空气动力学 HW3

3.8

解:

proof:

uniform flow

$\psi = V_\infty y$

$u = \frac{\partial \psi}{\partial y} = V_\infty$

$v = -\frac{\partial \psi}{\partial x} = 0$

$\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, incompressible

$$\nabla \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$= \vec{i}\cdot\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + \vec{j}\cdot\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \vec{k}\cdot\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$= 0\vec{i} + 0\vec{j} + 0\vec{k}$, irrotational

3.10

解:

proof:

$\phi = V_\infty x = V_\infty r\cos\theta$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = \frac{\partial V_\infty}{\partial x} + 0 + 0$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$= \frac{1}{r}\frac{\partial}{\partial r}(rV_\infty\cos\theta) + \frac{1}{r^2}\cdot V_\infty r\cdot(-\cos\theta) + 0$$

$= 0$ ，满足 Laplace 方程

$\psi = V_\infty y = V_\infty r\cos\theta$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = 0 + \frac{\partial V_\infty}{\partial y} + 0$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\partial^2\psi}{\partial z^2}$$

$$= \frac{1}{r}\frac{\partial}{\partial r}(rV_\infty\cos\theta) + \frac{1}{r^2}V_\infty r(-\cos\theta) + 0$$

$= 0$ ，满足 Laplace 方程

3.14

解: Doublet flow : $\psi = \frac{-k}{2\pi}\cdot\frac{\sin\theta}{r}$

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\cdot\frac{-k}{2\pi}\cdot\frac{\cos\theta}{r} = \frac{\partial\phi}{\partial r}$$

$$\Rightarrow \phi = \frac{+k}{2\pi}\cdot\frac{\cos\theta}{r}$$

3.15

解:

无升力圆柱绕流

$$\psi = V_\infty r\sin\theta\left(1 - \frac{R^2}{r^2}\right)$$

$$V_r = \frac{1}{r}\cdot\frac{\partial\psi}{\partial\theta} = \frac{1}{r}V_\infty r\left(1 - \frac{R^2}{r^2}\right)\cdot\cos\theta$$

$$= V_\infty\left(1 - \frac{R^2}{r^2}\right)\cos\theta$$

$$V_\theta = -\frac{\partial\psi}{\partial r} = (-1)\cdot V_\infty\sin\theta\cdot\left(1 + \frac{R^2}{r^2}\right)$$

$$C_p = 1 - \frac{V^2}{V_\infty^2} = 1 - \frac{V_r^2 + V_\theta^2}{V_\infty^2}$$

$$= 1 - \frac{\cancel{V_\infty^2}\cos^2\theta\cdot\left(1 - \frac{R^2}{r^2}\right)^2 + \cancel{V_\infty^2}\sin^2\theta\left(1 + \frac{R^2}{r^2}\right)^2}{\cancel{V_\infty^2}}$$

$$= 1 - \left[1 + \frac{R^4}{r^4} + 2\frac{R^2}{r^2}(\sin^2\theta - \cos^2\theta)\right]$$

$$= 2\frac{R^2}{r^2}(\cos^2\theta - \sin^2\theta) - \frac{R^4}{r^4} \quad \text{ANS}$$

on the surface

$$C_p|_{R=r} = 2(\cos^2\theta - \sin^2\theta) - 1$$

$$= 2\cos^2\theta - 2\sin^2\theta - 1$$

$$= 1 - 4\sin^2\theta$$

3.17

解:

有升力圆柱绕流

$$\psi = V_\infty r\sin\theta\left(1-\frac{R^2}{r^2}\right)+\frac{\Gamma}{2\pi}\ln\frac{r}{R}$$

$$V_r = \frac{1}{r}\cdot\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\cdot V_\infty r\cos\theta\left(1-\frac{R^2}{r^2}\right)$$

$$= V_\infty\cos\theta\cdot\left(1-\frac{R^2}{r^2}\right)$$

$$V_\theta = -\frac{\partial\psi}{\partial r} = (-1)\cdot\left[V_\infty\sin\theta\left(1+\frac{R^2}{r^2}\right)+\frac{\Gamma}{2\pi}\cdot\frac{1}{r}\right]$$

$\therefore \frac{V_r}{V_\infty} = \cos\theta\left(1-\frac{R^2}{r^2}\right)$, independent of V_∞;

$\frac{V_\theta}{V_\infty} = (-1)\left[\sin\theta\cdot\left(1+\frac{R^2}{r^2}\right)+\frac{\Gamma}{2\pi r}\cdot\frac{1}{V_\infty}\right]$,

it's dependent of V_∞.

So the streamlines shape will change.

