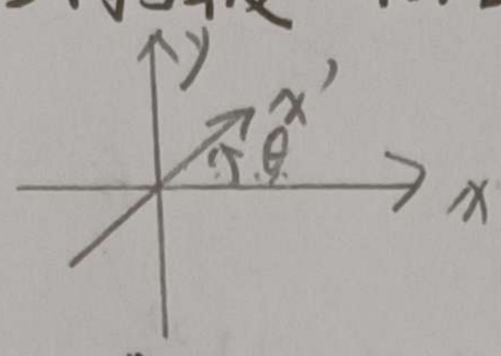
航空结构强度 HW2 12012127 邹佳驹

2.6

解:



counterclockwise 30°

$\theta = 30^\circ$

$$[\sigma'] = \begin{bmatrix} \cos30^\circ & \sin30^\circ \\ -\sin30^\circ & \cos30^\circ \end{bmatrix} \begin{bmatrix} 10 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} \cos30^\circ & -\sin30^\circ \\ \sin30^\circ & \cos30^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt3}{2} & \frac12 \\ -\frac12 & \frac{\sqrt3}{2} \end{bmatrix} \begin{bmatrix} 10 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} \frac{\sqrt3}{2} & -\frac12 \\ \frac12 & \frac{\sqrt3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 5\sqrt3+\frac32 & \frac32\sqrt3+1 \\ -5+\frac32\sqrt3 & -\frac32+\sqrt3 \end{bmatrix} \begin{bmatrix} \frac{\sqrt3}{2} & -\frac12 \\ \frac12 & \frac{\sqrt3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 10.598 & -1.964 \\ -1.964 & 1.4019 \end{bmatrix}$$

2.8

解:

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} x^2y & (a^2-y^2)x & 0 \\ (a^2-y^2)x & (y^3-3a^2y)/3 & 0 \\ 0 & 0 & 2az^2 \end{bmatrix}$$

$$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{yx}}{\partial y} + \frac{\partial\sigma_{zx}}{\partial z} + f_x = 0$$

$$\Rightarrow f_x = (-1)\,(2x\cdot y + (-2y)x + 0) = \underset{\sim}{0}$$

$$\frac{\partial\sigma_{xy}}{\partial x} + \frac{\sigma_{yy}}{\partial y} + \frac{\sigma_{zy}}{\partial z} + f_y = 0$$

$$\Rightarrow f_y = (-1)\left[(a^2-y^2) + \frac13(3y^2-3a^2)\right] = \underset{\sim}{0}$$

$$\frac{\partial\sigma_{xz}}{\partial x} + \frac{\partial\sigma_{yz}}{\partial y} + \frac{\partial\sigma_{zz}}{\partial z} + f_z = 0$$

