

第 6 周习题 常微分方程 B

March 22, 2022

1. For each of the following equations, decide whether the equation is linear or nonlinear. If the equation is linear, state whether the equation is homogeneous or inhomogeneous.

(1) $y'' + 3y' + 5y = 3 \cos t$

(2) $t^2 y'' + (\sin t)y' = y - \cos t$

(3) $y'' + 3y' + 4 \sin y = 0$

(4) $(1 - t^2)y'' = 3y$

2. For each of the following equations, find its general solution.

(1) $y'' + 3y' - 4y = 0$

(2) $2y'' - y' - y = 0$

3. Find the solution of the initial value problem

$$2y'' - 3y' + y = 0, \quad y(0) = 2, \quad y'(0) = \frac{1}{2}.$$

4. In each of the following exercises, use the definition to explain that $y_1(t)$ and $y_2(t)$ are linearly independent solutions of the given differential equation. In addition, calculate the Wronskian and use it to explain the independence of the given solutions.

(1) $y'' - y' - 2y = 0$, $y_1(t) = e^{-t}$, $y_2(t) = e^{2t}$

(2) $y'' + 6y' + 9y = 0$, $y_1(t) = e^{-3t}$, $y_2(t) = te^{-3t}$

5. If the Wronskian of f and g is $t \cos t - \sin t$, and if $u = f + 2g$, $v = f - g$, find the Wronskian of u and v .

6. If the differential equation $ty'' + 2y' + te^t y = 0$ has y_1 and y_2 as a fundamental set of solutions and if $W[y_1, y_2](1) = 2$, find the value of $W[y_1, y_2](5)$.

7. Can $y = \sin(t^2)$ be a solution on an interval containing $t = 0$ of an equation $y'' + p(t)y' + q(t)y = 0$ with continuous coefficients? Explain your answer. (*Hint. Note that if $y = \sin(t^2)$ is a solution, then it satisfies the initial conditions $y(0) = y'(0) = 0$. Apply the Existence and Uniqueness Theorem.*)