Variable Thermal Conductivity

- Variable Thermal Conductivity
  - In many situations, the thermal conductivity of the medium through which the energy is transferred varies significantly
  - When the variation of thermal conductivity with temperature $\lambda(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 can be determined from

$$\lambda_{avg} = \frac{\int_{T_1}^{T_2} \lambda(T)\, dT}{T_2 - T_1}$$

  - This relation is based on the requirement that the rate of heat transfer through a medium with constant average thermal conductivity λ_{avg} equals the rate of heat transfer through the same medium with variable conductivity $\lambda(T)$.

Variable Thermal Conductivity

- Heat Transfer Rate
  - The rate of steady heat transfer through a plane wall, cylindrical layer, or spherical layer for the case of variable thermal conductivity can be determined by replacing the constant thermal conductivity λ by the λ_{avg} expression.

$$\Phi_{wall} = \lambda_{avg} A \frac{T_1 - T_2}{L} = \frac{A}{L}\int_{T_2}^{T_1} \lambda(T)\,dT$$

$$\Phi_{cylinder} = 2\pi\lambda_{avg} L \frac{T_1 - T_2}{\ln(r_2/r_1)} = \frac{2\pi L}{\ln(r_2/r_1)}\int_{T_2}^{T_1} \lambda(T)\,dT$$

$$\Phi_{sphere} = 4\pi\lambda_{avg} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} = \frac{4\pi r_1 r_2}{r_2 - r_1}\int_{T_2}^{T_1} \lambda(T)\,dT$$

Variable Thermal Conductivity

- Heat Transfer Rate
  - Steady heat conduction through a plane wall.

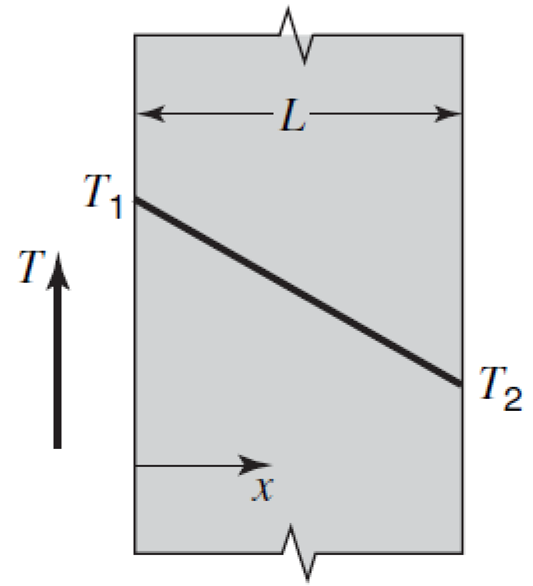
$$\Phi_{wall} = -\lambda(T) A \frac{dT}{dx}$$

$$\Phi_{wall} dx = -\lambda(T) A dT$$

$$\int_0^L \Phi_{wall} dx = -A \int_{T_1}^{T_2} \lambda(T) dT$$

$$\Phi_{wall} L = A \int_{T_2}^{T_1} \lambda(T) dT$$

$$\Phi_{wall} = \frac{A}{L} \int_{T_2}^{T_1} \lambda(T) dT = \lambda_{avg} A \frac{T_1 - T_2}{L}$$



Variable Thermal Conductivity

- Heat Transfer Rate
  - Steady heat conduction through a cylindrical wall.

