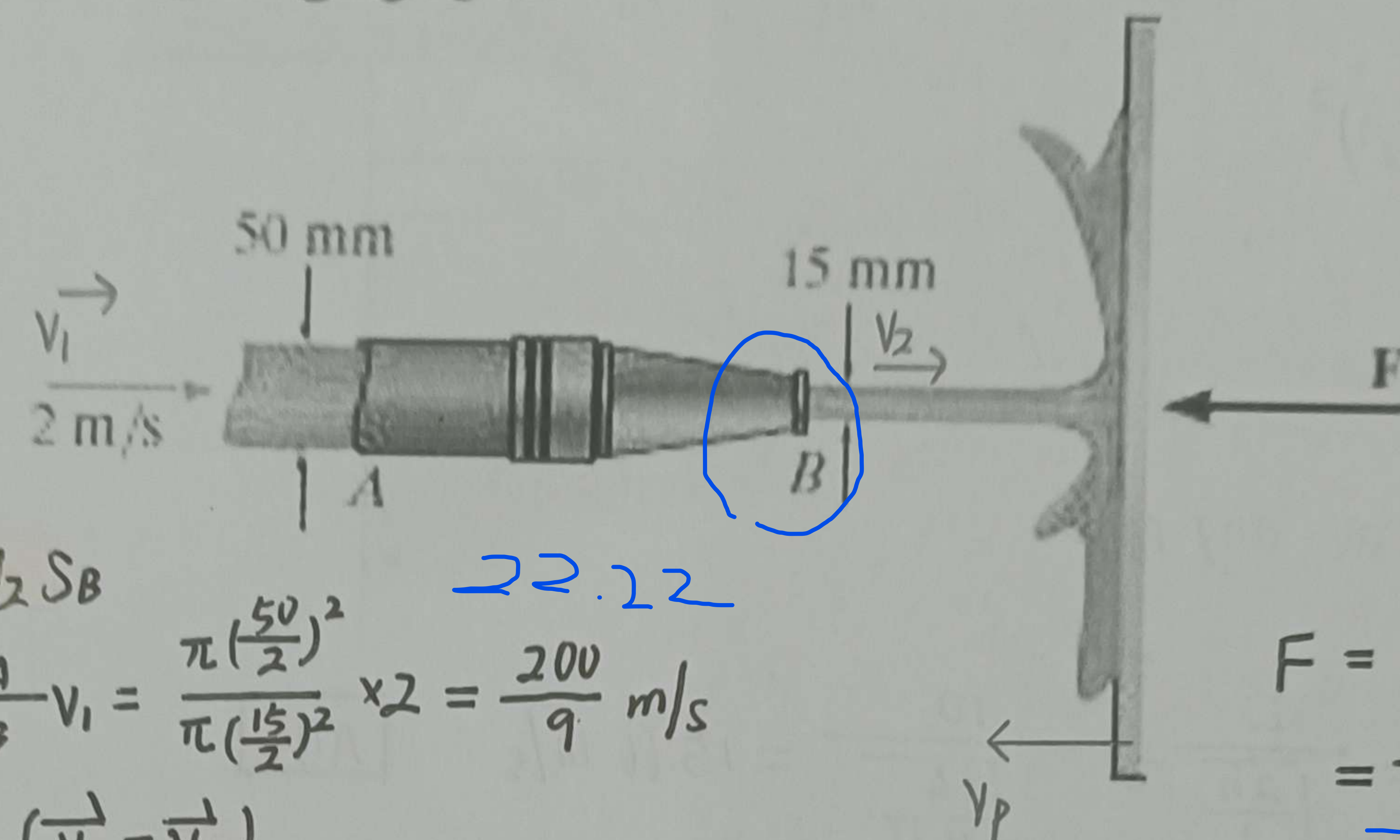
Homework 4

Questions No.	1	2	3	4	Total
Score	20%	20%	30%	30%	100%

Q4.1 Water flows through the hose with a velocity of $2m/s$. Determine the force **F** needed to keep the circular plate moving to the left at $2m/s$.



