Homework 6

Questions No.	1	2	3	Total
Score	30%	30%	40%	100%

$\nabla\times\vec V$

Q6.1 Determine an expression for the vorticity of the flow field described by

$$V = -xy^3\boldsymbol{i} + y^4\boldsymbol{j}$$

Is the flow irrotational?

解: $\nabla\times\vec V = \begin{vmatrix} \vec i & \vec j & \vec k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & 0 \end{vmatrix} = +\vec i\cdot\left(0-\frac{\partial v}{\partial z}\right) - \vec j\left(0-\frac{\partial u}{\partial z}\right) + \vec k\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$

$= \vec k\cdot(0+3xy^2) = 3xy^2\,\vec k$, not irrotational

Q6.2 The velocity potential for a flow is given by

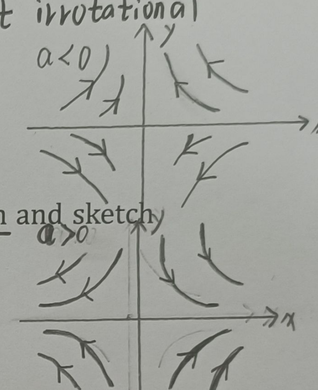
$$\Phi = \frac{a}{2}(x^2 - y^2)$$

Where a is a constant. Determine the corresponding stream function ψ and sketch the flow pattern.

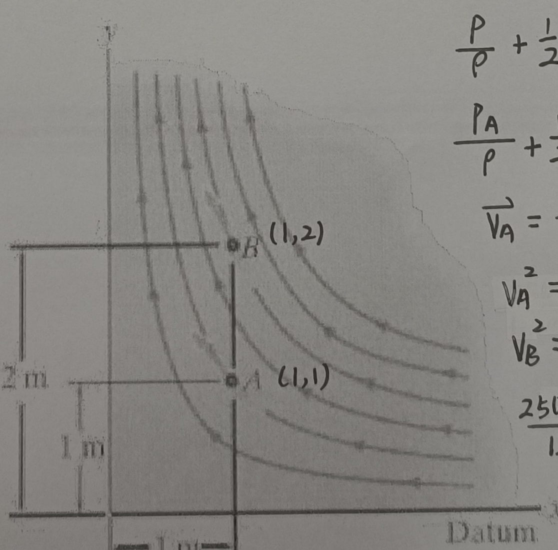
解: $u = \frac{\partial\Phi}{\partial x} = \frac{a}{2}\cdot 2x = ax = \frac{\partial\psi}{\partial y}$

$v = \frac{\partial\Phi}{\partial y} = \frac{a}{2}(0-2y) = -ay = \frac{-\partial\psi}{\partial x}$

$\Rightarrow \psi = axy + C$



Q6.3 The velocity field $\boldsymbol{V} = \{-6x\boldsymbol{i} + 6y\boldsymbol{j}\}\ m/s$ defines the two-dimensional ideal fluid flow in the vertical shown in Fig. Determine the volumetric dilatation rate $\nabla\vec V$ and the rotation of a fluid element located at point B (1m, 2m). If the pressure at point A (1m, 1m) is $250kPa$, what is the pressure at point B? Take $\rho = 1200kg/m^3$.



DDL :5/23/2022

解: volumetric dilatation rate

$\nabla\vec V = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

$= -6 + 6 + 0 = 0$ $\boxed{\text{ANS}}$

$\nabla\times\vec V = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$

$= \vec i\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) - \vec j\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)$

$+ \vec k\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

$= \vec i(0-0) - \vec j(0-0) + \vec k(0-0)$

$= \vec 0$, irrotational.

point B is irrotational $\boxed{\text{ANS}}$

$\vec W = \vec 0$

BE:

$\frac{P}{\rho} + \frac{1}{2}v^2 + gz = C$

$\frac{P_A}{\rho} + \frac{1}{2}V_A^2 + gz_A = \frac{P_B}{\rho} + \frac{1}{2}V_B^2 + gz_B$

$\vec V_A = -6\vec i + 6\vec j$, $\vec V_B = -6\vec i + 12\vec j$

$V_A^2 = 6^2 + 6^2 = 72$

$V_B^2 = 6^2 + 12^2 = 180$

$\frac{250\times10^3}{1200} + \frac{1}{2}\times72 + g = \frac{P_B}{1200} + \frac{180}{2} + 2g$

$P_B = 173.4\ kPa$