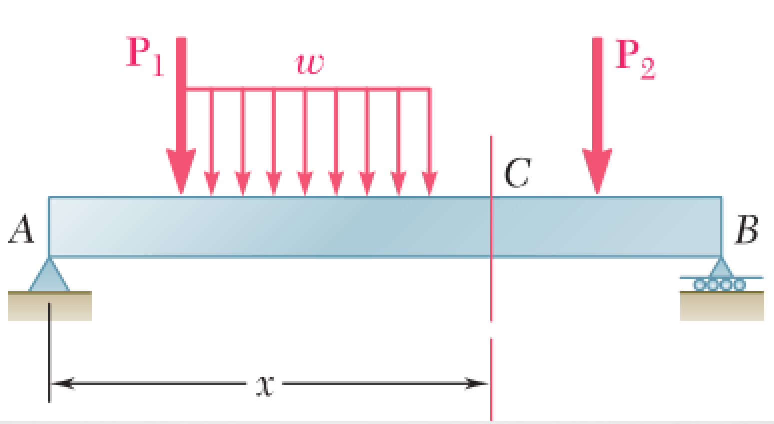
Positive shear

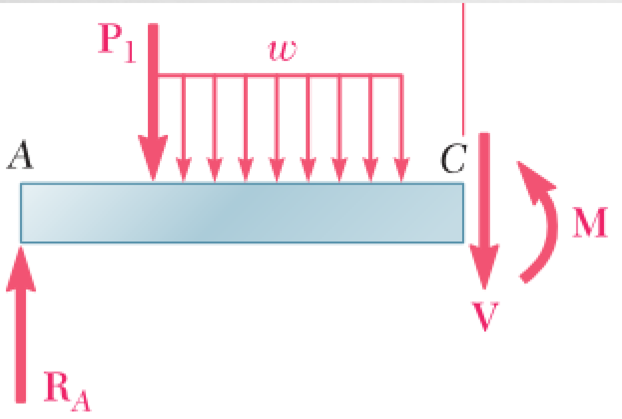


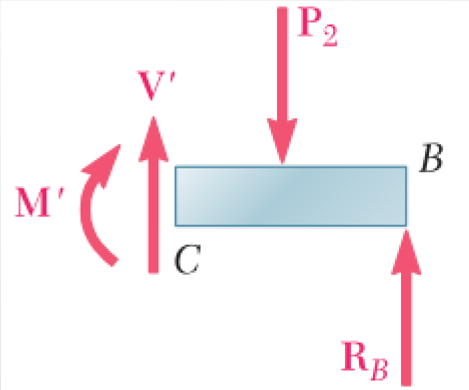
Positive bending moment

§ 6.3 Shear and moment diagrams

- The shear and moment diagrams will be obtained by determining the values of *V* and *M* at selected points of the beam.



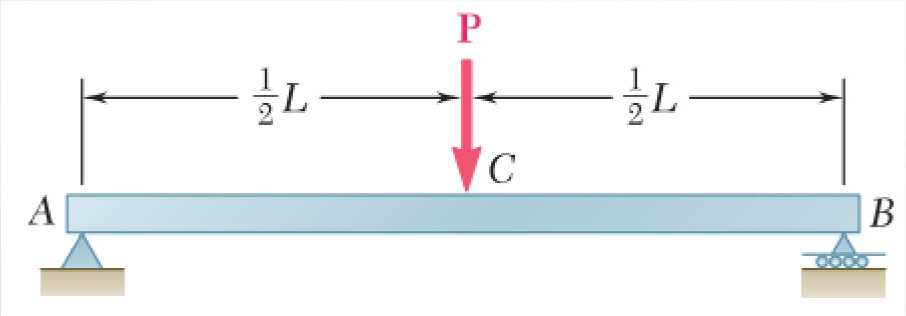


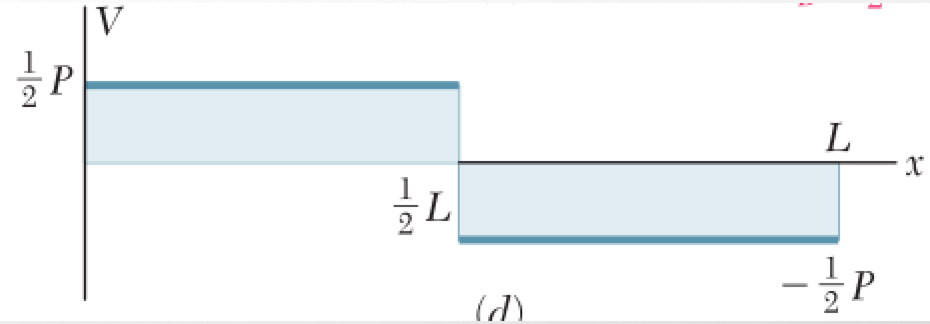


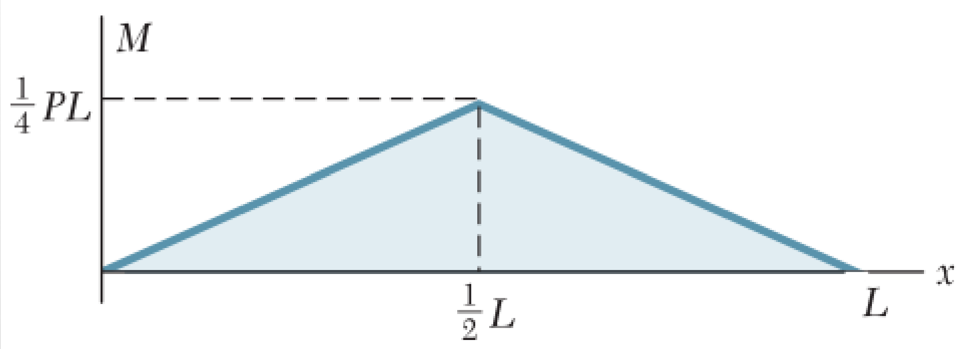
Example 6.1

(Beer, Page 320)

Draw the shear and bending-moment diagrams for a simply supported beam AB of span L subjected to a single concentrated load P at its mid-point C.



