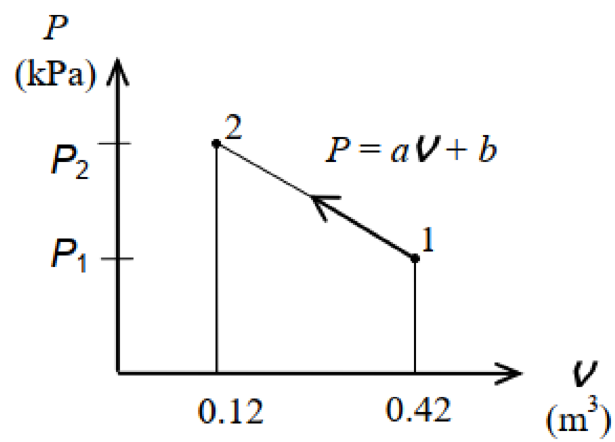
Homework 3 ANS

1. A gas is compressed from an initial volume of 0.42 m^3 to a final volume of 0.12m^3. During the quasi-equilibrium process, pressure changes with volume according to the relation $P = aV + b$, where $a = -1200kPa/m^3$ and b = 600 kPa. Calculate the work done during this process (a) by plotting the process on a *P-V* diagram and finding the area under the process curve and (b) by performing the necessary integrations.

ANS: **(a)** The pressure of the gas changes linearly with volume, and thus the process curve on a P-V diagram will be a straight line. The boundary work during this process is simply the area under the process curve, which is a trapezoidal. Thus,



$$P_1 = aV_1 + b = \left(-1200\text{kPa}/\text{m}^3\right)\left(0.42\text{m}^3\right) + (600\text{kPa}) = 96\text{kPa}$$

$$P_2 = aV_2 + b = \left(-1200\text{kPa}/\text{m}^3\right)\left(0.12\text{m}^3\right) + (600\text{kPa}) = 456\text{kPa}$$

$$W_{b,\text{ out}} = \text{Area} = \frac{P_1 + P_2}{2}\left(V_2 - V_1\right)$$

$$= \frac{(96+456)\text{kPa}}{2}(0.12-0.42)\text{m}^3\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right) = \mathbf{-82.8kJ}$$

(b) The boundary work can also be determined by integration to be

$$W_{b,\text{ out}} = \int_1^2 PdV = \int_1^2 (aV+b)dV = a\frac{V_2^2 - V_1^2}{2} + b\left(V_2 - V_1\right)$$

$$= \left(-1200\text{kPa}/\text{m}^3\right)\frac{\left(0.12^2 - 0.42^2\right)\text{m}^6}{2} + (600\text{kPa})(0.12-0.42)\text{m}^3 = \mathbf{-82.8kJ}$$

2. A piston–cylinder device contains 0.15 kg of air initially at 2 MPa and 350°C. The air is first expanded isothermally to 500 kPa, then compressed polytropically with a polytropic exponent of 1.2 to the initial pressure, and finally compressed at the constant pressure to the initial state. Determine the boundary work for each process and the net work of the cycle.

ANS: For the isothermal expansion process:

$$V_1 = \frac{mRT}{P_1} = \frac{(0.15\text{kg})(0.287\text{kJ}/\text{kg}\cdot\text{K})(350+273\text{K})}{(2000\text{kPa})} = 0.01341\text{m}^3$$

$$V_2 = \frac{mRT}{P_2} = \frac{(0.15\text{kg})(0.287\text{kJ}/\text{kg}\cdot\text{K})(350+273\text{K})}{(500\text{kPa})} = 0.05364\text{m}^3$$

$$W_{b,1-2} = P_1V_1\ln\left(\frac{V_2}{V_1}\right) = (2000\text{kPa})\left(0.01341\text{m}^3\right)\ln\left(\frac{0.05364\text{m}^3}{0.01341\text{m}^3}\right) = \mathbf{37.18kJ}$$

For the polytropic compression process:

$$P_2V_2^n = P_3V_3^n \qquad (500\text{kPa})\left(0.05364\text{m}^3\right)^{1.2} = (2000\text{kPa})V_3^{1.2} \qquad V_3 = 0.01690\text{m}^3$$

$$W_{b,2-3} = \frac{P_3V_3 - P_2V_2}{1-n} = \frac{(2000\text{kPa})\left(0.01690\text{m}^3\right) - (500\text{kPa})\left(0.05364\text{m}^3\right)}{1-1.2} = \mathbf{-34.86kJ}$$

