Quiz 9

Date: 2022-04-11 Name: SID:

Find the general solutions of the following two equations by the **Method of Undetermined Coefficients.*

1. $y'' + 2y' - 3y = 5sin3t;$
2. $y'' + 16y = e^{-4t} + 3sin4t.$

Find the general solution of the following equation by the **Variation of Parameters.*

3. $y'' - y' - 2y = e^{2t}.$

1. $y'' + 2y' - 3y = 5\sin 3t$.

$\lambda^2 + 2\lambda - 3 = 0$. $(\lambda+3)(\lambda-1) = 0$

$y_0 = c_1 e^{-3t} + c_2 e^{t}$

Let $y_1 = A\cos 3t + B\sin 3t$.

$y_1' = -3A\sin 3t + 3B\cos 3t$.

$y_1'' = -9A\cos 3t - 9B\sin 3t$.

$$\begin{cases} 6B - 12A = 0 \\ -12B - 6A = 5 \end{cases} \Rightarrow \begin{cases} A = -\frac{1}{6} \\ B = -\frac{1}{3} \end{cases}$$

$y = c_1 e^{-3t} + c_2 e^{t} - \frac{1}{6}\cos 3t - \frac{1}{3}\sin 3t$.

2. $y'' + 16y = e^{-4t} + 3\sin 4t$.

$\lambda^2 + 16 = 0$. $\lambda = \pm 4i$

$y_0 = C_1\cos 4t + C_2\sin 4t$

Let $y_1 = Ae^{-4t}$ $\quad$ $y_2 = Bt\cos 4t + Ct\sin 4t$

$y_1' = -4Ae^{-4t}$ $\quad$ $y_2' = B\cos 4t + C\sin 4t$

$\quad\quad\quad\quad\quad\quad - 4Bt\sin 4t + 4Ct\cos 4t$

$y_1'' = 16Ae^{-4t}$ $\quad$ $y_2'' = -8B\sin 4t + 8C\cos 4t$

$$-16B t\cos 4t - 16 C t \sin 4t$$

$$A = \frac{1}{32} \qquad B = -\frac{3}{8} \qquad C = 0$$

general solution is

$$y = C_1 \cos 4t + C_2 \sin 4t + \frac{1}{32} e^{-4t} - \frac{3}{8} t \cos 4t$$

3. $y'' - y' - 2y = e^{2t}$

$\lambda^2 - \lambda - 2 = 0 \qquad (\lambda - 2)(\lambda + 1) = 0$

$y = C_1 e^{2t} + C_2 e^{-t}$

Let $y = C_1(t) e^{2t} + C_2(t) e^{-t}$

$$\begin{cases} C_1' e^{2t} + C_2' e^{-t} = 0 \\ 2C_1' e^{2t} - C_2' e^{-t} = e^{2t} \end{cases} \Rightarrow \begin{cases} C_1' = \frac{1}{3} \\ C_2' = -\frac{1}{3} e^{3t} \end{cases}$$

$$\Rightarrow \begin{cases} C_1(t) = \frac{1}{3} t + C_1 \\ C_2(t) = -\frac{1}{9} e^{3t} + C_2 \end{cases}$$

general solution is

$$y = C_1 e^{2t} + C_2 e^{-t} + \frac{1}{3} t e^{2t} - \frac{1}{9} e^{2t}$$