Homework 5 ANS

1. Please consider the questions below.

(1) Are the efficiencies of all the work-producing devices, including the hydroelectric power plants, limited by the Kelvin–Planck statement of the second law? Explain.

ANS: No. The Kelvin-Plank limitation applies only to heat engines.

(2) In an effort to conserve energy in a heat-engine cycle, somebody suggests incorporating a refrigerator that will absorb some of the waste energy Q_L and transfer it to the energy source of the heat engine. Is this a smart idea? Explain.

ANS: No. At best (when everything is reversible), the increase in the work produced will be equal to the work consumed by the refrigerator. In reality, the work consumed by the refrigerator will always be greater than the additional work produced, resulting in a decrease in the thermal efficiency of the power plant.

2. An inventor claims to have developed a heat pump that produces a 200-kW heating effect for a 293K zone while only using 75kW work and a source at 273K. Justify its validity.

ANS: The coefficient of performance of this heat pump is, $\text{COP}_{\text{HP}} = \dfrac{\dot{Q}_H}{\dot{W}_{\text{net,in}}} = \dfrac{200\text{ kW}}{75\text{ kW}} = 2.67$

The maximum COP of a heat pump operating between the same temperature limits is

$$\text{COP}_{\text{HP,max}} = \frac{1}{1 - T_L / T_H} = \frac{1}{1 - (273\text{ K})/(293\text{ K})} = 14.7$$

Since the actual COP is less than the maximum COP, the claim is **valid**.

3. A refrigerator with refrigerant-134a as the working fluid is used to keep the refrigerated space at -35°C by rejecting waste heat to cooling water that enters the condenser at 18°C at a rate of 0.25 kg/s and leaves at 26°C. The refrigerant enters the condenser at 1.2 MPa and 50°C and leaves at the same pressure subcooled by 5°C. If the compressor consumes 3.3 kW of power, determine (a) the mass flow rate of the refrigerant, (b) the refrigeration load, (c) the COP, and (d) the minimum power input to the compressor for the same refrigeration load.

ANS: (a) The rate of heat transferred to the water is the energy change of the water from inlet to exit

$$\dot{Q}_H = \dot{m}_w (h_{w,2} - h_{w,1}) = (0.25 \text{ kg/s})(109.01 - 75.54) \text{ kJ/kg} = 8.367 \text{ kW}$$

The energy decrease of the refrigerant is equal to the energy increase of the water. That is,

$$\dot{Q}_H = \dot{m}_R (h_1 - h_2) \longrightarrow \dot{m}_R = \frac{\dot{Q}_H}{h_1 - h_2} = \frac{8.367 \text{ kW}}{(278.28 - 110.19) \text{ kJ/kg}} = \mathbf{0.0498 kg/s}$$

(b) The refrigeration load is $\dot{Q}_L = \dot{Q}_H - \dot{W}_{\text{in}} = 8.37 - 3.30 = \mathbf{5.07 kW}$

(c) The COP of the refrigerator is determined from its definition, $\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{5.07 \text{ kW}}{3.3 \text{ kW}} = \mathbf{1.54}$

(d) The COP of a reversible refrigerator operating between the same temperature limits is

$$\text{COP}_{\text{max}} = \frac{1}{T_H / T_L - 1} = \frac{1}{(18 + 273)/(-35 + 273) - 1} = 4.49$$

Then, the minimum power input to the compressor for the same refrigeration load would be

$$\dot{W}_{\text{in,min}} = \frac{\dot{Q}_L}{\text{COP}_{\text{max}}} = \frac{5.07 \text{ kW}}{4.49} = \mathbf{1.13 kW}$$

4. A completely reversible heat pump produces heat at a rate of 300 kW to warm a house maintained at 24°C. The exterior air, which is at 7°C, serves as the source. Calculate the rate of entropy change of the two reservoirs and determine if this heat pump satisfies the second law.

ANS: Since the heat pump is completely reversible, the combination of the coefficient of performance expression, first Law, and thermodynamic temperature scale gives

$$\mathrm{COP}_{\mathrm{HP,rev}} = \frac{1}{1 - T_L / T_H} = \frac{1}{1 - (280\ \mathrm{K})/(297\ \mathrm{K})} = 17.47$$

The power required to drive this heat pump is $\dot{W}_{\mathrm{net,in}} = \dfrac{\dot{Q}_H}{\mathrm{COP}_{\mathrm{HP,rev}}} = \dfrac{300\ \mathrm{kW}}{17.47} = 17.17\ \mathrm{kW}$

