6.89 A square box beam is made of two 20 × 80-mm planks and two 20 × 120-mm planks nailed together as shown. Knowing that the spacing between the nails is $s = 30$ mm and that the vertical shear in the beam is $V = 1200$ N, determine (*a*) the shearing force in each nail, (*b*) the maximum shearing stress in the beam.

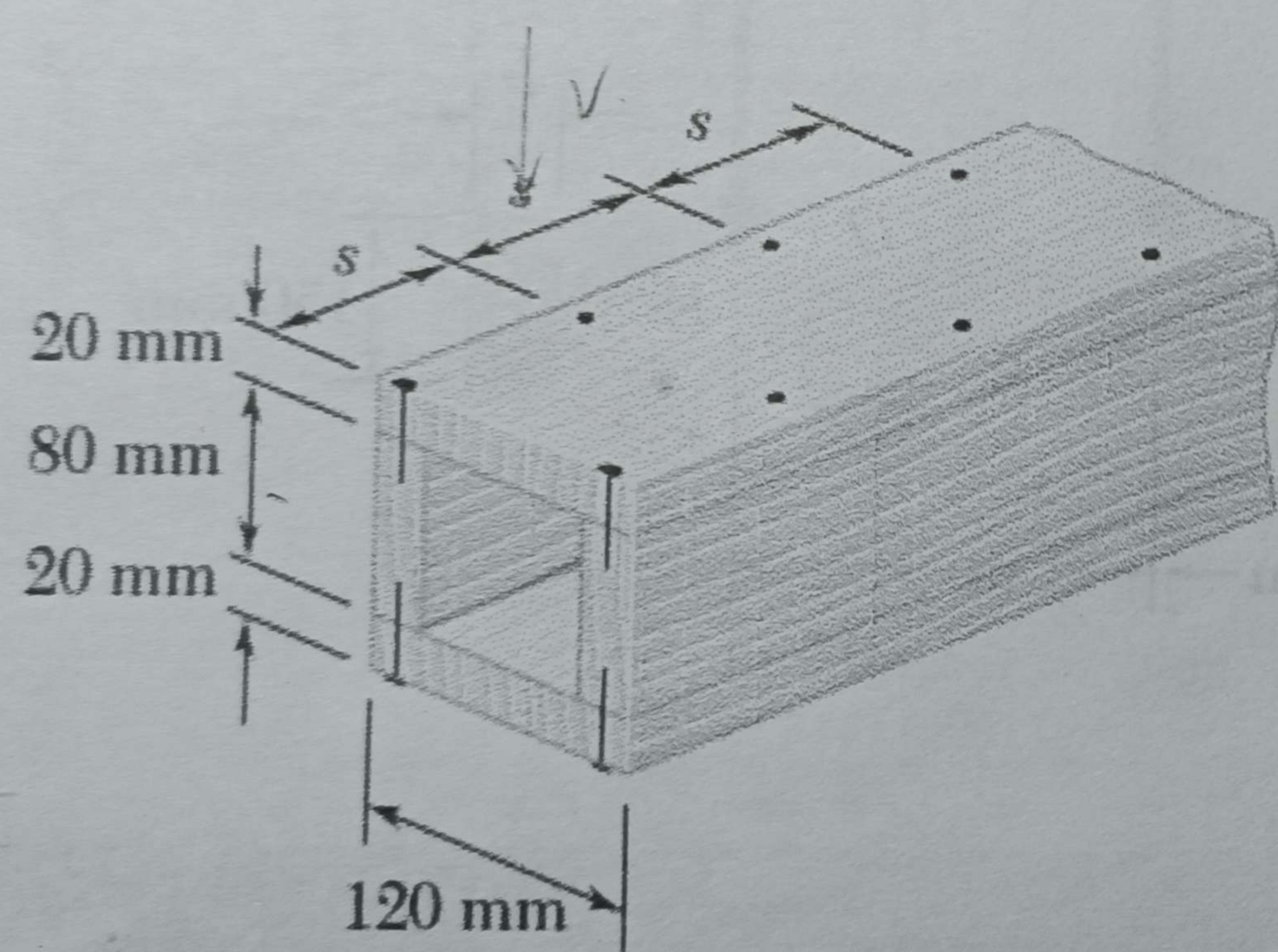
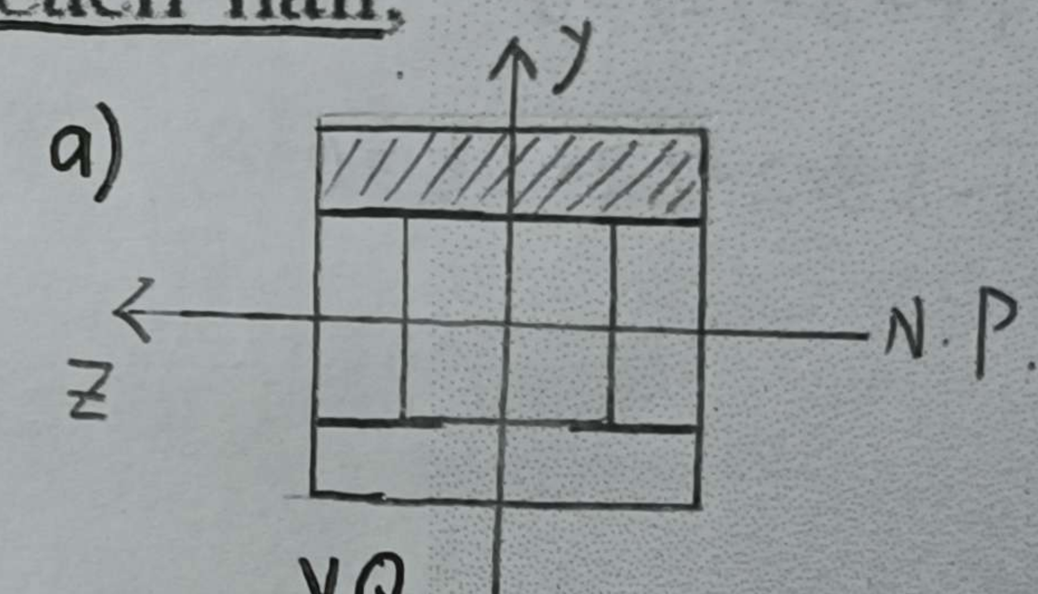


