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MAE-308 Heat transfer

DDL：4.12

1. Obtain relations for the characteristic lengths of a large plane wall of thickness $2L$, a very long cylinder of radius r_o, and a sphere of radius r_o when considering lumped system analysis.

1. 解：plane wall： $l_c = \frac{V}{A_s} = \frac{2L \cdot A}{2 \cdot A} = L$ $\boxed{\text{ANS}}$

cylinder： $l_c = \frac{V}{A_s} = \frac{\pi r_o^2 \cdot L}{2\pi r_o \cdot L} = \frac{r_o}{2}$ $\boxed{\text{ANS}}$ sphere $= \frac{V}{A_s} = \frac{\frac{4}{3}\pi r_o^3}{4\pi r_o^2} = \frac{1}{3} r_o$ $\boxed{\text{ANS}}$

2. Steel rods (ρ = 7832 kg/m3, c_p = 434 J/kg·K, and k = 63.9 W/m·K) are heated in a furnace to 850°C and then quenched in a water bath at 50°C for a period of 40 seconds as part of a hardening process. The convection heat transfer coefficient is 650 W/m²·K. If the steel rods have diameter of 40 mm and length of 2 m, determine their average temperature when they are taken out of the water bath.

$k = \lambda$, h, $d = 40mm$, $r = 20mm = 20 \times 10^{-3} m = 0.02 m$, $L = 2m$

2. 解：

$B_i = \frac{h l_c}{\lambda} = \frac{650 \times \frac{0.02}{2}}{63.9} = 0.1017$

Ask for average T considering lumped system analysis.

$b = \frac{hA_s}{\rho V c_p} = \frac{h}{\rho l_c \cdot c_p} = \frac{650}{7832 \times 0.01 \times 434} = 0.01912$

$T_f = T_\infty + (T_i - T_\infty) e^{-bt} = 50 + (850-50) \cdot e^{-0.01912 \times 40} = 422.34°C$ $\boxed{\text{ANS}}$

3. In an experiment, the temperature of a hot gas stream is to be measured by a thermocouple with a spherical junction. Due to the nature of this experiment, the response time of the thermocouple to register 99 percent of the initial temperature difference must be within 5 s. The properties of the thermocouple junction are k = 35 W/m·K, ρ = 8500 kg/m³, and c_p = 320 J/kg·K. If the heat transfer coefficient between the thermocouple junction and the gas is 250 W/m2·K, determine the diameter of the junction.

λ, $h = 250 W/m^2 \cdot k$

3. 解：

Considering lumped system analysis

$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p} t}$

99 percent

$\Rightarrow \frac{T(5) - T_\infty}{T_i - T_\infty} = 0.01$

$\frac{hA_s}{\rho V c_p} t = \ln 100$

$\frac{h \cdot 4\pi r^2}{\rho \cdot \frac{4}{3}\pi r^3 \cdot c_p} \cdot t = \ln 100$

$r = \frac{3ht}{\rho c_p \cdot \ln 100} = \frac{3 \times 250 \times 5}{8500 \times 320 \times \ln 100} = 2.9938 \times 10^{-4} m$

