Homework problems 15-19
Due in class, Friday, 16 October 2020

15. The 2014-T6 aluminium rod has a diameter of 30 mm and supports the load shown. Determine the displacement of end *A* with respect to end *E*. Neglect the size of the couplings. The modulus of elasticity is $E_{al} = 73.1$ GPa.

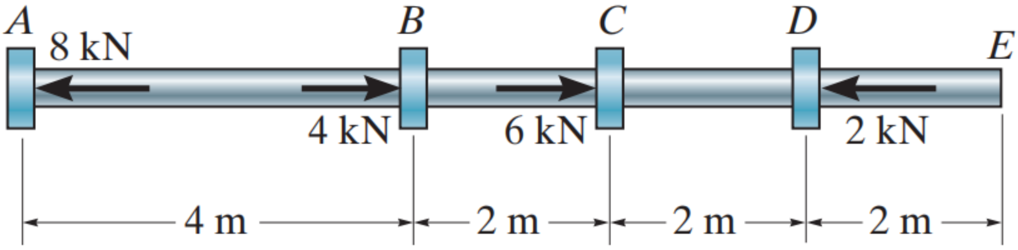
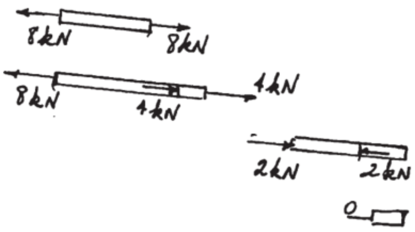


Figure 15

SOLUTION



$$\delta_{A/E} = \Sigma\frac{PL}{AE} = \frac{1}{AE}[8(4) + 4(2) - 2(2) + 0(2)](10^3)$$

$$= \frac{36(10^3)}{\frac{\pi}{4}(0.03)^2(73.1)(10^9)} = 0.697\,(10^{-3}) = 0.697 \text{ mm} \qquad \textbf{Ans.}$$

16. The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the yield stress for the steel is $(\sigma)_{st}$ = 640 MPa, and for the bronze $(\sigma)_{br}$ = 520 MPa, determine the magnitude of the largest elastic load P that can be applied to the assembly. E_{st} = 200 GPa, E_{br} = 100 GPa.

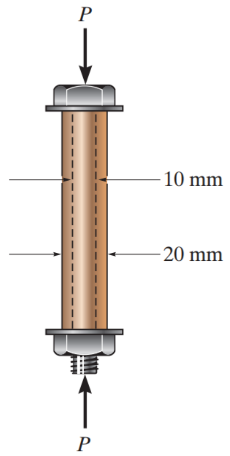


Figure 16

SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - P = 0 \qquad$ **(1)**

Assume failure of bolt:

$$P_{st} = (\sigma_Y)_{st}(A) = 640(10^6)\left(\frac{\pi}{4}\right)(0.01^2)$$
$$= 50265.5 \text{ N}$$

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br}L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

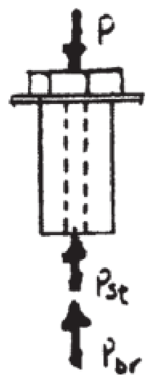
$$P_{st} = 0.6667\, P_{br}$$

$$50\,265.5 = 0.6667\, P_{br}$$

$$P_{br} = 75398.2 \text{ N}$$

From Eq. (1)

$$P = 50265.5 + 75398.2$$
$$= 125663.7 \text{ N} = 126 \text{ kN} \qquad \text{(controls)}$$ **Ans.**



Assume failure of sleeve:

$$P_{br} = (\sigma_Y)_{br}(A) = 520(10^6)\left(\frac{\pi}{4}\right)(0.02^2 - 0.01^2) = 122\,522.11 \text{ N}$$

$$P_{st} = 0.6667\, P_{br}$$
$$= 0.6667(122\,522.11)$$
$$= 81\,681.4 \text{ N}$$

From Eq. (1),

$$P = 122\,522.11 + 81\,681.4$$
$$= 204\,203.52 \text{ N}$$

17. The rigid bar is supported by the two short white spruce wooden posts and a spring. If each of the posts has an unloaded length of 1 m and a cross-sectional area of 600 mm², and the spring has a stiffness of k = 2 MN/m and an unstretched length of 1.02 m, determine the force and in each post after the load is applied to the bar. E_{wood} = 9.65 GPa.

