空气动力学 HW4

3.23

解: $\nabla\cdot\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

$$u = V_\infty + V_\infty \frac{h}{\beta}\cdot\frac{2\pi}{l}\left(\cos\frac{2\pi x}{l}\right)\cdot e^{\frac{-2\pi\beta y}{l}}$$

$$\frac{\partial u}{\partial x} = 0 + V_\infty\frac{h}{\beta}\cdot\frac{2\pi}{l}\cdot e^{\frac{-2\pi\beta y}{l}}\cdot\left(-\sin\frac{2\pi x}{l}\right)\cdot\frac{2\pi}{l}$$

$$= -V_\infty\frac{h}{\beta}\cdot\frac{4\pi^2}{l^2}\cdot e^{\frac{-2\pi\beta y}{l}}\cdot\sin\frac{2\pi x}{l}$$

$$v = -V_\infty h\frac{2\pi}{l}\left(\sin\frac{2\pi x}{l}\right)e^{\frac{-2\pi\beta y}{l}}$$

$$\frac{\partial v}{\partial y} = +V_\infty h\frac{2\pi}{l}\left(\sin\frac{2\pi x}{l}\right)\cdot\frac{+2\pi\beta y}{l}\ e^{\frac{-2\pi\beta y}{l}-1}$$

$$= V_\infty h\frac{4\pi^2}{l^2}\beta y\cdot\sin\frac{2\pi x}{l}\cdot e^{\frac{-2\pi\beta y-l}{l}}$$

$$\nabla\cdot\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$= -V_\infty\frac{h}{\beta}\frac{4\pi^2}{l^2}e^{\frac{-2\pi\beta y}{l}}\cdot\sin\frac{2\pi x}{l}$$

$$+ V_\infty h\frac{4\pi^2}{l^2}\beta y\cdot\sin\frac{2\pi x}{l}\cdot e^{\frac{-2\pi\beta y-l}{l}}$$

$$= V_\infty h\frac{4\pi^2}{l^2}\sin\frac{2\pi x}{l}\cdot e^{\frac{-2\pi\beta y}{l}}\left(\beta y\cdot e^{-1}-\frac{1}{\beta}\right)$$

if $\nabla\cdot\vec{V}=0$

$\sin\frac{2\pi x}{l}=0$ or $\frac{\beta y}{e}=\frac{1}{\beta}$

except these points, $\nabla\cdot\vec{V}\neq 0$

it's compressible.

4.2

解: $c = 2m$ NACA2412

$V_\infty = 50m/s$ $\rho_\infty = 1.23\ kg/m^3$

$L' = 1353N$

$$C_l = \frac{L'}{\frac{1}{2}\rho_\infty V_\infty^2\cdot c} = \frac{1353}{\frac{1}{2}\times1.23\times50^2\times2} = 0.44$$

查表, $\alpha_{L=0} = -2.1^\circ$

$$\alpha = \frac{C_l}{2\pi} + \alpha_{L=0}$$

$$= \frac{0.44}{2\pi} + \frac{-2.1}{180}\cdot\pi$$

$$= 0.03338\ rad$$

$$= 1.912^\circ$$

4.3

解:

$$\frac{D\Gamma}{Dt} = \frac{D}{Dt}\left(-\int_c\vec{V}d\vec{s}\right)$$

$$= (-1)\cdot\frac{D}{Dt}\left(\int_c\vec{V}d\vec{s}\right)$$

$$= (-1)\cdot\left[\int_c\frac{D\vec{V}}{Dt}d\vec{s} + \int_c\vec{V}\frac{D}{Dt}d\vec{s}\right]$$

$$= (-1)\cdot\left(-\int_c\frac{\nabla p}{\rho}d\vec{s} + \int_c\vec{V}d\vec{V}\right)$$

$$= (-1)\cdot\left[-\iint_S\frac{\nabla\times(\nabla p)}{\rho}d\vec{s} + \int_c d\left(\frac{\vec{V}}{2}\right)^2\right]$$

