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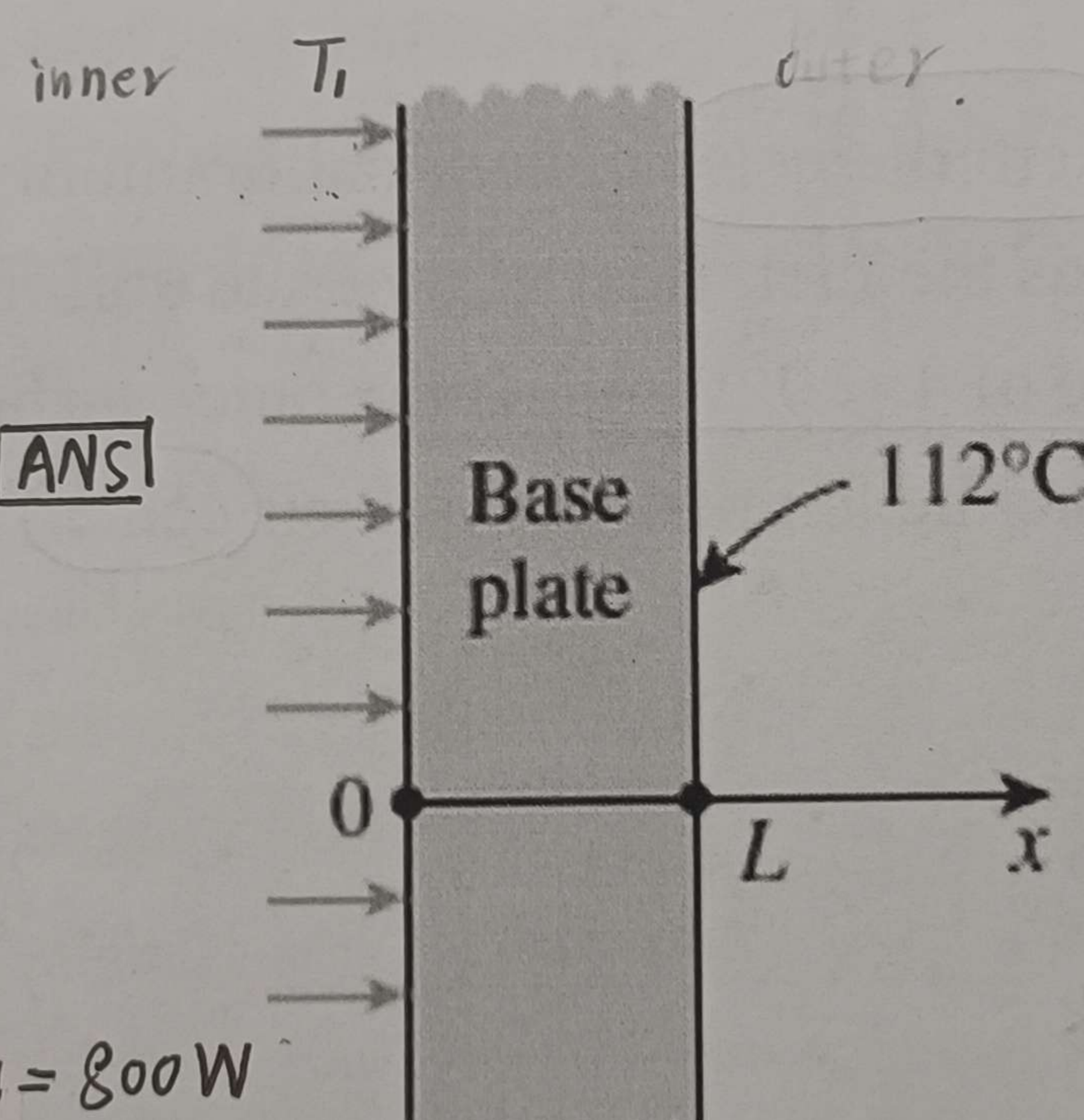
MAE-308 Heat transfer

DDL: 3.29

1.Consider the base plate of an 800W household iron with a thickness of L= 0.6 cm, base area of A = 160cm², and thermal conductivity of k = 60 W/m·K. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside. When steady operating conditions are reached, the outer surface temperature of the plate is measured to be 112°C, Disregarding any heat loss through the upper part of the iron, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the plate, (b) obtain a relation for the variation of temperature in the base plate by solving the differential equation, and (c) evaluate the inner surface temperature.