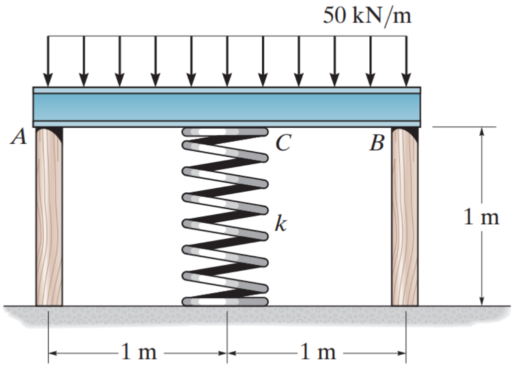


Figure 17

SOLUTION

Equations of Equilibrium:

$\circlearrowleft + \Sigma M_C = 0;\quad F_B(1) - F_A(1) = 0 \qquad F_A = F_B = F$

$+\uparrow \Sigma F_y = 0;\quad 2F + F_{sp} - 100(10^3) = 0 \qquad$ **(1)**

Compatibility:

$(+\downarrow)$

$$\delta_A + 0.02 = \delta_{sp}$$

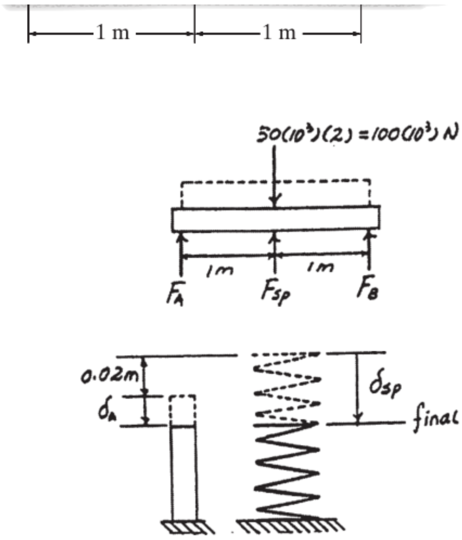
$$\frac{F(1)}{600(10^{-6})9.65(10^9)} + 0.02 = \frac{F_{sp}}{2.0(10^6)}$$

$$0.1727F + 20(10^3) = 0.5\,F_{sp} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$F_A = F_B = F = 25581.7\text{ N} = 25.6\text{ kN} \qquad \textbf{Ans.}$$

$$F_{sp} = 48\,836.5\text{ N}$$



18. The three A-36 steel wires have unloaded lengths of L_{AC} = 1.60 m and $L_{AB} = L_{AD}$ = 2.00 m. (a) If each wire has a diameter of 2 mm, determine the force in each wire after the 150-kg mass is suspended from the ring at *A*. (b) If both wire *AB* and *AD* have a diameter of 2 mm, determine the required diameter of wire *AC* so that each wire is subjected to the same force when the 150-kg mass is suspended from the ring at *A*.

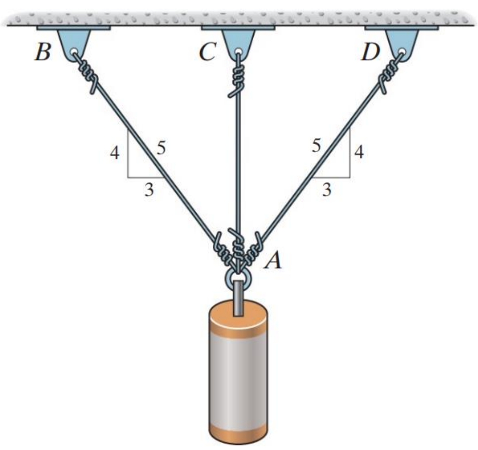
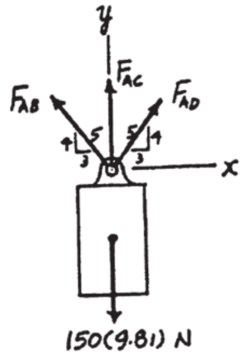


Figure 18

SOLUTION

Equations of Equilibrium:



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{3}{5}F_{AD} - \frac{3}{5}F_{AB} = 0 \qquad F_{AD} = F_{AB} = F$$