$$= (-1)(0+0)$$

$$= 0$$

4.4

解:

$$M'_{LE} = -\int_0^c \xi \, dL' = -\int_0^c \xi \rho_\infty V_\infty \gamma(\xi) d\xi$$

$$= -\rho_\infty V_\infty \int_0^c \xi \gamma(\xi) d\xi$$

Since $\xi = \frac{c}{2}(1-\cos\theta)$

$$d\xi = \frac{c}{2}\sin\theta \, d\theta$$

$$\gamma(\xi) \rightarrow \gamma(\theta) = 2\alpha V_\infty \frac{1+\cos\theta}{\sin\theta}$$

$$\therefore M'_{LE} = -\rho_\infty V_\infty \int_0^\pi \frac{c}{2}(1-\cos\theta) \cdot 2\alpha V_\infty \frac{1+\cos\theta}{\sin\theta} \cdot \frac{c}{2}\sin\theta d\theta$$

$$= -\rho_\infty V_\infty^2 \frac{c^2}{2}\alpha \int_0^\pi (1-\cos^2\theta) d\theta$$

$$= -\rho_\infty V_\infty^2 \cdot \frac{c^2}{2} \cdot \alpha \cdot \frac{\pi}{2}$$

$$= -q_\infty c^2 \frac{\pi\alpha}{2}$$

4.5

解: thin airfoil theory - symmetric

$$c_l = 2\pi\alpha = 2\pi \cdot \frac{1.5^\circ}{180^\circ} \cdot \pi = 0.1645$$

$$c_{m,LE} = -\frac{\pi}{2}\alpha = -\frac{\pi}{2} \cdot \frac{1.5}{180}\pi = -0.04112$$

4.6

解:

$$\frac{dz}{dx} = \begin{cases} 0.25c \cdot \left(\frac{0.8}{c} - \frac{2x}{c^2}\right) & 0 \le \frac{x}{c} \le 0.4 \\ 0.111c\left(\frac{0.8}{c} - \frac{2x}{c^2}\right) & 0.4 \le \frac{x}{c} \le 1 \end{cases}$$

Since

$$x = \frac{c}{2}(1-\cos\theta_0)$$

$$\frac{dz}{dx} = \begin{cases} -0.05 + 0.25\cos\theta_0 & 0 \le \theta_0 \le 1.369 \\ -0.0222 + 0.111\cos\theta_0 & 1.369 \le \theta_0 \le \pi \end{cases}$$

a) $$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{1.369}(-0.05+0.25\cos\theta)(\cos\theta-1)d\theta$$

$$-\frac{1}{\pi}\int_{1.369}^{\pi}(-0.0222+0.111\cos\theta)(\cos\theta-1)d\theta$$

$$= -\frac{1}{\pi}\left[\int_0^{1.369}(-0.3\cos\theta+0.05+0.25\cos^2\theta)d\theta + \int_{1.369}^{\pi}(-0.1332\cos\theta+0.0222+0.111\cos^2\theta)d\theta\right]$$

$$= -\frac{1}{\pi}\left[\left(-0.3\sin\theta+0.05\theta+\frac{1}{16}\sin2\theta+\frac{1}{8}\theta\right)\Big|_0^{1.369} + \left(-0.1332\sin\theta+0.0222\theta+0.02775\sin2\theta+0.0555\theta\right)\Big|_{1.369}^{\pi}\right]$$

$$= -\frac{1}{\pi} \cdot [-0.02979 + 0.2441 - (-0.01323)]$$

$$= -0.07243 \text{ rad} = -4.150^\circ$$

b) $$c_l = 2\pi(\alpha - \alpha_{L=0})$$

$$= 2\pi(3+4.15) \cdot \frac{\pi}{180}$$

$$= 0.7841$$

4.7

解:

$$C_{m,\frac{c}{4}} = C_{m,LE} + \frac{1}{4}C_l$$

$$C_{m,LE} = \frac{-\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right)$$

