力学与航空航天工程系
DEPARTMENT OF MECHANICS AND AEROSPACE ENGINEERING

MAE 312 Aircraft Flight Dynamics Assignment

Student Name1: 刘鸿磊
Name2: 邹佳驹

Student ID1: 12010202
ID2: 12012127

Data: 2022.12.25

Table of Content

1.Jet-driven aircraft-GulfStream G650

GulfStream Aerospace is the world's leading luxury, large business jets manufacturer. The Gulfstream G650 is an aircraft that can fly at near-sonic speeds that are unmatched by other traditional business jets. If time is money, then time saved is a new asset, and the G650 is designed with time in mind, but also with a greater focus on cabin comfort and practicality, reflecting the value of the G650 in use. The G650 is the most technologically advanced commercial aircraft in the sky.

GulfStream G650

1.1Basic configuration data

Weight

Basic operating weight: 24494kg

Max zero fuel weight: 27442kg

Max taking-off weight: 45178kg

Max landing weight: 37875kg

Max fuel: 20048kg

Max payload: 816kg

Outlook

Total interior length:16.33m

Cabin length(excluding baggage):14.27m

Cabin volumn:60.54m^3

Exterior Height:7.82m

Exterior Length:30.40m

Wing

Overall Wingspan:30.35m

Wing Area:131m

Capacity passengers:19 max

1.2Aerodynamics and propulsion characteristics

Aerodynamics

Max Range: 12964km

Max speed: 0.95Mach

High-speed cruise: 0.9Mach

Long-range cruise: 0.85Mach

Initial cruise altitude:12497m

Maximum cruise altitude:15545m

Propulsion

Two Rolls-Royce BR725

Rated take-off thrust (each):75.20kN

The BR725 redefines the top-end of the market for the most powerful business jet power-plants. It enables a whole new class of business aircraft to travel further and faster.

Design Insights

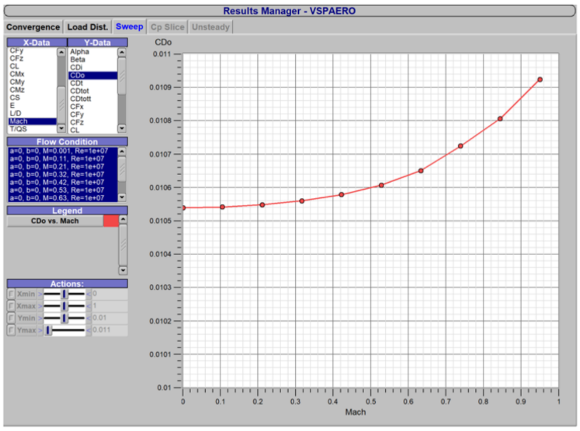
- The BR725 combines proven features from the BR 700 and Trent families.
- The BR725 features a 50 inch diameter titanium swept fan based on world-class Trent fan design for improved aerodynamic efficiency and lower noise – a first time in the business aviation market.
- 24 blades are driven by a three-stage, low-pressure turbine, for improved flow, increased efficiency, reduced noise and lower emissions.

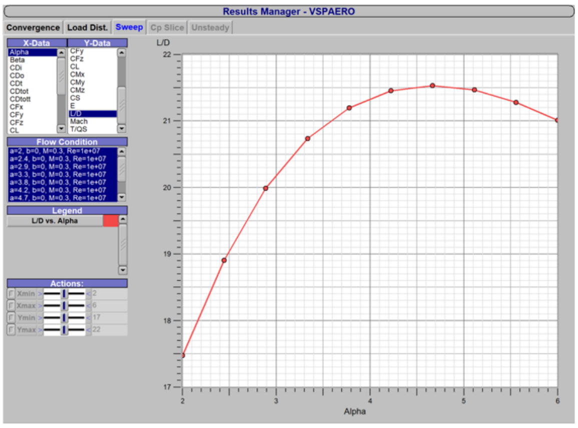
- ➢ Ten-stage high-pressure compressor incorporates the latest aerodynamic improvements and five stages of blisks for improved performance and optimized weight.
- ➢ The two-stage shrouded high-pressure turbine uses advanced aerodynamic design and latest material for high efficiency, enhanced performance retention and longer life.
- ➢ A high-efficiency thrust reverser system enables increased reverse thrust and lower drag.