The rate at which heat is removed from the low-temperature energy reservoir is

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{\mathrm{net,in}} = 300\ \mathrm{kW} - 17.17\ \mathrm{kW} = 282.8\ \mathrm{kW}$$

The rate at which the entropy of the high temperature reservoir changes is

$$\Delta \dot{S}_H = \frac{\dot{Q}_H}{T_H} = \frac{300\ \mathrm{kW}}{297\ \mathrm{K}} = \mathbf{1.01\ kW/K}$$

and that of the low-temperature reservoir is $\Delta \dot{S}_L = \dfrac{\dot{Q}_L}{T_L} = \dfrac{-17.17\ \mathrm{kW}}{280\ \mathrm{K}} = \mathbf{-1.01\ kW/K}$

The net rate of entropy change of everything in this system is

$$\Delta \dot{S}_{\mathrm{total}} = \Delta \dot{S}_H + \Delta \dot{S}_L = 1.01 - 1.01 = \mathbf{0\ kW/K}$$

as it must be since the heat pump is completely reversible.

5. A piston–cylinder device contains 1.2 kg of saturated water vapor at 200°C. Heat is now transferred to steam, and steam expands reversibly and isothermally to a final pressure of 800 kPa. Determine the heat transferred and the work done during this process.

ANS: From the steam tables (Tables A-4 through A-6),

$$\left.\begin{array}{l} T_1 = 200°\text{C} \\ sat.vapor \end{array}\right\} \begin{array}{l} u_1 = u_{g@200°\text{C}} = 2594.2 \text{ kJ/kg} \\ s_1 = s_{g@200°\text{C}} = 6.4302 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 800 \text{ kPa} \\ T_2 = T_1 \end{array}\right\} \begin{array}{l} u_2 = 2631.1 \text{ kJ/kg} \\ s_2 = 6.8177 \text{ kJ/kg} \cdot \text{K} \end{array}$$

The heat transfer for this reversible isothermal process can be determined from

$$Q = T\Delta S = Tm(s_2 - s_1) = (473 \text{ K})(1.2 \text{ kg})(6.8177 - 6.4302)\text{kJ/kg} \cdot \text{K} = \mathbf{219.9 \ \ kJ}$$

We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this closed system can be expressed as

$$Q_{\text{in}} - W_{\text{b,out}} = \Delta U = m(u_2 - u_1)$$
$$W_{\text{b,out}} = Q_{\text{in}} - m(u_2 - u_1)$$

Substituting, the work done during this process is determined to be

$$W_{\text{b,out}} = 219.9 \text{ kJ} - (1.2 \text{ kg})(2631.1 - 2594.2) \text{ kJ/kg} = \mathbf{175.6 \ \ kJ}$$

6. A 30-kg iron block and a 40-kg copper block, both initially at 80°C, are dropped into a large lake at 15°C. Thermal equilibrium is established after a while as a result of heat transfer between the blocks and the lake water. Determine the total entropy change for this process.

ANS: The thermal-energy capacity of the lake is very large, and thus the temperatures of both the iron and the copper blocks will drop to the lake temperature (15°C) when the thermal equilibrium is established. Then the entropy changes of the blocks become

$$\Delta S_{\text{iron}} = mc_{\text{avg}} \ln\left(\frac{T_2}{T_1}\right) = (30\text{ kg})(0.45\text{ kJ/kg}\cdot\text{K})\ln\left(\frac{288\text{ K}}{353\text{ K}}\right) = -2.746\text{ kJ/K}$$

$$\Delta S_{\text{copper}} = mc_{\text{avg}} \ln\left(\frac{T_2}{T_1}\right) = (40\text{ kg})(0.386\text{ kJ/kg}\cdot\text{K})\ln\left(\frac{288\text{ K}}{353\text{ K}}\right) = -3.141\text{ kJ/K}$$

We take both the iron and the copper blocks as the system. This is a closed system since no mass crosses the system boundary during the process. The energy balance for this system can be expressed as

$$Q_{\text{out}} = [mc(T_1 - T_2)]_{\text{iron}} + [mc(T_1 - T_2)]_{\text{copper}}$$

$$Q_{\text{out}} = (30\text{ kg})(0.45\text{ kJ/kg}\cdot\text{K})(353-288)\text{K} + (40\text{ kg})(0.386\text{ kJ/kg}\cdot\text{K})(353-288)\text{K} = 1881\text{ kJ}$$

Thus $\Delta S_{\text{lake}} = \dfrac{Q_{\text{lake,in}}}{T_{\text{lake}}} = \dfrac{1881\text{ kJ}}{288\text{ K}} = 6.528\text{ kJ/K}$

