3 Assignment 3

Question No.	1	2	3	4	5	Total
Score	24%	16%	20%	22%	18%	100%

Q 3.1. The sound field radiated by a wave packet can be described by the following inhomogeneous wave equation:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right) p' = Ae^{-\sigma x_1^2}\delta(x_2)\delta(x_3)e^{i(\omega t-\alpha x_1)}$$

where c is the speed of sound, x_1, x_2 and x_3 represent the three-dimensional cartesian coordinates, t represents time, A is a dimensional amplitude, and σ and α are positive constants.

(i) Show that the far-field sound pressure radiated by the wave packet is

$$\frac{A}{4c^2x\sqrt{\pi\sigma}}e^{i(\omega t-kx)}e^{-(k\cos\theta-\alpha)^2/(4\sigma)}$$

where $k = \omega/c$, $x = |\boldsymbol{x}|$, and θ is the polar angle between the observer and the x_1 axis. [10%]

Hint 1: For $x \gg y$, $|\boldsymbol{x}-\boldsymbol{y}| = x(1-\boldsymbol{x}\cdot\boldsymbol{y}/x^2+\cdots)$

Hint 2: $\displaystyle\int_{-\infty}^{\infty} e^{-\sigma y^2}e^{i\gamma y}\,\mathrm{d}y = \sqrt{\frac{\pi}{\sigma}}\,e^{-\gamma^2/(4\sigma)}$

(ii) What is the phase speed of the wave packet (NOT the sound wave)? [4%]

Hint 3: Phase speed $v_p = \mathrm{d}x/\mathrm{d}t$ when the phase $\omega t - \beta x$ = constant, where β is a phase constant.

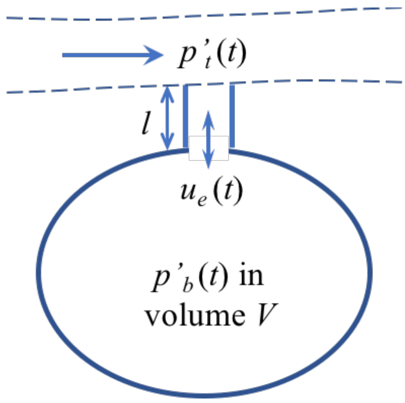
(iii) If $\sigma \ll 1$, show that the phase speed of the wave packet must be supersonic in order for it to radiate sound, and determine the polar angle θ of the sound radiated. [6%]

Total: [20%]

Q 3.2. A turbulent jet of flow speed U blows across the neck of a Helmholtz resonator (neck cross-sectional area S and length l, which connects to an otherwise closed small cavity of volume V). If the rms pressure perturbation at the neck opening is $\frac{1}{2}k\rho_0U^2$, show that the sound power radiated from the neck scales as

$$\rho_0U^3M\frac{S^2}{l^2}\left(1-\frac{SD^2}{\alpha^2M^2Vl}\right)^{-2}$$

where α is a non-dimensional number such that $\alpha U/D$ is the characteristic frequency of the turbulent fluctuations in the jet.



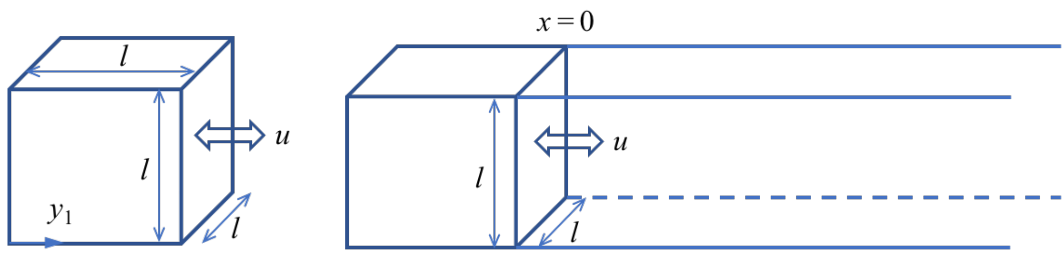
Total: [22%]

Q 3.3. A square piston forming one side of an otherwise rigid, fixed cube of side l vibrates with a small amplitude ε and radian frequency ω ($kl \ll 1$), where $k = \omega/c$ and c is the sound speed. Determine the radiated acoustic pressure at a distance r ($kr \gg 1$) from the cube when it is positioned in

(i) an unbounded fluid, [10%]

(ii) an infinitely long tube with square cross-section of side l.

In (ii) the cube is oriented so that the vibration of the position is parallel to the walls of the tube. Show that the power radiated in case (ii) is larger than in case (i) by a factor $4\pi(kl)^{-2}$. [12%]



Total: [22%]

Q 3.4. For a flame with the rate of heat addition per unit volume $w(\boldsymbol{x}, t)$, the wave equation is given by

rho 与 p 关系的成立？

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\rho'(\boldsymbol{x}, t) = \frac{\gamma - 1}{c^2}\frac{\partial w}{\partial t}$$

(i) By assuming that the flame source distribution is spatially compact, show that the sound radiated in the acoustic far field scales as

$$\rho' \sim \rho_0 \left(\frac{l}{x}\right) M^2$$

where ρ_0 is the ambient density, x is the distance from the source to the observer, l is the length scale of the flame, $M = u/c$, u is the velocity scale of the flame front and c represents the speed of sound. [13%]

Hint:

$$\frac{\gamma-1}{c^2}\frac{\partial}{\partial t}\int w\left(\boldsymbol{y}, t-\frac{x}{c}\right)\mathrm{d}^3\boldsymbol{y} = \rho_0\frac{\partial^2}{\partial t^2}\int \beta\left(\boldsymbol{y}, t-\frac{x}{c}\right)\mathrm{d}^3\boldsymbol{y}$$

where β is the fractional growth of flame volume per unit volume.

(ii) Use this result to find the scaling of the far-field acoustic power radiated by the source. [7%]

Total: [20%]

Q 3.5. A small loudspeaker producing a rate of volume outflow $Q(t)$ is at a position described by cylindrical co-ordinates (r_s, ϕ_s, z_s) within a wedge-shaped region bounded by rigid surface $\phi = 0$ and $\phi = \pi/3$, $0 \leqslant \phi_s \leqslant \pi/3$. Determine the location of all necessary images of the loudspeaker, and write down an expression for the pressure perturbation at position (r, ϕ, z). [9%]

How much enhancement is acoustic power output relative to that expected in an unbounded region is obtained in the limit of small r_s? Explain why the power output of the source increases. [7%]

Total: [16%]