力学与航空航天工程系
DEPARTMENT OF MECHANICS AND AEROSPACE ENGINEERING

MECHANICS OF MATERIALS

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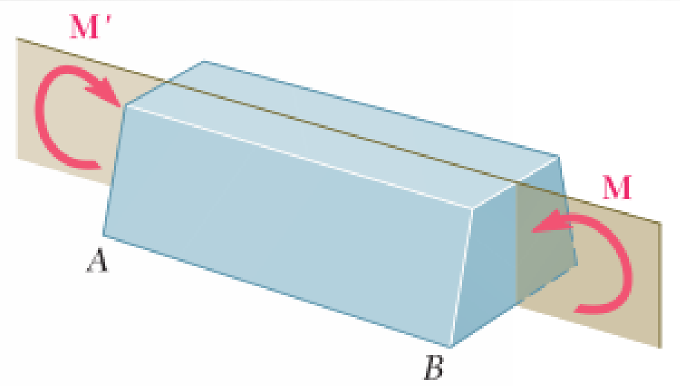
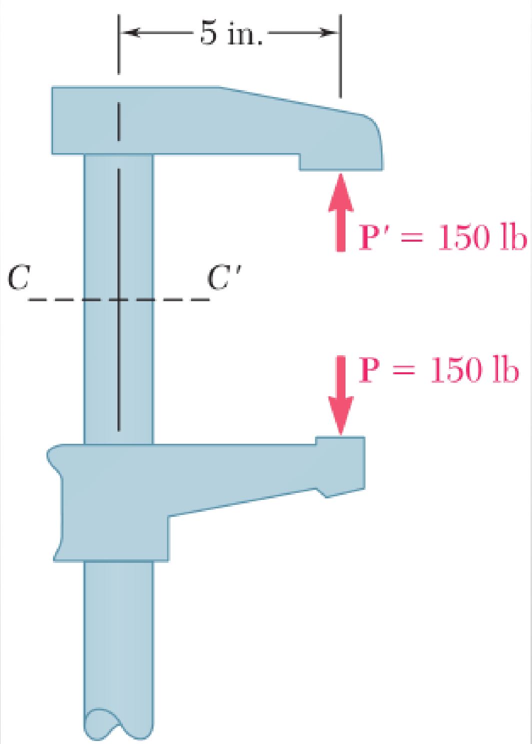
Lesson 5： Pure bending

- Pure bending, internal force analysis, bending moment, stress and strain analysis.

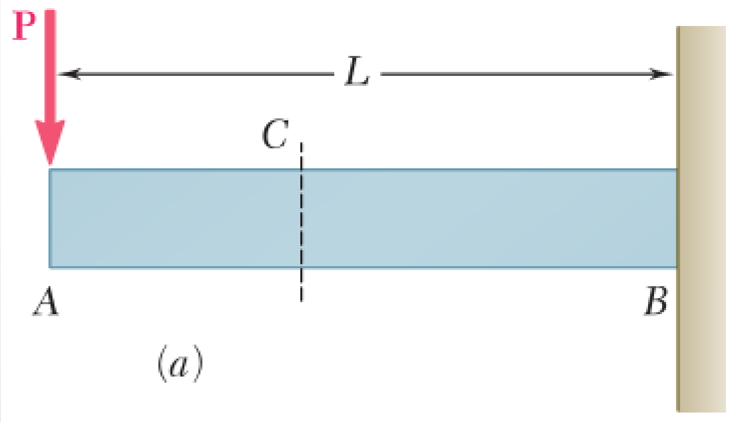
§ 5.1 Introduction

- **Pure bending**: analysis of prismatic members subjected to equal and opposite couples M and M′ acting in the same longitudinal plane





