力学与航空航天工程系
DEPARTMENT OF MECHANICS AND AEROSPACE ENGINEERING

MECHANICS OF MATERIALS

YAHUI XUE (薛亚辉)

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Vocabulary

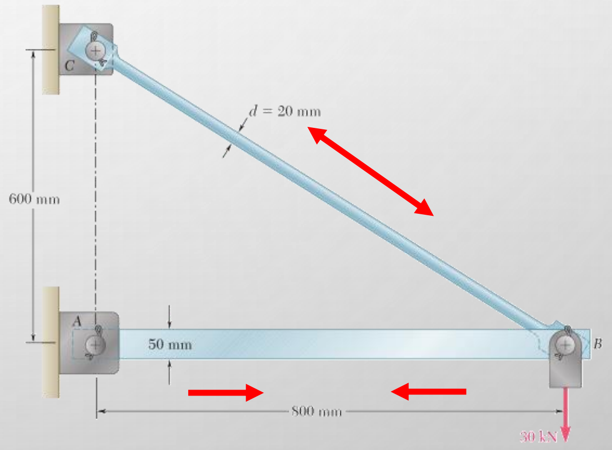
- Elastic (弹性的)
- Plastic （塑性的）
- Indeterminate （非静定的）
- Moment （弯矩）
- Stress（应力）
- Strain （应变）
- Modulus of elasticity (弹性模量)

- Saint Venant’s principle (圣维南原理)
- Resultant （合力）
- Fracture （断裂）
- Yield （屈服）
- Ductile（软的）
- Fatigue （疲劳）
- Dilation（膨胀）

- Compatibility （协调性）
- Poisson （泊松）
- Resilience （弹性恢复）
- Elastoplastic （弹塑性）
- Strain hardening (应变硬化)

Lesson 2： Axial Load

- Internal force and stress on the cross section of a rob under axial loading
- Strength condition of the bar
- Deformation calculation of a rod
- Statically indeterminate problems
- Plastic deformation

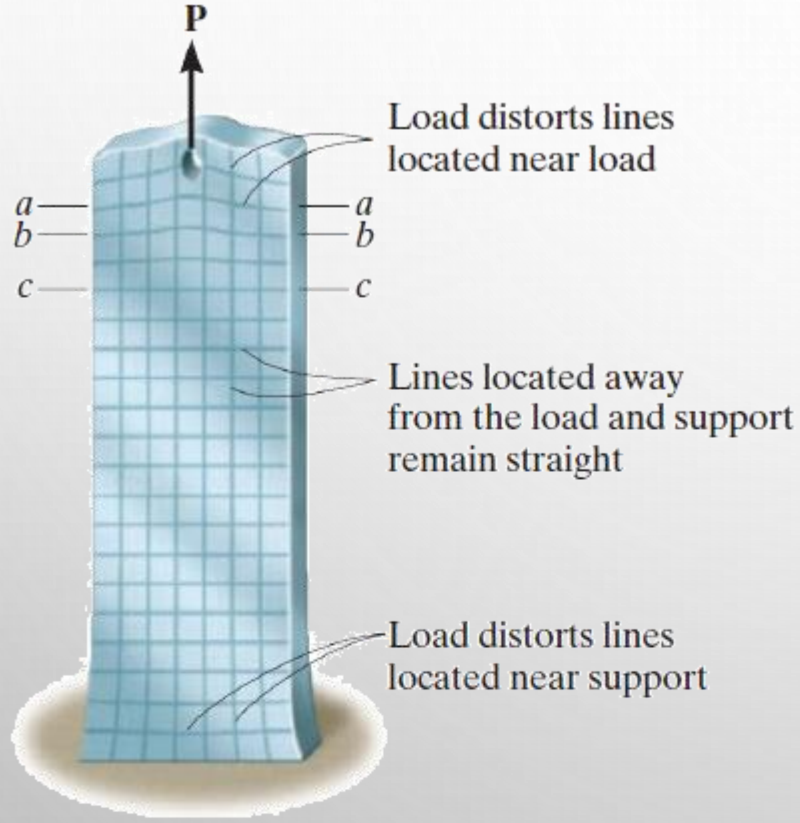


§ 2.1 Introduction

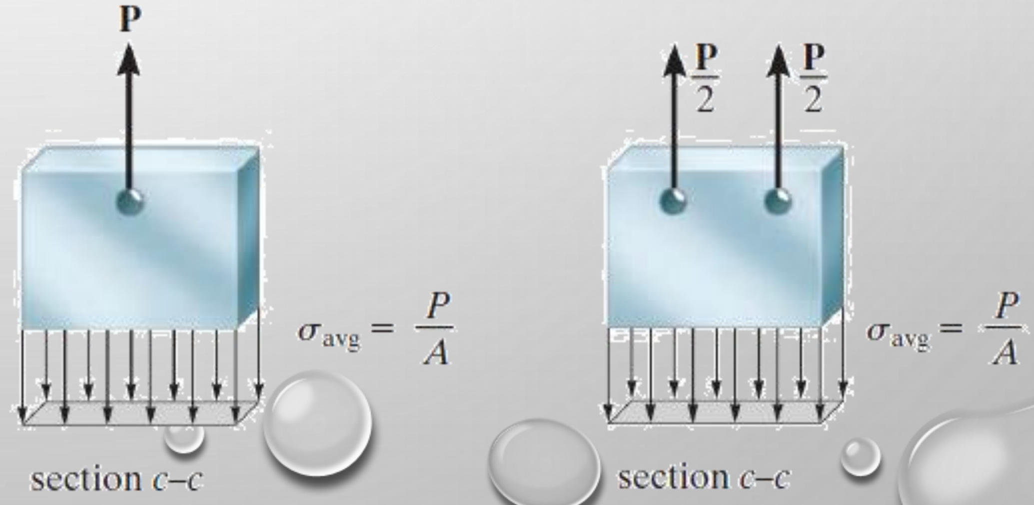
- This chapter is devoted to the study of deformations occurring in structural components subjected to axial loading.

  - deformations
  - stress-strain diagram
  - modulus of elasticity
  - plastic deformation
  - statically indeterminate problems

§ 2.2 Saint Venant’s principle



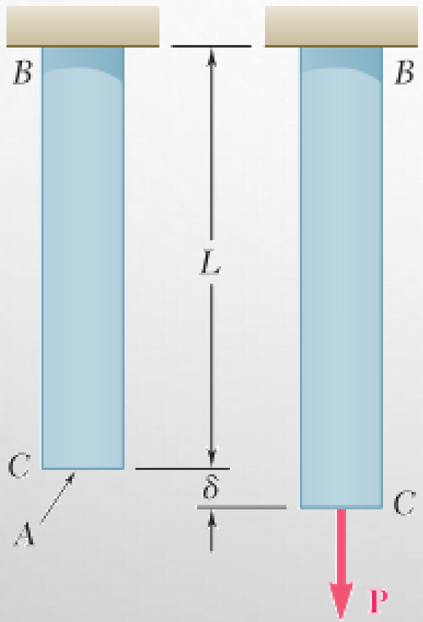
It states that **the stress and strain** produced at points in a body sufficiently removed from the region of external load application will be **the same as the stress and strain** produced by any other applied external loading that **has the same statically equivalent resultant** and **is applied to the body within the same region**.



§ 2.3 Elastic deformation of an axially loaded member

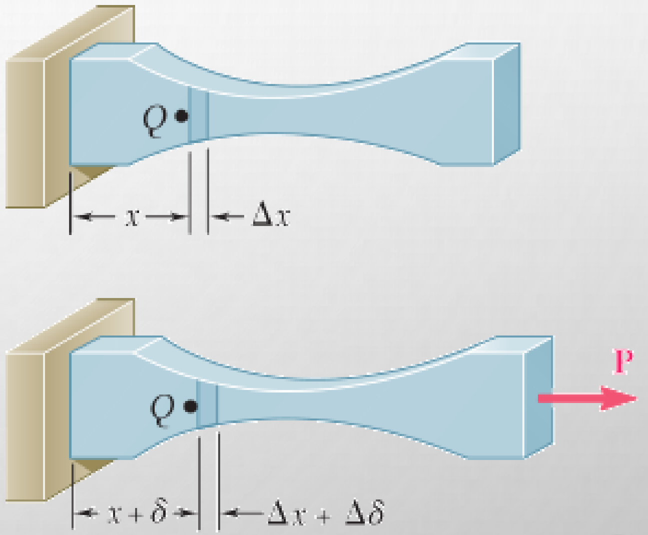
- **Average normal strain**

$$\varepsilon_{\text{avg}} = \frac{\delta}{L}$$



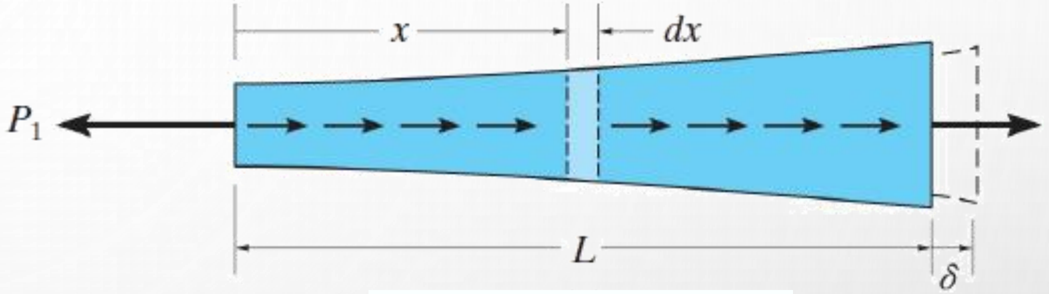
- **Local normal strain**

$$\varepsilon = \frac{\Delta\delta}{\Delta x}$$

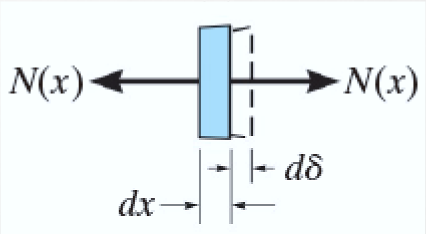


§ 2.3 Elastic deformation of an axially loaded member

- Find the relative displacement (δ) of one end of the bar with respect to the other end as caused by the loading.



$$\sigma(x)=\frac{N(\mathrm{x})}{A(x)} \qquad \varepsilon(x)=\frac{d\delta}{dx} \qquad \sigma(x)=E(x)\varepsilon(\mathrm{x})$$



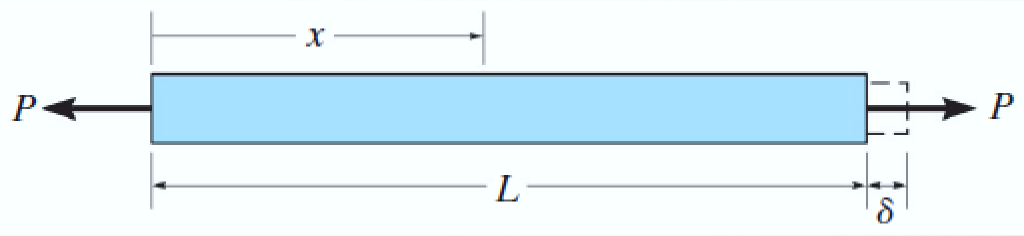
Hooke’s law; E, modulus of elasticity

$$\frac{d\delta}{dx}=\frac{N(\mathrm{x})}{E(x)A(x)} \Rightarrow \delta=\int_0^L \frac{N(\mathrm{x})}{E(x)A(x)}dx$$

