



南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Please write carefully and clearly in complete sentences. Your explanations are your only representative when your work is being graded.

1. Find the order of

$$(y')^3 y'' = y^3 e^{\cos t}.$$

2. Since the solution $x(t) = \frac{1}{x-1}$ solves the initial value problem $x' = x^2$, $x(0) = 1$, the interval of existence of the solution is $(-\infty, 0) \cup (0, +\infty)$. True or False?
3. The existence and uniqueness theorem applies to the IVP $ty' - 2y^2 = t^2$ with $y(0) = 1$. True or False?
4. Suppose that $f(t, y)$ is continuous and has continuous partial derivatives. Is it possible that $y_1(t) = \cos t$ and $y_2(t) = 1 - \sin t$ are both solutions to $y' = f(t, y)$ near $t = 0$?
5. Do the solutions e^{3t} and $6e^{3t}$ form a fundamental set of solutions for the ODE $y'' - 9y = 0$?
6. Find a form of a particular solution to $y'' + 2y' + 2y = e^{-2t} \sin t$.
7. Solve the following ODEs:
- (1) $y' + 2ty = 2t$;
 - (2) $\frac{dy}{dx} = \frac{x-y+5}{x-y-2}$;
 - (3) $ydx + (y-x)dy = 0$;
 - (4) $y'' + 2y' - 3y = 0$;
 - (5) $y'' + 12y' + 36y = 0$, $y(0) = 1$, $y'(0) = 7$.
8. Study the autonomous ODE

$$y' = y(y^2 - y - 6)$$

- (1) Find the equilibriums.

- (2) Sketch a phase line.
 - (3) Sketch the equilibrium in the ty plane. These equilibrium solution divide the ty plane into regions. Sketch at least one solution trajectory on each of these regions.
9. A 40-gal tank initially contains 20 gal of pure water. Salt-water solution containing 1 lb of salt for each gallon of water begins entering the tank at a rate of 4 gal/min. Simultaneously, a drain is opened at the bottom of the tank, allowing the salt-water solution to leave the tank at a rate of 2 gal/min.
- (1) Find the salt content $y(t)$ in the tank at time t .
 - (2) When the tank is full, what is the salt content (in pounds)?
10. Find the general solutions of the following ODEs.
- (1) $x^2y' = x^2y^2 + xy + 1$.
 - (2) $y'' + 4y' + 4y = e^{2t} + \cos 2t$.