2.Flight Performance

Parameters Obtaining $C_{D0}, K\ and\ C_{Lmax}$

By using OpenVsp to simulate the model, we obtain:





$$C_{D0} = 0.0106$$

$$(\frac{L}{D})max = 21.5 = \frac{1}{\sqrt{4KC_{D0}}}$$

$$K = 0.05102$$

The airfoil of the wing is NACA1223, we can obtain C_{Lmax} as following:

$$C_{lmax} = 1.3368\ at\ \alpha = 19\ ^{\circ}$$

$$a_0 = 4.031rad^{-1}$$

$$\begin{aligned} a_{\text{comp}} &= \frac{a_0 \cos n}{\sqrt{1-(M\cos n)^2+\left(\frac{a_0\cos n}{\pi AR}\right)^2}+\frac{a_0\cos n}{\pi AR}} \\ &= \frac{4.031\times\cos 60^{\circ}}{\sqrt{1-(0.8\times\cos 60^{\circ})^2+\left(\frac{4.031\times\cos 60^{\circ}}{\pi\times 8.01185}\right)^2}+\frac{4.031\times\cos 60^{\circ}}{\pi\times 8.0185}} \\ &= 2.0154 \\ C_{\text{L.max}} &= a_{\text{comp}}\cdot(\alpha-\alpha_0) = 0.668332 \end{aligned}$$

2.1Take-off distance

We choose the max take-off weight to calculate:

$$W_{\max} = 45178 \times 9.81 = 443196.18 \text{ N}$$

Assume ambient temperature 20℃, and we can obtain density:

$$\rho_{\infty} = \rho_0 \frac{273}{273+t} = 1.293 \times \frac{273}{273+20} = 1.205 \text{ kg/m}^3$$

Rated take-off thrust (each): 75.20kN

$$T_{\max} = 75.2 \times 10^3 \times 2 = 150400 \text{ N}$$

S_g

$$S_g = \frac{1.21\left(\frac{W_{\max}}{S}\right)}{\rho_{\infty} \cdot g \cdot C_{\text{Lmax}} \cdot \frac{T_{\max}}{W_{\max}}}$$
$$= \frac{1.21 \times \frac{443196.18}{131}}{1.205 \times 9.81 \times 0.668332 \times \frac{150400}{443196.18}} = 1526.9 \text{ m}$$

S_a

$$V_{\text{stall}} = \sqrt{\frac{2}{\rho_{\infty}} \cdot \frac{W_{\max}}{S} \cdot \frac{1}{C_{\text{Lmax}}}} = \sqrt{\frac{2}{1.205} \times \frac{443196.18}{131} \times \frac{1}{0.668332}} = 91.66 \text{ m/s}$$

$$R = \frac{6.96 \cdot V_{\text{stall}}^2}{9} = \frac{6.96 \times 91.66^2}{9.81} = 5961 \text{ m}$$

$$h_{OB} = 35ft = 10.688 \text{ m}$$

$$\theta_{OB} = \cos^{-1}\left(1 - \frac{h_{OB}}{R}\right) = \cos^{-1}\left(1 - \frac{10.688}{5961}\right) = 3.43^{\circ}$$

$$S_a = R \cdot \sin\theta_{OB} = 5961 \times \sin 3.43^{\circ} = 356.6 \text{ m}$$

$$S = S_a + S_g = 1526.9 + 356.6 = 1883.5\text{m}$$

$$\text{Error} = \frac{1883.5 - 1786}{1786} \times 100\% = 5.46\%$$

The calculated results do not differ much from the actual results, indicating that the calculation is valid and the source of error may come from the deviation of the maximum lift coefficient.

2.2Level Turn

The altitude can range from 12497m to 15545m, and we choose the altitude as 13000m to continue our calculation.

$$\text{At altitude} = 13000\ \text{m},\ \rho_\infty = 0.2658\ \text{kg/m}^3$$

We choose the basic operating weight 24494kg to calculate.

$$W = 24494 \times 9.81 = 240286.14\ \text{N}$$

Minimum turning radius:

$$(n)_{R_{\min}} = \sqrt{2 - \frac{4KC_{D0}}{\left(\frac{T}{W}\right)^2}} = \sqrt{2 - \frac{4 \times 0.05102 \times 0.0106}{\frac{150400}{240286.14}}} = 1.413$$

$$R_{\min} = \frac{(V_\infty)^2_{R_{\min}}}{g\sqrt{(n)^2_{R_{\min}} - 1}} = \frac{106.1^2}{9.81 \times \sqrt{1.413^2 - 1}} = 1149.5\ \text{m}$$