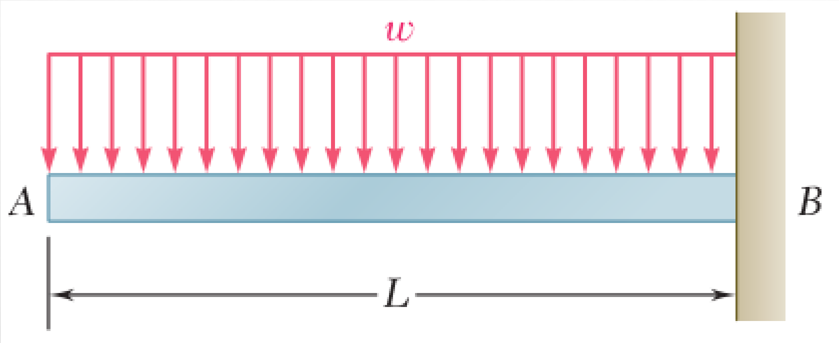


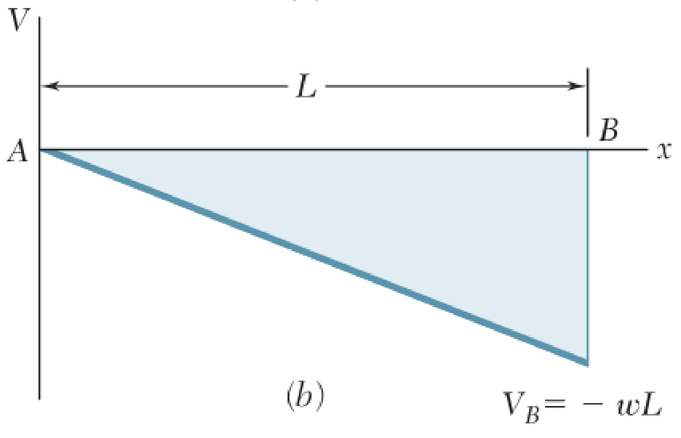


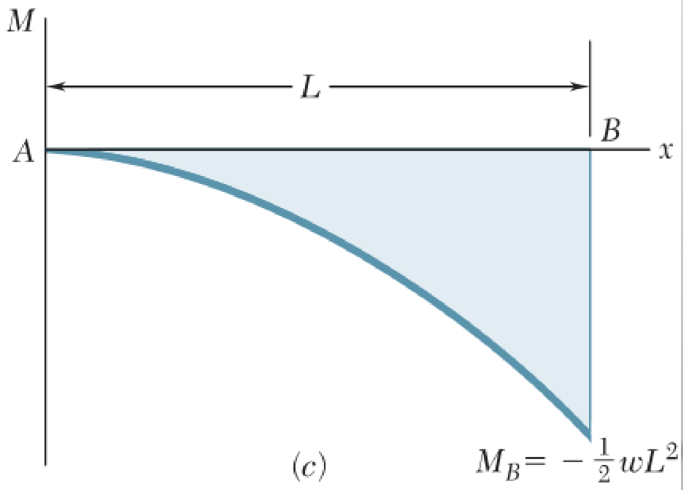
Example 6.2

(Beer, Page 321)

Draw the shear and bending-moment diagrams for a cantilever beam AB of span L supporting a uniformly distributed load w.







§ 6.4 Relation between load and shear

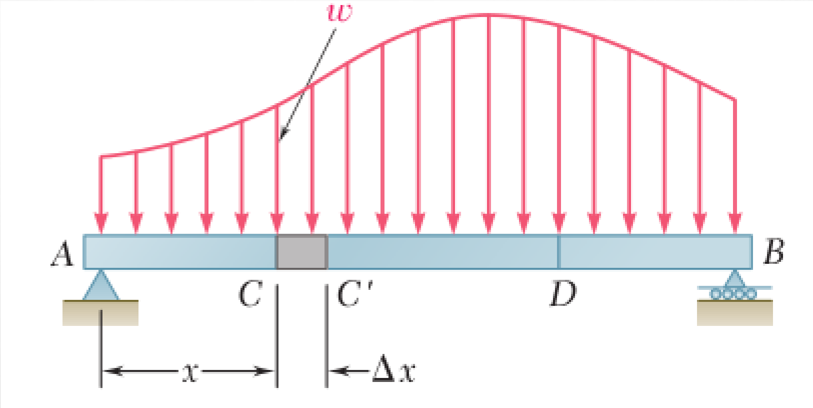
- Consider a simply supported beam AB carrying a distributed load *w* per unit length

