Aeroacoustics HW 2 12012127 邹佳驹

Q 2.1

Solution

$$\nabla^2 p' - \frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2} = 0$$

94'

rigid walls:

$\frac{\partial p'}{\partial x_1} = 0$ on $x_1 = 0, a$ ①

$\frac{\partial p'}{\partial x_2} = 0$ on $x_2 = 0, b$ ②

$\frac{\partial p'}{\partial x_3} = 0$ on $x_3 = 0, L$ ③

velocity vanishes ($u = 0$)

Assume one seperable solution

$$p'(\underline{x}, t) = f(x_1)\, g(x_2)\, h(x_3)\, e^{i\omega t}$$

$$\Rightarrow \frac{\ddot f}{f} = -\frac{\ddot g}{g} - \frac{\ddot h}{h} - \frac{\omega^2}{c^2} = \text{const} = -\alpha_1^2$$

x_1 only; x_2 and x_3 only

$\Rightarrow f(x_1) = A_1\cos(\alpha_1 x_1) + B_1\sin(\alpha_1 x_1)$

BC ① $\Rightarrow$ $B_1 = 0$, $\sin\alpha_1 a = 0$

$\alpha_1 = \frac{m\pi}{a}$ m : integer

similarly:

$\Rightarrow g(x_2) = A_2\cos(\alpha_2 x_2) + B_2\sin(\alpha_2 x_2)$

BC ② $\Rightarrow$ $B_2 = 0$, $\sin\alpha_2 x_2 = 0$

$\alpha_2 = \frac{n\pi}{b}$ n: integer

$\Rightarrow$ combine $f(x_1)$ and $g(x_2)$

$$\frac{d^2 h(x_3)}{dx_3^2} + \left[\frac{\omega^2}{c^2} - \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)\right] h(x_3) = 0$$

$$\Rightarrow h(x_3) = A_{mn} e^{-ik_{mn}x_3} + B_{mn} e^{ik_{mn}x_3}$$

BC ③ $\Rightarrow \begin{cases} A_{mn}(-ik_{mn}) + B_{mn}(ik_{mn}) = 0 \\ A_{mn} e^{-ik_{mn}L}\cdot(-ik_{mn}) + B_{mn} e^{ik_{mn}L}(ik_{mn}) = 0 \end{cases}$

$\Rightarrow \begin{cases} A_{mn} = B_{mn} \\ e^{-ik_{mn}L} = e^{ik_{mn}L} \Rightarrow \sin k_{mn}L = 0 \end{cases}$

$k_{mn} = \frac{q\pi}{L}$, q: integer

$$\Rightarrow p'(\underline{x}, t) = \sum_m \sum_n \sum_q A_{mnq} \cos\frac{m\pi x_1}{a} \cos\frac{n\pi x_2}{b} \cos\frac{q\pi x_3}{L}\, e^{i\omega t}$$

where

$$k_{mn} = \frac{q\pi}{L} = \left[\frac{\omega^2}{c^2} - \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)\right]^{1/2}$$

$$\Rightarrow \omega = c\pi\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{q}{L}\right)^2\right]^{1/2}$$

three lowest resonant freqs

$L = 1m$ $c = 340 m/s$

$a = 0.3m$

$b = 0.2m$

(m, n, q)	ω [rad/s]
1, 0, 0	$340\pi \cdot \frac{1}{0.3} = 3560.47$
0, 1, 0	$340\pi \cdot \frac{1}{0.2} = 5340.71$
0, 0, 1	$340\pi \cdot 1 = 1068.14$

$$\omega = 340\pi \cdot \left(\frac{m^2}{0.3^2} + \frac{n^2}{0.2^2} + \frac{q^2}{1^2}\right)^{1/2}$$

min

$(0, 0, 1) \Rightarrow 1068.14$ rad/s

$(0, 0, 2) \Rightarrow 2136.28$ rad/s

$(0, 0, 3) \Rightarrow 3204.42$ rad/s

6'

Q2.2

(i) Sound propagating through the perforated sheet

$$p'_i(x,t) = I e^{i\omega(t - x/c)}$$

$$p'_r(x,t) = R e^{i\omega(t+x/c)}$$

$$p'_t(x,t) = T e^{i\omega(t-x/c)}$$

Across the sheet hole: $p'\downarrow$: $0.1\rho_0 c u_h$

the pressure on the left :

$$p_1' = (R+I)e^{i\omega t}$$

the pressure on the right:

$$p_2' = T e^{i\omega t}$$

$$\Rightarrow \quad R+I-T = 0.1\rho_0 c u_h = 0.1\rho_0 c \frac{u_n}{\alpha} \quad ①$$

Particle velocity on both sides :

$$u_n = \frac{(I-R)}{\rho_0 c} = \boxed{\frac{T}{\rho_0 c}} \quad ②$$

?

