Homework problems 49-52
Due in class, Friday, 18 December 2020

49. Determine the slope and deflection of end A of the cantilevered beam using the method of integration. E = 200 GPa and $I = 65.0(10^6)$ mm^4.

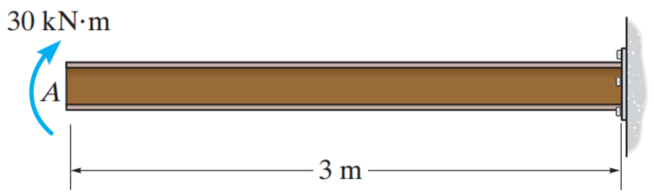


Figure 49

Use left segment,

$M(x) = 30\ \text{kN}\cdot\text{m}$

$$EI\frac{d^2v}{dx^2} = 30$$

$$EI\frac{dv}{dx} = 30x + C_1$$

$$EIv = 15x^2 + C_1 x + C_2$$

At $x = 3$ m, $\dfrac{dv}{dx} = 0.$

$C_1 = -90\ \text{kN}\cdot\text{m}^2$

At $x = 3$ m, $v = 0.$

$C_2 = 135\ \text{kN}\cdot\text{m}^3$

$$\frac{dv}{dx} = \frac{1}{EI}(30x - 90)$$

$$v = \frac{1}{EI}(15x^2 - 90x + 135)$$

For end A, $x = 0$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{90(10^3)}{200(10^9)[65.0(10^{-6})]} = -0.00692\ \text{rad}$$

Ans.

$$v_A = v|_{x=0} = \frac{135(10^3)}{200(10^9)[65.0(10^{-6})]} = 0.01038\ \text{m} = 10.4\ \text{mm}$$

Ans.

50. The pipe assembly consists of three equal-sized pipes with flexibility stiffness EI and torsional stiffness GJ. Determine the vertical deflection at A.

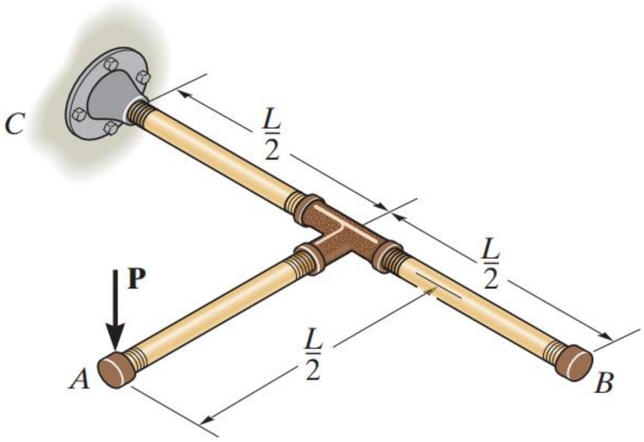
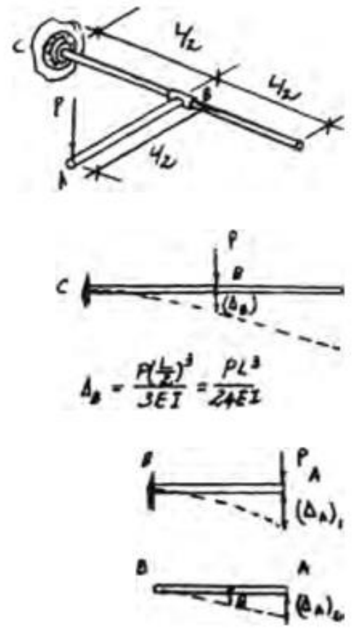


Figure 50



$$\Delta_B = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI}$$

$$(\Delta_A)_1 = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI}$$

$$\theta = \frac{TL}{JG} = \frac{(PL/2)\left(\frac{L}{2}\right)}{JG} = \frac{PL^2}{4JG}$$

$$(\Delta_A)_2 = \theta\left(\frac{L}{2}\right) = \frac{PL^3}{8JG}$$

$$\Delta_A = \Delta_B + (\Delta_A)_1 + (\Delta_A)_2$$

$$= \frac{PL^3}{24EI} + \frac{PL^3}{24EI} + \frac{PL^3}{8JG}$$

$$= PL^3\left(\frac{1}{12EI} + \frac{1}{8JG}\right)$$

51. Using the method of integration to determine the moment reactions at the supports A and B, then draw the shear and moment diagrams. Solve by expressing the internal moment in the beam in terms of A_y and M_A. EI is constant.

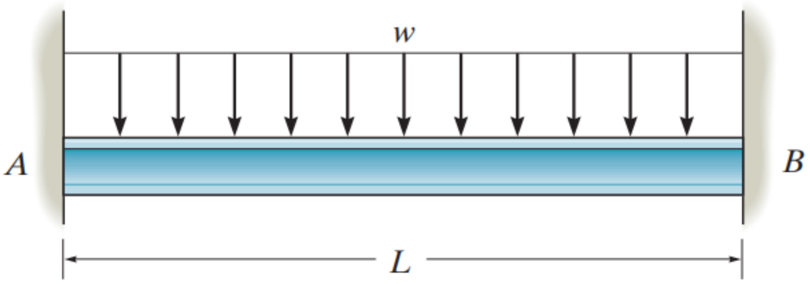
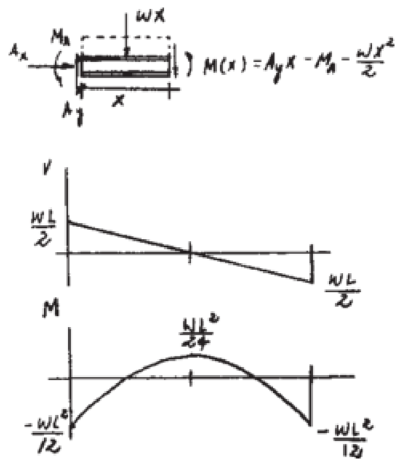


