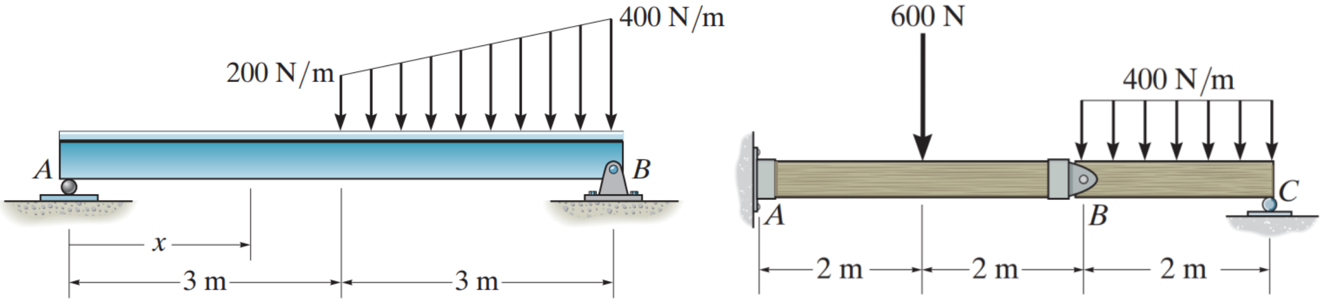
Homework problems 26-31
Due in class, Friday, 6 November 2020

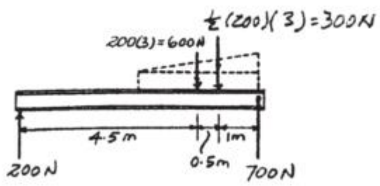
26. Draw the shear and moment diagrams for the beam.



(a) (b)

Figure 27

SOLUTION



Support Reactions: As shown on FBD.

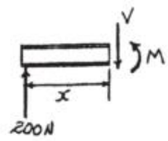
Shear and Moment Functions:

For $0 \le x < 3$ m:

$+\uparrow\Sigma F_y = 0;\quad 200 - V = 0 \qquad V = 200\text{ N}$ **Ans.**

$\circlearrowleft+\Sigma M_{NA} = 0;\quad M - 200\,x = 0$

$$M = \{200\,x\}\text{ N}\cdot\text{m}$$ **Ans.**

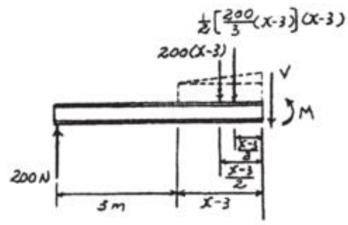


For $3\text{ m} < x \le 6$ m:

$$+\uparrow\Sigma F_y = 0;\quad 200 - 200(x-3) - \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3) - V = 0$$

$$V = \left\{-\frac{100}{3}x^2 + 500\right\}\text{ N}$$ **Ans.**

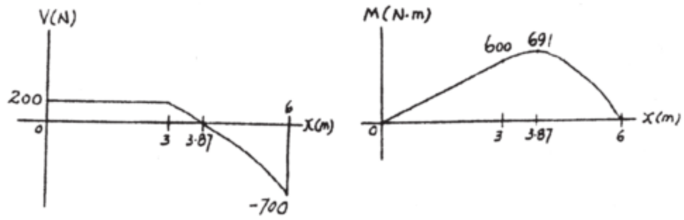
Set $V = 0, x = 3.873$ m



$$\circlearrowleft+\Sigma M_{NA} = 0;\quad M + \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3)\left(\frac{x-3}{3}\right) + 200(x-3)\left(\frac{x-3}{2}\right) - 200x = 0$$

$$M = \left\{-\frac{100}{9}x^3 + 500x - 600\right\}\text{ N}\cdot\text{m}$$ **Ans.**

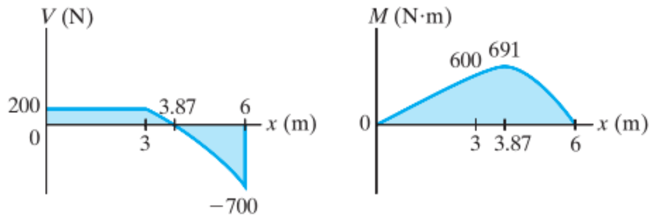
Substitute $x = 3.87$ m, $M = 691\text{ N}\cdot\text{m}$



