《Fundamentals of Electric Circuits》 homework CH.15

15.9 Determine the I aplace transforms of these functions:

(a) $f(t)= (t - 4)u(t - 2)$

(b) $g(t) = 2e^{-4t}u(t - 1)$

(c) $h(t) = 5\cos(2t - 1)u(t)$

(d) $p(t)=6[u(t - 2) - u(t - 4)]$ (20')

解: a) $\mathcal{L}\{(t-4)u(t-2)\} = \mathcal{L}\{u(t-2)(t-2)\} - 2\mathcal{L}\{u(t-2)\} = e^{-2s}\frac{1}{s^2} - 2\cdot\frac{e^{-2s}}{s} = \frac{e^{-2s}}{s}(\frac{1}{s}-2)$ $\boxed{ANS}$

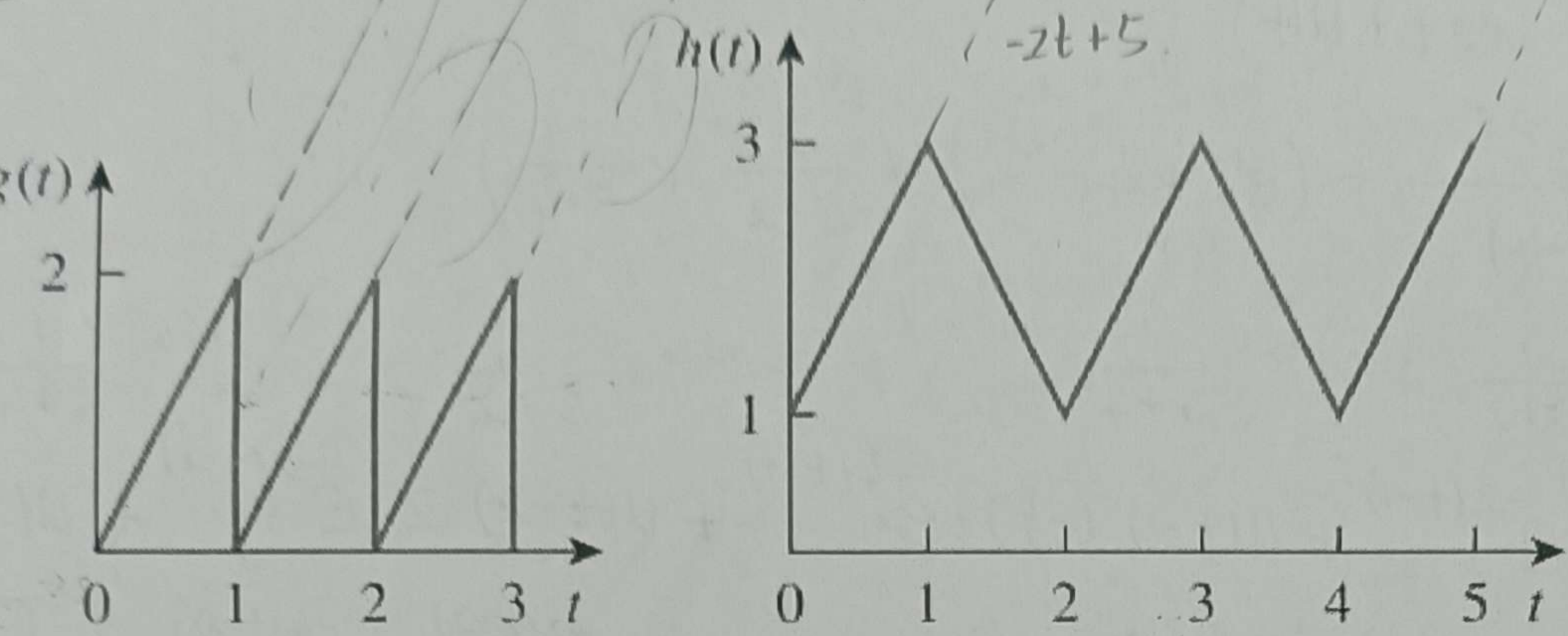
b) $\mathcal{L}\{2e^{-4t}u(t-1)\} = 2\mathcal{L}\{e^{-4t}u(t-1)\} = 2\cdot\frac{e^{-(s+4)}}{s+4}$ $\boxed{ANS}$

c) $\mathcal{L}\{5\cos(2t-1)u(t)\} = 5\cdot\frac{s\cos 1 - 2\sin 1}{s^2+4}$ $\boxed{ANS}$

d) $\mathcal{L}\{p(t)\} = 6\mathcal{L}\{u(t-2)\} - 6\mathcal{L}\{u(t-4)\} = 6\cdot(\frac{e^{-2s}}{s} - \frac{e^{-4s}}{s}) = \frac{6e^{-2s}}{s}(1-e^{-2s})$ $\boxed{ANS}$