For the constant pressure compression process:

$$W_{b,3-1} = P_3\left(V_1 - V_3\right) = (2000\text{kPa})(0.01341-0.01690)\text{m}^3 = \mathbf{-6.97kJ}$$

For the polytropic compression process:

$$W_{\text{net}} = W_{b,1-2} + W_{b,2-3} + W_{b,3-1} = 37.18 + (-34.86) + (-6.97) = \mathbf{-4.65kJ}$$

3. A 0.5-m^3 rigid tank contains refrigerant-134a initially at 160 kPa and 40 percent quality. Heat is now transferred to the refrigerant until the pressure reaches 700 kPa. Determine (a) the mass of the refrigerant in the tank and (b) the amount of heat transferred. Also, show the process on a *P-V* diagram with respect to saturation lines.

ANS: **(a)** We take the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$Q_{in} = \Delta U = m(u_2 - u_1) \quad (\text{since } W = \text{KE} = \text{PE} = 0)$$

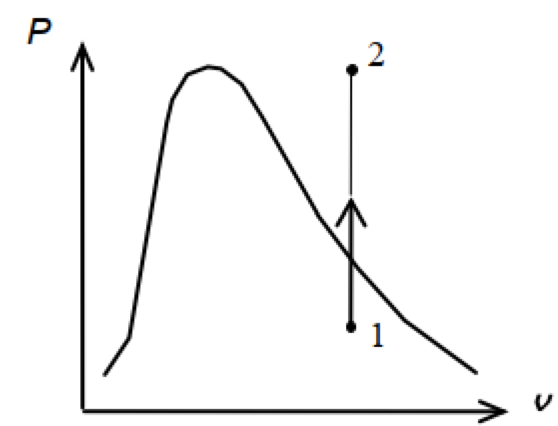
Using data from refrigerant tables(Tables A-11 through A-13), the properties of R-134a are determined to be

$$\left.\begin{array}{l} P_1 = 160\text{kPa} \\ x_1 = 0.4 \end{array}\right\} v_f = 0.0007435, \quad v_g = 0.12355\text{m}^3/\text{kg}; \quad u_f = 31.06, \quad u_{fg} = 190.31\text{kJ}/\text{kg}$$

$$v_1 = v_f + x_1 v_{fg} = 0.0007435 + 0.4(0.12355 - 0.0007435) = 0.04987\text{m}^3/\text{kg}$$

$$u_1 = u_f + x_1 u_{fg} = 31.06 + 0.4(190.31) = 107.19\text{kJ}/\text{kg}$$

$$\left.\begin{array}{l} P_2 = 700\text{kPa} \\ (V_2 = V_1) \end{array}\right\} u_2 = 377.23\text{kJ}/\text{kg} \quad (\text{Superheated vapor})$$



Then the mass of the refrigerant is determined to be

$$m = \frac{V_1}{v_1} = \frac{0.5\text{m}^3}{0.04987\text{m}^3/\text{kg}} = \mathbf{10.03kg}$$

(b) Then the heat transfer to the tank becomes

$$Q_{in} = m(u_2 - u_1) = (10.03\text{kg})(377.23 - 107.19)\text{kJ}/\text{kg} = \mathbf{2708kJ}$$

4. Is it possible to compress an ideal gas isothermally in an adiabatic piston–cylinder device? Explain.

ANS: No, it isn't. This is because the first law relation Q - W = ΔU reduces to W = 0 in this case since the system is adiabatic (Q = 0) and ΔU = 0 for the isothermal processes of ideal gases. Therefore, this adiabatic system cannot receive any net work at constant temperature.

5. An ideal gas contained in a piston-cylinder device undergoes a polytropic process in which the polytropic exponent *n* is equal to *k*, i.e. the ratio of specific heats as we introduced in class. Show that this process is adiabatic.

