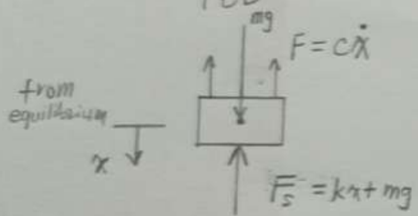
Homework

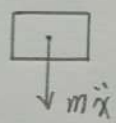
Two identical dashpots are arranged parallel to each other, as shown. Show that if the damping coefficient $c < \sqrt{mk}$, then the block of mass m will vibrate as an underdamped system.

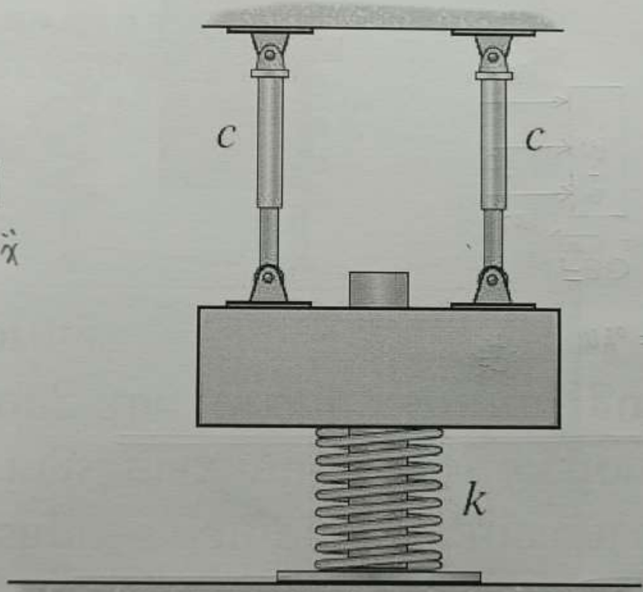
解:

FBD



KD





$$+\ mg - 2c\dot{x} - (kx + mg) = m\ddot{x}$$

$$m\ddot{x} + 2c\dot{x} + kx = 0$$

$$C_c = 2m\,w_n = 2m\sqrt{k/m}$$

$$\xi = \frac{2c}{C_c} = \frac{2c}{2m\sqrt{k/m}} = \frac{c}{\sqrt{mk}}$$

if $c < \sqrt{mk}$, $\xi = \frac{c}{\sqrt{mk}} < 1$

∴ it's underdamped system.

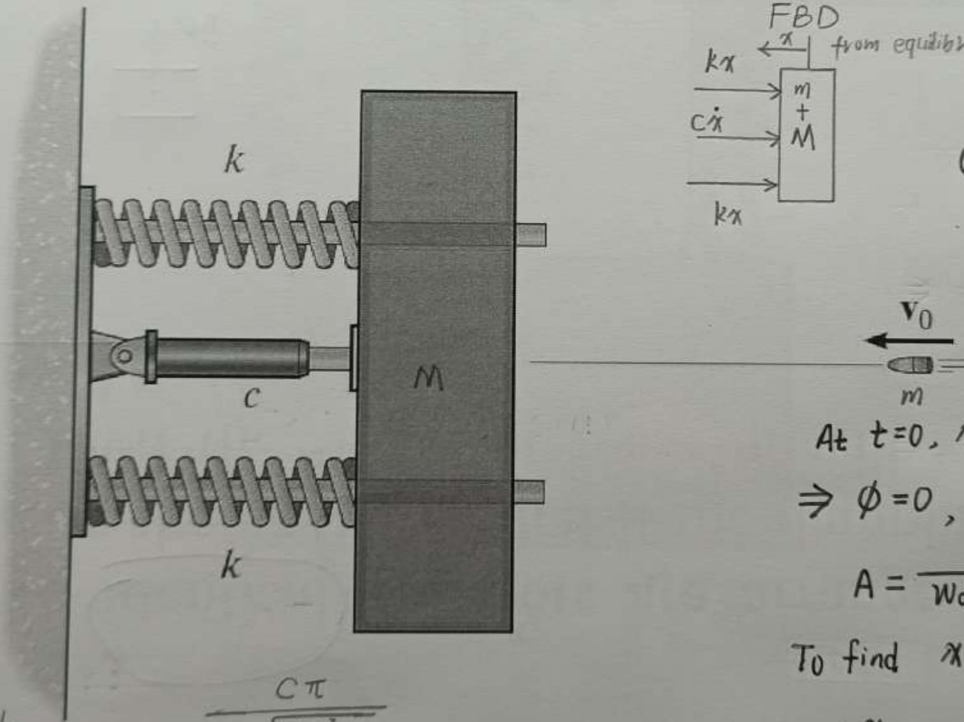
Homework

A bullet of mass m has a velocity $\mathbf{v}_0$ just before it strikes the target of mass M. If the bullet embeds in the target, and the dashpot's damping coefficient is $0 < c << c_c$, determine the springs' maximum compression. The target is free to move along the two horizontal guides that are "nested" in the springs.