$$\Rightarrow f_z = (-1)\,(0+0+4az) = \underset{\sim}{-4az}$$

$$\therefore \vec{f} = 0\,\vec{i} + 0\,\vec{j} - 4az\,\vec{k}$$

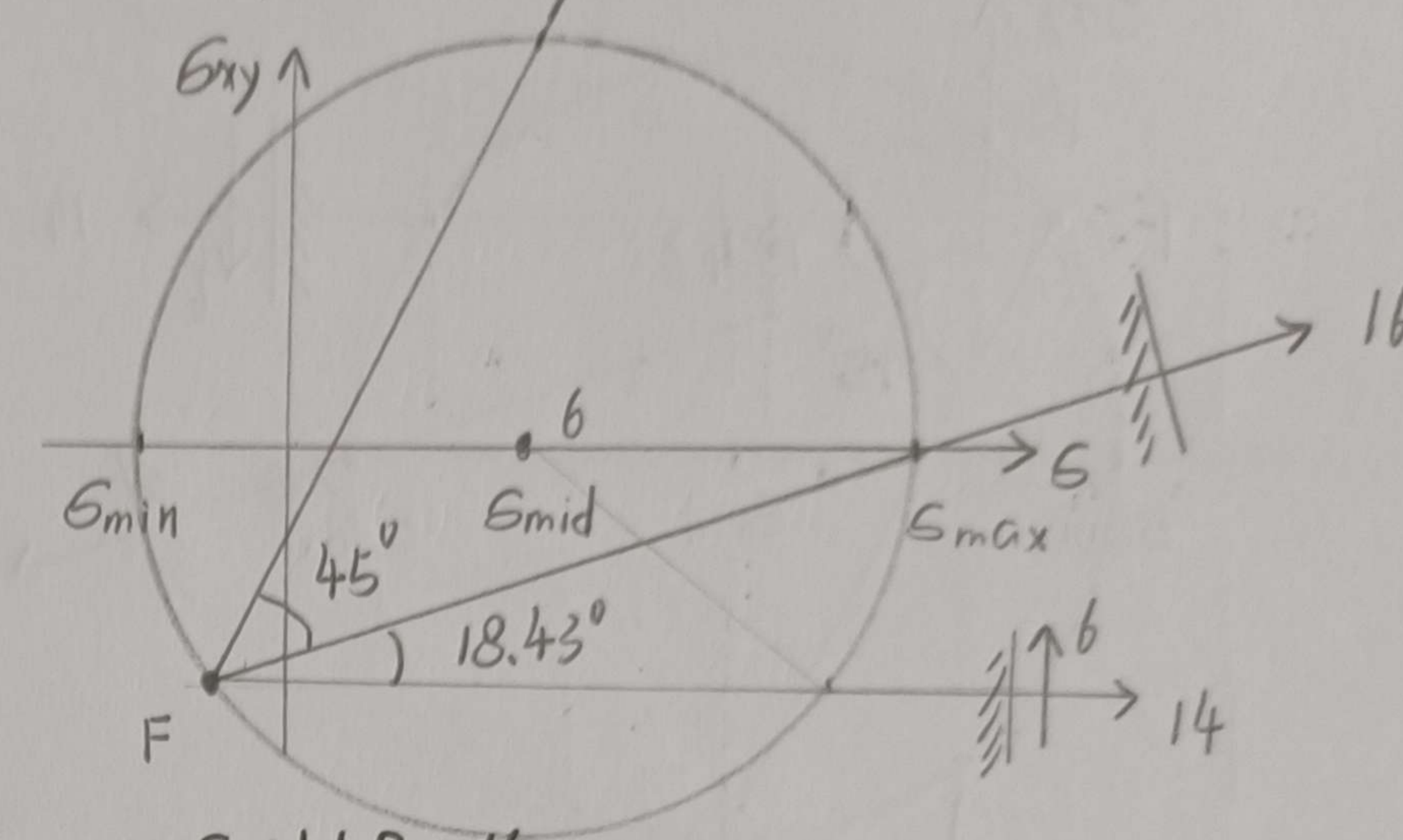
for the equilibrium.

2.10

解: 由图: $\sigma_{xx} = 14$ $\sigma_{yy} = -2$ $\sigma_{xy} = 6$

$$\sigma_{mid} = \frac{\sigma_{xx}+\sigma_{yy}}{2} = 6$$

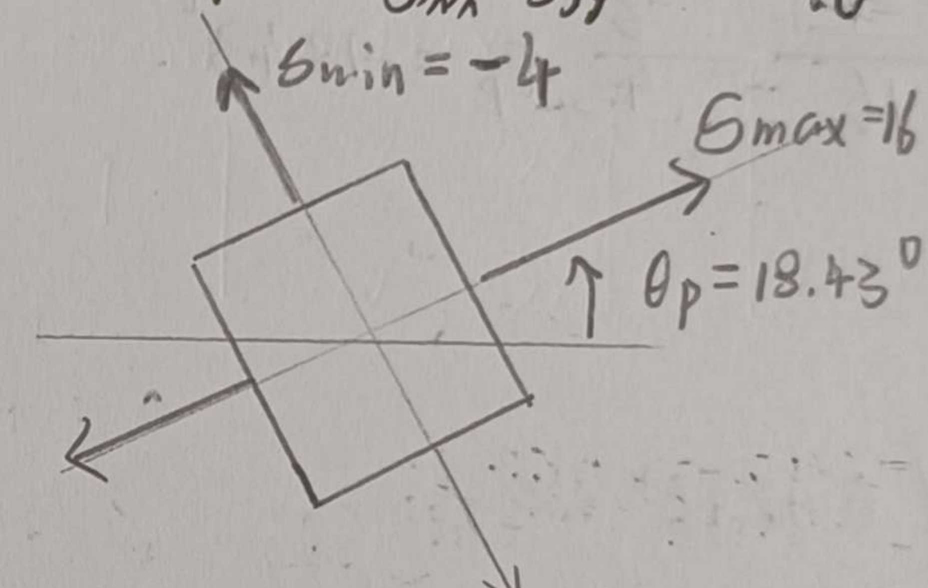
$$R = \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = \sqrt{8^2+6^2} = 10$$



