空气动力学 HW 13

9.12

解:

compression corner

Oblique shock wave

θ-β-M relation:

$$\tan\theta = 2\cot\beta \frac{M_1^2\sin^2\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}$$

$$\theta = 30.6^\circ,\ M_1 = 3 \Rightarrow \beta = 53.1^\circ$$

$$M_{n.1} = M_1\cdot\sin\beta = 3\times\sin 53.1^\circ = 2.4$$

$$M_{n.2}^2 = \frac{1+\frac{\gamma-1}{2}M_{n.1}^2}{\gamma M_{n.1}^2 - \frac{\gamma-1}{2}} = 0.2737,\ M_{n.2} = 0.5231$$

$$M_2 = \frac{M_{n.2}}{\sin(\beta-\theta)} = \frac{0.5231}{\sin(53.1^\circ-30.6^\circ)} = 1.367$$

expansion corner

$$\theta = \nu(M_3)-\nu(M_2)$$

$$\nu(\theta) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}$$

$$\nu(M_2) = \sqrt{\frac{2.4}{0.4}}\tan^{-1}\sqrt{\frac{0.4}{2.4}(1.367^2-1)} - \tan^{-1}\sqrt{1.367^2-1} = 8.1^\circ$$

$$\nu(M_3) = \theta+\nu(M_2) = 30.6^\circ+8.1^\circ = 38.7^\circ$$

$$\Rightarrow M_3 = 2.48 \quad \boxed{\text{ANS}}$$

$$\left.\begin{aligned}\frac{P_3}{P_2} &= \left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_3^2}\right)^{\frac{\gamma}{\gamma-1}}\\ \frac{P_2}{P_1} &= 1+\frac{2\gamma}{\gamma+1}(M_{n.1}^2-1)\end{aligned}\right\} \Rightarrow P_3 = 1.20\ \text{atm} \quad \boxed{\text{ANS}}$$

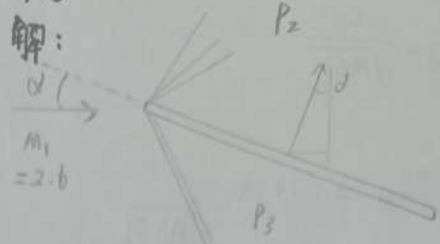
$$\begin{cases}\dfrac{T_3}{T_2} = \dfrac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_3^2}\\ \dfrac{T_2}{T_1} = \dfrac{P_2}{P_1}\cdot\dfrac{\rho_1}{\rho_2} = \left[1+\frac{2\gamma}{\gamma+1}(M_{n.1}^2-1)\right]\cdot\dfrac{2+(\gamma-1)M_{n.1}^2}{(\gamma+1)M_{n.1}^2}\end{cases}$$

$$\Rightarrow T_3 = 358\ \text{k} \quad \boxed{\text{ANS}}$$

There is shock wave, which is not isentropic, thus the state can't return to the origional.

9.13

解:




a) $\alpha = 5^\circ$

$$\theta = \nu(M_2)-\nu(M_1)$$

$$\nu(M_1) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_1^2-1)} - \tan^{-1}\sqrt{M_1^2-1} = 41.4^\circ$$

$$\nu(M_2) = \theta+\nu(M_1) = 46.41^\circ$$

$$\Rightarrow M_2 = 2.83$$

$$\frac{P_2}{P_1} = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}} = 0.7024$$

$$C_{p.u} = \frac{2}{\gamma M_1^2}\left(\frac{P_2}{P_1}-1\right) = -0.06289$$

θ-β-M relation:

$$\tan\theta = 2\cot\beta\frac{M_1^2\sin^2\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}$$

$$\theta = 5^\circ,\ M_1 = 2.6 \Rightarrow \beta = 26.5^\circ$$

$$M_{n.1} = M_1\cdot\sin\beta = 2.6\times\sin 26.5^\circ = 1.16$$

$$\frac{P_3}{P_1} = 1+\frac{2\gamma}{\gamma+1}(M_{n1}^2-1) = 1.403$$

$$C_{p.l} = \frac{2}{\gamma M_1^2}\left(\frac{P_3}{P_1}-1\right) = 0.08516$$

$$C_n = \frac{1}{c}\int_0^c (C_{p.l}-C_{p.u})\,dx = 0.14805$$

$$C_L = C_n\cdot\cos\alpha = 0.1475$$

$$C_d = C_n\cdot\sin\alpha = 0.0129 \quad \boxed{\text{ANS}}$$

b) 同理, $\nu(M_2) = \theta+\nu(M_1) = 15^\circ+41.1^\circ = 56.41^\circ,\ M_2 = 3.37$

$$\left.\begin{aligned}C_{p.u} &= -0.1448\\ C_{p.l} &= 0.3231\end{aligned}\right\}\Rightarrow C_n = 0.4679$$

$$\Rightarrow C_L = 0.4520$$

$$C_d = 0.1211 \quad \boxed{\text{ANS}}$$

c) 同理可得, $C_L = 1.19$

$$C_d = 0.687 \quad \boxed{\text{ANS}}$$

12.1

解: $C_l = \frac{4\alpha}{\sqrt{M_\infty^2-1}}$ , $C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2-1}}$

a) $\alpha = 5^\circ$

$$C_l = \frac{4}{\sqrt{2.6^2-1}} \times \frac{5}{180}\cdot\pi = 0.1454$$

$$C_d = \frac{4}{\sqrt{2.6^2-1}} \times \left(\frac{5}{180}\pi\right)^2 = 0.0127$$