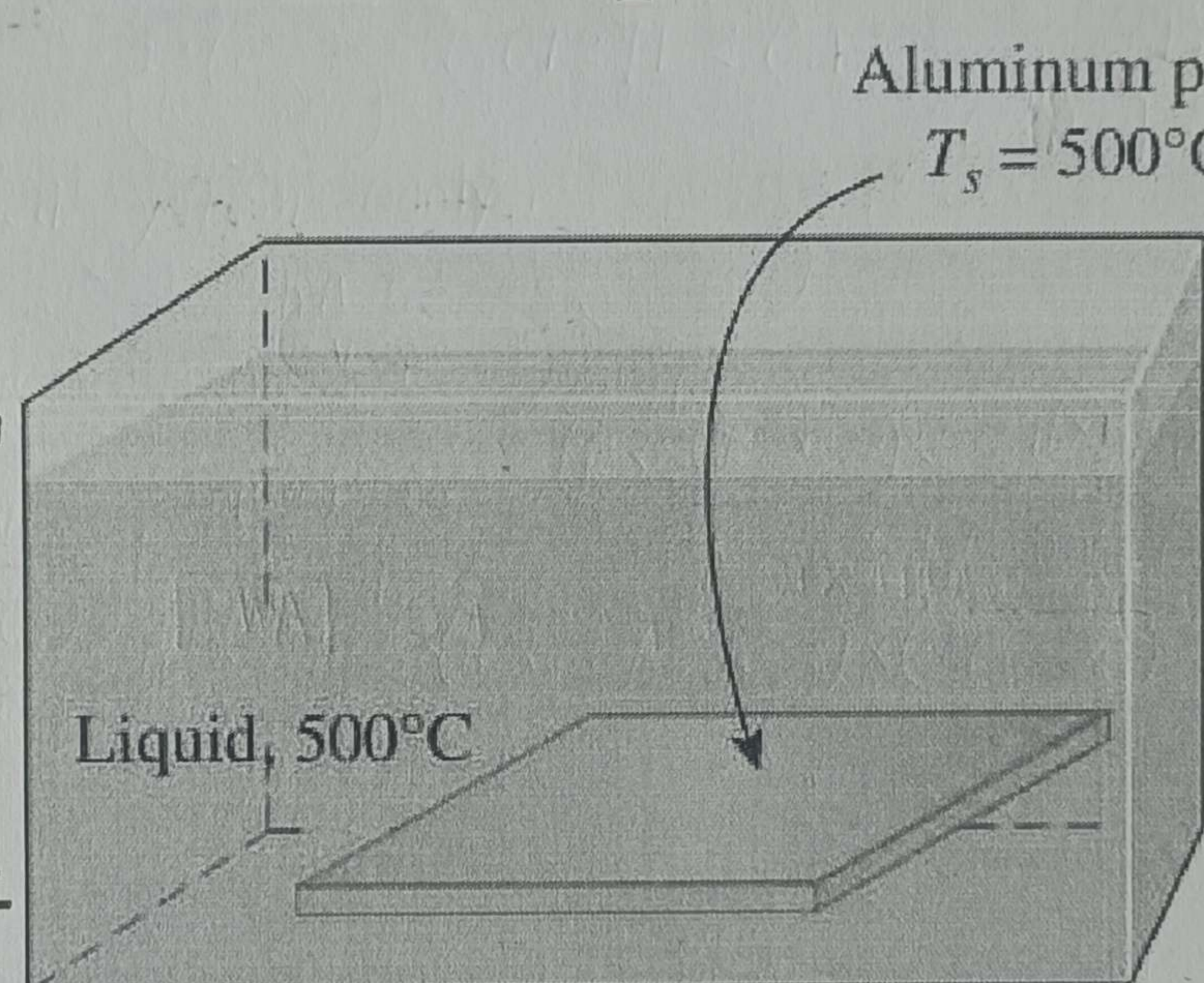
$d = 5.9875 \times 10^{-4} m = 0.59875 mm$ $\boxed{\text{ANS}}$

$B_i = \frac{h \cdot l_c}{\lambda} = \frac{250 \times \frac{1}{3} \times 2.9938 \times 10^{-4}}{35} = 7.1281 \times 10^{-4} < 0.1$, this analysis is reasonable.

4. A 10-cm thick aluminum plate (ρ = 2702 kg/m3, c_p = 903 J/kg·K, k = 237 W/m·K, and $\alpha = 97.1 \times 10^{-6}$ m²/s) is being heated in liquid with temperature of 500°C. The aluminum plate has a uniform initial temperature of 25°C. If the surface temperature of the aluminum plate is approximately the liquid temperature, determine the temperature at the center plane of the aluminum plate after 15 seconds of heating. Solve this problem using analytical one-term approximation method (not the Heisler charts).