Eccentric load



Cantilever beam

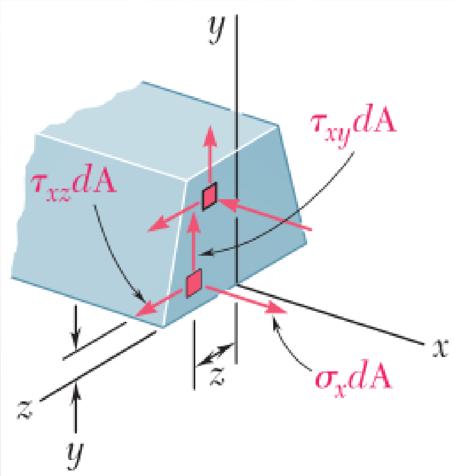
§ 5.2 Pure bending

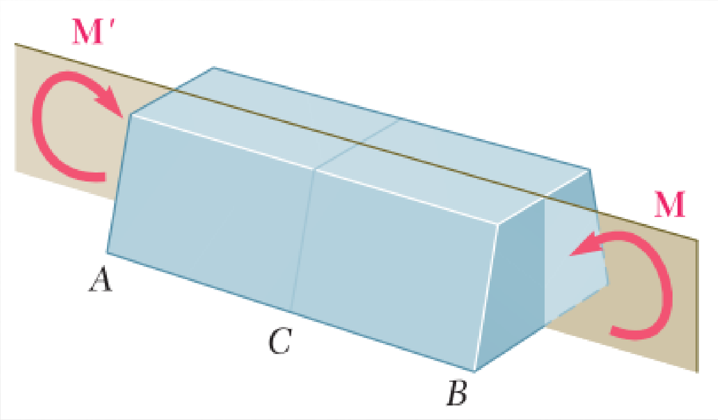
- Consider a prismatic member AB possessing a plane of symmetry and subjected to equal and opposite couples M and M′ acting in that plane