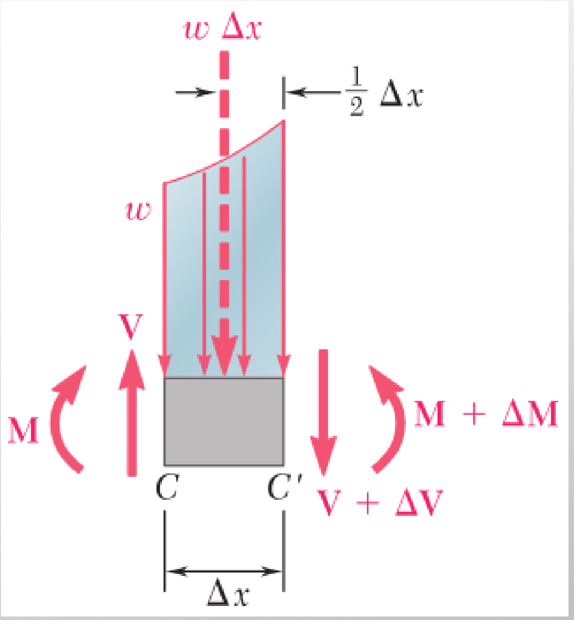
$$\sum F_y = 0$$

$$V - (V + \Delta V) - w\Delta x = 0$$

$$\boxed{\frac{dV}{dx} = -w}$$

$$V_D - V_C = -\int_{x_C}^{x_D} w dx$$



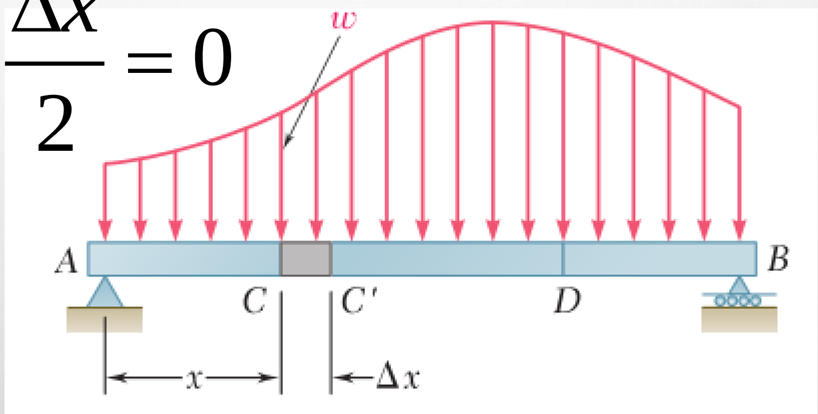


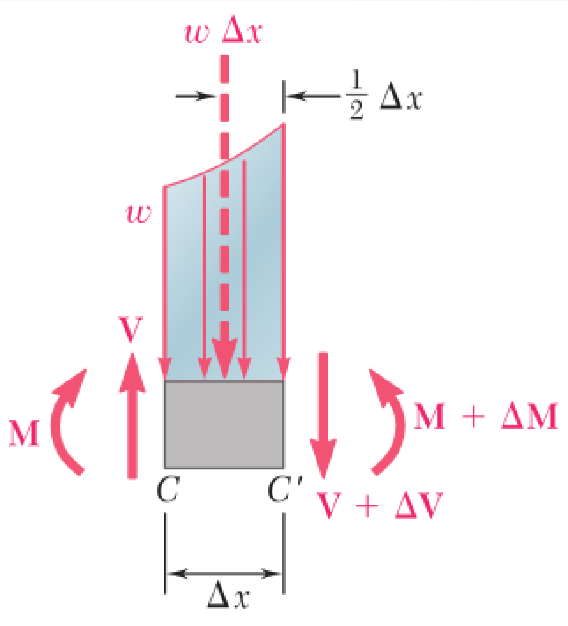
§ 6.5 Relation between shear and bending moment

- Consider a simply supported beam AB carrying a distributed load *w* per unit length

$$\sum M_{C'} = 0$$

$$(M + \Delta M) - M - V\Delta x + w\Delta x \frac{\Delta x}{2} = 0$$





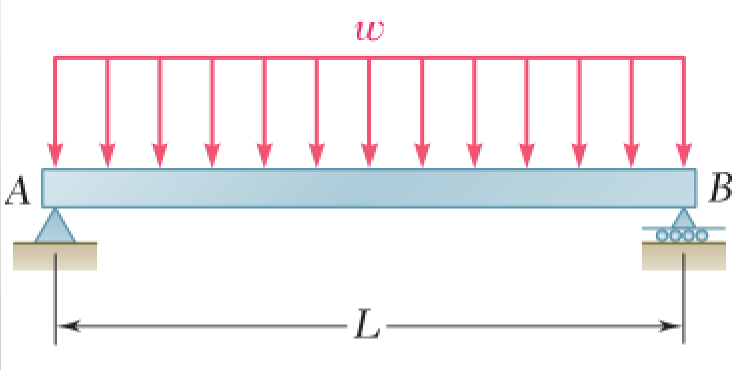
$$\boxed{\frac{dM}{dx} = V}$$

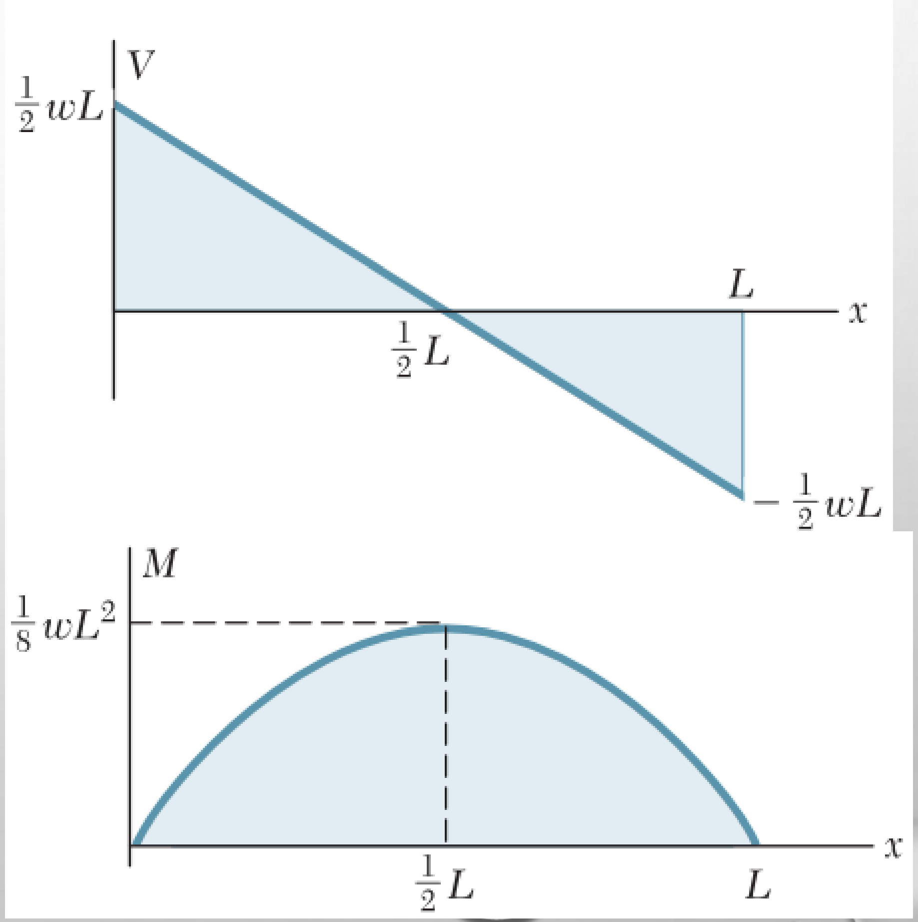
$$M_D - M_C = \int_{x_C}^{x_D} V dx$$

Example 6.3

(Beer, Page 331)

Draw the shear and bending-moment diagrams for the simply supported beam and determine the maximum value of the bending moment.