§ 2.3 Elastic deformation of an axially loaded member

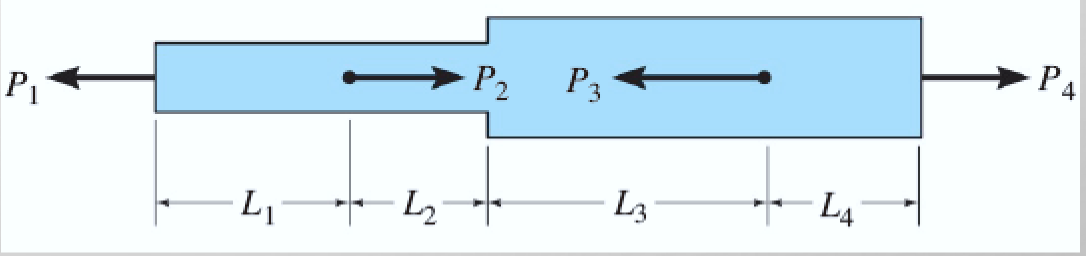
- For constant load and cross-sectional area,

$$\delta = \frac{NL}{EA} \qquad (N=P)$$



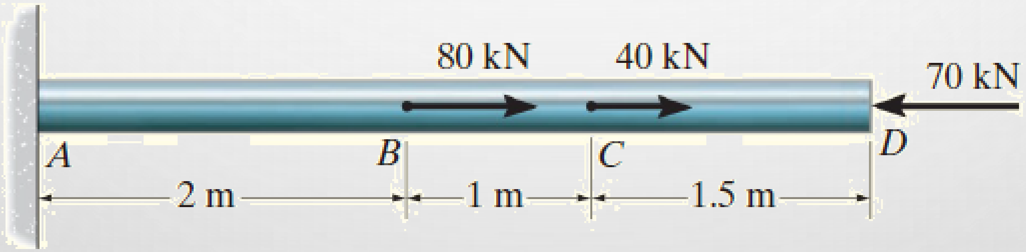
- For several different axial forces or abruptly changed cross-sectional area or modulus of elasticity,

$$\delta = \sum \frac{NL}{EA} \qquad (N=P)$$



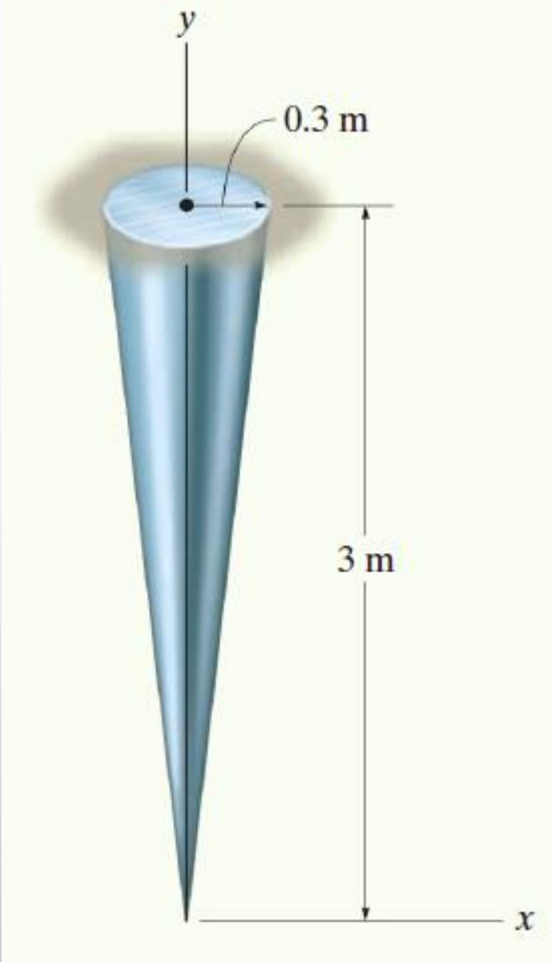
Example 2.1

The uniform A-36 steel bar has a diameter of 50 mm and is subjected to the loading shown. Determine the displacement at D, and the displacement of point B relative to C.



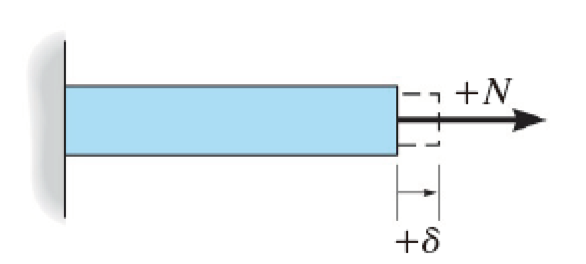
Example 2.2

A member is made of a material that has a specific weight of g = 6 kN/m^3 and modulus of elasticity of 9 GPa. If it is in the form of a cone, determine how far its end is displaced due to gravity when it is suspended in the vertical position.

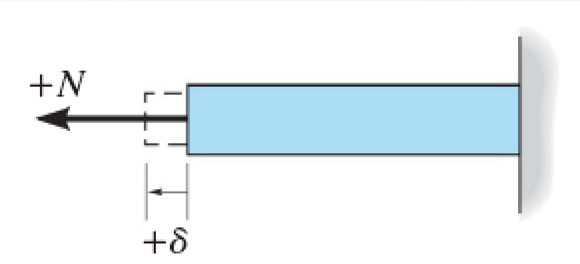


§ 2.4 Sign convention

- Consider both the force and displacement to be positive if they cause tension and elongation;



- A negative force and displacement will cause compression and contraction



§ 2.5 The tension and compression test

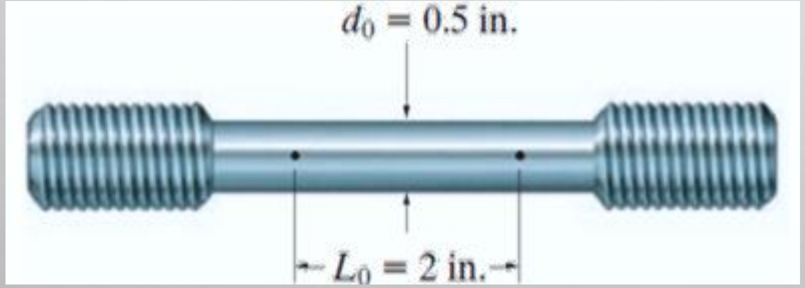
- Nominal or engineering stress and strain