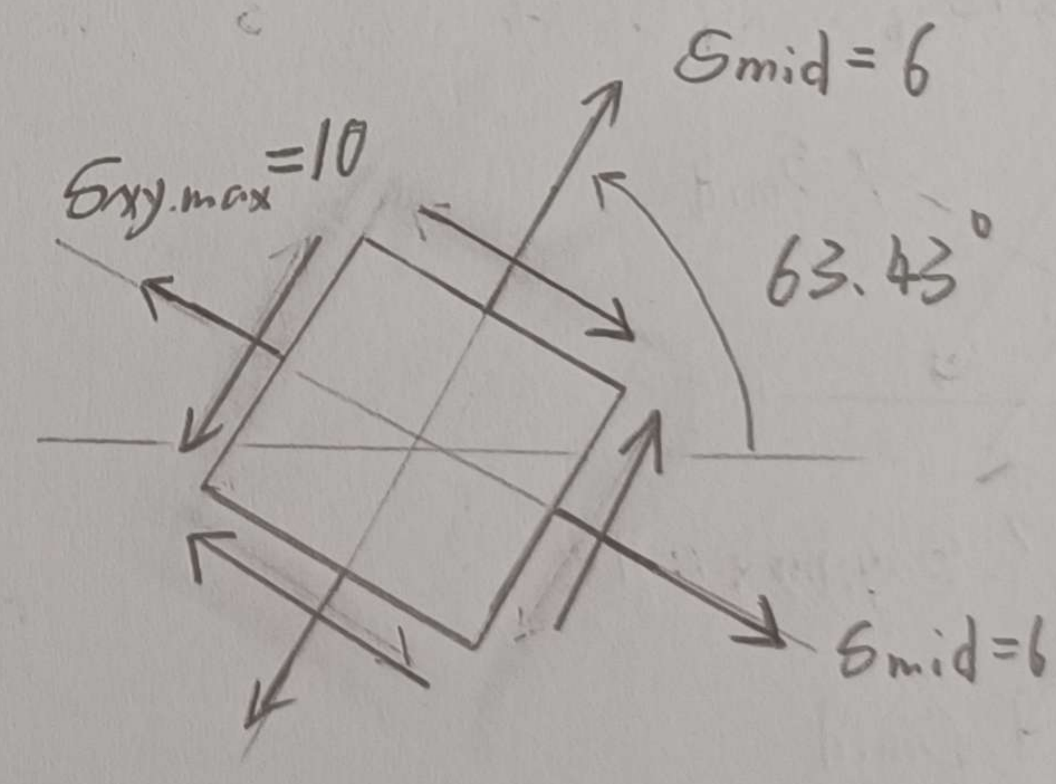
$\sigma_{max} = \sigma_{mid} + R = 16$

$\sigma_{min} = \sigma_{mid} - R = -4$

$$\tan2\theta_p = \frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}} = \frac{12}{16} = \frac34,\ \theta_p = 18.43^\circ$$