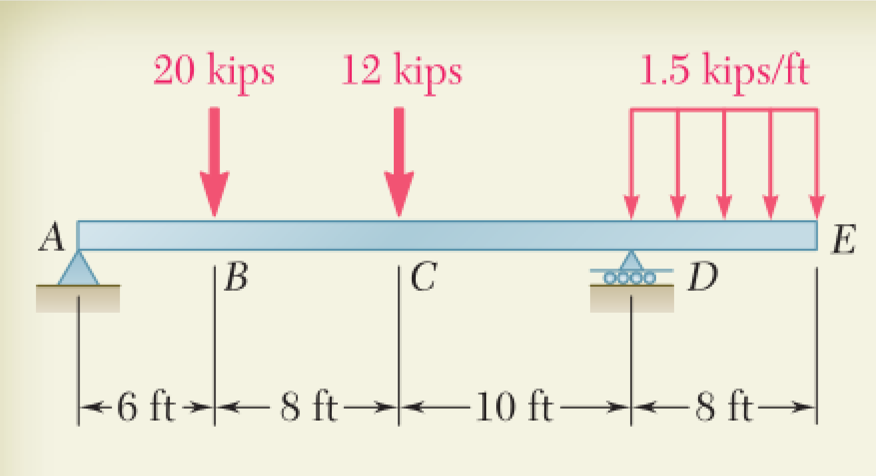


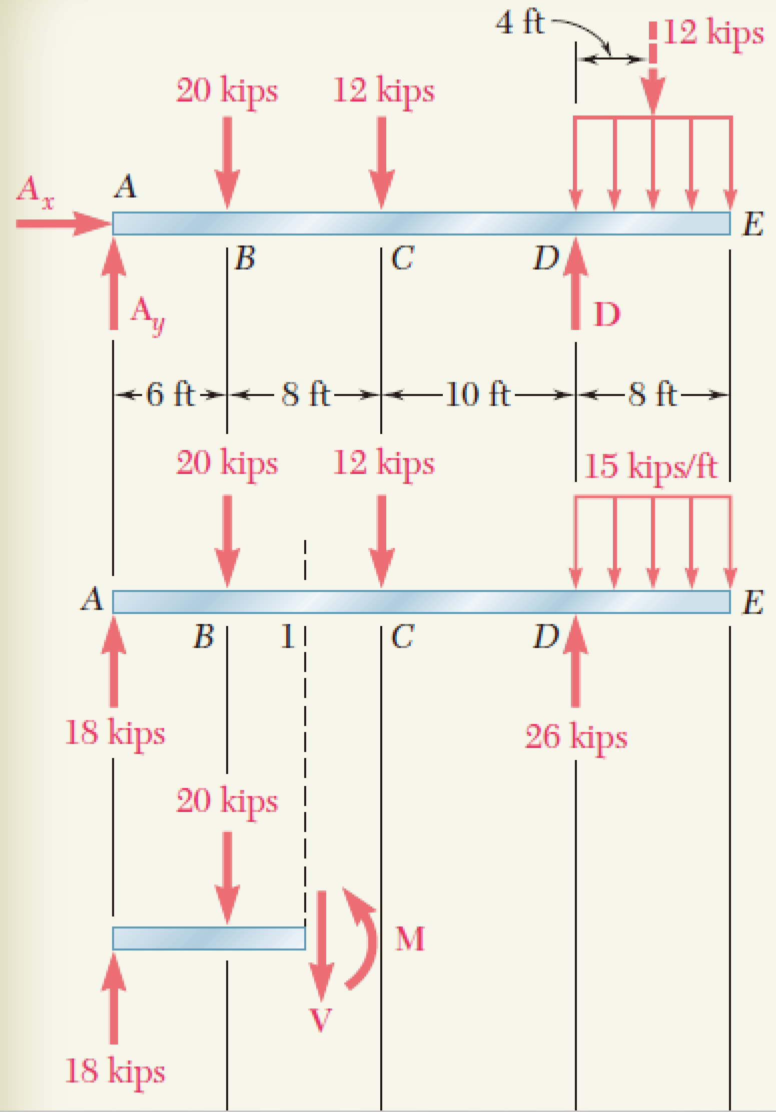


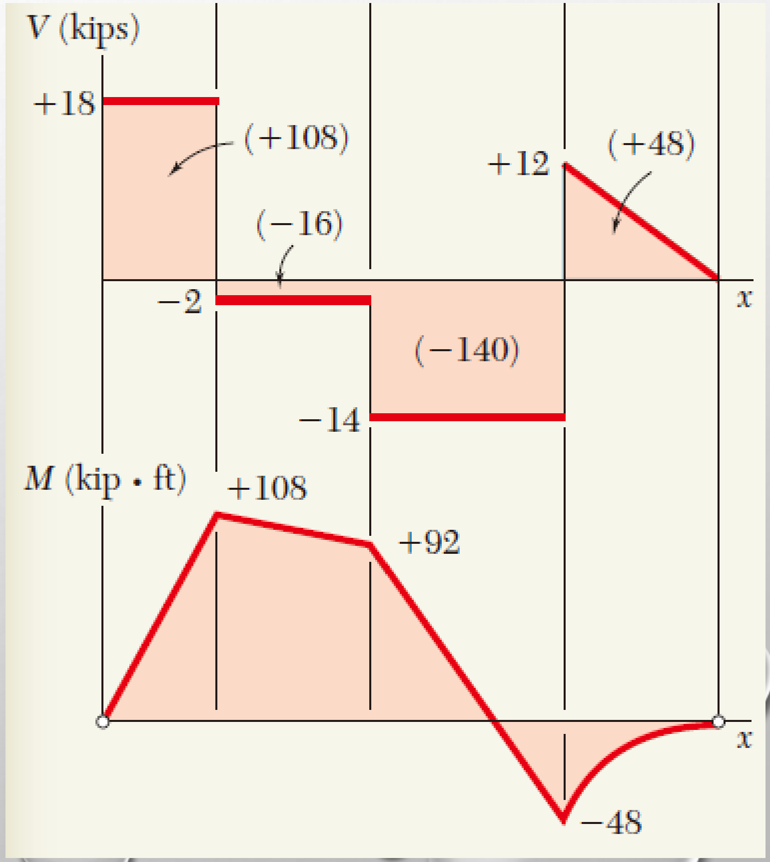
Example 6.4

(Beer, Page 333)

Draw the shear and bending-moment diagrams for the beam and loading.







