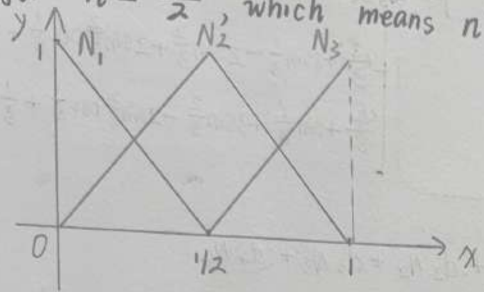
H18·vi h = mesh parameter $= \frac{1}{4}$

$re_{,x}$ at the midpoints $= \frac{1}{192}$

as calculated in part (b)

for $h = \frac{1}{2}$, which means $n = 2$



we calculated this case in the class:

$$K = \begin{bmatrix} 2 & -2 \\ -2 & 4 \end{bmatrix}$$

$$F_A = (N_A, ax) + N_A(0)\cdot 0 - a(N_A, N_{n+1})\cdot 0$$

$$= (N_A, ax)$$

$$\Rightarrow F_1 = \frac{a}{24}, \quad F_2 = \frac{a}{4}$$

$$Kd = F \Rightarrow \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{6}a \\ \frac{7}{48}a \end{bmatrix}$$

$$u^h = d_1 N_1 + d_2 N_2 + g N_3$$

$$= \frac{1}{6}aN_1 + \frac{7}{48}a N_2$$

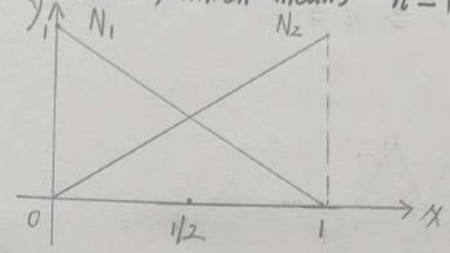
$$u^h_{,x} = \frac{1}{6}a N_{1,x} + \frac{7}{48}a N_{2,x}$$

$$= \begin{cases} -\frac{a}{24} & (0, 1/2) \\ -\frac{7a}{24} & (1/2, 1) \end{cases}$$

$$re_{,x}(1/4) = \frac{\left|-\frac{a}{24} + \frac{1}{2}a\times\frac{1}{16}\right|}{a/2} = \frac{1}{48}$$

$$re_{,x}(3/4) = \frac{\left|-\frac{7a}{24} + \frac{1}{2}a\times\frac{9}{16}\right|}{a/2} = \frac{1}{48}$$

for $h = 1$, which means $n = 1$



$N_1 = -x + 1$, $N_2 = x$ at $[0, 1]$

$N_{1,x} = -1$, $N_{2,x} = 1$

$$K = a(N_1, N_1) = \int_0^1 (-1)(-1)\,dx = 1$$

$$F_1 = (N_1, ax) = \int_0^1 (-x+1)\,ax\,dx = \frac{1}{6}a$$

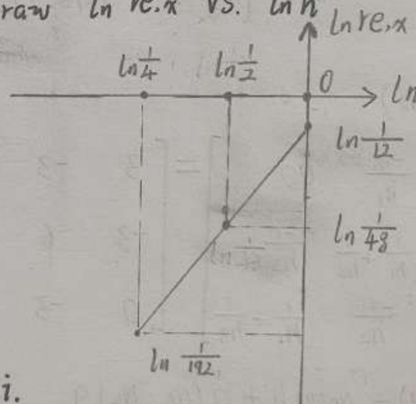
$$\Rightarrow d_1 = \frac{1}{6}a$$

$$u^h = d_1 N_1 + g N_2 = \frac{1}{6}a N_1$$

$$u^h_{,x} = \frac{1}{6}a N_{1,x} = -\frac{1}{6}a \quad [0, 1]$$

$$re_{,x}(1/2) = \frac{\left|-\frac{1}{6}a + \frac{1}{2}a\times\frac{1}{4}\right|}{a/2} = \frac{1}{12}$$