解:

a) $\frac{d^2T}{dx^2}=0$

$x=0,\ T=T_1$

$x=L,\ T=112°C$ $\boxed{ANS}$

$-\lambda A\frac{dT}{dx}=800W$

b) $T=ax+b$

BC: $T_1=b$

$112°C=aL+b$

$\Phi=-\lambda A\frac{dT}{dx}=-\lambda A\cdot a=800W$

$\Rightarrow a=\frac{-2500}{3}$ k/m

$b=112+273+\frac{2500}{3}\times0.6\times10^{-2}=390k$

$\therefore T=\frac{-2500}{3}x+390=-833.3x+390$ K

$\boxed{ANS}$

c) At $x=0$

$T=390k=117°C$ $\boxed{ANS}$

2. In a food processing facility, a spherical container of inner radius r_1 = 40 cm, outer radius r_2 = 41 cm, and thermal conductivity k=1.5 W/m·K is used to store hot water and to keep it at 100°C at all times. To accomplish this, the outer surface of the container is wrapped with a 800W electric strip heater and then insulated. The temperature of the inner surface of the container is observed to be nearly 120°C at all times. Assuming 10 percent of the heat generated in the heater is lost through the insulation, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the container, (b) obtain a relation for

10%

the variation of temperature in the container material by solving the differential equation, and (c) evaluate the outer surface temperature of the container. Also determine how much water at 100°C this tank con supply steadily if the cold water enters at 20°C.

解: a) $\frac{d}{dr}\left(\lambda r^2 \frac{dT}{dr}\right) = 0$

$r = r_1$, $T = 120°C$

$r = r_2$, $T = T_2$ $\boxed{ANS}$

$\Phi = -\lambda A_2 \frac{dT}{dr} = -90\% \times 800W$

b) $T = \frac{-C_1}{r} + C_2$

$\Phi = -\lambda A_2 \frac{dT}{dr} = -\lambda A_i C_1 \cdot \frac{1}{r^2} = -90\% \times 800W$