$\boxed{\text{ANS}}$

b) $\alpha = 15^\circ$

$$C_l = \frac{4}{\sqrt{2.6^2-1}} \times \frac{15}{180}\pi = 0.4363$$

$$C_d = \frac{4}{\sqrt{2.6^2-1}} \times \left(\frac{15}{180}\pi\right)^2 = 0.1142$$

$\boxed{\text{ANS}}$

c) $\alpha = 30^\circ$

$$C_l = \frac{4}{\sqrt{2.6^2-1}} \times \frac{30}{180}\pi = 0.8727$$

$$C_d = \frac{4}{\sqrt{2.6^2-1}} \times \left(\frac{30}{180}\pi\right)^2 = 0.4569$$

$\boxed{\text{ANS}}$

When the AoA is small, the results obtained by the linearized theory is accurate.
With AoA increasing, accuracy decreases.

12.2

解: $C_p = \frac{2\theta}{\sqrt{M_\infty^2-1}} = \frac{2}{\gamma M_\infty^2}\left(\frac{P}{P_\infty}-1\right)$

$$\Rightarrow \frac{P_t}{P_\infty} = \frac{\gamma\theta\cdot M_\infty^2}{\sqrt{M_\infty^2-1}}+1 \ , \ \frac{P_b}{P_\infty} = \frac{-\theta\gamma M_\infty^2}{\sqrt{M_\infty^2-1}}+1$$

a) $\theta = \alpha = 5^\circ$

$$\frac{P_t}{P_\infty} = \frac{1.4\times\frac{5\pi}{180}\times 2.6^2}{\sqrt{2.6^2-1}}+1 = 1.344$$

$$\frac{P_b}{P_\infty} = \frac{-1.4\times\frac{5\pi}{180}\times 2.6^2}{\sqrt{2.6^2-1}}+1 = 0.656$$

$\boxed{\text{ANS}}$

b) $$\frac{P_t}{P_\infty} = \frac{1.4\times\frac{15\pi}{180}\times 2.6^2}{\sqrt{2.6^2-1}}+1 = 2.032$$

$$\frac{P_b}{P_\infty} = \frac{-1.4\times\frac{15\pi}{180}\times 2.6^2}{\sqrt{2.6^2-1}}+1 = -0.0324$$

$\boxed{\text{ANS}}$

c) $$\frac{P_t}{P_\infty} = \frac{1.4\times\frac{30\pi}{180}\times 2.6^2}{\sqrt{2.6^2-1}}+1 = 3.065$$

$$\frac{P_b}{P_\infty} = \frac{-1.4\times\frac{30\pi}{180}\times 2.6^2}{\sqrt{2.6^2-1}}+1 = -1.065$$

$\boxed{\text{ANS}}$

With AoA increasing, the accuracy of linearized theory is decreasing.

12.4

解: $C_d = \frac{D'}{q_\infty\cdot c}$

$$\Rightarrow D' = C_d\cdot\frac{1}{2}\rho_\infty V_\infty^2\cdot c$$

$$= \frac{4\alpha^2}{\sqrt{M_\infty^2-1}}\cdot\frac{1}{2}M_\infty^2\cdot\gamma\cdot P_\infty\cdot c$$

$$= \frac{M_\infty^2}{\sqrt{M_\infty^2-1}}\cdot(2\alpha^2\gamma P_\infty\cdot c)$$

∴ the drag force ↑ with $M_\infty$ ↑

12.5

解: $\frac{L}{D} = \frac{C_L}{C_d} = \frac{1}{\alpha}$

$$\frac{d\left(\frac{C_l}{C_d}\right)}{d\alpha} = -\frac{1}{\alpha^2} < 0$$

$\frac{L}{D}$ decreases as $\alpha$ increasing

∴ $\left(\frac{L}{D}\right)_{max}$ is close to infinity $(+\infty)$

when $\alpha$ is near to zero

14.1

解:

a. Using Newtonian theory

1) $\alpha = 5^\circ$

$C_{p.u} = 0,\ C_{p.l} = 2\sin^2\alpha = 0.01519$

$C_n = \frac{1}{c}\int_0^c (C_{p.l} - C_{p.u})\,dx = 0.01519$

$C_l = C_n\cos\alpha = 0.01513$, $\text{Error} = \frac{0.1475-0.01513}{0.1475} = 90\%$

