

第 5 周习题 常微分方程 B

March 17, 2022

Part I

作业题目: (教材 Boyce, DiPrima, Meade, 11 edition)

第 2.8 节—题 1, 2, 5, 6, 12

Part II

1. Show that $y(t) = 0$ and $y(t) = t^3$ are both solutions of the initial value problem

$$y' = 3y^{2/3}, \quad y(0) = 0.$$

Explain why this fact does not contradict the existence and uniqueness theorem. Can you find a third solution to the initial value problem?

2. Show that the following initial value problem has no solution:

$$ty' = 2y - t, \quad y(0) = 2.$$

Explain why this lack of solution does not contradict the existence theorem.

3. Suppose that $y = \phi(t)$ is a solution to the initial value problem

$$y' = \frac{x^3 - x}{1 + t^2 x^2}, \quad y(0) = 1/2.$$

Show that $0 < \phi(t) < 1$ for all t for which $\phi(t)$ is defined.

4. Suppose that $y = \phi(t)$ is a solution to the initial value problem

$$y' = y - t^2 + 2t, \quad y(0) = 1.$$

Show that $\phi(t) > t^2$ for all t for which $\phi(t)$ is defined.