解: $V_1 S_A = V_2 S_B$

$\Rightarrow V_2 = \frac{S_A}{S_B} V_1 = \frac{\pi(\frac{50}{2})^2}{\pi(\frac{15}{2})^2} \times 2 = \frac{200}{9}\ m/s$ 22.22

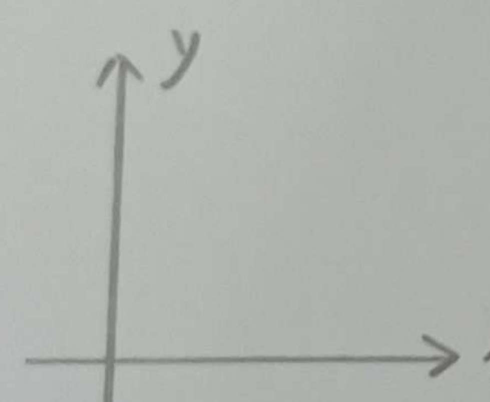
$\Sigma \vec{F} = \dot{m}(\vec{V_p} - \vec{V_2})$

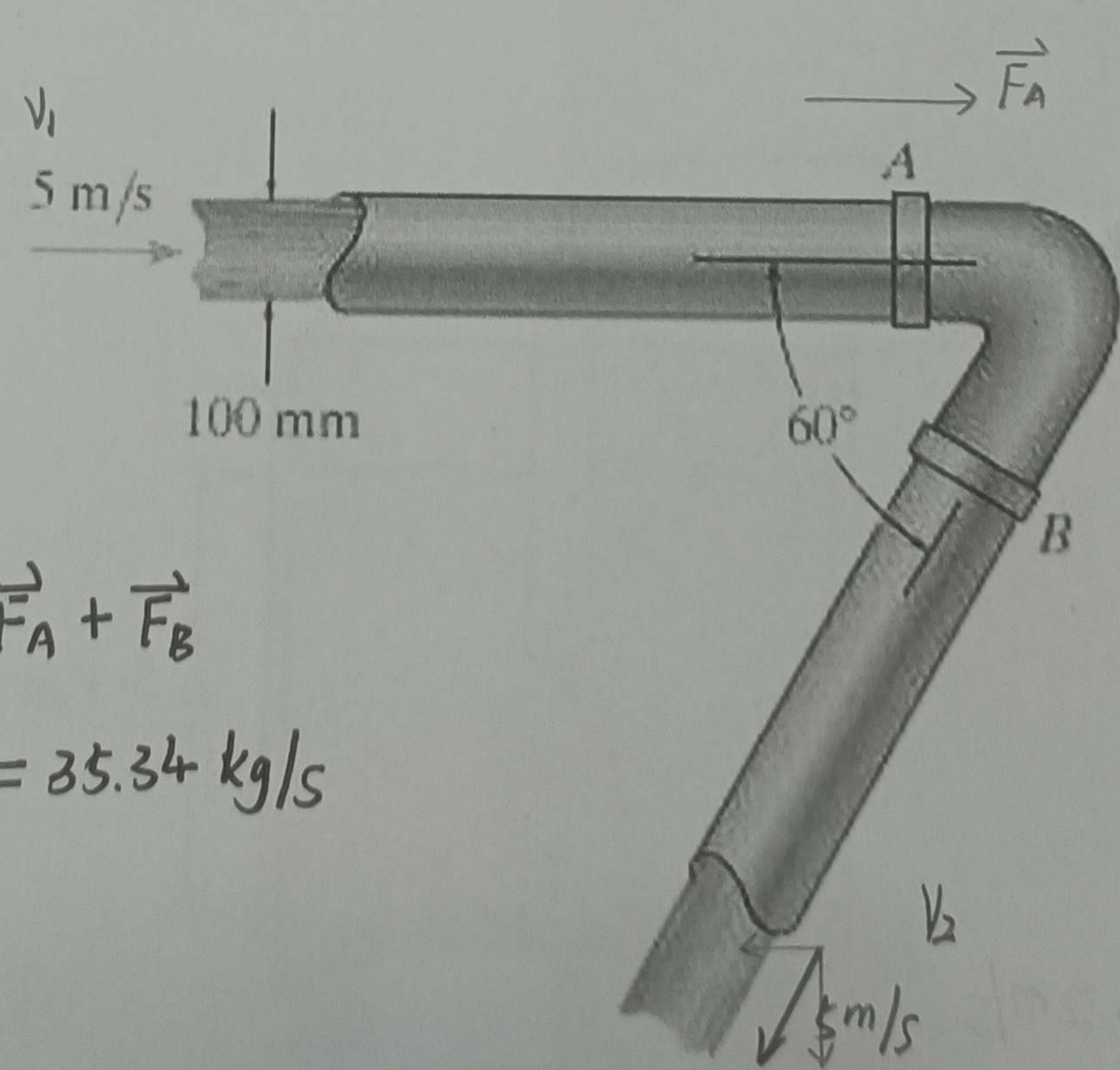
$\dot{m} = \rho V_1 S_A = 1000\ kg/m^3 \times 2m/s \times \pi(\frac{50}{2}\times10^{-3})^2 m^2 = \frac{5}{4}\pi\ kg/s$

