Reference Answers for Homework 7

1. Consider two ideal-gas systems that are at the same pressure as the environment. The first system is at the same temperature as the environment, whereas the second system is at a lower temperature than the environment. Which system do you think has larger internal energy? How about exergy?

ANS: Because the internal energy of ideal gas depends on the temperature only, so the system at the temperature of the surroundings has larger internal energy than that at a lower temperature than the surroundings. However, the former system has zero exergy; but the latter has some exergy, since we can run a heat engine between the latter system and the environment to do some work.

2. What is the difference between the reversible work and the exergy? How about the reversible work and the exergy change? Please use a designed example to explain your answer.

ANS: Both reversible work and exergy are related to the maximum useful work of a system. However, for reversible work, it considers a real process that had occurred; while for exergy, it considers an imaginary process that the system change from a given state to a specific environment. In a given environment, the exergy change equals to the reversible work. For the example, just make sure that your example is physically sound.

3. A piston–cylinder device initially contains 2 L of air at 100 kPa and 25°C. Air is now compressed to a final state of 600 kPa and 150°C. The useful work input is 1.2 kJ. Assuming the surroundings are at 100 kPa and 25°C, determine

(a) the exergy of the air at the initial and the final states;

(b) the minimum work that must be supplied to accomplish this compression process;

(c) the second-law efficiency of this process.

ANS **(a)** Note that $X_1 = \Phi_1 = 0$ since air initially is at the dead state. The mass of air is

$$m = \frac{P_1 V_1}{RT_1} = \frac{(100\text{ kPa})(0.002\text{ m}^3)}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\text{ K})} = 0.00234\text{ kg}$$

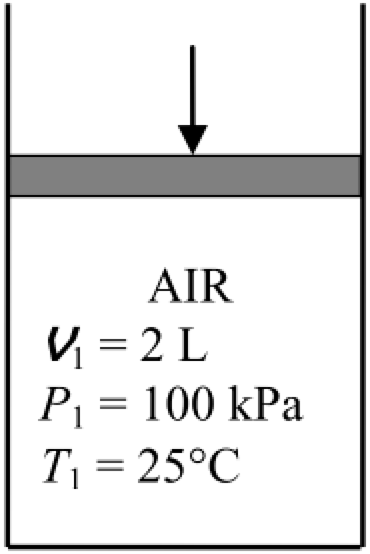
Also, $\dfrac{P_2 V_2}{T_2} = \dfrac{P_1 V_1}{T_1} \longrightarrow V_2 = \dfrac{P_1 T_2}{P_2 T_1} V_1 = \dfrac{(100\text{ kPa})(423\text{ K})}{(600\text{ kPa})(298\text{ K})}(2\text{ L}) = 0.473\text{ L}$

and

$$s_2 - s_0 = c_{p,\text{avg}} \ln\frac{T_2}{T_0} - R\ln\frac{P_2}{P_0}$$

$$= (1.009\text{ kJ/kg}\cdot\text{K})\ln\frac{423\text{ K}}{298\text{ K}} - (0.287\text{ kJ/kg}\cdot\text{K})\ln\frac{600\text{ kPa}}{100\text{ kPa}}$$

$$= -0.1608\text{ kJ/kg}\cdot\text{K}$$



Thus, the exergy of air at the final state is

$$X_2 = \Phi_2 = m\left[c_{v,\mathrm{avg}}(T_2 - T_0) - T_0(s_2 - s_0)\right] + P_0(V_2 - V_0)$$
$$= (0.00234\ \mathrm{kg})\left[(0.722\ \mathrm{kJ/kg \cdot K})(423 - 298)\mathrm{K} - (298\ \mathrm{K})(-0.1608\ \mathrm{kJ/kg \cdot K})\right]$$
$$+ (100\ \mathrm{kPa})(0.000473 - 0.002)\mathrm{m}^3[\mathrm{kJ/m}^3 \cdot \mathrm{kPa}]$$
$$= \mathbf{0.171\ kJ}$$

