空气动力学 HW 12

11.1

解:

$$u = \frac{\partial\phi}{\partial x} = V_\infty + \frac{70}{\sqrt{1-M_\infty^2}} e^{-2\pi\sqrt{1-M_\infty^2}\cdot y} (\cos 2\pi x)\cdot 2\pi$$

$$v = \frac{\partial\phi}{\partial y} = 0 + \frac{70}{\sqrt{1-M_\infty^2}} e^{-2\pi\sqrt{1-M_\infty^2}\cdot y} \cdot(-2\pi)\sqrt{1-M_\infty^2}\cdot \sin 2\pi x$$

$$M_\infty = \frac{V_\infty}{a} = \frac{V_\infty}{\sqrt{\gamma R T_\infty}} = \frac{700}{\sqrt{1.4\times1716\times519}} = 0.6269$$

At $(0.2\text{ft}, 0.2\text{ft})$

$$u = 700 + \frac{70}{\sqrt{1-0.6269^2}} e^{-2\pi\sqrt{1-0.6269^2}\times 0.2} \cos 0.4\pi \times 2\pi$$

$$= 765.53 \text{ ft/s}$$

$$v = \frac{70}{\sqrt{1-0.6269^2}} e^{-2\pi\sqrt{1-0.6269^2}\times0.2} \cdot(-2\pi)\sqrt{1-0.6269^2}\cdot\sin 0.4\pi$$

$$= -157.14 \text{ ft/s}$$

$$V = \sqrt{u^2+v^2} = 781.49 \text{ ft/s}$$

$$\frac{T_0}{T_\infty} = 1+\frac{\gamma-1}{2}M_\infty^2 \Rightarrow T_0 = (1+0.2\times0.6269^2)\times 519 = 559.8^0 R$$

$$a_0 = \sqrt{\gamma R T_0} = \sqrt{1.4\times1716\times559.8} = 1159.7 \text{ ft/s}$$

$$a^2 = a_0^2 - \frac{\gamma-1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right] \quad \triangle$$

$$a = \sqrt{1159.7^2 - 0.2\times781.49^2} = 1105.8 \text{ ft/s}$$

$$M = \frac{V}{a} = \frac{781.49}{1105.8} = 0.7067 \quad \boxed{\text{ANS}}$$

$$\frac{p}{p_\infty} = \left(\frac{1+\frac{\gamma-1}{2}M_\infty^2}{1+\frac{\gamma-1}{2}M^2}\right)^{\frac{\gamma}{\gamma-1}} = 0.9331$$

$$\Rightarrow p = 0.9331 \text{ atm} \quad \boxed{\text{ANS}}$$

$$T = \frac{a^2}{\gamma R} = 509.0 \ {}^\circ R \quad \boxed{\text{ANS}}$$

11.2

解: According to Figure 4.5

At AoA $= 5^\circ$, $C_{l,0} = 0.75$

$$C_{l,\text{comp}} = \frac{C_{l,0}}{\sqrt{1-M_\infty^2}} = \frac{0.75}{\sqrt{1-0.6^2}} = 0.9375 \quad \boxed{\text{ANS}}$$

11.3

解: $C_{p,0} = -0.54$

a) $$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}} = \frac{-0.54}{\sqrt{1-0.58^2}} = -0.663 \quad \boxed{\text{ANS}}$$

b) $$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2} + \left[M_\infty^2/(1+\sqrt{1-M_\infty^2})\right]C_{p,0}/2}$$

$$= \frac{-0.54}{\sqrt{1-0.58^2} + \frac{0.58^2}{1+\sqrt{1-0.58^2}}\times\frac{(-0.54)}{2}}$$

$$= -0.7063 \quad \boxed{\text{ANS}}$$

c) $$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2} + C_{p,0}M_\infty^2\left(1+\frac{\gamma-1}{2}M_\infty^2\right)/\left(2\sqrt{1-M_\infty^2}\right)}$$

$$= \frac{-0.54}{\sqrt{1-0.58^2} + \frac{(-0.54)\times0.58^2\times(1+0.2\times0.58^2)}{2\times\sqrt{1-0.58^2}}}$$

$$= -0.7763 \quad \boxed{\text{ANS}}$$

11.4

解: $C_{p.0} = -0.41$

$$C_p = \frac{C_{p.0}}{\sqrt{1-M_{cr}^2}} \quad ①$$

$$C_{p.cr} = \frac{2}{\gamma M_{cr}^2}\left[\left(\frac{1+\frac{\gamma-1}{2}M_{cr}^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma}{\gamma-1}} -1\right] . \quad ②$$

Using matlab to draw curve ① and ②

We obtain $M_{cr} = 0.74$. at which the two curves intersect.