$C_d = C_n\sin\alpha = 1.3239\times10^{-3}$, $\text{Error} = \frac{0.0129-0.0013239}{0.0129} = 90\%$

2) $\alpha = 15^\circ$，同理可得

$C_l = 0.1294$, $\text{Error} = \frac{0.452-0.1294}{0.452} = 71\%$

$C_d = 0.03468$, $\text{Error} = \frac{0.1211-0.03468}{0.1211} = 71\%$

3) $\alpha = 30^\circ$

$C_l = 0.4330$, $\text{Error} = \frac{1.19-0.433}{1.19} = 64\%$

$C_d = 0.25$, $\text{Error} = \frac{0.687-0.25}{0.687} = 64\%$

b. Using Modified newtonian theory

$C_p = C_{p.max}\sin^2\alpha$

$C_{p.max} = \frac{2}{\gamma M_\infty^2}\left(\frac{P_{0.2}}{P_\infty} - 1\right)$

At $M_1 = 2.6$, $\frac{P_{0.2}}{P_\infty} = \frac{P_{0.2}}{P_1} = 9.181 \Rightarrow C_{p.max} = 1.7289$

1) $\alpha = 5^\circ$

$C_{p.u} = 0,\ C_{p.l} = 1.7289\sin^2 5^\circ = 0.01313$

$C_n = \frac{1}{c}\int_0^c (C_{p.l} - C_{p.u})\,dx = 0.01313$

$C_l = C_n\cos\alpha = 0.01308$, $\text{Error} = \frac{0.1475-0.01308}{0.1475} = 91\%$

$C_d = C_n\sin\alpha = 0.001144$, $\text{Error} = \frac{0.0129-0.001144}{0.0129} = 91\%$

2) $\alpha = 15^\circ$. 同理可得:

$C_l = 0.1119$, $\text{Error} = \frac{0.452-0.1119}{0.452} = 75\%$

$C_d = 0.02997$, $\text{Error} = \frac{0.1211-0.02997}{0.1211} = 75\%$

3) $\alpha = 30^\circ$

$C_l = 0.3743$, $\text{Error} = \frac{1.19-0.3743}{1.19} = 69\%$

$C_d = 0.2161$, $\text{Error} = \frac{0.687-0.2161}{0.687} = 69\%$

At low supersonic Mach, the accuracy of newtonian and modified newtonian theories increases with AoA increasing. But it's not recommended to use these theories to solve low supersonic Mach problems due to the large error.

14.2

解:

Using straight newtonian theory

$C_{p.u} = 0$

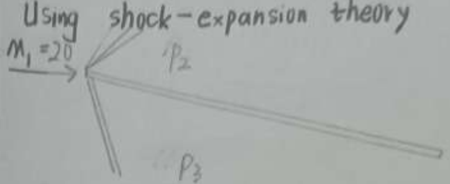
$C_{p.l} = 2\sin^2\alpha = 2\sin^2 20^\circ = 0.2340$

$C_n = \frac{1}{c}\int_0^c (C_{p.l} - C_{p.u})\,dx = 0.2340$

$C_l = C_n\cos\alpha = 0.220$ $\boxed{\text{ANS}}$

$C_d = C_n\sin\alpha = 0.08$

Using shock-expansion theory




expansion wave

$\theta = \nu(M_2) - \nu(M_1)$

$\nu(M_1) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_1^2-1)} - \tan^{-1}\sqrt{M_1^2-1} = 116.20^\circ$

$\nu(M_2) = \theta + \nu(M_1) = 20^\circ + 116.20^\circ = 136.20^\circ$

it's beyond the max expansion angle and $M_2$ can't be found, so there is a void on the upper surface.

Namely, $P_2 = 0$

$C_{p.u} = \frac{2}{\gamma M_1^2}\left(\frac{P_2}{P_1} - 1\right) = -3.5714\times10^{-3}$

oblique shock wave

$\theta$-$\beta$-$M$ relation

$\tan\theta = 2\cot\beta\,\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}$

$\theta = 20^\circ,\ M_1 = 20 \Rightarrow \beta = 24.9^\circ$

$M_{n,1} = M_1\sin\beta = 8.42$

$\frac{P_3}{P_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n,1}^2 - 1) = 82.55$

$C_{p.l} = \frac{2}{\gamma M_1^2}\left(\frac{P_3}{P_1} - 1\right) = 0.2912$

$C_n = \frac{1}{c}\int_0^c (C_{p.l} - C_{p.u})\,dx = 0.2948$

$C_l = C_n\cdot\cos\alpha = 0.2770$ $\boxed{\text{ANS}}$

$C_d = C_n\cdot\sin\alpha = 0.1008$

$\text{Error}_{cl} = \frac{-0.22+0.277}{0.277} = 21\%$

$\text{Error}_{cd} = \frac{0.1008-0.08}{0.1008} = 21\%$