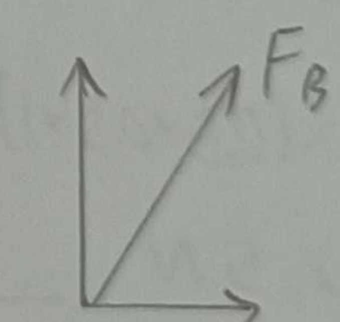
$F = \dot{m}(V_p + V_2)$
$= \frac{5}{4}\pi \times (2 + \frac{200}{9})$
$= 95.12\ N \leftarrow$

Q4.2 Oil flows through the 100-mm-diameter pipe with a velocity of $5m/s$. If the pressure in the pipe at A and B is $80kPa$, determine the x and y components of force the flow exerts on the elbow. The flow occurs in the horizontal plane. Take $\rho_0 = 900kg/m^3$.

解:







$\Sigma \vec{F} = \dot{m}(\vec{V_2} - \vec{V_1}) = \vec{F} + \vec{F_A} + \vec{F_B}$

$\dot{m} = \rho_0 V_1 A = 900 \times 5 \times \pi \times (50\times10^{-3})^2 = 35.34\ kg/s$

$\vec{V_1} = 5\vec{i} + 0\vec{j}$

$\vec{V_2} = -2.5\vec{i} - 2.5\sqrt{3}\vec{j}$

$\vec{F_A} = P_A A \vec{i} = 80\times10^3 \times \pi(50\times10^{-3})^2 \vec{i} = 200\pi\ \vec{i}\ N$

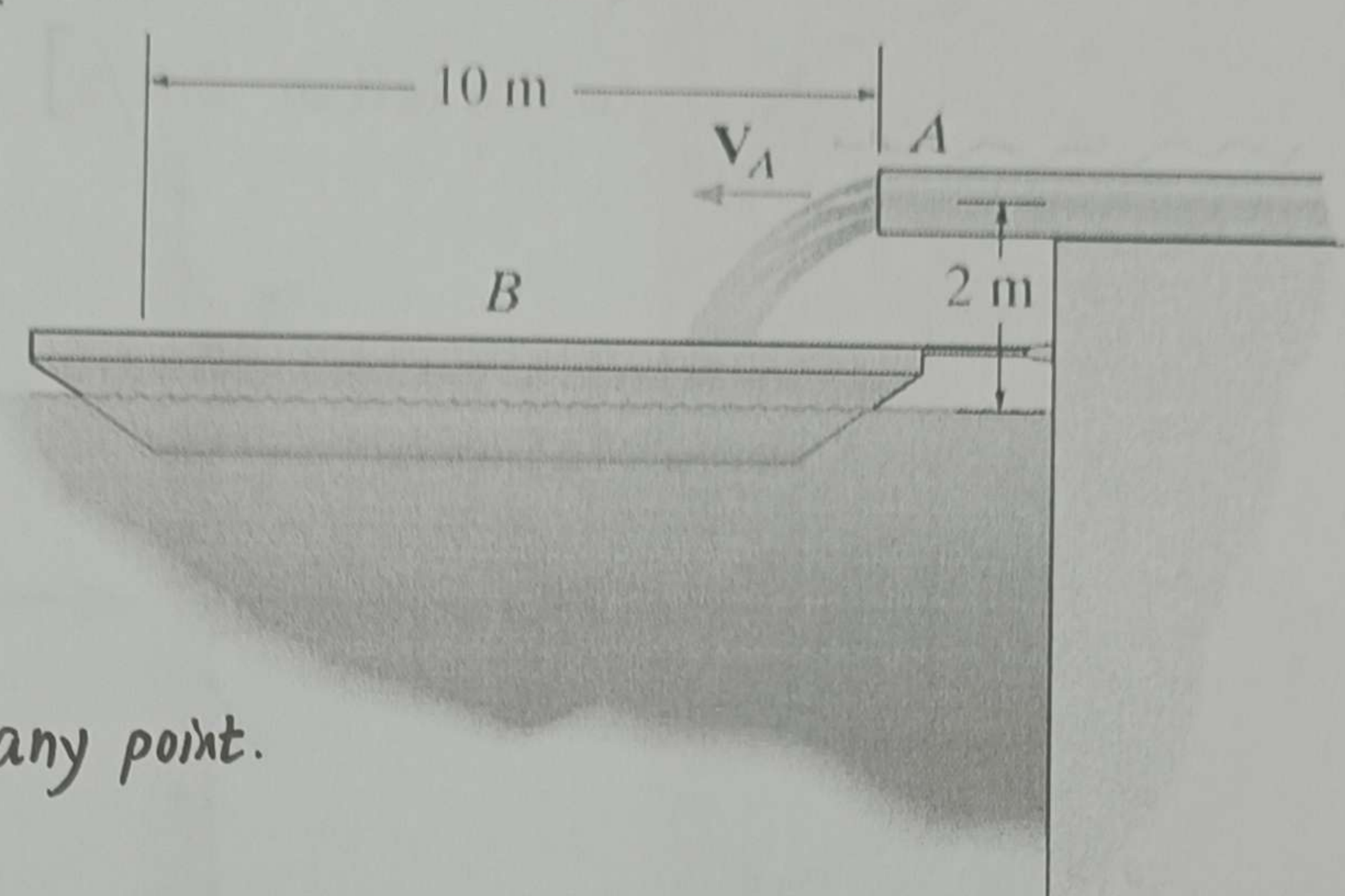
$\vec{F_B} = +200\pi \times \frac{1}{2}\vec{i} + 200\pi \times \frac{\sqrt{3}}{2}\vec{j} = +100\pi\vec{i} + 100\pi\sqrt{3}\vec{j}\ N$

