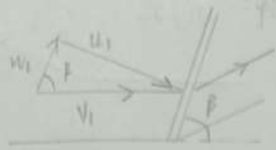
空气动力学 HW10

9.2

解： wave angle $\beta = 30^\circ$

$M_1 = 4$



$P_1 = 2.65 \times 10^4$ Pa

$T_1 = 223.3$ K

$\sin\beta = \frac{u_1}{V_1} = \frac{M_{n.1}}{M_1} \Rightarrow M_{n.1} = 4\sin 30^\circ = 2$

$\frac{P_2}{P_1} = 1 + \frac{2r}{r+1}(M_{n.1}^2 - 1)$

$\Rightarrow P_2 = \left[1 + \frac{2r}{r+1}(M_{n.1}^2 - 1)\right] \cdot P_1$

$= \left[1 + \frac{2.8}{2.4} \times (4-1)\right] \times 2.65 \times 10^4$

$= 11.925 \times 10^4$ Pa $\boxed{\text{ANS}}$ ✓

$\frac{T_2}{T_1} = \frac{P_2}{P_1} \cdot \frac{\rho_1}{\rho_2}$

$= \left[1 + \frac{2r}{r+1}(M_{n.1}^2 - 1)\right] \cdot \frac{2 + (r-1)M_{n.1}^2}{(r+1)M_{n.1}^2}$

$\Rightarrow T_2 = \left(1 + \frac{2.8}{2.4} \times 3\right) \times \frac{2 + 0.4 \times 4}{2.4 \times 4} \times 223.3$

$= 376.8$ K $\boxed{\text{ANS}}$ ✓

$M_{n.2} = \sqrt{\frac{1 + \frac{r-1}{2}M_{n.1}^2}{rM_{n.1}^2 - \frac{r-1}{2}}} = \sqrt{\frac{1 + 0.2 \times 4}{1.4 \times 4 - 0.2}} = 0.5774$

$\tan\theta = 2\cot\beta \cdot \frac{M_1^2 \sin^2\beta - 1}{M_1^2 \cdot (r + \cos 2\beta) + 2}$

$= 2 \cdot \frac{1}{\tan 30^\circ} \cdot \frac{16 \times \frac{1}{4} - 1}{16 \times (1.4 + \frac{1}{2}) + 2} = 0.3208$

$\Rightarrow \theta = 17.79^\circ$

$\sin(\beta - \theta) = \frac{u_2}{V_2} = \frac{M_{n.2}}{M_2}$

$\Rightarrow M_2 = \frac{M_{n.2}}{\sin(\beta - \theta)} = \frac{0.5774}{\sin(30^\circ - 17.79^\circ)} = 2.73$ $\boxed{\text{ANS}}$

$\frac{T_{0.2}}{T_2} = 1 + \frac{r-1}{2}M_2^2 \Rightarrow T_{0.2} = (1 + 0.2 \times 2.73^2) \times 376.8$

$= 938.4$ K $\boxed{\text{ANS}}$

$\frac{P_{0.2}}{P_2} = \left(\frac{T_0}{T_2}\right)^{\frac{r}{r-1}} \Rightarrow P_{0.2} = \left(\frac{938.4}{376.8}\right)^{\frac{1.4}{0.4}} \times 11.925 \times 10^4$

$= 290.7 \times 10^4$ Pa $\boxed{\text{ANS}}$

$S_2 - S_1 = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = \frac{rR}{r-1}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$

$= \frac{1.4 \times 287}{0.4}\ln\frac{376.8}{223.3} - 287\ln\frac{11.925 \times 10^4}{2.65 \times 10^4} = 93.88$ J/(kg·K)

9.4

解： wave angle $\beta = 36.87^\circ$

$\sin\beta = \frac{M_{n.1}}{M_1} \Rightarrow M_{n.1} = \sin\beta \cdot M_1$

$= \sin 36.87^\circ \times 3 = 1.8$

$M_{n.2} = \sqrt{\frac{1 + \frac{r-1}{2}M_{n.1}^2}{rM_{n.1}^2 - \frac{r-1}{2}}} = \sqrt{\frac{1 + 0.2 \times 1.8^2}{1.4 \times 1.8^2 - 0.2}}$

$= 0.6165$

$\tan\theta = 2\cot\beta \cdot \frac{M_1^2 \sin^2\beta - 1}{M_1^2(r + \cos 2\beta) + 2}$

$= 2\cot 36.87^\circ \cdot \frac{9\sin^2 36.87^\circ - 1}{9 \times [1.4 + \cos(36.87^\circ \times 2)] + 2}$

$= 0.3489$

$\Rightarrow \theta = 19.23^\circ$

$\sin(\beta - \theta) = \frac{M_{n.2}}{M_2} \Rightarrow M_2 = \frac{M_{n.2}}{\sin(\beta - \theta)} = 1.848$

