**Homework problems 20-25**
**Due in class, Wednesday, 28 October 2020**

20. Determine the shear stress at point $A$ on the surface of the shaft. Represent the state of stress on a volume element at this point. The shaft has a radius of 50 mm.

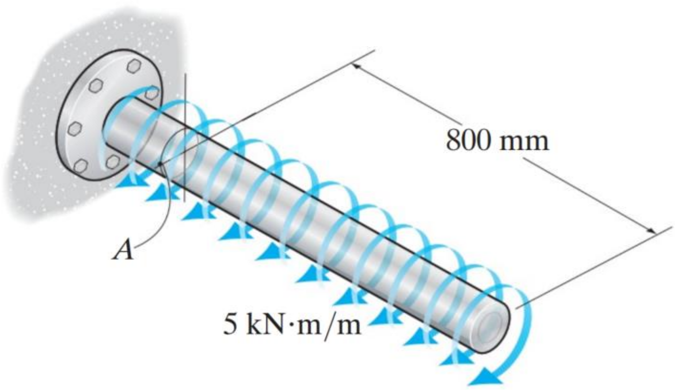



Figure 20

21. The shaft is subjected to a distributed torque along its length of $t = (10x^2)$ N·m/m, where $x$ is in meters. If the maximum stress in the shaft is to remain constant at 60 MPa, determine the variation of the radius $c$ of the shaft along the length.

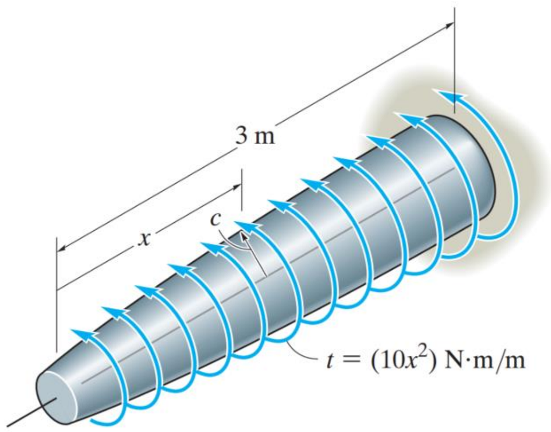
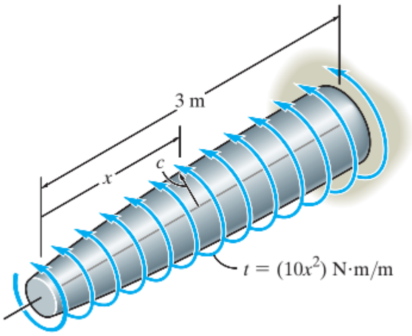



Figure 21

**5–30.** The shaft is subjected to a distributed torque along its length of $t = (10x^2)$ N·m/m, where $x$ is in meters. If the maximum stress in the shaft is to remain constant at 80 MPa, determine the required variation of the radius $c$ of the shaft for $0 \le x \le 3$ m.

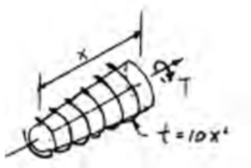



$$T = \int t\,dx = \int_0^x 10\,x^2 dx = \frac{10}{3}x^3$$

$$\tau = \frac{Tc}{J}; \qquad 80(10^6) = \frac{(\frac{10}{3})x^3\,c}{\frac{\pi}{2}c^4}$$

$$c^3 = 26.526(10^{-9})\,x^3$$

$$c = (2.98\,x)\text{ mm}$$ **Ans.**




22. The solid steel shaft $AC$ has a diameter of 25 mm and is supported by smooth bearings at $D$ and $E$. It is coupled to a motor at $C$, which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears $A$ and $B$ remove 1 kW and 2 kW, respectively, determine the maximum shear stress in the shaft within regions $AB$ and $BC$. The shaft is free to turn in its support bearings $D$ and $E$.

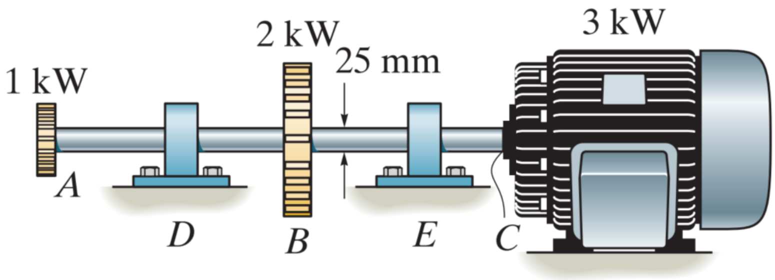



Figure 22

$$T_C = \frac{P}{\omega} = \frac{3(10^3)}{50(2\pi)} = 9.549\text{ N}\cdot\text{m}$$

$$T_A = \frac{1}{3}T_C = 3.183\text{ N}\cdot\text{m}$$

$$(\tau_{AB})_{\max} = \frac{T_C}{J} = \frac{3.183\,(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 1.04\text{ MPa}$$ **Ans.**

$$(\tau_{BC})_{\max} = \frac{T_C}{J} = \frac{9.549\,(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 3.11\text{ MPa}$$ **Ans.**

23. The 60-mm-diameter shaft is made of 6061-T6 aluminum. If the allowable shear stress is tallow = 80 MPa, and the angle of twist of disk $A$ relative to disk $C$ is limited so that it does not exceed 0.06 rad, determine the maximum allowable torque **T**. Shear modulus $G$ = 26 GPa.