$120°C = \frac{-C_1}{0.4} + C_2 = 393K$

$T_2 = \frac{-C_1}{0.41} + C_2$

$\Rightarrow C_1 = \frac{+120}{\pi}$, $C_2 = 393 + \frac{300}{\pi}$

$\therefore T = \frac{-120}{\pi r} + 393 + \frac{300}{\pi}$ $\boxed{ANS}$

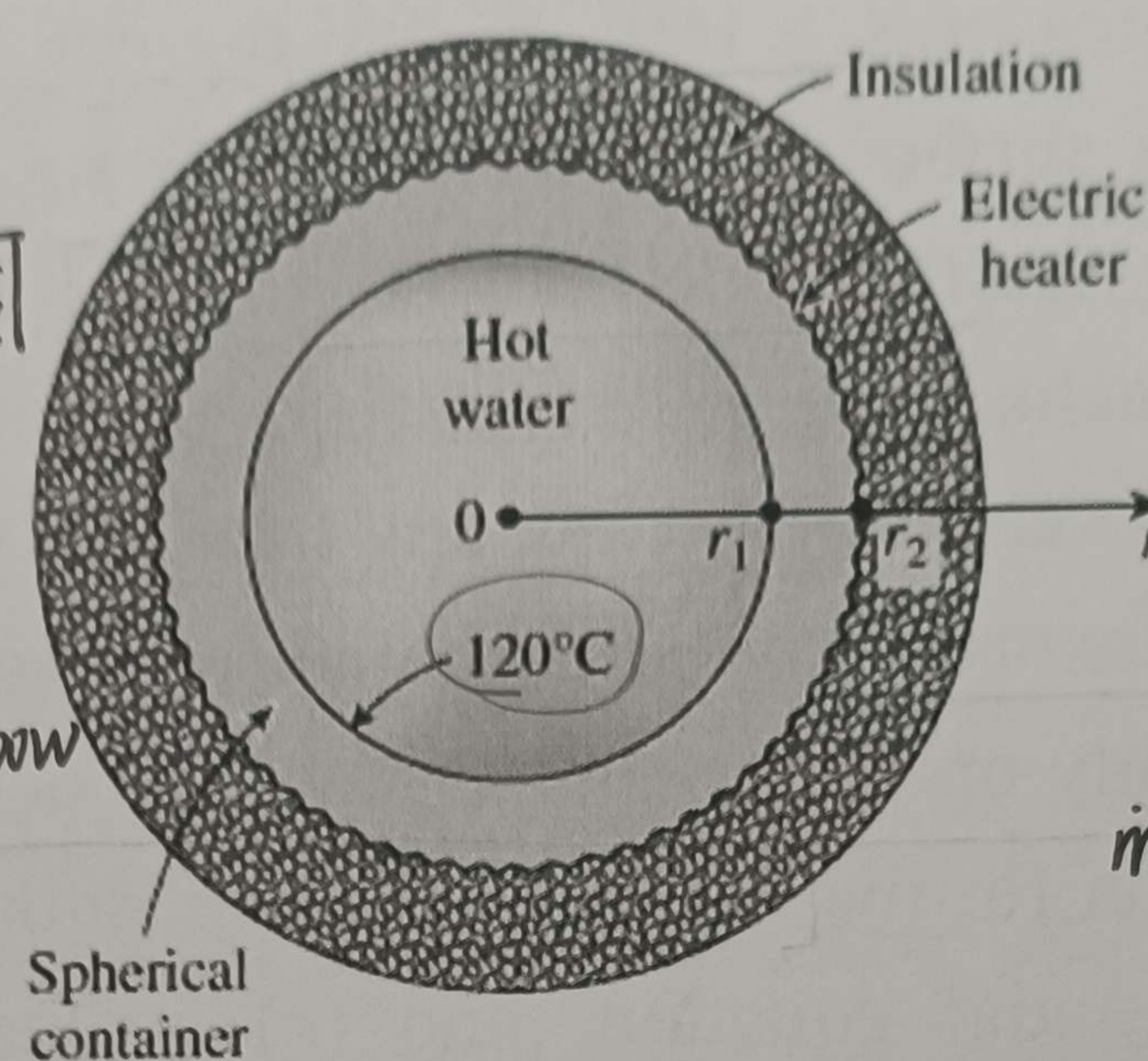
$= \frac{-38.20}{r} + 488.49$

c) $r = r_2$

800W

$T = \frac{-120}{\pi \times 0.41} + 393 + \frac{300}{\pi} = 395.33\ K$

$= 122.33°C$ $\boxed{ANS}$



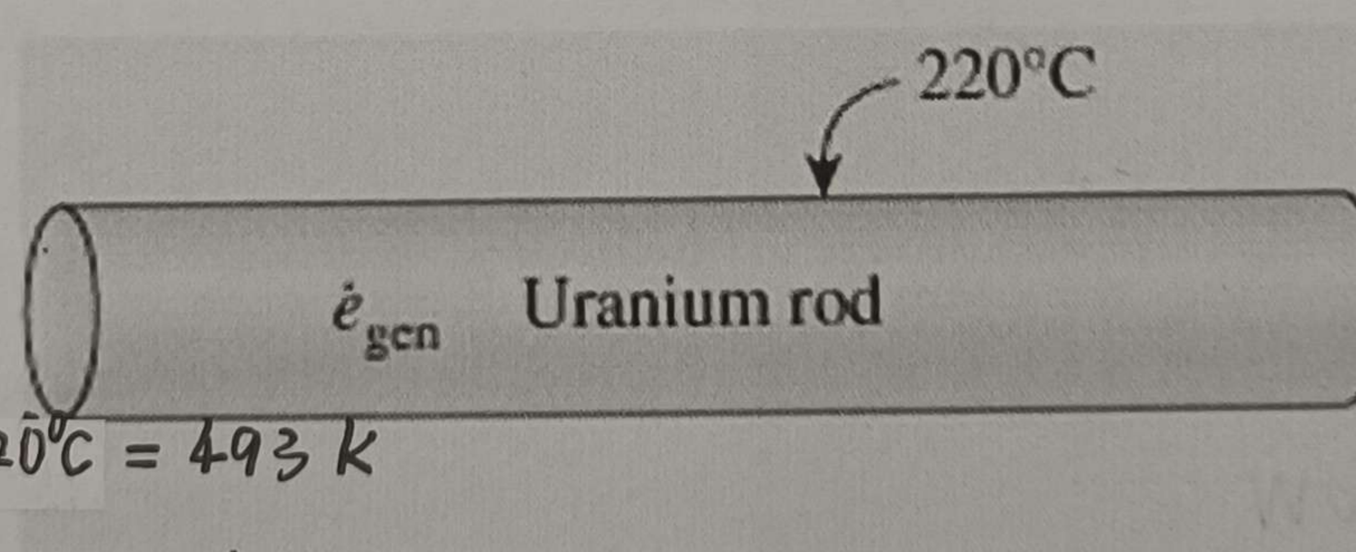
· Cold water : 20°C → 100°C

$\dot{m}_{water} = \frac{\eta \cdot \dot{Q}}{C_p \Delta T}$

$= \frac{90\% \times 800}{4.186 \times 10^3 \times (100-20)} = 2.15 kg/s$ $\boxed{ANS}$

3. In a nuclear reactor, 1 cm diameter cylindrical uranium rods cooled by water from outside serve as the fuel. Heat is generate uniformly in the rods ($k = 29.5$ W/m·K) at a rate of 4×10^7 W/m³. If the outer surface temperature of rods is 220°C, determine the temperature at their center.