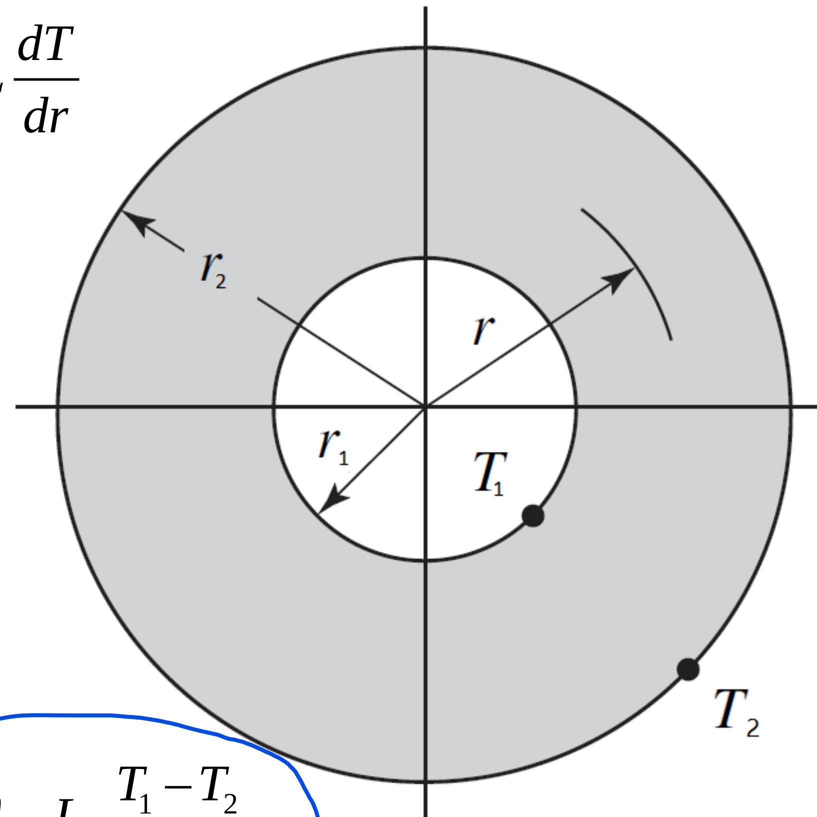
$$\Phi_{cylinder} = -\lambda(T)A\frac{dT}{dr} = -\lambda(T)2\pi rL\frac{dT}{dr}$$

$$\Phi_{cylinder}\frac{dr}{r} = -2\pi L\lambda(T)dT$$

$$\int_{r_1}^{r_2}\Phi_{cylinder}\frac{dr}{r} = -2\pi L\int_{T_1}^{T_2}\lambda(T)dT$$

$$\Phi_{cylinder}\ln\left(\frac{r_2}{r_1}\right) = 2\pi L\int_{T_2}^{T_1}\lambda(T)dT$$

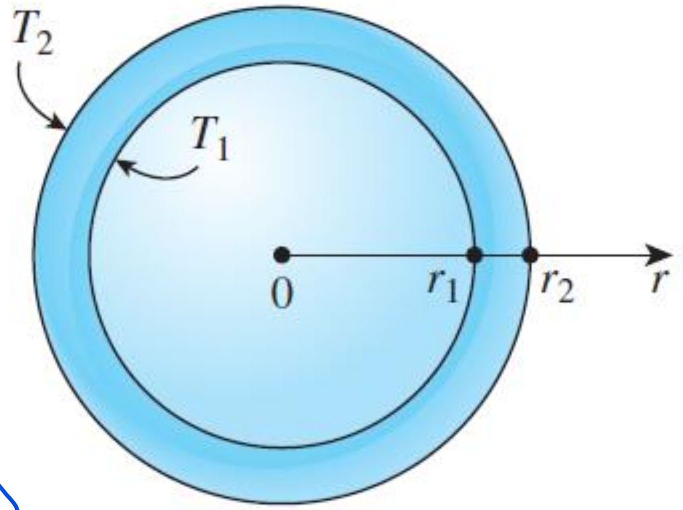
$$\Phi_{cylinder} = \frac{2\pi L}{\ln(r_2/r_1)}\int_{T_2}^{T_1}\lambda(T)dT = 2\pi\lambda_{avg}L\frac{T_1 - T_2}{\ln(r_2/r_1)}$$



Variable Thermal Conductivity

- Heat Transfer Rate
  - Steady heat conduction through a spherical wall.

$$\Phi_{sphere} = -\lambda(T) A \frac{dT}{dr} = -\lambda(T) 4\pi r^2 \frac{dT}{dr}$$



$$\Phi_{sphere} = \frac{4\pi r_1 r_2}{r_2 - r_1} \int_{T_2}^{T_1} \lambda(T) dT = 4\pi\lambda_{avg} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1}$$