ANS: $dU = dW + dQ$ and $dQ = 0$ for adiabatic case $\Longrightarrow$ $c_v dT = -PdV$

Let $k = \dfrac{c_p}{c_v}$

$$\left.\begin{array}{l} PV = mRT \qquad PdV + VdP = mRdT \\ mR = c_p - c_v = (k-1)c_v \end{array}\right\} PdV + VdP = (k-1)c_v dT$$

$$PdV + VdP + (k-1)PdV = 0$$

$$\frac{dP}{P} + \frac{kdV}{V} = 0$$

$$\int \frac{dP}{P} + \int \frac{kdV}{V} = \int 0 \quad \Longrightarrow \quad \ln(P) + k\ln(V) = \ln(PV^k) = C$$

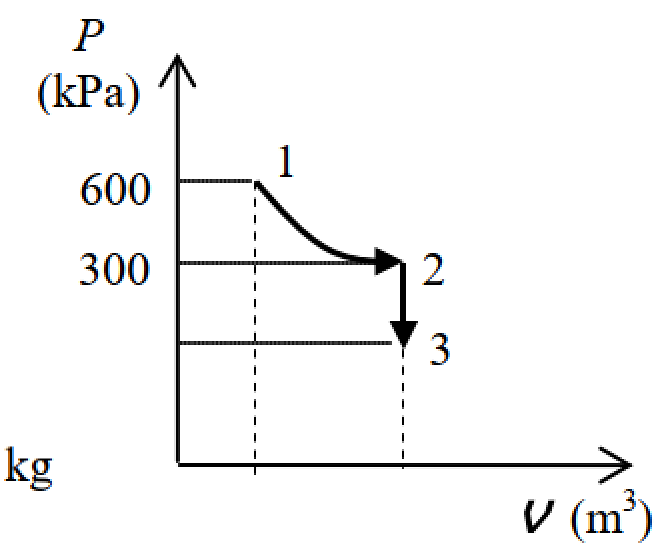
So when *n*=*k*, this process is adiabatic.

6. Air is contained in a piston-cylinder device at 600 kPa and 927°C, and occupies a volume of 0.8 m^3. The air undergoes and isothermal (constant temperature) process until the pressure in reduced to 300 kPa. The piston is now fixed in place and not allowed to move while a heat transfer process takes place until the air reaches 27°C.

(a) Sketch the system showing the energies crossing the boundary and the *P-V* diagram for the combined processes.

(b) For the combined processes determine the net amount of heat transfer, in kJ, and its direction.

ANS: (a) The processes 1-2 (isothermal) and 2-3 (constant-volume) are sketched on the P-V diagram as shown.



(b) We take air as the system. This is a closed system since no mass crosses the boundaries of the system. The energy balance for this system fort he process 1-3 can be expressed as

$$-W_{b,\text{out},1-2} + Q_{\text{in}} = \Delta U = mc_v(T_3 - T_1)$$

The mass of the air is

$$m = \frac{P_1 V_1}{RT_1} = \frac{(600\,\text{kPa})(0.8\,\text{m}^3)}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(1200\,\text{K})} = 1.394\,\text{kg}$$

The work during process 1-2 is determined from boundary work relation for an isothermal process to be

$$W_{b,out,1-2} = mRT_1 \ln\frac{V_2}{V_1} = mRT_1 \ln\frac{P_1}{P_2}$$

$$= (1.394\,\text{kg})(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(1200\,\text{K})\ln\frac{600\,\text{kPa}}{300\,\text{kPa}}$$

$$= 332.8\,\text{kJ}$$

Since $\frac{V_2}{V_1} = \frac{P_1}{P_2}$ for an isothermal process.

Substituting these values into energy balance equation,

$$
\begin{aligned}
Q_{\text{in}} &= W_{b,\text{out},1\text{-}2} + mc_v(T_3 - T_1) \\
&= 332.8\ \text{kJ} + (1.394\ \text{kg})(0.718\ \text{kJ/kg}\cdot\text{K})(300 - 1200)\text{K} \\
&= \mathbf{-568\ kJ}
\end{aligned}
$$

Thus,

$$Q_{\text{out}} = \mathbf{568\ kJ}$$

7. Consider a well-insulated horizontal rigid cylinder that is divided into two compartments by a piston that is free to move but does not allow either gas to leak into the other side. Initially, one side of the piston contains 1 m^3 of N_2 gas at 500 kPa and 120°C while the other side contains 1 m^3 of He gas at 500 kPa and 40°C. Now thermal equilibrium is established in the cylinder as a result of heat transfer through the piston. Using constant specific heats at room temperature, determine the final equilibrium temperature in the cylinder. What would your answer be if the piston were not free to move?

