Midterm exam

2020/11/11, Wednesday, 8:00 am – 10:00 am

1. The stress–strain diagram for an aluminum alloy is shown in Figure 1. A specimen of such material has an original diameter of 40 mm and an original length of 20 cm. When the applied load on the specimen is 500 kN, the diameter contracts to 39.95225 mm. (a) What is the length of the specimen after the load is applied? (b) Determine the shear modulus for the aluminum. $G = E/2(1+v)$. (c) If the specimen is subjected to a tensile strain of 0.03 and then unloaded, what is the final length of the specimen? (20')

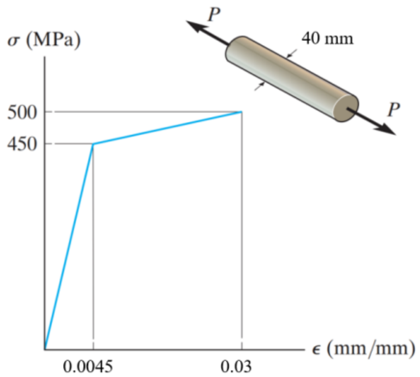


Figure 1

$$1.(a)\quad E = \frac{450\times 10^6}{0.0045} = 10^{11}(\mathrm{Pa}) \qquad (2')$$

$$\varepsilon = \frac{F}{EA} = 3.98\times 10^{-3} \qquad (2')$$

$$L = L_0(1+\varepsilon) = 20.0796(\mathrm{cm}) \qquad (2')$$

$$(b)\quad v = -\frac{\varepsilon_{\mathrm{lat}}}{\varepsilon_{\mathrm{long}}} = 0.3 \qquad (3')$$

$$G = \frac{E}{2(1+v)} = 38.5(\mathrm{GPa}) \qquad (3')$$

$$(c)\quad \frac{450}{0.0045} = \frac{500}{x} \Rightarrow x = 0.005 \qquad (4')$$

$$\varepsilon' = 0.03 - x = 0.025 \qquad (2')$$

$$L = L_0(1+\varepsilon') = 20.5(\mathrm{cm}) \qquad (2')$$

2. A 100-kg mass is suspended by steel wires, each has a diameter of 2 mm and Young's modulus of E_{st} = 200 GPa, from the ring at A. (a) If the ring A is connected to two wires, AB and AD as shown, determine the vertical displacement of the ring caused by the suspension of the mass. The unloaded lengths are $L_{AB} = L_{AD}$ = 2.00 m. (b) If the ring A is connected to three wires as shown, determine the force in each wire. The unloaded lengths are L_{AC} = 1.60 m and $L_{AB} = L_{AD}$ = 2.00 m. (20')

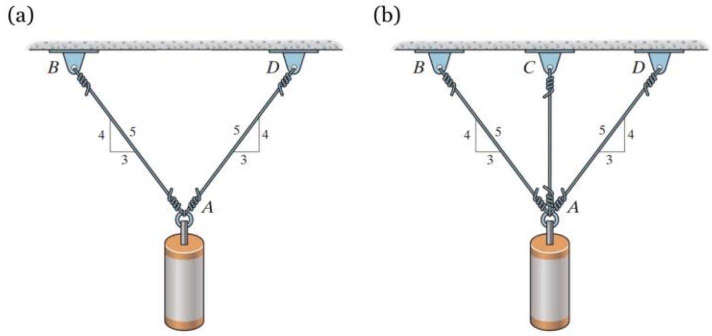
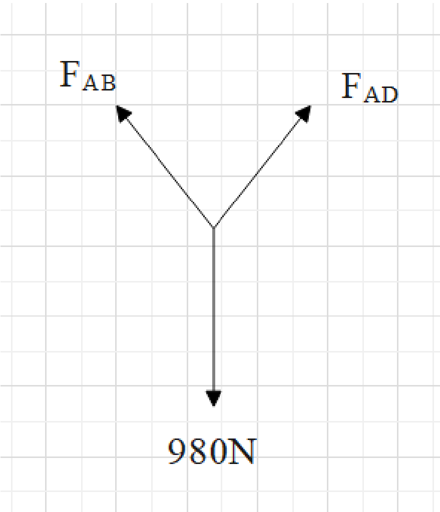
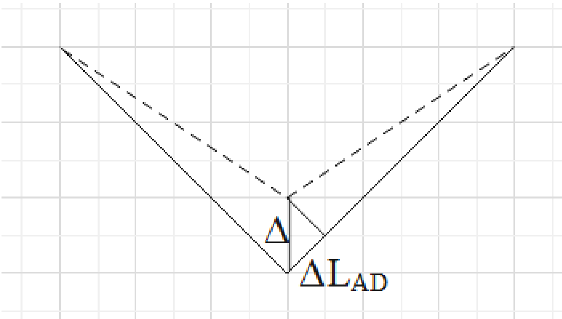