$$\sigma = \frac{P}{A_0}, \quad \varepsilon = \frac{\delta}{L_0}$$

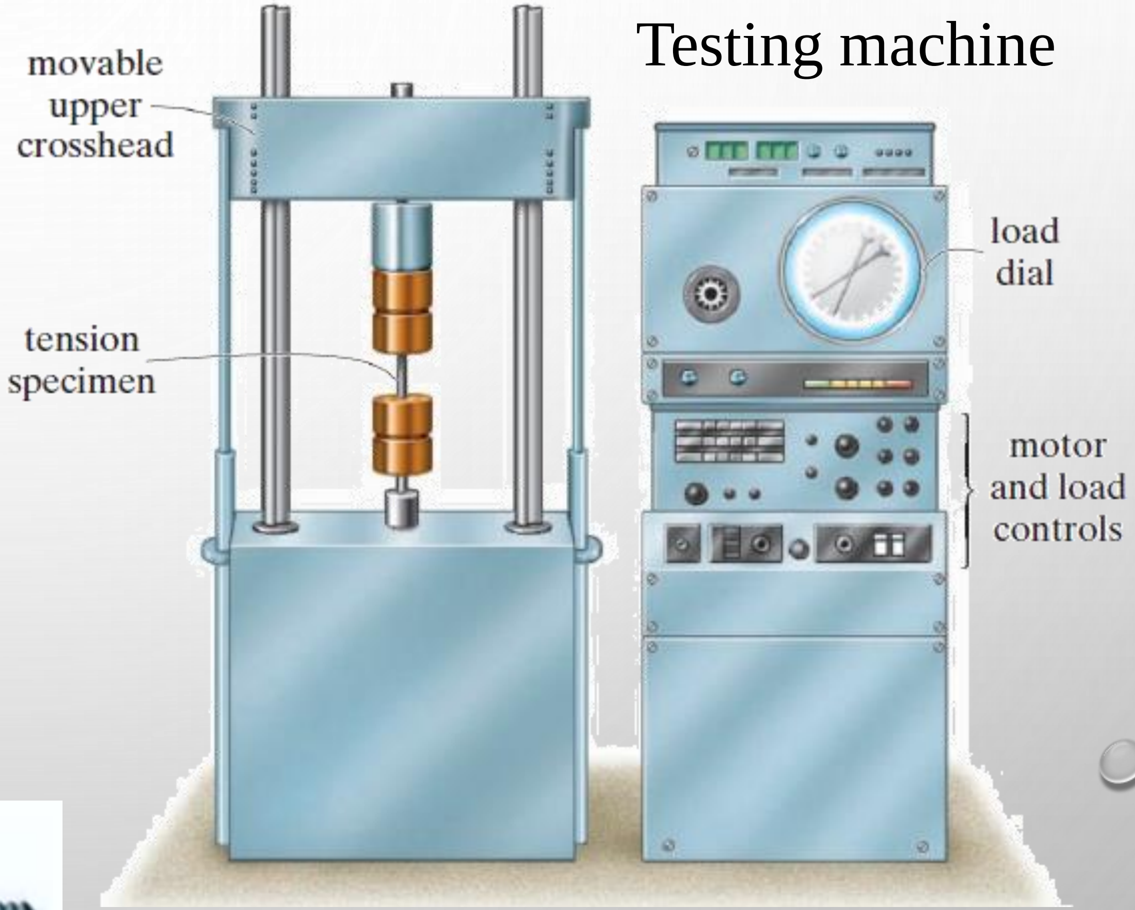
- True or actual stress and strain

$$\sigma = \frac{P}{A}, \quad \varepsilon = \frac{\delta}{L}$$

- Standard specimen

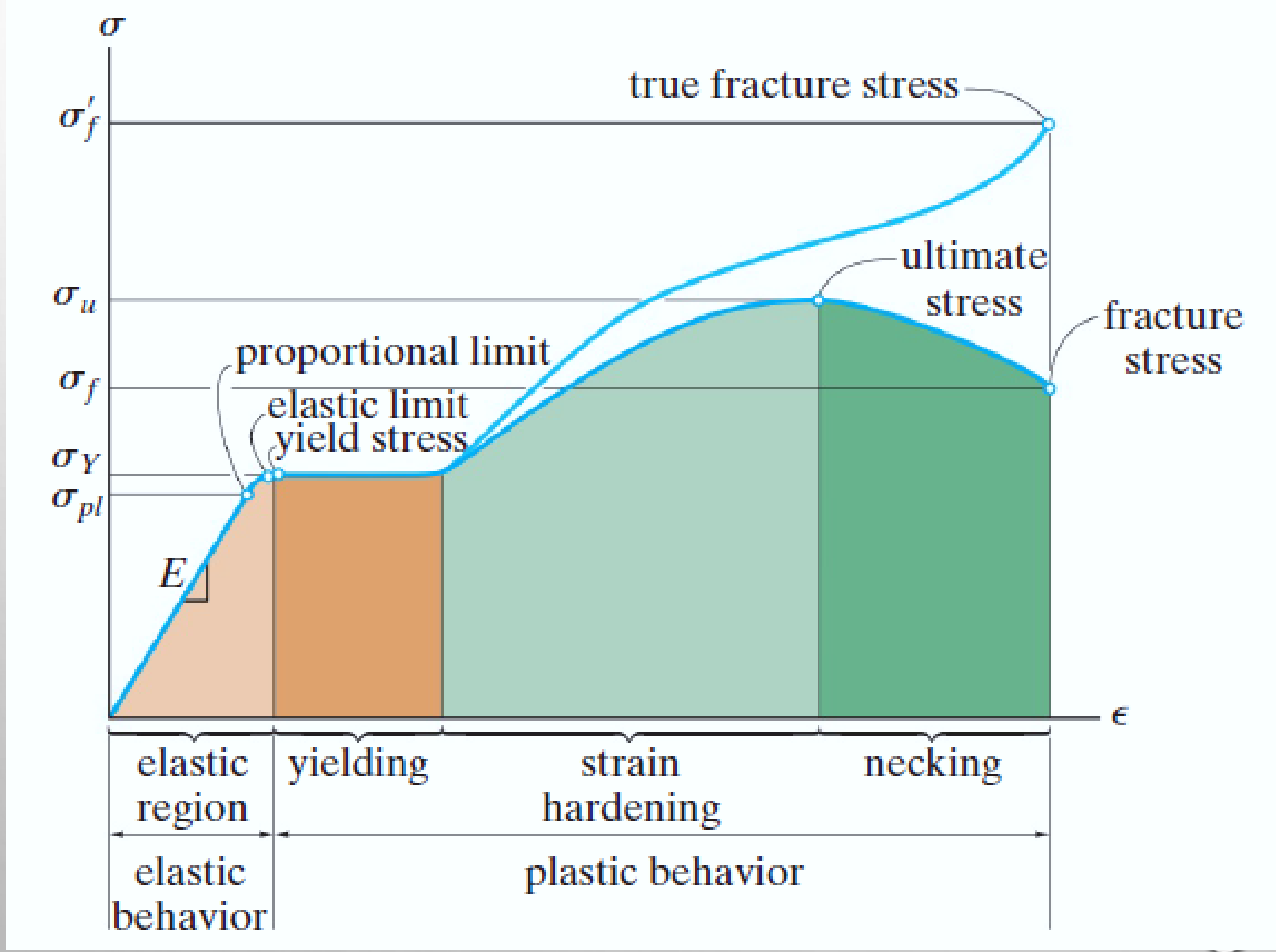


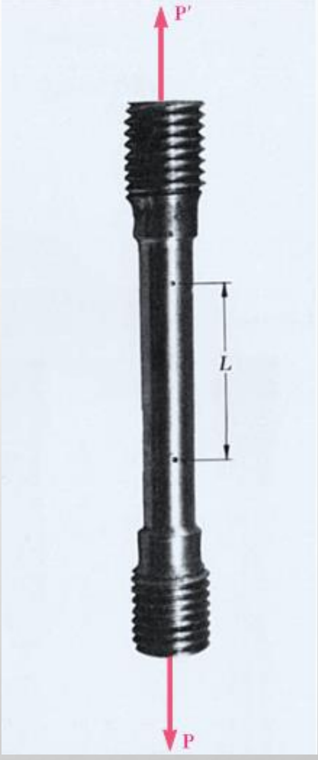
Testing machine

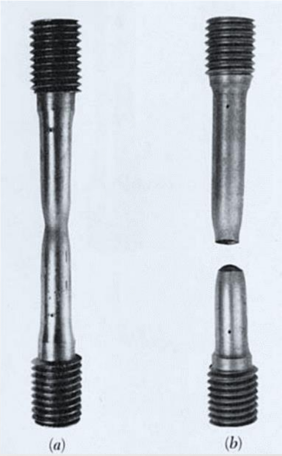


Tensile Test Stainless Steel Specimen

§ 2.6 Stress-strain diagram

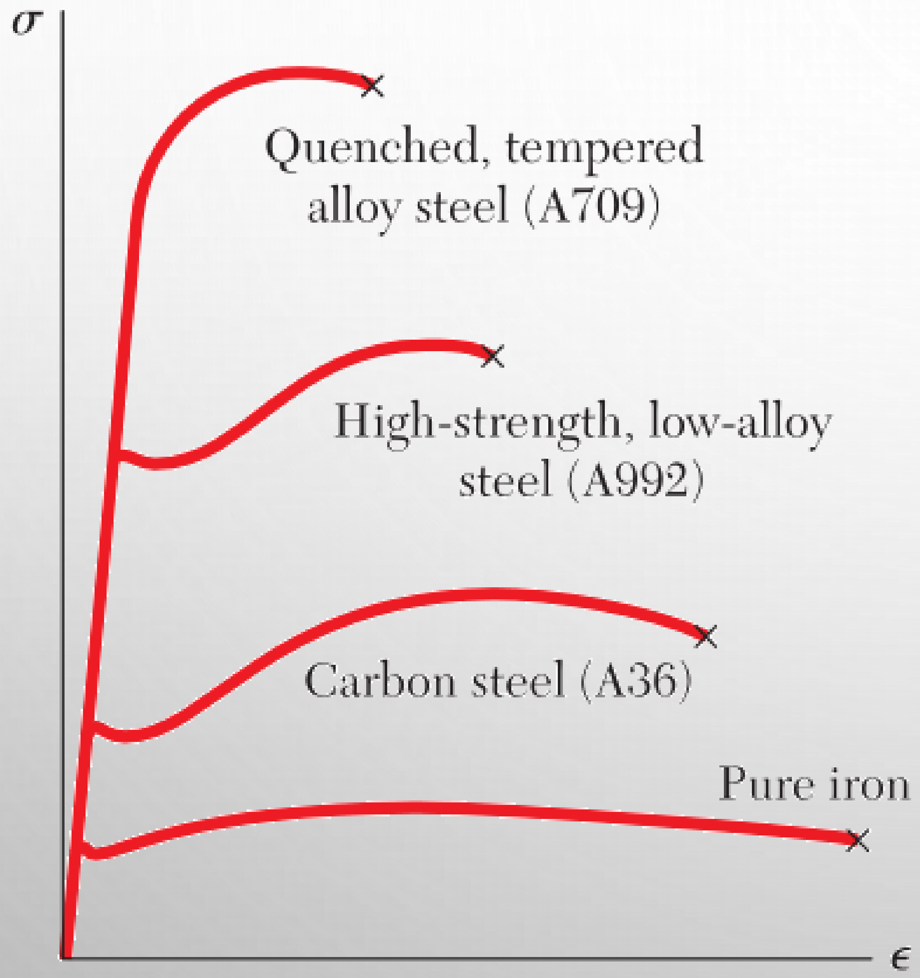






Ductile material

§ 2.7 Strength condition of the bar

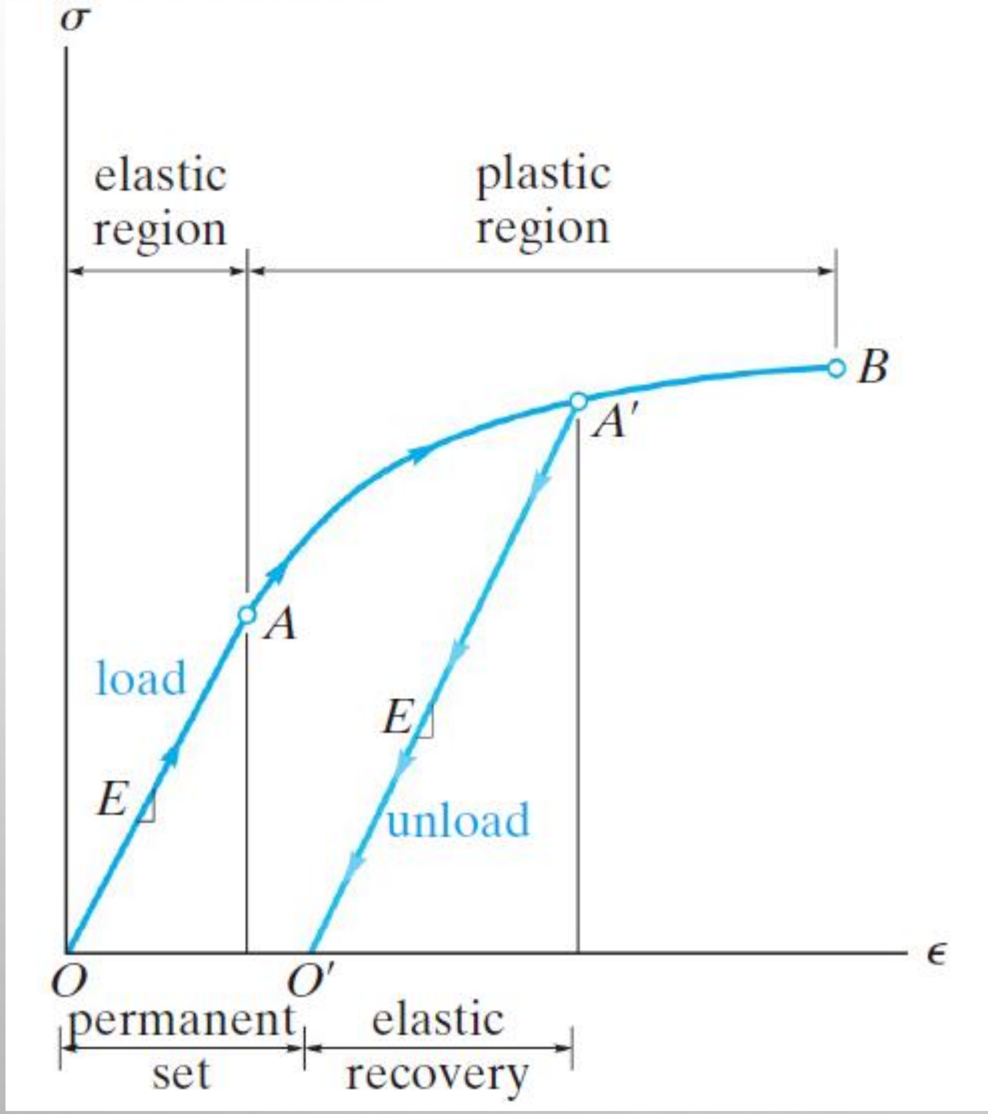


- The modulus of elasticity is a mechanical property that indicates the **stiffness** of a material.
- The **strength** of a material depends on its ability to sustain a load without undue deformation or failure.

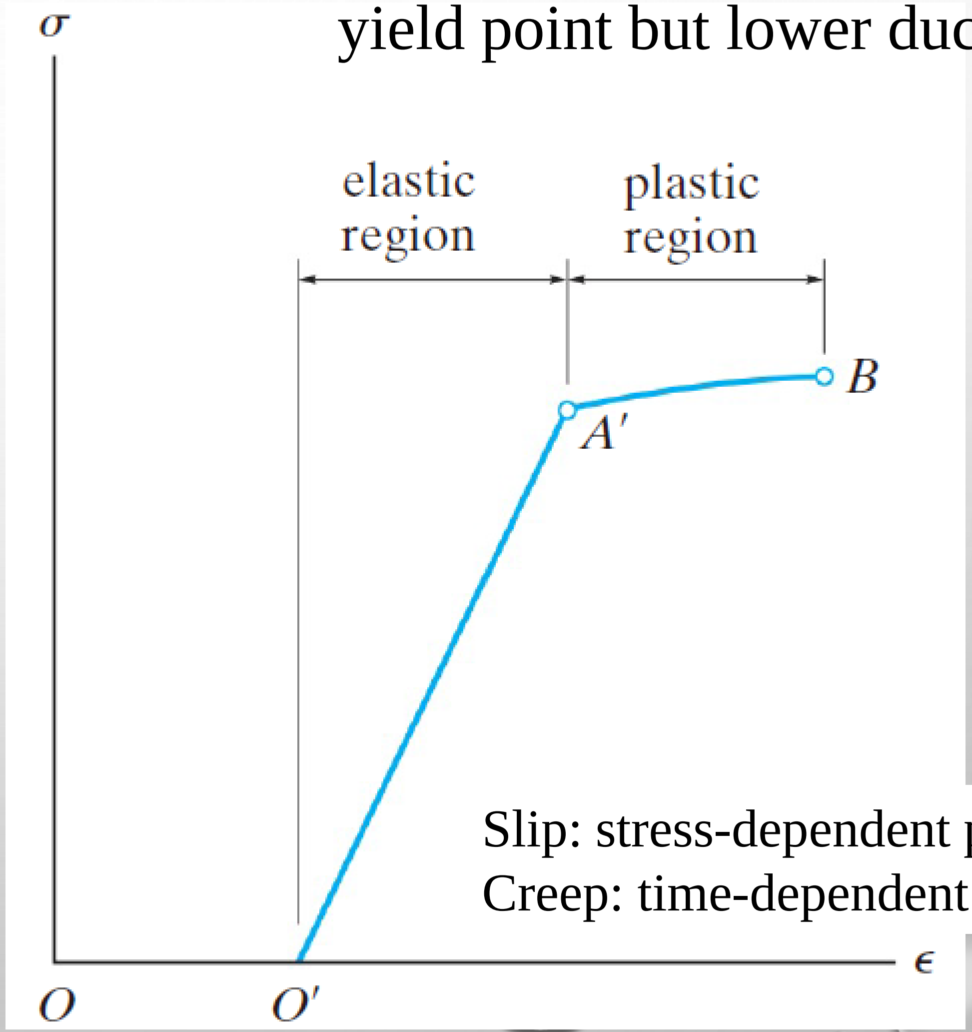
$$\sigma < [\sigma]_{\text{allowable}}$$

$$\text{Factor of safety} = \frac{[\sigma]_{\text{ultimate}}}{[\sigma]_{\text{allowable}}}$$

§ 2.8 Elastic versus plastic behavior



Re-load diagram has higher yield point but lower ductility

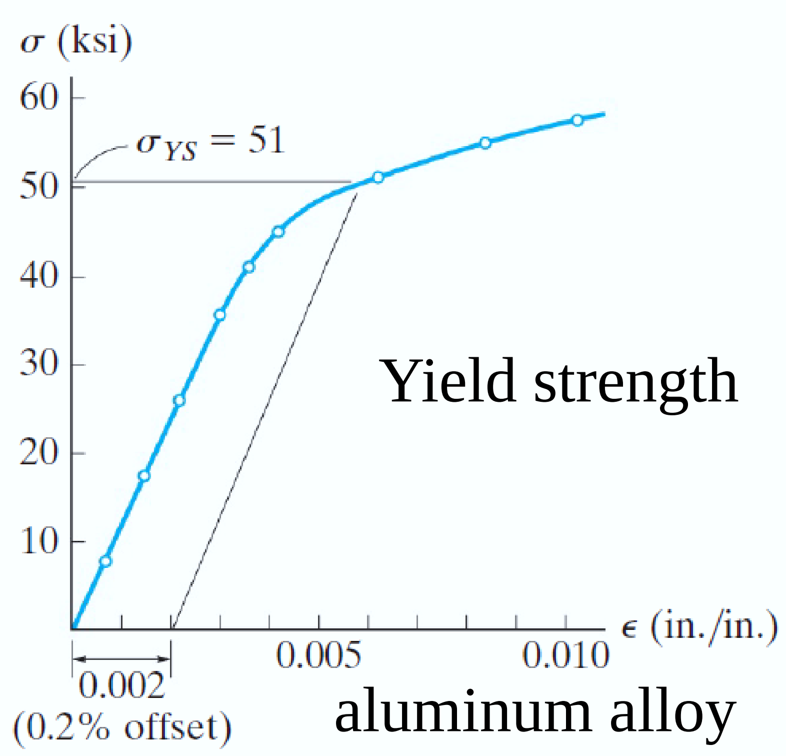


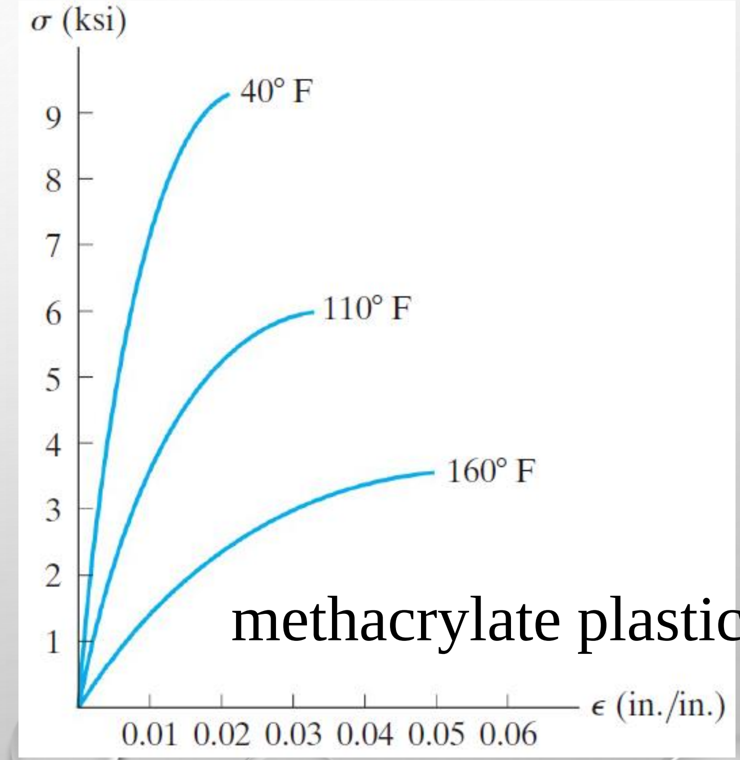
Slip: stress-dependent part
Creep: time-dependent part

