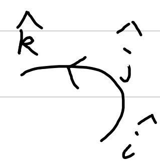
Solution

Q1

(a) $\vec{M}_E = \vec{r}_{EC} \times \vec{P}$

$$\vec{r}_{EC} = \vec{r}_C - \vec{r}_E = (16\hat{i} + 0\hat{j} + 10\hat{k}) - (24\hat{k})$$
$$= (16\hat{i} - 14\hat{k})\ dm = (1.6\hat{i} - 1.4\hat{k})\ m$$

$\vec{P} = -100\hat{j}\ N$



$$\vec{M}_E = \vec{r}_{EC} \times \vec{P} = (1.6\hat{i} - 1.4\hat{k}) \times (-100\hat{j})\ N\cdot m$$
$$= (-160\,\hat{i} \times \hat{j} + 140\hat{k} \times \hat{j})\ N\cdot m$$
$$= (-160\hat{k} - 140\hat{i})\ N\cdot m$$
$$= (-140\hat{i} - 160\hat{k})\ N\cdot m$$

$\therefore \boxed{\vec{M}_E = (-140\hat{i} - 160\hat{k})\ N\cdot m}$

(b) $\vec{AE} = (-7\hat{i} + 24\hat{k})\ dm$, $\quad |\vec{AE}| = 25\ dm$

$$\hat{u}_{AE} = \vec{AE}/|\vec{AE}| = -\frac{7}{25}\hat{i} + \frac{24}{25}\hat{k}$$

$$M_{AE} = \vec{M}_E \cdot \hat{u}_{AE} = (-140\hat{i} - 160\hat{k}) \cdot \left(-\frac{7}{25}\hat{i} + \frac{24}{25}\hat{k}\right)\ N\cdot m$$
$$= \left(140 \times \frac{7}{25} - 160 \times \frac{24}{25}\right)\ N\cdot m$$
$$= -114.4\ N\cdot m$$

$\therefore$ The magnitude of the moment about AE is <u>114.4 N·m</u>

(A 卷 Q2) Define CS: x ← ↓ y

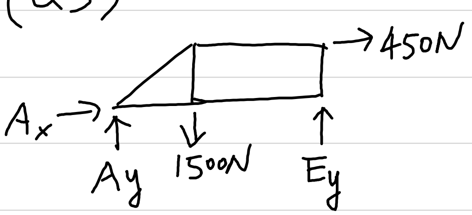
$$\vec{R} = \Sigma\vec{F} = 50\times2\,\hat{j} + 300\,\hat{j} + 200\times\frac{4}{5}\,\hat{j} + 200\times\frac{3}{5}\,\hat{i}$$
$$= (120\hat{i} + 560\hat{j})\ N$$

$$|\vec{R}| = \sqrt{120^2+560^2}\ N = 573N \quad , \quad \theta = \tan^{-1}\left(\frac{560}{120}\right) = 77.9^\circ$$

$\therefore\ R = 573N,\ 77.9^\circ$

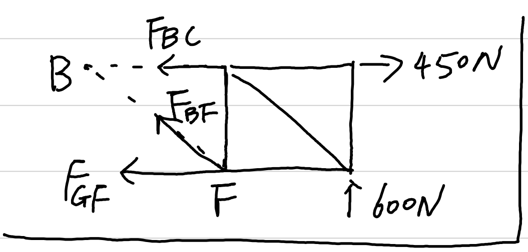
$$(+\curvearrowright)\ M_0 = \left\{50\times2\times(2+2+1) + 300\times6 + 200\times\frac{3}{5}\times(2+2) + 200\times\frac{4}{5}\times2\right\} N\cdot m$$
$$= (500+1800+480+320)\ N\cdot m = \underline{3100\ N\cdot m}$$

(Q3)