Ans:
For $0 \leq x < 3$ m: $V = 200$ N, $M = \{200x\}$ N·m,

For 3 m $< x \leq 6$ m: $V = \left\{-\frac{100}{3}x^2 + 500\right\}$ N,

$M = \left\{-\frac{100}{9}x^3 + 500x - 600\right\}$ N·m



SOLUTION

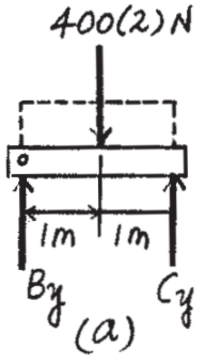
Support Reactions: Referring to the free-body diagram of segment BC shown in Fig. a,

$\circlearrowleft + \Sigma M_B = 0; \qquad C_y(2) - 400(2)(1) = 0$

$C_y = 400\text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad B_y + 400 - 400(2) = 0$

$B_y = 400\text{ N}$



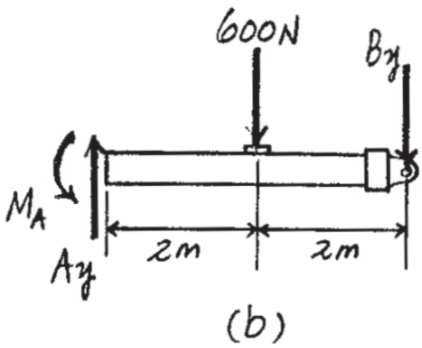
Using the result of $\mathbf{B}_y$ and referring to the free-body diagram of segment AB, Fig. b,

$+\uparrow \Sigma F_y = 0; \qquad A_y - 600 - 400 = 0$

$A_y = 1000\text{ N}$

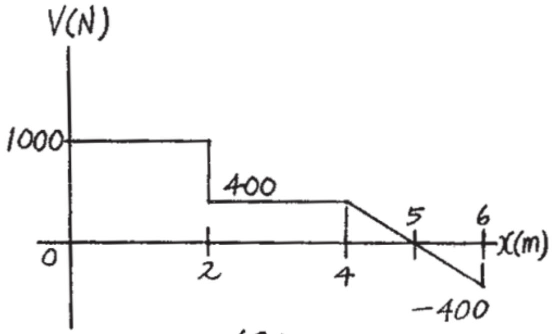
$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - 600(2) - 400(4) = 0$

$M_A = 2800\text{ N}$

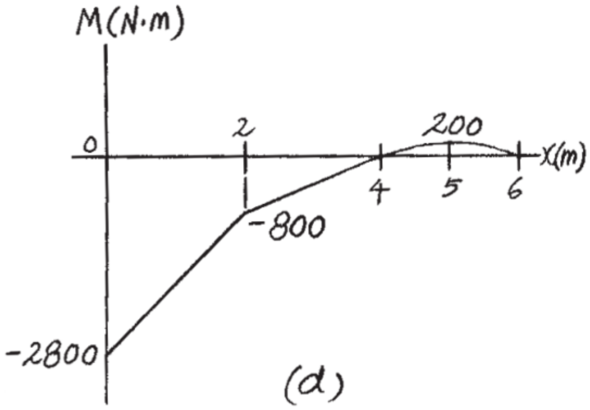


Shear and Moment Diagrams: As shown in Figs. c and d.

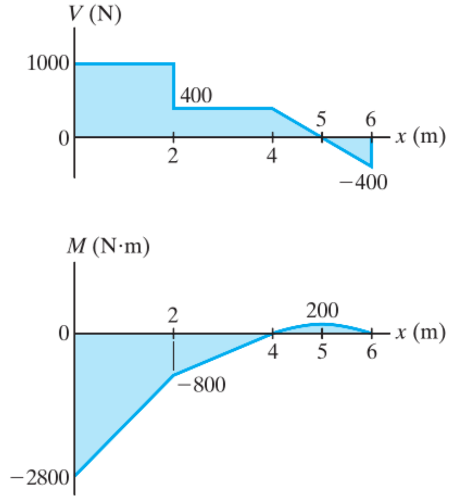
(b)



(c)



Ans:



27. If the built-up beam is subjected to an internal moment of M = 75 kN·m, determine (a) the maximum tensile and compressive stress acting in the beam; (b) the amount of this internal moment resisted by plate *A*.