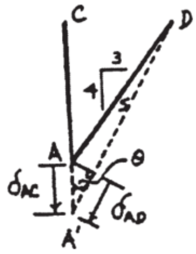
$$+\uparrow \Sigma F_y = 0; \qquad 2\left(\frac{4}{5}F\right) + F_{AC} - 150(9.81) = 0$$

$$1.6F + F_{AC} - 1471.5 = 0 \qquad \textbf{(1)}$$

Compatibility:

$$\delta_{AD} = \delta_{AC} \cos\theta$$

Since the displacement is very small, $\cos\theta = \dfrac{4}{5}$



$$\delta_{AD} = \frac{4}{5}\delta_{AC}$$

$$\frac{F(2)}{AE} = \frac{4}{5}\left[\frac{F_{AC}(1.6)}{AE}\right]$$

$$F = 0.640\,F_{AC} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$F_{AC} = 727\text{ N} \qquad \textbf{Ans.}$$

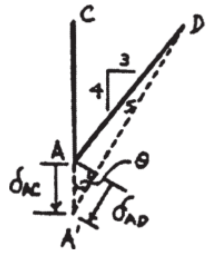
$$F_{AB} = F_{AD} = F = 465\text{ N} \qquad \textbf{Ans.}$$

SOLUTION

Equations of Equilibrium: Each wire is required to carry the same amount of load. Hence

$$F_{AB} = F_{AC} = F_{AD} = F$$

Compatibility:



$$\delta_{AD} = \delta_{AC} \cos\theta$$

Since the displacement is very small, $\cos\theta = \dfrac{4}{5}$

$$\delta_{AD} = \frac{4}{5}\delta_{AC}$$

$$\frac{F(2)}{\frac{\pi}{4}(0.002^2)E} = \frac{F(1.6)}{\frac{\pi}{4}d_{AC}^2 E}$$

$$d_{AC} = 0.001789\text{ m} = 1.79\text{ mm} \qquad \textbf{Ans.}$$

19. The 50-mm-diameter cylinder is made from Am 1004-T61 magnesium and is placed in the clamp when the temperature is $T_1 = 20$ °C. If the 304-stainless-steel carriage bolts of the clamp each have a diameter of 10 mm, and they hold the cylinder snug with negligible force against the rigid jaws, determine the force in the cylinder when the temperature rises to $T_2 = 130$ °C. $E_{mg} = 44.7$ GPa, $\alpha_{mg} = 26(10^{-6}/°\text{C})$; $E_{st} = 193$ GPa, $\alpha_{st} = 17(10^{-6}/°\text{C})$;.

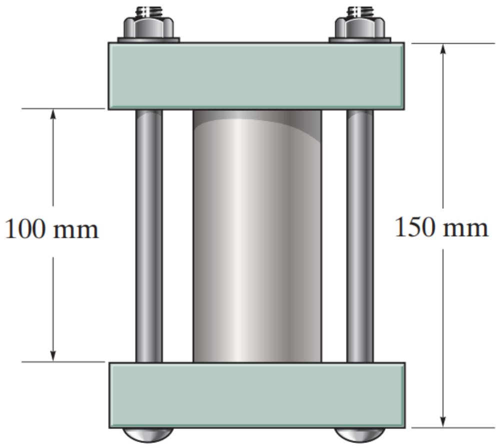
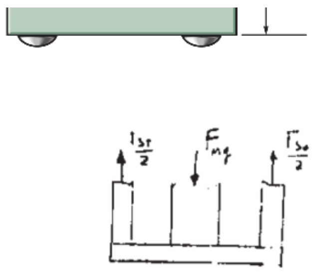


Figure 19

SOLUTION



$+\uparrow \Sigma F_y = 0; \qquad F_{st} = F_{mg} = F$

$$\delta_{mg} = \delta_{st}$$

$$\alpha_{mg} L_{mg} \Delta T - \frac{F_{mg} L_{mg}}{E_{mg} A_{mg}} = \alpha_{st} L_{st} \Delta T + \frac{F_{st} L_{st}}{E_{st} A_{st}}$$

$$26(10^{-6})(0.1)(110) - \frac{F(0.1)}{44.7(10^9)\frac{\pi}{4}(0.05)^2} = 17(10^{-6})(0.150)(110) + \frac{F(0.150)}{193(10^9)(2)\frac{\pi}{4}(0.01)^2}$$

$F = 904$ N **Ans.**