15.22 Find the Laplace transforms of the functions in Fig. 15.34. (20')

$2t+1$ $\quad -2t+5$



(a)

Figure 15.34

15.22 解:

a) $g_1(t) = 2t[u(t)-u(t-1)]$
$= 2[tu(t) - tu(t-1)]$
$= 2[tu(t) - (t-1)u(t-1) - u(t-1)]$

$G_1(s) = 2(\frac{1}{s^2} - e^{-s}\frac{1}{s^2} - \frac{e^{-s}}{s})$
$= 2\cdot\frac{1-e^{-s}-se^{-s}}{s^2}$

$G(s) = \frac{G_1(s)}{1-e^{-s}} = \frac{2(1-e^{-s}-se^{-s})}{s^2(1-e^{-s})}$ $\boxed{ANS}$

b) $h_1(t) = (2t+1)[u(t)-u(t-1)] + (5-2t)[u(t-1)-u(t-2)]$
$= 2t\cdot u(t) + (4-4t)u(t-1) + (2t-5)u(t-2) + u(t)$
$= 2t\cdot u(t) - 4(t-1)u(t-1) + 2(t-2)u(t-2) - u(t-2) + u(t)$

$H_1(s) = \frac{2}{s^2} + \frac{1}{s} - 4\cdot e^{-s}\cdot\frac{1}{s^2} + 2\cdot e^{-2s}\cdot\frac{1}{s^2} - \frac{e^{-2s}}{s}$
$= \frac{2+s-4e^{-s}+2e^{-2s}-se^{-2s}}{s^2}$

$H(s) = \frac{H_1(s)}{1-e^{-2s}} = \frac{2+s-4e^{-s}+2e^{-2s}-se^{-2s}}{s^2(1-e^{-2s})}$ $\boxed{ANS}$

15.25 Let

$$F(s) = \frac{5(s+1)}{(s+2)(s+3)}$$

(a) Use the initial and final value theorems to find $f(0)$ and $f(\infty)$.

(b) Verify your answer in part (a) by finding $f(t)$, using partial fractions.

15.25 解: a) $f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\frac{5s(s+1)}{(s+2)(s+3)} = 5$

$(s+2)(s+3) = 0$, $s_1 = -2$, $s_2 = -3$

$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0}\frac{5s(s+1)}{(s+2)(s+3)} = 0$

b) $F(s) = \frac{A}{s+2} + \frac{B}{s+3} = \frac{5s+5}{(s+2)(s+3)}$

$As+3A+Bs+2B = 5s+5$

$\begin{cases} A+B=5 \\ 3A+2B=5 \end{cases} \Rightarrow \begin{cases} A=-5 \\ B=10 \end{cases}$, $F(s) = \frac{-5}{s+2} + \frac{10}{s+3}$

$f(t) = -5\cdot e^{-2t} + 10e^{-3t}$, $f(0) = 10$, $f(\infty) = 0$

15.31 Find $f(t)$ for each $F(s)$:

(a) $\frac{10s}{(s+1)(s+2)(s+3)}$

(b) $\frac{2s^2+4s+1}{(s+1)(s+2)^3}$

(c) $\frac{s+1}{(s+2)(s^2+2s+5)}$ (20')

$(s+2)(Bs+c)$
$Bs^2+Cs+2Bs+2C$

15.34 Find the time functions that have the following Laplace transforms:

15.31 解: a) $F(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3} = \frac{10s}{(s+1)(s+2)(s+3)}$

$s=-1$, $A+0+0 = \frac{-10}{1\times2} = -5$

$s=-2$, $0+B+0 = \frac{-20}{-1\times1} = 20$

$s=-3$, $0+0+C = \frac{-30}{-2\times(-1)} = -15$

$F(s) = \frac{-5}{s+1} + \frac{20}{s+2} + \frac{-15}{s+3}$

$f(t) = (-5e^{-t} + 20e^{-2t} + (-15)e^{-3t})u(t)$ $\boxed{ANS}$

b) $F(s) = \frac{A}{s+1} + \frac{B}{(s+2)^3} + \frac{C}{(s+2)^2} + \frac{D}{(s+2)} = \frac{2s^2+4s+1}{(s+1)(s+2)^3}$