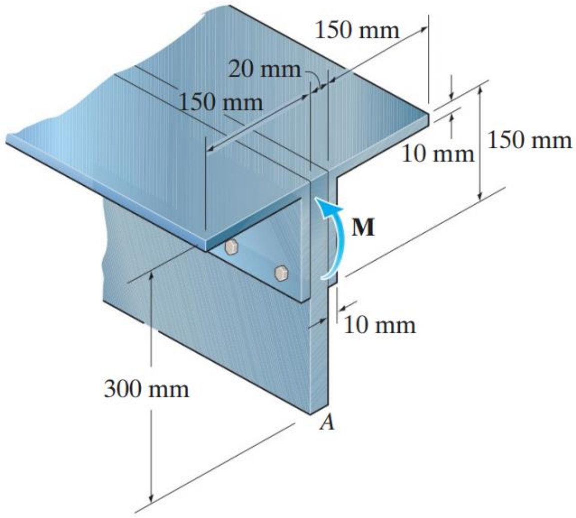


Figure 27

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.15(0.3)(0.02) + 2[0.225(0.15)(0.01)] + 2[0.295(0.01)(0.14)]}{0.3(0.02) + 2(0.15)(0.01) + 2(0.01)(0.14)} = 0.2035 \text{ m}$$

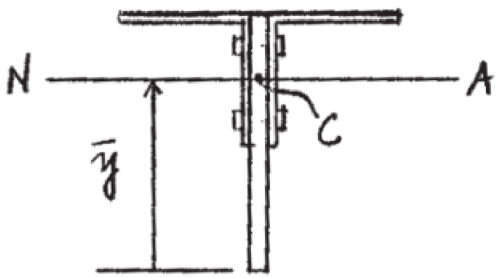
Thus, the moment of inertia of the cross section about the neutral axis is

$$\begin{aligned} I &= \Sigma \bar{I} + Ad^2 \\ &= \frac{1}{12}(0.02)(0.3^3) + 0.02(0.3)(0.2035 - 0.15)^2 \\ &\quad + 2\left[\frac{1}{12}(0.01)(0.15^3) + 0.01(0.15)(0.225 - 0.2035)^2\right] \\ &\quad + 2\left[\frac{1}{12}(0.14)(0.01^3) + 0.14(0.01)(0.295 - 0.2035)^2\right] \\ &= 92.6509(10^{-6}) \text{ m}^4 \end{aligned}$$

Maximum Bending Stress: The maximum compressive and tensile stress occurs at the top and bottom-most fiber of the cross section.

$$(\sigma_{\max})_c = \frac{My}{I} = \frac{75(10^3)(0.3 - 0.2035)}{92.6509(10^{-6})} = 78.1 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\sigma_{\max})_t = \frac{Mc}{I} = \frac{75(10^3)(0.2035)}{92.6509(10^{-6})} = 165 \text{ MPa} \qquad \textbf{Ans.}$$

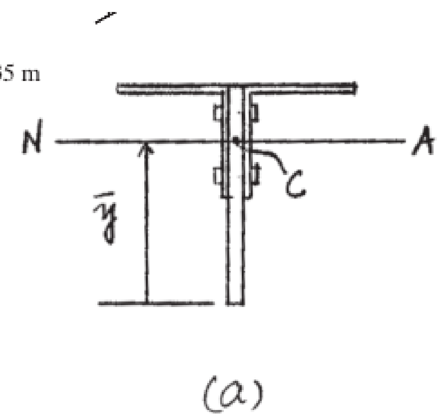


(a)

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{0.15(0.3)(0.02) + 2[0.225(0.15)(0.01)] + 2[0.295(0.01)(0.14)]}{0.3(0.02) + 2(0.15)(0.01) + 2(0.01)(0.14)} = 0.2035 \text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$\begin{aligned} I &= \bar{I} + Ad^2 \\ &= \frac{1}{12}(0.02)(0.3^3) + 0.02(0.3)(0.2035 - 0.15)^2 \\ &+ 2\left[\frac{1}{12}(0.01)(0.15^3) + 0.01(0.15)(0.225 - 0.2035)^2\right] \\ &+ 2\left[\frac{1}{12}(0.14)(0.01^3) + 0.14(0.01)(0.295 - 0.2035)^2\right] \\ &= 92.6509(10^{-6}) \text{ m}^4 \end{aligned}$$



Bending Stress: The distance from the neutral axis to the top and bottom of plate A is $y_t = 0.3 - 0.2035 = 0.0965$ m and $y_b = 0.2035$ m.

