Reference Answers of Homework 10

1. A steam power plant operates on a simple ideal Rankine cycle between the pressure limits of 3 MPa and 50 kPa. The temperature of the steam at the turbine inlet is 300°C, and the mass flow rate of steam through the cycle is 35 kg/s. Show the cycle on a T-s diagram with respect to saturation lines, and determine:

(a) the thermal efficiency of the power plant;

(b) the net power output of the power plant;

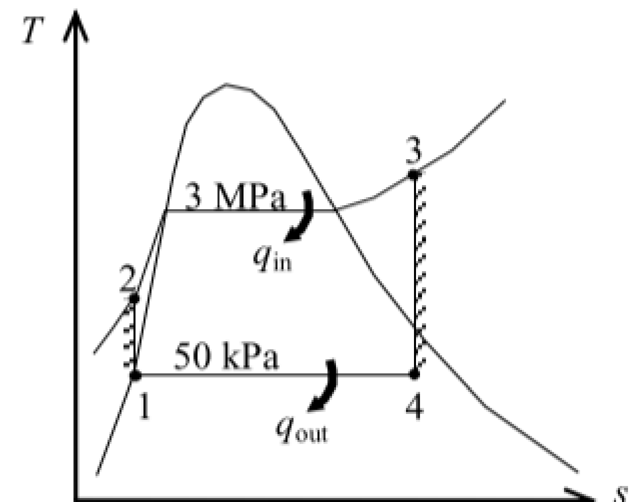
(c) the exergy destruction associated with each process of this cycle, assuming a source temperature of 1500 K and a sink temperature of 290 K.

ANS: (a)

$$h_1 = h_{f @ 50\,\mathrm{kPa}} = 340.54\ \mathrm{kJ/kg}$$
$$\boldsymbol{v}_1 = \boldsymbol{v}_{f @ 50\,\mathrm{kPa}} = 0.001030\ \mathrm{m^3/kg}$$

$$\begin{aligned} w_{p,\mathrm{in}} &= \boldsymbol{v}_1 (P_2 - P_1) \\ &= (0.001030\ \mathrm{m^3/kg})(3000-50)\ \mathrm{kPa}\left(\frac{1\ \mathrm{kJ}}{1\ \mathrm{kPa\cdot m^3}}\right) \\ &= 3.04\ \mathrm{kJ/kg} \end{aligned}$$

$$h_2 = h_1 + w_{p,\mathrm{in}} = 340.54 + 3.04 = 343.58\ \mathrm{kJ/kg}$$



$$\left.\begin{aligned} P_3 &= 3\ \mathrm{MPa} \\ T_3 &= 300\ ^\circ\mathrm{C} \end{aligned}\right\} \begin{aligned} h_3 &= 2994.3\ \mathrm{kJ/kg} \\ s_3 &= 6.5412\ \mathrm{kJ/kg\cdot K} \end{aligned}$$

$$\left.\begin{aligned} P_4 &= 50\ \mathrm{kPa} \\ s_4 &= s_3 \end{aligned}\right\} x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.5412 - 1.0912}{6.5019} = 0.8382$$

$$\begin{aligned} h_4 &= h_f + x_4 h_{fg} = 340.54 + (0.8382)(2304.7) \\ &= 2272.3\ \mathrm{kJ/kg} \end{aligned}$$

$$q_{\text{in}} = h_3 - h_2 = 2994.3 - 343.58 = 2650.7 \text{ kJ/kg}$$
$$q_{\text{out}} = h_4 - h_1 = 2272.3 - 340.54 = 1931.8 \text{ kJ/kg}$$
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 2650.7 - 1931.8 = 718.9 \text{ kJ/kg}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1931.8}{2650.7} = \mathbf{27.1\%}$$

(b) $$\dot{W}_{\text{net}} = \dot{m} w_{\text{net}} = (35 \text{ kg/s})(718.9 \text{ kJ/kg}) = \mathbf{25.2 \text{ MW}}$$

(c)

$$\chi_{des,1-2} = 0 \qquad \chi_{des,3-4} = 0$$

$$s_3 = s_4 = 6.5412 \text{ kJ/kg} \cdot \text{K}$$

$$s_1 = s_2 = 1.0912 \text{ kJ/kg} \cdot \text{K}$$

$$\chi_{des,2-3} = T_0 \left[(s_3 - s_2) - \frac{q_{in}}{T_{source}} \right] = 1068 \text{ kJ/kg}$$

$$\chi_{des,4-1} = T_0 \left[(s_1 - s_4) + \frac{q_{out}}{T_{sink}} \right] = 351.3 \text{ kJ/kg}$$

2. Consider the Carnot and the simple Rankine cycles with steam as the working fluid. Steam enters the turbine in both cases at 5 MPa as a saturated vapor, and the condenser pressure is 50 kPa. In the Rankine cycle, the condenser exit state is saturated liquid and in the Carnot cycle, the boiler inlet state is saturated liquid.
(a) draw the T-s diagrams for both cycles;
(b) compare the net work output and the thermal efficiency for both cycles.

