SUSTC
Midterm test

Ordinary Differential Equations - B
2020

(1) (15 points) For this problem, you do not need to show your work. Just answer **True** or **False**.

× (a) Consider the linear differential equation $x' = 2x + t^2e^t$ and assume that the function φ is a solution. Then, any other solution to the equation is of the form $C\varphi$ where C is a real constant.

True False

✓ (b) Suppose y_P is a particular solution to the equation $y' = t^4y + e^{\cos t}(*)$. Then every solution to the equation (*) is the sum of a solution to the equation $y' = t^4y$ plus y_P.

True False

× (c) A differential form $dF = P(x,y)dx + Q(x,y)dy$ is exact in a rectangle if $\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$.

True False

× ✓ (d) Let f be continuous on the entire plane and $x' = f(t,x)$ be a differential equation. Then given any solution its interval of existence is $(-\infty, \infty)$.

True False

✓ (e) Let $x' = f(x)$ be a differential equation. x_0 is an equilibrium point if $f(x_0) = 0$

True False

✓ (f) 0 is an asymptotically stable equilibrium point of $x' = -\frac{1}{5}\sin(e^x - 1)$.

True False

✓ (g) Assume that φ is a solution of an autonomous differential equation defined in $\mathbb{R}$. Then, ψ such that $\psi(t) = \varphi(t + C)$, $t \in \mathbb{R}$, is a solution of the same equation.

True False

✓ (h) Consider the following differential equation: $x'' + 2cx' + \omega_0^2 x = 0$, with $c > 0$ and $\omega_0 > 0$. Then $x(t) \to 0$ as $t \to +\infty$.

True False

× (i) Let $f(t)$ and $g(t)$, $t \in \mathbb{R}$, be two differentiable functions. Suppose there is $t_0 \in \mathbb{R}$ such that the Wronskian of f and g is 0, then it is 0 for all $t \in \mathbb{R}$.

True False

× (j) Let y be a piecewise differentiable function of exponential order. Then for large enough values of s, the Laplace transform of y' satisfies $\mathcal{L}(y') = s\mathcal{L}(y)$.

True False

(2) (5 points)

(a) Find the general solution of $y' = y + 2xe^x$.

(b) Find the solution to the initial value problem

$$y' = y + 2xe^x, \qquad y(0) = 3.$$

(3) (5 points) Consider the differential equation $5ydx + 4xdy = 0$.

(a) Determine conditions on a and b so that $\mu(x, y) = x^a y^b$ is an integrating factor.

(b) Find a particular integrating factor and use it to solve (implicitly) the differential equation.

(4) (5 points) Consider the autonomous equation $y' = 6 + y - y^2$.

(a) Sketch the graph of the right-hand side of the equation.

(b) Sketch the phase line. Classify each equilibrium point as either unstable or asymptotically stable.

(c) Sketch the equilibrium solutions in the ty-plane. These equilibrium solutions divide the ty-plane into regions. Sketch at least one solution trajectory in each of these regions.

(5) (5 points) Consider the second order differential equation $x'' + x = \frac{1}{\cos t}$

(a) Find the general solution to the equation $x'' + x = 0$.

(b) Find the solution to the initial value problem

$$x'' + x = \frac{1}{\cos t} \quad \text{with} \quad x(0) = 1 \text{ and } x'(0) = 2.$$