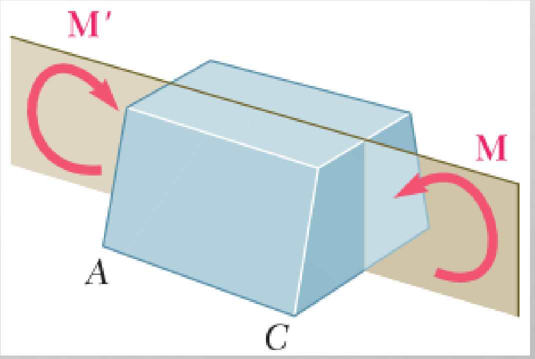
$$\int \sigma_x dA = 0$$

$$\int \sigma_x z dA = 0$$

$$\int (-\sigma_x y) dA = M$$

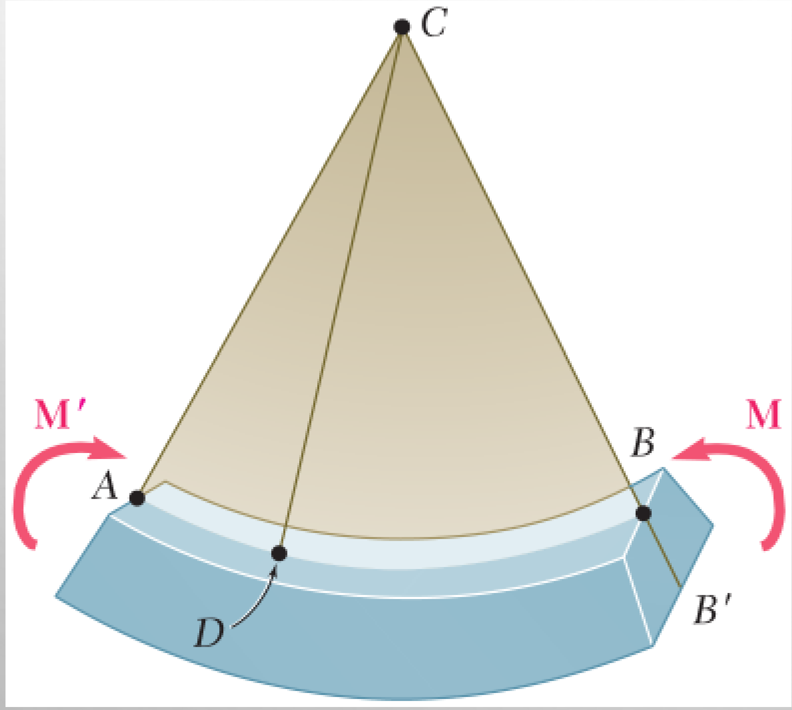


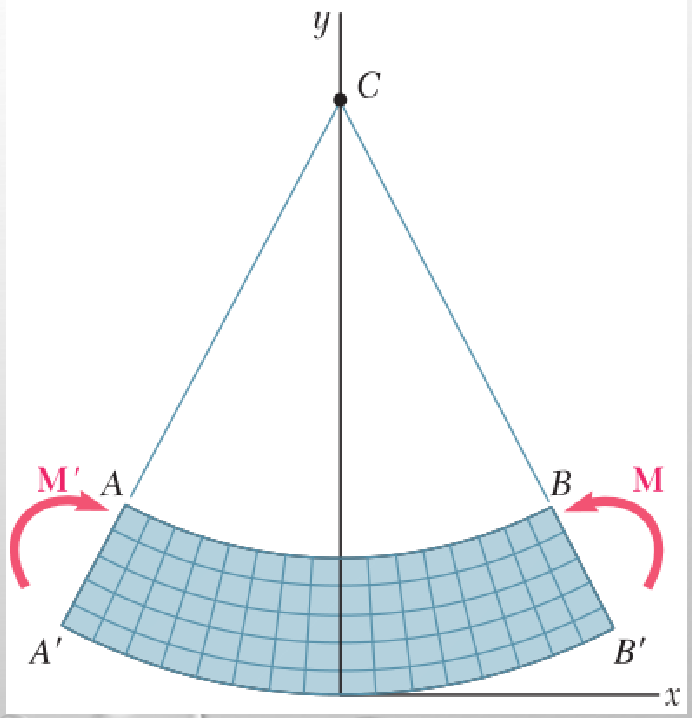




§ 5.3 Deformation of a member in pure bending

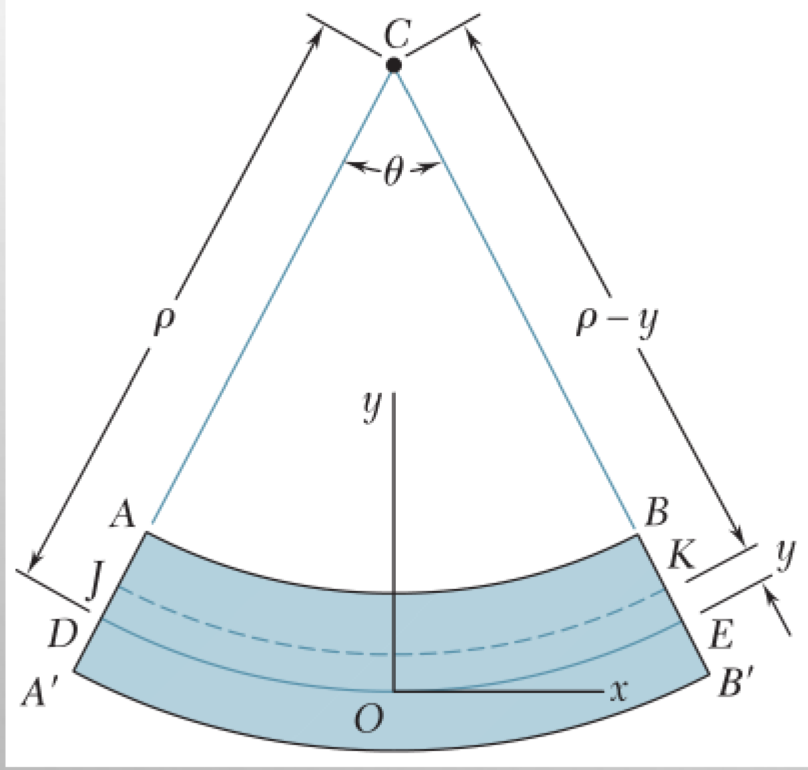
- A prismatic member possessing a plane of symmetry and subjected at its ends to equal and opposite couples M and M′ acting in the plane of symmetry