$\theta_p + 45^\circ = 63.43^\circ$ to get $\sigma_{xy\,max}$ at σ_{mid}

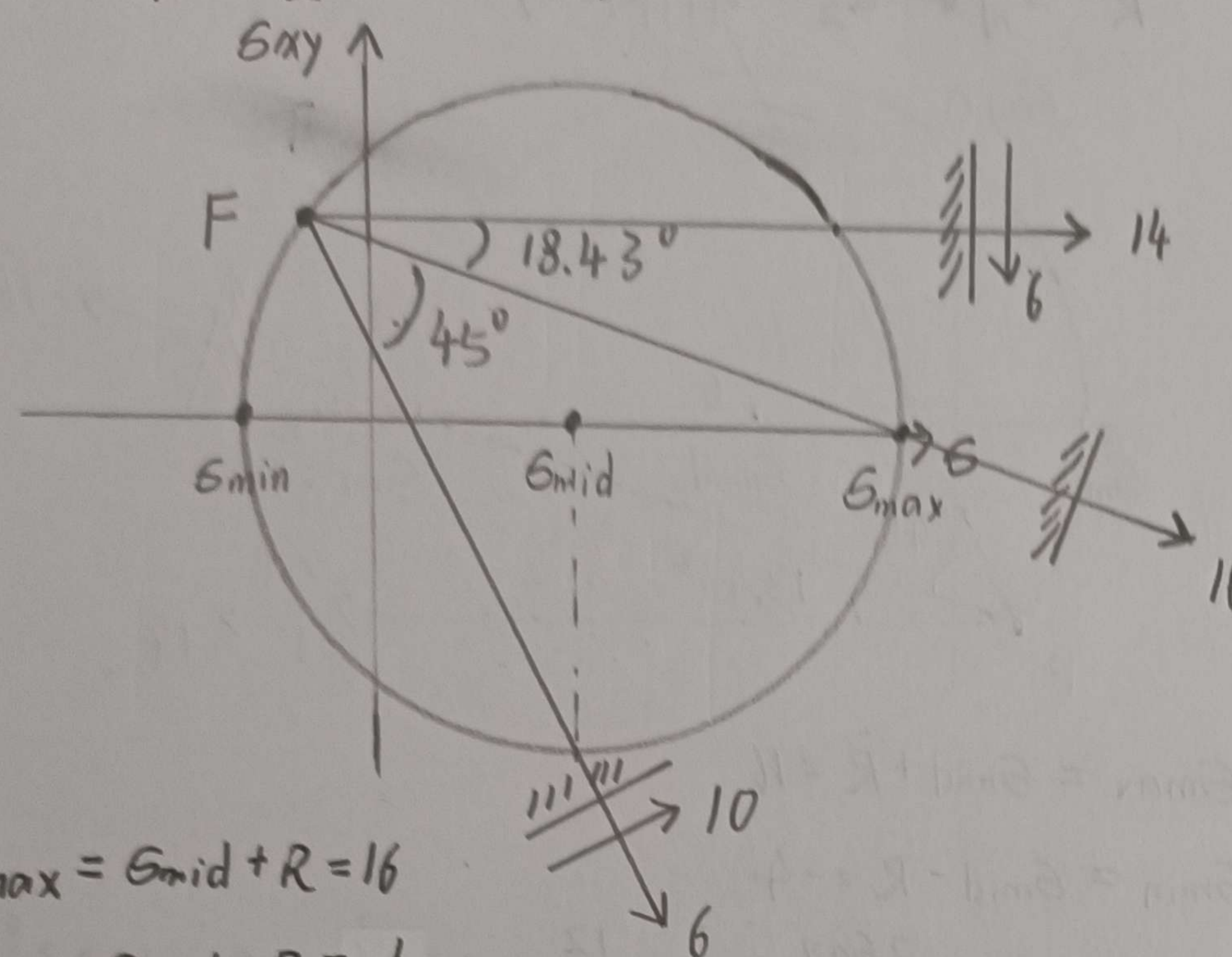


2.11

解：由图：$\sigma_{xx}=14$，$\sigma_{yy}=-2$，$\sigma_{xy}=-6$

$$\sigma_{mid}=\frac{\sigma_{xx}+\sigma_{yy}}{2}=6$$

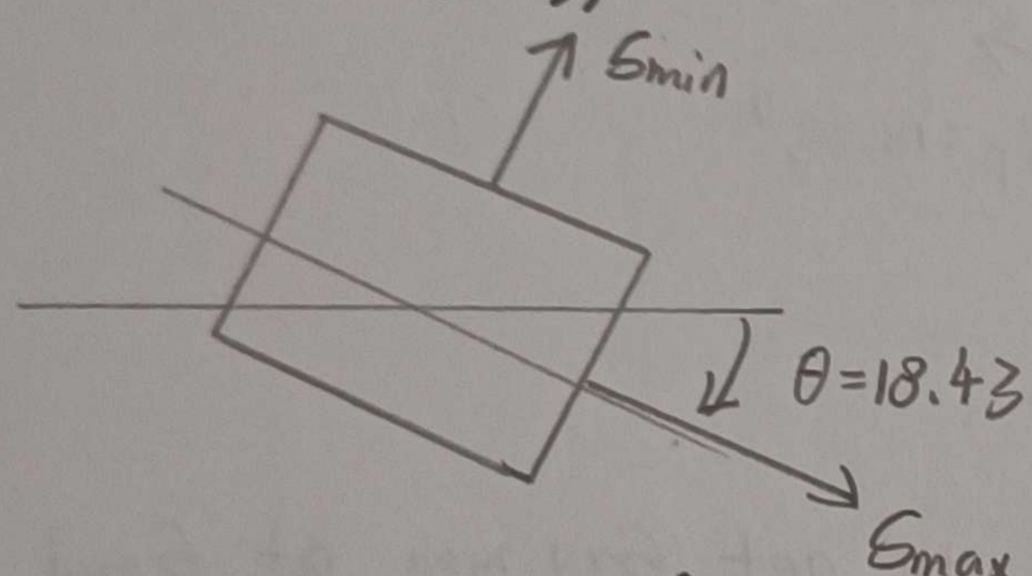
$$R=\sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2+\sigma_{xy}^2}=10$$