$\Rightarrow \vec{F} = 35.34 \times (-7.5\vec{i} - 2.5\sqrt{3}\vec{j}) - 200\pi\vec{i} - 100\pi\vec{i} - 100\pi\sqrt{3}\vec{j} = -1208\vec{i} + (-697.2)\vec{j}$

由牛三，$F_x' = 1208\ N$，$F_y' = 697.2\ N$.

Q4.3 The barge is being loaded with an industrial waste liquid having a density of $1200\ kg/m^3$. Determine the maximum force in the tie rope needed to hold the barge stationary. The waste can enter the barge at any point within the 10-m region. Also, what is the speed of the waste exiting the pipe at A when this occurs? The pipe has a diameter of 100mm.

解: consider horizental direction.



$$\Sigma \vec{F} = \dot{m}(\vec{V_2} - \vec{V_1})$$

$$\dot{m} = \rho V_A S = 1200\, V_A \cdot \pi\,(50\times10^{-3})^2$$
$$= 3\pi \cdot V_A \quad kg/s$$

$$F = \dot{m}\cdot V_A = 3\pi V_A^2$$

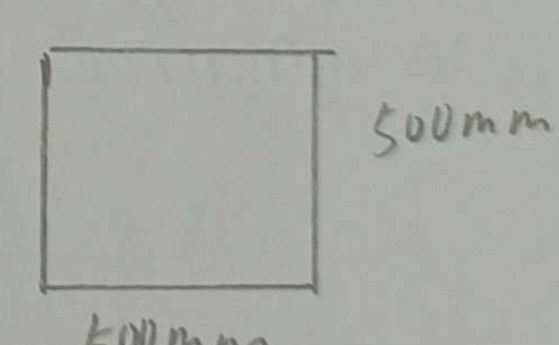
As the waste can enter at any point during the 10m region.

