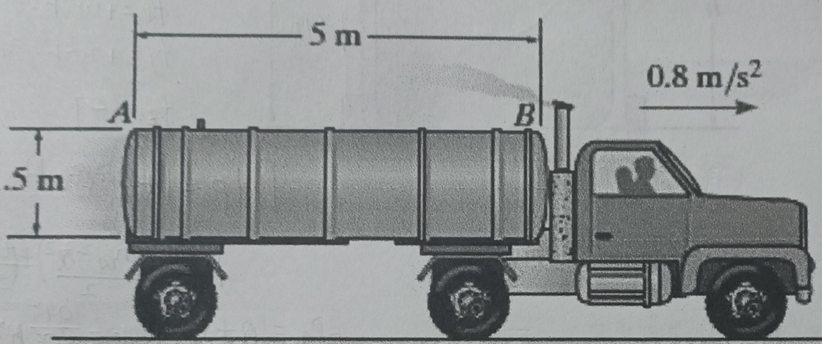
Homework 2

Questions No.	1	2	3	4	Total
Score	25%	15%	20%	40%	100%

Q2.1 The closed cylindrical tank is filled with milk for which $\rho_m = 1030\ kg/m^3$. If the inner diameter of the tank is 1.5 m, determine the difference in pressure within the tank between corners A and B when the truck accelerates at $0.8 m/s^2$.

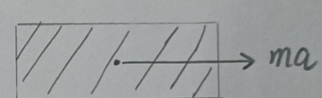


解:

$m = \rho_m V = 1030 \times 5 \times \pi \times (\frac{1.5}{2})^2$

$= 9101\ kg$

since tank full of milk

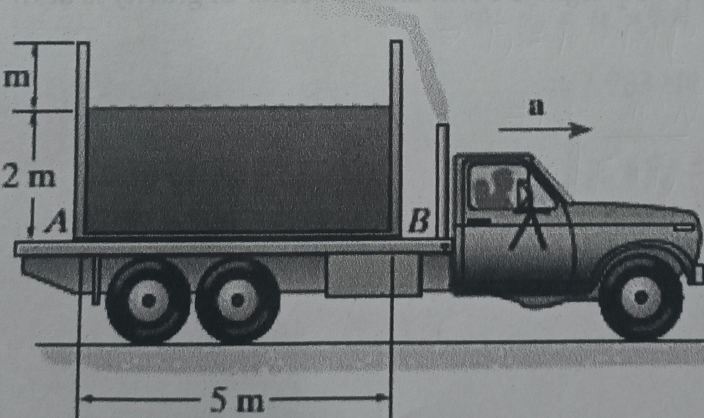


$\Delta P \cdot A = ma$

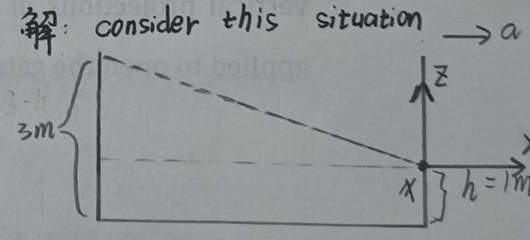
$\Delta P = \frac{ma}{A} = \frac{9101 \times 0.8}{\pi \times (\frac{1.5}{2})^2}$

$= 4120\ Pa$

Q2.2 The truck carries an open container of water as shown. Determine the maximum constant acceleration a it can have without causing the water to spill out of the container.



解: consider this situation



$\frac{(h+3) \times 5}{2} = 10 \Rightarrow h = 1m$

$f_x = 0,\ f_y = a,\ f_z = -g$

$dp = f_x dx + f_y dy + f_z dz$

$p = 0 + \int_y^0 a dy + \int_0^z -g dz$

$= -ay - gz + C$

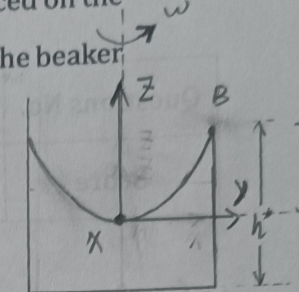
BC: $P = P_0,\ x = y = z = 0 \Rightarrow C = P_0$

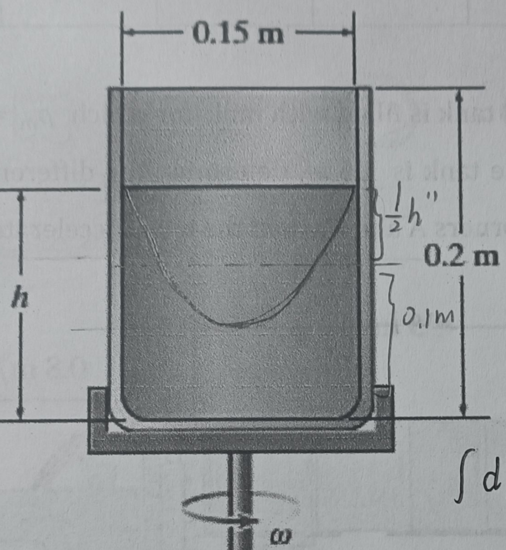
$P = P_0,\ x = 0,\ y = -5,\ z = 2 \Rightarrow 5a - 2g = 0$

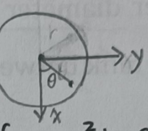
$a = \frac{2}{5} g$

Q2.3 The beaker is filled to a height of $h = 0.1\ m$ with kerosene and placed on the platform. To what height h=h', does the kerosene rise against the wall of the beaker when the platform has an angular velocity of $\omega = 15\ rad/s$