§ 2.9 Ductile and brittle materials

- **Ductile Materials**: Any material that can be subjected to large strains before it fractures

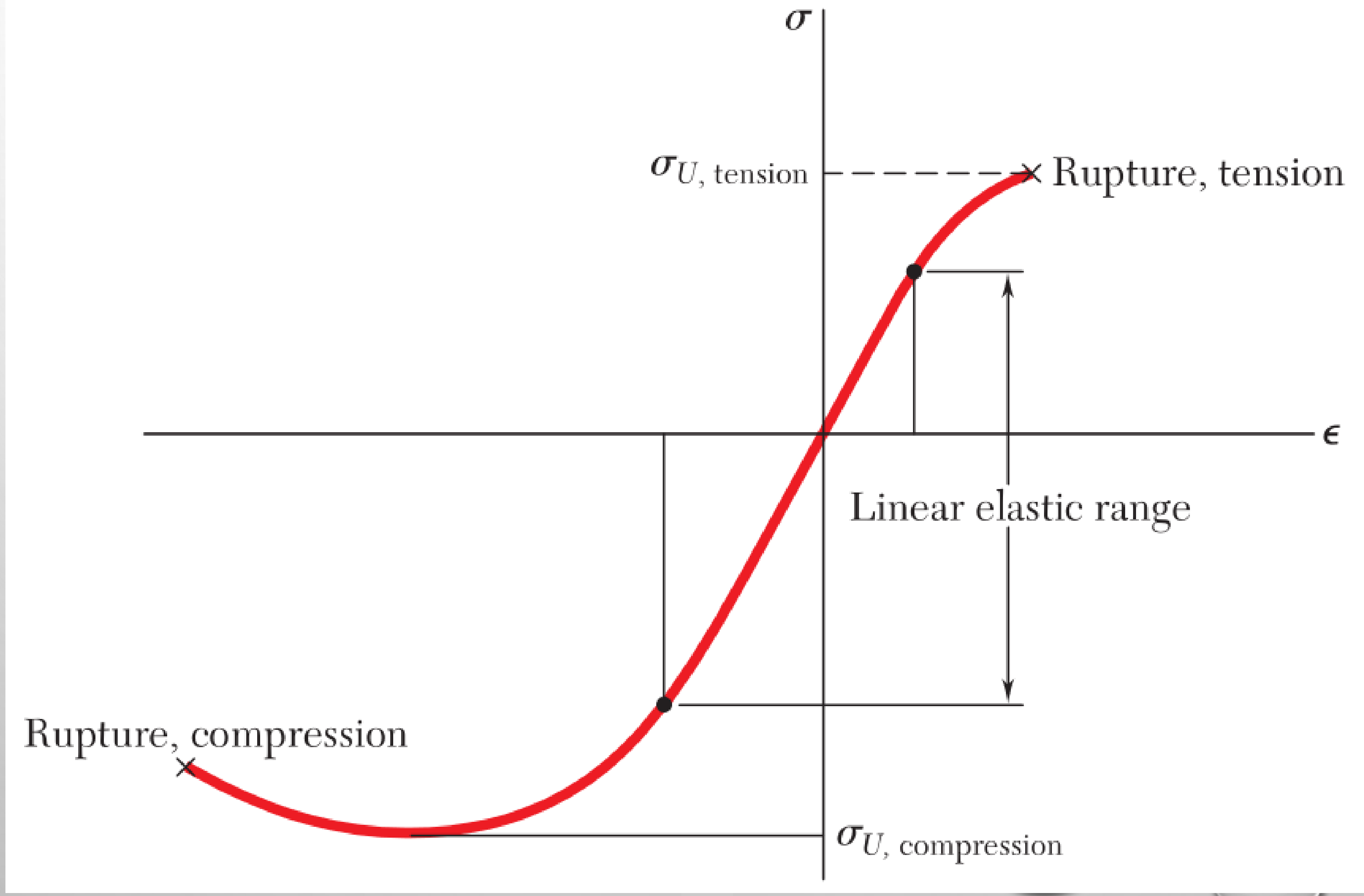
- **Brittle Materials**: Materials that exhibit little or no yielding before failure





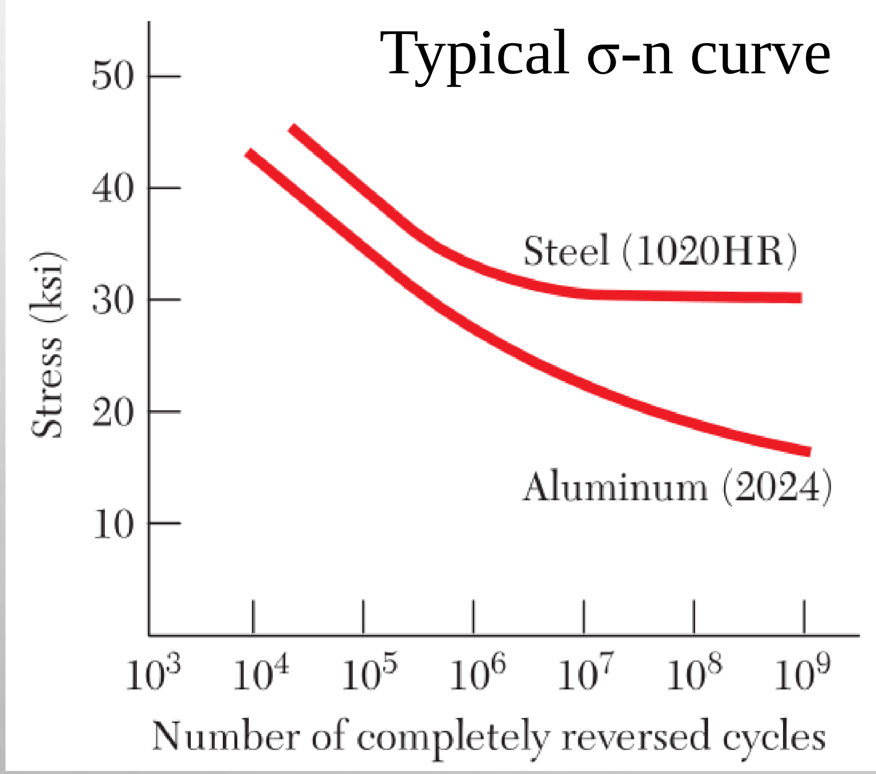
§ 2.9 Ductile and brittle materials

- Stress-strain diagram for concrete



§ 2.10 Repeated loadings; Fatigue

- Rupture will occur at a stress much lower than the static breaking strength when loadings are repeated thousands or millions of times. This phenomenon is known as fatigue.



For aluminum and copper, one defines the fatigue limit as the stress corresponding to failure after a specified number of loading cycles, such as 500 million.

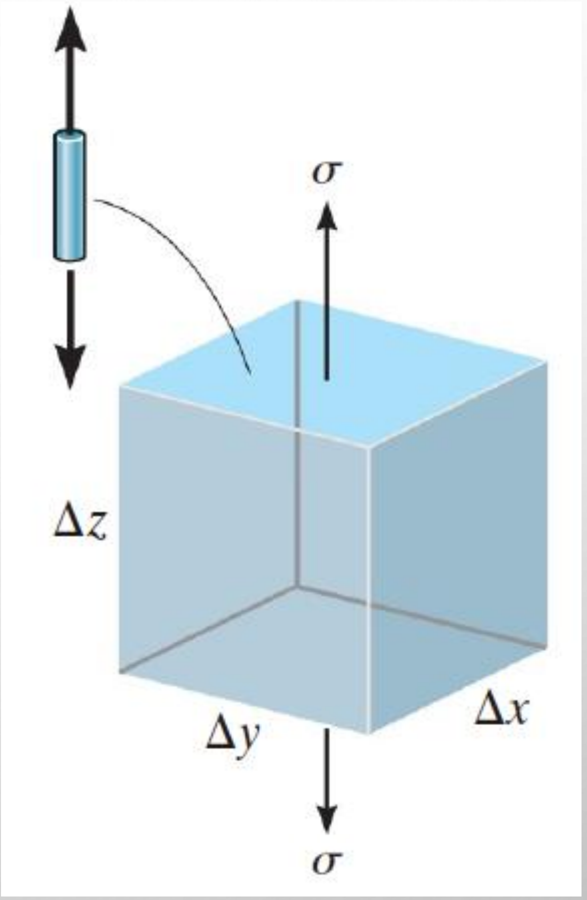
§ 2.10 Strain energy

- **Strain energy**: Internal energy stored in the material as deformed by an external load.

$$\Delta U = \frac{1}{2}\Delta F \Delta D = \frac{1}{2}\sigma\varepsilon\Delta x\Delta y\Delta z = \frac{1}{2}\sigma\varepsilon\Delta V$$

- **Strain energy density**: the strain energy per unit volume of material

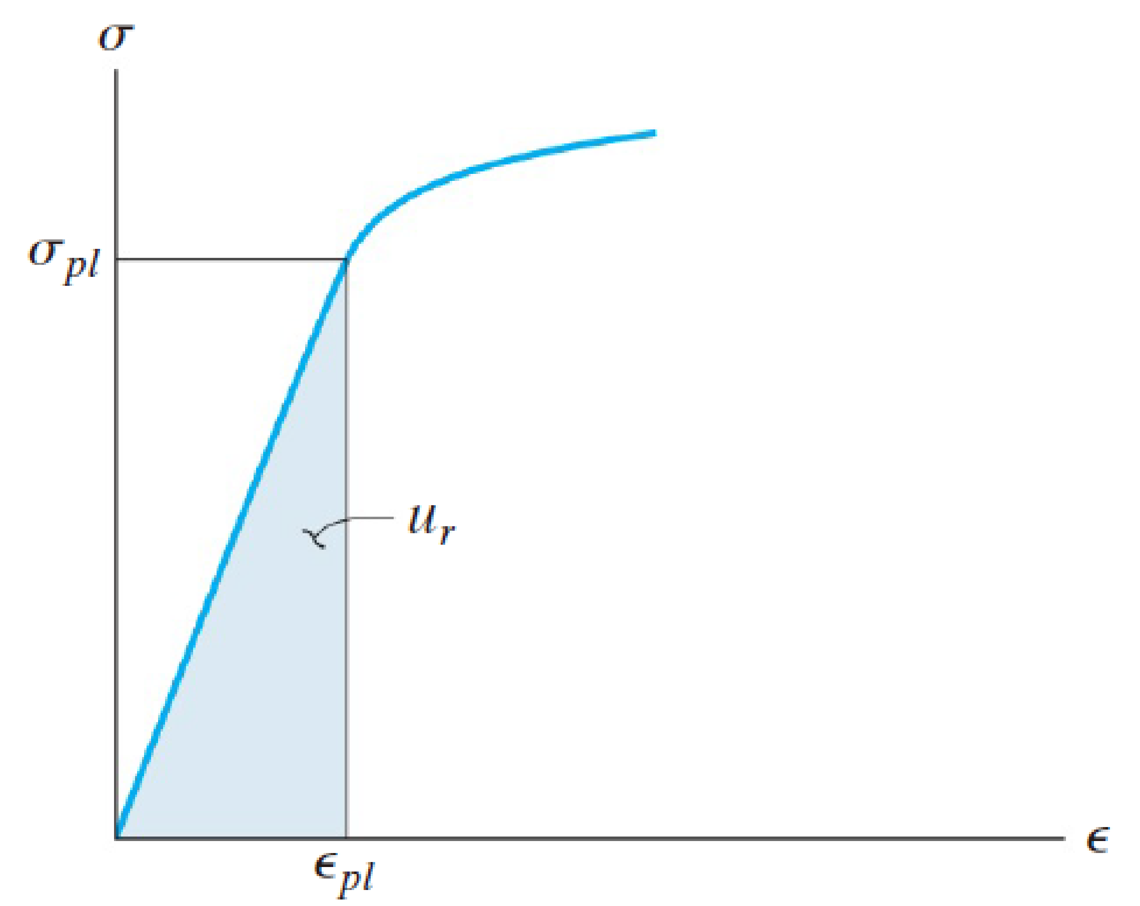
$$u = \frac{\Delta U}{\Delta V} = \frac{1}{2}\sigma\varepsilon$$



