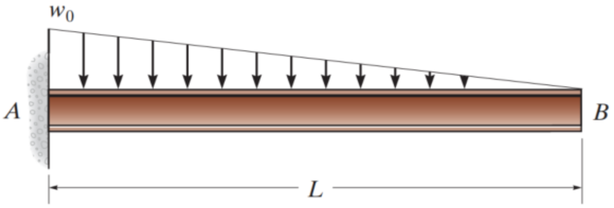
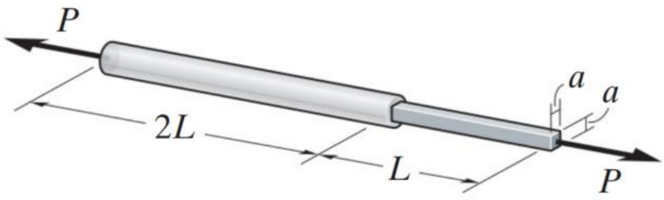
Homework problems 56-59
Due in class, Wednesday, 30 December 2020

56. (a) Determine the bending strain energy in the beam. EI is constant. (b) The A-36 steel bar consists of two segments, one of circular cross section of radius r, and one of square cross section. If the bar is subjected to the axial loading of P, determine the dimensions a of the square segment so that the strain energy within the square segment is the same as in the circular segment.



(a)

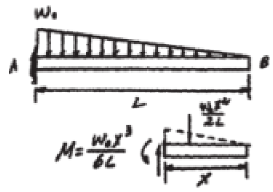


(b)

Figure 56

SOLUTION

$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^L \left(\frac{w_0 x^3}{6L}\right)^2 dx = \frac{w_0^2 L^5}{504\ EI}$$ **Ans.**



SOLUTION

Axial Strain Energy: Applying Eq. 14–16 to the circular segment gives

$$(U_i)_c = \frac{N^2 L_c}{2AE} = \frac{P^2(2L)}{2(\pi r^2)E} = \frac{P^2 L}{\pi r^2 E}$$

Applying Eq. 14–16 to the square segment gives

$$(U_i)_s = \frac{N^2 L_s}{2AE} = \frac{P^2 L}{2(a^2)E} = \frac{P^2 L}{2a^2 E}$$

Require

$$(U_i)_c = (U_i)_s$$

$$\frac{P^2 L}{\pi r^2 E} = \frac{P^2 L}{2a^2 E}$$

$$a = \sqrt{\frac{\pi}{2}}\, r$$ **Ans.**

57. The composite aluminum 2014-T6 bar is made from two segments having diameters of 7.5 mm and 15 mm. Determine the maximum axial stress developed in the bar if the 10-kg collar is dropped from a height of h = 100 mm. E_{Al} = 73.1 GPa.

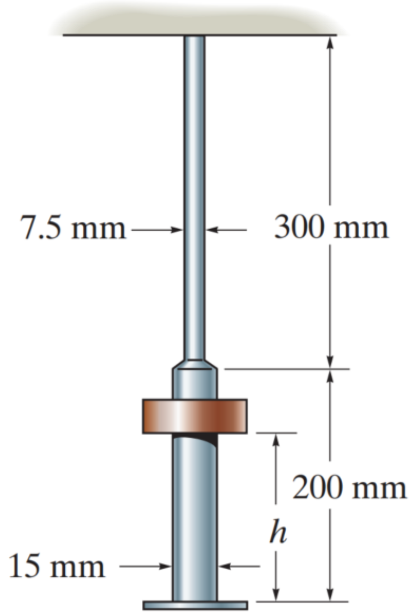


Figure 57

SOLUTION

$$\Delta_{st} = \Sigma \frac{WL}{AE} = \frac{10(9.81)(0.3)}{\frac{\pi}{4}(0.0075)^2(73.1)(10^9)} + \frac{10(9.81)(0.2)}{\frac{\pi}{4}(0.015)^2(73.1)(10^9)}$$

$$= 10.63181147\,(10^{-6})\text{ m}$$

$$n = \left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right] = \left[1 + \sqrt{1 + 2\left(\frac{0.1}{10.63181147(10^{-6})}\right)}\right] = 138.16$$

$$\sigma_{max} = n\,\sigma_{st} \quad \text{Here } \sigma_{st} = \frac{W}{A} = \frac{10\,(9.81)}{\frac{\pi}{4}(0.0075^2)} = 2.22053\text{ MPa}$$

$$\sigma_{max} = 138.16\,(2.22053)$$

$$= 307\text{ MPa} < \sigma_Y = 414\text{ MPa} \qquad \textbf{OK} \qquad \textbf{Ans.}$$