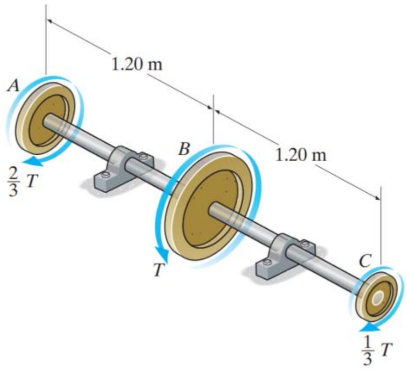
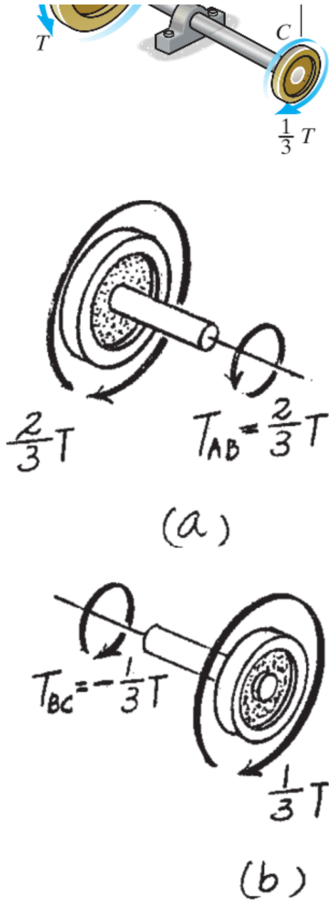



Figure 23

## SOLUTION




**Internal Loading:** The internal torques developed in segments $AB$ and $BC$ of the shaft are shown in Figs. $a$ and $b$, respectively.

**Allowable Shear Stress:** Segment $AB$ is critical since it is subjected to a greater internal torque. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi$ m$^4$. We have

$$\tau_{allow} = \frac{T_{AB}\,c}{J}; \qquad 80(10^6) = \frac{(\frac{2}{3}T)(0.03)}{0.405(10^{-6})\pi}$$

$$T = 5089.38\ \text{N}\cdot\text{m} = 5.09\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

**Angle of Twist:** The internal torques developed in segments $AB$ and $BC$ of the shaft are $T_{AB} = \frac{2}{3}(5089.38) = 3392.92$ N·m and $T_{BC} = -\frac{1}{3}(5089.38) = -1696.46$ N·m. We have

$$\phi_{A/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG_{al}} + \frac{T_{BC}L_{BC}}{JG_{al}}$$

$$\phi_{A/C} = \frac{3392.92(1.20)}{0.405(10^{-6})\pi(26)(10^9)} + \frac{-1696.46(1.20)}{0.405(10^{-6})\pi(26)(10^9)}$$

$$= 0.06154\ \text{rad} = 3.53° \qquad \textbf{Ans.}$$


## SOLUTION

**Internal Loading:** The internal torques developed in segments $AB$ and $BC$ of the shaft are shown in Figs. $a$ and $b$, respectively.

**Allowable Shear Stress:** Segment $AB$ is critical since it is subjected to a greater internal torque. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi \text{ m}^4$. We have

$$\tau_{\text{allow}} = \frac{T_{AB}\, c}{J}; \qquad 80(10^3) = \frac{(\frac{2}{3}T)(0.03)}{0.405(10^{-6})\pi}$$

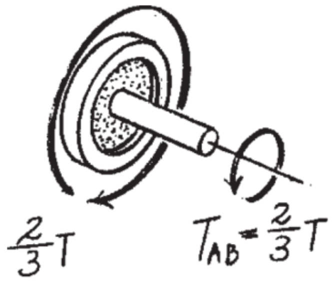
$$T = 5089.38 \text{ N}\cdot\text{m} = 5.089 \text{ kN}\cdot\text{m}$$

**Angle of Twist:** It is required that $\phi_{A/C} = 0.06$ rad. We have

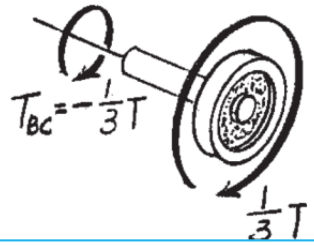
$$\phi_{A/C} = \Sigma\frac{T_i L_i}{J_i G_i} = \frac{T_{AB}L_{AB}}{JG_{al}} + \frac{T_{BC}L_{BC}}{JG_{al}}$$

$$0.06 = \frac{(\frac{2}{3}T)(1.2)}{0.405(10^{-6})\pi(26)(10^9)} + \frac{(-\frac{1}{3}T)(1.2)}{0.405(10^{-6})\pi(26)(10^9)}$$

$$T = 4962.14 \text{ N}\cdot\text{m} = 4.96 \text{ kN}\cdot\text{m (controls)} \qquad \textbf{Ans.}$$




