Quiz 10

Date: 2022-04-18 Name: SID:

Q1. Find the general solutions of the following equation by the *Method of Undetermined Coefficients.*

$$y''' - 2y'' + y' = 3t^3 + 2e^t.$$

Q2. Find the general solution of the following equation by the *Variation of Parameters.*

$$y''' - y' = e^{-t}.$$

1. $y''' - 2y'' + y' = 3t^3 + 2e^t$

$\lambda^3 - 2\lambda^2 + \lambda = 0$ $\quad\lambda = 0, 1, 1$

$y = c_1 + c_2 e^t + c_3 t e^t$

let $g_1(t) = 3t^3$ $\quad g_2(t) = 2e^t$

$Y_1(t) = t(At^3 + Bt^2 + Ct + D)$

$Y_1' = 4At^3 + 3Bt^2 + 2Ct + D$

$Y_1'' = 12At^2 + 6Bt + 2C$

$Y_1''' = 24At + 6B$

$$\begin{cases} 4A = 3 \\ 3B - 24A = 0 \\ 24A - 12B + 2C = 0 \\ D + 6B - 4C = 0 \end{cases}$$

$$\Rightarrow \begin{cases} A = \frac{3}{4} \\ B = 6 \\ C = 27 \\ D = 72 \end{cases}$$

$Y_1 = \frac{3}{4}t^4 + 6t^3 + 27t^2 + 72t$

$Y_2(t) = Et^2 e^t$

$Y_2' = (2Et + Et^2)e^t$

$Y_2'' = e^t(Et^2 + 4Et + 2E)$

$Y_2''' = e^t(Et^2 + 6Et + 6E)$

$E = 1$

$Y_2(t) = t^2 e^t$

Thus general solution is

$$y = c_1 + c_2 e^t + c_3 t e^t + t^2 e^t + \frac{3}{4}t^4 + 6t^3 + 27t^2 + 72t$$

2. $y''' - y' = e^{-t}$ $\lambda^3 - \lambda = 0$ $\lambda = 0, -1, 1$.

$$y = c_1 + c_2 e^{-t} + c_3 e^{t}$$

$$W(t) = \begin{vmatrix} 1 & e^{-t} & e^{t} \\ 0 & -e^{-t} & e^{t} \\ 0 & e^{-t} & e^{t} \end{vmatrix} = -2.$$

$$W_1(t) = \begin{vmatrix} 0 & e^{-t} & e^{t} \\ 0 & -e^{-t} & e^{t} \\ 1 & e^{-t} & e^{t} \end{vmatrix} = 2$$

$$W_2(t) = \begin{vmatrix} 1 & 0 & e^{t} \\ 0 & 0 & e^{t} \\ 0 & 1 & e^{t} \end{vmatrix} = -e^{t}$$

$$W_3(t) = \begin{vmatrix} 1 & e^{-t} & 0 \\ 0 & -e^{-t} & 0 \\ 0 & e^{-t} & 1 \end{vmatrix} = -e^{-t}$$

$$Y(t) = \int e^{-t} \frac{2}{-2} dt + e^{-t} \int e^{-t} \cdot \frac{-e^{t}}{-2} dt$$

$$+ e^{t} \int e^{-t} \frac{-e^{-t}}{-2} dt$$

$$= e^{-t} + \frac{1}{2} t e^{-t} - \frac{1}{4} e^{-t} = \frac{1}{2} t e^{-t} + \frac{3}{4} e^{-t}$$

$$y(t) = c_1 + c_2 e^{-t} + c_3 e^{t} + \frac{1}{2} t e^{-t} + \frac{3}{4} e^{-t}.$$