?

from ① + ② : $R = 0.05\rho_0 c u_h$

$$T = I - 0.05\rho_0 c u_h$$

$$u_n = \frac{I - 0.05\rho_0 c u_h}{\rho_0 c}$$

$$\therefore Z = \frac{p_s'}{u_n} = \frac{(R+I)\cdot \rho_0 c}{I - 0.05\rho_0 c u_h}$$

$$= \frac{(0.05\rho_0 c u_h + I)\cdot \rho_0 c}{I - 0.05\rho_0 c u_h}$$

×

page 3→4.

(ii) $T = 600\,K$

$$c = \sqrt{\gamma R T} = \sqrt{1.4\times 287\times 600} = 491\ m/s$$

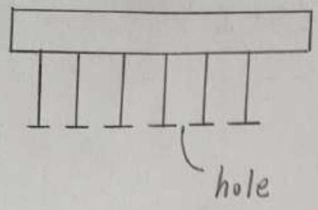
$$f = 1450\ Hz, \quad \omega = 2\pi f$$

$$\Rightarrow \lambda = \frac{2\pi c}{\omega} = \frac{c}{f} = 0.339\ m$$

determine the honeycomb depth ?

$$\frac{1}{4}\lambda.$$

process is in the next page



$x = d$

$x = 0$ surface

plane acoustic waves propagating in the honeycomb

$$\left.\begin{aligned} p_i'(x,t) &= I e^{iw(t-x/c)} \\ p_r'(x,t) &= R e^{iw(t+x/c)} \end{aligned}\right\} \quad \begin{aligned} &\text{Surface pressure } (x=0) \\ &I e^{iwt} + R e^{iwt} \end{aligned}$$

across the hole

$$\left.\begin{aligned} p_A'(x,t) &= A e^{iw(t-x/c)} \\ p_B'(x,t) &= B e^{iw(t+x/c)} \end{aligned}\right\} \quad \begin{aligned} &\text{Surface pressure } (x=0) \\ &A e^{iwt} + B e^{iwt} \end{aligned}$$

pressure loss:

$$(I+R)e^{iwt} = (A+B)e^{iwt} + 0.1\rho_0 c u_h \quad ①$$

continuity of velocity at the surface $(x=0)$

$p' = \rho_0 c u$ for $+x$ wave

$= -\rho_0 c u$ for $-x$ wave

$$\left(\frac{1}{\rho_0 c} I e^{iwt} - \frac{1}{\rho_0 c} R e^{iwt}\right) = \left(\frac{1}{\rho_0 c} A e^{iwt} - \frac{1}{\rho_0 c} B e^{iwt}\right) \cdot \alpha \Leftarrow (u_n = u_h \cdot \alpha)$$

$$I - R = (A-B)\alpha \quad ②$$

At the end of the liner $(x=d)$ (velocity=0)

$$\frac{1}{\rho_0 c} A e^{iw(t-d/c)} - \frac{1}{\rho_0 c} B e^{iw(t+d/c)} = 0$$

$$A e^{iw(t-d/c)} - B e^{iw(t+d/c)} = 0$$

$$A e^{iw(\frac{-d}{c})} - B e^{iw\frac{d}{c}} = 0 \qquad \frac{w}{c} = k$$

$$A e^{-ikd} - B e^{ikd} = 0$$

$$A = B e^{2ikd} \quad ③$$

$$p_s' = e^{i\omega t}(I+R)$$
$$u_n = u_h \cdot \alpha$$
$\Rightarrow$ surface impedance $Z = \frac{p_s'}{u_n}$

$$= \frac{e^{i\omega t}(I+R)}{\alpha \cdot u_h}$$
$$= \frac{I+R}{\alpha \cdot V_h}$$

$$u_n = \frac{1}{\rho_0 c}(I-R)e^{i\omega t}$$
$$u_h = \underline{\frac{1}{\rho_0 c}(A-B)}e^{i\omega t} = \underset{\text{magnitude}}{\underline{V_h}} \cdot e^{i\omega t}$$

$$= \frac{I+R}{\alpha \cdot \frac{1}{\rho_0 c}(A-B)}$$
$$= \frac{\rho_0 c(I+R)}{\alpha(A-B)}$$

