第 3 周习题 Part II
常微分方程 B

March 3, 2022

1. Determine which of the following equations are exact and solve the ones that are.

   (1) $(1 - y\sin x)dx + \cos x\ dy = 0$

   (2) $\dfrac{x}{\sqrt{x^2+y^2}}dx + \dfrac{y}{\sqrt{x^2+y^2}}dy = 0$

   (3) $y' = \dfrac{3x^2+y}{3y^2+x}$

2. Solve the initial value problem.

$$(2x - y) + (2y - x)y' = 0, \quad y(1) = 3.$$

3. Show that $\mu(x, y) = \dfrac{1}{x^2+y^2}$ is an integrating factor for the equation

$$x\ dx - (x^2 + y^2 - y)dy = 0.$$

Find another integrating factor of the equation that depends *only on one variable*. Solve the equation by using either of the integrating factors.

4. Solve the following equations.

   (1) $(3x^2y + 2xy + y^3) + (x^2 + y^2)y' = 0$

   (2) $(y + 2xe^{-y/x})dx - x\ dy = 0$