Homework 4 ANS

1. Nitrogen gas at 60 kPa and 7°C enters an adiabatic diffuser steadily with a velocity of 275 m/s and leaves at 85 kPa and 27°C. Determine (a) the exit velocity of the nitrogen and (b) the ratio of the inlet to exit area *A1/A2*.

ANS: (a) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take diffuser as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{m}(h_1 + V_1^2 / 2) = \dot{m}(h_2 + V_2^2 / 2) \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta \text{pe} \cong 0)$$

By substitute the properties into the equations,

$$0 = \frac{(8723 - 8141)\,\text{kJ/kmol}}{28\,\text{kg/kmol}} + \frac{V_2^2 - (275\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right)$$

It yields $\quad \mathbf{V_2 = 185\ m/s}$

(*b*) The ratio of the inlet to exit area is determined from the conservation of mass relation,

$$\frac{1}{v_2} A_2 V_2 = \frac{1}{v_1} A_1 V_1 \longrightarrow \frac{A_1}{A_2} = \frac{v_1}{v_2}\frac{V_2}{V_1} = \left(\frac{RT_1 / P_1}{RT_2 / P_2}\right)\frac{V_2}{V_1}$$

$$\frac{A_1}{A_2} = \left(\frac{T_1 / P_1}{T_2 / P_2}\right)\frac{V_2}{V_1} = \frac{(280\text{ K}/60\text{ kPa})(185\text{ m/s})}{(300\text{ K}/85\text{ kPa})(200\text{ m/s})} = \mathbf{0.887}$$

2. Helium is to be compressed from 105 kPa and 295 K to 700 kPa and 460 K. A heat loss of 15 kJ/kg occurs during the compression process. Neglecting kinetic energy changes, determine the power input required for a mass flow rate of 60 kg/min.

ANS: There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the compressor as the system. The energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{\text{in}} = \dot{Q}_{\text{out}} + \dot{m}c_p(T_2 - T_1)$$

$$= (60/60 \text{ kg/s})(15 \text{ kJ/kg}) + (60/60 \text{ kg/s})(5.1926 \text{ kJ/kg}\cdot\text{K})(460 - 295)\text{K}$$

$$= \mathbf{872 kW}$$

3. Steam enters a steady-flow turbine with a mass flow rate of 13 kg/s at 600°C, 8 MPa, and a negligible velocity. The steam expands in the turbine to a saturated vapor at 300 kPa where 10 percent of the steam is removed for some other use. The remainder of the steam continues to expand to the turbine exit where the pressure is 10 kPa and quality is 90 percent. If the turbine is adiabatic, determine the rate of work done by the steam during this process.

ANS: We take the entire turbine. Noting that one fluid stream enters the turbine and two fluid streams leave, the energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}_{\text{out}}$$

$$\dot{W}_{\text{out}} = \dot{m}_1 (h_1 - 0.1h_2 - 0.9h_3)$$

$$= (13 \text{ kg/s})(3642.4 - 0.1 \times 2724.9 - 0.9 \times 2225.1) \text{ kJ/kg}$$

$$= 17{,}776 \text{ kW} = \mathbf{17.8 MW}$$

4. Air enters the compressor of a gas-turbine plant at ambient conditions of 100 kPa and 25°C with a low velocity and exits at 1 MPa and 347°C with a velocity of 90 m/s. The compressor is cooled at a rate of 1500 kJ/min, and the power input to the compressor is 250 kW. Determine the mass flow rate of air through the compressor.

ANS: We take the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{W}_{\text{in}} + \dot{m}(h_1 + V_1^2/2) = \dot{Q}_{\text{out}} + \dot{m}(h_2 + V_2^2/2) \quad (\text{since } \Delta\text{pe} \cong 0)$$

$$\dot{W}_{\text{in}} - \dot{Q}_{\text{out}} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}\right)$$

By substituting, the mass flow rate is determined to be

$$250\ \text{kJ/s} - (1500/60\ \text{kJ/s}) = \dot{m}\left[628.07 - 298.2 + \frac{(90\ \text{m/s})^2 - 0}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right)\right] \rightarrow \dot{m} = \mathbf{0.674\ kg/s}$$

5. Hot exhaust gases of an internal combustion engine are to be used to produce saturated water vapor at 2 MPa pressure. The exhaust gases enter the heat exchanger at 400°C at a rate of 32kg/min while water enters at 15°C. The heat exchanger is not well insulated, and it is estimated that 10 percent of heat given up by the exhaust gases is lost to the surroundings. If the mass flow rate of the exhaust gases is 15 times that of the water, determine (a) the temperature of the exhaust gases at the heat exchanger exit and (b) the rate of heat transfer to the water. Use the constant specific heat properties of air for the exhaust gases.