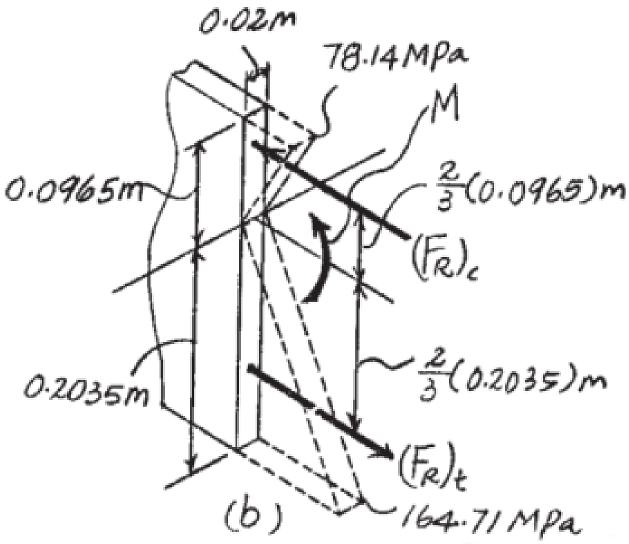
$$\sigma_t = \frac{My_t}{I} = \frac{75(10^3)(0.0965)}{92.6509(10^{-6})} = 78.14 \text{ MPa (C)}$$

$$\sigma_b = \frac{My_b}{I} = \frac{75(10^3)(0.2035)}{92.6509(10^{-6})} = 164.71 \text{ MPa (T)}$$

The bending stress distribution across the cross section of plate A is shown in Fig. b. The resultant forces of the tensile and compressive triangular stress blocks are

$$(F_R)_t = \frac{1}{2}(164.71)(10^6)(0.2035)(0.02) = 335\,144.46 \text{ N}$$

$$(F_R)_c = \frac{1}{2}(78.14)(10^6)(0.0965)(0.02) = 75\,421.50 \text{ N}$$



Thus, the amount of internal moment resisted by plate A is

$$\begin{aligned} M &= 335144.46\left[\frac{2}{3}(0.2035)\right] + 75421.50\left[\frac{2}{3}(0.0965)\right] \\ &= 50315.65 \text{ N}\cdot\text{m} = 50.3 \text{ kN}\cdot\text{m} \end{aligned}$$ **Ans.**

28. A shaft is made of a polymer having an elliptical cross section. If it resists an internal moment of M = 50 N·m, determine the maximum bending stress in the material (a) using the flexure formula, where $I_z = 1/4 \times \pi(0.08 \text{ m})(0.04 \text{ m})^3$, (b) using integration. Sketch a three-dimensional view of the stress distribution acting over the cross-sectional area.

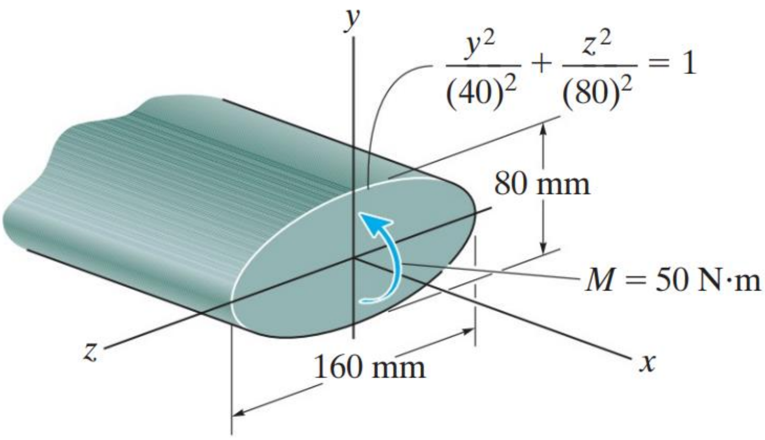


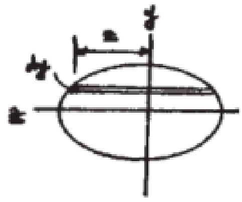
Figure 28

SOLUTION

(a)

$$I = \frac{1}{4}\pi\, ab^3 = \frac{1}{4}\pi(0.08)(0.04)^3 = 4.021238(10^{-6})\ \text{m}^4$$