解:



cylinder rod

$T = -\frac{\dot{\Phi} r^2}{4\lambda} + C_2$

At surface, $T = -\frac{\dot{\Phi}\left(\frac{d}{2}\right)^2}{4\lambda} + C_2 = 220°C = 493\ K$

$C_2 = 501.47\ K$

At center, $T = 0 + C_2 = 501.47K = 228.47°C$ $\boxed{ANS}$

4. A pipe is used for transporting hot fluid in which the inner surface is at 150°C. The pipe has a wall thickness of 5 mm (0.5cm) and an inner diameter of 15 cm. The pipe wall has a variable thermal conductivity given as $k(T) = k_0 (1+\beta T)$, where $k_0 = 8.5$ W/m·K, $\beta = 0.001$ K^{-1}, and T is in K. The pipe is situated in surroundings ~~of freezing~~ air at 0°C with a convection heat (h) transfer coefficient of 60 W/m²·K on the pipe's outer surface. Solar radiation is incident on the pipe's outer surface at a rate of 100 W/m², and both the emissivity and solar absorptivity of the outer surface are 0.9. Determine the outer surface temperature of the pipe.

解:

Conduction:

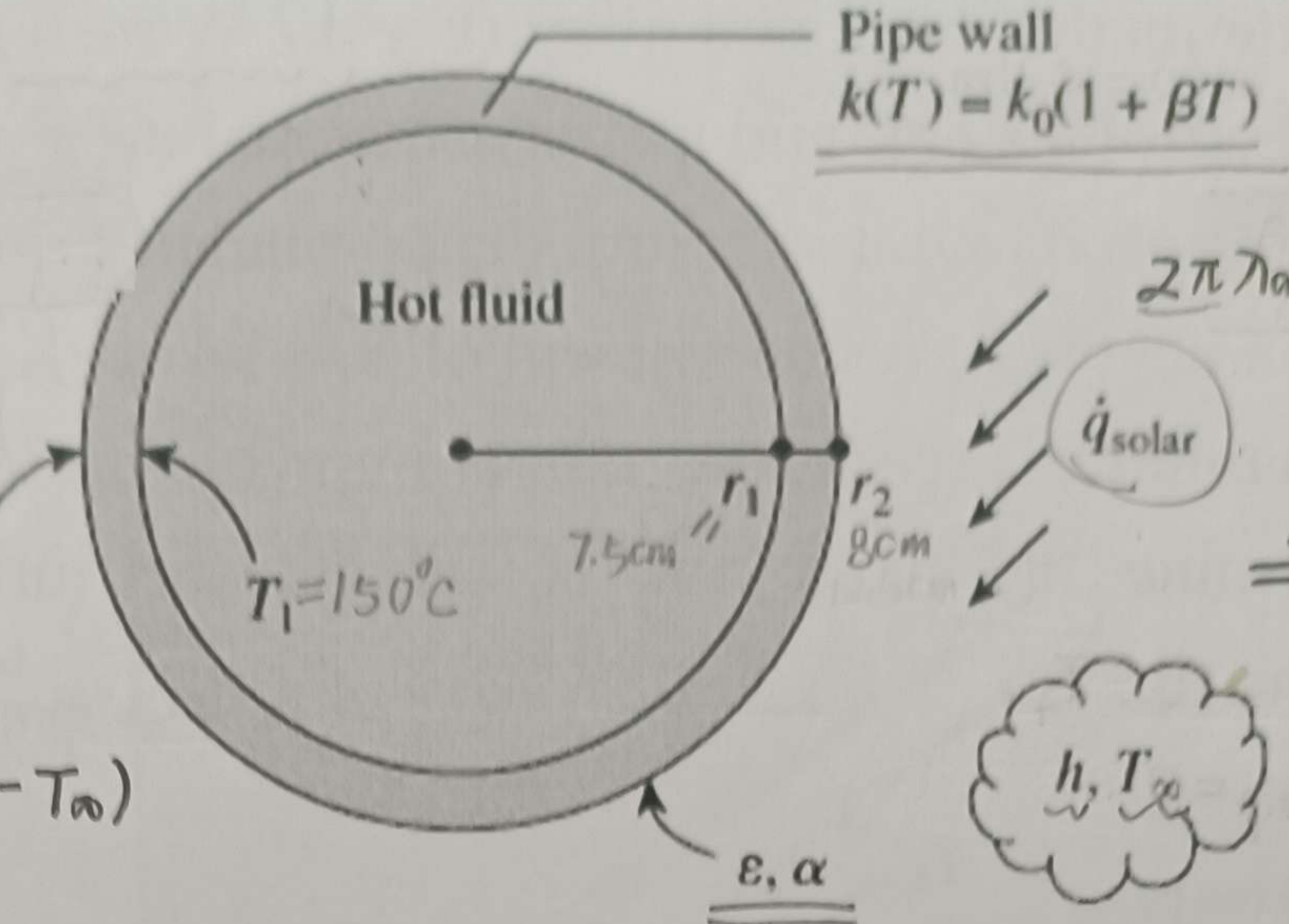
$\lambda(T)=k_0(1+\beta T)$, $\lambda_{avg}=k_0\left(1+\beta\frac{T_1+T_2}{2}\right)$