4.解: $\tau = \frac{\alpha t}{L^2} = \frac{97.1\times10^{-6}\times 15}{(5\times10^{-2})^2} = 0.5826$, $X = \frac{x}{L}$

$\theta = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 X)$



the surface temperature of the aluminum plate is approximately the liquid temperature

$\Rightarrow h \to \infty$, $Bi \to \infty$

at surface $X = \pm 1$

$\theta = \frac{T(\pm L, 15) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\lambda_1$

$0 = A_1 e^{-\lambda_1^2 \tau} \cos\lambda_1$

$\Rightarrow \cos\lambda_1 = 0$

$\lambda_1 = \frac{\pi}{2}$

$A_1 = \frac{4\sin\lambda_1}{2\lambda_1 + \sin 2\lambda_1} = \frac{4}{\pi + 0} = \frac{4}{\pi}$

at center $X = 0$

$\theta = \frac{T(0,15) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cdot 1 = \frac{4}{\pi} \cdot e^{-\frac{\pi^2}{4}\tau} \Rightarrow T(0,15) = \frac{4}{\pi} \cdot e^{-\frac{\pi^2}{4}\cdot\tau} \cdot (25-500) + 500 = 356.35°C$ $\boxed{\text{ANS}}$

5. A long Pyroceram rod (ρ = 2600 kg/m3, c_p = 808 J/kg·K, k = 3.98 W/m·K, and $\alpha = 1.89\times10^{-6}$ m²/s) with diameter of 10 mm has an initial uniform temperature of 1000°C. The Pyroceram rod is allowed to cool in ambient temperature of 25°C and convection heat transfer coefficient of 80 W/m2 ·K. If the Pyroceram rod is allowed to cool for 3 minutes, determine the temperature at the center of the rod using (a) Table 4–2 and (b) the Heisler chart (Figure 4–18).

$r = 5mm = 0.005m$, T_i , T_∞ , h , $t = 180s$

5.解:

a) $\tau = \frac{\alpha t}{r_0^2} = \frac{1.89\times10^{-6}\times180}{0.005^2} = 13.608$

