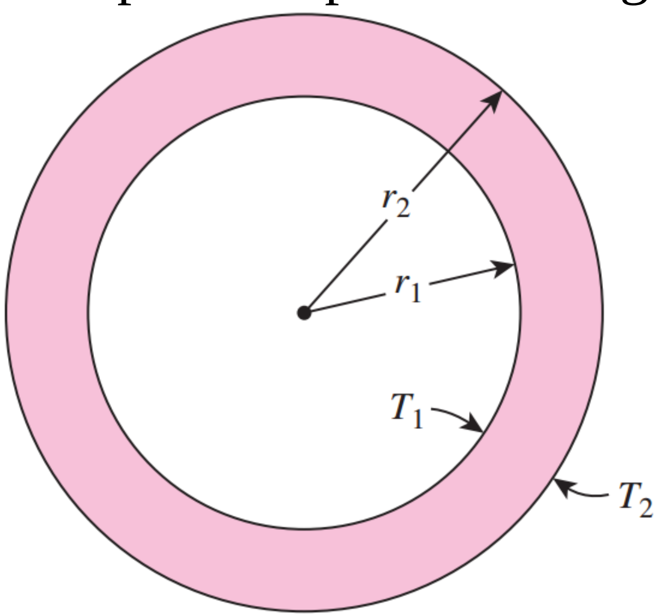
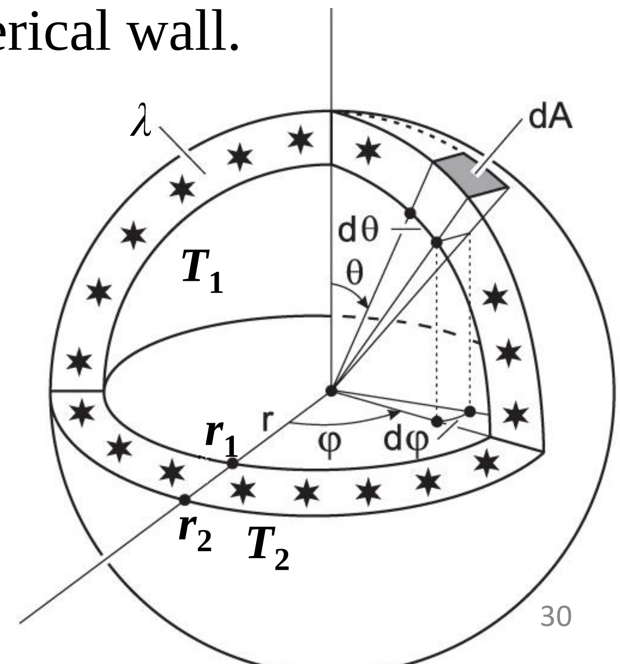
One Dimensional Conduction

- Steady Problem without Heat Source
  - Conduction through a spherical wall (shell)

    Consider a spherical wall with the inner and outer radii of r_1 and r_2, respectively, and the thermal conductivity λ. The temperatures at the inner and outer walls are T_1 and T_2, respectively. Determine the temperature profile through the spherical wall.





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Solution:

Governing equation:

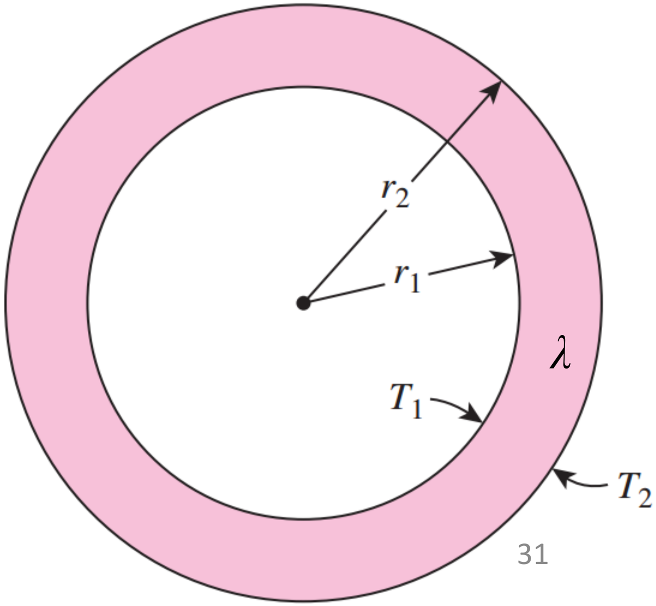
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda r^2\frac{\partial T}{\partial r}\right)=0$$

Boundary Condition:

$$r = r_1,\ T = T_1;\ r = r_2,\ T = T_2$$

Integrate the governing equation twice and implement the boundary condition

$$T = T_2 + \left(T_1 - T_2\right)\frac{1/r - 1/r_2}{1/r_1 - 1/r_2}$$



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<u>*Solution:*</u>

Heat flux:

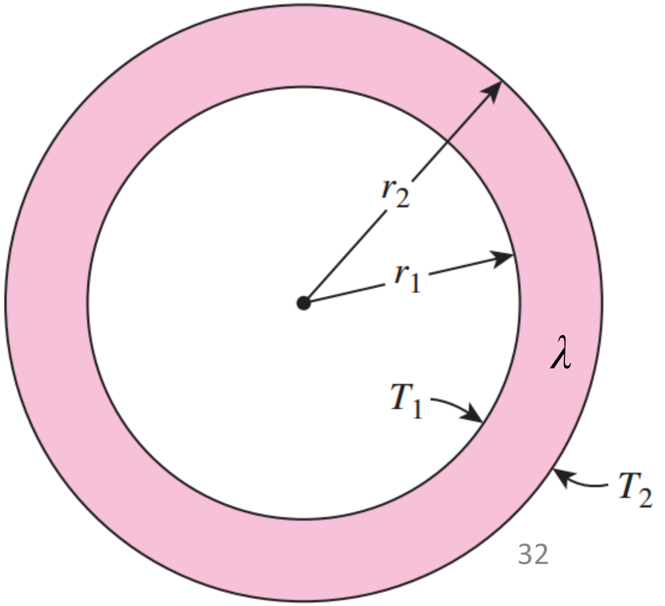
$$q = -\lambda \frac{dT}{dr} = \frac{\lambda}{r^2} \frac{T_1 - T_2}{1/r_1 - 1/r_2}$$

Heat transfer rate:

$$\Phi = qA = 4\pi r^2 \frac{\lambda}{r^2} \frac{T_1 - T_2}{1/r_1 - 1/r_2} = 4\pi\lambda \frac{T_1 - T_2}{1/r_1 - 1/r_2}$$

Thermal resistance

$$R = \frac{T_1 - T_2}{\Phi} = \frac{1}{4\pi\lambda}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$



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Thermal resistance:

$$R_t = R_1 + R_2 + R_3 = \frac{1}{4\pi\lambda_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi\lambda_2}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \frac{1}{4\pi\lambda_3}\left(\frac{1}{r_3} - \frac{1}{r_4}\right)$$

$$\Phi = \frac{T_1 - T_4}{R_t}$$

$$= \frac{4\pi\left(T_1 - T_4\right)}{\frac{1}{\lambda_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{\lambda_2}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \frac{1}{\lambda_3}\left(\frac{1}{r_3} - \frac{1}{r_4}\right)}$$