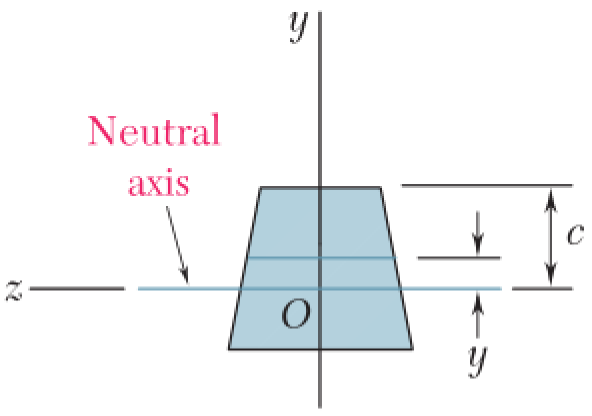




§ 5.3 Deformation of a member in pure bending

- Neutral plane





- Strain

$$L = \rho\theta$$

$$L' = (\rho - y)\theta$$

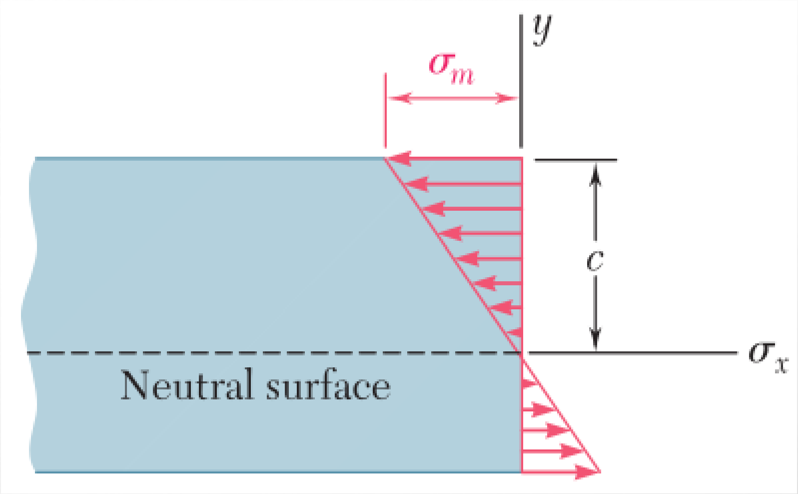
$$\varepsilon = \frac{\delta}{L} = -\frac{y}{\rho}$$

$$\varepsilon_{\max} = \frac{c}{\rho} \qquad \varepsilon_x = -\frac{y}{c}\varepsilon_{\max}$$

§ 5.4 Stress and deformations in the elastic range

- Hooke's law for uniaxial stress applies

$$\sigma_x = E\varepsilon_x \quad \sigma_x = -\frac{y}{c}\sigma_{\max}$$



- Force balance

$$\int \sigma_x dA = 0 \qquad -\frac{\sigma_{\max}}{c}\int y dA = 0$$

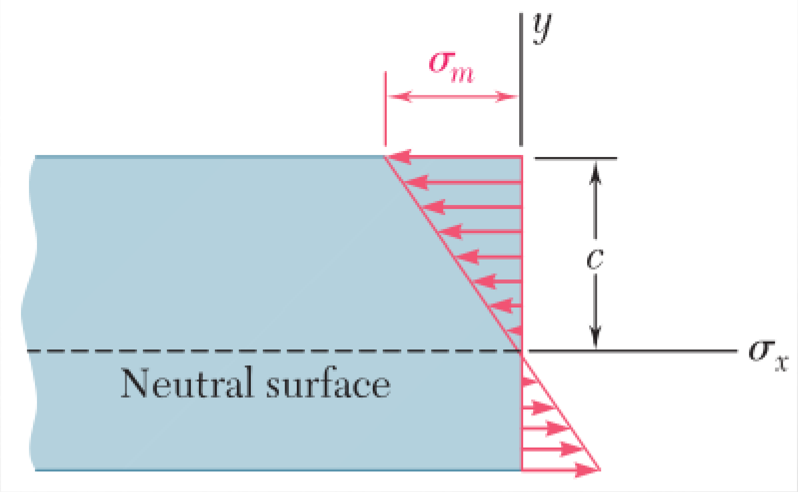
the neutral axis passes through the centroid of the section