解:







$$dp = \rho(f_x dx + f_y dy + f_z dz)$$

$$f_x = +w^2 r \cdot \cos\theta = +w^2 x$$

$$f_y = +w^2 r \cdot \sin\theta = +w^2 y$$

$$f_z = -g$$

$$\int dp = \int +\rho w^2 x\, dx + \int (+\rho w^2 y\, dy) + \int -g\, dz \quad \rho$$

$$P = \rho w^2 \cdot \frac{r^2}{2} + (-gz) + C$$

$$BC: \quad z = h'',\ r = \frac{0.15}{2} m,\ P = P_0 \qquad z = 0,\ r = 0,\ P = P_0 \quad \Rightarrow C = P_0, \quad h'' = \frac{\rho w^2 r^2}{2g}$$

Q2.4 The sluice gate for a water channer is $1.5\ m$ wide and in the closed position, as shown. Determine the magnitude of the resultant force of the water acting on the gate. Solve the problem by considering the fluid acting on the horizontal and vertical projections of the gate. Determine the smallest torque T that must be applied to open the gate if its weight is 30kN and its center of gravity is at G?

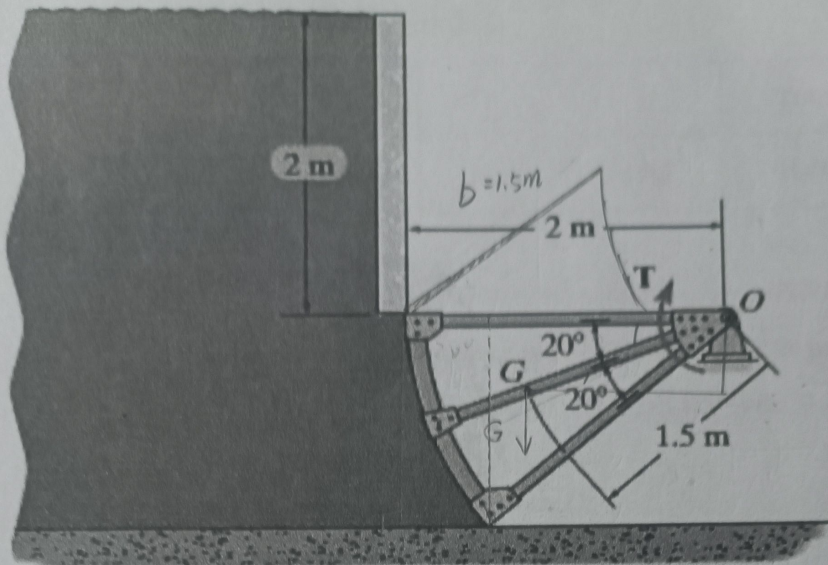
$$h' = 0.1m + \frac{1}{2}h''$$

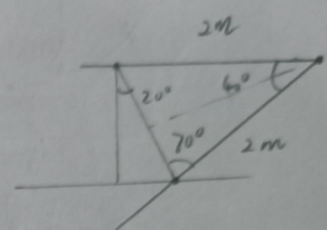
$$= 0.1 + \frac{\rho \times 15^2 \times 0.075^2}{4g}$$

$$= 0.1 + \frac{81}{256} \cdot \frac{\rho}{g}$$

$$= 0.1 + \frac{81}{256} \times \frac{0.8}{9.81}$$

$$= 0.1258\ m$$





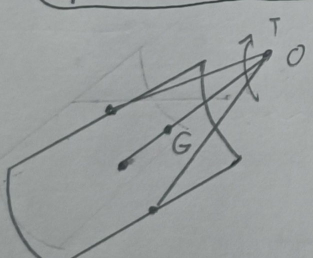
DDL: 25th, March

解: $F = \rho g \bar{h} A$

$A = 1.5 \times 2 \times \frac{40}{180}\pi \ m^2 = \frac{2}{3}\pi \ m^2$

$\bar{h} = 2 + \frac{1}{2} \times \cos 20^\circ (2 \times 2 \sin 20^\circ) = 2.643 \ m$

$\therefore F = 1.0 \times 10^3 \times 9.81 \times 2.643 \times \frac{2}{3}\pi = 5.430 \times 10^4 \ N$



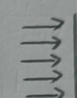
F is normal to the surface towards the point O directly

对 M_O: $G \cdot d_{OG} \cdot \cos 20^\circ - T = 0$

$\therefore T = 30 \times 10^3 \times 1.5 \cos 20^\circ = 4.229 \times 10^4 \ N \cdot m$

将F分解为F_x, F_y

$F_x = \rho g \bar{h}_1 A_1$



$\bar{h}_1 = 2 + \frac{1}{2} \times 2 \sin 40^\circ = 2.643 \ m$

$A_1 = 2 \sin 40^\circ \times 1.5 = 1.928 \ m^2$

$\Rightarrow F_x = 1.0 \times 10^3 \times 9.81 \times 2.643 \times 1.928 = 4.999 \times 10^4 \ N \rightarrow$

$F_y = -\rho g V$

$V_1 = (\pi \times 4 \times \frac{40^\circ}{360^\circ} - \frac{1}{2} \times 2\cos 40^\circ \cdot 2 \sin 40^\circ) \times 1.5 = 0.6172 \ m^3$

$\Rightarrow F_y = 10^3 \times 9.81 \times [0.6172 + 2 \times 1.5 \times (2 - 2\cos 40^\circ)] = 1.983 \times 10^4 \ N \uparrow$

$\Rightarrow F = \sqrt{F_x^2 + F_y^2} = 5.378 \times 10^4 \ N$