The velocity with minimum turning radius:

$$(V_\infty)_{R_{\min}} = \sqrt{\frac{4K\left(\frac{W}{S}\right)}{\rho_\infty\left(\frac{T}{W}\right)}} = \sqrt{\frac{4 \times 0.05102 \times \frac{240286.14}{131}}{0.2658 \times \frac{150400 \times 0.2}{240286.14}}} = 106.1\ \text{m/s}$$

Maximum turning rate:

$$(n)_{w_{\max}} = \left(\frac{\frac{T}{W}}{\sqrt{KC_{D.0}}} - 1\right)^{\frac{1}{2}} = \left(\frac{\frac{150400 \times 0.2}{240286.14}}{\sqrt{0.05102 \times 0.0106}} - 1\right)^{\frac{1}{2}} = 2.09$$

$$w_{\max} = \frac{1}{2}\rho_\infty (V_\infty)^2_{w_{\max}} \sqrt{\frac{\rho_\infty}{\frac{W}{S}}\left[\frac{\frac{T}{W}}{2K} - \left(\frac{C_{D.0}}{K}\right)^{\frac{1}{2}}\right]}$$

$$= \frac{1}{2} \times 0.2658 \times 174^2 \times \sqrt{\frac{0.2658}{\frac{240286.14}{131}} \times \left[\frac{\frac{150400 \times 0.2}{240286.14}}{2 \times 0.05102} - \left(\frac{0.0106}{0.05102}\right)^{\frac{1}{2}}\right]} = 42.53\text{degree/s}$$

The velocity with maximum turning rate:

$$(V_\infty)_{w_{\max}} = \left[\frac{2\left(\frac{W}{S}\right)}{\rho_\infty}\right]^{\frac{1}{2}} \left(\frac{K}{C_{D.0}}\right)^{\frac{1}{4}} = \left(\frac{2 \times \frac{240286.14}{131}}{0.2658}\right)^{\frac{1}{2}} \times \left(\frac{0.05102}{0.0106}\right)^{\frac{1}{4}} = 174\ \text{m/s}$$

2.3 Climb
Still at 13000m with basic operating weight 24494kg.

The maximum climbing angle:

$$\sin\theta_{\max} = \frac{T}{W} - \sqrt{4C_{D.0}\cdot K} = \frac{150400}{240286.14} - \sqrt{4\times 0.0106\times 0.05102} = 0.5794$$
$$\theta_{\max} = \sin^{-1}(0.5794) = 35.41^\circ$$

The velocity with the maximum climbing angle:

$$V_{(\theta_{\max})} = \sqrt{\frac{2}{\rho_\infty}\left(\frac{K}{C_{D.0}}\right)^{\frac{1}{2}}\frac{W}{S}\cos\theta_{\max}}$$
$$= \sqrt{\frac{2}{0.2658}\times\left(\frac{0.05102}{0.0106}\right)^{\frac{1}{2}}\times\frac{240286.14}{131}}\times\cos 35.41^\circ = 157.09\ \mathrm{m/s}$$

The maximum rate of climbing:

$$\left(\frac{L}{D}\right)_{\max} = \frac{1}{\sqrt{4C_{D.0}k}} = \frac{1}{\sqrt{4\times 0.0106\times 0.05102}} = 21.5$$

$$Z = 1+\sqrt{1+\frac{3}{\left(\frac{L}{D}\right)_{\max}^2\left(\frac{T}{W}\right)^2}} = 1+\sqrt{1+\frac{3}{21.5^2\times\left(\frac{150400}{240286.14}\right)^2}} = 2.0$$

$$\left(\frac{R}{C}\right)_{\max} = \left[\frac{\left(\frac{W}{S}\right)\cdot Z}{3\rho_\infty C_{D.0}}\right]^{\frac{1}{2}}\left(\frac{T}{W}\right)^{\frac{3}{2}}\left[1-\frac{Z}{6}-\frac{3}{2\left(\frac{T}{W}\right)^2\left(\frac{L}{D}\right)_{\max}^2 Z}\right]$$
$$= \left(\frac{\frac{240286.14}{131}\times 2}{3\times 0.2658\times 0.0106}\right)^{\frac{1}{2}}\times\left(\frac{150400}{240286.14}\right)^{\frac{3}{2}}\left[1-\frac{2}{6}-\frac{3}{2\times\left(\frac{150400}{240286.14}\right)^2\times 21.5^2\times 2}\right] = 216.1\ \mathrm{m/s}$$

2.4 Cruise

Still at 13000m with basic operating weight 24494kg.