(a)




24. A rod is made from two segments: $AB$ is steel and $BC$ is brass. It is fixed at its ends and subjected to a torque of T = 680 N·m. If the steel portion has a diameter of 30 mm, determine (a) the required diameter of the brass portion so the reactions at the walls will be the same, and (b) the absolute maximum shear stress in the shaft. $G_{st}$ = 75 GPa, $G_{st}$ = 39 GPa.

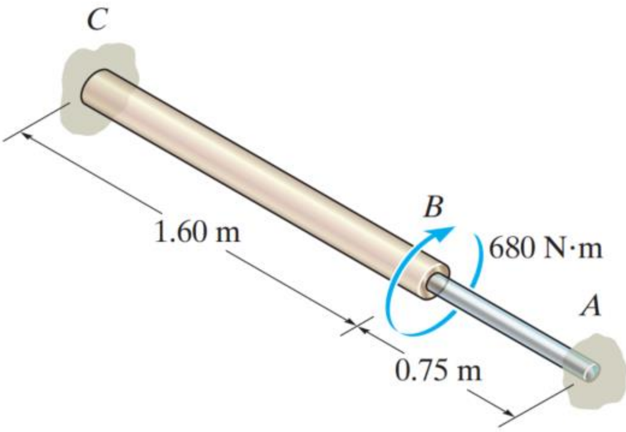



Figure 24

## SOLUTION

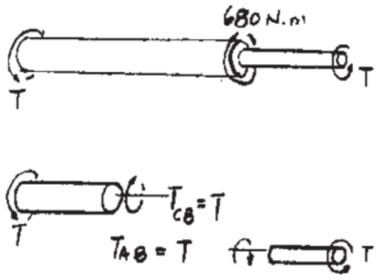
**Compatibility Condition:**

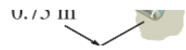
$$\phi_{B/C} = \phi_{B/A}$$

$$\frac{T(1.60)}{\frac{\pi}{2}(c^4)(39)(10^9)} = \frac{T(0.75)}{\frac{\pi}{2}(0.015^4)(75)(10^9)}$$

$c = 0.02134$ m

$d = 2c = 0.04269 \text{ m} = 42.7 \text{ mm}$ **Ans.**



## SOLUTION

**Equilibrium,**

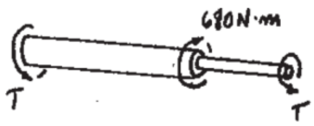
$$2T = 680$$

$$T = 340 \text{ N} \cdot \text{m}$$

$\tau_{\substack{abs\\max}}$ occurs in the steel. See solution to Prob. 5–88.

$$\tau_{\substack{abs\\max}} = \frac{Tc}{J} = \frac{340(0.015)}{\frac{\pi}{2}(0.015)^4}$$

$$= 64.1 \text{ MPa}$$ **Ans.**




25. The plastic hexagonal tube is subjected to a torque of 150 N·m. Determine the minimum mean dimension $a$ of its sides if the allowable shear stress is $\tau_{allow}$ = 60 MPa. Each side has a thickness of $t$ = 3 mm.

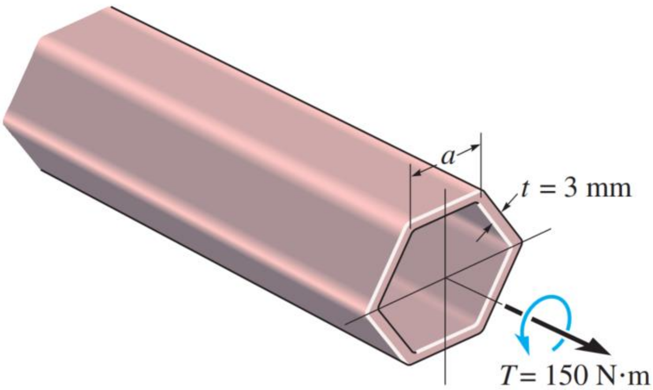
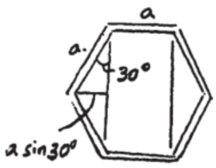



Figure 25

## SOLUTION




$$A_m = 4\left[\frac{1}{2}(a\cos 30^\circ)(a\sin 30^\circ)\right] + (a)(2a)\cos 30^\circ = 2.5981\,a^2$$

$$\tau_{avg} = \tau_{allow} = \frac{T}{2\,t\,A_m}$$

$$60\left(10^6\right) = \frac{150}{(2)(0.003)\left(2.5981\,a^2\right)}$$

$$a = 0.01266\text{ m} = 12.7\text{ mm}$$ **Ans.**