Quiz 3

Date: 2022-02-28 Name: SID:

Q1. Assume that $y = at + 10$ is a solution of the equation $y' = 5ty + bt^3$, then please give all solutions of it. (3 pts) $a = a(t)$, $b = b(t)$.

Q2. Solve the equation $\frac{dy}{dt} = \frac{t^2 - 2y^2}{2ty}$. (3 pts)

Q3. Solve the equation using the method of variation of parameters $y' - y = 2 + 3t$. (4 pts)

Solution

Q1. $y' - 5ty = 0$

$$y = c \cdot e^{-\int p\,dt} = c \cdot e^{\frac{5}{2}t^2}$$

So $y = c e^{\frac{5}{2}t^2} + a(t) + 10$

$\boxed{\text{C. 通解 + 特解}}$

Q2. $$\frac{dy}{dt} = \frac{x^2 - 2y^2}{2xy} = \frac{1 - 2\left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)}$$

$v = \frac{y}{x}$ $\quad y = vx$

$$v + x\frac{dv}{dx} = \frac{1 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1 - 4v^2}{2v}$$

① $1 - 4v^2 = 0$ $\quad v = \pm\frac{1}{2}$ $\quad y = \pm\frac{1}{2}x$

② $1 - 4v^2 \neq 0$

$$\frac{2v}{1 - 4v^2}dv = \frac{1}{x}dx$$

$$-\frac{1}{4}\ln|1 - 4v^2| = \ln|x| + C_0$$

$$\ln|1-4v^2| = \ln x^{-4} - 4C_0$$

$$1-4v^2 = \frac{C}{x^4} \qquad C = e^{-4C_0}$$

$$x^2 - 4y^2 = \frac{C}{x^2}$$

Q3. $y' - y = 2 + 3t$

$y' - y = 0 \qquad y = Ce^{t}$

$y = C(t)e^{t}$

$C'(t)e^{t} = 2 + 3t$

$C'(t) = 2e^{-t} + 3te^{-t}$

$C(t) = -2e^{-t} - 3te^{-t} - 3e^{-t} + C$

$y = -2 - 3t - 3 + Ce^{t}$

$= -3t - 5 + C\cdot e^{t}$

Quiz 3

Date: 2022-03-04 Name: SID:

Q1. Assume that $y = a(t) + 10$ is a solution of the equation $y' = 5ty + b(t)t^3$, then please give all solutions of it. (3 pts)

Q2. Solve the equation $\frac{dy}{dx} = \frac{xy - 2y^2}{2x^2}$. (3 pts)

Q3. Solve the equation using the method of variation of parameters $y' - y = 1 + 5t$. (4 pts)

Q1. $y' - 5ty = 0$

$$y = c \cdot e^{-\int p\, dt} = c \cdot e^{\frac{5}{2}t^2}$$

$$y = c\, e^{\frac{5}{2}t^2} + a(t) + 10$$

Q2.

$$\frac{dy}{dx} = \frac{xy - 2y^2}{2x^2} = \frac{\left(\frac{y}{x}\right) - 2\left(\frac{y}{x}\right)^2}{2}$$

Let $v = \frac{y}{x}$. $y = vx$.

$$v + x\frac{dv}{dx} = \frac{1}{2}v - v^2$$

$$x\frac{dv}{dx} = -\frac{1}{2}v - v^2$$

① $-\frac{1}{2}v - v^2 = 0 \Rightarrow v = 0$ or $v = -\frac{1}{2}$

$y = 0$ or $y = -\frac{1}{2}x$

②
$$\frac{dv}{\left(-\frac{1}{2} - v\right)v} = \frac{dx}{x}$$

$$\left(\frac{1}{v} - \frac{1}{\frac{1}{2}+v}\right) dv = -\frac{1}{2}\frac{dx}{x}$$

$$\frac{v^2}{\left(\frac{1}{2}+v\right)^2} = \frac{C}{x} \qquad \frac{y^2}{\frac{1}{2}x^2 + y^2} = \frac{C}{x}$$

Q3.

$$y' - y = 1 + 5t$$

$$y' - y = 0 \qquad y = ce^{t}$$

$$y = C(t)e^{t}$$

$$C'(t)e^{t} = 1 + 5t$$

$$C'(t) = e^{-t} + 5te^{-t}$$

$$C(t) = -e^{-t} - 5te^{-t} - 5e^{-t} + C$$

$$y = -1 - 5t - 5 + ce^{t}$$

$$= -5t - 6 + c \cdot e^{t}$$