Fig. P6.89

解：a)



$$q = \frac{VQ}{I}$$

$$I = I_{out} - I_{in} = \frac{1}{12} \times 120^4 - \frac{1}{12} \times 80^4 = 1.387 \times 10^7 \ mm^4$$

$$Q = \int y \, dA = \bar{y} \cdot A_1 = 50 \times 20 \times 120 = 1.2 \times 10^5 \ mm^3$$

$$\Delta H = \frac{q}{2} \cdot \Delta x = \frac{1}{2} \times \frac{1200 \times 1.2 \times 10^5}{1.387 \times 10^7 \times 10^{-3}} \times 30 \times 10^{-3} = 155.7 \ N \quad \boxed{ANS}$$

b) $$q = \frac{VQ}{I}$$

$$\tau = \frac{VQ}{It} = \frac{1200 \times 1.52 \times 10^5}{1.397 \times 10^7 \times 40 \times 10^{-6}} = 3.288 \times 10^5 \ N/m^2 \quad \boxed{ANS}$$

$$(Q = \int y \, dA = 20 \times 120 \times 50 + 2 \times 20 \times 40 \times 20 = 1.52 \times 10^5 \ mm^3)$$

Homework-VII

Problem 2

6.92 For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used, $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa.

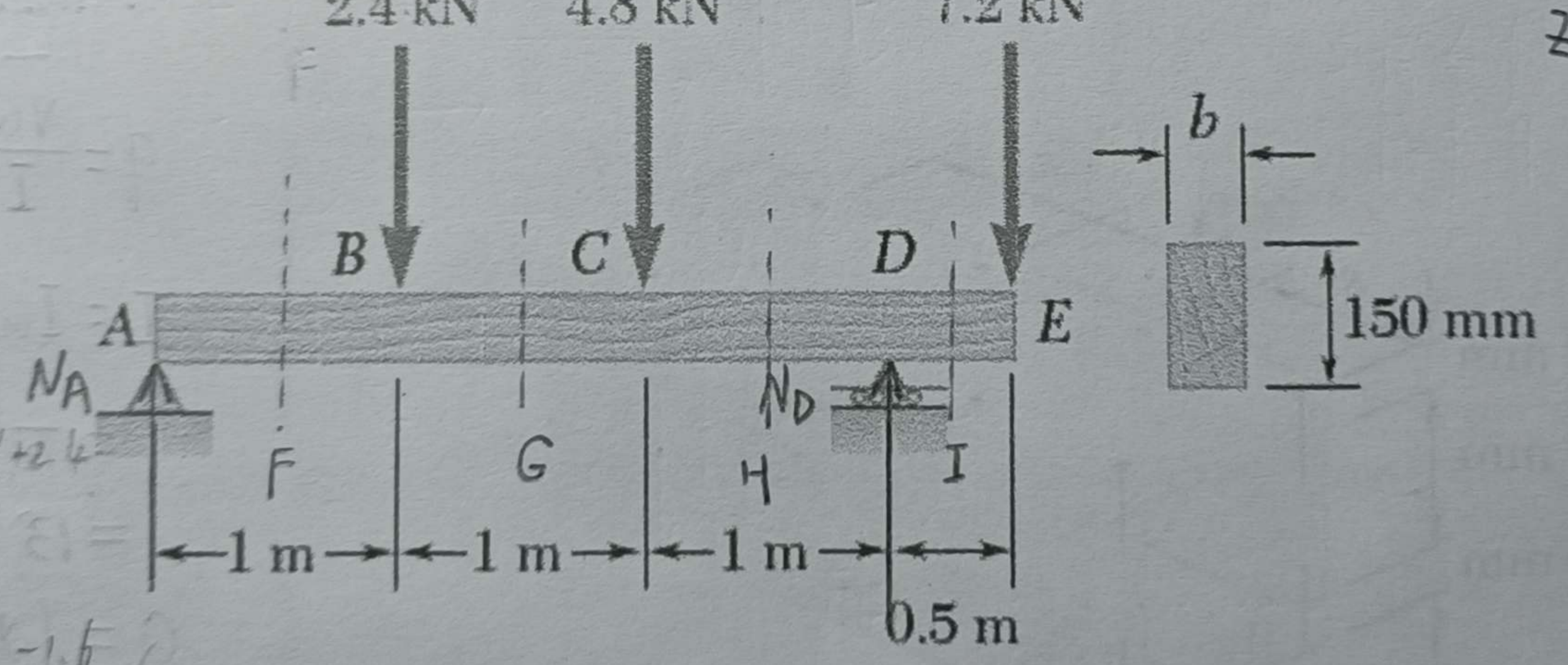


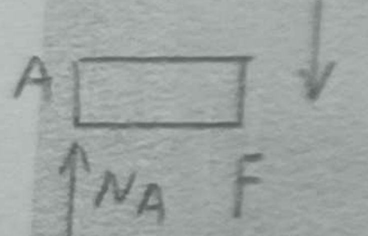
Fig. P6.92

解: $+\uparrow \sum F_y = 0:\ N_A + N_D - (2.4 + 4.8 + 7.2) = 0$

$\circlearrowleft \sum M_A = 0:\ N_D \cdot 3 - 2.4 \times 1 - 4.8 \times 2 - 7.2 \times 3.5 = 0$