16'

from ①: $I+R = A+B+0.1\rho_0 c V_h$
$$= A+B+0.1\rho_0 c \cdot \frac{1}{\rho_0 c}(A-B)$$
$$= A+B+0.1(A-B)$$
where $A = Be^{2ikd}$ ③

$$= \frac{\rho_0 c}{\alpha} \cdot \frac{A+B+0.1(A-B)}{A-B}$$
$$= \frac{\rho_0 c}{\alpha} \cdot \left(0.1 + \frac{e^{2ikd}+1}{e^{2ikd}-1}\right)$$
$$= \frac{\rho_0 c}{\alpha}\left[0.1 - i\cot(kd)\right]$$

(ii) considering the absorption coefficient

the liner is backed by a rigid wall, no T

$\therefore \alpha = 1 - \left|\frac{R}{I}\right|^2$ max.

$$\left|\frac{R}{I}\right| = \left|\frac{Z-\rho_0 c}{Z+\rho_0 c}\right| = \left|1 - \frac{2\rho_0 c}{Z+\rho_0 c}\right| = \left|1 - \frac{2}{\frac{1}{\alpha}(0.1 - i\cot kd)+1}\right|$$

to make $\left|\frac{R}{I}\right|_{min}$, $\cot kd = 0$

$$kd = n\pi + \frac{\pi}{2} \quad n = 0, 1, \cdots$$
$$d = \frac{n}{k}\pi + \frac{\pi}{2k}$$
$$k = \frac{w}{c} = \frac{2\pi f}{c} = \frac{2\pi f}{\sqrt{\gamma RT}} = \frac{2\pi \times 1450}{\sqrt{1.4 \times 287 \times 600}}$$
$$= 18.56$$

$n = 0$.
$$d = \frac{\pi}{2k} = \frac{\pi \cdot c}{2 \cdot 2\pi f} = \frac{c}{4f} = \frac{\lambda}{4} = 0.0847 \text{ m}$$

Q2.3

Solution

(i) proof

incident : $p_i'(x,y,t) = I e^{iw(t - x\cos\theta/c - y\sin\theta/c)}$

reflected : $p_r'(x,y,t) = R e^{iw(t + x\cos\theta'/c - y\sin\theta'/c)}$

At $x = 0$,

surface pressure : $p'(0,y,t) = I\, e^{iw(t - y\sin\theta/c)} + R e^{iw(t - y\sin\theta'/c)}$

velocity continuity:

$$u(0,y,t) = \frac{1}{\rho_0 c}\left[I\cos\theta\, e^{iw(t - y\sin\theta/c)} - R\cos\theta' e^{iw(t - y\sin\theta'/c)}\right]$$

$\Rightarrow$

$$I e^{iw(t - y\sin\theta/c)} + R e^{iw(t - y\sin\theta'/c)} = Z\cdot\frac{1}{\rho_0 c}\left[I\cos\theta\, e^{iw(t - y\sin\theta/c)} - R\cos\theta' e^{iw(t - y\sin\theta'/c)}\right]$$

① apply along all the wall (for all y)

② Z is independent of y

$\Rightarrow \theta = \theta'$

$$\Rightarrow \frac{R}{I} = \frac{\cos\theta Z - \rho_0 c}{\cos\theta Z + \rho_0 c}$$

7'

(ii) Z: purely real and positive constant ($\neq 0$)

$0 \le \theta \le \pi/2$, $0 \le \cos\theta \le 1$

$$\left|\frac{R}{I}\right| = \left|\frac{Z\cos\theta - \rho_0 c}{Z\cos\theta + \rho_0 c}\right| = \left|\frac{\frac{Z\cos\theta}{\rho_0 c} - 1}{\frac{Z\cos\theta}{\rho_0 c} + 1}\right| \quad (*)$$

set $\frac{Z\cos\theta}{\rho_0 c} = r$, $y(r) = \left|\frac{r-1}{r+1}\right|$

$r \in [0, \frac{Z}{\rho_0 c}]$.