§ 6.6 Design of prismatic beams for bending

- The design of a beam is usually controlled by the maximum absolute value $|M|_{max}$ of the bending moment that will occur in the beam.

$$\sigma_{max} = \frac{|M|_{max}\, c}{I} \qquad \sigma_{max} = \frac{|M|_{max}}{S}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}}$$

allowable stress

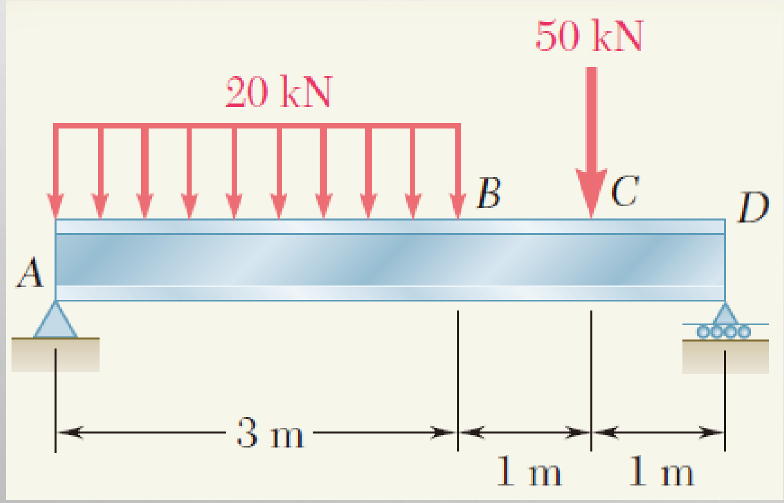
In most cases the dominant criterion in the design of a beam for strength is the maximum value of the normal stress in the beam.

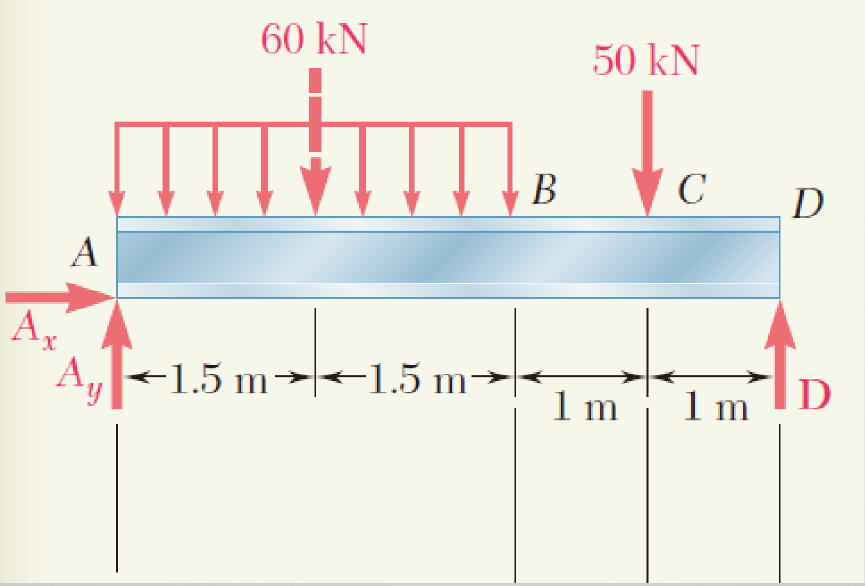
Example 6.5

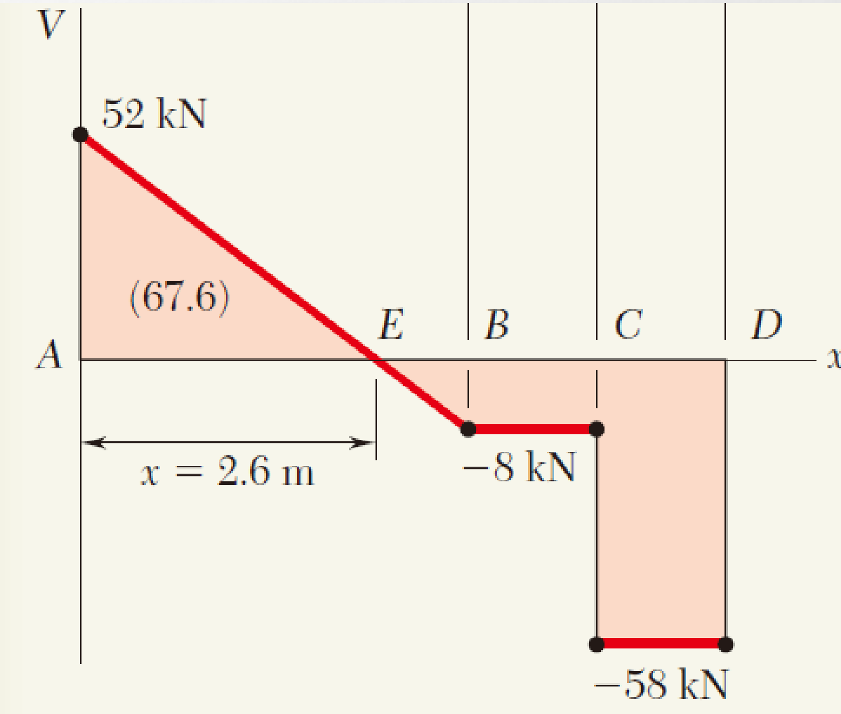
(Beer, Page 343)

A 5-m-long, simply supported steel beam AD is to carry the distributed and concentrated loads shown. Knowing that the allowable normal stress for the grade of steel to be used is 160 MPa, select the wide-flange shape that should be used.

Shape	S, mm^3
W410 × 38.8	629
W360 × 32.9	475
W310 × 38.7	547
W250 × 44.8	531
W200 × 46.1	451