(b) The minimum work input is the reversible work, which can be determined from the exergy change

$$W_{\mathrm{rev,in}} = X_2 - X_1 = 0.171 - 0 = \mathbf{0.171\ kJ}$$

(c) The second-law efficiency of this process is $\eta_{\mathrm{II}} = \dfrac{W_{\mathrm{rev,in}}}{W_{\mathrm{u,in}}} = \dfrac{0.171\ \mathrm{kJ}}{1.2\ \mathrm{kJ}} = \mathbf{14.3\%}$

4. Stainless steel ball bearings (ρ = 8085 kg/m^3 and c_p = 0.480 kJ/kg·°C) having a diameter of 1.2 cm are to be quenched in water at a rate of 1400 per minute. The balls leave the oven at a uniform temperature of 900°C and are exposed to air at 30°C for a while before they are dropped into the water. If the temperature of the balls drops to 850°C prior to quenching, determine

(a) the rate of heat transfer from the balls to the air, and

(b) the rate of exergy destruction due to heat loss from the balls to the air.

ANS

(a) We take a single bearing ball as the system. The energy balance for this closed system can be expressed as

$$-Q_{out} = \Delta U_{ball} = m(u_2 - u_1)$$

$$Q_{out} = mc(T_1 - T_2)$$

The total amount of heat transfer from a ball is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (8085\ \text{kg/m}^3)\frac{\pi(0.012\ \text{m})^3}{6} = 0.007315\ \text{kg}$$

$$Q_{out} = mc(T_1 - T_2) = (0.007315\ \text{kg})(0.480\ \text{kJ/kg.°C})(900 - 850)\text{°C} = 0.1756\ \text{kJ/ball}$$

Then the rate of heat transfer from the balls to the air becomes

$$\dot{Q}_{total} = \dot{n}_{ball} Q_{out\,(per\,ball)} = (1400\ \text{balls/min}) \times (0.1756\ \text{kJ/ball}) = 245.8\ \text{kJ/min} = \mathbf{4.10\ kW}$$

Therefore, heat is lost to the air at a rate of 4.10 kW.

(b) The exergy destruction can be determined from its definition $X_{destroyed} = T_0 S_{gen}$. The entropy generated during this process can be determined by applying an entropy balance on an extended system that includes the ball and its immediate surroundings so that the boundary temperature of the extended system is at 30°C at all times:

$$-\frac{Q_{\text{out}}}{T_b} + S_{\text{gen}} = \Delta S_{\text{system}} \rightarrow S_{\text{gen}} = \frac{Q_{\text{out}}}{T_b} + \Delta S_{\text{system}}$$

where

$$\Delta S_{\text{system}} = m(s_2 - s_1) = mc_{\text{avg}} \ln\frac{T_2}{T_1} = (0.007315\,\text{kg})(0.480\,\text{kJ/kg.K}) \ln\frac{850+273}{900+273} = -0.0001530\,\text{kJ/K}$$

Then the rate of entropy generation becomes

$$S_{\text{gen}} = \frac{Q_{\text{out}}}{T_b} + \Delta S_{\text{system}} = \frac{0.1756\,\text{kJ}}{303\,\text{K}} - 0.0001530\,\text{kJ/K} = 0.0004265\,\text{kJ/K} \quad \text{(per ball)}$$

For all the balls,

$$\dot{S}_{\text{gen}} = S_{\text{gen}}\dot{n}_{\text{ball}} = (0.0004265\,\text{kJ/K}\cdot\text{ball})(1400\,\text{balls/min}) = 0.597\,\text{kJ/min.K} = \mathbf{0.00995\,kW/K}$$

Finally,

$$\dot{X}_{\text{destroyed}} = T_0\dot{S}_{\text{gen}} = (303\,\text{K})(0.00995\,\text{kW/K}) = \mathbf{3.01\,kW/K}$$

5. Combustion gases enter a gas turbine at 900°C, 800 kPa, and 100 m/s and leave at 650°C, 400 kPa, and 220 m/s. Taking c_p = 1.15 kJ/kg·°C and k = 1.3 for the combustion gases, determine