Figure 2





2. (a) Due to sysmetry $F_{AB} = F_{AD}$ $(2')$

$$\Sigma F_y = 0 \Rightarrow 2F_{AD} \cdot \frac{4}{5} = 980 \quad (2')$$

$$F_{AD} = 612.5(\text{N}) \quad (2')$$

$$\Delta L_{AD} = \frac{F_{AD} \cdot L_{AD}}{EA} = 1.95(\text{mm}) \quad (2')$$

$$\Delta = \frac{\Delta L_{AD}}{\frac{4}{5}} = 2.44(\text{mm}) \quad (2')$$

(b) $\Sigma F_y = 0 \quad F_{AC} + 2F_{AD} \cdot \frac{4}{5} = 980$ $(2')$

Compatibility $\Delta L_{AC} \cdot \frac{4}{5} = \Delta L_{AD}$ $(3')$ $\Rightarrow F_{AD} = 0.64F_{AC}$ $(3')$

$\Rightarrow \quad F_{AC} = 484.2(\text{N}) \quad F_{AD} = 309.9(\text{N})$ $(2')$

3. The A-36 steel shaft has a diameter of 60 mm and is subjected to the distributed and concentrated loadings as shown. The shaft is fixed at *A*. (a) Determine the absolute maximum shear stress in the shaft. (b) Determine the angle of twist at *B*. G_{st} = 75 GPa. (15')

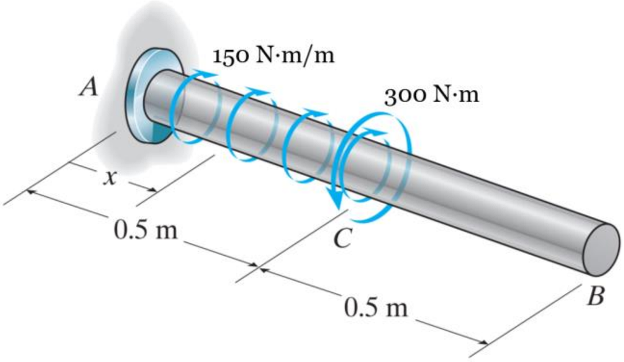


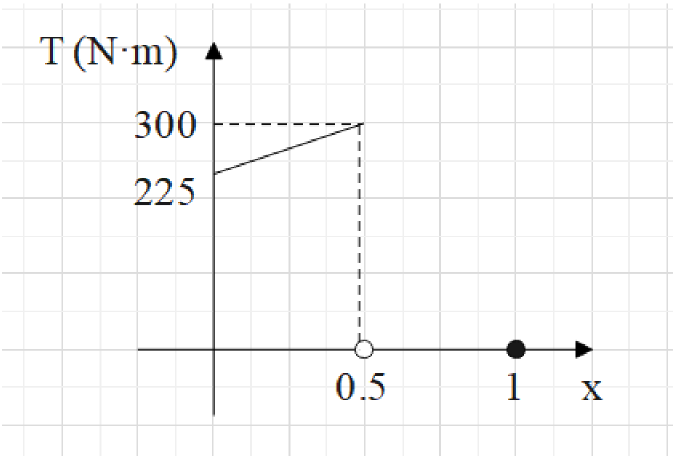
Figure 3

$$3.(a)\ \ T(x) = 225 + 150x,\ \ x \in [0\,,0.5]$$

$$T(x) = 0,\qquad\qquad x \in (0.5\,,1]$$

$$T(x)_{max} = 300(\mathrm{N}\cdot\mathrm{m}) \qquad (5')$$

$$\tau_{max} = \frac{T_{max}}{J}\cdot c = 7.07(\mathrm{MPa}) \qquad (3')$$



$$(b)\ \Phi_B = \Phi_c = \int \frac{T(x)}{G(x)J(x)}dx \qquad (3')$$

$$= \frac{1}{75\times10^9\cdot\frac{\pi}{2}\left(\frac{D}{2}\right)^4}\int_0^{0.5}(225+150x)dx \qquad (2')$$

$$= 0.00138(\mathrm{rad}) = 0.0788^\circ \qquad (2')$$

4. The beam *ABC* is fixed at *A* and subjected to a distributed load and a concentrated load. The beam has cross section as shown. (a) Draw the shear and moment diagram along the beam. (b) Determine the maximum bending stress at section *B*. (15')