$s=-1$, $A+0+0+0 = \frac{2-4+1}{+1} = -1$

$s=-2$, $0+B+0+0 = \frac{1}{-1} = -1$

$A(s+2)^3 + B(s+1) + C(s+2)(s+1) + D(s+2)^2(s+1) = 2s^2+4s+1$

$D=+1$

$F(s) = \frac{-1}{s+1} + \frac{-1}{(s+2)^3} + \frac{3}{(s+2)^2} + \frac{-1}{(s+2)}$

$f(t) = (-e^{-t} - e^{-2t}\cdot\frac{1}{2}t^2 + 3e^{-2t}\cdot t - e^{-2t})u(t)$ $\boxed{ANS}$

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c) $F(s) = \frac{A}{(s+2)} + \frac{Bs+C}{s^2+2s+5}$

$As^2+2As+5A+Bs^2+(C+2B)s+2C = s+1$

$\begin{cases} A+B=0 \\ 2A+C+2B=1 \\ 5A+2C=1 \end{cases} \Rightarrow \begin{cases} A=-\frac{1}{5} \\ B=\frac{1}{5} \\ C=1 \end{cases}$

$F(s) = \frac{-\frac{1}{5}}{s+2} + \frac{\frac{1}{5}(s+1)}{(s+1)^2+2^2} + \frac{2\times\frac{2}{5}}{(s+1)^2+2^2}$

$f(t) = (-\frac{1}{5}e^{-2t} + \frac{1}{5}e^{-t}\cos 2t + \frac{2}{5}e^{-t}\sin 2t)u(t)$ $\boxed{ANS}$

(a) $F(s) = 10 + \frac{s^2+1}{s^2+4}$

(b) $G(s) = \frac{e^{-s} + 4e^{-2s}}{s^2 + 6s + 8}$

(c) $H(s) = \frac{(s+1)e^{-2s}}{s(s+3)(s+4)}$

(20')

解: a) $F(s) = 10 + \frac{s^2+4-3}{s^2+4} = 10+1-3\cdot\frac{1}{s^2+4} = 11 - \frac{2}{s^2+4}\cdot\frac{3}{2}$

$f(t) = 11\,\delta(t) - \left(\frac{3}{2}\sin 2t\right)\cdot u(t)$

b) $G(s) = \frac{e^{-s}+4e^{-2s}}{(s+2)(s+4)} = (e^{-s}+4e^{-2s})\cdot\left(\frac{\frac{1}{2}}{s+2} + \frac{-\frac{1}{2}}{s+4}\right)$

$= e^{-s}\cdot\frac{1}{2}\cdot\frac{1}{s+2} + e^{-s}\cdot\frac{1}{s+4}\cdot(-\frac{1}{2}) + e^{-2s}\cdot\frac{1}{s+2}\cdot 2 + e^{-2s}\cdot\frac{1}{s+4}(-2)$

$g(t) = u(t-1)\cdot\frac{1}{2}\cdot e^{-2(t-1)} + u(t-1)\cdot(-\frac{1}{2})\cdot e^{-4(t-1)} + u(t-2)\cdot 2\cdot e^{-2(t-2)} + u(t-2)\cdot(-2)\cdot e^{-4(t-2)}$

$= u(t-1)\cdot\frac{1}{2}\left(e^{-2(t-1)} - e^{-4(t-1)}\right) + u(t-2)\cdot 2\cdot\left(e^{-2(t-2)} - e^{-4(t-2)}\right)$ $\boxed{\text{ANS}}$

c) $H(s) = e^{-2s}\cdot\left(\frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4}\right) = e^{-2s}\cdot\frac{s+1}{s(s+3)(s+4)}$

$A+0+0 = \frac{1}{12}$

$B+0+0 = \frac{-2}{-3\times 1} = \frac{2}{3}$

$C+0+0 = \frac{-3}{-4\times(-1)} = -\frac{3}{4}$

$H(s) = e^{-2s}\left(\frac{1}{12}\cdot\frac{1}{s} + \frac{2}{3}\cdot\frac{1}{s+3} - \frac{3}{4}\cdot\frac{1}{s+4}\right)$

$h(t) = u(t-2)\left(\frac{1}{12}\cdot 1 + \frac{2}{3}e^{-3(t-2)} - \frac{3}{4}e^{-4(t-2)}\right)$ $\boxed{\text{ANS}}$