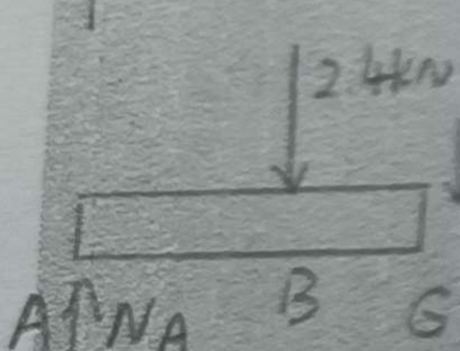
$\Rightarrow N_A = 2\,kN$

$N_D = 12.4\,kN$



$V_F = 2\,kN$

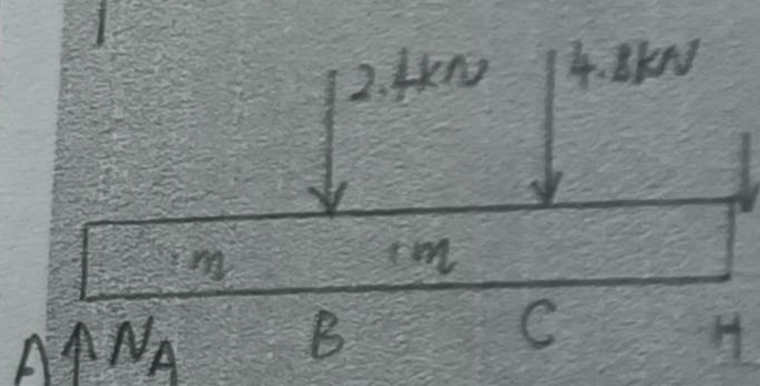
$M_F = -2x\ \ kN \cdot m$



$V_G = -0.4\,kN$

$M_G = 2.4(x-1) - 2x$

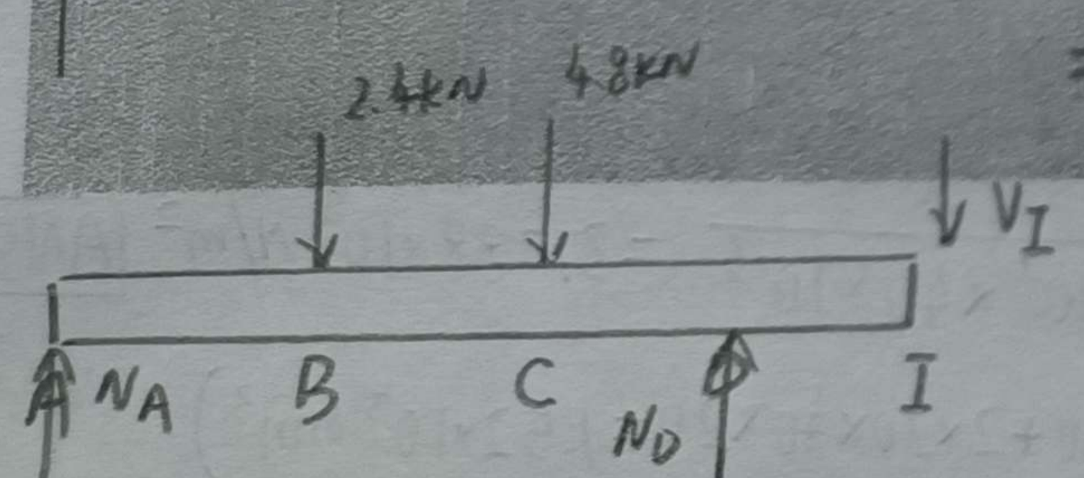
$= 0.4x - 2.4\ \ kN \cdot m$



$V_H = -5.2\,kN$

$M_H = 4.8(x-2) + 2.4(x-1) - 2x$

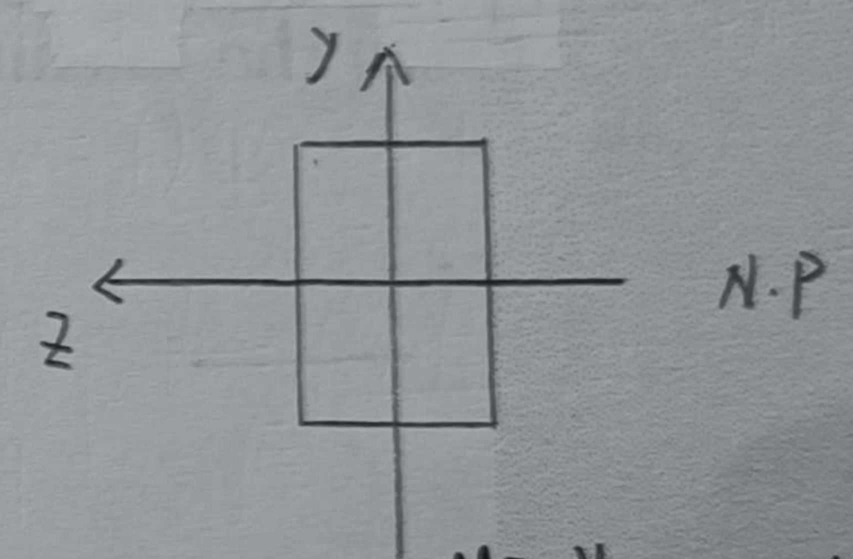
$= 5.2x - 12\ \ kN \cdot m$



$V_I = 14.4 - 7.2 = 7.2\,kN$

$M_I = 4.8(x-2) + 2.4(x-1) - 12.4(x-3) - 2x$

$= -7.2x + 25.2\ \ kN \cdot m$



$$\sigma_{all} = \frac{M_z \cdot y}{I_z} = \frac{M_z \cdot 75}{\frac{1}{12} \times b \times 150^3 \times 10^{-6}}$$