① if $Z/\rho_0 c < 1$, $\left|\frac{R}{I}\right|_{min} = \frac{-Z + \rho_0 c}{Z + \rho_0 c}$

② if $Z/\rho_0 c > 1$, $\left|\frac{R}{I}\right|_{min} = 0$

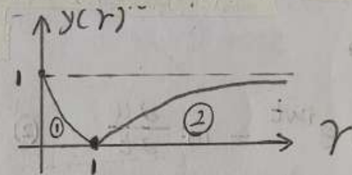
③ if $\cos\theta = 0$, $\theta = \pi/2$

$\Rightarrow \left|\frac{R}{I}\right|_{max} = 1$

8'

or : heavy or rigid surface

Z is very large $\Rightarrow \left|\frac{R}{I}\right| \simeq 1$



$\left|\frac{r-1}{r+1}\right|$ $\quad r = \frac{Z\cos\theta}{\rho_0 c}$

(iii) Z is purely imaginary (Ai)

$$\left|\frac{R}{I}\right| = \left|\frac{A\cos\theta\cdot i - \rho_0 c}{A\cos\theta\cdot i + \rho_0 c}\right|$$

$$= \frac{|A\cos\theta\cdot i - \rho_0 c|}{|A\cos\theta\cdot i + \rho_0 c|} = 1$$

2'

Q 2.4

Solution

(i)

in $x<0$,

$$p_1'(x,t) = I e^{iw(t-x/c_1)} + R e^{iw(t+x/c_1)}$$

in $x>0$,

$$p_t'(x,t) = T e^{iw(t-x/c_2)}$$

Particle velocity

on the left wall: $(I-R)\dfrac{e^{iwt}}{\rho_1 c_1}$

right : $T\dfrac{e^{iwt}}{\rho_2 c_2}$

same

$$\Rightarrow u = (I-R)\frac{e^{iwt}}{\rho_1 c_1} = T\frac{e^{iwt}}{\rho_2 c_2} \quad ①$$

Wall motion equation

$$(I+R)e^{iwt} - Te^{iwt} = m\frac{\partial u}{\partial t} \quad ②$$

$$\frac{\partial u}{\partial t} = \frac{I-R}{\rho_1 c_1} e^{iwt}\cdot iw = \frac{T}{\rho_2 c_2} e^{iwt}\cdot iw$$

$$\Rightarrow I+R-T = m\frac{I-R}{\rho_1 c_1} iw = m\cdot\frac{T}{\rho_2 c_2} iw$$

$$\Rightarrow \frac{R}{I} = \frac{miw+\rho_2 c_2-\rho_1 c_1}{\rho_1 c_1+miw+\rho_2 c_2}$$

$$\left|\frac{R}{I}\right| = \sqrt{\frac{(\rho_2 c_2-\rho_1 c_1)^2+(mw)^2}{(\rho_2 c_2+\rho_1 c_1)^2+(mw)^2}} \quad \boxed{///}$$

$$R = \frac{\rho_2 c_2-\rho_1 c_1+miw}{\rho_2 c_2+\rho_1 c_1+miw} I \quad 12'$$

$$T = \frac{2\rho_2 c_2}{\rho_2 c_2+\rho_1 c_1+miw} I$$

(ii) two dimensionless numbers

①

Energy transmission coefficient

$$\left|\frac{T}{I}\right| = \frac{4\rho_2^2 c_2^2}{(\rho_2 c_2+\rho_1 c_1)^2+(mw)^2}$$

$$\left|\frac{T}{I}\right| \to 0, \text{ then } \left|\frac{R}{I}\right| \to 1$$

$$\to 1 \qquad\qquad \to 0$$

physical significance: energy transmitted through the wall over the energy of the incident beam.

② Absorption coefficient

$$\alpha = 1 - \left|\frac{R}{I}\right|^2 - \left|\frac{T}{I}\right|^2 \quad ?$$

physical significance: the proportion of the incident energy dissipated by the wall (absorber).

…… 13' 镜讨论

$$\begin{cases} Z_1 = \dfrac{\rho_2 c_2}{\rho_1 c_1} \\[2ex] Z_2 = \dfrac{mw}{\rho_1 c_1} \propto \dfrac{m}{\rho_1 \lambda} \end{cases}$$

