装订线内不答题！

1.

解:

1) $h_0 = h + \frac{1}{2}V^2 = C_p T + \frac{1}{2}V^2$

$= \frac{\gamma R}{\gamma - 1} T + \frac{1}{2}V^2$

$= \frac{1.4 \times 287}{1.4-1} \times 300 + \frac{1}{2} \times 180^2$

$= 317550 \ (J/kg)$

2) $h_0 = C_p \cdot T_0 = \frac{\gamma R}{\gamma - 1} T_0$

$\Rightarrow T_0 = \frac{\gamma - 1}{\gamma R} h_0 = \frac{1.4-1}{1.4 \times 287} \times 317550 = 316.13 K$

$\frac{P_0}{P} = (\frac{T_0}{T})^{\frac{\gamma}{\gamma-1}}$

$\therefore P_0 = P \cdot (\frac{T_0}{T})^{\frac{\gamma}{\gamma-1}} = 1.5 \ atm \times (\frac{316.13}{300})^{\frac{1.4}{1.4-1}} = 1.802 \ atm$

$= 1.802 \times 1.01 \times 10^5 \ Pa$

2.

解:

(1) thin airfoil

$\frac{1}{2\pi} \int_0^c \frac{\gamma(\xi) d\xi}{x - \xi} = V_\infty (\alpha - \frac{dz}{dx})$

Since thin flat plate: $\frac{dz}{dx} = 0$ symmetric. $\alpha_{L=0} = 0$

$\therefore C_L = 2\pi \alpha = 2\pi \times \frac{3}{180} \times \pi = 0.3290$

(2) $C_{m.LE} = -\frac{\pi}{2} \alpha = -\frac{\pi}{2} \times \frac{3}{180} \times \pi = -0.08225$

$C_{m.x} = C_{m.LE} + \frac{x}{c} C_L$

$\therefore C_{m.TE} = C_{m.LE} + \frac{c}{c} C_L = C_{m.LE} + C_L = 0.2468$

3.

解:

(1) $C_{D,i} = \dfrac{C_L^2}{\pi AR e}$, $e = \dfrac{1}{1+\delta}$

$C_L = \sqrt{C_{D,i}\ \pi\ AR\ e} = \sqrt{0.02 \times \pi \times 10 \times \dfrac{1}{1+0.06}} = 0.7699$ At AoA $= 2^\circ$

$C_L = a(\alpha - \alpha_{L=0})$

$a = \dfrac{a_0}{1+\dfrac{a_0(1+\tau)}{\pi AR}} = \dfrac{C_L}{\alpha - \alpha_{L=0}} = \dfrac{0.7699}{2-(-2)} = 0.1925\ /^\circ$

$= 11.03\ \text{rad}^{-1}$

At $\alpha = 1^\circ$,

$C_L = a(\alpha - \alpha_{L=0}) = 0.1925 \times [1-(-2)] = 0.5774$

(2) Similar Wing.

Same a_0

$a = \dfrac{a_0}{1+\dfrac{a_0(1+\tau)}{\pi AR}} \Rightarrow a_0 = \dfrac{\pi AR a}{\pi AR - a(1+\tau)} = \dfrac{\pi \times 10 \times 11.03}{\pi \times 10 - 11.03 \times (1+0.06)} = 17.57\ \text{rad}^{-1}$

For new Wing.

$a_{new} = \dfrac{a_0}{1+\dfrac{a_0(1+\tau_{new})}{\pi\ AR_{new}}} = \dfrac{17.57}{1+\dfrac{17.57 \times 1.12}{\pi \times 15}} = 12.39\ \text{rad}^{-1}$

$C_{L\,new} = a_{new}(\alpha - \alpha_{L=0}) = 12.39 \times [1-(-2)] \times \dfrac{\pi}{180} = 0.6489$

$C_{D,i} = \dfrac{C_{L\,new}^2}{\pi AR \cdot e_{new}} = \dfrac{0.6489^2}{\pi \times 15 \times \dfrac{1}{1+0.12}} = 0.01$

4. 解:

(1) $C_p = \dfrac{p - p_\infty}{q_\infty} = \dfrac{\frac{1}{2}\rho_\infty V_\infty^2 - \frac{1}{2}\rho_\infty V^2}{\frac{1}{2}\rho_\infty V_\infty^2} = 1 - \left(\dfrac{V}{V_\infty}\right)^2$

$V^2 = V_\theta^2 + V_r^2$

on the surface: $V_\theta = -2V_\infty \sin\theta - \dfrac{2\pi R V_\infty}{2\pi R} = -2V_\infty \sin\theta - V_\infty$

$r = R$

$V_r = 0$

$\dfrac{V}{V_\infty} = -2\sin\theta - 1$

$\therefore C_p = 1 - (2\sin\theta + 1)^2$

$= 1 - (4\sin^2\theta + 1 + 4\sin\theta) = -4\sin^2\theta - 4\sin\theta$

(2) $P = P_\infty$, $C_p = 0$. on the surface.

namely, $-4\sin^2\theta - 4\sin\theta = 0$

$\sin^2\theta + \sin\theta = 0$

$\sin\theta(1 + \sin\theta) = 0$

① $\sin\theta = 0$, $\theta = 0, \pi$ points $(R, 0)$, (R, π)

② $1 + \sin\theta = 0$

$\sin\theta = -1$, $\theta = \frac{3}{2}\pi$ point $(R, \frac{3}{2}\pi)$

3.