$(+\curvearrowleft)\ \Sigma M_A = 0:\ -1500\times3 + 9E_y - 450\times2 = 0$

$E_y = 600\ N$



$(+\curvearrowleft)\ \Sigma M_F = 0:\ 2F_{BC} - 450\times2 + 600\times3 = 0$

$\underline{F_{BC} = -450N\ \text{(compression)}}$

$+\uparrow \Sigma F_y = 0:\quad \frac{2}{\sqrt{13}}F_{BF} + 600 = 0 \Rightarrow \underline{F_{BF} = -1082\ N\ \text{(compression)}}$

$\overset{+}{\rightarrow} \Sigma F_x = 0:\ -(-450) + 450 - F_{GF} - \frac{3}{\sqrt{13}}(-1082) = 0$

$450 + 450 - F_{GF} + 900.3 = 0$

$\Rightarrow \underline{F_{GF} = 1800\ N,\ \text{(Tension)}}$

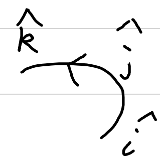
Solution

Q1

(a) $\vec{M}_E = \vec{r}_{EC} \times \vec{P}$

$$\vec{r}_{EC} = \vec{r}_C - \vec{r}_E = (16\hat{i} + 0\hat{j} + 10\hat{k}) - (24\hat{k})$$
$$= (16\hat{i} - 14\hat{k})\ dm = (1.6\hat{i} - 1.4\hat{k})\ m$$

$\vec{P} = -100\hat{j}\ N$



$$\vec{M}_E = \vec{r}_{EC} \times \vec{P} = (1.6\hat{i} - 1.4\hat{k}) \times (-100\hat{j})\ N\cdot m$$
$$= (-160\ \hat{i}\times\hat{j} + 140\hat{k}\times\hat{j})\ N\cdot m$$
$$= (-160\hat{k} - 140\hat{i})\ N\cdot m$$
$$= (-140\hat{i} - 160\hat{k})\ N\cdot m$$

$$\therefore \boxed{\vec{M}_E = (-140\hat{i} - 160\hat{k})\ N\cdot m}$$

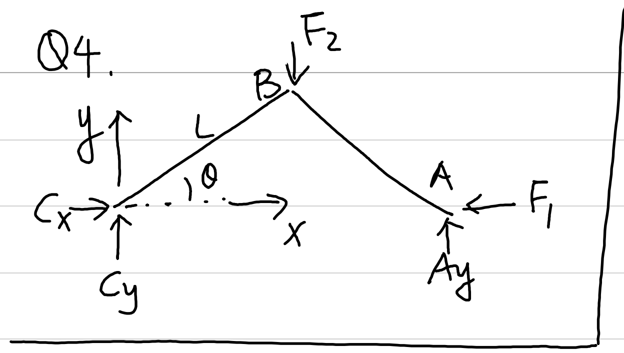
(b) $\vec{AE} = (-7\hat{i} + 24\hat{k})\ dm$, $\quad |\vec{AE}| = 25\ dm$

$$\hat{u}_{AE} = \frac{\vec{AE}}{|\vec{AE}|} = -\frac{7}{25}\hat{i} + \frac{24}{25}\hat{k}$$

$$M_{AE} = \vec{M}_E \cdot \hat{u}_{AE} = (-140\hat{i} - 160\hat{k}) \cdot \left(-\frac{7}{25}\hat{i} + \frac{24}{25}\hat{k}\right)\ N\cdot m$$
$$= \left(140\times\frac{7}{25} - 160\times\frac{24}{25}\right)\ N\cdot m$$
$$= -114.4\ N\cdot m$$

$\therefore$ The magnitude of the moment about AE is <u>114.4 N·m</u>

Q4.



Define x-y axes as shown, point C as origin.