$$b = \left(\frac{75 M_z}{\frac{1}{12}\sigma_{all} \times 150^3 \times 10^{-6}}\right) = \frac{75 \times 3.6 \times 10^3}{\frac{1}{12} \times 12 \times 10^6 \times 150^3 \times 10^{-6}} =$$

$$\tau_{all} = \frac{VQ}{I_z b} = \frac{V \cdot b \cdot 75 \times 10^{-3} \times (\frac{1}{2} \times 75 \times 10^{-3})}{\frac{1}{12} b \times 150^3 \times 10^{-6}}$$

$$b = \left(\frac{75^2 \times \frac{1}{2} \times 10^{-3} \times 7.2 \times 10^3}{\frac{1}{12}\tau_{all} \times 150^3 \times 10^{-6} \times 825 \times 10^3}\right) = 0.08727\,m$$

$\therefore b_{min} = 0.08727\,m$ $\boxed{ANS}$

ork-VII

6.33 The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the nails (a) at A, (b) at B. (*Given:* $I_x = 1.504 \times 10^9\ \text{mm}^4$.)

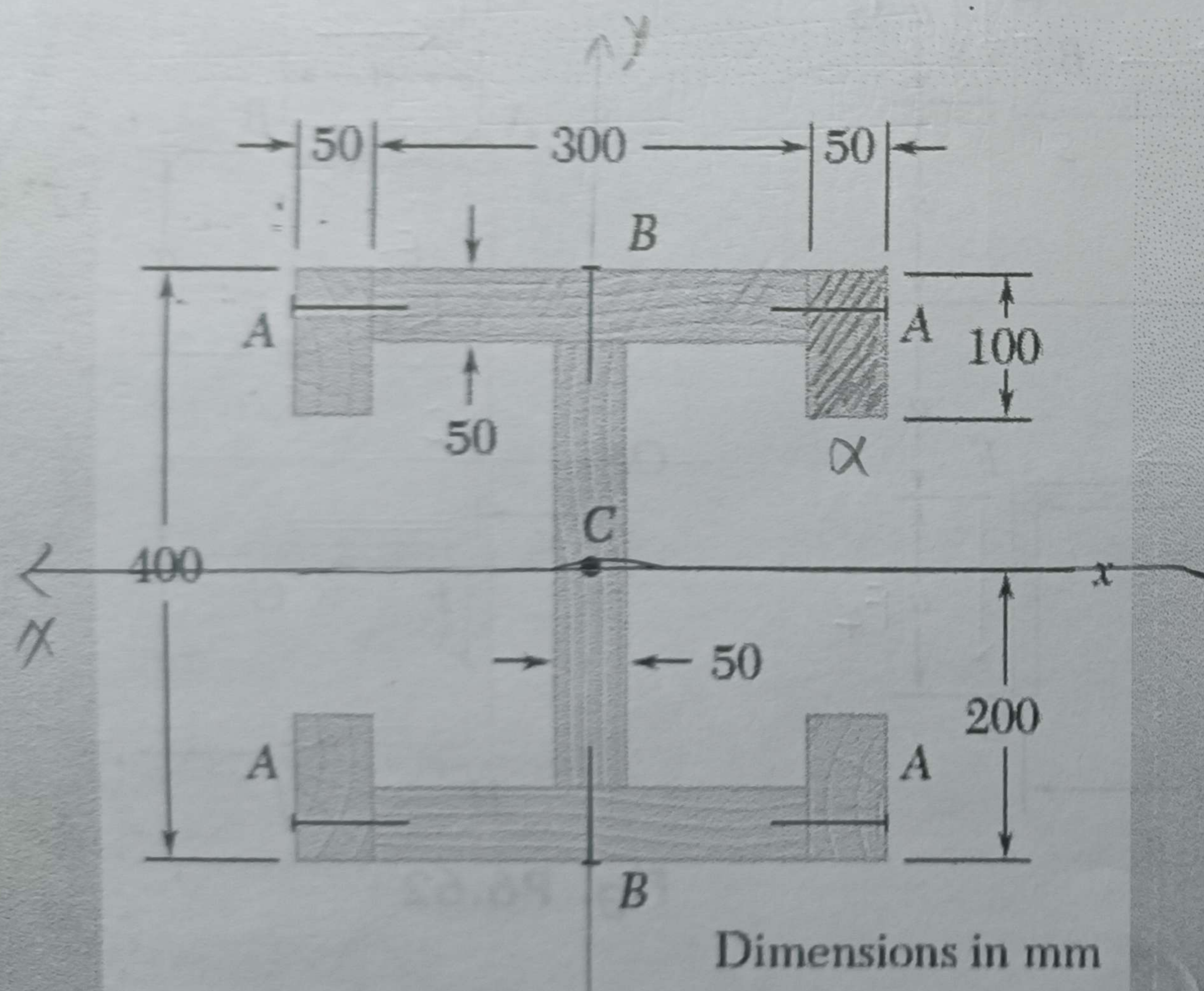


Fig. P6.33

解: a) $q_A = \dfrac{VQ_A}{I_x}$