3.18

解: $L' = 6\,N/m$

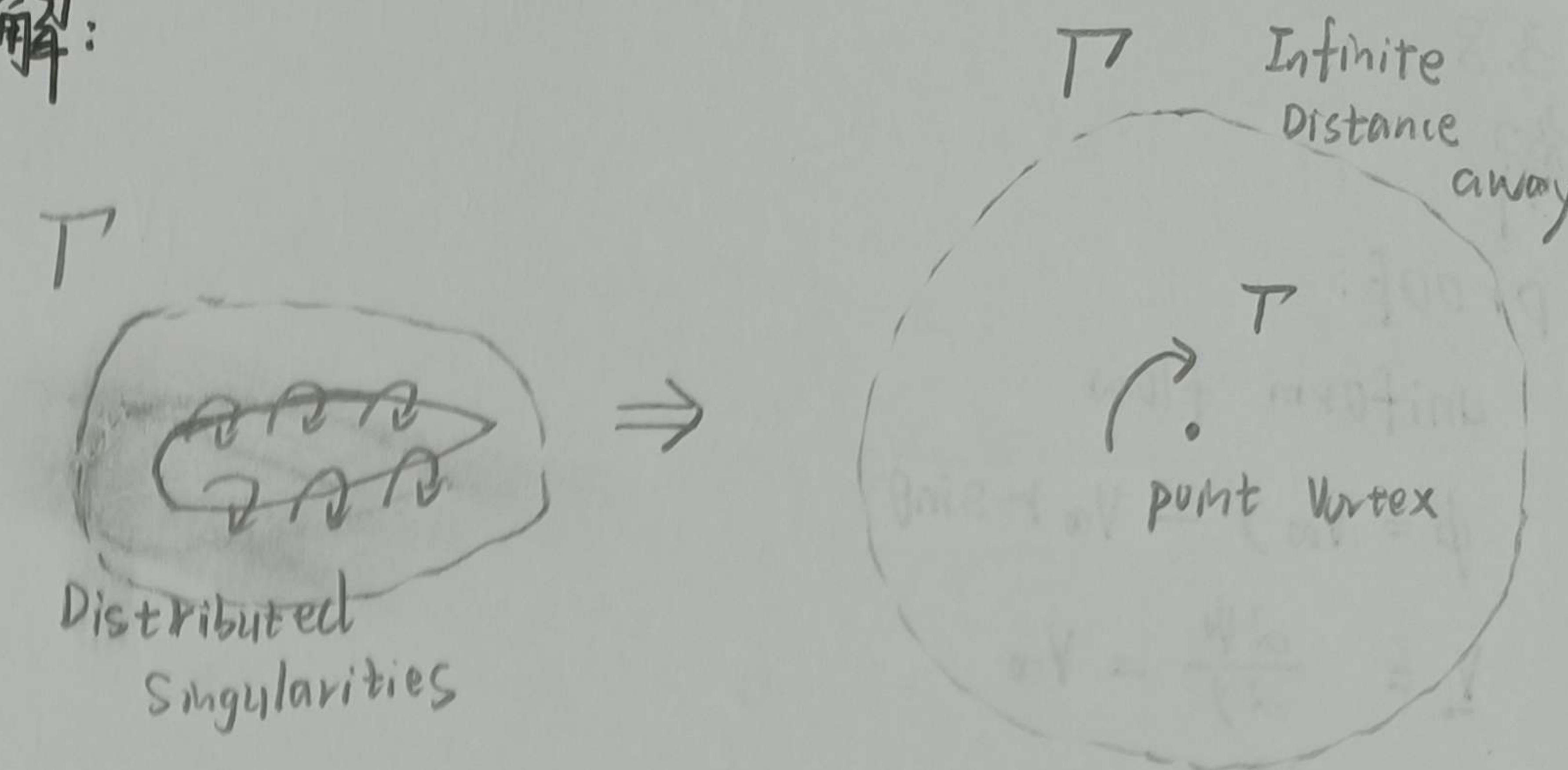
$V_\infty = 30\,m/s$

$$L' = \rho_\infty V_\infty \Gamma$$

$$\therefore \Gamma = \frac{L'}{\rho_\infty V_\infty} = \frac{6}{1.23\times 30} = 0.1626\ m^2/s$$

3.20

解:



airfoil 上的 flow 可合成为点源或点涡的分布,

涡强度的叠加是为总环量 Γ,

Γ 的值在所有包含 airfoil 的闭合曲线上相同,

airfoil 成为纸面上一个 speck, 分布的点涡形成一个强度为 Γ 的较大涡,

这与圆柱体的单点涡等价,

airfoil 上的升力与圆柱体相同, 即有 $L' = \rho_\infty V_\infty \Gamma$

3.21

解：

Since the volume flow rates between the streamlines are the same.

$$\Rightarrow \quad \psi_3 - \psi_2 = \psi_2 - \psi_1$$

$$\psi_3 = 2\psi_2 - 0$$

$$\psi = V_\infty r \sin\theta - \frac{K}{2\pi} \cdot \frac{\sin\theta}{r} = V_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right)$$

for ψ_2 passes through $\left(\frac{6}{5}R, \frac{\pi}{2}\right)$

$$\psi_2 = V_\infty \cdot \frac{6}{5}R\left(1 - \frac{25}{36}\right) = V_\infty R \cdot \frac{11}{30}$$

$$\therefore \psi_3 = \frac{11}{15} V_\infty R = V_\infty r \sin\frac{\pi}{2}\left(1 - \frac{R^2}{r^2}\right)$$

$$r^2 - \frac{11}{15}R \cdot r - R^2 = 0$$

$$r = \frac{\frac{11}{15}R \pm \sqrt{\left(\frac{11}{15}R\right)^2 + 4R^2}}{2} = \frac{\frac{11}{15}R \pm \sqrt{\frac{1021}{225}}R}{2}$$

$\therefore r = 1.432R$ （舍去负值）