§ 2.11 Modulus of resilience

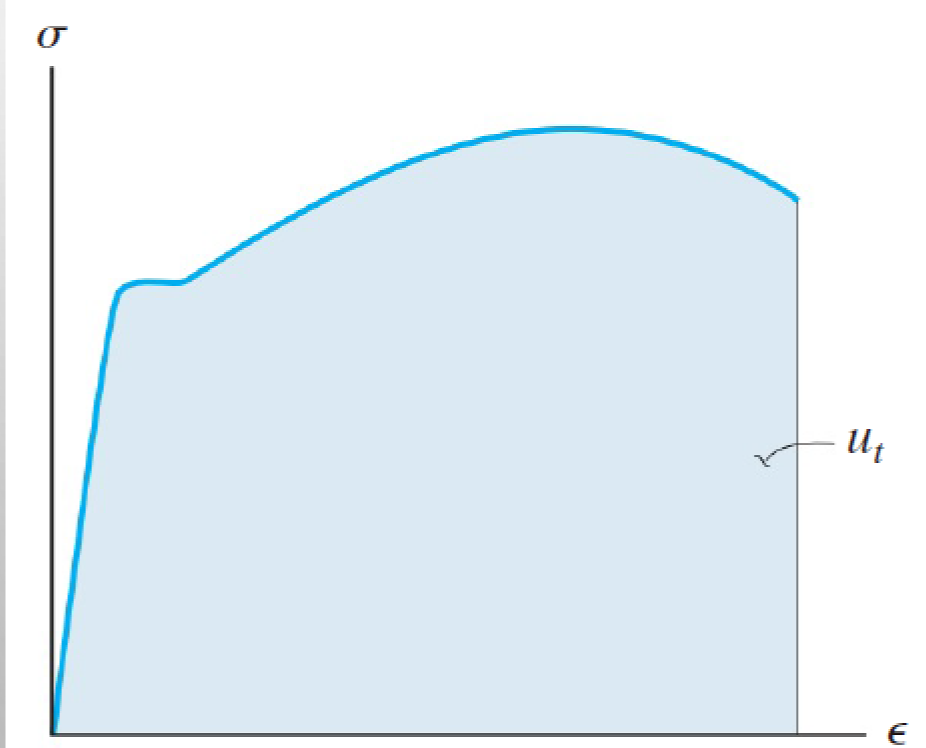
- The strain energy density when the stress in a material reaches the proportional limit.

$$u_{\mathrm{r}} = \frac{1}{2}\sigma_{\mathrm{pl}}\varepsilon_{\mathrm{pl}} = \frac{1}{2}\frac{\varepsilon_{\mathrm{pl}}^{2}}{E}$$

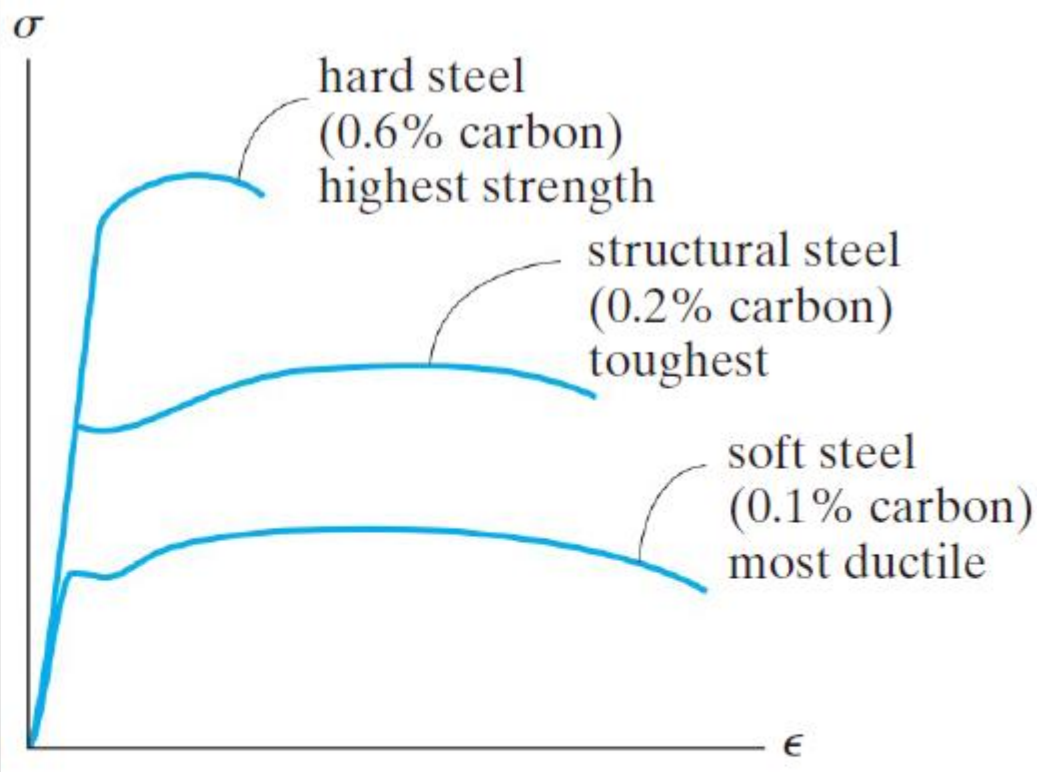


§ 2.12 Modulus of toughness

- This quantity represents the entire area under the stress–strain diagram, and therefore it indicates the maximum amount of strain energy per unit volume the material can absorb just before it fractures.

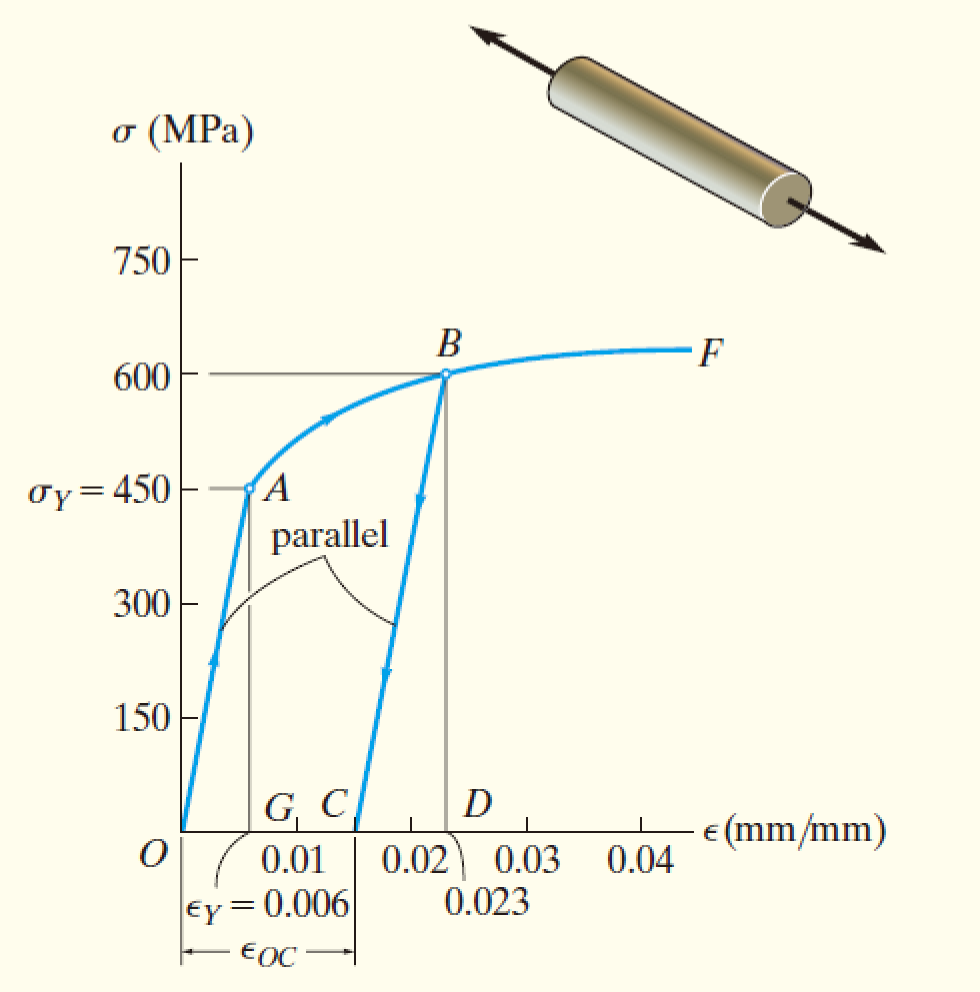


Modulus of toughness u_t



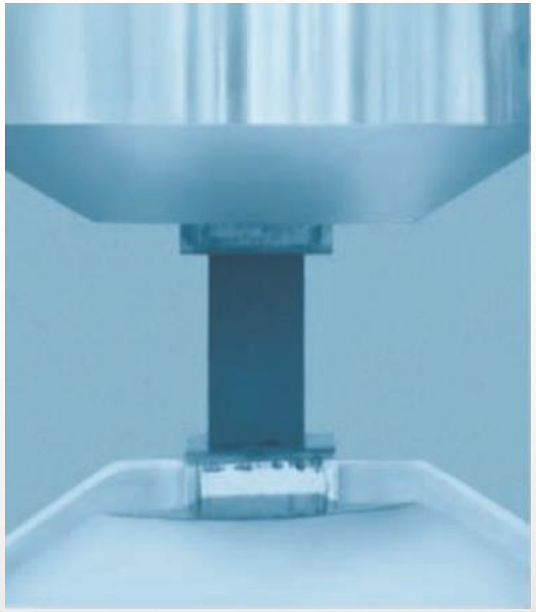
Example 2.3

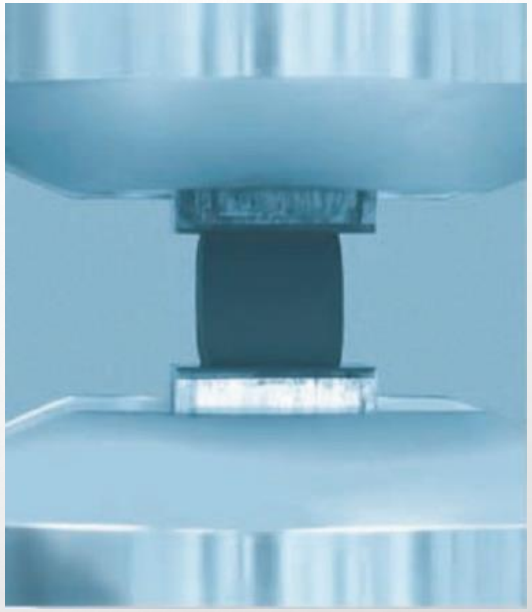
The stress–strain diagram for an aluminum alloy that is used for making aircraft parts is as shown. If a specimen of this material is stressed to σ= 600 MPa, determine the permanent set that remains in the specimen when the load is released. Also, find the modulus of resilience both before and after the load application.



