

Q<sub>1</sub>

(1) For point O we can get (ignore the couples  $M_1, M_2, M_3, M_4$ )

$$\begin{aligned}\sum F_x = 0 &\Rightarrow F_1 + F_2 + F_3 + F_4 = G \\ \sum M_x = 0 &\Rightarrow (F_3 + F_4)\left(\frac{H}{2} + d \cos \theta\right) - (F_1 + F_2)\left(\frac{H}{2} + d \cos \theta\right) = 0 \\ \sum M_y = 0 &\Rightarrow (F_1 + F_4)\left(\frac{L}{2} + d \sin \theta\right) - (F_2 + F_3)\left(\frac{L}{2} + d \sin \theta\right) = 0\end{aligned}$$

$$\Rightarrow F_1 = F_3 = N \quad F_2 = F_4 = \frac{G}{2} - N = 0.02 \text{ kgf} - N \quad (\text{for all } N \text{ is OK})$$

For example you can get  $F_1 = F_2 = F_3 = F_4 = 0.01 \text{ kgf}$

(2) For point O we can get (ignore the gravity)

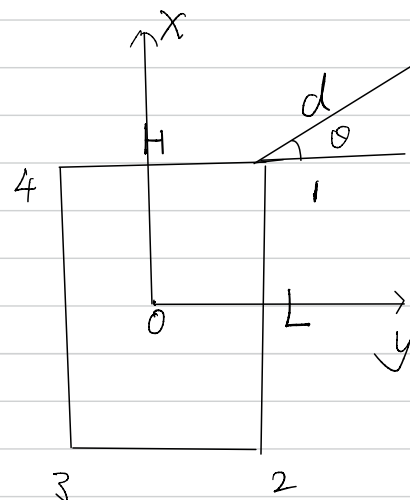
$$\begin{aligned}F_x &= 0 \\ F_y &= 0 \\ F_z &= F_1 + F_2 + F_3 + F_4 = -0.208 \text{ kgf}\end{aligned} \Rightarrow \vec{F} = (0\vec{i} + 0\vec{j} - 0.208\vec{k}) \text{ kgf}$$

$$\begin{aligned}M_x &= (F_3 + F_4)\left(\frac{H}{2} + d \cos \theta\right) - (F_1 + F_2)\left(\frac{H}{2} + d \cos \theta\right) = (0.06 + 0.064 - 0.04 - 0.044) \times (150 + 125\sqrt{3}) \\ &= (6 + 5\sqrt{3}) \text{ kgf} \cdot \text{mm} = 14.66 \text{ kgf} \cdot \text{mm}\end{aligned}$$

$$M_y = (F_1 + F_4)\left(\frac{L}{2} + d \sin \theta\right) - (F_2 + F_3)\left(\frac{L}{2} + d \sin \theta\right) = 0$$

$$M_z = (M_2 + M_4) - (M_1 + M_3) = (0.11 + 0.16) - (0.1 + 0.15) = 0.02 \text{ kgf} \cdot \text{mm}$$

$$\Rightarrow \vec{M} = (14.66\vec{i} + 0\vec{j} + 0.02\vec{k}) \text{ kgf} \cdot \text{mm}$$



2. two force bar

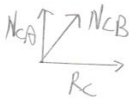
point B



$$\begin{cases} \sum F_x = 0 & \frac{0.75}{0.85} N_{BA} + \frac{0.75}{1.25} N_{BC} = 0 \\ \sum F_y = 0 & \frac{0.4}{0.85} N_{BA} = 10 + \frac{1}{1.25} N_{BC} \end{cases}$$

$$\begin{cases} N_{BC} = -\frac{12.5}{14} \text{ kN} = -8.93 \text{ kN} \quad (C) \\ N_{BA} = \frac{85}{14} \text{ kN} = 6.07 \text{ kN} \quad (T) \end{cases}$$

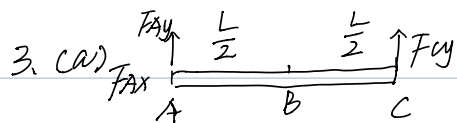
point C



$$\sum F_y = 0$$

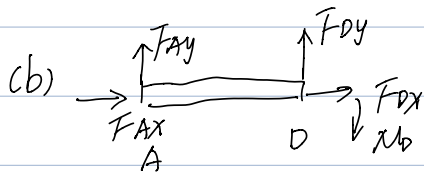
$$N_{CA} + \frac{1}{1.25} N_{CB} = 0$$

$$N_{CA} = \frac{50}{7} \text{ kN} = 7.14 \text{ kN} \quad (T)$$



$$\begin{cases} \sum F_x = 0: & F_{Ax} = 0 \\ \sum F_y = 0: & F_{Ay} = -F_{Cy} \\ \sum M_B = 0: & F_{Ay} \cdot \frac{L}{2} + M_0 = F_{Cy} \left( \frac{L}{2} \right) \end{cases}$$

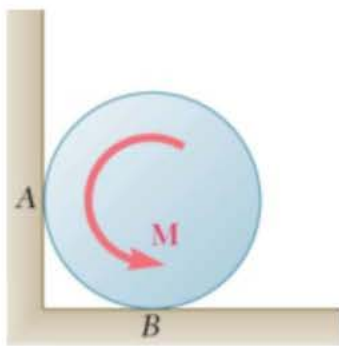
$$\begin{cases} F_{Ax} = 0 \\ F_{Ay} = -\frac{M_0}{L} \\ F_{Cy} = \frac{M_0}{L} \end{cases}$$



$$\begin{cases} \sum F_x = 0: & F_{Ax} = F_{Dx} = 0 \\ \sum F_y = 0: & F_{Dy} = -F_{Ay} = \frac{M_0}{L} \\ \sum M(D) = 0: & M_0 + F_{Ay} \cdot \frac{L}{4} = 0 \end{cases}$$

$$\begin{cases} F_{Dx} = 0 \\ F_{Dy} = \frac{M_0}{L} \\ M_D = \frac{M_0}{4} \end{cases}$$

## PROBLEM 8.22



The cylinder shown is of weight  $W$  and radius  $r$ , and the coefficient of static friction  $\mu_s$  is the same at  $A$  and  $B$ . Determine the magnitude of the largest couple  $\mathbf{M}$  that can be applied to the cylinder if it is not to rotate.