58. Using the principle of virtual work to determine the displacement at point C and the slope at B. EI is constant.

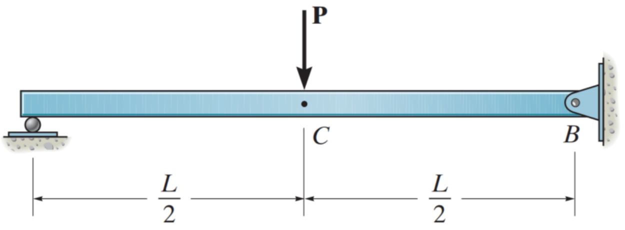


Figure 58

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

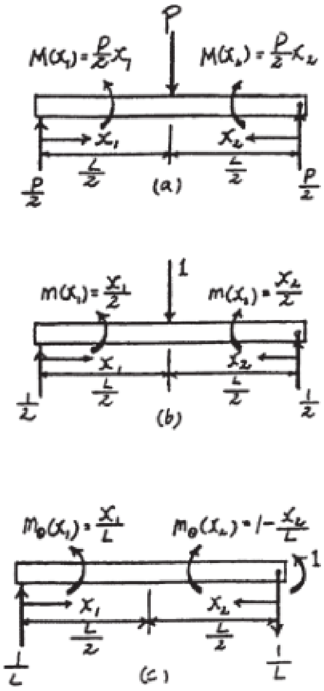
$$1 \cdot \Delta_C = 2\left[\frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{x_1}{2}\right)\left(\frac{P}{2}x_1\right)dx_1\right]$$

$$\Delta_C = \frac{PL^3}{48EI}\downarrow$$ **Ans.**

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$1 \cdot \theta_B = \frac{1}{EI}\left[\int_0^{\frac{L}{2}}\left(\frac{x_1}{L}\right)\left(\frac{P}{2}x_1\right)dx_1 + \int_0^{\frac{L}{2}}\left(1-\frac{x_2}{L}\right)\left(\frac{P}{2}x_2\right)dx_2\right]$$

$$\theta_B = \frac{PL^2}{16EI}$$ **Ans.**



59. The beam is made of oak, for which E_o = 11 GPa. Using the method of virtual forces to determine the slope and displacement at point A.

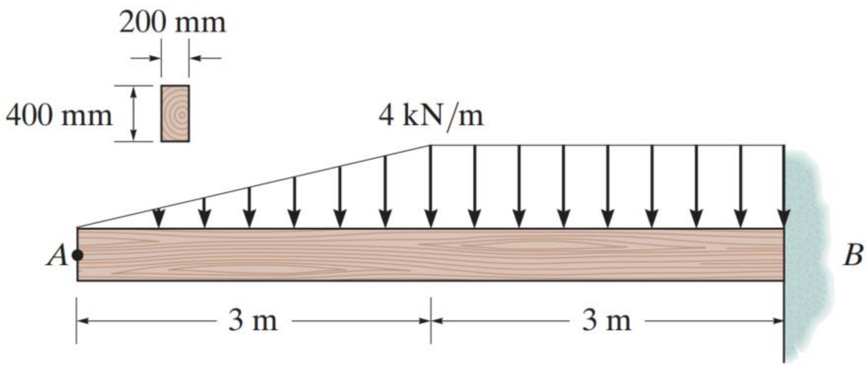
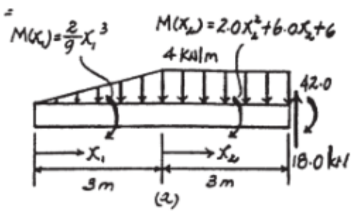


Figure 59

SOLUTION



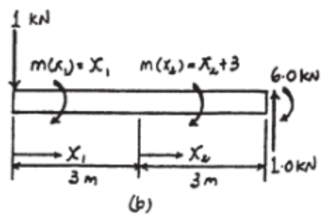
Virtual Work Equation: For the displacement at point A, apply Eq. 14–42.

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}dx$$

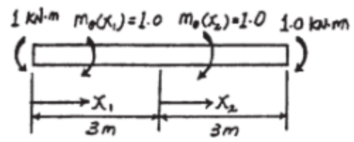
$$1\text{ kN} \cdot \Delta_A = \frac{1}{EI}\int_0^{3\text{ m}} x_1\left(\frac{2}{9}x_1^3\right)dx_1 + \frac{1}{EI}\int_0^{3\text{ m}} (x_2+3)\left(2.00x_2^2+6.00x_2+6.00\right)dx_2$$

$$\Delta_A = \frac{321.3\text{ kN}\cdot\text{m}^3}{EI} = \frac{321.3\,(10^3)}{11(10^9)\left[\frac{1}{12}(0.2)(0.4^3)\right]}$$

$$= 0.02738\text{ m} = 27.4\text{ m}\ \downarrow$$ **Ans.**



For the slope at A, apply Eq. 14–43.



$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}dx$$

$$1\text{ kN}\cdot\text{m}\cdot\theta_A = \frac{1}{EI}\int_0^{3\text{ m}} 1.00\left(\frac{2}{9}x_1^3\right)dx_1 + \int_0^{3\text{ m}} 1.00\left(2.00x_2^2+6.00x_2+6.00\right)dx_2$$

$$\theta_A = \frac{67.5\text{ kN}\cdot\text{m}^2}{EI} = \frac{67.5\,(1000)}{11(10^9)\left[\frac{1}{12}(0.2)(0.4^3)\right]}$$

$$= 5.75\,(10^{-3})\text{ rad}$$ **Ans.**