Minimum thrust required:

$$(T_R)_{\min} = \frac{W}{\left(\frac{L}{D}\right)_{\max}} = \frac{240286.14}{21.5} = 11176 \text{ N}$$

The velocity with the minimum required thrust:

$$V_{(T_R)_{\min}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{C_{D.0}}}\frac{W}{S}\right)^{\frac{1}{2}} = \left(\frac{2}{0.2658} \times \sqrt{\frac{0.05102}{0.0106}} \times \frac{240286.14}{131}\right)^{\frac{1}{2}} = 174 \text{ m/s}$$

We can obtain the velocity through the given thrust by the following equation:

$$V_\infty = \left[\frac{\frac{T_G}{W}\frac{W}{S} \pm \frac{W}{S}\sqrt{\left(\frac{T_G}{W}\right)^2 - 4C_{D,0}K}}{\rho_\infty C_{D,0}}\right]^{1/2} \qquad (Eq.8.5)$$

We cannot substitute T_G with 150400N since 150400N is the maximum take-off thrust.

There are 4 types of maximum thrust:

1) the maximum take-off thrust
2) the maximum continuous thrust
3) the maximum climbing thrust
4) the maximum cruise thrust

The maximum cruise thrust is the smallest one among the above thrusts, and we take it as 20000N for calculation.

$$V_\infty = \left(\frac{\frac{20000}{131} + \sqrt{\left(\frac{20000}{131}\right)^2 - 4 \times 0.0106 \times 0.05102 \times \left(\frac{240286.14}{131}\right)^2}}{0.2658 \times 0.0106}\right)^{\frac{1}{2}} = 314.84 \text{ m/s}$$

The Mach number:

$$\text{Mach} = \frac{V_\infty}{a} = 0.926$$

This is close to the actual maximum speed 0.95Mach.

2.5 Range

The thrust specific fuel consumption is $18.6\frac{g}{kN*s}$ at 0.85 Mach.

We choose the maximum take-off weight as the initial weight, and the maximum zero fuel weight as the final weight.

$$W_i = 45178 \times 9.81 \text{ N} = 443196 \text{ N}$$
$$W_f = 27442 \times 9.81 \text{ N} = 269206 \text{ N}$$

Then we can calculate the range:

$$R_{\max} = \frac{2}{c_t} \cdot \sqrt{\frac{2}{\rho_\infty S}} \cdot \frac{3}{4}\left(\frac{1}{3KC_{D\cdot 0}^3}\right)^{\frac{1}{4}} \cdot \left(W_j^{\frac{1}{2}} - W_f^{\frac{1}{2}}\right)$$
$$= \frac{2}{\frac{18.6\times10^{-3}\times9.81}{10^3}} \times \sqrt{\frac{2}{0.2658 \times 131}} \times \frac{3}{4} \times \left(\frac{1}{3 \times 0.05102 \times 0.0106^3}\right)^{\frac{1}{4}} \times (\sqrt{443196} - \sqrt{269206})$$
$$= 14004774.33 \text{ m}$$
$$= 14005 \text{ km} \quad \text{close to } 12964 \text{ km}$$

2.6 Endurance

$$E_{\max} = \frac{1}{c_t}\left(\frac{L}{D}\right)_{\max} \ln\frac{W_i}{W_f}$$
$$= \frac{1}{\frac{18.6\times10^{-3}\times9.81}{10^3}} \times 21.5 \times \ln\frac{443196}{269206}$$
$$= 58742 \text{ s} = 16.32 \text{ h}$$

2.7 Gliding
Minimum descent angle:

$$\tan\theta_{\min} = \frac{1}{\left(\frac{L}{D}\right)_{\max}} = \sqrt{4C_{D0}K} = \sqrt{4\times 0.0106\times 0.05102} = 0.04651$$

$$\theta_{\min} = \tan^{-1}(0.04651) = 2.6629^{\circ}$$

The velocity with the minimum descent angle:

$$V_{\theta_{\min}} = \sqrt{\frac{2W\cos(\theta_{\min})}{\rho_\infty S\sqrt{\frac{C_{D,0}}{K}}}} = \sqrt{\frac{2\times 1834.244\times\cos(2.6629^{\circ})}{1.225\times\sqrt{0.0106/0.05102}}} = 81.012\ \mathrm{m/s}$$