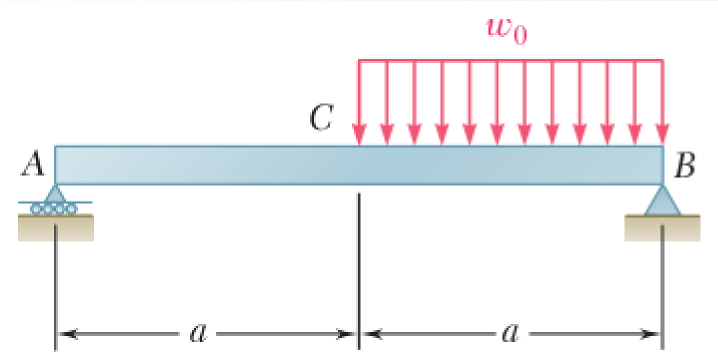






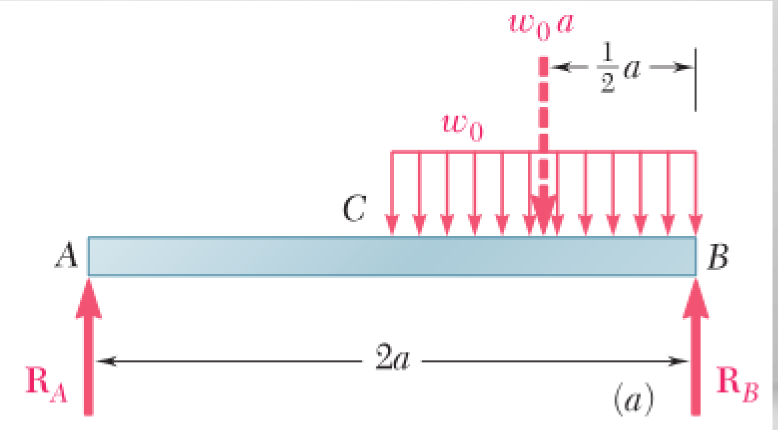
§ 6.7 Using singularity functions

- The use of singularity functions makes it possible to represent the shear V and the bending moment M by single mathematical expressions.



$$w(x) = w_0 \langle x-a \rangle^0$$

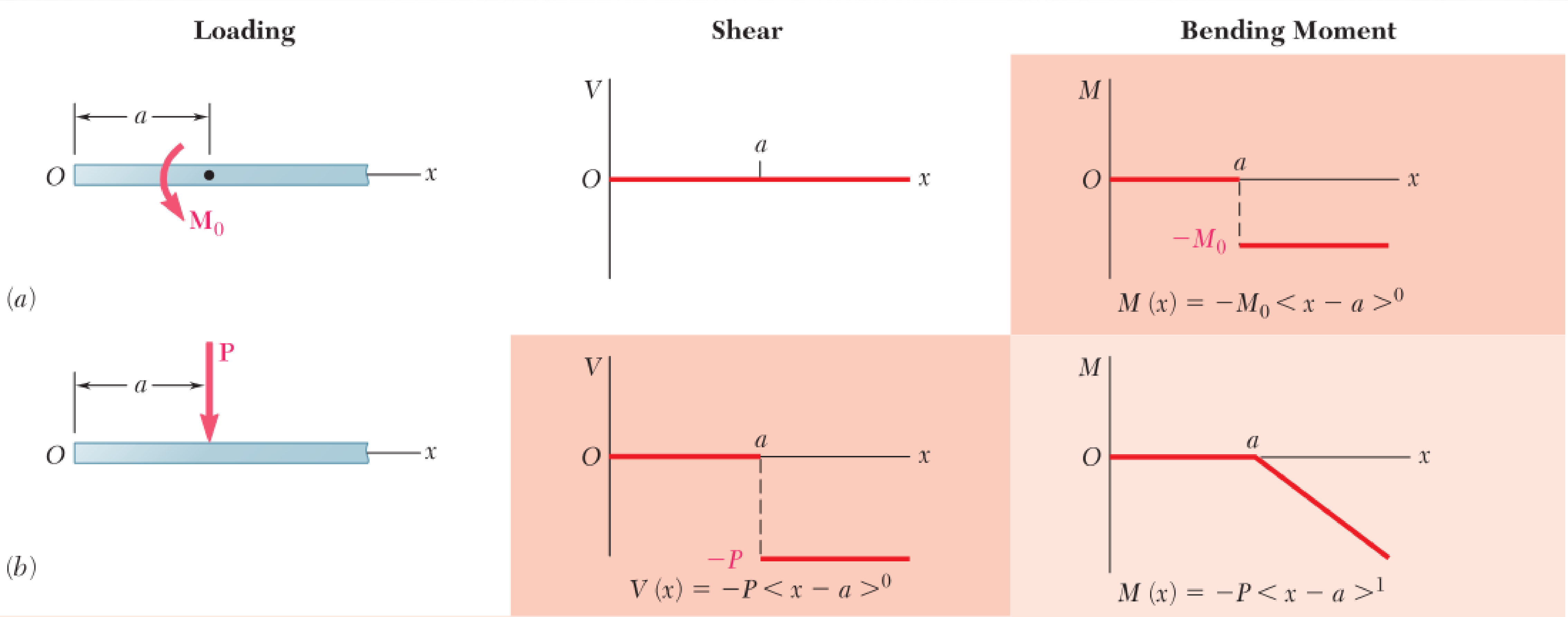
$$V(x) = \frac{1}{4} w_0 a - w_0 \langle x-a \rangle$$



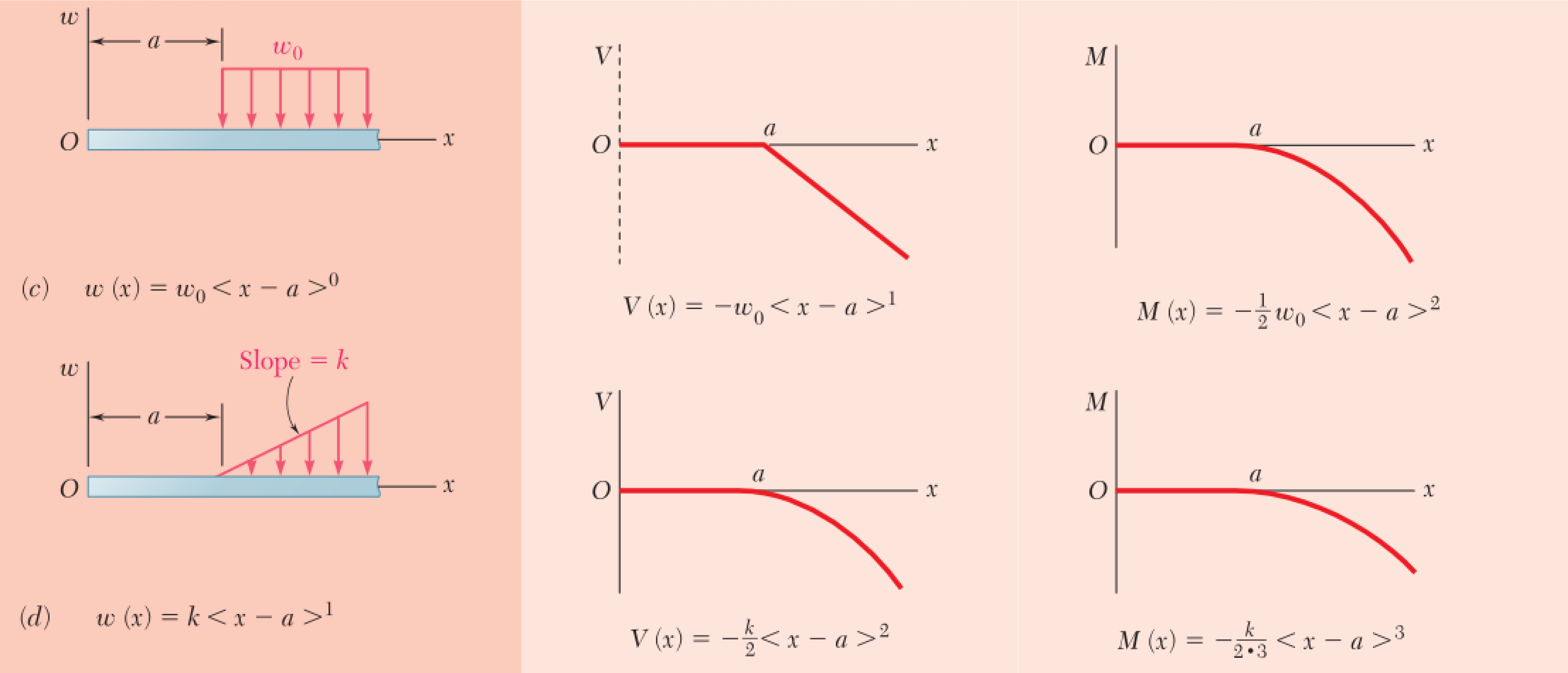
$$M(x) = \frac{1}{4} w_0 a x - \frac{1}{2} w_0 \langle x-a \rangle^2$$