Q2.5

Solution

(i) $\dfrac{\sin\beta}{c(x)} = \text{constant} \qquad (5.1)$

ray path $y = y(x)$

$$y' = \frac{dy}{dx} = \tan\beta$$

$$\Rightarrow \frac{y'}{c(x)\cdot(1+y'^2)^{1/2}} = \text{constant}$$

$$\frac{y'}{(1+y'^2)^{1/2}} \cdot \frac{1}{C_0 e^{\alpha x}} = \text{constant}$$

$$= \frac{\sin\beta}{C_0} \quad (BC)$$

>0

$$\Rightarrow y'^2 = \frac{\sin^2\beta\, e^{2\alpha x}}{1-\sin^2\beta\, e^{2\alpha x}}$$

$$y' = \pm \frac{\sin\beta\, e^{\alpha x}}{\sqrt{1-\sin^2\beta\, e^{2\alpha x}}}$$

$$y = \pm \sin\beta \int \frac{e^{\alpha x}}{\sqrt{1-\sin^2\beta\, e^{2\alpha x}}}\,dx$$

$$= \pm \sin\beta \cdot \left[\frac{\sin^{-1}(\sin\beta\, e^{\alpha x})}{\alpha \sin\beta} + C\right]$$

$$= \pm \frac{\sin^{-1}(\sin\beta\, e^{\alpha x})}{\alpha} \pm C\cdot\sin\beta.$$

At $x=0,\ y=0$ (origin)

$$\Rightarrow C = \frac{-\beta}{\alpha \sin\beta}$$

$$\therefore y = \pm \frac{\sin^{-1}(\sin\beta\, e^{\alpha x})}{\alpha} \pm \frac{-\beta}{\alpha}$$

$$= \pm\left[\frac{\sin^{-1}(\sin\beta\, e^{\alpha x}) - \beta}{\alpha}\right]$$

8'

(ii)

ray passes through the origin with angle $\beta > 0$ to the positive x axis

$\Rightarrow 1 > \sin\beta > 0$

① if $\alpha > 0$, $e^{\alpha x} > 1$ for $x > 0$

Let $\sin\beta\, e^{\alpha x} = 1 \Rightarrow x = \frac{1}{\alpha}\underbrace{\ln\frac{1}{\sin\beta}}_{>0} > 0.$

where $\sin\beta < 1$

$$y = \pm\left(\frac{\frac{\pi}{2}-\beta}{\alpha}\right)$$

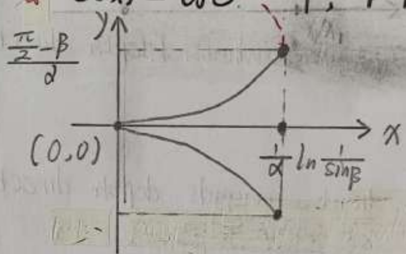
which are the end points.

Due to $\dfrac{\sin\beta}{c(x)} = \text{constant}$

$x \to \infty$

$y \to \frac{\pi-\beta}{\alpha}$

$c(x) = C_0 e^{\alpha x} \uparrow$, $\beta \uparrow$, bent backward



8'

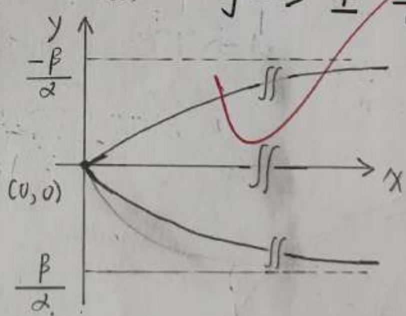
② if $\alpha < 0$, $e^{\alpha x} < 1$ for $x > 0$

similarly, $\sin\beta\, e^{\alpha x} \to 0$ as $x \to +\infty$

resulting $\sin^{-1}(\sin\beta\, e^{\alpha x}) \to 0$

$$y \to \pm\frac{-\beta}{\alpha}$$

$\beta \div 0$ 题.