$\Phi_{cylinder} = 2\pi\lambda_{avg}\cdot L\cdot\frac{T_1-T_2}{\ln r_2-\ln r_1}$

Radiation:

$\Phi_{radia} = -\varepsilon A\sigma(T_\infty^4-T_2^4) = +\varepsilon\cdot 2\pi r_2 L\sigma(T_2^4-T_\infty^4)$

$\Phi_{cond} = \Phi_{conv} + \Phi_{radia} - \Phi_{solar}$



$2\pi\lambda_{avg}\cdot L\cdot\frac{T_1-T_2}{\ln r_2-\ln r_1} = h2\pi r_2\cdot L(T_2-T_\infty) + \varepsilon\cdot 2\pi r_2 L\cdot\sigma(T_2^4-T_\infty^4) - \alpha\cdot 2\pi r_2\cdot L\cdot q_{solar}$

$\Rightarrow T_2 = 418.75\ K = 145.75°C$ $\boxed{ANS}$

Convection:

$\Phi_{conv} = -hA(T_\infty-T_2) = h\cdot 2\pi r_2\cdot L(T_2-T_\infty)$

5. Consider a very long rectangular fin attached to a flat surface such that the temperature at the end of the fin is essentially that of the surrounding air, i.e. 20°C. Its width is 5.0 cm; thickness is 1.0 mm; thermal conductivity is 200 W/m·K; and base temperature is 40°C. The heat transfer coefficient is 20 W/m2·K. Estimate the fin temperature at a distance of 5.0 cm from the base and the rate of heat loss from the entire fin.

$T_{tip} = T_\infty = 20°C$

$P \approx 2W = 0.1m$

$A_c = wt = 5\times10^{-2}\times10^{-3}$

$\boxed{ANS}$

解:

very long fin

$T_{tip} = T_\infty = 20°C$

$m = \sqrt{\frac{hP}{\lambda A_c}} = \sqrt{\frac{h\cdot 2w}{\lambda\cdot wt}} = \sqrt{\frac{20\times 2}{200\times 1\times 10^{-3}}} = 10\sqrt{2}$

$T(x) = (T_b - T_\infty)e^{-mx} + T_\infty \Rightarrow T(0.05) = (40-20)e^{-10\sqrt{2}\times 0.05} + 20+273 = 302.86K = 29.86°C$

$\Phi = -\lambda A_c\frac{dT}{dx}\Big|_{x=0} = \sqrt{hP\lambda A_c}(T_b-T_\infty) = \sqrt{20\times0.1\times200\times5\times10^{-5}}\times 20 = 2\sqrt{2}\ W = 2.828\ W$ $\boxed{ANS}$

6. A hot surface at 100°C is to be cooled by attaching 3-cm-long, 0.25-cm-diameter aluminum pin fins ($k = 237$ W/m·K) to it, with a center-to-center distance of 0.6 cm. The temperature of the surrounding medium is 30°C, and the heat transfer coefficient on the surfaces is 35 W/m²·K. Determine the rate of heat transfer from the surface for a 1-m×1-m section of the plate. Also determine the overall effectiveness of the fins.

$\varepsilon_{overall}$

解:

find the number of fins: $n = \frac{1\times1}{0.006\times0.006} = 27778$

for one fin:

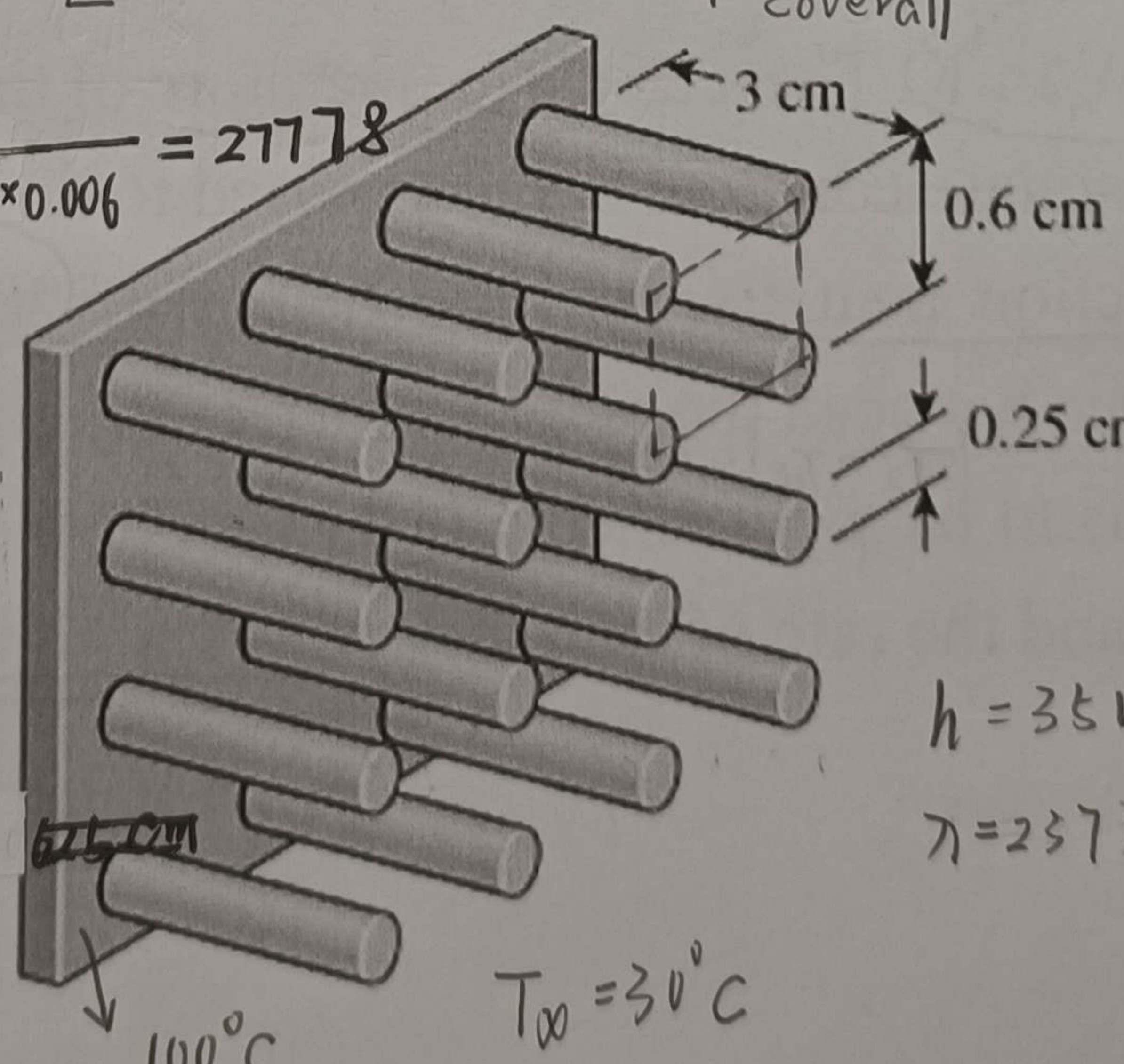
$\Phi_{fin} = \sqrt{hP\lambda A_c}(T_b-T_\infty)\tanh(mL_c)$

$m = \sqrt{\frac{hP}{\lambda A_c}} = \sqrt{\frac{35\times\pi\times0.25\times10^{-2}}{237\times\pi\times(0.125\times10^{-2})^2}}$