$Bi = \frac{h r_0}{\lambda} = \frac{80\times0.005}{3.98} = 0.1005 = 0.1$

$\boxed{A_1 = 1.0246, \lambda_1 = 0.4417}$

$R = \frac{r}{r_0} = 0$ at center

$\theta = A_1 e^{-\lambda_1^2 \tau} J_0(0)$, $J_0(0) = 1$

$= 1.0246 \times e^{-0.4417^2 \times 13.608}$

$= 0.07203 = \frac{T(0,180) - T_\infty}{T_i - T_\infty}$

$\Rightarrow T(0,180) = 95.234°C$ $\boxed{\text{ANS}}$

b) $\frac{1}{Bi} = \frac{1}{0.1005} = 10$, $\tau = 13.608$

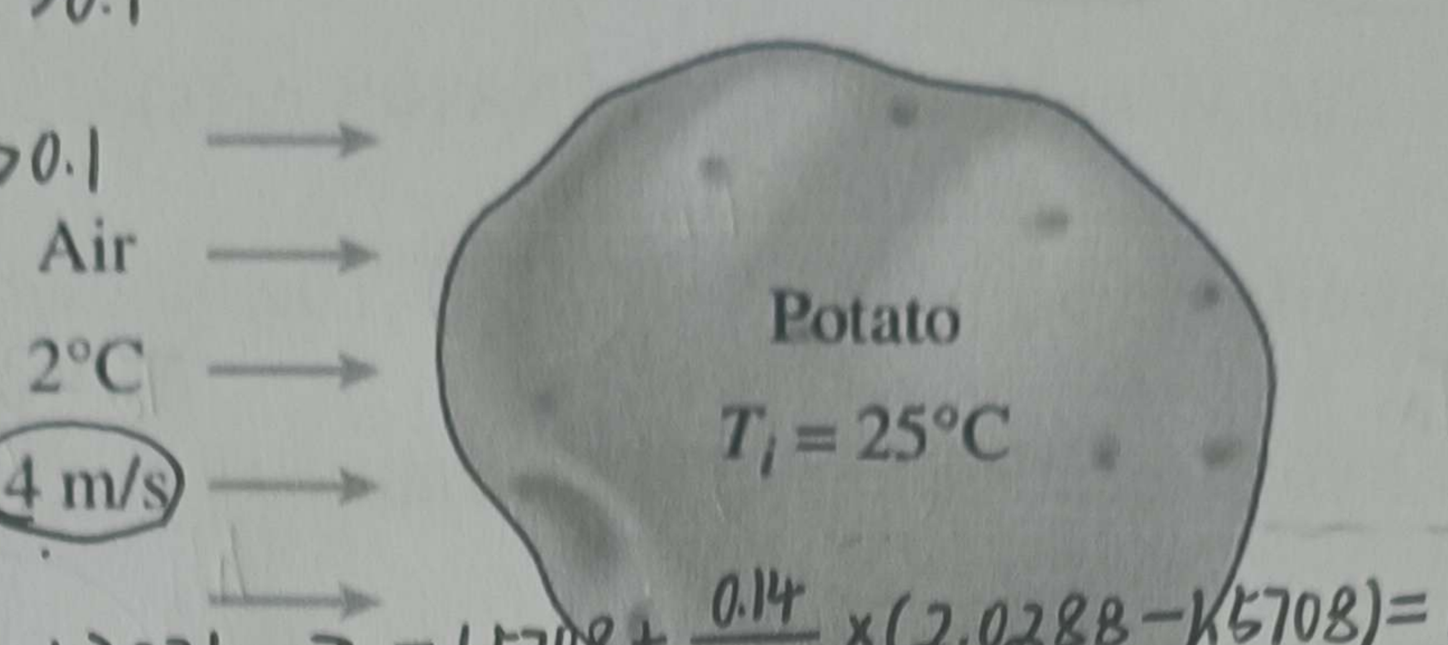
$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.075$

$T_0 = 0.075 \times (1000 - 25) + 25 = 98.125°C$ $\boxed{\text{ANS}}$

take for the center temperature of the potatoes to drop to 6°C

解: $B_i = \frac{h L_c}{\lambda} = \frac{19\times\frac{0.03}{3}}{0.5} = 0.38 > 0.1$

$B_i = \frac{h r_0}{\lambda} = \frac{19\times 0.03}{0.5} = 1.14 > 0.1$



consider 1-D transient analysis

$\tau = \frac{\alpha t}{r_0^2} = \frac{0.13\times10^{-6}\times t}{0.03^2}$

consider one-term approximation

$A_1 = 1.2732 + \frac{0.14}{2-1}\times(1.4793-1.2732) = 1.3021$, $\lambda_1 = 1.5708 + \frac{0.14}{1}\times(2.0288-1.5708) = 1.6349$

$\theta = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 R)}{\lambda_1 R}$

$\theta = A_1 e^{-\lambda_1^2\tau} = \frac{T(0,t)-T_\infty}{T_i - T_\infty} = \frac{6-2}{25-2} = \frac{4}{23}$

at center $R = \frac{r}{r_0} = 0$ $\lim_{x\to0}\frac{\sin x}{x} = \lim_{x\to0}\frac{\cos x}{1} = 1$