$$\frac{1}{2}gt^2 = h \qquad V_{A\max}\cdot t = L$$

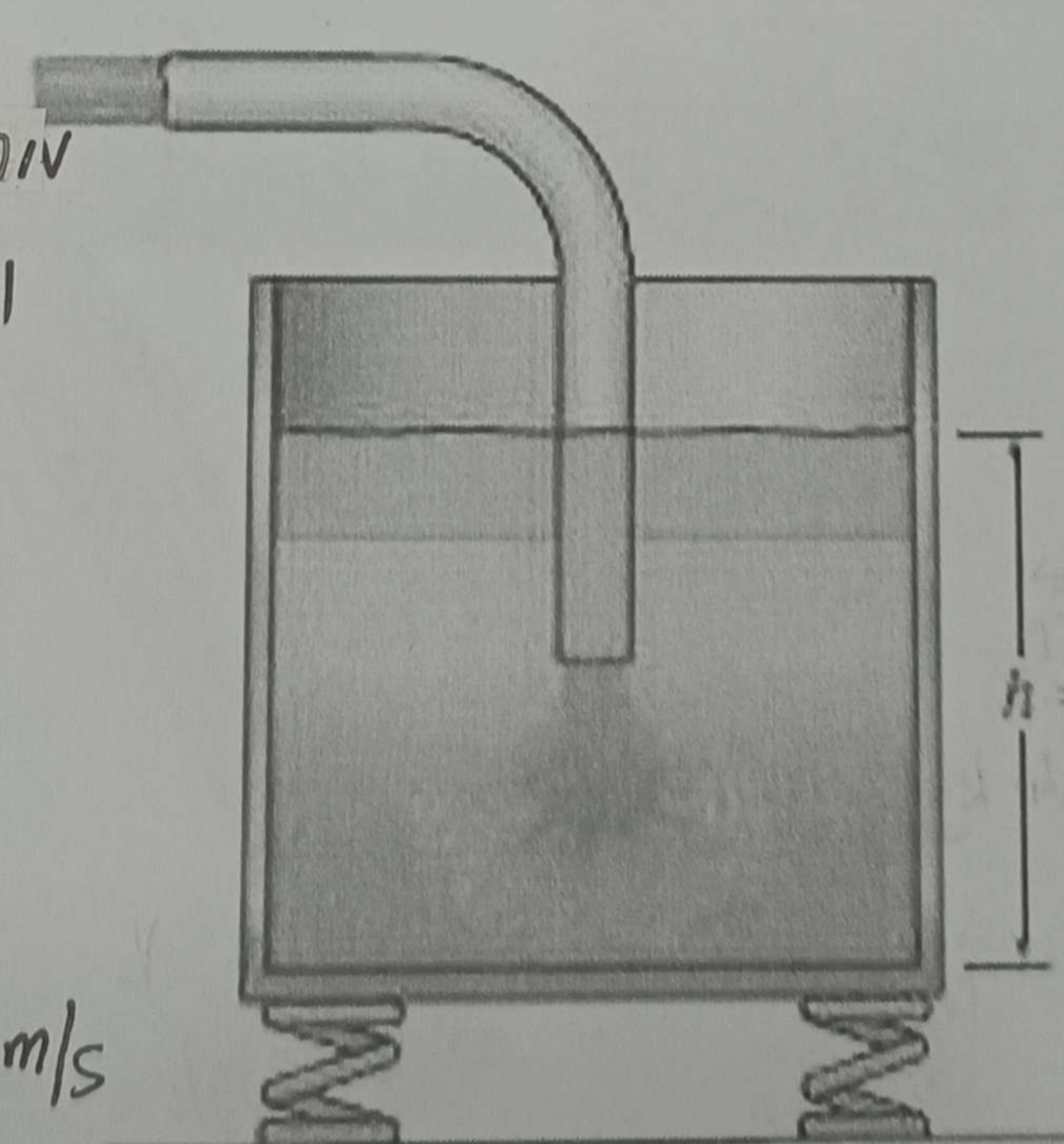
$$\Rightarrow V_{A.max} = \frac{L}{\sqrt{\frac{2h}{g}}} = \frac{10}{\sqrt{\frac{4}{9.81}}} = 15.66\ m/s \quad \boxed{ANS}$$

$$F = 3\pi V_A^2 = 2311\ N. \quad \boxed{ANS}$$

Q4.4 Water flows into the tank at the rate of $0.05m^3/s$ from the 100-mm-diameter pipe. If the tank is $500mm$ on each side, determine the compression in each of the four springs that support its corners when the water reaches a depth of $h = 1m$. Each spring has a stiffness of $k = 8kN/m$. When empty, the tank compresses each spring $30mm$.



解:



$$G_{tank} = 4\Delta x_1 \cdot k = 120\times10^{-3}\times 8\times10^3 = 960N$$

$$G_{water} = \rho V g = 1000\times(0.5\times0.5\times1)\times9.81$$
$$= 2452.5\,N$$

the water flow.

$$\Sigma \vec{F} = \dot{m}(\vec{V_2} - \vec{V_1})$$

$$\dot{m} = \rho \dot{\forall} = 1000\times0.05 = 50\,kg/s$$

$$V_2 = \frac{V_1\cdot A_1}{A_2} = \frac{6.366\times\pi(50\times10^{-3})^2}{(500\times10^{-3})^2} = 0.2\,m/s$$

$$V_1 = \frac{\dot{\forall}}{S} = \frac{0.05}{\pi\times(50\times10^{-3})^2} = 6.366\ m/s$$

DDL 2021/4/29

$$F = 50\times(6.366+0.2) = 328.3\,N$$

$$\Delta x_{total} = \frac{F + G_{water} + G_{tank}}{4k} = \frac{328.3+2452.5+960}{8000\times4} = 116.9\ mm$$