南方科技大学 SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Please write carefully and clearly in complete sentences. Your explanations are your only representative when your work is being graded.

1. Solve the following problems:

   (a) $y^{(4)} + 2y'' + y = 8\sin t - 16\cos t$;

   (b) $y''' - 3y'' + 3y' - y = 6e^t$;

   (c) $y' + y^2 \sin x = 0$.

2. Use the method of undetermined coefficients to find a particular solution of

$$\mathbf{x}' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2e^{-t} \\ 3t \end{bmatrix}.$$

3. Determine the type and stability of the critical point $(4, 3)$ of the locally linear system

$$\begin{aligned} \frac{dx}{dt} &= 33 - 10x - 3y + x^2, \\ \frac{dy}{dt} &= -18 + 6x + 2y - xy. \end{aligned}$$

4. Construct a suitable Liapunov function to determine the stability of the origin for the system

$$\begin{aligned} \frac{dx}{dt} &= x^3 - 2y^3, \\ \frac{dy}{dt} &= xy^2 + x^2 y + \frac{1}{2}y^3. \end{aligned}$$

5. Consider the system

$$\mathbf{x}' = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \mathbf{x}$$

   Find a fundamental matrix.

6. Transform $u'' - 2u' - 3u = 0$ into a system of first order equations.

7. It two functions $f(t)$ and $g(t)$ are linearly independent on the interval $(-1, 1)$, then the Wronskian $W[f, g](t) \neq 0$ for all $(-1, 1)$. True or False?

8. Write down the definition of the matrix exponential e^{At}.

9. All solutions of $y' + 2y = \sin t$ converges to a particular solution as $t \to \infty$. True or False.

10. Can be a saddle point asymptotically stable in a particular direction? Yes or No?

11. Suppose that $f(x, y)$ and $f_y(x, y)$ are continuous on the xy-plane. Is it possible that $y_1(x) = \sin x$ and $y_2(x) = \cos x$ are solutions of $y'(x) = f(x, y)$? Yes or no?

12. Use Laplace transform to solve the following initial value problem

$$\begin{cases} y^{(4)} + 2y'' + y = 4te^t, \\ y(0) = y'(0) = y''(0) = y'''(0) = 0. \end{cases}$$