Quiz 5

Date: 2022-03-14 Name: SID:

Solve the following equations.

(1) $\frac{dy}{dx} = \frac{3x-y-2}{x+y+1}$;

(2) $y' = 1 + t^2 - 2ty + y^2$, with a particular solution $y_1(t) = t$.

(1). $$\frac{dy}{dx} = \frac{3x-y-2}{x+y+1}$$

Let $x = \xi + \alpha$. $y = \eta + \beta$.

$$\begin{cases} 3\alpha - \beta - 2 = 0 \\ \alpha + \beta + 1 = 0 \end{cases} \Rightarrow \begin{cases} \alpha = \frac{1}{4} \\ \beta = -\frac{5}{4} \end{cases}$$

$$\frac{d\eta}{d\xi} = \frac{\xi - \eta}{\xi + \eta} = \frac{3 - v}{1+v} \quad \left(v = \frac{\eta}{\xi}\right)$$

$$v + \xi \frac{dv}{d\xi} = \frac{3-v}{1+v}$$

$$\xi \frac{dv}{d\xi} = \frac{3-2v-v^2}{1+v}$$

① $3-2v-v^2=0$ $\quad v = -3$ or $1 \quad = \frac{y+\frac{5}{4}}{x-\frac{1}{4}}$

i.e. $y = -3x - \frac{1}{2}$ or $y = x - \frac{3}{2}$.

② $3-2v-v^2 \neq 0$.

$$\frac{1+v}{3-2v-v^2} dv = \frac{1}{\xi} d\xi$$

$$-\frac{1}{2} \ln|3-2v-v^2| = \ln \xi + C_0$$

$$3-2v-v^2 = \frac{C}{\xi^2}.$$

$3\xi^2 - 2\xi\eta - \eta^2 = C$.

$3(x-\frac{1}{4})^2 - 2(x-\frac{1}{4})(y+\frac{5}{4}) - (y+\frac{5}{4})^2 = C$

(2) $y' = 1+t^2-2ty+y^2$, $\quad y_1(t) = t$.

Let $y = u+t$.

$y' = u'+1 = 1+t^2-2t^2-2tu+u^2+t^2+2tu$

$u' = u^2$ Bernoulli

~~① $u=0$ $y=t$ ② $u\neq 0$.~~

let $z = u^{1-n} = u^{-1}$

$\frac{dz}{dt} = -\frac{1}{u^2}\frac{du}{dt} = -1$

$z = -t+C = \frac{1}{u} = \frac{1}{y-t}$

$(-t+C)y + (t^2-Ct) = 1$

$y = \frac{1+Ct-t^2}{C-t}$

① $u=0$ $y=t$.

② $u\neq 0$.

$u = -\frac{1}{t}+C$

$y = -\frac{1}{t}+t+C$