ANS: We take the entire heat exchanger as the system. The mass and energy balances for this steady-flow system can be expressed in the rate form as

$$\dot{m}_{\text{exh}}h_{\text{exh,in}} + \dot{m}_{\text{w}}h_{\text{w,in}} = \dot{m}_{\text{exh}}h_{\text{exh,out}} + \dot{m}_{\text{w}}h_{\text{w,out}} + \dot{Q}_{\text{out}} \quad (\text{since } \dot{W} = \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

or
$$\dot{m}_{\text{exh}}c_pT_{\text{exh,in}} + \dot{m}_w h_{\text{w,in}} = \dot{m}_{\text{exh}}c_pT_{\text{exh,out}} + \dot{m}_w h_{\text{w,out}} + \dot{Q}_{\text{out}}$$

Noting that the mass flow rate of exhaust gases is 15 times that of the water, substituting gives

$$\begin{aligned}&15\dot{m}_{\text{w}}(1.045\ \text{kJ/kg.°C})(400°\text{C}) + \dot{m}_{\text{w}}(62.98\ \text{kJ/kg})\\ &\qquad = 15\dot{m}_{\text{w}}(1.045\ \text{kJ/kg.°C})T_{\text{exh,out}} + \dot{m}_{\text{w}}(2798.3\ \text{kJ/kg}) + \dot{Q}_{out}\end{aligned} \qquad (1)$$

The heat given up by the exhaust gases and heat picked up by the water are

$$\dot{Q}_{exh} = \dot{m}_{\text{exh}}c_p(T_{\text{exh,in}} - T_{\text{exh,out}}) = 15\dot{m}_{\text{w}}(1.045\ \text{kJ/kg.°C})(400 - T_{\text{exh,out}})°\text{C} \qquad (2)$$

$$\dot{Q}_w = \dot{m}_{\text{w}}(h_{\text{w,out}} - h_{\text{w,in}}) = \dot{m}_{\text{w}}(2798.3 - 62.98)\text{kJ/kg} \qquad (3)$$

The heat loss is
$$\dot{Q}_{out} = f_{\text{heat loss}}\dot{Q}_{exh} = 0.1\dot{Q}_{exh}$$

The solution may be obtained by a trial-error approach.

$$T_{\text{exh,out}} = \mathbf{206.1°C},\ \dot{Q}_{\text{w}} = \mathbf{97.26kW},\ \dot{m}_{\text{w}} = 0.03556\ \text{kg/s},\ \dot{m}_{\text{exh}} = 0.5333\ \text{kg/s}$$

6. A 4-L pressure cooker has an operating pressure of 175 kPa. Initially, one-half of the volume is filled with liquid and the other half with vapor. If it is desired that the pressure cooker not run out of liquid water for 1 h, determine the highest rate of heat transfer allowed.

ANS: We take the cooker as the system. The mass and energy balances for this uniform-flow system can be expressed as

$$\textit{Mass balance:} \qquad m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}} \rightarrow m_e = m_1 - m_2$$

$$\textit{Energy balance:} \qquad Q_{\text{in}} - m_e h_e = m_2 u_2 - m_1 u_1 \;\; (\text{since } W \cong ke \cong pe \cong 0)$$

The initial mass, initial internal energy, and final mass in the tank are

$$m_1 = m_f + m_g = \frac{V_f}{v_f} + \frac{V_g}{v_g} = \frac{0.002\ \text{m}^3}{0.001057\ \text{m}^3/\text{kg}} + \frac{0.002\ \text{m}^3}{1.0036\ \text{m}^3/\text{kg}} = 1.893 + 0.002 = 1.895\ \text{kg}$$

$$U_1 = m_1 u_1 = m_f u_f + m_g u_g = (1.893)(486.82) + (0.002)(2524.5) = 926.6\ \text{kJ}$$

$$m_2 = \frac{V}{v_2} = \frac{0.004\ \text{m}^3}{1.0037\ \text{m}^3/\text{kg}} = 0.004\ \text{kg}$$

Then from the mass and energy balances,

$$m_e = m_1 - m_2 = 1.895 - 0.004 = 1.891\ \text{kg}$$

$$\begin{aligned} Q_{\text{in}} &= m_e h_e + m_2 u_2 - m_1 u_1 \\ &= (1.891\ \text{kg})(2700.2\ \text{kJ/kg}) + (0.004\ \text{kg})(2524.5\ \text{kJ/kg}) - 926.6\ \text{kJ} = 4188\ \text{kJ} \end{aligned}$$

Thus,

$$\dot{Q} = \frac{Q}{\Delta t} = \frac{4188\ \text{kJ}}{3600\ \text{s}} = \mathbf{1.163kW}$$

7. An air compressor compresses 15 L/s of air at 120 kPa and 20°C to 800 kPa and 300°C while consuming 6.2 kW of power. How much of this power is being used to increase the pressure of the air versus the power needed to move the fluid through the compressor?

ANS: The specific volume of the air at the inlet is

$$\boldsymbol{v}_1 = \frac{RT_1}{P_1} = \frac{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273\,\text{K})}{120\,\text{kPa}} = 0.7008\,\text{m}^3/\text{kg}$$

The mass flow rate of the air is $\dot{m} = \frac{\boldsymbol{V}_1}{\boldsymbol{v}_1} = \frac{0.015\,\text{m}^3/\text{s}}{0.7008\,\text{m}^3/\text{kg}} = 0.02140\,\text{kg/s}$

Combining the flow work expression with the ideal gas equation of state gives the flow work as

$$w_{\text{flow}} = P_2\boldsymbol{v}_2 - P_1\boldsymbol{v}_1 = R(T_2 - T_1) = (0.287\,\text{kJ/kg}\cdot\text{K})(300-20)\text{K} = 80.36\,\text{kJ/kg}$$

The flow power is

$$\dot{W}_{\text{flow}} = \dot{m}w_{\text{flow}} = (0.02140\,\text{kg/s})(80.36\,\text{kJ/kg}) = \mathbf{1.72kW}$$

The remainder of compressor power input is used to increase the pressure of the air:

$$\dot{W} = \dot{W}_{\text{total,in}} - \dot{W}_{\text{flow}} = 6.2 - 1.72 = \mathbf{4.48kW}$$

So their power ratio is 4.48/1.72 = 2.60.