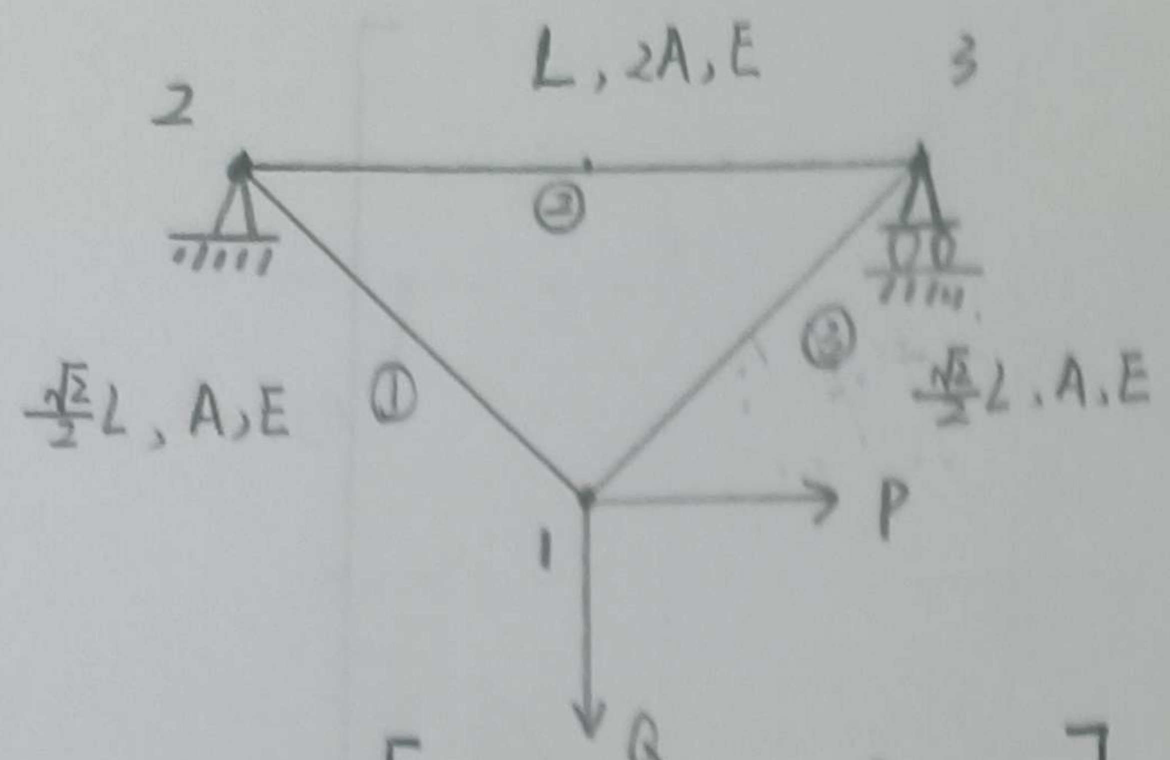
FEA for Prob 3.3

解:



Since $[k] = \frac{AE}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$ $\quad l=\cos\theta$, $m=\sin\theta$

for ①: $\theta=135°$, $l=-\frac{\sqrt{2}}{2}$, $m=\frac{\sqrt{2}}{2}$

$$[k_①] = \frac{\sqrt{2}AE}{L}\begin{array}{c} \begin{matrix} u_1 & v_1 & u_2 & v_2 \end{matrix} \\ \begin{bmatrix} \frac12 & -\frac12 & -\frac12 & \frac12 \\ -\frac12 & \frac12 & \frac12 & -\frac12 \\ -\frac12 & \frac12 & \frac12 & -\frac12 \\ \frac12 & -\frac12 & -\frac12 & \frac12 \end{bmatrix} \end{array}$$

for ②, $\theta=0°$, $l=1$, $m=0$

$$[k_②] = \frac{2AE}{L}\begin{array}{c} \begin{matrix} u_2 & v_2 & u_3 & v_3 \end{matrix} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

for ③, $\theta=45°$, $l=\frac{\sqrt{2}}{2}$, $m=\frac{\sqrt{2}}{2}$

$$[k_③] = \frac{\sqrt{2}AE}{L}\begin{array}{c} \begin{matrix} u_1 & v_1 & u_3 & v_3 \end{matrix} \\ \begin{bmatrix} \frac12 & \frac12 & -\frac12 & -\frac12 \\ \frac12 & \frac12 & -\frac12 & -\frac12 \\ -\frac12 & -\frac12 & \frac12 & \frac12 \\ -\frac12 & -\frac12 & \frac12 & \frac12 \end{bmatrix} \end{array}$$

Superposition.

$$\frac{\sqrt{2}AE}{L}\begin{bmatrix} 1 & 0 & -\frac12 & \frac12 & -\frac12 & -\frac12 \\ 0 & 1 & \frac12 & -\frac12 & -\frac12 & -\frac12 \\ -\frac12 & \frac12 & \sqrt{2}+\frac12 & -\frac12 & -\sqrt{2} & 0 \\ \frac12 & -\frac12 & -\frac12 & \frac12 & 0 & 0 \\ -\frac12 & -\frac12 & -\sqrt{2} & 0 & \sqrt{2}+\frac12 & \frac12 \\ -\frac12 & -\frac12 & 0 & 0 & \frac12 & \frac12 \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{bmatrix}$$

BC: $u_2 = v_2 = 0$, $v_3 = 0$, $F_{1x} = P$, $F_{1y} = -Q$

$F_{3x} = 0$

Condensed FE equation.

$$\frac{\sqrt{2}AE}{L}\begin{bmatrix} 1 & 0 & -\frac12 & \frac12 & -\frac12 & -\frac12 \\ 0 & 1 & \frac12 & -\frac12 & -\frac12 & -\frac12 \\ -\frac12 & \frac12 & \frac12+\sqrt{2} & -\frac12 & -\sqrt{2} & 0 \\ \frac12 & -\frac12 & -\frac12 & \frac12 & 0 & 0 \\ -\frac12 & -\frac12 & -\sqrt{2} & 0 & \sqrt{2}+\frac12 & \frac12 \\ -\frac12 & -\frac12 & 0 & 0 & \frac12 & \frac12 \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ 0 \\ 0 \\ u_3 \\ 0 \end{bmatrix} = \begin{bmatrix} P \\ -Q \\ F_{2x} \\ F_{2y} \\ 0 \\ F_{3y} \end{bmatrix}$$

$$\Rightarrow u_1 = \frac{L[(1+4\sqrt{2})P - Q]}{4AE\cdot 2}$$

$$= \frac{L[(1+4\sqrt{2})P - Q]}{8AE}$$

$\boxed{\text{ANS}}$

$$v_1 = \frac{L[P - (4\sqrt{2}+1)Q]}{8AE}$$

$$u_3 = \frac{L(P-Q)}{4AE}$$

$$F_{2x} = -P$$

$$F_{2y} = \frac12(P+Q)$$

$$F_{3y} = \frac12(Q-P)$$