$Q_A = \int_A y\,dA = 50\times100\times(200-50) = 7.5\times10^5\ \text{mm}^3$

$\Delta H_A = q_A \cdot \Delta x_A = \dfrac{8\times10^3\times7.5\times10^5}{1.504\times10^9}\times 60$

$= 239.4\ N$ $\boxed{\text{ANS}}$

b) $q_B = \dfrac{VQ_B}{I_x}$

$Q_B = \int_B y\,dA = 2\times7.5\times10^5 + 300\times50\times(200-25)$

$= 4.125\times10^6\ \text{mm}^3$

$\Delta H_B = q_B \cdot \Delta x_B = \dfrac{8\times10^3\times4.125\times10^6\times25}{1.504\times10^9}$

$= 548.5\ N$ $\boxed{\text{ANS}}$

$\frac{(t\cdot s\cdot a)}{I} ds = \frac{Vta}{I}\cdot 2a^2 = \frac{Vta^3}{I}$

$q_2 ds) = 2\left[\int_0^a \frac{V s\cdot t\cdot(2a-\frac{s}{2})}{I} ds + \int_0^a \frac{V\cdot s\cdot t\cdot(a-\frac{s}{2})}{I} ds\right] = \frac{7}{3}\frac{Vt}{I}a^3$

Ans

work-VII

6.61 and 6.62 Determine the location of the shear center O of a th[in-]walled beam of uniform thickness having the cross section shown