$$C_l = 2\pi\left(A_0 + \frac{1}{2}A_1\right)$$

$$\Rightarrow C_{m,\frac{c}{4}} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right) + \frac{\pi}{2}\left(A_0 + \frac{1}{2}A_1\right)$$

$$= \frac{\pi}{4}(A_2 - A_1)$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos n\theta\, d\theta$$

$$A_1 = \frac{2}{\pi}\left[\int_0^{1.369}(-0.05 + 0.25\cos\theta)\cos\theta\, d\theta + \int_{1.369}^{\pi}(-0.0222 + 0.111\cos\theta)\cos\theta\, d\theta\right] = \frac{2}{\pi}(0.1467 + 0.1092)$$

$$A_2 = \frac{2}{\pi}\left[\int_0^{1.369}(-0.05 + 0.25\cos\theta)\cos 2\theta\, d\theta + \int_{1.369}^{\pi}(-0.0222 + 0.111\cos\theta)\cos 2\theta\, d\theta\right] = \frac{2}{\pi}(0.07838 - 0.03480)$$

$$\therefore C_{m,\frac{c}{4}} = \frac{\pi}{4}\cdot\frac{2}{\pi}(0.07838 - 0.03480 - 0.1467 - 0.1092) = -0.1016 \quad \boxed{\text{ANS}}$$

$$\frac{x_{cp}}{c} = -\frac{C_{m,LE}}{C_l} = -\frac{C_{m,\frac{c}{4}} - \frac{1}{4}C_l}{C_l} = -\frac{C_{m,\frac{c}{4}}}{C_l} + 0.25 = 0.3854$$

4.9

解:

$$M'_{LE} = -\int_0^c \xi\, dL' = -\rho_\infty V_\infty \int_0^c \xi\gamma(\xi)\, d\xi$$

Since $\xi = \frac{c}{2}(1-\cos\theta)$

$$d\xi = \frac{c}{2}\sin\theta\, d\theta$$

$$\gamma(\xi) \to \gamma(\theta) = 2V_\infty\left(A_0\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty}A_n\sin n\theta\right)$$

$\therefore M'_{LE} =$

$$-\rho_\infty V_\infty\int_0^\pi \frac{c}{2}(1-\cos\theta)\, 2V_\infty\left(A_0\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty}A_n\sin n\theta\right)\frac{c}{2}\sin\theta\, d\theta$$

$$= -\rho_\infty V_\infty^2\cdot\frac{c^2}{2}\int_0^\pi(1-\cos\theta)\left[A_0(1+\cos\theta) + \sum_{n=1}^{\infty}A_n\sin\theta\cdot\sin n\theta\right]d\theta$$

$$= -\rho_\infty V_\infty^2\cdot\frac{c^2}{2}\int_0^\pi\left[A_0(1-\sin^2\theta) + \sum_{n=1}^{\infty}A_n\sin\theta\cdot\sin n\theta - \frac{1}{2}\sum_{n=1}^{\infty}A_n\sin 2\theta\sin n\theta\right]d\theta$$

$$\int_0^\pi A_0\sin^2\theta\, d\theta = A_0\cdot\frac{\pi}{2}$$

$$\int_0^\pi A_n\sin\theta\cdot\sin n\theta\, d\theta = \begin{cases}\frac{\pi}{2}A_1 & n=1\\ 0 & n\neq 1\end{cases}$$

$$\int_0^\pi A_n\sin 2\theta\sin n\theta\, d\theta = \begin{cases}\frac{\pi}{2}A_2 & n=2\\ 0 & n\neq 2\end{cases}$$

$$\text{原式} = -q_\infty c^2\left(A_0\pi - \frac{\pi}{2}A_0 + \frac{\pi}{2}A_1 - \frac{\pi}{4}A_2\right) = -q_\infty c^2\left(\frac{\pi}{2}A_0 + \frac{\pi}{2}A_1 - \frac{\pi}{4}A_2\right)$$

$$C_{m,LE} = \frac{M'_{LE}}{q_\infty c^2} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{1}{2}A_2\right)$$