(a) the exergy of the combustion gases at the turbine inlet;

(b) the work output of the turbine under reversible conditions;

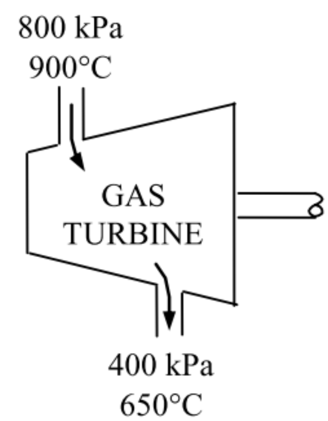
(c) Can this turbine be adiabatic?

(Assume the surroundings to be at 25°C and 100 kPa.)

ANS

(a) The exergy of the gases at the turbine inlet is simply the flow exergy,

$$\psi_1 = h_1 - h_0 - T_0(s_1 - s_0) + \frac{V_1^2}{2} + gz_1^{\nearrow 0}$$



where

$$s_1 - s_0 = c_p \ln\frac{T_1}{T_0} - R\ln\frac{P_1}{P_0}$$
$$= (1.15\text{ kJ/kg}\cdot\text{K})\ln\frac{1173\text{ K}}{298\text{ K}} - (0.265\text{ kJ/kg}\cdot\text{K})\ln\frac{800\text{ kPa}}{100\text{ kPa}}$$
$$= 1.025\text{ kJ/kg}\cdot\text{K}$$

Thus,

$$\psi_1 = (1.15\text{ kJ/kg.K})(900-25)°\text{C} - (298\text{ K})(1.025\text{ kJ/kg}\cdot\text{K}) + \frac{(100\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{705.8\text{ kJ/kg}}$$

(b) The reversible work is determined by the exergy change at the inlet and outlet,

$$w_{rev,out} = h_1 - h_2 + T_0(s_2 - s_1) - \Delta ke = c_p(T_1 - T_2) + T_0(s_2 - s_1) - \Delta ke$$

where $\Delta ke = \dfrac{V_2^2 - V_1^2}{2} = \dfrac{(220\text{ m/s})^2 - (100\text{ m/s})^2}{2}\left(\dfrac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 19.2\text{ kJ/kg}$

and $s_2 - s_1 = c_p \ln\dfrac{T_2}{T_1} - R\ln\dfrac{P_2}{P_1} = (1.15\text{ kJ/kg}\cdot\text{K})\ln\dfrac{923\text{ K}}{1173\text{ K}} - (0.265\text{ kJ/kg}\cdot\text{K})\ln\dfrac{400\text{ kPa}}{800\text{ kPa}} = -0.09196\text{ kJ/kg}\cdot\text{K}$

Therefore,

$$\begin{aligned} w_{rev,out} &= h_1 - h_2 + T_0(s_2 - s_1) - \Delta ke = c_p(T_1 - T_2) + T_0(s_2 - s_1) - \Delta ke \\ &= (1.15\text{ kJ/kg}\cdot\text{K})(900 - 650)^\circ\text{C} + (298\text{ K})(-0.09196\text{ kJ/kg}\cdot\text{K}) - 19.2\text{ kJ/kg} \\ &= \mathbf{240.9\text{ kJ/kg}} \end{aligned}$$

(c) The entropy change of this turbine is negative, but the entropy generation is always larger than zero, so there must involve heat transfer during this process. Therefore, this turbine cannot be adiabatic.

6. A vertical piston–cylinder device initially contains 0.12 m^3 of helium at 20°C. The mass of the piston is such that it maintains a constant pressure of 200 kPa inside. A valve is now opened, and helium is allowed to escape until the volume inside the cylinder is decreased by one-half. Heat transfer takes place between the helium and its surroundings at 20°C and 95 kPa so that the temperature of helium in the cylinder remains constant. Determine

(a) the maximum work potential of the helium at the initial state, and

(b) the exergy destroyed during this process.