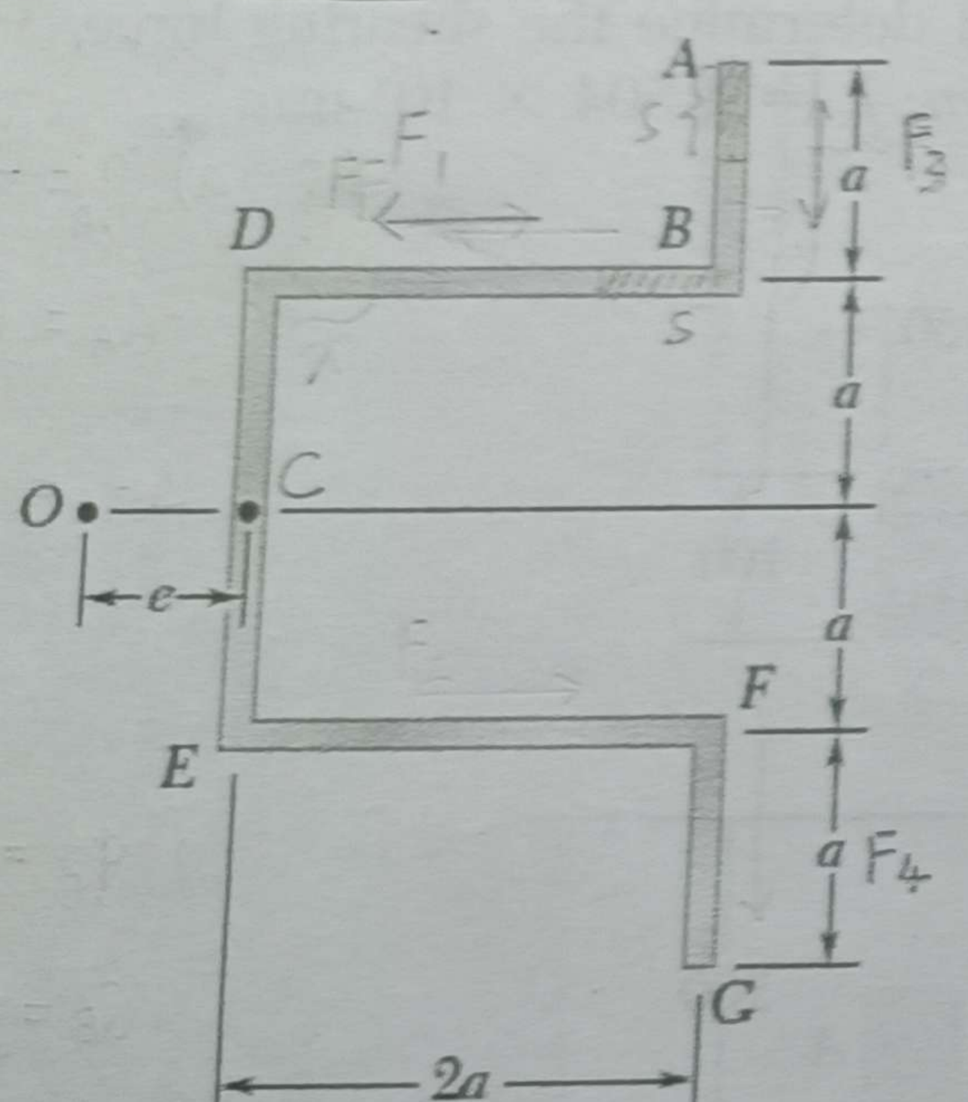


Fig. P6.61

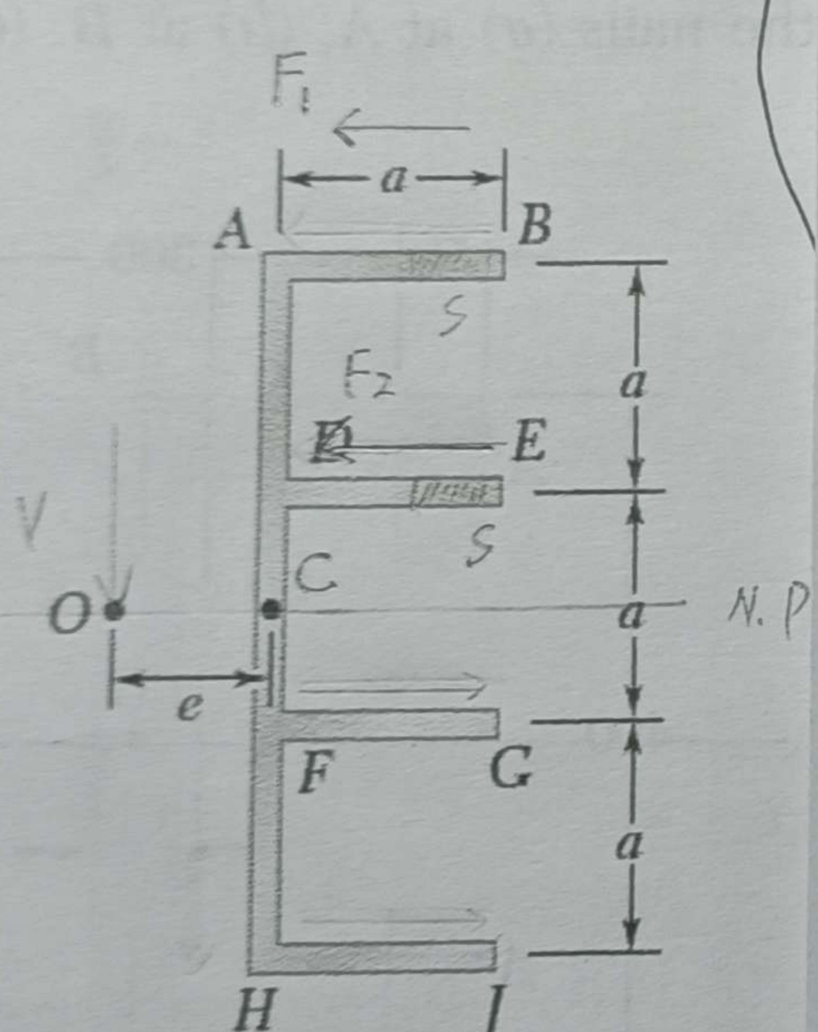


Fig. P6.62

6.61

解：$I_{AB} = \frac{1}{12}ta^3 + ta\cdot(\frac{3}{2}a)^2 = \frac{7}{3}ta^3 = I_{FG}$

$I_{DB} = \frac{1}{12}2a\cdot t^3 + t\cdot 2a\cdot a^2 = 2ta^3 = I_{EF}$

$I_{DE} = \frac{1}{12}t(2a)^3 = \frac{2}{3}ta^3$

$I = 2I_{AB} + 2I_{DB} + I_{DE} = \frac{28}{3}ta^3$

$\cancel{\tau_{AB} = \frac{VQ}{It} = \frac{V}{It}\cdot \ldots}$

$q_{AB} = \frac{VQ}{I} = \frac{V}{I}t\cdot s\cdot(2a-\frac{s}{2}) = \frac{Vt2a}{I}\cdot s - \frac{Vt}{2I}s^2$

$F_3 = \int_0^a q_{AB}\,ds$

$= \int_0^a(\frac{Vt\cdot 2a}{I}\cdot s - \frac{Vt}{2I}s^2)\,ds$

$= \frac{Vt}{I}2a\cdot\frac{1}{2}a^2 - \frac{Vt}{2I}\cdot\frac{1}{3}a^3 = \frac{5}{56}V$

$q_{BD} = \frac{VQ}{I} = \frac{V}{I}\cdot st\cdot a$

$F_1 = \int_0^{2a} q_{BD}\,ds$

$= \int_0^{2a}\frac{V}{I}tas\,ds$

$= \frac{V}{I}ta\cdot\frac{1}{2}\cdot 4a^2 = \frac{3}{14}V$

由 $M_C = 0$:

$F_1\cdot 2a - 2F_3\cdot 2a = V\cdot e$