$\frac{P_{0.1}}{P_1} = \left(1 + \frac{r-1}{2}M_1^2\right)^{\frac{r}{r-1}}$

$\Rightarrow P_{0.1} = (1 + 0.2 \times 9)^{\frac{1.4}{0.4}} \times 1 = 36.73$ atm

a) From Appendix B, $M_{n.2} = 0.6165$

$\frac{P_{0.2}}{P_{0.1}} = 0.8127$

$P_{0.2} = 0.8127 \times 36.73 = 29.85$ atm $\boxed{\text{ANS}}$

b) From Appendix B, $M_{n.2} = 0.6165$

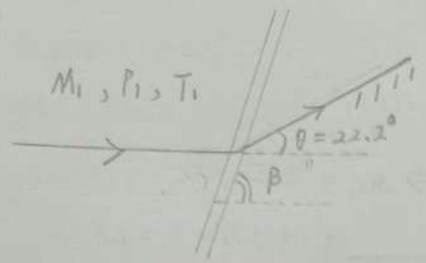
$\frac{P_{0.2}}{P_1} = 4.670$

$P_{0.2} = 4.670 \times 1 = 4.670$ atm $\boxed{\text{ANS}}$

Compare: error $= \frac{29.85 - 4.670}{29.85} \times 100\%$

$= 84.36\%$

9.5
解:



$$\tan\theta = 2\cot\beta \cdot \frac{M_1^2 \sin^2\beta - 1}{M_1^2 \cdot (\gamma + \cos 2\beta) + 2}$$

$$\tan 22.2^\circ = 2\cot\beta \cdot \frac{2.5^2 \sin^2\beta - 1}{2.5^2 (1.4 + \cos 2\beta) + 2}$$

$\Rightarrow$ wave angle $\beta = 45.89^\circ$ $\boxed{\text{ANS}}$ ✓

$$M_{n1} = \sin\beta \cdot M_1 = \sin 45.89^\circ \times 2.5 = 1.795$$

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n.1}^2 - 1)$$

$\Rightarrow P_2 = 1 \times \left[1 + \frac{2.8}{2.4} \times (1.795^2 - 1)\right] = 3.59$ $\boxed{\text{ANS}}$ ✓

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} \cdot \frac{\rho_1}{\rho_2} = \left[1 + \frac{2\gamma}{\gamma+1}(M_{n.1}^2 - 1)\right] \cdot \frac{2 + (\gamma-1) M_{n.1}^2}{(\gamma+1) M_{n.1}^2}$$

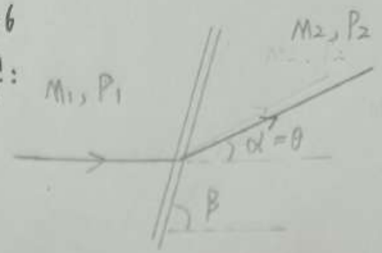
$\Rightarrow T_2 = 300 \times \left[1 + \frac{2.8}{2.4} \times (1.795^2 - 1)\right] \times \frac{2 + 0.4 \times 1.795^2}{2.4 \times 1.795^2} = 458.4\,k$ ✓ $\boxed{\text{ANS}}$

$$M_{n.2} = \sqrt{\frac{1 + \frac{\gamma-1}{2} M_{n.1}^2}{\gamma M_{n.1}^2 - \frac{\gamma-1}{2}}} = 0.6176$$

$M_2 = \frac{M_{n.2}}{\sin(\beta - \theta)} = \frac{0.6176}{\sin(45.89^\circ - 22.2^\circ)} = 1.537$ $\boxed{\text{ANS}}$ ✓

9.6
解:

$\theta_{max} = 28.5^\circ$
$\beta = 64.5^\circ$.
from figure 9.9.



$$\tan\theta = 2\cot\beta \frac{M_1^2 \sin^2\beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2}$$

$$= 2\cot\beta \cdot \frac{2.4^2 \sin^2\beta - 1}{2.4^2 (1.4 + \cos 2\beta) + 2}$$

From figure 9.9, (Text book Page 613)
At $M_1 = 2.4$, $\theta_{max}\big|_{\beta = 64.5^\circ} = 28.5^\circ$ $\boxed{\text{ANS}}$

$$\sin\beta = \frac{M_{n.1}}{M_1} \Rightarrow M_{n.1} = 2.4 \sin 64.5^\circ = 2.166$$

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n.1}^2 - 1)$$

$$\Rightarrow P_2 = P_1 \cdot \left[1 + \frac{2\gamma}{\gamma+1}(M_{n.1}^2 - 1)\right]$$

$$= 1 \times \left[1 + \frac{2.8}{2.4} \times (2.166^2 - 1)\right]$$

$= 5.307$ atm $\boxed{\text{ANS}}$

9.11

解: expansion wave.

$M_1 = 1.58$, $P_1 = 1$ atm

$P_2 = 0.1306$ atm

$$\frac{P_{0.1}}{P_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{0.2}}{P_2} = \left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}}$$

isentropic expansion : $P_{0.1} = P_{0.2}$

$$\therefore \frac{P_2}{P_1} = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow M_2 = 2.9$$

$\theta = \nu(M_2) - \nu(M_1)$

$$= \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_2^2-1)} - \tan^{-1}\sqrt{M_2^2-1} -$$

$$\left[\sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_1^2-1)} - \tan\sqrt{M_1^2-1}\right]$$

$= 0.5851$ rad

$= 33.52^\circ$