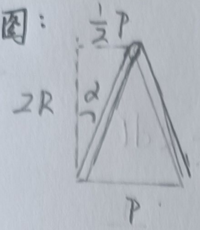
3.20

解：

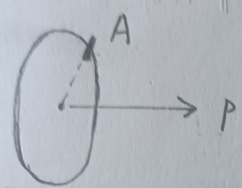
由 pitch 产生的夹角

$$\tan\alpha = \frac{\frac{1}{2}P}{2R} = \frac{\pi r}{2R}$$

如下图：



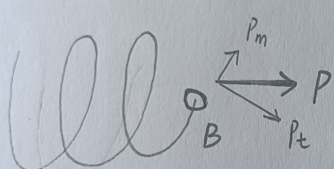
对于一个圆环而言



截面A处 仅有 $T = PR$

Since 截面A 平行于力P作用线.

对于螺旋线



截面B垂直于螺旋线

∴B与P间存在夹角α

$$\begin{aligned} P_m &= P\sin\alpha \\ P_t &= P\cos\alpha \end{aligned} \Rightarrow \begin{aligned} M &= P\sin\alpha \cdot R \\ T &= P\cos\alpha \cdot R \end{aligned}$$

$$U^* = \int_0^{2\pi n} \frac{T^2}{2GJ} R\,d\theta + \int_0^{2\pi n} \frac{M^2}{2EI} R\,d\theta$$

$$\frac{\partial U^*}{\partial P} = \int_0^{2\pi n} \frac{T}{GJ}\frac{\partial T}{\partial P} R\,d\theta + \int_0^{2\pi n} \frac{M}{EI}\frac{\partial M}{\partial P} R\,d\theta$$

$$= \frac{P\cos^2\alpha R^3}{GJ}\cdot 2\pi n + \frac{P\sin^2\alpha R^3}{EI}\cdot 2\pi n$$

$$= 2\pi n P R^3 \left(\frac{\cos^2\alpha}{GJ} + \frac{\sin^2\alpha}{EI}\right)$$

$$k = \frac{1}{2\pi n R^3\left(\frac{\cos^2\alpha}{GJ} + \frac{\sin^2\alpha}{EI}\right)}$$

$\cancel{\alpha \to 0}$, $k = \dfrac{GJ}{2\pi n R^3} = \dfrac{G\cdot\frac{\pi}{2}r^4}{2\pi n R^3} = \dfrac{Gr^4}{4nR^3}$

$$\tan\alpha = \frac{\pi r}{2R}$$

$$\sin\alpha = \frac{\pi r}{\sqrt{(2R)^2 + (\pi r)^2}}$$

$$\cos\alpha = \frac{2R}{\sqrt{(2R)^2 + (\pi r)^2}}$$