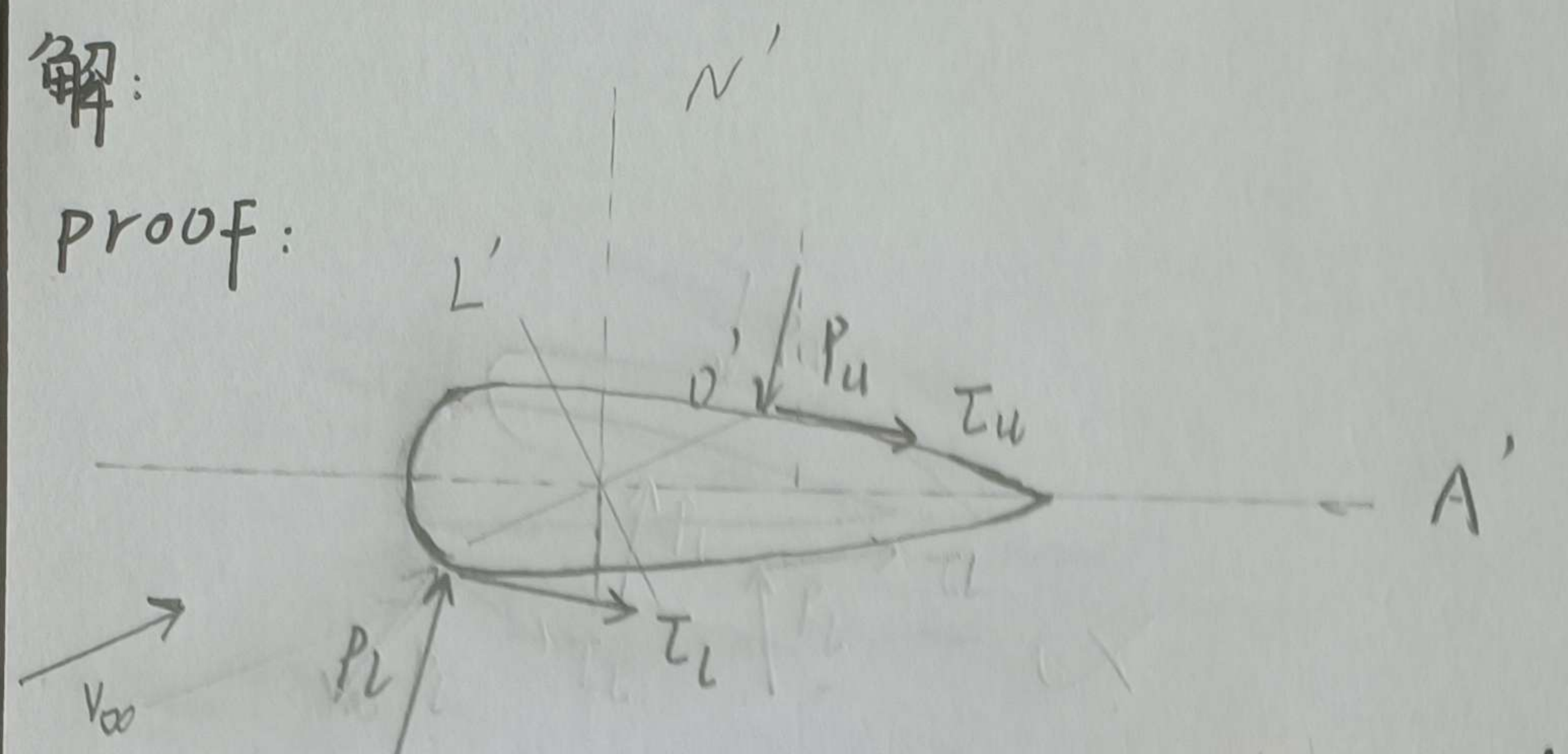
空气动力学

2.2

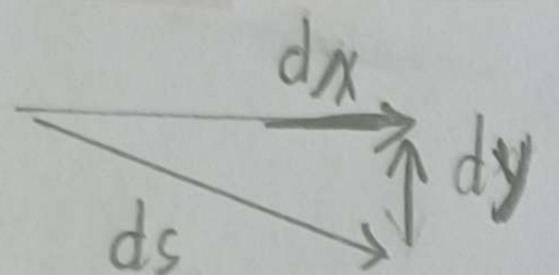
解:

proof:



$$dN' = P_l ds_l \cdot \cos\theta - P_u ds_u \cos\theta - \tau_l ds_l \sin\theta - \tau_u ds_u \sin\theta$$

$$dA' = \tau_u \cos\theta ds_u + \tau_l \cos\theta ds_l - P_u \sin\theta ds_u + P_l \sin\theta ds_l$$



$$dx = ds \cdot \cos\theta$$
$$dy = -ds \cdot \sin\theta$$

$$dN' = P_l dx - P_u dx + \tau_l dy_l + \tau_u dy_u$$

$$dA' = \tau_u dx + \tau_l dx + P_u dy_u - P_l dy_l$$

$$N' = \int_0^c (P_l - P_u) dx + \int_0^c \left(\tau_l \frac{dy_l}{dx} + \tau_u \frac{dy_u}{dx}\right) dx$$

$$A' = \int_0^c (\tau_u + \tau_l) dx + \int_0^c \left(P_u \frac{dy_u}{dx} - P_l \frac{dy_l}{dx}\right) dx$$

$$L' = N'\cos\alpha - A'\sin\alpha$$
$$= \cos\alpha \int_0^c (P_l - P_u) dx + \cos\alpha \int_0^c \left(\tau_l \frac{dy_l}{dx} + \tau_u \frac{dy_u}{dx}\right) dx$$
$$- \sin\alpha \int_0^c (\tau_u + \tau_l) dx + \sin\alpha \int_0^c \left(P_l \frac{dy_l}{dx} - P_u \frac{dy_u}{dx}\right) dx$$

① 忽略 shear stress 作用

② $\cos\alpha \to 1$, $\sin\alpha \to 0$

$$\Rightarrow L' \approx \cos\alpha \int_0^c (P_l - P_u) dx = \int_0^c (P_l - P_u) dx$$

2.4

解:

$$\frac{u}{dx} = \frac{v}{dy}$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{-x}{y}$$

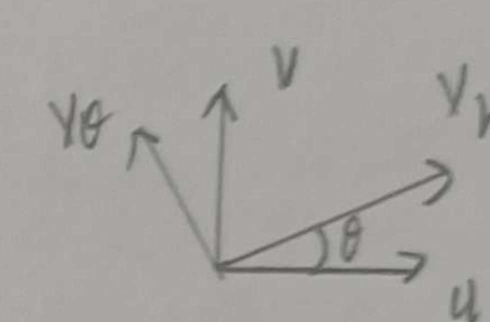
$$y\,dy = -x\,dx$$

$$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C \quad , C \text{ is const}$$

$$\psi = x^2 + y^2 = D \quad \boxed{\text{ANS}} \quad D \text{ is const}$$

2.8

解:



a) $$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial(rV_r)}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta}$$

$$V_r = u\cos\theta + v\sin\theta = \frac{cr\sin\theta}{r^2}\cdot\cos\theta + \frac{-cr\cos\theta}{r^2}\cdot\sin\theta = 0$$

$$V_\theta = v\cos\theta - u\sin\theta = \frac{-cr\cos^2\theta}{r^2} - \frac{cr\sin^2\theta}{r^2} = \frac{-c}{r}$$

$$\therefore \nabla\cdot\vec{V} = \frac{1}{r}\cdot 0 + \frac{1}{r}\cdot 0 = 0$$

b) $$\nabla \times \vec{V} = \begin{vmatrix} \vec{e}_r & \vec{e}_\theta & \vec{e}_z \\ \frac{1}{r}\frac{\partial r}{\partial r} & \frac{1}{r}\frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ 0 & \frac{-c}{r} & 0 \end{vmatrix}$$

$$\therefore \vec{\xi} = \nabla\times\vec{V} = \vec{e}_r\cdot 0 + \vec{e}_\theta\cdot 0 + \vec{e}_z\cdot 0$$

3.1

proof:

动量方程(in x direction):

$$\frac{\partial \rho u}{\partial t} + \nabla\cdot(\rho u\vec{V}) = \frac{-\partial p}{\partial x} + \rho f_x + f_{vis.x}$$

steady + incompressible + inviscid + no body force

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)\rho = -\frac{\partial p}{\partial x} \quad ①$$

$$\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)\rho = -\frac{\partial p}{\partial y} \quad ②$$

$$\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)\rho = -\frac{\partial p}{\partial z} \quad ③$$

irrotational : $\nabla\times\vec{V} = \vec{0}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u & v & w \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = \left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right)\vec{i} + \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)\vec{j} + \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)\vec{k}$$

$$\frac{\partial v}{\partial z} = \frac{\partial w}{\partial y},\quad \frac{\partial w}{\partial x} = \frac{\partial u}{\partial z},\quad \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} \quad (*)$$