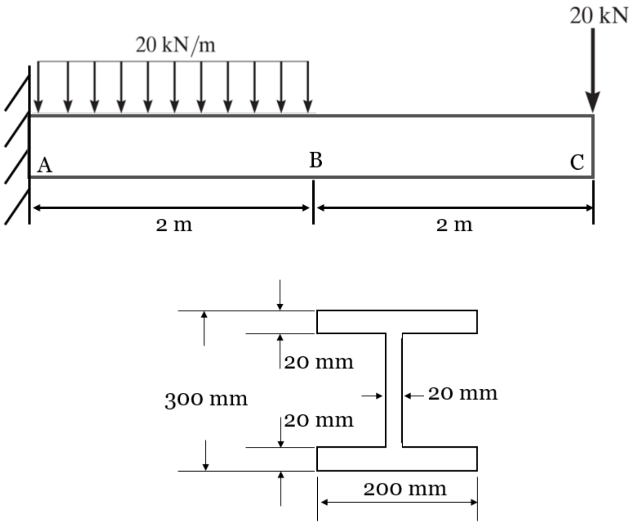
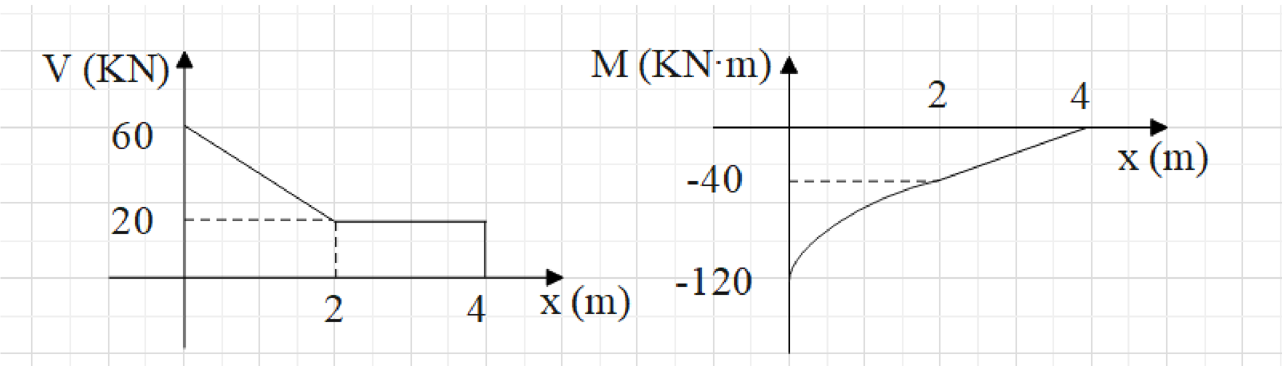


Figure 4

4. (a)



(8′)

$$(b) \quad At\ B \quad M = 40(\mathrm{kN \cdot m}) \qquad (1')$$

$$I_B = \sum(I_0 + Ad^2)$$

$$= \frac{1}{12} \times 20 \times 260^3 + 2 \times \left(\frac{1}{12} \times 200 \times 20^3 + 200 \times 20 \times 140^2\right)$$

$$= 1.8636 \times 10^{-4}(\mathrm{m}^4) \qquad (2')$$

$$\sigma_{max} = \frac{M}{I} \cdot y_{max} \qquad (2')$$

$$= \frac{40 \times 10^3}{1.8636 \times 10^{-4}} \times 0.15 = 32.2(\mathrm{MPa}) \qquad (2')$$

5. The cantilevered beam with rectangular cross section as shown is fixed at the left. A concentrated force P is applied at position x. Determine (a) the maximum bending stress; (b) the maximum shear stress; (c) the position x such that the maximum bending stress is equivalent to the maximum shear stress. (15')

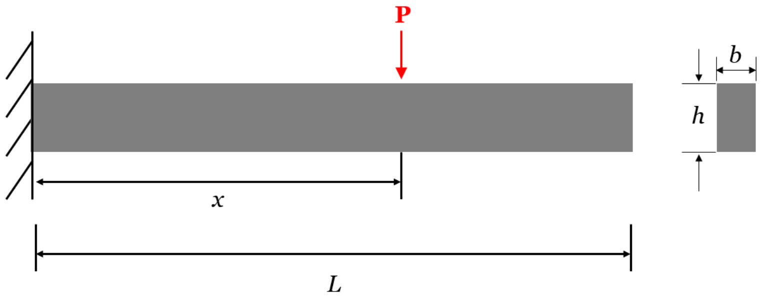


Figure 5

5. (a) $M_{max} = Px$ $(1')$ $V_{max} = P$ $(1')$

$$I = \frac{1}{12}bh^3 \qquad (1')$$

$$\sigma_{max} = \frac{M_{max}}{I} \cdot \left(\frac{h}{2}\right) = \frac{6px}{bh^2} \quad \text{at section } A - A \qquad (2')$$

(b) $\tau_{max} = \dfrac{V \cdot Q_{max}}{I \cdot b}$ $(2')$ $Q_{max} = \dfrac{1}{2}bh \cdot \dfrac{h}{4} = \dfrac{bh^2}{8}$ $(2')$

$$\Rightarrow \tau_{max} = \frac{3P}{2bh} \qquad (1')$$

(c) Let $\sigma_{max} = \tau_{max}$

$$\frac{6Px}{bh^2} = \frac{3P}{2bh} \quad (3') \qquad \Rightarrow x = \frac{h}{4} \quad (2')$$