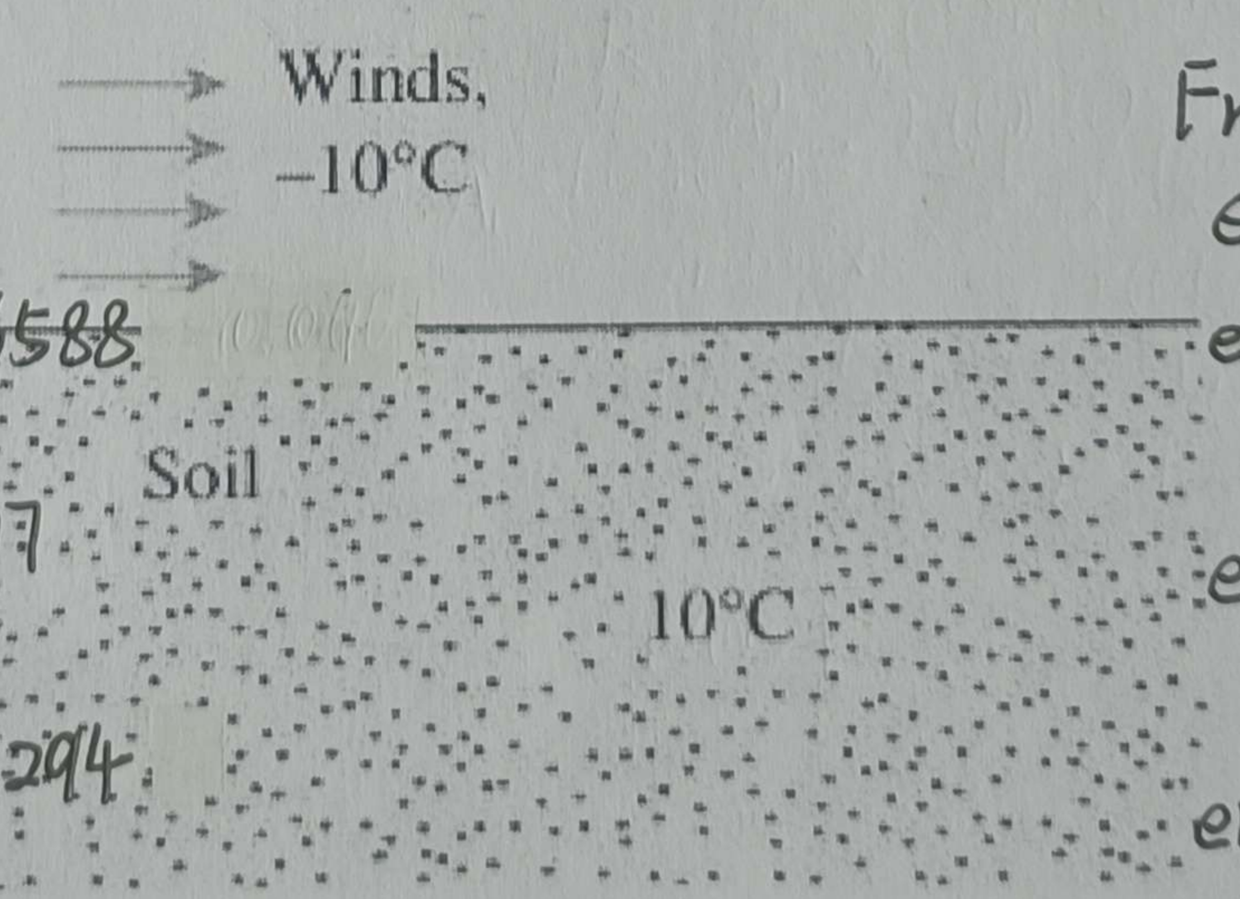
$\Rightarrow \tau = \frac{1}{\lambda_1^2}\ln\frac{23\cdot A_1}{4} = \frac{\alpha t}{r_0^2} \Rightarrow t = \frac{r_0^2}{\lambda_1^2\alpha}\cdot\ln\frac{23A_1}{4}$

$= 5214.34s$

$= 86.91min$ $\boxed{ANS}$

7. The soil temperature in the upper layers of the earth varies with the variations in the atmospheric conditions. Before a cold front moves in, the earth at a location is initially at a uniform temperature of 10°C (T_i). Then the area is subjected to a temperature (T_∞) of ~~210°C~~ −10°C and high winds that resulted in a convection heat transfer coefficient (h) of 40 W/m2 ·K on the earth's surface for a period (t) of 10 h. Taking the properties of the soil at that location to be $k = 0.9$ W/m·K and $\alpha = 1.6\times10^{-5}$ m²/s, determine the soil temperature at distances $\boxed{0, 10, 20, \text{and } 50\ cm}$ from the earth's surface at the end of this 10-h period.