The corresponding descent rate:

$$\frac{R}{D} = V_{\theta_{\min}}\sin\theta_{\min} = 81.012\times\sin(2.6629^{\circ}) = 3.7638\ \mathrm{m/s}$$

Minimum descent rate (assuption $cos\theta = 1$):

$$\begin{aligned}\left(\frac{R}{D}\right)_{\min} &= 4\sqrt{\frac{2\cos(\theta)^3W}{\rho_\infty S}}\left(\frac{3}{kC_{D.0}^{\frac{1}{3}}}\right)^{-\frac{3}{4}} \\ &= 4\times\sqrt{\frac{2\times 240281.14}{0.2658\times 131}}\left(\frac{3}{0.05102\times 0.0106^{\frac{1}{3}}}\right)^{-\frac{3}{4}} \\ &= 7.1\ \mathrm{m/s}\end{aligned}$$

The velocity with the minimum descent rate

$$V_{\left(\frac{R}{D}\right)_{\min}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{3C_{D.0}}}\frac{W}{S}\right)^{\frac{1}{2}} = \left(\frac{2}{1.225}\times\sqrt{\frac{0.05102}{3\times 0.0106}}\times 1834.244\right)^{\frac{1}{2}} = 61.59\ \mathrm{m/s}$$

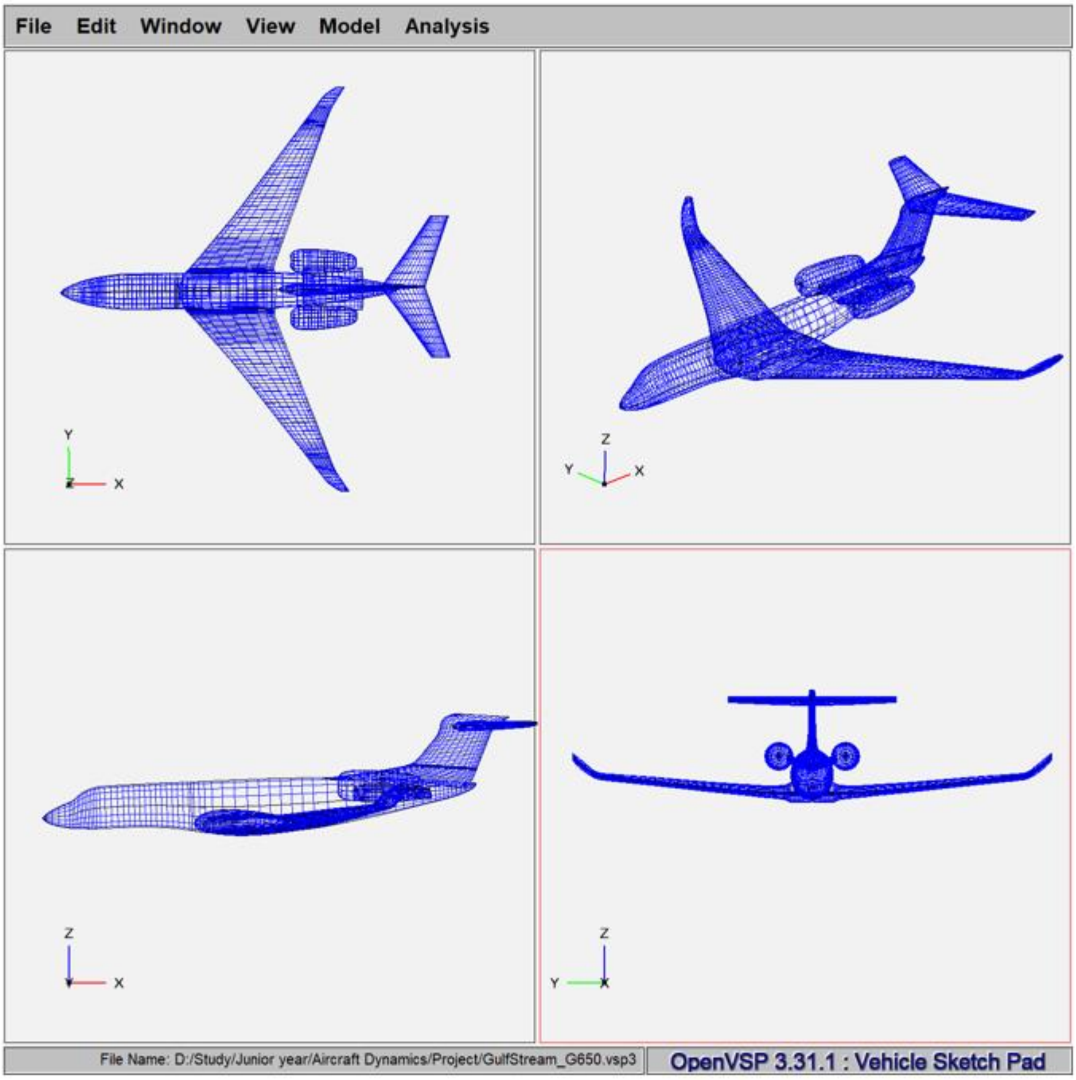
3.Reference

[1] G650ER - Gulfstream Aerospace. (n.d.). Retrieved December 25, 2022, from https://www.gulfstream.com/en/aircraft/gulfstream-g650er/

[2] BR725. Rolls. (n.d.). Retrieved December 25, 2022, from https://www.rolls-royce.com/products-and-services/civil-aerospace/business-aviation/br725.aspx

[3] Gulfstream G650. VSP Hangar | Gulfstream G650. (n.d.). Retrieved December 25, 2022, from https://hangar.openvsp.org/vspfiles/402

4.Appendix



The OpenVSP model

XFOIL Version 6.99

Calculated polar for: NACA 1223

1 1 Reynolds number fixed Mach number fixed

xtrf = 1.000 (top) 1.000 (bottom)
Mach = 0.000 Re = 0.300 e 6 Ncrit = 9.000

alpha	CL	CD	CDp	CM	Top_Xtr	Bot_Xtr
0.000	0.0865	0.01211	0.00518	-0.0119	0.5656	0.6266
1.000	0.1859	0.01220	0.00530	-0.0090	0.5207	0.6743