ANS

(a) From the ideal gas equation, the initial and the final masses in the cylinder are determined to be

$$m_1 = \frac{P_1 \mathbf{V}}{RT_1} = \frac{(200\text{ kPa})(0.12\text{ m}^3)}{(2.0769\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{ K})} = 0.03944\text{ kg}$$

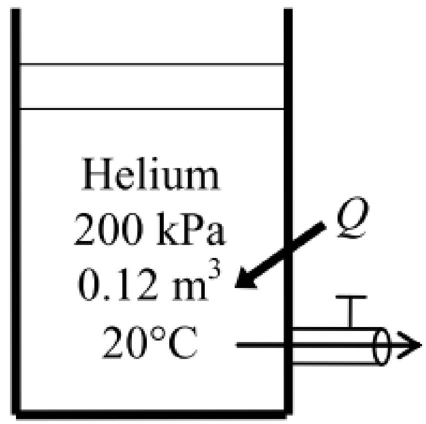
$$m_e = m_2 = m_1/2 = 0.03944/2 = 0.01972\text{ kg}$$

The work potential of helium at the initial state is simply the initial exergy of helium, and is determined from the exergy in a closed-system,

$$\Phi_1 = m_1\phi = m_1\left[(u_1 - u_0) - T_0(s_1 - s_0) + P_0(\boldsymbol{v}_1 - \boldsymbol{v}_0)\right]$$

where $\boldsymbol{v}_1 = \dfrac{RT_1}{P_1} = \dfrac{(2.0769\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\ \text{K})}{200\ \text{kPa}} = 3.043\ \text{m}^3/\text{kg}$

$$\boldsymbol{v}_0 = \frac{RT_0}{P_0} = \frac{(2.0769\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\ \text{K})}{95\ \text{kPa}} = 6.406\ \text{m}^3/\text{kg}$$



and $s_1 - s_0 = c_p \ln\dfrac{T_1}{T_0} - R\ln\dfrac{P_1}{P_0}$

$$= (5.1926\ \text{kJ/kg}\cdot\text{K})\ln\frac{293\ \text{K}}{293\ \text{K}} - (2.0769\ \text{kJ/kg}\cdot\text{K})\ln\frac{200\ \text{kPa}}{95\ \text{kPa}}$$

$$= -1.546\ \text{kJ/kg}\cdot\text{K}$$

Thus, $\Phi_1 = (0.03944\ \text{kg})\{(3.1156\ \text{kJ/kg}\cdot\text{K})(20-20)^\circ\text{C} - (293\ \text{K})(-1.546\ \text{kJ/kg}\cdot\text{K})$

$$+ (95\ \text{kPa})(3.043 - 6.406)\text{m}^3/\text{kg}[\text{kJ/kPa}\cdot\text{m}^3]\}$$

$$= \mathbf{5.27\ kJ}$$

(b) We take the cylinder as the system, which is a control volume. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

$$m_e = m_1 - m_2$$

$$Q_{\text{in}} - m_e h_e + W_{\text{b,in}} = m_2 u_2 - m_1 u_1$$

Combining the two relations gives $Q_{\text{in}} = (m_1 - m_2)h_e + m_2 u_2 - m_1 u_1 - W_{\text{b,in}} = 0$

The exergy destroyed can be determined the definition $X_{destroyed} = T_0 S_{gen}$.
Noting that the pressure and temperature of helium in the cylinder are maintained constant during this process and heat transfer is zero, it gives

$$\begin{aligned}
-m_e s_e + S_{\text{gen}} &= \Delta S_{\text{cylinder}} = (m_2 s_2 - m_1 s_1)_{\text{cylinder}} \\
S_{\text{gen}} &= m_2 s_2 - m_1 s_1 + m_e s_e \\
&= m_2 s_2 - m_1 s_1 + (m_1 - m_2) s_e \\
&= (m_2 - m_1 + m_1 - m_2) s_1 \\
&= 0
\end{aligned}$$

since the initial, final, and the exit states are identical and thus $s_e = s_2 = s_1$. Therefore, this discharge process is reversible, and thus

$$X_{\text{destroyed}} = T_0 S_{\text{gen}} = \mathbf{0}$$

7. Hot exhaust gases leaving an internal combustion engine at 400°C and 150 kPa at a rate of 0.8 kg/s is to be used to produce saturated steam at 200°C in an insulated heat exchanger. Water enters the heat exchanger at the ambient temperature of 20°C, and the exhaust gases leave the heat exchanger at 350°C. Determine