解: $\eta = \frac{x}{\sqrt{4\alpha t}}$



$\eta_1 = 0$,

$\eta_2 = \frac{0.1}{\sqrt{4\times1.6\times10^{-5}\times3600\times10}} = \frac{\sqrt{10}}{48} = 0.06588$

$\eta_3 = \frac{0.2}{\sqrt{4\times1.6\times10^{-5}\times3600\times10}} = \frac{\sqrt{10}}{24} = 0.1317$

$\eta_4 = \frac{0.5}{\sqrt{4\times1.6\times10^{-5}\times3600\times10}} = \frac{5\sqrt{10}}{48} = 0.3294$

From Table 4-4

$erfc(\eta_1) = 1$

$erfc(\eta_2) = 0.9324 + \frac{0.00588}{0.02}\times(0.9099-0.9324) = 0.9258$

$erfc(\eta_3) = 0.8652 + \frac{0.0117}{0.02}\times(0.8431-0.8652) = 0.8523$

$erfc(\eta_4) = 0.6509 + \frac{0.0094}{0.02}\times(0.6306-0.6509) = 0.6414$

$\frac{T(x,t)-T_s}{T_i - T_s} = 1 - erfc(\eta)$

$\Rightarrow T(x,t) = T_s + (T_i - T_s)[1 - erfc(\eta)]$

$\Rightarrow T_1 = -10°C$

$T_2 = -8.516°C$

$T_3 = -7.046°C$

$T_4 = -2.828°C$

$\boxed{ANS}$

short cylinder

8. A 20-cm-long cylindrical aluminum block ($\rho = 2702$ kg/m^3, $c_p = 0.896$ kJ/kg·K, $k = 236$ W/m·K, and $\alpha = 9.75\times10^{-5}$ m^2/s), 15 cm in diameter, is initially at a uniform temperature of 20°C. The block is to be heated in a furnace at 1200°C until its center temperature rises to $\boxed{300°C}$. If the heat transfer coefficient on all surfaces of the block is 80 W/m^2·K, determine how long the block should be kept in the furnace. Also, determine the amount of heat transfer from the aluminum block if it is allowed to cool in the room until its temperature drops to 20°C throughout.

$r = 7.5\,cm = 0.075\,m$ $\quad T_i \quad T_\infty \quad T_f \quad r = 0 \quad h \quad$ Ask t

解: $\underset{\text{short cylinder}}{\theta(r, x, t)} = \underset{\text{plane wall}}{\theta(x, t)} \cdot \underset{\text{infinite cylinder}}{\theta(r, t)}$

for plane wall : $2L = 20cm$

$$B_i = \frac{hL}{\lambda} = \frac{80\times0.1}{236} = 0.03390$$

consider one-term solution approximation

from table 4-2

$$A_1 = 1.0033 + \frac{0.0339-0.02}{0.04-0.02}\times(1.0066-1.0033) = 1.0056$$

$$\lambda_1 = 0.1410 + \frac{0.0339-0.02}{0.04-0.02}\times(0.1987-0.141) = 0.1811$$

$$\theta_{\text{plane wall}} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 X)$$

at center $X = 0$, $\theta_{\text{plane wall}} = A_1 e^{-\lambda_1^2\tau}$, $\tau = \frac{\alpha t}{L^2} = \frac{9.75\times10^{-5}}{0.1^2}t = 9.75\times10^{-3}t$

for infinite cylinder : $r_0 = 0.075\,m$

$$B_i = \frac{hr_0}{\lambda} = \frac{80\times0.075}{236} = 0.02542$$

similarly, $A_1 = 1.005 + \frac{0.02542-0.02}{0.04-0.02}\times(1.0099-1.005) = 1.0063$