§ 2.13 Poisson's Ratio

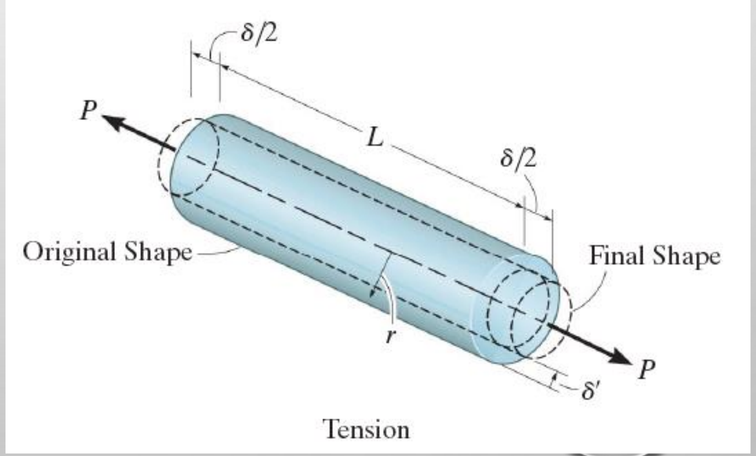
$$\varepsilon_{\text{long}} = \frac{\delta}{L} \qquad \varepsilon_{\text{lat}} = \frac{\delta'}{R}$$





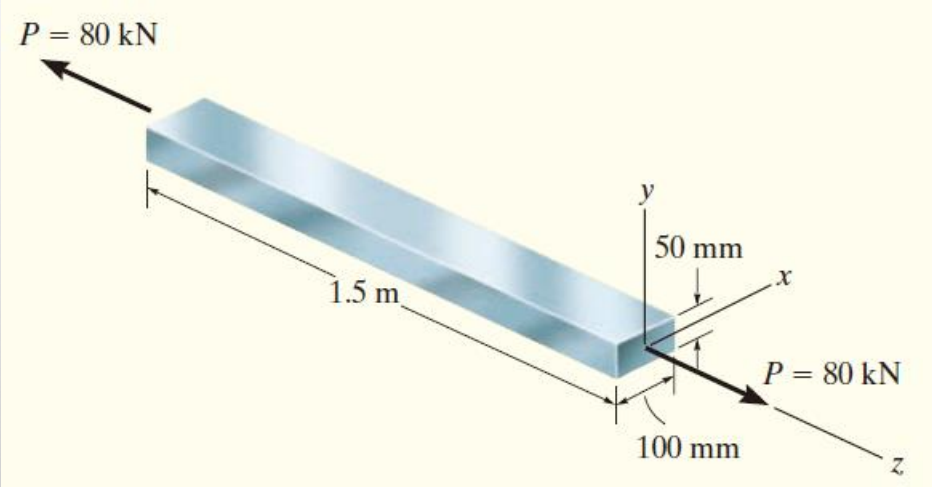
$$\nu = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}}$$

$$0 < \nu < 0.5$$



Example 2.4

A bar made of A-36 steel (E_{st} = 200 Gpa, ν_{st} = 0.32) has the dimensions as shown. If an axial force of P = 80 kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section. The material behaves elastically.

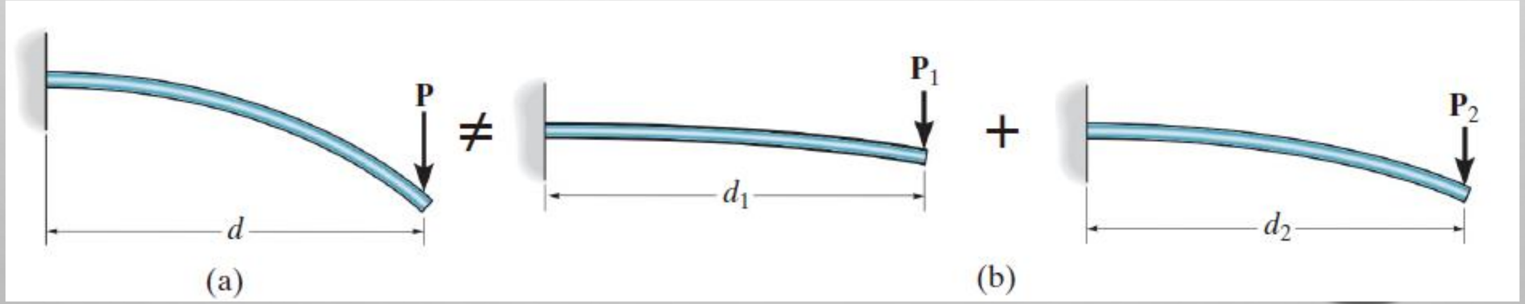


§ 2.14 Principle of superposition

This principle states that the resultant stress or displacement at the point can be determined by algebraically summing the stress or displacement caused by each load component applied separately to the member.

1. The loading N must be linearly related to the stress S ***or displacement*** *D* ***that is to be determined***

2. *The loading must not significantly change the original geometry or configuration of the member.*



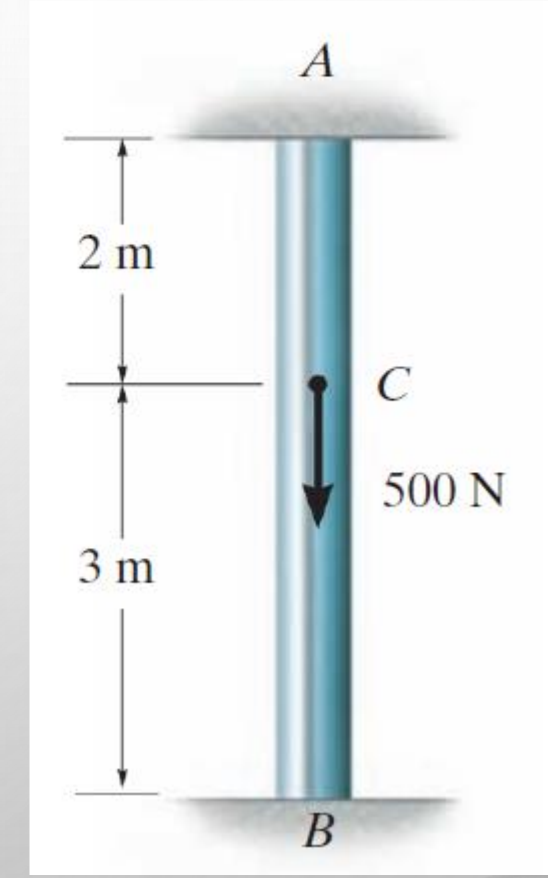
§ 2.15 Statically indeterminate problem

- A problem is called statically indeterminate when the **equilibrium** equation is not sufficient to determine both reactions on the bar.

$$\Sigma F = 0$$

- A **compatibility** or kinematic condition is needed to specify the **displacement**.

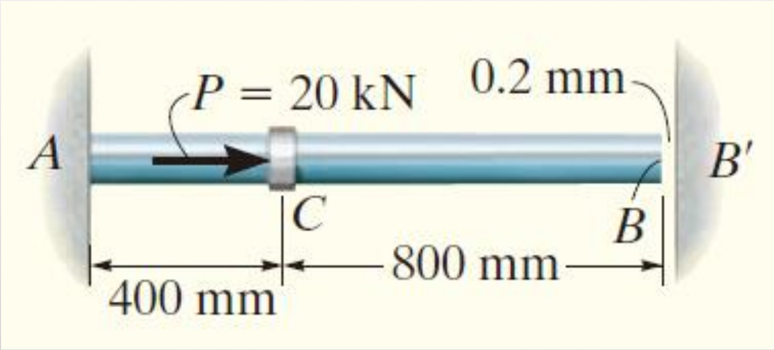
$$\delta = \frac{NL}{EA} \qquad \delta_{A/B} = 0$$



Example 2.5

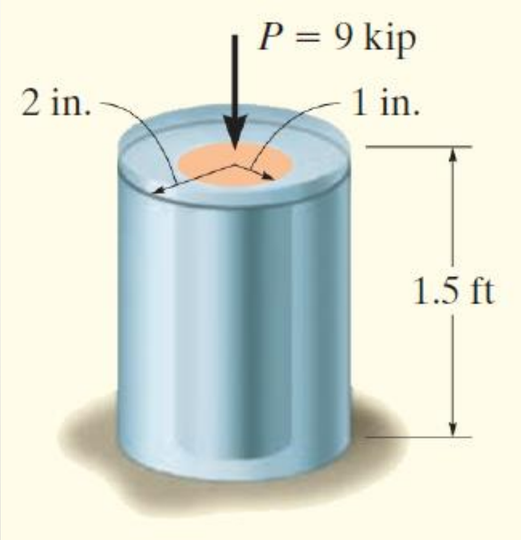
The steel rod has a diameter of 10 mm. It is fixed to the wall at A, and before it is loaded, there is a gap of 0.2 mm between the wall at B′ and the rod. Determine the reactions on the rod if it is subjected to an axial force of P = 20 kN. Neglect the size of the collar at C. Take E_{st} = 200 GPa.

- Equilibrium
- Compatibility
- Load–Displacement



Example 2.6

The aluminum post is reinforced with a brass core. If this assembly supports an axial compressive load of P = 9 kip, applied to the rigid cap, determine the average normal stress in the aluminum and the brass. Take $E_{al} = 10 \times 10^3$ ksi and $E_{br} = 15 \times 10^3$ ksi.



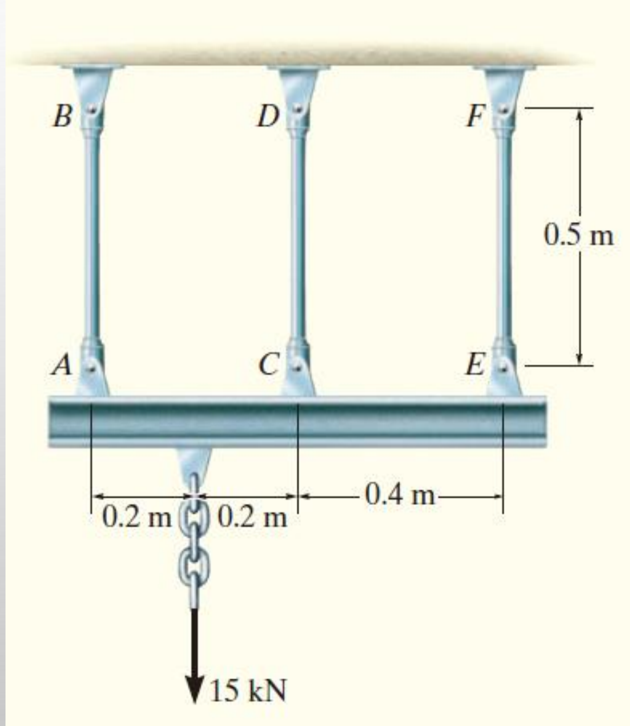
- Equilibrium
- Compatibility
- Load–Displacement

Kip, one thousand pound force.

Example 2.7

The three A992 steel bars are pin connected to a rigid member. If the applied load on the member is 15 kN, determine the force developed in each bar. Bars AB and EF each have a cross-sectional area of 50 mm^2, and bar CD has a cross-sectional area of 30 mm^2.

- Equilibrium
- Compatibility
- Load–Displacement

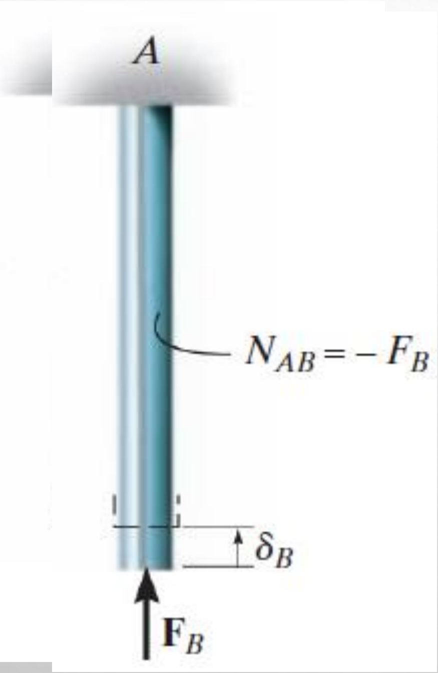


§ 2.16 The force method of analysis

- Solve statically indeterminate problems by writing the compatibility equation using the principle of superposition

$$\delta = \frac{NL}{EA}$$

$$\delta_P - \delta_B = 0$$



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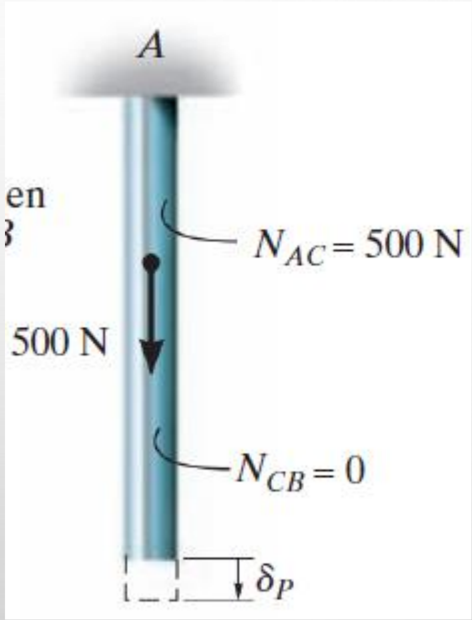