Figure 51

SOLUTION



$$M(x) = A_y x - M_A - \frac{wx^2}{2}$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x) = A_y x - M_A - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{A_y x^2}{2} - M_A x - \frac{wx^3}{6} + C_1 \quad \textbf{(1)}$$

$$EIv = \frac{A_y x^3}{6} - \frac{M_A x^2}{2} - \frac{wx^4}{24} + C_1 x + C_2 \quad \textbf{(2)}$$

Boundary Conditions:

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = 0$$

From Eq. (1)

$$C_1 = 0$$

$$v = 0 \quad \text{at} \quad x = 0$$

From Eq. (2)

$$C_2 = 0$$

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = L$$

From Eq. (1)

$$0 = \frac{A_y L^2}{2} - M_A L - \frac{wL^3}{6} \quad \textbf{(3)}$$

$$v = 0 \quad \text{at} \quad x = L$$

From Eq. (2)

$$0 = \frac{A_y L^3}{6} - \frac{M_A L^2}{2} - \frac{wL^4}{24} \quad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

$$A_y = \frac{wL}{2}$$

$$M_A = \frac{wL^2}{12} \quad \textbf{Ans.}$$

Due to symmetry:

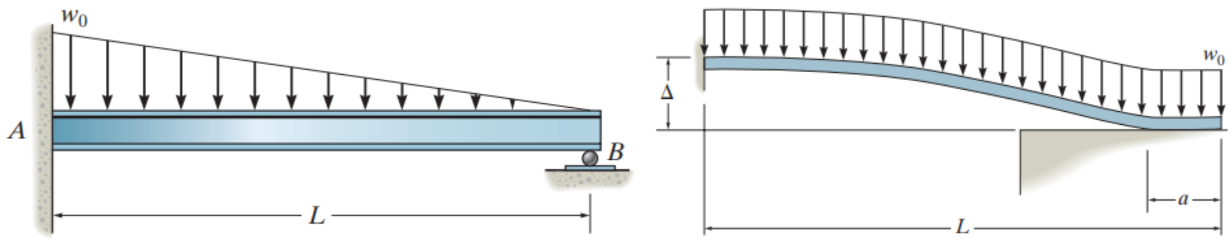
$$M_B = \frac{wL^2}{12} \quad \textbf{Ans.}$$

Ans:

$M_A = \frac{wL^2}{12}$,

$M_B = \frac{wL^2}{12}$

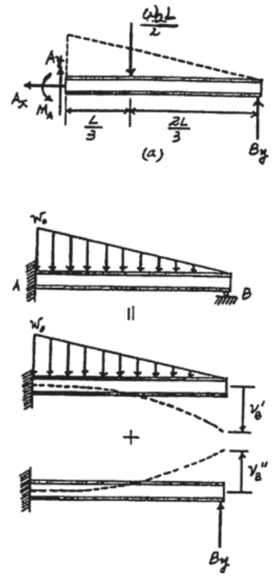
52. (a) Determine the reactions at the supports A and B. EI is constant. (b) The beam is made from a soft linear elastic material having a constant EI. If it is originally a distance Δ from the surface of its end support, determine the length a that rests on this support when it is subjected to the uniform load w_o, which is great enough to cause this to happen.



(a) (b)

Figure 52

SOLUTION



Support Reactions: FBD(a).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + M_A - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \qquad \textbf{(2)}$$

Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B{}' = \frac{w_0 L^4}{30EI}\downarrow \qquad v_B{}'' = \frac{B_y L^3}{3EI}\uparrow$$

The compatibility condition requires

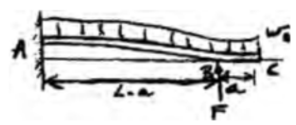
$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}''$$

$$0 = \frac{w_0 L^4}{30EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$B_y = \frac{w_0 L}{10} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs. (1) and (2) yields,

$$A_y = \frac{2w_0 L}{5} \qquad M_A = \frac{w_0 L^2}{15} \qquad \textbf{Ans.}$$



The curvature of the beam in region BC is zero, therefore there is no bending moment in the region BC, The reaction F is at B where it touches the support. The slope is zero at this point and the deflection is Δ where

$$\Delta = \frac{w_0(L-a)^4}{8EI} - \frac{R(L-a)^3}{3EI}$$

$$\theta_1 = \frac{w_0(L-a)^3}{6EI} - \frac{R(L-a)^2}{2EI}$$

Thus,

$$R = \left(\frac{8\Delta EI}{9w_0^3}\right)^{\frac{1}{4}} \qquad \textbf{Ans.}$$

$$L - a = \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}}$$

$$a = L - \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}} \qquad \textbf{Ans.}$$