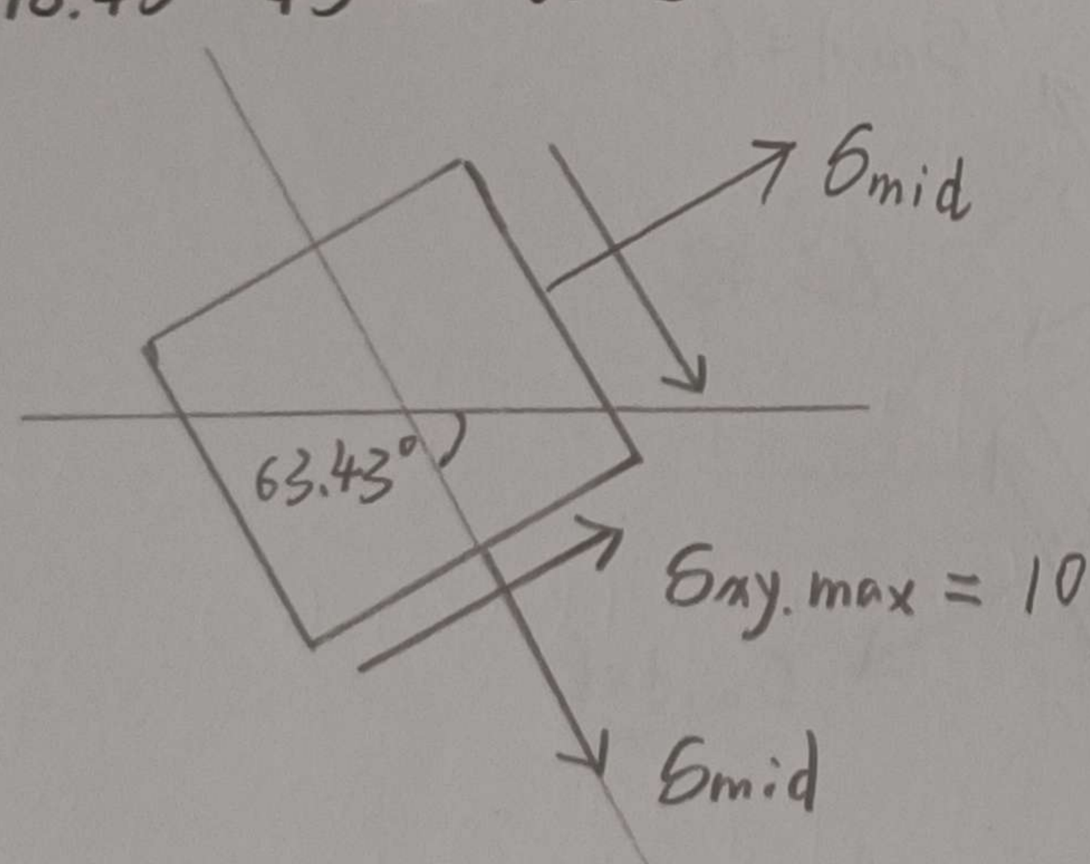
$\sigma_{max}=\sigma_{mid}+R=16$

$\sigma_{min}=\sigma_{mid}-R=-4$

$$\tan 2\theta_p=\frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}}=\frac{-12}{16}=\frac{-3}{4},\ \theta_p=-18.43^\circ$$



$-18.43^\circ-45^\circ=-63.43^\circ$



2.12

解：

$$\varepsilon'_{xx}=c^2\varepsilon_{xx}+s^2\varepsilon_{yy}+2cs\,\varepsilon_{xy}$$

$\theta_1=0^\circ$，$\theta_2=120^\circ$，$\theta_3=240^\circ$ 依次代入计算

$$\varepsilon_1'=\cos^2 0^\circ\varepsilon_{xx}+0+0=\varepsilon_{xx}=3\times10^{-6}$$

$$\varepsilon_2'=\cos^2 120^\circ\varepsilon_{xx}+\sin^2 120^\circ\varepsilon_{yy}+2\cos120^\circ\cdot\sin120^\circ\varepsilon_{xy}=5\times10^{-6}$$

$$\varepsilon_3'=\cos^2 240^\circ\varepsilon_{xx}+\sin^2 240^\circ\varepsilon_{yy}+2\cos240^\circ\sin240^\circ\varepsilon_{xy}=8\times10^{-6}$$

$\Rightarrow$ $\varepsilon_{xx}=3\times10^{-6}$

$$\frac{1}{4}\varepsilon_{xx}+\frac{3}{4}\varepsilon_{yy}+(-1)\frac{\sqrt3}{2}\varepsilon_{xy}=5\times10^{-6}$$

$$\frac{1}{4}\varepsilon_{xx}+\frac{3}{4}\varepsilon_{yy}+\frac{\sqrt3}{2}\varepsilon_{xy}=8\times10^{-6}$$

$\Rightarrow$ $\varepsilon_{xx}=3\times10^{-6}$