∴将①②③分别乘 dx, dy, dz，再代换 $(*)$

$$u\frac{\partial u}{\partial x}dx + v\frac{\partial v}{\partial x}dx + w\frac{\partial w}{\partial x}dx = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial x}dx$$

$$u\frac{\partial u}{\partial y}dy + v\frac{\partial v}{\partial y}dy + w\frac{\partial w}{\partial y}dy = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial y}dy$$

$$u\frac{\partial u}{\partial z}dz + v\frac{\partial v}{\partial z}dz + w\frac{\partial w}{\partial z}dz = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial z}dz$$

相加得:

$$u\left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz\right) +$$

$$v\left(\frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz\right) +$$

$$w\left(\frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz\right) = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz\right)$$

namely, $u\,du + v\,dv + w\,dw = -\frac{1}{\rho}\cdot dp$

$$\frac{1}{2}(u^2 + v^2 + w^2) = -\frac{1}{\rho}\cdot p + Const$$

$$\frac{1}{2}\rho V^2 + p = Const$$

∴ For irrotational flow,

BE is applicable between any two points.

空气动力学 HW 2

3.6

解: $p + \frac{1}{2}\rho V^2 = Const$

$$P_\infty + \frac{1}{2}\rho_\infty V_\infty^2 = P_{pitot}$$

$$V_\infty = \sqrt{\frac{2(P_{pitot} - P_\infty)}{\rho_\infty}}$$

$$= \sqrt{\frac{2\times(1.07\times 10^5 - 1.01\times 10^5)}{1.23}}$$

$$= 98.77\ m/s$$

3.7

解: $$C_p = \frac{P - P_\infty}{q_\infty} = \frac{\frac{1}{2}\rho_\infty V_\infty^2 - \frac{1}{2}\rho_\infty V^2}{\frac{1}{2}\rho_\infty V_\infty^2}$$

$$= 1 - \left(\frac{V}{V_\infty}\right)^2$$

$$= 1 - \left(\frac{130}{98.77}\right)^2$$

$$= -0.7324$$

3.9

解:

proof:

Source flow $V_r = \frac{c}{r}$, $V_\theta = 0$

for $r>0$, $\vec{V} = V_r \vec{e_r} + V_\theta \vec{e_\theta}$

$$\nabla\cdot\vec{V} = \frac{1}{r}\frac{\partial r V_r}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta}$$

$$= \frac{1}{r}\cdot 0 + 0 = 0$$

连续性方程: $\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0$

$$\frac{\partial \rho}{\partial t} + (\nabla\rho)\cdot\vec{V} + \rho\,\nabla\cdot\vec{V} = 0$$

$\therefore \frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \vec{V}\cdot(\nabla\rho) = 0$, incompressible excpt at origin ($r=0$)

$$\vec{\xi} = \nabla\times\vec{V} = \begin{vmatrix} \vec{e_r} & \vec{e_\theta} & \vec{e_z} \\ \frac{1}{r}\frac{\partial r}{\partial r} & \frac{1}{r}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial\theta} \\ \frac{c}{r} & 0 & 0 \end{vmatrix}$$

$= 0\vec{e_r} + 0\vec{e_\theta} + 0\vec{e_z} = \vec{0}$, irrotational

3.11

解：

proof:

$$\phi = \frac{\Lambda}{2\pi} \ln r$$

$$\nabla^2 \phi = \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2} \cdot \frac{\partial^2 \phi}{\partial \theta^2}$$

$$= \frac{1}{r} \cdot \frac{\partial}{\partial r}\left(\frac{\Lambda}{2\pi}\right) + 0$$

$$= 0$$

$\therefore \phi$ satisfies the Laplace equation

$$\psi = \frac{\Lambda}{2\pi} \theta$$

$$\nabla^2 \psi = \frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2} \cdot \frac{\partial^2 \psi}{\partial \theta^2}$$

$$= \frac{1}{r} \cdot \frac{\partial}{\partial r} 0 + \frac{1}{r^2} \cdot \frac{\partial}{\partial \theta}\left(\frac{\Lambda}{2\pi}\right)$$

$$= 0 + 0 = 0$$

$\therefore \psi$ satisfies the Laplace equation