ANS: The mass of each gas in the cylinder is

$$m_{N_2} = \left(\frac{P_1 V_1}{RT_1}\right)_{N_2} = \frac{(500\text{ kPa})(1\text{ m}^3)}{(0.2968\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(393\text{ K})} = 4.287\text{ kg}$$

$$m_{He} = \left(\frac{P_1 V_1}{RT_1}\right)_{He} = \frac{(500\text{ kPa})(1\text{ m}^3)}{(2.0769\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(313\text{ K})} = 0.7691\text{ kg}$$

Taking the entire contents of the cylinder as our system, the 1st law relation can be written as

$$0 = \Delta U = (\Delta U)_{N_2} + (\Delta U)_{He}$$

$$0 = [mc_v(T_2 - T_1)]_{N_2} + [mc_v(T_2 - T_1)]_{He}$$

Substituting,

$$(4.287\text{ kg})(0.743\text{ kJ/kg}\cdot{}^\circ\text{C})(T_f - 120)^\circ\text{C} + (0.7691\text{ kg})(3.1156\text{ kJ/kg}\cdot{}^\circ\text{C})(T_f - 40)^\circ\text{C} = 0$$

It gives

T_f = **85.7°C** where T_f is the final equilibrium temperature in the cylinder.

The answer would be the **same** if the piston were not free to move since it would effect only pressure, and not the specific heats.

8. The early steam engines were driven by the atmospheric pressure acting on the piston fitted into a cylinder filled with saturated steam. A vacuum was created in the cylinder by cooling the cylinder externally with cold water, and thus condensing the steam. Consider a piston–cylinder device with a piston surface area of 0.1 m^2 initially filled with 0.05 m^3 of saturated water vapor at the atmospheric pressure of 100 kPa. Now cold water is poured outside the cylinder, and the steam inside starts condensing as a result of heat transfer to the cooling water outside. If the piston is stuck at its initial position, determine the friction force acting on the piston and the amount of heat transfer when the temperature inside the cylinder drops to 30°C.

ANS: We take the contents of the cylinder (the saturated liquid-vapor mixture) as the system, which is a closed system. Noting that the volume remains constant during this phase change process, the energy balance for this system can be expressed as

$$-Q_{\text{out}} = \Delta U = m(u_2 - u_1)$$

The saturation properties of water at 100 kPa and at 30 °C are (Tables A-4 and A-5)

$$P_1 = 100\text{ kPa} \longrightarrow \begin{array}{l} v_f = 0.001043\text{ m}^3/\text{kg}, \; v_g = 1.6941\text{ m}^3/\text{kg} \\ u_f = 417.40\text{ kJ/kg}, \quad u_g = 2505.6\text{ kJ/kg} \end{array}$$

$$T_2 = 30^\circ\text{C} \longrightarrow \begin{array}{l} v_f = 0.001004\text{ m}^3/\text{kg}, v_g = 32.879\text{ m}^3/\text{kg} \\ u_f = 125.73\text{ kJ/kg}, \quad u_{fg} = 2290.2\text{ kJ/kg} \\ P_{\text{sat}} = 4.2469\text{ kPa} \end{array}$$

Then

$$P_2 = P_{sat@30^\circ C} = 4.2469 \text{ kPa}$$

$$\boldsymbol{v}_1 = \boldsymbol{v}_{g@100\,\text{kPa}} = 1.6941 \text{ m}^3/\text{kg}$$

$$u_1 = u_{g@100\,\text{kPa}} = 2505.6 \text{ kJ/kg}$$

and

$$m = \frac{\boldsymbol{V}_1}{\boldsymbol{v}_1} = \frac{0.05 \text{ m}^3}{1.6941 \text{ m}^3/\text{kg}} = 0.02951 \text{ kg}$$

$$\boldsymbol{v}_2 = \boldsymbol{v}_1 \longrightarrow x_2 = \frac{\boldsymbol{v}_2 - \boldsymbol{v}_f}{\boldsymbol{v}_{fg}} = \frac{1.6941 - 0.001}{32.879 - 0.001} = 0.05150$$

$$u_2 = u_f + x_2 u_{fg} = 125.73 + 0.05150 \times 2290.2 = 243.67 \text{ kJ/kg}$$

The friction force that develops at the piston-cylinder interface balances the force acting on the piston, and is equal to

$$F = A(P_1 - P_2) = (0.1 \text{ m}^2)(100 - 4.2469)\text{kPa}\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right) = \mathbf{9575 \;\; N}$$

The heat transfer is determined from the energy balance to be

$$\begin{aligned} Q_{\text{out}} &= m(u_1 - u_2) \\ &= (0.02951 \text{ kg})(2505.6 - 243.67)\text{kJ/kg} \\ &= \mathbf{66.8 \;\; kJ} \end{aligned}$$