材料力学　HW 10 Week 14.

Problem 1.

解:

a) $F.S = \dfrac{P_{cr}}{P_0}$, $I = \dfrac{1}{4}\pi r^4 = 7.854 \times 10^{-9}$

$P_{cr} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2 \times 200 \times 10^9 \times 7.854 \times 10^{-9}}{0.9^2} = 19.14\ kN$

$F.S = 2.552$　　$\boxed{ANS}$

b) $F.S = \dfrac{P_{cr}}{P_0}$

$F.S \cdot P_0 = \dfrac{\pi^2 E \cdot \frac{1}{4}\pi r^4}{L_e^2}$

$r^* = 4\sqrt{\dfrac{F.S \cdot P_0 L_e^2 \cdot 4}{E \pi^3}}$

for (2): $r_2 = 4\sqrt{\dfrac{2.552 \times 7.5 \times 10^3 \times (2 \times 0.9)^2 \times 4}{200 \times 10^9 \times \pi^3}}$

$L_e = 2L$

　　　$= 0.01414\ m = 14.14\ mm$, $d_2 = 28.28\ mm$

for (3): $r_3 = 7.071\ mm$, $d_3 = 14.14\ mm$

$L_e = \frac{1}{2}L$

for (4): $r_4 = 8.367\ mm$, $d_4 = 16.73\ mm$

$L_e = 0.7L$

for (5): $r_5 = 14.14\ mm$, $d_5 = 28.28\ mm$.

$L_e = 2L$

Problem 2.

解:

a) At point C.

$y_{max} = y_c = e\left(\sec\dfrac{PL}{2} - 1\right)$, $P = \sqrt{\dfrac{P}{EI}}$

$= e\left(\sec\dfrac{L}{2}\sqrt{\dfrac{P}{EI}} - 1\right)$

$P = \sqrt{\dfrac{37 \times 10^3}{200 \times 10^9 \times \frac{1}{4}\pi \times (0.016)^4}} = 1.896$

$\therefore y_{max} = 1.659\ mm$

b) $\sigma_{max} = \dfrac{P}{A} + \dfrac{M_{max}\ C}{I}$

$= \dfrac{P}{A} + \dfrac{C}{I} \cdot P(y_{max} + e)$

$= \dfrac{37 \times 10^3}{\pi \times (0.016)^2} + \dfrac{0.016}{\frac{1}{4}\pi (0.016)^4} \times 37 \times 10^3 \times (1.659 + 1.2)$

$= 78.89\ MPa$

Problem 3

解: a) $y_{max} = e\left(\sec\frac{PLe}{2} - 1\right).$

$$P = \sqrt{\frac{P}{EI}} = \sqrt{\frac{15 \times 10^3}{70 \times 10^9 \times \frac{1}{12} \times (0.03)^4}} = 1.7817$$

$$\frac{PLe}{2} = 1.069 \quad ; \quad \sec\frac{PLe}{2} = \frac{1}{\cos\left(\frac{1.069 \times 180}{\pi}\right)} = 2.079$$

$$\therefore y_{max} = 4.316 \text{ mm}$$

b) $\delta_{max} = \frac{P}{A} + \frac{C}{I} \cdot P(y_{max} + e)$

$$= \frac{15 \times 10^3}{(0.03)^2} + \frac{0.015}{\frac{1}{12} \times 0.03^4} \times 15 \times 10^3 (8.316)$$

$$= 44.39 \text{ MPa}$$