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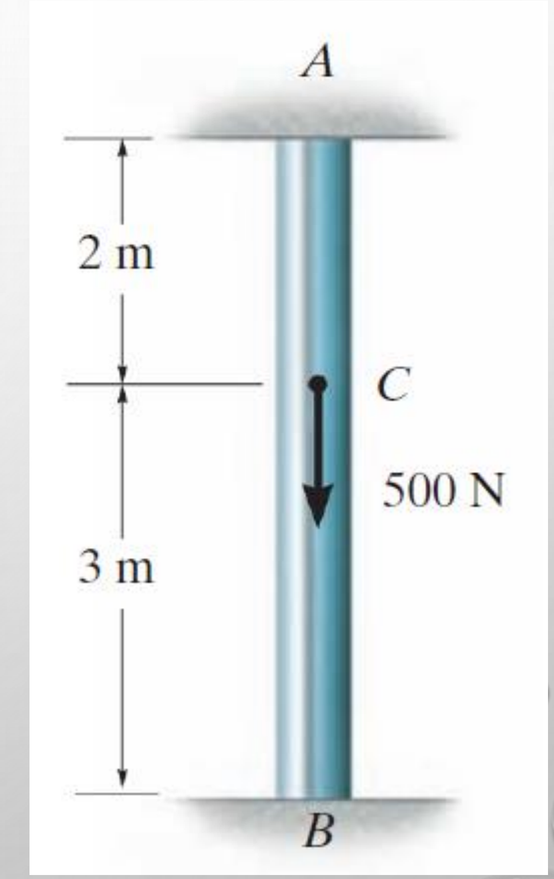
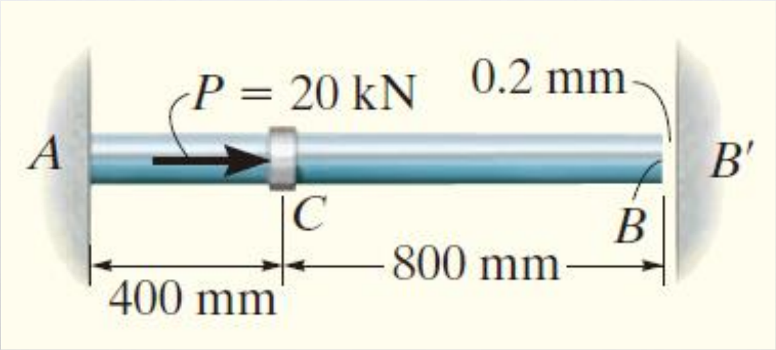


Example 2.8

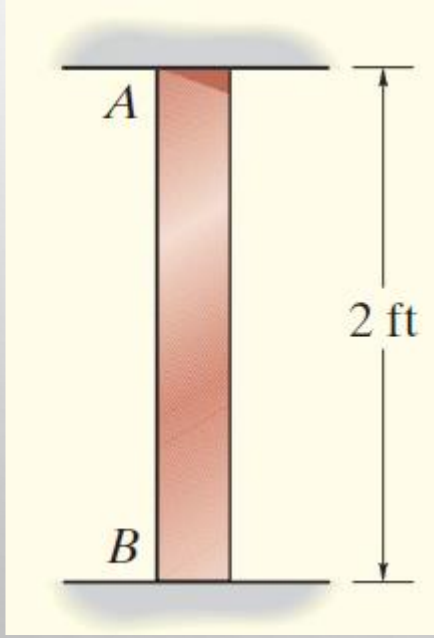
The steel rod has a diameter of 10 mm. It is fixed to the wall at A, and before it is loaded, there is a gap of 0.2 mm between the wall at B′ and the rod. Determine the reactions on the rod if it is subjected to an axial force of P = 20 kN. Neglect the size of the collar at C. Take E_{st} = 200 GPa.

- Compatibility
- Load–Displacement
- Equilibrium



§ 2.17 Thermal stress

- Thermal stresses that must be considered in design for a statically indeterminate member



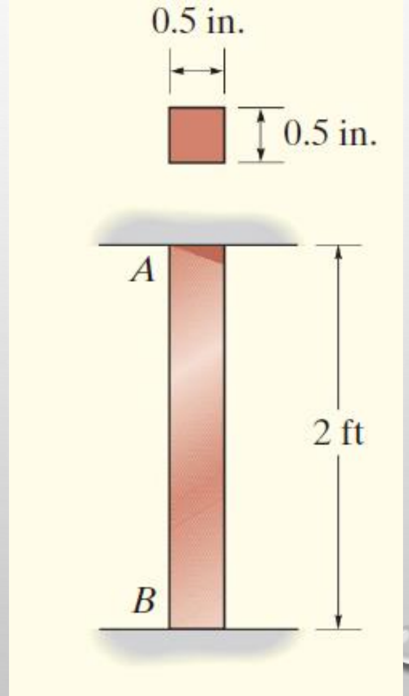
$$\delta_T = \alpha \Delta \mathrm{T} \cdot \mathrm{L}$$

α，linear coefficient of thermal expansion

$$\delta_T = \frac{NL}{EA} \qquad \sigma_T = \frac{E\delta}{L}$$

Example 2.9

The A-36 steel bar is constrained to just fit between two fixed supports when T1 = 60° F. If the temperature is raised to T2 = 120° F, determine the average normal thermal stress developed in the bar. $\alpha=6.6\times10^{-6}/°$ F.

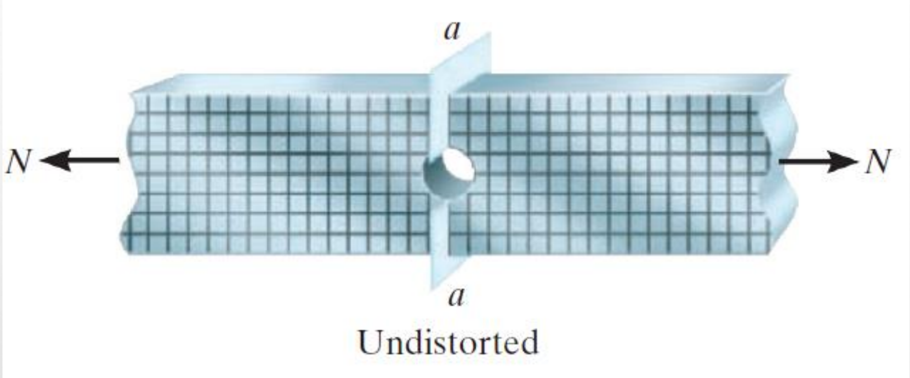


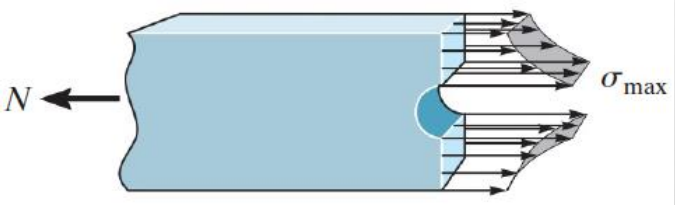
§ 2.18 Stress concentration

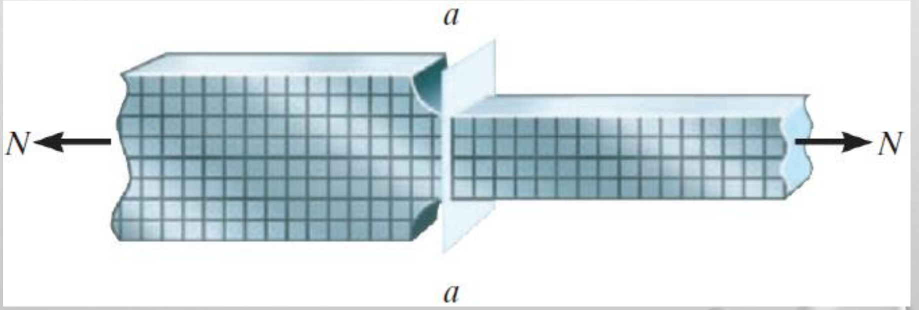
- Stress concentration coefficient, K

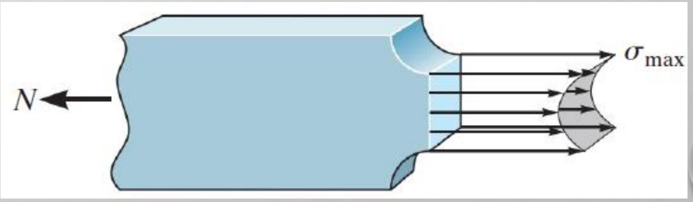
$$N = \int_A \sigma \, dA$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$







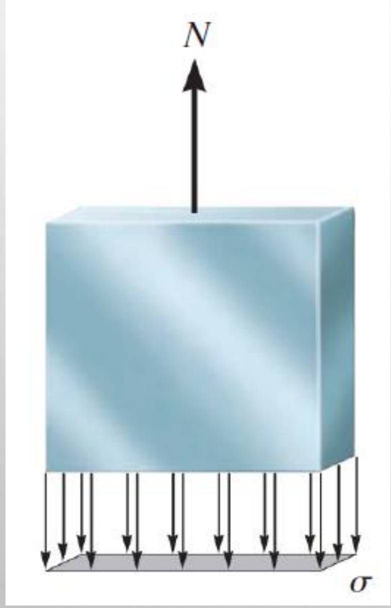


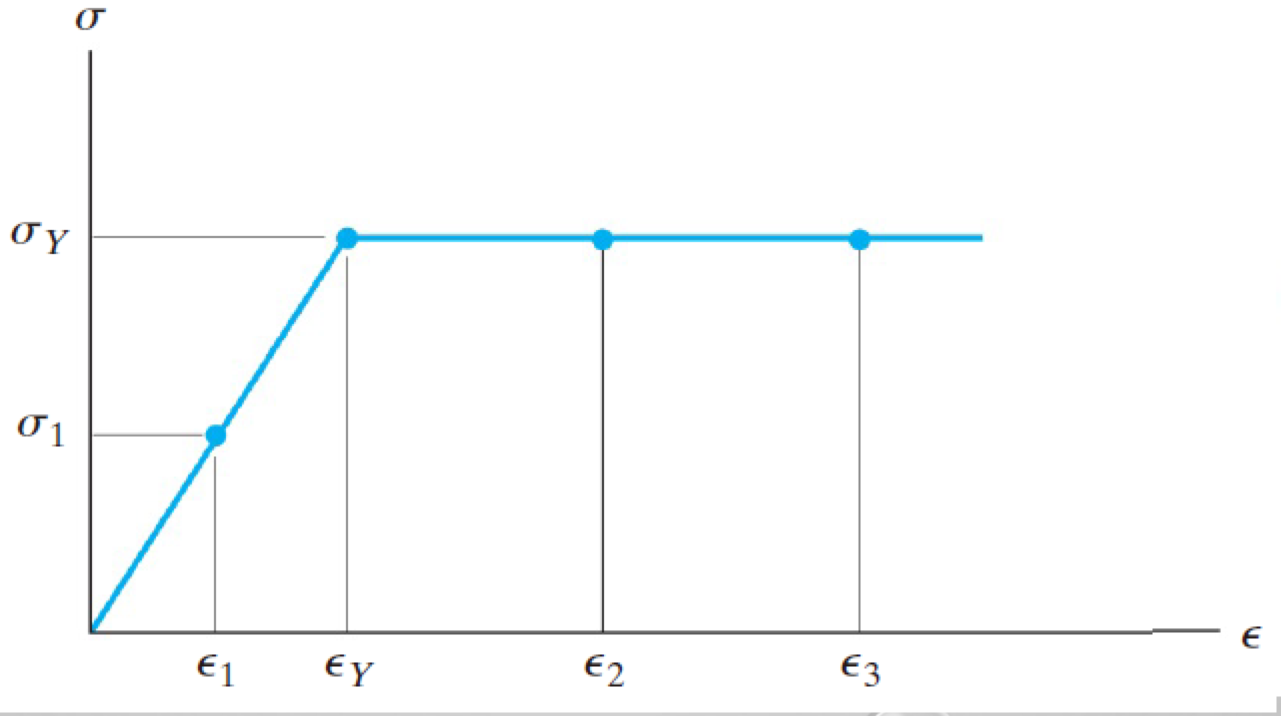
§ 2.19 Elastoplastic behavior

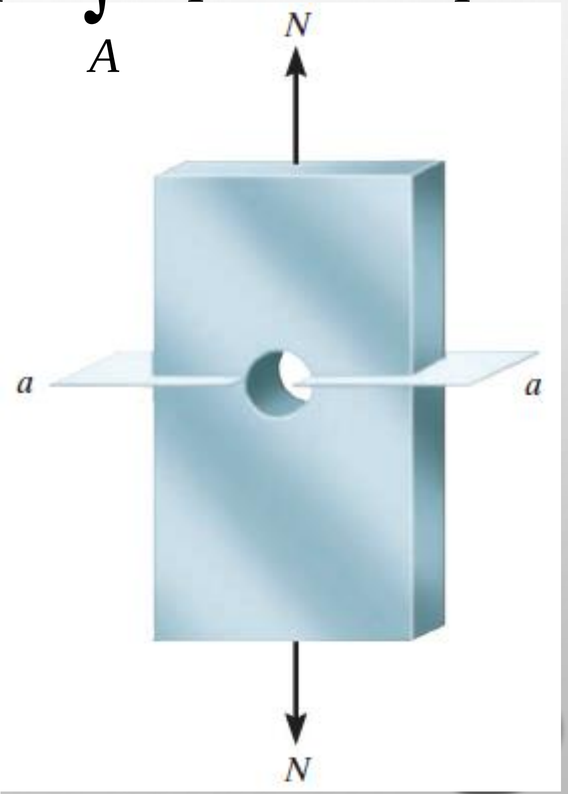
- Elastic perfectly plastic or elastoplastic