Then the total entropy change for this process is

$$\Delta S_{\text{total}} = \Delta S_{\text{iron}} + \Delta S_{\text{copper}} + \Delta S_{\text{lake}} = (-2.746) + (-3.141) + 6.528 = \mathbf{0.642\ kJ/K}$$

7. Air enters a compressor steadily at the conditions of 100 kPa and 22°C and leaves at 800 kPa. Heat is lost from the compressor in an amount of 120 kJ/kg and the air experiences an entropy decrease of 0.40 kJ/kg·K. Using constant specific heats at 300 K, determine

(a) the exit temperature of the air

(b) the work input to the compressor

(c) the entropy generation during this process

ANS: (a) The exit temperature of the air may be determined from the relation for the entropy change of air

$$\Delta S_{\text{air}} = c_p \ln\frac{T_2}{T_{a1}} - R\ln\frac{P_2}{P_1}$$

$$-0.40\text{ kJ/kg.K} = (1.005\text{ kJ/kg.K})\ln\frac{T_2}{(22+273)\text{ K}} - (0.287\text{ kJ/kg.K})\ln\frac{800\text{ kPa}}{100\text{ kPa}}$$

$$T_2 = 358.8\text{ K} = \mathbf{85.8°C}$$

(b) The work input to the compressor is obtained from an energy balance on the compressor

$$w_{in} = c_p(T_2 - T_1) + q_{out} = (1.005\text{ kJ/kg.°C})(85.8-22)\text{°C} + 120\text{ kJ/kg} = \mathbf{184.1\,kJ/kg}$$

(c) The entropy generation associated with this process may be obtained by adding the entropy change of air as it is compressed in the compressor and the entropy change of the surroundings

$$\Delta s_{\text{surr}} = \frac{q_{\text{out}}}{T_{\text{surr}}} = \frac{120\text{ kJ/kg}}{(22+273)\text{ K}} = 0.4068\text{ kJ/kg.K}$$

$$s_{\text{gen}} = \Delta s_{\text{total}} = \Delta s_{\text{air}} + \Delta s_{\text{surr}} = -0.40 + 0.4068 = \mathbf{0.0068\text{ kJ/kg.K}}$$

8. A well-insulated heat exchanger is to heat water (c_p=4.18 kJ/kg·°C) from 25 to 60°C at a rate of 0.50 kg/s. The heating is to be accomplished by geothermal water (c_p=4.31 kJ/kg·°C) available at 140°C at a mass flow rate of 0.75 kg/s. Determine

(a) the rate of heat transfer

(b) the rate of entropy generation in the heat exchanger

ANS: (a) Then the rate of heat transfer to the cold water in the heat exchanger is

$$\dot{Q}_{\text{in, water}} = [\dot{m}c_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (0.50 \text{ kg/s})(4.18 \text{ kJ/kg.°C})(60°\text{C} - 25°\text{C}) = \mathbf{73.15 \text{ kW}}$$

Noting that heat transfer to the cold water is equal to the heat loss from the geothermal water, the outlet temperature of the geothermal water is determined from

$$\dot{Q}_{\text{out}} = [\dot{m}c_p(T_{\text{in}} - T_{\text{out}})]_{\text{geot.water}} \longrightarrow T_{\text{out}} = T_{\text{in}} - \frac{\dot{Q}_{\text{out}}}{\dot{m}c_p} = 140°\text{C} - \frac{73.15 \text{ kW}}{(0.75 \text{ kg/s})(4.31 \text{ kJ/kg.°C})} = 117.4°\text{C}$$

(b) The rate of entropy generation within the heat exchanger is determined by applying the rate form of the entropy balance on the entire heat exchanger:

$$\dot{m}_{\text{water}} s_1 + \dot{m}_{\text{geo}} s_3 - \dot{m}_{\text{water}} s_2 - \dot{m}_{\text{geo}} s_4 + \dot{S}_{\text{gen}} = 0$$

$$\dot{S}_{\text{gen}} = \dot{m}_{\text{water}}(s_2 - s_1) + \dot{m}_{\text{geo}}(s_4 - s_3)$$

Noting that both fresh and geothermal water are incompressible substances, thus

$$\dot{S}_{gen} = \dot{m}_{\text{water}} c_p \ln\frac{T_2}{T_1} + \dot{m}_{\text{geo}} c_p \ln\frac{T_4}{T_3}$$

$$= (0.50 \text{ kg/s})(4.18 \text{ kJ/kg.K}) \ln\frac{60+273}{25+273} + (0.75 \text{ kg/s})(4.31 \text{ kJ/kg.K}) \ln\frac{117.4+273}{140+273} = \mathbf{0.050 \text{ kW/K}}$$