11.5

解:

$$C_p = \frac{P-P_\infty}{q_\infty} = \frac{2}{\gamma M_\infty^2}\left(\frac{P}{P_\infty}-1\right)$$

$$= \frac{2}{\gamma M_\infty^2}\cdot\left[\left(\frac{1+\frac{\gamma-1}{2}M_\infty^2}{1+\frac{\gamma-1}{2}M^2}\right)^{\frac{\gamma}{\gamma-1}} -1\right]$$

if $M_\infty = M_{cr}$, $M=1$

$$\therefore \quad \frac{P}{P_\infty} = \left(\frac{1+\frac{\gamma-1}{2}M_{cr}^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \left(\frac{1+0.2\times 0.8^2}{1+0.2}\right)^{\frac{1.4}{0.4}}$$

$$= 0.8053 \quad \boxed{\text{ANS}}$$

11.6

解: $M_\infty = 0.5$

$M_A = 0.86$

$$C_{p.A} = \frac{P_A - P_\infty}{q_\infty} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{1+\frac{\gamma-1}{2}M_\infty^2}{1+\frac{\gamma-1}{2}M_A^2}\right)^{\frac{\gamma}{\gamma-1}} -1\right]$$

$$= \frac{2}{1.4\times 0.5^2} \times \left[\left(\frac{1+0.2\times 0.5^2}{1+0.2\times 0.86^2}\right)^{\frac{1.4}{0.4}} -1\right]$$

$$= -1.5318 \quad \boxed{\text{ANS}}$$

From the compressible date on the book.

$$C_{p.A} = \frac{2}{\gamma M_\infty^2}\left(\frac{P_A}{P_\infty}-1\right)$$

$$\left.\begin{array}{l} M_\infty = 0.5, \quad \frac{P_0}{P_\infty} = 1.186 \\ M_A = 0.86, \quad \frac{P_0}{P_A} = 1.621 \end{array}\right\} \Rightarrow \frac{P_A}{P_\infty} = \frac{1.186}{1.621}$$

$$\therefore C_{p.A} = \frac{2}{1.4\times 0.5^2} \times \left(\frac{1.186}{1.621}-1\right) = -1.5334 \quad \boxed{\text{ANS}}$$

11.7

解:

$$C_{p.A} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{1+\frac{\gamma-1}{2}M_\infty^2}{1+\frac{\gamma-1}{2}M_A^2}\right)^{\frac{\gamma}{\gamma-1}} -1\right] = \frac{(C_{p.0})_{min}}{\sqrt{1-M_\infty^2}}$$

with date of figure 11.5 a)

$$C_{p.A} = \frac{2}{1.4\times 0.3^2}\left[\left(\frac{1+0.2\times 0.3^2}{1+0.2\times 0.435^2}\right)^{\frac{1.4}{0.4}} -1\right] = -1.037$$

$$(C_{p.0})_{min} = C_{p.A}\cdot\sqrt{1-M_\infty^2} = -0.9894$$

with $M_\infty = 0.61$

$$C_{p.A}' = \frac{2}{1.4\times 0.61^2}\left[\left(\frac{1+0.2\times 0.61^2}{1+0.2\times M_A^2}\right)^{\frac{1.4}{0.4}} -1\right] = \frac{-0.9894}{\sqrt{1-0.61^2}}$$

$$\Rightarrow M_A = 1.006 \approx 1.0$$

11.9

解:

For circular cylinder surface

incompressible flow:

$$C_{p.0} = \frac{P-P_\infty}{q_\infty} = \frac{P-P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\frac{1}{2}\rho_\infty V_\infty^2 - \frac{1}{2}\rho_\infty V^2}{\frac{1}{2}\rho_\infty V_\infty^2}$$

$$= 1-\left(\frac{V}{V_\infty}\right)^2 = 1-4\sin^2\theta$$

$$(C_{p.0})_{min} = 1-4\times1^2 = -3$$

$$C_p = \frac{C_{p.0}}{\sqrt{1-M_\infty^2}}$$

$$C_{p.cr} = \frac{(C_{p.0})_{min}}{\sqrt{1-M_{cr}^2}} = \frac{2}{\gamma M_{cr}^2}\left[\left(\frac{1+\frac{\gamma-1}{2}M_{cr}^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma}{\gamma-1}}-1\right]$$

$$\frac{-3}{\sqrt{1-M_{cr}^2}} = \frac{2}{1.4\times M_{cr}^2}\left[\left(\frac{1+0.2M_{cr}^2}{1+0.2}\right)^{\frac{1.4}{0.4}}-1\right]$$

$\Rightarrow M_{cr} = 0.4181$ $\boxed{\text{ANS}}$

$$\text{Error} = \frac{0.4181-0.404}{0.404} = 3.49\%$$