$$\int -\sigma_x y dA = 0 \qquad \frac{\sigma_{\max}}{c}\int y^2 dA = M$$

§ 5.4 Stress and deformations in the elastic range

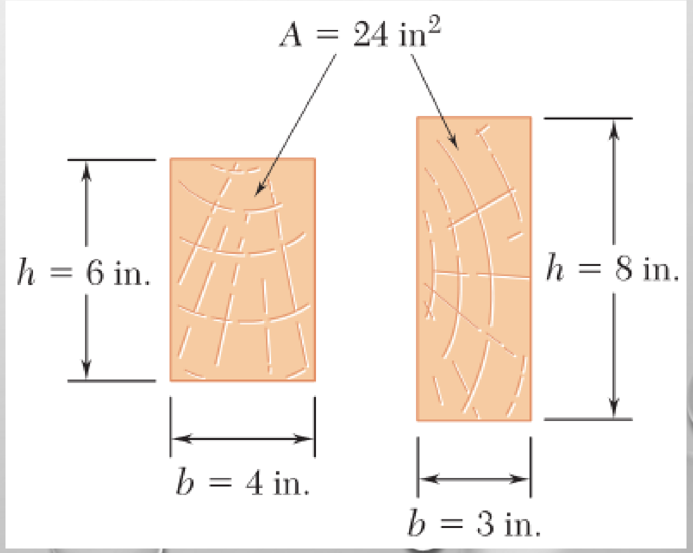
- Elastic flexure formulas

$$I = \int y^2 dA \quad \sigma_{\max} = \frac{Mc}{I} \quad \sigma_x = -\frac{My}{I}$$



- Elastic section modulus； I, moment of inertia

$$\sigma_{\max} = \frac{M}{S} \quad S = \frac{I}{c} \quad I = \int y^2 dA = \frac{1}{12} bh^3 \quad c = \frac{1}{2} h$$

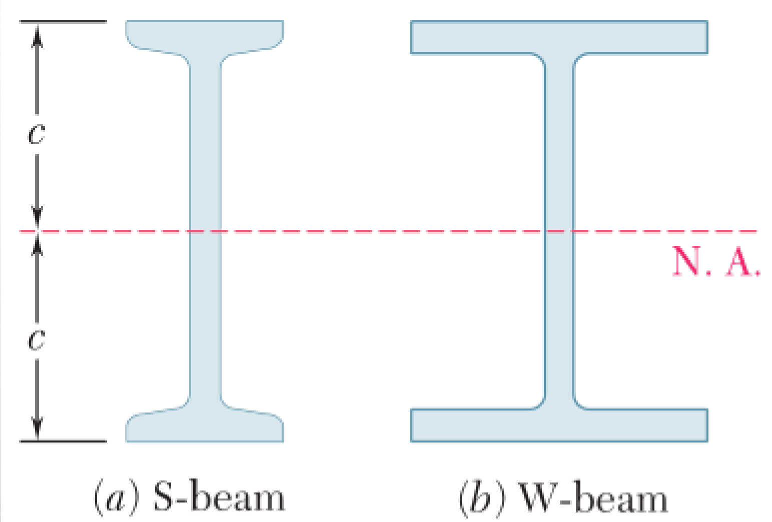


§ 5.5 Structural steel; Curvature

- American standard beams (S-beams) and wide-flange beams (W-beams)

$$I = \int y^2 dA$$

$$S = \frac{I}{c}$$



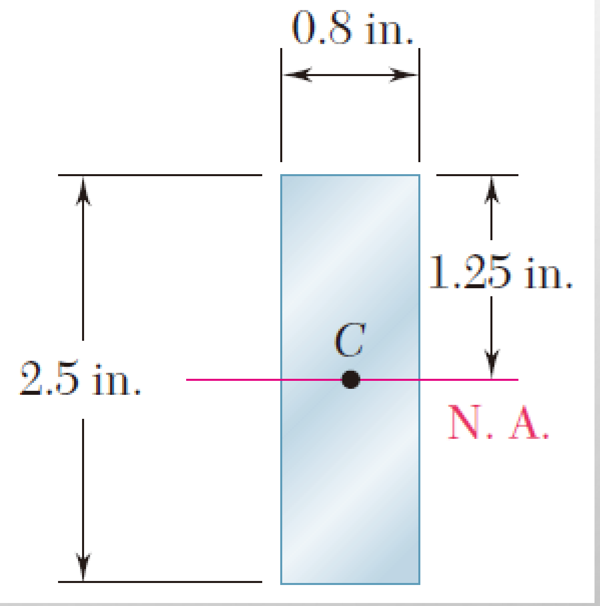
- Curvature vs. bending moment

$$\varepsilon_{\max} = \frac{c}{\rho} \qquad \sigma_{\max} = \frac{Mc}{I} \qquad \frac{1}{\rho} = \frac{M}{EI}$$