$\Rightarrow$ under damped

$$\xi = \frac{c}{2(M+m)\omega_n}$$

解: $mV_0 = (M+m)V_i$



$$-(kx\cdot 2 + c\dot{x}) = (m+M)\ddot{x}$$

$$(M+m)\ddot{x} + c\dot{x} + 2kx = 0$$

$0 < c << c_c$: under damped

$$x(t) = Ae^{-\xi\omega_n t}\sin(\omega_d t + \phi)$$

$$\dot{x}(t) = Ae^{-\xi\omega_n t}\left[\omega_d\cos(\omega_d t+\phi) - \xi\omega_n\sin(\omega_d t+\phi)\right]$$

At $t=0$, $x(0)=0$, $\dot{x}(0) = V_i = \frac{m}{M+m}V_0$

$\Rightarrow \phi = 0$,

$$A = \frac{mV_0}{\omega_d(M+m)}$$

To find x_{max}, $\sin(\omega_d t) = 1 \Rightarrow t = \frac{\pi}{\omega_d 2}$

$$\therefore x_{max} = A\cdot e^{-\xi\omega_n\cdot\frac{\pi}{2\omega_d}}$$

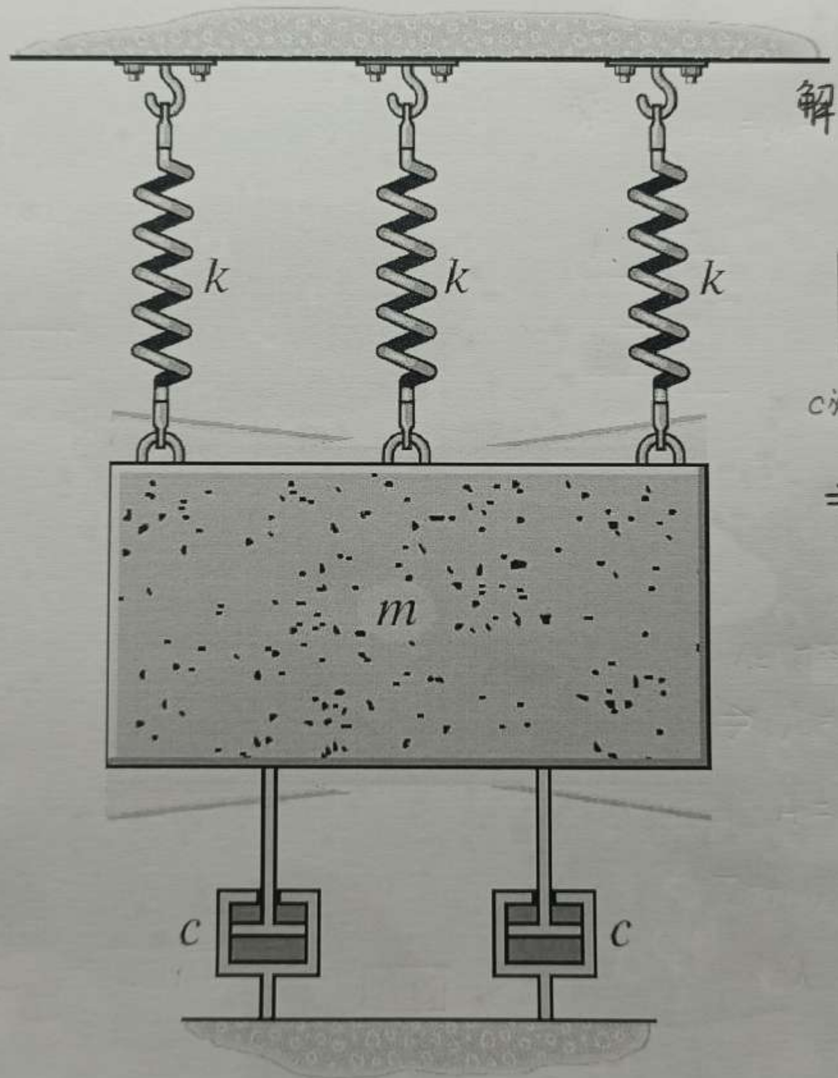
$$= \frac{mV_0}{\omega_d(M+m)}e^{-\frac{c\pi}{4(M+m)\omega_d}} \quad \boxed{\text{ANS}}$$

where $\omega_d = \omega_n\sqrt{1-\xi^2} = \sqrt{\frac{16k^2-c^2}{(M+m)^2\cdot 4}} = \frac{\sqrt{16k^2-c^2}}{2(M+m)}$

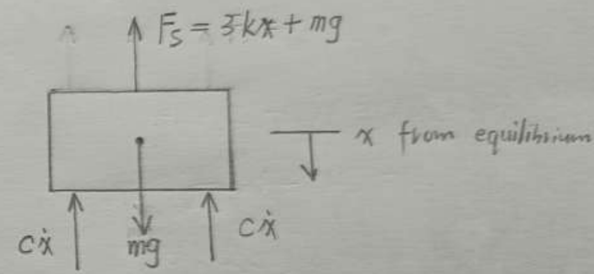
$$\frac{2mV_0}{\cancel{(M+m)\times}\sqrt{16k^2-c^2}}\cdot e^{-\frac{c\pi}{2\sqrt{16k^2-c^2}}} \quad \boxed{\text{ANS}}$$

Homework

Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs? Take $k = 100\ \mathrm{N/m}$, $c = 200\ \mathrm{N \cdot s/m}$, $m = 25\ \mathrm{kg}$.



解: 3k 等效于 1×3k



$\Rightarrow +mg - (3kx + mg) - 2c\dot{x} = m\ddot{x}$

$m\ddot{x} + 2c\dot{x} + 3kx = 0$

$25\ddot{x} + 400\dot{x} + 300x = 0$

$\ddot{x} + 16\dot{x} + 12x = 0$ $\boxed{\text{ANS}}$

$C_c = 2 \times 1 \times \sqrt{12/1} = 4\sqrt{3}$

$\xi = \frac{c}{C_c} = \frac{16}{4\sqrt{3}} = \frac{4}{\sqrt{3}} > 1$

it's over damped.