(a) the rate of steam production,

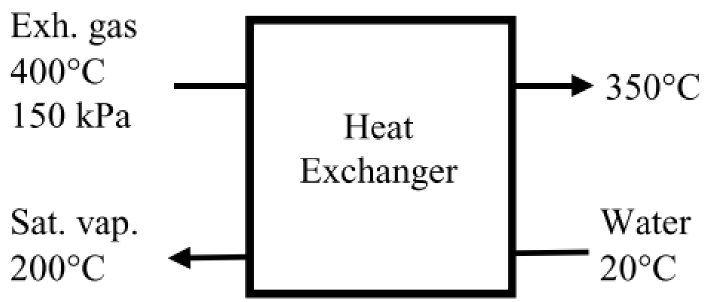
(b) the rate of exergy destruction in the heat exchanger, and

ANS

(a) We denote the inlet and exit states of exhaust gases by (1) and (2) and that of the water by (3) and (4). The properties of water are (Table A-4)

$$\left.\begin{array}{l} T_3 = 20°\text{C} \\ x_3 = 0 \end{array}\right\} \begin{array}{l} h_3 = 83.91 \text{ kJ/kg} \\ s_3 = 0.29649 \text{ kJ/kg.K} \end{array}$$

$$\left.\begin{array}{l} T_4 = 200°\text{C} \\ x_4 = 1 \end{array}\right\} \begin{array}{l} h_4 = 2792.0 \text{ kJ/kg} \\ s_4 = 6.4302 \text{ kJ/kg.K} \end{array}$$



An energy balance on the heat exchanger gives

$$\dot{m}_a h_1 + \dot{m}_w h_3 = \dot{m}_a h_2 + \dot{m}_w h_4$$

$$\dot{m}_a c_p (T_1 - T_2) = \dot{m}_w (h_4 - h_3)$$

$$(0.8 \text{ kg/s})(1.063 \text{ kJ/kg°C})(400 - 350)°\text{C} = \dot{m}_w (2792.0 - 83.91) \text{kJ/kg}$$

$$\dot{m}_w = \mathbf{0.01570 kg/s}$$

(b) The specific exergy changes of each stream as it flows in the heat exchanger is

$$\Delta s_a = c_p \ln\frac{T_2}{T_1} = (0.8\,\text{kg/s})(1.063\,\text{kJ/kg.K})\ln\frac{(350+273)\,\text{K}}{(400+273)\,\text{K}} = -0.08206\,\text{kJ/kg.K}$$

$$\begin{aligned}\Delta\psi_a &= c_p(T_2 - T_1) - T_0\Delta s_a \\ &= (1.063\,\text{kJ/kg.°C})(350 - 400)\text{°C} - (20+273\,\text{K})(-0.08206\,\text{kJ/kg.K}) \\ &= -29.106\,\text{kJ/kg}\end{aligned}$$

$$\begin{aligned}\Delta\psi_w &= h_4 - h_3 - T_0(s_4 - s_3) \\ &= (2792.0 - 83.91)\text{kJ/kg} - (20+273\,\text{K})(6.4302 - 0.29649)\text{kJ/kg.K} \\ &= 910.913\,\text{kJ/kg}\end{aligned}$$

The exergy destruction is determined from an exergy balance on the heat exchanger to be

$$-\dot{X}_{\text{dest}} = \dot{m}_a\Delta\psi_a + \dot{m}_w\Delta\psi_w = (0.8\,\text{kg/s})(-29.106\,\text{kJ/kg}) + (0.01570\,\text{kg/s})(910.913)\,\text{kJ/kg} = -8.98\,\text{kW}$$

$$\dot{X}_{\text{dest}} = \mathbf{8.98\,kW}$$