空气动力学 HW 7

7.8

解：

ME:

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$u_1 = 0 : \quad p_1 = p_2 + \rho_2 u_2^2$$

state equation: $p = \rho R T$

$$p_1 = \rho_1 R T_1$$

$$p_2 = \rho_2 R T_2$$

EE:

$$h_1 + \frac{1}{2} u_1^2 = h_2 + \frac{1}{2} u_2^2$$

$$h_1 = h_2 + \frac{1}{2} u_2^2$$

$$c_p T_1 = c_p T_2 + \frac{1}{2} u_2^2$$

$$u_2 = \sqrt{c_p (T_1 - T_2) \cdot 2}$$

$$= \sqrt{\frac{2 \gamma R}{\gamma - 1} (T_1 - T_2)}$$

$$= \sqrt{\frac{2 \times 1.4 \times 287}{1.4 - 1} \times (1000 - 600)}$$

$$= 896.44 \text{ m/s}$$

7.9

解：

$$h_\infty + \frac{1}{2}V_\infty^2 = h_1 + \frac{1}{2}V_1^2$$

$$C_p T_\infty + \frac{1}{2}V_\infty^2 = C_p \cdot T_1 + \frac{1}{2}V_1^2$$

$$\frac{P_\infty}{P_1} = \left(\frac{T_\infty}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_\infty = \frac{P_\infty}{R\rho_\infty} = \frac{0.61\times1.01\times10^5}{287\times0.819} = 262.11\ K$$

$$T_1 = \frac{T_\infty}{\left(\frac{P_\infty}{P_1}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{262.11}{\left(\frac{0.61}{0.5}\right)^{\frac{0.4}{1.4}}}$$

$$= 247.63\ K$$

$$C_p = \frac{\gamma R}{\gamma-1} = \frac{1.4}{0.4}\times287 = 1004.5\ J/(kg\cdot k)$$

$$\therefore\quad V_1 = \sqrt{2C_p(T_\infty - T_1) + V_\infty^2}$$

$$= \sqrt{2\times1004.5\times(262.11-247.63)+300^2}$$

$$= 345.09\ m/s$$

7.10

解: incompressible BE

$$p_\infty + \frac{1}{2}\rho_\infty V_\infty^2 = p_1 + \frac{1}{2}\rho_\infty V_1^2$$

$$V_{1in} = \sqrt{\frac{(p_\infty - p_1 + \frac{1}{2}\rho_\infty V_\infty^2)\cdot 2}{\rho_\infty}}$$

$$= \sqrt{\frac{(0.61-0.5)\times 1.01\times 10^5\times 2 + 0.819\times 300^2}{0.819}}$$

$$= 342.24 \text{ m/s}$$

$$\text{error} = \frac{V_1 - V_{1in}}{V_1}\times 100\% = \frac{345.09-342.24}{345.09}\times 100\%$$

$$= 0.8259\%$$

7.11

解： $h_\infty + \frac{1}{2}V_\infty^2 = h_2 + \frac{1}{2}V_2^2$

$$c_p T_\infty + \frac{1}{2}V_\infty^2 = c_p T_2 + \frac{1}{2}V_2^2$$

$$T_\infty = 262.11\ K$$

$$T_2 = \frac{T_\infty}{\left(\frac{P_\infty}{P_2}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{262.11}{\left(\frac{0.61}{0.3}\right)^{\frac{0.4}{1.4}}} = 214.00\ K$$

$$\therefore V_2 = \sqrt{2c_p(T_\infty - T_2) + V_\infty^2}$$

$$= \sqrt{2 \times 1004.5 \times (262.11 - 214) + 300^2}$$

$$= 432.03\ m/s$$

7.12

解：
$$V_{1in} = \sqrt{\frac{2(P_\infty - P_1 + \frac{1}{2}\rho_\infty V_\infty^2)}{\rho_\infty}}$$

$$= \sqrt{\frac{(0.61-0.3)\times 1.01\times 10^5 \times 2 + 0.819\times 300^2}{0.819}}$$

$$= 407.99 \ m/s$$

$$error = \frac{V_1 - V_{1in}}{V_1} \times 100\% = \frac{432.03 - 407.99}{432.03} \times 100\%$$

$$= 5.564\%$$