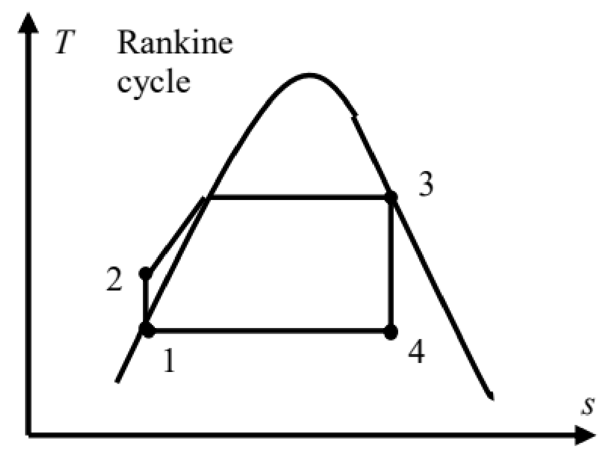
ANS: Rankine cycle analysis: From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{f@\,50\,\text{kPa}} = 340.54\ \text{kJ/kg}$$

$$v_1 = v_{f@\,50\,\text{kPa}} = 0.001030\ \text{m}^3/\text{kg}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1) = 5.10\ \text{kJ/kg}$$

$$h_2 = h_1 + w_{p,\text{in}} = 340.54 + 5.10 = 345.64\ \text{kJ/kg}$$



$$\left.\begin{array}{l} P_3 = 5\ \text{MPa} \\ x_3 = 1 \end{array}\right\} \begin{array}{l} h_3 = 2794.2\ \text{kJ/kg} \\ s_3 = 5.9737\ \text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_4 = 50\ \text{kPa} \\ s_4 = s_3 \end{array}\right\} x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{5.9737 - 1.09120}{6.5019} = 0.7509$$

$$h_4 = h_f + x_4 h_{fg} = 340.54 + (0.7509)(2304.7) = 2071.2\ \text{kJ/kg}$$

$$q_{in} = h_3 - h_2 = 2794.2 - 345.64 = 2448.6 \text{ kJ/kg}$$
$$q_{out} = h_4 - h_1 = 2071.2 - 340.54 = 1730.7 \text{ kJ/kg}$$
$$w_{net} = q_{in} - q_{out} = 2448.6 - 1730.7 = \mathbf{717.9 \text{ kJ/kg}}$$

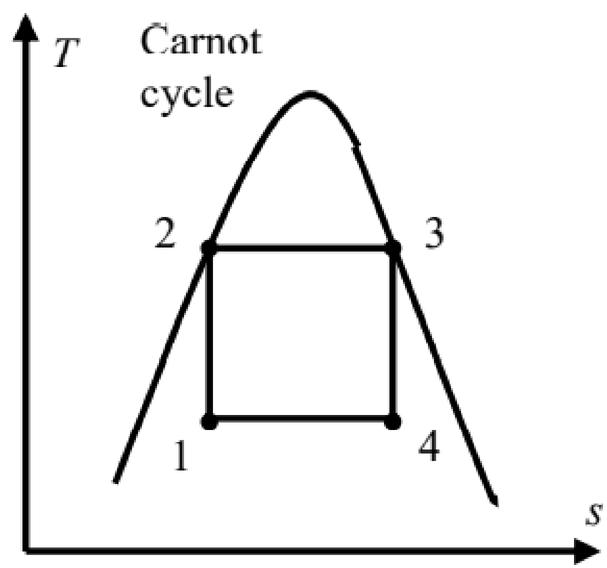
$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1730.7}{2448.6} = 0.2932 = \mathbf{29.3\%}$$

Carnot Cycle analysis:

$$\left.\begin{array}{l} P_3 = 5 \text{ MPa} \\ x_3 = 1 \end{array}\right\} \begin{array}{l} h_3 = 2794.2 \text{ kJ/kg} \\ T_3 = 263.9°\text{C} \end{array}$$

$$\left.\begin{array}{l} T_2 = T_3 = 263.9°\text{C} \\ x_2 = 0 \end{array}\right\} \begin{array}{l} h_2 = 1154.5 \text{ kJ/kg} \\ s_2 = 2.9207 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\left.\begin{array}{l} P_1 = 50 \text{ kPa} \\ s_1 = s_2 \end{array}\right\} \begin{array}{l} x_1 = \dfrac{s_1 - s_f}{s_{fg}} = \dfrac{2.9207 - 1.0912}{6.5019} = 0.2814 \\ h_1 = h_f + x_1 h_{fg} \\ \quad = 340.54 + (0.2814)(2304.7) = 989.05 \text{ kJ/kg} \end{array}$$



$$q_{in} = h_3 - h_2 = 2794.2 - 1154.5 = 1639.7 \text{ kJ/kg}$$
$$q_{out} = h_4 - h_1 = 2071.2 - 340.54 = 1082.2 \text{ kJ/kg}$$
$$w_{net} = q_{in} - q_{out} = 1639.7 - 1082.2 = \mathbf{557.5 \text{ kJ/kg}}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1082.2}{1639.7} = 0.3400 = \mathbf{34.0\%}$$