Quiz 8

Date: 2022-04-05 Name: SID:

Q1. Find the solution for each of the following initial value problem.

(1) $y'' + 2y' + 3y = 0, y(0) = 1, y'(0) = 1;$

(2) $4y'' + 12y' + 9y = 0, y(1) = e^{-3/2}, y'(1) = -e^{-3/2}.$

Q2. Use the method of reduction of order to find the general solution of the given differential equation.

$t^2y'' - 4ty' + 6y = 0, t > 0; y_1(t) = t^2.$

Q1.

(1) $y''+2y'+3y=0$. $\lambda^2+2\lambda+3=0$.

$$\lambda=\frac{-2\pm\sqrt{-8}}{2}=-1\pm\sqrt{2}i$$

solution is $y=c_1e^{-t}\cos\sqrt{2}t+c_2e^{-t}\sin\sqrt{2}t$

$y(0)=1 \Rightarrow c_1=1$.

$y'=-e^{-t}(c_1\cos\sqrt{2}t+c_2\sin\sqrt{2}t)$

$+e^{-t}(-\sqrt{2}c_1\sin\sqrt{2}t+\sqrt{2}c_2\cos\sqrt{2}t)$

$$\begin{cases}1=c_1\\ 1=-c_1+\sqrt{2}c_2\end{cases} \Rightarrow \begin{cases}c_1=1\\ c_2=\sqrt{2}.\end{cases}$$

$y=e^{-t}\cos\sqrt{2}t+\sqrt{2}e^{-t}\sin\sqrt{2}t$.

(2) $4y''+12y'+9y=0$. $4\lambda^2+12\lambda+9=0$

$(2\lambda+3)^2=0$. $\lambda=-\frac{3}{2}$

solution is $y=c_1e^{-\frac{3}{2}t}+c_2te^{-\frac{3}{2}t}$.

$y'=(-\frac{3}{2}c_1+c_2)e^{-\frac{3}{2}t}-\frac{3}{2}c_2te^{-\frac{3}{2}t}$

$$\begin{cases}y(1)=(c_1+c_2)e^{-\frac{3}{2}}=e^{-\frac{3}{2}}\\ y'(1)=(-\frac{3}{2}c_1-\frac{1}{2}c_2)e^{-\frac{3}{2}}=-e^{-\frac{3}{2}}\end{cases}$$

$$\Rightarrow \begin{cases} c_1 + c_2 = 1 \\ 3c_1 + c_2 = 2 \end{cases} \Rightarrow \begin{cases} c_1 = \frac{1}{2} \\ c_2 = \frac{1}{2} \end{cases}$$

$$y = \frac{1}{2} e^{-\frac{3}{2}t} + \frac{1}{2} t e^{-\frac{3}{2}t}$$

Q2. $t^2 y'' - 4ty' + 6y = 0 \quad t > 0. \quad y_1(t) = t^2$

let $y = v y_1 = v t^2$

$y' = t^2 v' + 2tv$

$y'' = t^2 v'' + 4tv' + 2v$

$t^4 v'' + 4t^3 v' + 2t^2 v - 4t^3 v' - 8t^2 v + 6t^2 v = 0$

$t^4 v'' = 0 \quad \Rightarrow v'' = 0$

$v' = c_1 \quad v = c_1 t + c_2$

general solution is

$$y = c_1 t^3 + c_2 t^2$$