Plastic load

$$N_P = \int_A \sigma_Y dA = \sigma_Y A$$

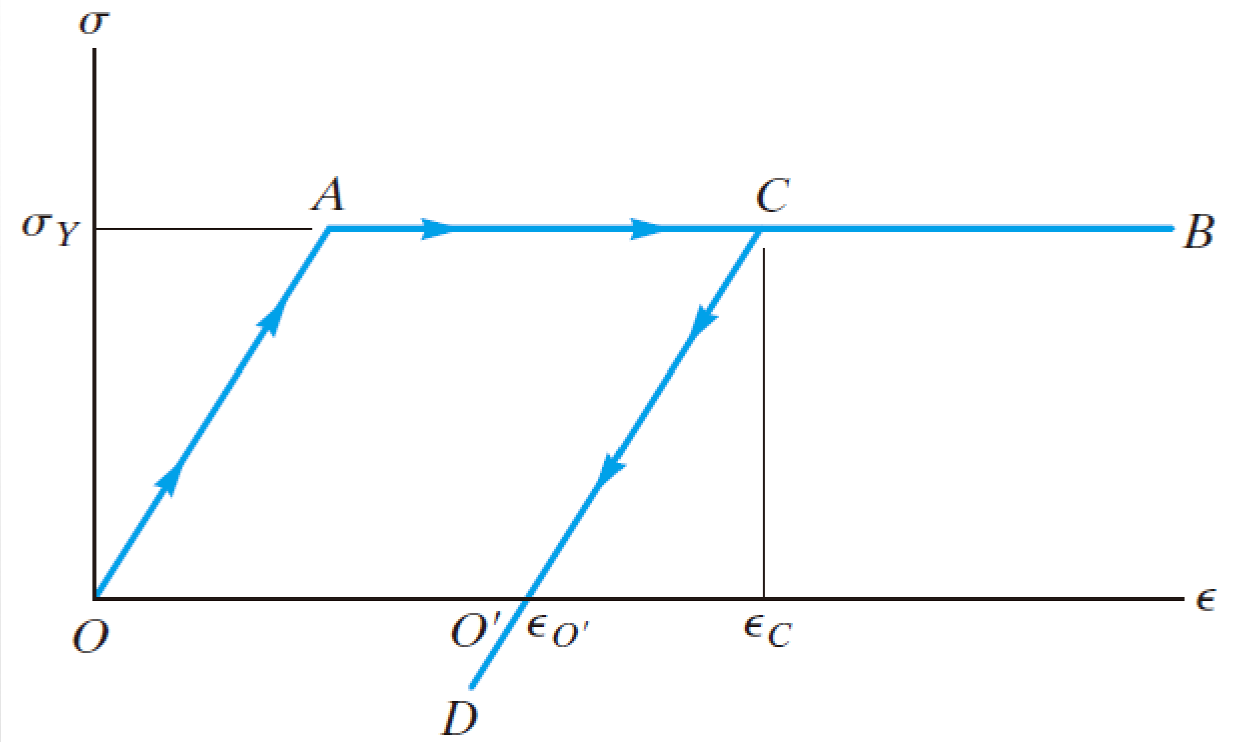






§ 2.20 Residual stress

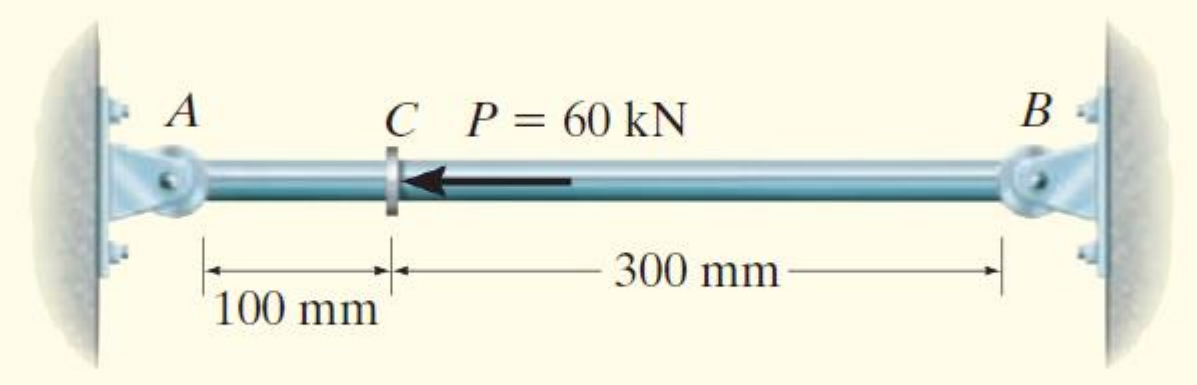
- Residual stress constrains statically indeterminate members from the full recovery after removal of the external load that causes the support forces to respond to the elastic recovery.



The complete cycle of loading and then unloading of the member can be considered as the superposition of a positive load (loading) on a negative load (unloading).

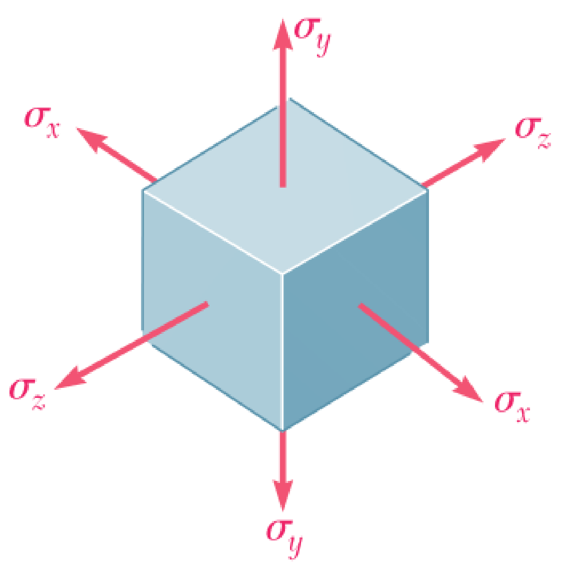
Example 2.10

The rod has a radius of 5 mm and is made of an elastic perfectly plastic material for which $\sigma_Y = 420$ MPa, E = 70 GPa. If a force of P = 60 kN is applied to the rod and then removed, determine the residual stress in the rod.



§ 2.21 Multiaxial loading; Generalized Hook's law

- Normal stresses σ_x , σ_y , and σ_z are all different from zero.



Generalized Hooke's Law by principle of superposition

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

§ 2.22 Dilation; Bulk modulus

- The effect of the normal stresses σ_x , σ_y , and σ_z on the volume of an element of isotropic material.

$$v = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$$

$$\approx 1+\varepsilon_x+\varepsilon_y+\varepsilon_z$$

Volume change,

$$e = v - 1 = \varepsilon_x+\varepsilon_y+\varepsilon_z$$

$$e = \frac{1-2\nu}{E}\left(\sigma_x+\sigma_y+\sigma_z\right)$$

For a body subjected to a uniform hydrostatic pressure *p*,

$$e = -\frac{3\left(1-2\nu\right)}{E}p$$

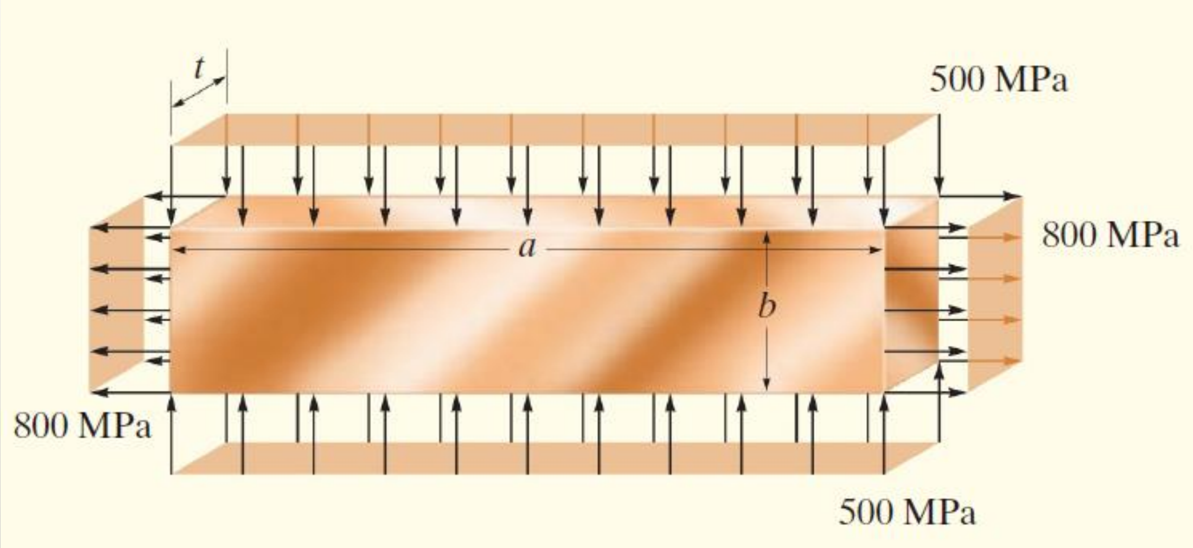
$$k = \frac{E}{3\left(1-2\nu\right)}$$

Bulk modulus

$$0 < \nu < 0.5$$

Example 2.11

The copper bar is subjected to a uniform loading as shown. If it has a length a = 300 mm, width b = 50 mm, and thickness t = 20 mm before the load is applied, determine its new length, width, and thickness after application of the load. Take E_{cu} = 120 GPa, ν_{cu} = 0.34.



§ 2.21 Summary

- **Saint Venant's principle**
- **Force analysis**：Internal force, stress, and deformation, strain analysis of a member under axial load;
- **Stress-strain diagram:** elastic, plastic, necking, strength condition, strain hardening
- **Strain energy:** modulus of resilience, modulus of toughness
- **Other concepts:** thermal stress, stress concentration, residual stress, Multiaxial loading, Generalized Hook's law, Dilation; Bulk modulus

Average Mechanical Properties of Typical Engineering Materials[a]

(SI Units)

Materials		Density ρ (Mg/m^3)	Moduls of Elasticity E (GPa)	Modulus of Rigidity G (GPa)	Yield Strength (MPa) σ_Y Tens.	Comp.[b]	Shear	Ultimate Strength (MPa) σ_u Tens.	Comp.[b]	Shear	% Elongation in 50 mm specimen	Poisson's Ratio ν	Coef. of Therm. Expansion α $(10^{-6})/°C$
Metallic													
Aluminum Wrought Alloys	2014-T6	2.79	73.1	27	414	414	172	469	469	290	10	0.35	23
	6061-T6	2.71	68.9	26	255	255	131	290	290	186	12	0.35	24
Cast Iron Alloys	Gray ASTM 20	7.19	67.0	27	–	–	–	179	669	–	0.6	0.28	12
	Malleable ASTM A-197	7.28	172	68	–	–	–	276	572	–	5	0.28	12
Copper Alloys	Red Brass C83400	8.74	101	37	70.0	70.0	–	241	241	–	35	0.35	18
	Bronze C86100	8.83	103	38	345	345	–	655	655	–	20	0.34	17
Magnesium Alloy	[Am 1004-T61]	1.83	44.7	18	152	152	–	276	276	152	1	0.30	26
Steel Alloys	Structural A-36	7.85	200	75	250	250	–	400	400	–	30	0.32	12
	Structural A992	7.85	200	75	345	345	–	450	450	–	30	0.32	12
	Stainless 304	7.86	193	75	207	207	–	517	517	–	40	0.27	17
	Tool L2	8.16	200	75	703	703	–	800	800	–	22	0.32	12
Titanium Alloy	[Ti-6Al-4V]	4.43	120	44	924	924	–	1,000	1,000	–	16	0.36	9.4
Nonmetallic													
Concrete	Low Strength	2.38	22.1	–	–	–	12	–	–	–	–	0.15	11
	High Strength	2.37	29.0	–	–	–	38	–	–	–	–	0.15	11
Plastic Reinforced	Kevlar 49	1.45	131	–	–	–	–	717	483	20.3	2.8	0.34	–
	30% Glass	1.45	72.4	–	–	–	–	90	131	–	–	0.34	–
Wood Select Structural Grade	Douglas Fir	0.47	13.1	–	–	–	–	2.1[c]	26[d]	6.2[d]	–	0.29[e]	–
	White Spruce	3.60	9.65	–	–	–	–	2.5[c]	36[d]	6.7[d]	–	0.31[e]	–

[a] Specific values may vary for a particular material due to alloy or mineral composition,mechanical working of the specimen,or heat treatment. For a more exact value reference books for the material should be consulted.

[b] The yield and ultimate strengths for ductile materials can be assumed equal for both tension and compression.

[c] Measured perpendicular to the grain.

[d] Measured parallel to the grain.

[e] Deformation measured perpendicular to the grain when the load is applied along the grain.