$= 15.37$

$L_c = L + \frac{r}{2} = 3.125\ cm$

$\therefore \Phi_{fin} = 0.54925\ W$



$h = 35\ W/m^2K$

$\lambda = 237\ W/m\cdot K$

15257.16356

$T_\infty = 30°C$

$\Phi_{unfin} = h A_{unfin}(T_b - T_\infty)$

$= 35\times[1\times1 - n\times\pi\times(0.125\times10^{-2})^2]\times 70$

$= 2115.93\ W$

$\Phi_{total} = n\Phi_{fin} + \Phi_{unfin} = 17373\ W$

$\boxed{ANS}$

$\Phi_{nofin} = h A_{nofin}(T_b - T_\infty)$

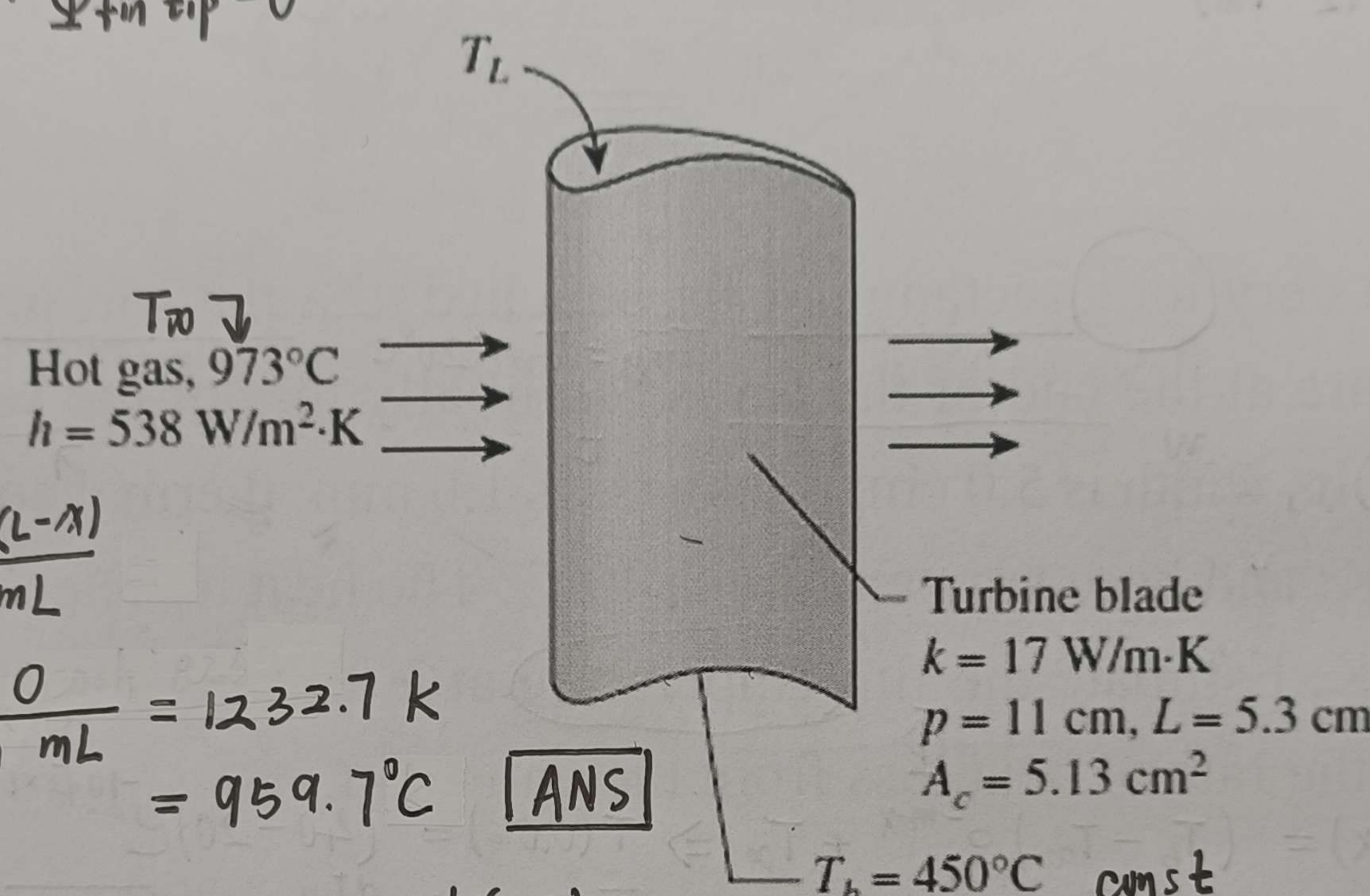
$= 35\times1\times70 = 2450\ W$

$\varepsilon_{overall} = \frac{\Phi_{total}}{\Phi_{nofin}} = \frac{17373}{2450} = 7.09$ $\boxed{ANS}$

7. A turbine blade made of a metal alloy (k = 17 W/m·K) has a length of 5.3 cm, a perimeter of 11 cm, and a cross-sectional area of 5.13 cm². The turbine blade is exposed to hot gas from the combustion chamber at 973°C with a convection heat transfer coefficient of 538 W/m²·K. The base of the turbine blade maintains a constant temperature of 450°C and the tip is adiabatic. Determine the heat transfer rate to the turbine blade and temperature at the tip. T_L

λ L $P = 0.11m$ A_c h Φ

解: tip adiabatic : $\Phi_{fin\ tip} = 0$



$$\frac{T(x)-T_\infty}{T_b-T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$m = \sqrt{\frac{hP}{\lambda A_c}} = \sqrt{\frac{538\times 0.11}{17\times 5.13\times 10^{-4}}} = 82.38$$