Example 5.1

(Beer, Page 232)

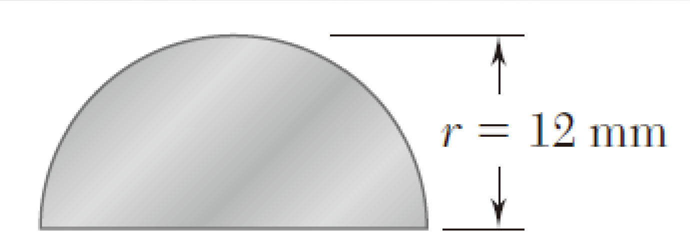
A steel bar of 0.8 × 2.5-in. rectangular cross section is subjected to two equal and opposite couples acting in the vertical plane of symmetry of the bar. Determine the value of the bending moment M that causes the bar to yield. Assume σ_Y = 36 ksi.

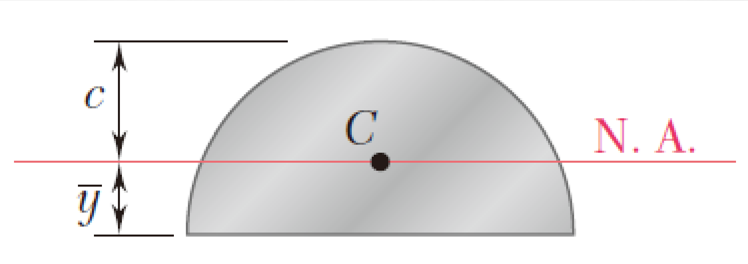


Example 5.2

(Beer, Page 233)

An aluminum rod with a semicircular cross section of radius r = 12 mm is bent into the shape of a circular arc of mean radius r = 2.5 m. Knowing that the flat face of the rod is turned toward the center of curvature of the arc, determine the maximum tensile and compressive stress in the rod. Use E = 70 GPa.





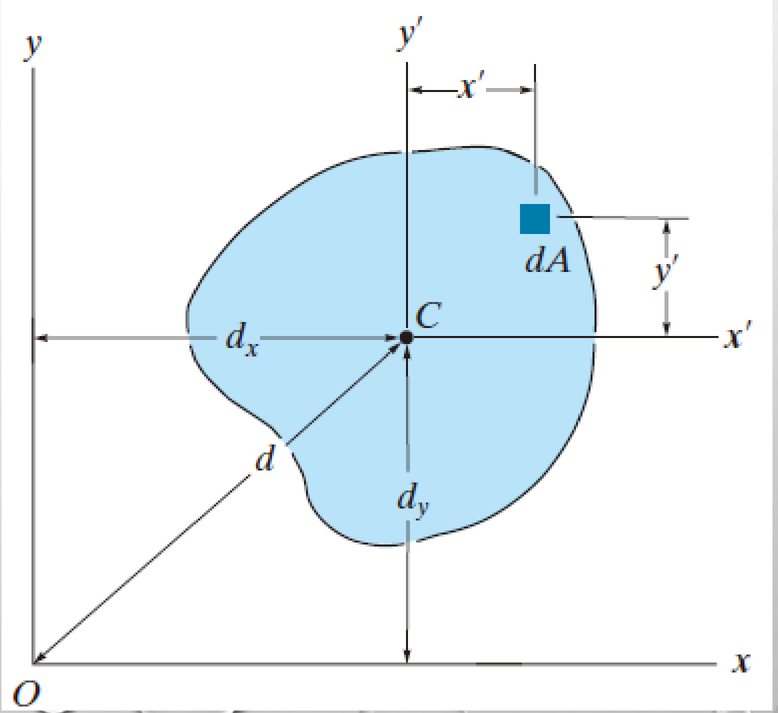
§ 5.6 Parallel-Axis Theorem for an Area

- If the moment of inertia for an area is known about a centroidal axis, we can determine the moment of inertia of the area about a corresponding parallel axis using the parallel-axis theorem.