$$\langle x-a \rangle^n = \begin{cases} (x-a)^n, x \ge a \\ 0 \qquad\quad , x < a \end{cases}$$

§ 6.7 Using singularity functions

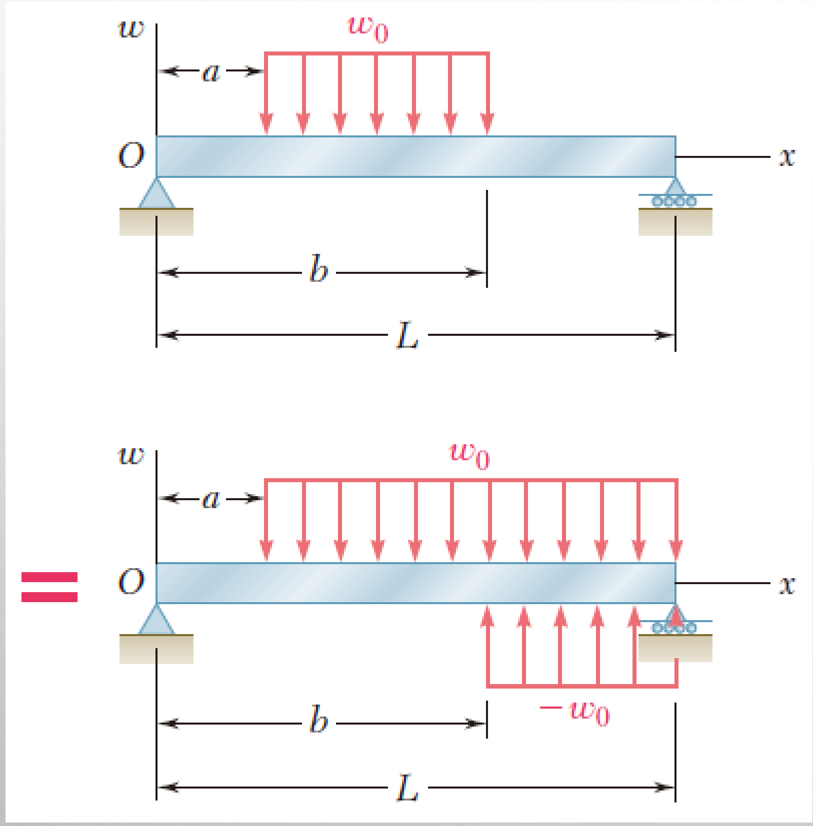


§ 6.7 Using singularity functions



§ 6.7 Using singularity functions

- Use of open-ended loadings to create a closed-ended loading

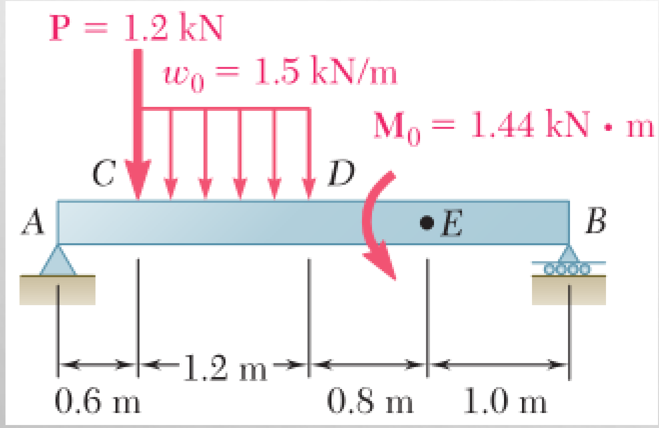


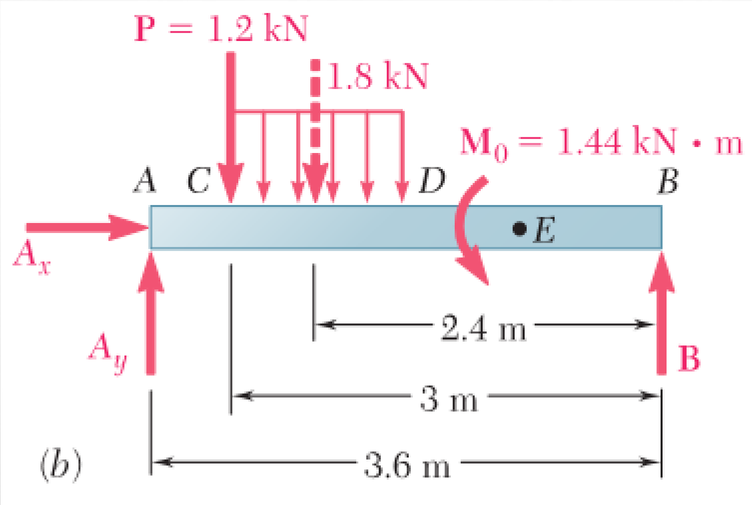
$$w(x) = w_0 \langle x-a \rangle^0 - w_0 \langle x-b \rangle^0$$