$e = \frac{2a\times\frac{3}{14}V - 4a\times\frac{5}{56}V}{V} = \frac{1}{14}a$

6.62解：

$I_{AB} = \frac{1}{12}at^3 + t\cdot a\cdot(\frac{3}{2}a)^2 = \frac{1}{12}at^3 + \frac{9}{4}ta^3 = I_{HJ}$

$I_{DE} = \frac{1}{12}at^3 + ta\cdot(\frac{a}{2})^2 = \frac{1}{12}at^3 + \frac{1}{4}ta^3 = I_{FG}$

$I_{AH} = \frac{1}{12}t\cdot(3a)^3 = \frac{9}{4}ta^3$

$I = \frac{1}{6}at^3 + \frac{9}{2}ta^3 + \frac{1}{6}at^3 + \frac{1}{2}ta^3 + \frac{9}{4}ta^3$

$= \frac{1}{3}at^3 + \frac{29}{4}ta^3$

Since thin-walled

$I \approx \frac{29}{4}ta^3$

$q_{AB} = \frac{V}{I}\cdot st\cdot\frac{3}{2}a$

$F_1 = \int_0^a q_{AB}\,ds = \int_0^a \frac{V}{I}\frac{3}{2}at\cdot s\,ds$

$= \frac{V}{I}\cdot\frac{3}{2}at\cdot\frac{1}{2}a^2 = \frac{3Vt}{4I}a^3 = \frac{3}{29}V$

$q_{DE} = \frac{V}{I}st\cdot\frac{1}{2}a$

$F_2 = \int_0^a q_{DE}\,ds = \int_0^a\frac{Vat}{2I}\cdot s\,ds$

$= \frac{Vat}{2I}\cdot\frac{1}{2}a^2 = \frac{Vta^3}{4I} = \frac{1}{29}V$

由 $M_C = 0$

$Ve = F_1\cdot 3a + F_2\cdot a$

$e = \frac{\frac{9}{29}Va + \frac{a}{29}V}{V} = \frac{10}{29}a$