Draw $\ln re_{,x}$ vs. $\ln h$



d) i.

$$\frac{\ln\frac{1}{48} - \ln\frac{1}{192}}{\ln\frac{1}{2} - \ln\frac{1}{4}} = \frac{\ln 4}{\ln 2} = \frac{\ln\frac{1}{12} - \ln\frac{1}{48}}{\ln 1 - \ln\frac{1}{2}} = \text{slope} = 2$$

significance: the relative error in $u_{,x}$

the slope ↑, the error ↑

$$re_{,x} = C\cdot h^2 \Leftarrow \ln re_{,x} = C_1 + 2\ln h$$

ii.

y-intercept significance:

when mesh parameter = 1, the relative error in $u_{,x}$.

HW2 Correction from submitted file

3. Solution

a) $\hat{f} \simeq \sum_{B=1}^{n+1} f_B N_B \quad , (N_A, f) \rightarrow (N_A, \hat{f})$

$= 0 \cdot N_1 + \sin\frac{1}{3} N_2 + \sin\frac{2}{3} N_3 + \sin 1 \cdot N_4$

$\hat{F}_1 = \frac{1}{18} \sin\frac{1}{3} + h$

$\hat{F}_2 = \frac{2}{9} \sin\frac{1}{3} + \frac{1}{18} \sin\frac{2}{3}$

$\hat{F}_3 = \frac{1}{18} \sin\frac{1}{3} + \frac{2}{9} \sin\frac{2}{3} + \frac{1}{18} \sin 1 + 3g$

$K = \begin{bmatrix} 3 & -3 & 0 \\ -3 & 6 & -3 \\ 0 & -3 & 6 \end{bmatrix}, \quad |K| = 27$

$K^{-1} = \begin{bmatrix} 1 & +\frac{2}{3} & \frac{1}{3} \\ +\frac{2}{3} & \frac{2}{3} & +\frac{1}{3} \\ \frac{1}{3} & +\frac{1}{3} & \frac{1}{3} \end{bmatrix}$ (Correction)

$\hat{d} = K^{-1} \hat{F} \Rightarrow \begin{bmatrix} \hat{d}_1 \\ \hat{d}_2 \\ \hat{d}_3 \end{bmatrix} = \begin{bmatrix} F_1 + \frac{2}{3}F_2 + \frac{1}{3}F_3 \\ \frac{2}{3}F_1 + \frac{2}{3}F_2 + \frac{1}{3}F_3 \\ \frac{1}{3}F_1 + \frac{1}{3}F_2 + \frac{1}{3}F_3 \end{bmatrix}$

$\hat{u}^h = \hat{d}_1 N_1 + \hat{d}_2 N_2 + \hat{d}_3 N_3 + g N_4$

$\Rightarrow \hat{u}^h(0) = \hat{d}_1$

$\hat{u}^h(1/3) = \hat{d}_2$

$\hat{u}^h(2/3) = \hat{d}_3$

$\hat{u}^h(1) = g$

b) $F_A = (N_A, \sin x) + N_A(0) h - a(N_A, N_4) g$

$F_1 = 1 - 3\sin\frac{1}{3} + h$

$F_2 = 3(2\sin\frac{1}{3} - \sin\frac{2}{3})$

$F_3 = 3(-2\sin\frac{2}{3}\cos\frac{1}{3} + 2\sin\frac{2}{3} + g)$

$d_1 = F_1 + \frac{2}{3}F_2 + \frac{1}{3}F_3$

$d_2 = \frac{2}{3}F_1 + \frac{2}{3}F_2 + \frac{1}{3}F_3$

$d_3 = \frac{1}{3}F_1 + \frac{1}{3}F_2 + \frac{1}{3}F_3$

$u^h(0) = d_1$

$u^h(1/3) = d_2$

$u^h(2/3) = d_3$

$u^h(1) = g$