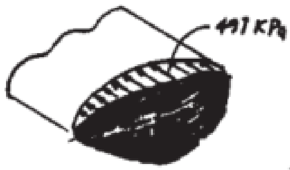
$$\sigma_{\max} = \frac{Mc}{I} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\ \text{kPa} \qquad \textbf{Ans.}$$



(b)

$$M = \frac{\sigma_{\max}}{c}\int_A y^2 dA$$

$$= \frac{\sigma_{\max}}{c}\int y^2 2z\,dy$$



$$z = \sqrt{0.0064 - 4y^2} = 2\sqrt{(0.04)^2 - y^2}$$

$$2\int_{-0.04}^{0.04} y^2 z\,dy = 4\int_{-0.04}^{0.04} y^2\sqrt{(0.04)^2 - y^2}\,dy$$

$$= 4\left[\frac{(0.04)^4}{8}\sin^{-1}\left(\frac{y}{0.04}\right) - \frac{1}{8}y\sqrt{(0.04)^2 - y^2}(0.04^2 - 2y^2)\right]\Bigg|_{-0.04}^{0.04}$$

$$= \frac{(0.04)^4}{2}\sin^{-1}\left(\frac{y}{0.04}\right)\Bigg|_{-0.04}^{0.04}$$

$$= 4.021238(10^{-6})\ \text{m}^4$$

$$\sigma_{\max} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\ \text{kPa} \qquad \textbf{Ans.}$$

29. The member has a square cross section and is subjected to the moment M = 850 N·m as shown. Determine the stress at each corner and sketch the stress distribution. Set $\theta = 30°$.

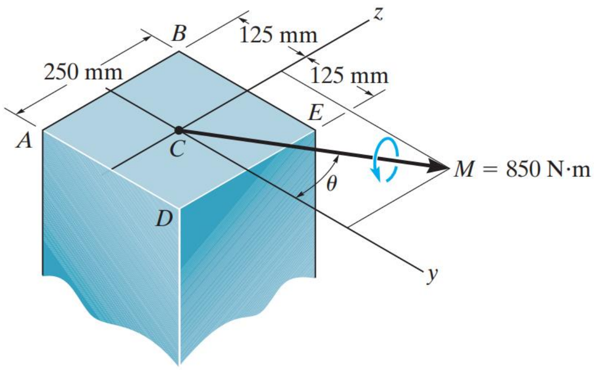


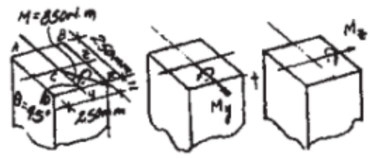
Figure 29

SOLUTION

$M_y = 850 \cos 45° = 601.04 \text{ N} \cdot \text{m}$

$M_z = 850 \sin 45° = 601.04 \text{ N} \cdot \text{m}$

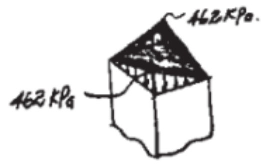
$I_z = I_y = \frac{1}{12}(0.25)(0.25)^3 = 0.3255208(10^{-3}) \text{ m}^4$



$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{601.04(-0.125)}{0.3255208(10^{-3})} + \frac{601.04(-0.125)}{0.3255208(10^{-3})} = 0 \qquad \textbf{Ans.}$$

$$\sigma_B = -\frac{601.04(-0.125)}{0.3255208(10^{-3})} + \frac{601.04(0.125)}{0.3255208(10^{-3})} = 462 \text{ kPa} \qquad \textbf{Ans.}$$



$$\sigma_D = -\frac{601.04(0.125)}{0.3255208(10^{-3})} + \frac{601.04(-0.125)}{0.3255208(10^{-3})} = -462 \text{ kPa} \qquad \textbf{Ans.}$$

$$\sigma_E = -\frac{601.04(0.125)}{0.3255208(10^{-3})} + \frac{601.04(0.125)}{0.3255208(10^{-3})} = 0 \qquad \textbf{Ans.}$$

The negative sign indicates compressive stress.