解:

1) $x = \rho\cos\theta$

$y = \rho\sin\theta$

$x^2+y^2 = \rho^2$

$V_r = u\cos\theta + v\sin\theta$

$V_\theta = -u\sin\theta + v\cos\theta$

$$u = \frac{\rho^2(\cos^2\theta - \sin^2\theta) - 2\rho^2\cos\theta\sin\theta}{\rho^4} = \frac{\cos2\theta - \sin2\theta}{\rho^2}$$

$$v = \frac{\rho^2(\cos^2\theta - \sin^2\theta) + 2\rho^2\cos\theta\sin\theta}{\rho^4} = \frac{\cos2\theta + \sin2\theta}{\rho^2}$$

$$\therefore V_r = \frac{\cos\theta\cos2\theta - \cos\theta\sin2\theta + \sin\theta\cos2\theta + \sin\theta\sin2\theta}{\rho^2} = \frac{\partial\phi}{\partial r}$$

$$V_\theta = \frac{\sin\theta\sin2\theta - \sin\theta\cos2\theta + \cos\theta\cos2\theta + \cos\theta\sin2\theta}{\rho^2} = \frac{1}{r}\frac{\partial\phi}{\partial\theta}$$

Simplify:

$$\left.\begin{aligned} V_r &= \frac{\cos\theta - \sin\theta}{r^2} = \frac{\partial\phi}{\partial r} \\ V_\theta &= \frac{\cos\theta + \sin\theta}{r^2} = \frac{1}{r}\frac{\partial\phi}{\partial\theta} \end{aligned}\right\} \Rightarrow \phi = \frac{\sin\theta - \cos\theta}{r} = \frac{r\sin\theta - r\cos\theta}{r^2} = \frac{y-x}{x^2+y^2}$$

2)

$$\left.\begin{aligned} V_r &= \frac{\cos\theta - \sin\theta}{r^2} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \\ V_\theta &= \frac{\cos\theta + \sin\theta}{r^2} = -\frac{\partial\psi}{\partial r} \end{aligned}\right\} \Rightarrow \psi = \frac{\cos\theta + \sin\theta}{r} = \frac{x+y}{x^2+y^2}$$

6.

解:

1) $z = \beta x\ (1-\frac{x}{c})^2$

$$\frac{dz}{dx} = \beta\left[(1-\frac{x}{c})^2 + x\cdot 2(1-\frac{x}{c})\cdot(-\frac{1}{c})\right]$$

$$= \beta\left[(1-\frac{x}{c})^2 + 2\frac{x}{c}(\frac{x}{c}-1)\right]$$

令 $x = \frac{c}{2}(1-\cos\theta)$ $\quad (1-\frac{x}{c})^2 = \left[1-(\frac{1}{2}-\frac{1}{2}\cos\theta)\right]^2 = (\frac{1}{2}+\frac{1}{2}\cos\theta)^2$

$$\frac{x}{c} = \frac{1}{2} - \frac{1}{2}\cos\theta$$

$$\therefore \frac{dz}{dx} = \beta\left[(\frac{1}{2}\cos\theta - \frac{1}{2})^2 + (1-\cos\theta)(-\frac{1}{2}\cos\theta - \frac{1}{2})\right]$$

+

$$= \beta\cdot(\frac{3}{4}\cos^2\theta - \frac{1}{2}\cos\theta - \frac{1}{4})$$

+

$C_L = a_0(\alpha - \alpha_{L=0})$, thin airfoil : $a_0 = 2\pi$.

$$\alpha_{L=0} = \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}(1-\cos\theta)d\theta$$

$$= \frac{\beta}{\pi}\int_0^\pi (\frac{3}{4}\cos^2\theta - \frac{1}{2}\cos\theta - \frac{1}{4})(1-\cos\theta)\,d\theta = \frac{\beta}{\pi}\cdot\frac{3}{8}\pi = \frac{3}{8}\beta$$

$\therefore C_l = 2\pi(\alpha - \frac{3}{8}\beta)$ $\quad$ -1

2) $C_{m.le} = -\frac{\pi}{2}(A_0 + A_1 - \frac{1}{2}A_2)$

$$\alpha - A_0 = \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}d\theta = \frac{\beta}{\pi}\int_0^\pi(\frac{3}{4}\cos^2\theta - \frac{1}{2}\cos\theta - \frac{1}{4})d\theta = \frac{1}{8}\pi\cdot\frac{\beta}{\pi} = \frac{\beta}{8},\ A_0 = \alpha - \frac{\beta}{8}$$

+

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos n\theta\, d\theta$$

+

$$\therefore A_1 = \frac{2\beta}{\pi}\int_0^\pi(\frac{3}{4}\cos^3\theta - \frac{1}{2}\cos^2\theta - \frac{1}{4}\cos\theta)d\theta = \frac{2\beta}{\pi}\cdot(-\frac{\pi}{4}) = -\frac{\beta}{2}$$ -1

+

$$A_2 = \frac{2\beta}{\pi}\int_0^\pi(\frac{3}{4}\cos^2\theta - \frac{1}{2}\cos\theta - \frac{1}{4})\cos 2\theta\, d\theta = \frac{2\beta}{\pi}\cdot\frac{3}{16}\pi = \frac{3}{8}\beta$$

$$\therefore C_{m.le} = -\frac{\pi}{2}(\alpha - \frac{\beta}{8} - \frac{\beta}{2} - \frac{3}{16}\beta) = -\frac{\pi}{2}(\alpha - \frac{13}{16}\beta) = -\frac{\pi}{2}\alpha + \frac{13}{32}\pi\beta$$ -1

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