2.000	0.2833	0.01249	0.00554	-0.0056	0.4781	0.7263
3.000	0.3788	0.01287	0.00600	-0.0019	0.4363	0.7799
4.000	0.4718	0.01353	0.00666	0.0022	0.3986	0.8331
5.000	0.5621	0.01445	0.00757	0.0070	0.3661	0.8835
6.000	0.6538	0.01564	0.00873	0.0112	0.3391	0.9283
7.000	0.7763	0.01732	0.01025	0.0085	0.3155	0.9581
8.000	0.9281	0.01903	0.01192	-0.0010	0.2933	0.9752
9.000	1.0723	0.02077	0.01353	-0.0098	0.2740	0.9926
10.000	1.1398	0.02199	0.01481	-0.0049	0.2589	1.0000
11.000	1.1452	0.02322	0.01602	0.0115	0.2470	1.0000
12.000	1.1749	0.02522	0.01808	0.0223	0.2340	1.0000
13.000	1.2156	0.02817	0.02115	0.0291	0.2203	1.0000
14.000	1.2547	0.03214	0.02526	0.0338	0.2063	1.0000
15.000	1.2878	0.03728	0.03053	0.0371	0.1927	1.0000
16.000	1.3169	0.04351	0.03682	0.0390	0.1795	1.0000
17.000	1.3281	0.05227	0.04586	0.0394	0.1661	1.0000
18.000	1.3352	0.06217	0.05592	0.0384	0.1536	1.0000
19.000	1.3368	0.07318	0.06707	0.0364	0.1419	1.0000
20.000	1.3236	0.08669	0.08084	0.0326	0.1307	1.0000

The NACA 1223 lift coefficient

Individual Report

You will need to evaluate the contribution of the team members on the project on 4 aspects: Research, Analysis, Documentation and Presentation. There are 10 marks for each aspect in each group, the sum of all the team members in one group should be 10 marks on each aspect. Each student should submit an individual report, please attach it in the last page of your soft copy report. *Note: Your report will not be disclosed to any other students in the class.*

Name	邹佳驹	Student ID	12012127	Group ID	1
Group Member	Research	Analysis	Documentation	Presentation	Total
1:刘鸿磊	5	5	5	5	20
2:邹佳驹	5	5	5	5	20
Total	10	10	10	10	40