30. The composite beam is made of steel (A) bonded to brass (B) and has the cross section shown. If it is subjected to a moment of M = 6.5 kN·m, determine the maximum bending stress in the brass and steel. Also, what is the stress in each material at the seam where they are bonded together? E_{br} = 100 GPa, E_{st} = 200 GPa.

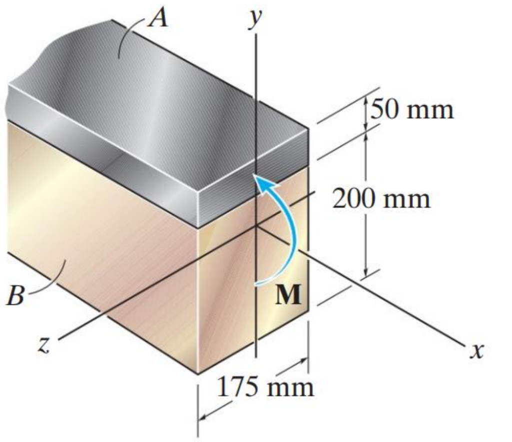
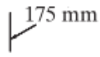
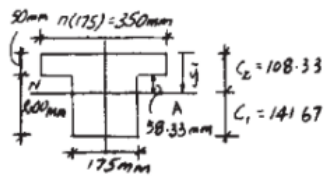


Figure 30



SOLUTION



$$n = \frac{E_{st}}{E_{br}} = \frac{200(10^9)}{100(10^9)} = 2$$

$$\bar{y} = \frac{(350)(50)(25) + (175)(200)(150)}{350(50) + 175(200)} = 108.33\,\text{mm}$$

$$I = \frac{1}{12}(0.35)(0.05^3) + (0.35)(0.05)(0.08333^2) + \frac{1}{12}(0.175)(0.2^3) +$$

$$(0.175)(0.2)(0.04167^2) = 0.3026042(10^{-3})\ \text{m}^4$$

Maximum stress in brass:

$$(\sigma_{br})_{max} = \frac{Mc_1}{I} = \frac{6.5(10^3)(0.14167)}{0.3026042(10^{-3})} = 3.04\ \text{MPa} \qquad \textbf{Ans.}$$

Maximum stress in steel:

$$(\sigma_{st})_{max} = \frac{nMc_2}{I} = \frac{(2)(6.5)(10^3)(0.10833)}{0.3026042(10^{-3})} = 4.65\ \text{MPa} \qquad \textbf{Ans.}$$

Stress at the junction:

$$\sigma_{br} = \frac{M\rho}{I} = \frac{6.5(10^3)(0.05833)}{0.3026042(10^{-3})} = 1.25\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_{st} = n\sigma_{br} = 2(1.25) = 2.51\ \text{MPa} \qquad \textbf{Ans.}$$

31. If the radius of each notch on the plate is r = 10 mm, determine the largest moment M that can be applied. The allowable bending stress is σ_{allow} = 180 MPa.

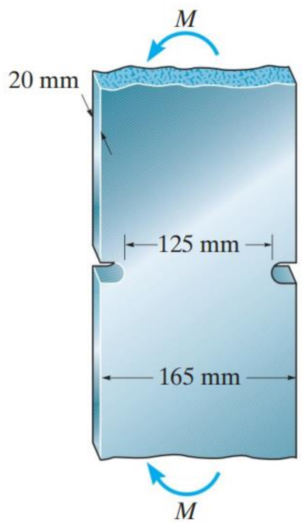


Figure 31

SOLUTION

Stress Concentration Factor: From the graph in the text with $\frac{b}{r} = \frac{20}{10} = 2$ and $\frac{r}{h} = \frac{10}{125} = 0.08$, then $K = 2.1$.

Allowable Bending Stress:

$$\sigma_{\max} = \sigma_{\text{allow}} = K\frac{Mc}{I}$$

$$180\left(10^6\right) = 2.1\left[\frac{M(0.0625)}{\frac{1}{12}(0.02)\left(0.125^3\right)}\right]$$

$$M = 4464\ \text{N}\cdot\text{m} = 4.46\ \text{kN}\cdot\text{m}$$ **Ans.**