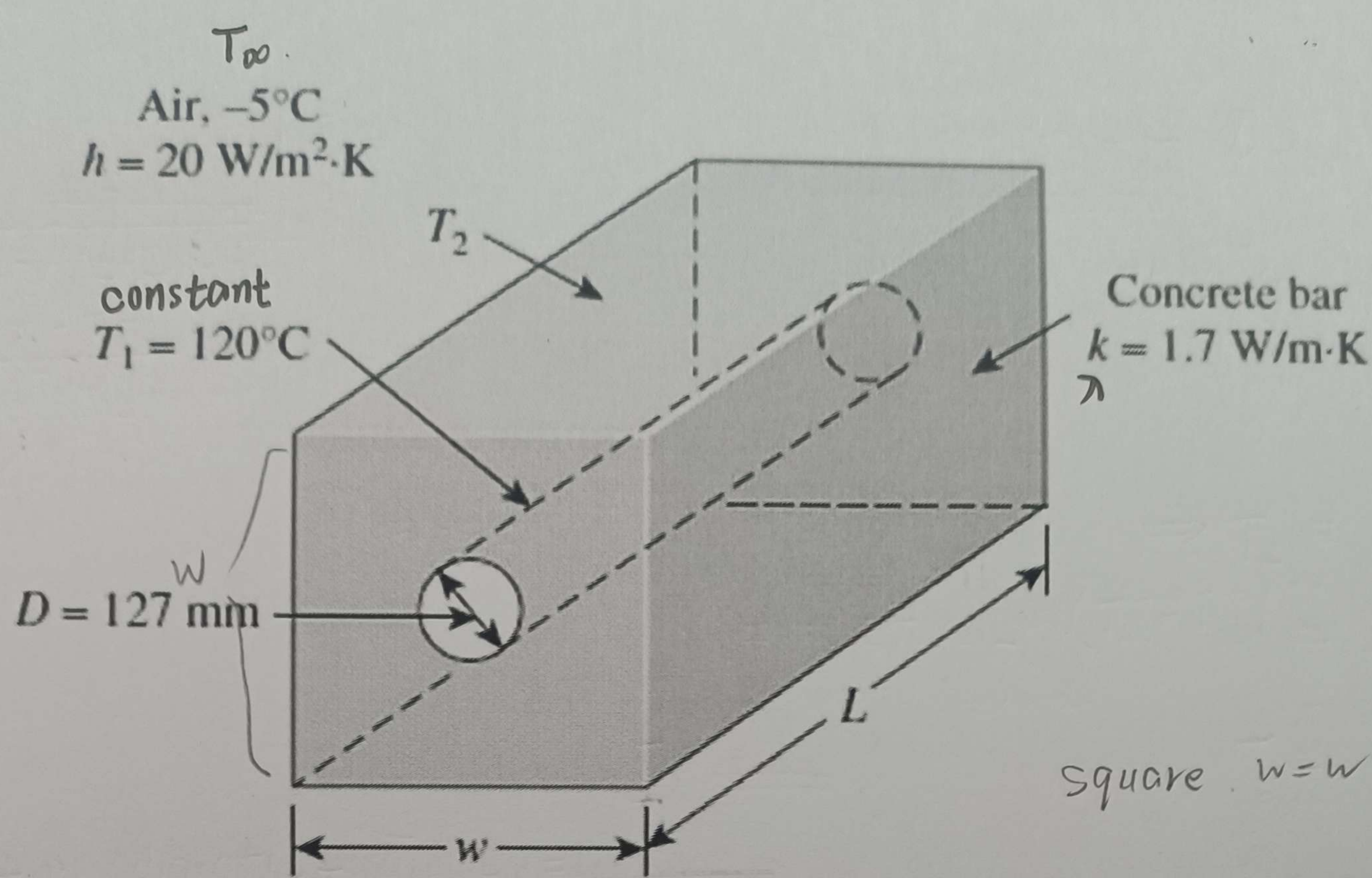
$$T(x) = T_\infty + (T_b - T_\infty)\frac{\cosh m(L-x)}{\cosh mL}$$

$$T(L) = T_\infty + (T_b - T_\infty)\frac{\cosh 0}{\cosh mL} = 1232.7\ K = 959.7^\circ C \quad \boxed{ANS}$$

$$\Phi = -\lambda A_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hP\lambda A_c}\,(-T_b + T_\infty)\tanh(mL) = +375.61\ W \quad \boxed{ANS}$$

8. In a combined heat and power (CHP) generation process, by-product heat is used for domestic or industrial heating purposes. Hot steam is carried from a CHP generation plant by a tube with diameter of 127 mm centered at a square cross section solid bar made of concrete with thermal conductivity of 1.7 W/m·K. The surface temperature of the tube is constant at 120°C, while the square concrete bar is exposed to air with temperature of −5°C and convection heat transfer coefficient of 20 W/m²·K. If the temperature difference between the outer surface of the square concrete bar and the ambient air is to be maintained at 5°C, determine the width of the square concrete bar and the rate of heat loss per meter length.

$|T_2 - T_\infty| = 5^\circ C$.



解: temperature difference

$$T_2 - T_\infty = 5^\circ C \Rightarrow T_2 = 0^\circ C$$

$$S = \frac{2\pi L}{\ln \frac{1.08w}{D}}$$

$$\Phi = S \lambda (T_1 - T_2)$$

Convection

$$\Phi = h \cdot A (T_2 - T_\infty)$$

$$\Rightarrow \frac{2\pi L}{\ln \frac{1.08w}{D}} \cdot \lambda (T_1 - T_2) = h \cdot wL \cdot 4 (T_2 - T_\infty)$$

$$w = 1.324 m \qquad \boxed{ANS}$$

$$\frac{\Phi}{L} = \frac{2\pi\lambda}{\ln \frac{1.08w}{D}} (T_1 - T_2) = 529.4 \, W/m \qquad \boxed{ANS}$$