$y_B = L\sin\theta$, $\delta y_B = L\cos\theta\,\delta\theta$

$x_A = 2L\cos\theta$, $\delta x_A = -2L\sin\theta\,\delta\theta$

$$\delta U = 0 \Rightarrow -F_2\,\delta y_B - F_1\,\delta x_A = 0$$

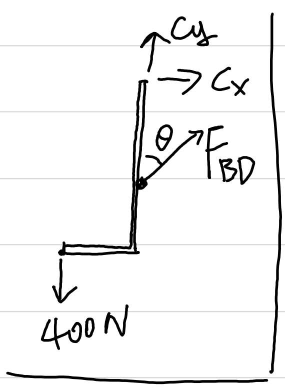
$$F_2 L\cos\theta\,\delta\theta - F_1(2L\sin\theta\,\delta\theta) = 0$$

$$(F_2\cos\theta - 2F_1\sin\theta)L\,\delta\theta = 0, \quad \forall\,\delta\theta$$

$$\Rightarrow \underline{F_2\cos\theta - 2F_1\sin\theta = 0}$$

or $\underline{F_2 = 2F_1\tan\theta}$

Q5. BD is two-force member



$$|CB| = \sqrt{0.51^2 - 0.24^2} = 0.45\text{ m}$$

$$\cos\theta = 0.45/0.51 = 15/17, \quad \sin\theta = \frac{0.24}{0.51} = \frac{8}{17}$$

$$\circlearrowleft+\ \Sigma M_C = 0:\ 400\times 0.135 + \frac{8}{17}F_{BD}\times 0.45 = 0$$

$$\Rightarrow \underline{F_{BD} = -255\text{ N} \quad (C)}$$

$$\xrightarrow{+} \Sigma F_x = 0:\ C_x + F_{BD}\times\frac{8}{17} = 0 \Rightarrow C_x - \frac{8}{17}\times 255 = 0$$

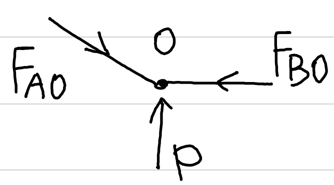
$$\Rightarrow \underline{C_x = 120\text{ N}}$$

$$+\uparrow \Sigma F_y = 0:\ C_y - 400 + \frac{15}{17}\times(-255) = 0$$

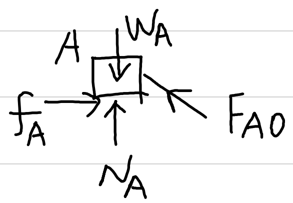
$$\Rightarrow \underline{C_y = 625\text{ N}}$$

$$\boxed{|F_{BD}| = 255\text{ N (c)}, \quad C_x = 120\text{ N} \rightarrow, \quad C_y = 625\text{N} \uparrow}$$

(Q6) Link AO, BO are two-force members.



$$\begin{cases} \frac{4}{5}F_{AO} - F_{BO} = 0 \\ P - \frac{3}{5}F_{AO} = 0 \end{cases} \Rightarrow \begin{cases} F_{AO} = \frac{5}{3}P \cdots ① \\ F_{BO} = \frac{4}{3}P \cdots ② \end{cases}$$



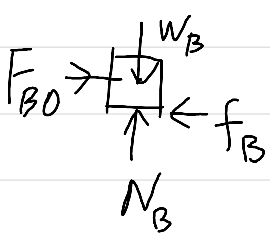
$N_A - W_A + \frac{3}{5}F_{AO} = 0 \Rightarrow N_A = 50 - P \cdots ③$

$f_A - \frac{4}{5}F_{AO} = 0 \Rightarrow f_A = \frac{4}{3}P \cdots ④$

If block A is about to slide: $f_A = \mu_s N_A \cdots ⑤$

③④⑤ $\Rightarrow \frac{4}{3}P = 0.3(50 - P) \Rightarrow 4P = 45 - 0.9P \Rightarrow 4.9P = 45$

$\Rightarrow P = 9.18\ N$



$N_B - W_B = 0 \Rightarrow N_B = W_B = 100N \cdots ⑥$

$F_{BO} - f_B = 0 \Rightarrow f_B = \frac{4}{3}P \cdots ⑦$

If block B is about to slide: $f_B = \mu_s N_B \cdots ⑧$

⑥⑦⑧ $\Rightarrow \frac{4}{3}P = 0.3 \times 100 \Rightarrow P = 22.5\ N$

Comparing the above two cases, we know that

$\underline{P_{max} = 9.18\ N}$, without cause movement.