第 4 周习题
常微分方程 B

March 8, 2022

1. Solve the following equations.

   (a) $y' = ry - ky^2$, where $r, k > 0$;

   (b) $y' + x^{-1}y = xy^2$;

   (c) $\dfrac{dy}{dx} = \dfrac{x+1}{x-y}$;

   (d) $y' = (4x - y + 1)^2$.

2. To find a particular solution of a Riccati equation that involves only powers of the variables, such as

$$y' = -\frac{4}{x^2} - \frac{1}{x}y + y^2, \tag{1}$$

a function of the form $y_1 = cx^a$ would always be a good guess, where a, c are constants to be determined. Find a particular solution of the form $y_1 = cx^a$ for (1) and solve the equation.

3. Consider the the initial value problem

$$y' = y(3 - ty), \quad y(0) = 0.5. \tag{2}$$

   (a) Use Euler's method to find approximate values of the solution of (2) at $t = 1, 1.5, 2, 2.5$, and 3: (i) With $h = 0.1$, (ii) With $h = 0.05$, (iii) With $h = 0.025$, (iv) With $h = 0.01$.

   (b) Note that (2) is a Bernoulli equation, find the exact solution $y = \phi(t)$ of the initial value problem and evaluate $\phi(t)$ at $t = 1, 1.5, 2, 2.5$, and 3. Compare these values with the results of (a).