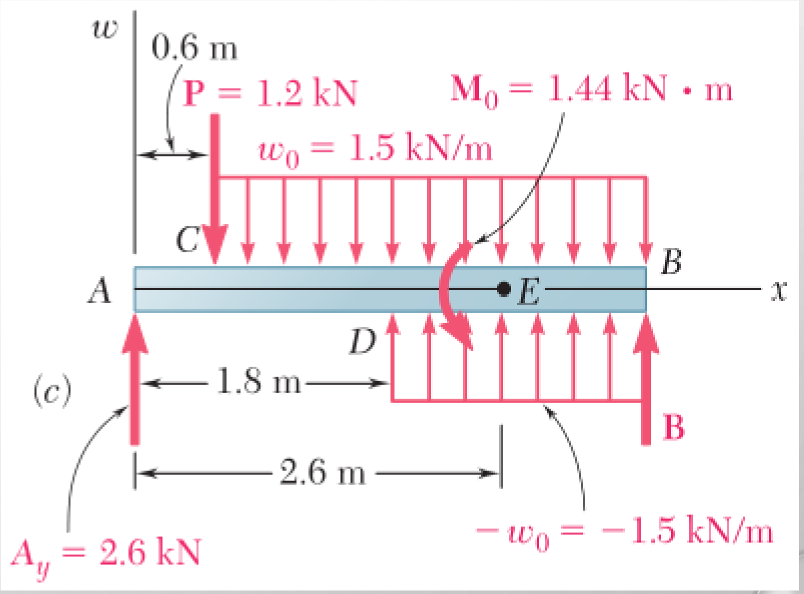
Example 6.6

(Beer, Page 354)

For the beam and loading shown (Fig. 5.20a) and using singularity functions, express the shear and bending moment as functions of the distance x from the support at A.



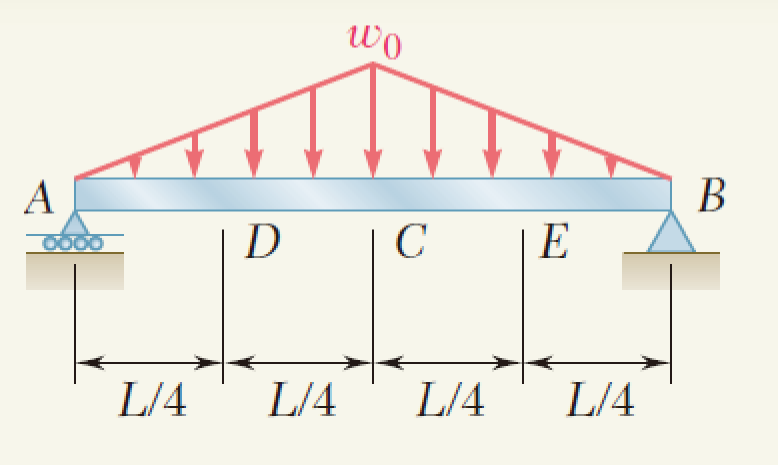


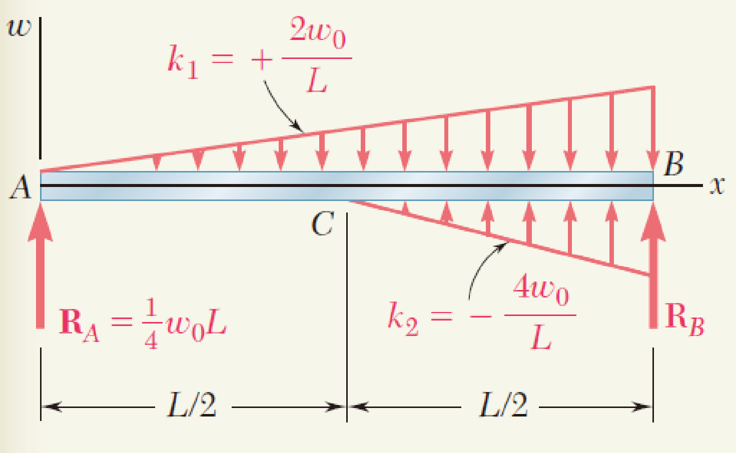


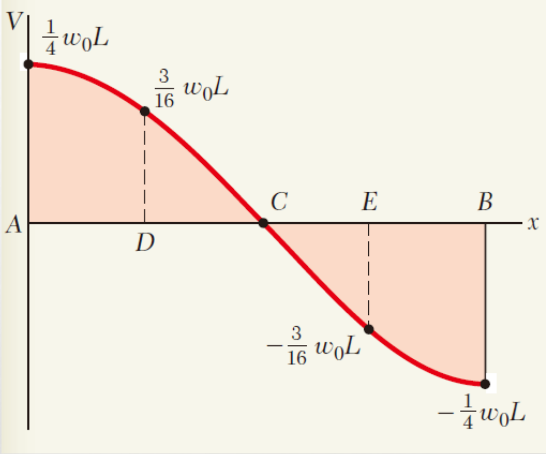
Example 6.7

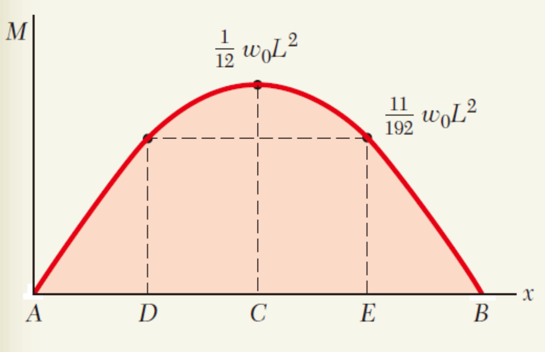
(Beer, Page 355)

For the beam and loading shown, determine (a) the equations defining the shear and bending moment at any point, (b) the shear and bending moment at points C, D, and E.









§ 6.8 Summary

- **The design of prismatic beams**
- **Normal stresses due to bending**
- **Relations among load, shear, and bending moment**
- **Design of prismatic beams**
- **Singularity functions**