$$I_x = \int_A (y' + d_y)^2 dA$$

$$= \int_A y'^2 dA + 2d_y \int_A y' dA + d_y^2 \int_A dA$$

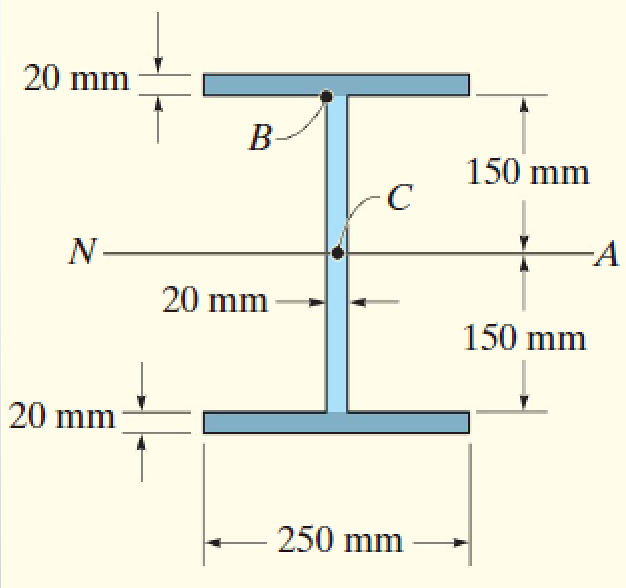
$$= \bar{I}_{x'} + A d_y^2$$



Example 5.3

(Hibbeler, Page 298)

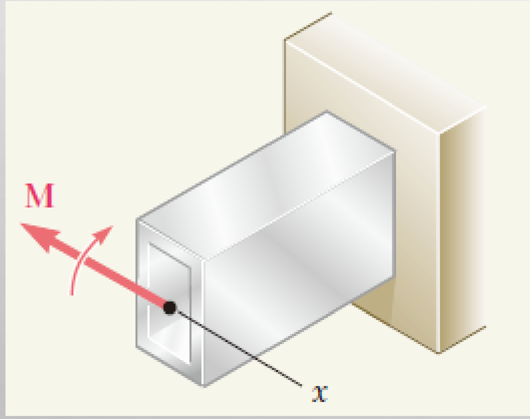
Determine the total moment of inertia about the neutral axis using the parallel-axis theorem.