$\lambda_1 = 0.1995 + \frac{0.02542-0.02}{0.02}\times(0.2814-0.1995) = 0.2217$

$\theta_{\text{cylinder}} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 R)$ $\quad \theta = A_1 e^{-\lambda_1^2\tau}$, $\tau = \frac{\alpha t}{r_0^2} = \frac{9.75\times10^{-5}t}{0.075^2} = 0.0173t$

at center $R = 0$, $J_0(0) = 1$, θ_{cylinder}

$$\Rightarrow \frac{T_f - T_\infty}{T_i - T_\infty} = (A_1 e^{-\lambda_1^2\tau})_{wall}\cdot(A_1 e^{-\lambda_1^2\tau})_{cylinder}$$

$$\frac{300-1200}{20-1200} = 1.0056e^{-0.1811^2\times0.00975t}\times1.0063e^{-0.2217^2\times0.0173t} \Rightarrow t = 241.64\,s \;\boxed{ANS}$$

$$\left(\frac{Q}{Q_{max}}\right)_{wall} = 1-\theta_{x=0}\frac{\sin\lambda_1}{\lambda_1} = 1 - A_1 e^{-\lambda_1^2\tau}\cdot\frac{\sin\lambda_1}{\lambda_1},\quad \left(\frac{Q}{Q_{max}}\right)_{cylinder} = 1-2\theta_{R=0}\frac{J_1(\lambda_1)}{\lambda_1} = 1-2A_1e^{-\lambda_1^2\tau}\cdot\frac{J_1(\lambda_1)}{\lambda_1}$$

$$J_1(\lambda_1) = J_1(0.2217) = 0.0995 + \frac{0.2217-0.2}{0.3-0.2}\times(0.1483-0.0995) = 0.1101$$

$$(Q_{max})_{total} = C_p\rho V(T_i - T_\infty) = 896\times2702\times\pi\times0.075^2\times0.2\times(1200-20) = 10097\,kJ$$

$$Q_{total} = (Q_{max})_{total}\cdot\left(\left(\frac{Q}{Q_{max}}\right)_{wall} + \left(\frac{Q}{Q_{max}}\right)_{cylinder}\left[1-\left(\frac{Q}{Q_{max}}\right)_{wall}\right]\right) = 10097\times[0.07426+0.18615\times0.92574]$$

$$= 2489.78\,kJ \;\boxed{ANS}$$

Q 7.

解：it's third BC

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc(\eta) - e^{\left(\frac{hx}{\lambda} + \frac{h^2\alpha t}{\lambda^2}\right)} \cdot erfc\left(\eta + \frac{h\sqrt{\alpha t}}{\lambda}\right)$$

$x = 0$

$$\frac{T(0,10h)-10}{-10-10} = 0 - exp\left(\frac{40^2 \times 1.6\times10^{-5} \times 3.6\times10^{-5}}{0.9^2}\right)$$

$$\times erfc\left(\frac{40 \times \sqrt{1.6\times10^{-5} \times 3.6\times10^{-5}}}{0.9}\right) \approx -1$$

$$\Rightarrow T(0,10h) = -10^\circ C$$

$x = 0.1$

$$\frac{T(0.1,10h)-10}{-10-10} = erfc\left(\frac{0.1}{2\sqrt{1.6\times3.6}}\right)$$

$$-exp\left(\frac{40\times0.1}{0.9} + \frac{40^2\times1.6\times3.6}{0.9^2}\right)$$

$$\times erfc\left(\frac{0.1}{\sqrt{4\times1.6\times3.6}} + \frac{40\times\sqrt{1.6\times3.6}}{0.9}\right)$$

$$\Rightarrow T(0.1, 10h) = -8.52^\circ C$$

$x = 0.2$

$$\Rightarrow T(0.2, 10h) = -7.04^\circ C$$

$x = 0.5$

$$\Rightarrow T(0.5, 10h) = -2.82^\circ C$$