### SOLUTION

#### FBD cylinder:

For maximum  $M$ , motion impends at both  $A$  and  $B$

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: N_A - F_B = 0$$

$$N_A = F_B = \mu_s N_B$$

$$F_A = \mu_s N_A = \mu_s^2 N_B$$

$$\uparrow \Sigma F_y = 0: N_B + F_A - W = 0$$

$$N_B + \mu_s^2 N_B = W$$

or

$$N_B = \frac{W}{1 + \mu_s^2}$$

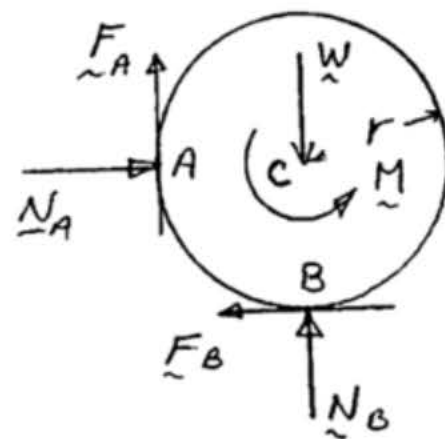
and

$$F_B = \frac{\mu_s W}{1 + \mu_s^2}$$

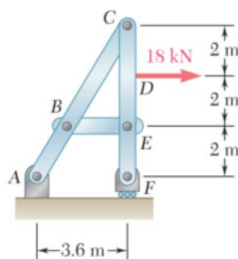
$$F_A = \frac{\mu_s^2 W}{1 + \mu_s^2}$$

$$\curvearrowleft \Sigma M_C = 0: M - r(F_A + F_B) = 0$$

$$M = r(\mu_s + \mu_s^2) \frac{W}{1 + \mu_s^2}$$



$$M_{\max} = Wr\mu_s \frac{1 + \mu_s}{1 + \mu_s^2} \blacktriangleleft$$

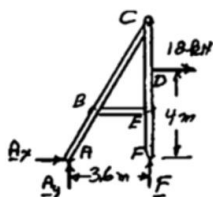


### PROBLEM 6.79

For the frame and loading shown, determine the components of all forces acting on member  $ABC$ .

### SOLUTION

Free body: Entire frame:



$$\rightarrow \Sigma F_x = 0: A_x + 18 \text{ kN} = 0$$

$$A_x = -18 \text{ kN}$$

$$A_x = 18.00 \text{ kN} \leftarrow$$

$$+\circlearrowleft \Sigma M_E = 0: -(18 \text{ kN})(4 \text{ m}) - A_y(3.6 \text{ m}) = 0$$

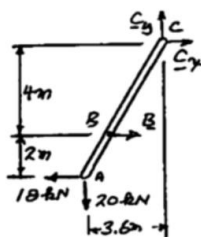
$$A_y = -20 \text{ kN} \quad A_y = 20.0 \text{ kN} \downarrow$$

$$+\uparrow \Sigma F_y = 0: -20 \text{ kN} + F = 0$$

$$F = +20 \text{ kN} \quad F = 20 \text{ kN} \uparrow$$

Free body: Member  $ABC$

Note:  $BE$  is a two-force member, thus  $\mathbf{B}$  is directed along line  $BE$ .



$$+\circlearrowleft \Sigma M_C = 0: B(4 \text{ m}) - (18 \text{ kN})(6 \text{ m}) + (20 \text{ kN})(3.6 \text{ m}) = 0$$

$$B = 9 \text{ kN}$$

$$\mathbf{B} = 9.00 \text{ kN} \rightarrow$$

$$\rightarrow \Sigma F_x = 0: C_x - 18 \text{ kN} + 9 \text{ kN} = 0$$

$$C_x = 9 \text{ kN}$$

$$\mathbf{C}_x = 9.00 \text{ kN} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: C_y - 20 \text{ kN} = 0$$

$$C_y = 20 \text{ kN}$$

$$\mathbf{C}_y = 20.0 \text{ kN} \uparrow$$

Q6. solution:

$$(a) \vec{P} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

(b) for arbitrary two-force member to be in equilibrium.

$$\sum F_x = 0, -F_a \sin \alpha + F_b \sin \beta = 0.$$

$$\sum F_y = 0, -F_a \cos \beta + F_b \cos \beta = 0.$$

$$\sum M_B = 0, F_a \cdot L \cdot \sin \alpha = 0.$$

$$\sum M_A = 0, F_b \cdot L \cdot \sin \beta = 0.$$

For arbitrary two-force member,

$L, F_a, F_b$  are arbitrary value.

$$\text{so. } \alpha = \beta = 0.$$

$$\Rightarrow \sum F_y = 0, -F_a + F_b = 0.$$

$$\Rightarrow F_a = F_b.$$

Quod Erat Demonstrandum.

or Q.E.D.

