Quiz 6

Date: 2022-03-21 Name: SID:

Q1. Consider the initial value problem $y' = 3y^{2/3}, y(2) = 27$.

(1) Give a solution; (2pts)

(2) Is your solution above unique on the region $R = \{(t, y); (t-1)^2 + (y-27)^2 \leq 100\}$? Please give your reason. (4pts)

Q2. Suppose that y is a solution to the initial value problem

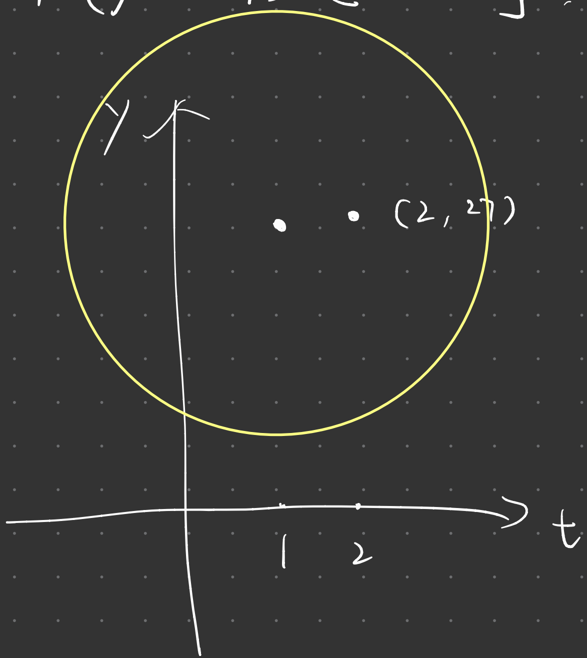
$y' = y^2 - cos^2 t - sint, y(0) = 2$. Show that $y(t) > cost$ for all t for which y is defined. (4pts)

1\. (1) $y' = 3y^{\frac{2}{3}}$ $\quad y(2) = 27$

general solution is $y = (t+C)^3 \Rightarrow C = 1$

solution is $y = (t+1)^3$

(2) $R = \{(t,y) \mid (t-1)^2 + (y-27)^2 \le 100\}$.



$f(t,y) = 3y^{\frac{2}{3}}$.

$\frac{\partial f}{\partial y} = 2y^{-\frac{1}{3}}$.

are contiontimous on R.

$\Rightarrow y = (t+1)^3$ is the unique solution on R.

2\. Notice that $y_1(t) = \cos t$ is a solution with initial value $y_1(0) = 1 < 2 = y(0)$.

$f = y^2 - \cos^2 t - \sin t$. $\quad \frac{\partial f}{\partial y} = 2y$

are both continuous on the whole plane.

Apply the uniqueness theorem, the solution y cannot intersect $y_1(t) = \cos t$.

Hence $y(t) > y_1(t) = \cos t$ for $\forall t$.