6. The rectangular plate has a thickness of 20 mm (along the direction perpendicular to the paper) and the force P = 4 kN acts along the centerline of this thickness such that d = 150 mm. Plot the distribution of normal stress acting along section a–a. (15')

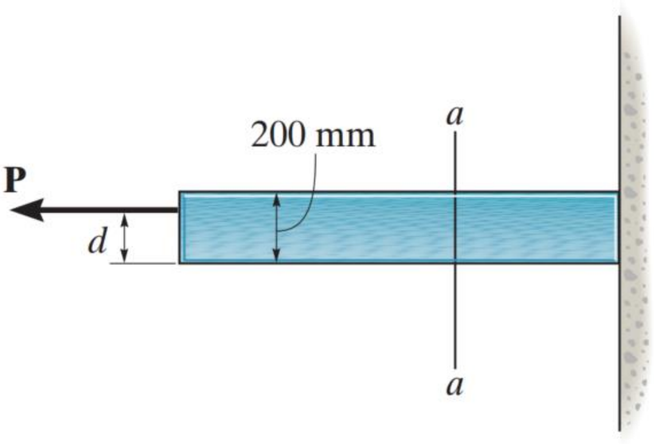
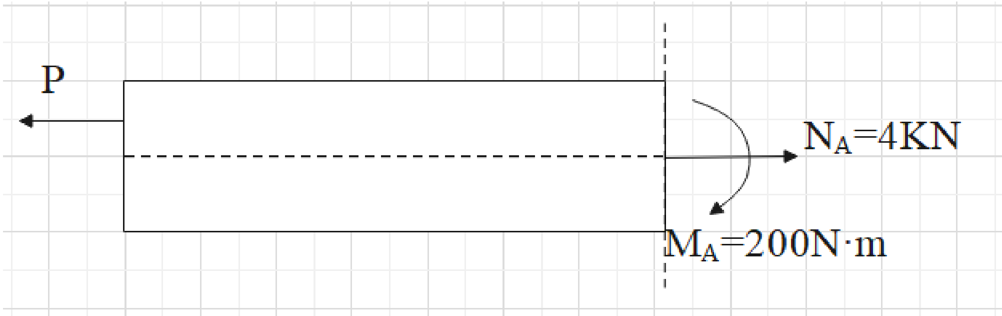


Figure 6

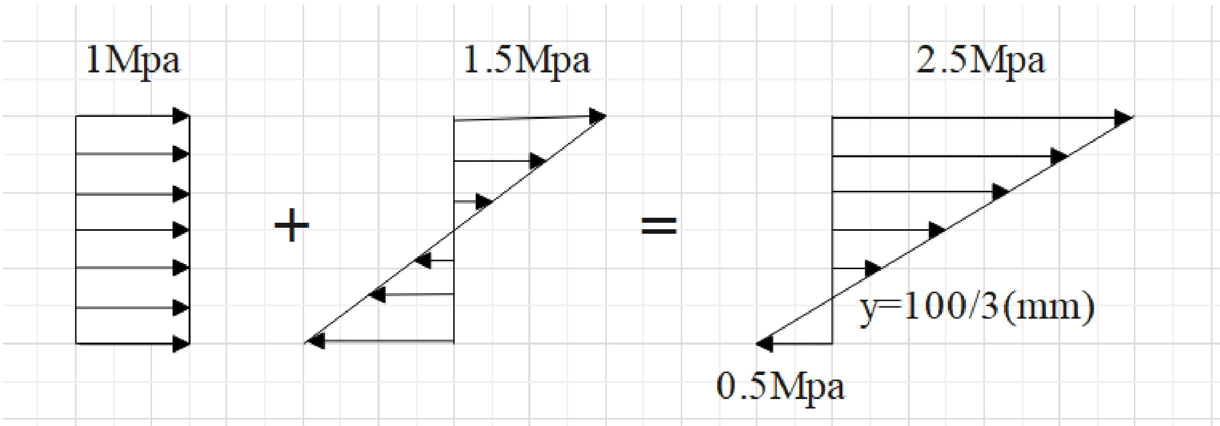


6. $M_A = P \times 0.05 = 200(\mathrm{N \cdot m})$

Normal stress $\sigma = \frac{N}{A} = 1(\mathrm{MPa})$ (5')

Bending stress $I = \frac{1}{12} \times 20 \times 10^{-3} \times 0.2^3 = 1.35 \times 10^{-5}(\mathrm{m}^4)$

$\sigma_{max} = \frac{M}{1} \times 0.1 = 1.5(\mathrm{MPa})$ (5')



(5')