Q 2.6

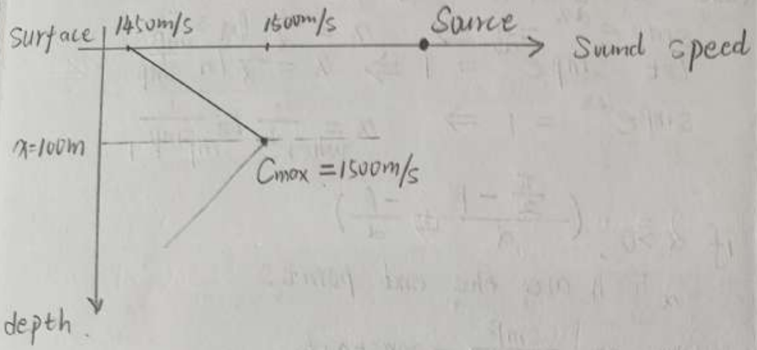
21

Solution

(i) from sea surface to depth of $x = 100\,m$

$$C(x) = 1450 + \frac{1}{2}x \qquad m/s$$

when $x > 100\,m$, $C(x)\downarrow$

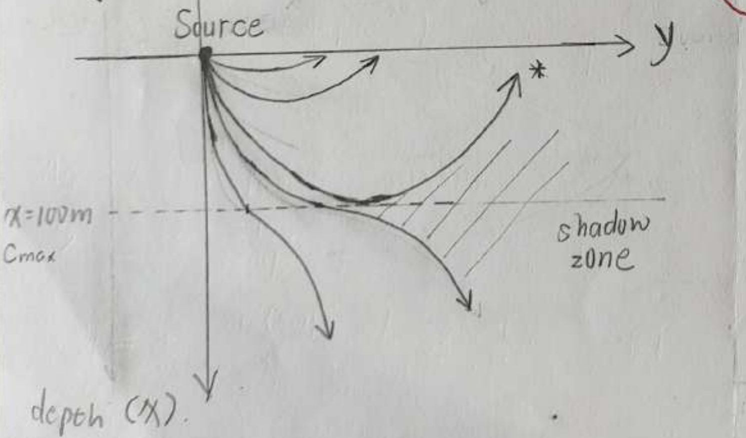


for a sound source on the surface

- Ray $\downarrow \rightarrow C\uparrow \rightarrow$ refracted back
  向下传
- Ray $\uparrow \rightarrow C\downarrow \rightarrow$ bent towards depth direction
  向上传

Across the level of $x = 100\,m$:

- Ray $\downarrow \rightarrow C\downarrow \rightarrow$ bent towards depth direction
- Ray $\uparrow \rightarrow C\uparrow \rightarrow$ refracted back (no exist)



(ii) consider the ray with * notation

$$\frac{\sin\theta_0}{C(0)} = \text{constant} = \frac{\sin\frac{\pi}{2}}{C_{max}}$$

$$\Rightarrow \sin\theta_0 = \frac{C(0)}{C_{max}} = \frac{1450}{1500}$$

$$\theta_0 = 75.16^\circ$$

(iii) from Snell's law

$$\frac{y'}{C(x)(1+y'^2)^{1/2}} = \text{constant} = \frac{\sin\theta_0}{\beta} \qquad \beta = C(0)$$

$$C(x) = \alpha x + \beta \quad \text{with } \alpha = 1/2,\ \beta = 1450$$

$$\Rightarrow y'^2 = \frac{\sin^2\theta_0(\alpha x+\beta)^2}{\beta^2 - \sin^2\theta_0(\alpha x+\beta)^2}$$

$$y' = \pm \frac{\sin\theta_0(\alpha x+\beta)}{\sqrt{\beta^2 - \sin^2\theta_0(\alpha x+\beta)^2}}$$

$$y = \mp \frac{1}{\alpha\sin\theta_0}\sqrt{\beta^2 - \sin^2\theta_0(\alpha x+\beta)^2} + C$$

At $x = 0$, $y = 0$ (source)

$$\Rightarrow C = \pm\frac{\beta\cos\theta_0}{\alpha\sin\theta_0}$$

$$\therefore y = \mp\frac{1}{\alpha\sin\theta_0}\sqrt{\beta^2 - \sin^2\theta_0(\alpha x+\beta)^2} \pm \frac{\beta}{\alpha\tan\theta_0}$$

$$\Rightarrow \left(x+\frac{\beta}{\alpha}\right)^2 + \left(y \mp \frac{\beta}{\alpha\tan\theta_0}\right)^2 = \frac{\beta^2}{\alpha^2\sin^2\theta_0}$$

we choose "−" in this case

At $x = 0$

$$\frac{\beta^2}{\alpha^2} + \left(y - \frac{\beta}{\alpha\tan\theta_0}\right)^2 = \frac{\beta^2}{\alpha^2\sin^2\theta_0}$$

$$\left(y - \frac{\beta}{\alpha\tan\theta_0}\right)^2 = \frac{\beta^2}{\alpha^2}\cdot\frac{1}{\tan^2\theta_0}$$

$$\Rightarrow y = 0 \quad \text{or} \quad y = \frac{2\beta}{\alpha\tan\theta_0}$$

$$= \frac{2\times 1450}{1/2\cdot\tan 75.16^\circ}$$

$$= 1536.76\ m$$