

Quiz 4

Date: 2022-03-07

Name:

SID:

Determine whether each of the following equations is exact. If it is exact, find the solution. If it is not exact, find an integrating factor and solve it. (10 pts)

1. $(6x^2 - 2xy + 4) + (6y^2 - x^2 + 2)y' = 0;$ ✓

2. $y' = e^{3x} + y - 1.$ ✗

1. $\frac{\partial P}{\partial y} = -2x = \frac{\partial Q}{\partial x}$

$$\frac{\partial F}{\partial x} = P = 6x^2 - 2xy + 4$$

$$F = 2x^3 - x^2y + 4x + \phi(y)$$

$$\frac{\partial F}{\partial y} = -x^2 + \phi'(y) = 6y^2 - x^2 + 2$$

$$\phi'(y) = 6y^2 + 2$$

$$\phi(y) = 2y^3 + 2y$$

Solution is $F(x, y) = C$

$$2x^3 - x^2y + 4x + 2y^3 + 2y = C$$

2. $(e^{3x} + y - 1)dx - dy = 0$

$$h(x) = \frac{1}{Q} \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right) = -1$$

$$u(x) = e^{\int h(x) dx} = e^{-x}$$

$$(e^{2x} + e^{-x}y - e^{-x})dx - e^{-x}dy = 0$$

$$\frac{\partial F}{\partial x} = e^{2x} + e^{-x}y - e^{-x}$$

$$F = \frac{1}{2}e^{2x} - e^{-x}y + e^{-x} + \phi(y)$$

$$\frac{\partial F}{\partial y} = -e^{-x} + \phi'(y) = -e^{-x}$$

$$\phi'(y) = 0 \quad \phi(y) = 0$$

$$\frac{1}{2}e^{2x} - e^{-x}y + e^{-x} = C$$