$\varepsilon_{yy}=\frac{23}{3}\times10^{-6}=7.67\times10^{-6}$

$\varepsilon_{xy}=\sqrt3\times10^{-6}=1.73\times10^{-6}$

strain matrix

$$[\varepsilon]=\begin{bmatrix}\varepsilon_{xx}&\varepsilon_{yx}\\ \varepsilon_{xy}&\varepsilon_{yy}\end{bmatrix}=\begin{bmatrix}3\times10^{-6}&1.73\times10^{-6}\\ 1.73\times10^{-6}&7.67\times10^{-6}\end{bmatrix}$$

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2.13

解:

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2}{\partial x \partial y}\gamma_{xy}$$

$$A\cdot 2 + A\cdot 2 = C$$

$$\therefore C = 4A \quad \boxed{\text{ANS}}$$

$$\left.\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} = Ay^2 \\ \varepsilon_{yy} &= \frac{\partial v}{\partial y} = Ax^2 \end{aligned}\right\} \Rightarrow \begin{cases} u = Ay^2\cdot x + C_1(y) \\ v = Ax^2\cdot y + C_2(x) \end{cases} \quad (*).$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = Cxy$$

$$A\cdot x\cdot 2y + C_1'(y) + Ay\cdot 2x + C_2'(x) = Cxy = 4Axy$$

$$C_1'(y) + C_2'(x) = 0$$

$$\therefore \begin{cases} u = Axy^2 + C_1(y) \\ v = Ayx^2 + C_2(x) \end{cases} \quad \boxed{\text{ANS}}$$

其中 $C_1'(y) + C_2'(x) = 0$

eg: $C_1(y) = my + n$ (m, n, p 为 const).

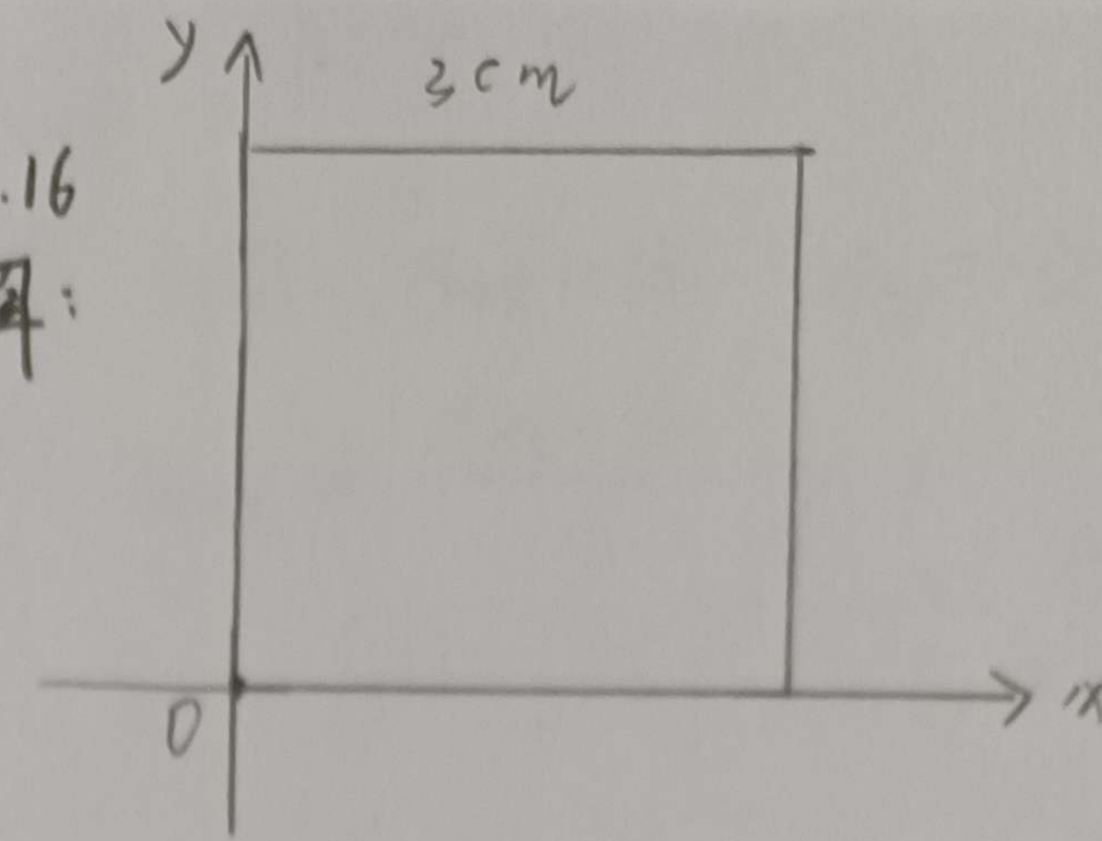
$C_2(x) = -mx + p$

particular solution: $u = Axy^2 + my + n$

$v = Ayx^2 + p - mx$

2.16

解:



$\varepsilon_{xx} = \frac{\partial u}{\partial x} = 3\times10^{-6}$, $u = 3\times10^{-6}x + C_1(y)$ cm

$\varepsilon_{yy} = \frac{\partial v}{\partial y} = 5\times10^{-6}$, $v = 5\times10^{-6}y + C_2(x)$ cm

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = -4\times10^{-6}$$

$$C_1'(y) + C_2'(x) = -8\times10^{-6}$$

$$C_1(0) = C_2(0) = 0$$

set $C_1(y) = Ay$

$C_2(x) = Bx$

$A + B = -8\times10^{-6}$

set $A = B = -4\times10^{-6}$

$u = 3\times10^{-6}x - 4\times10^{-6}y$ cm

$v = 5\times10^{-6}y - 4\times10^{-6}x$ cm

At $(2, 1)$, $u = 2\times10^{-6}$ cm $\boxed{\text{ANS}}$

$v = -3\times10^{-6}$ cm