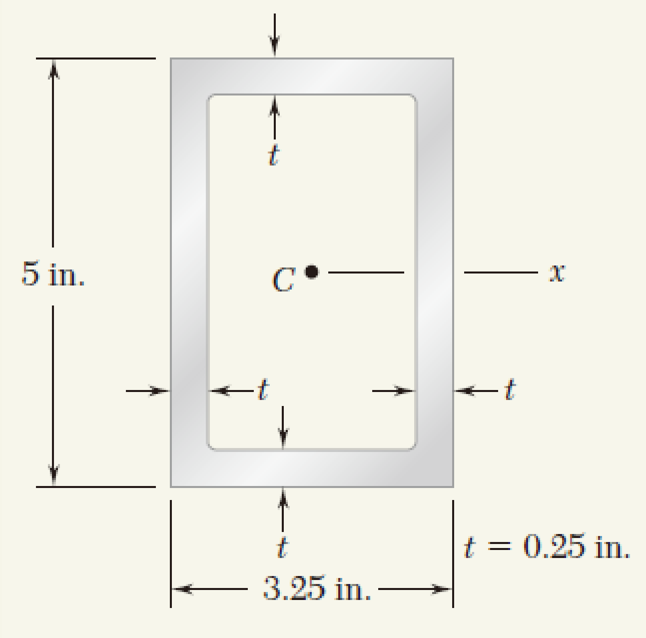


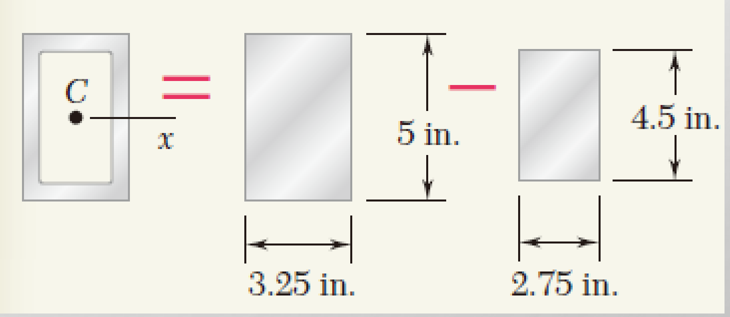
Example 5.4

(Beer, Page 235)

The rectangular tube shown is extruded from an aluminum alloy for which σ_Y = 40 ksi, σ_U = 60 ksi, and E = 10.6×10^6 psi. Neglecting the effect of fillets, determine (a) the bending moment M for which the factor of safety will be 3.00, (b) the corresponding radius of curvature of the tube.





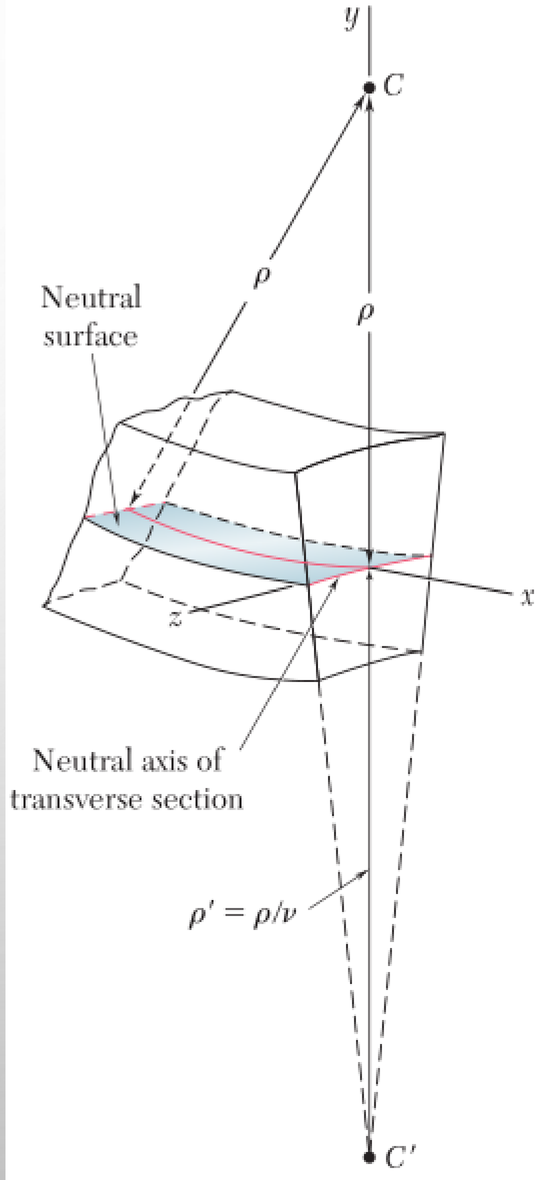


§ 5.6 Deformation in a transverse cross section

- The transverse cross-section remains plain, but there exists deformation.

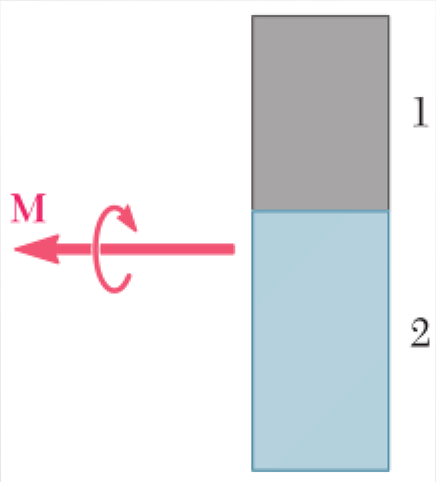
$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = \frac{\nu y}{\rho}$$

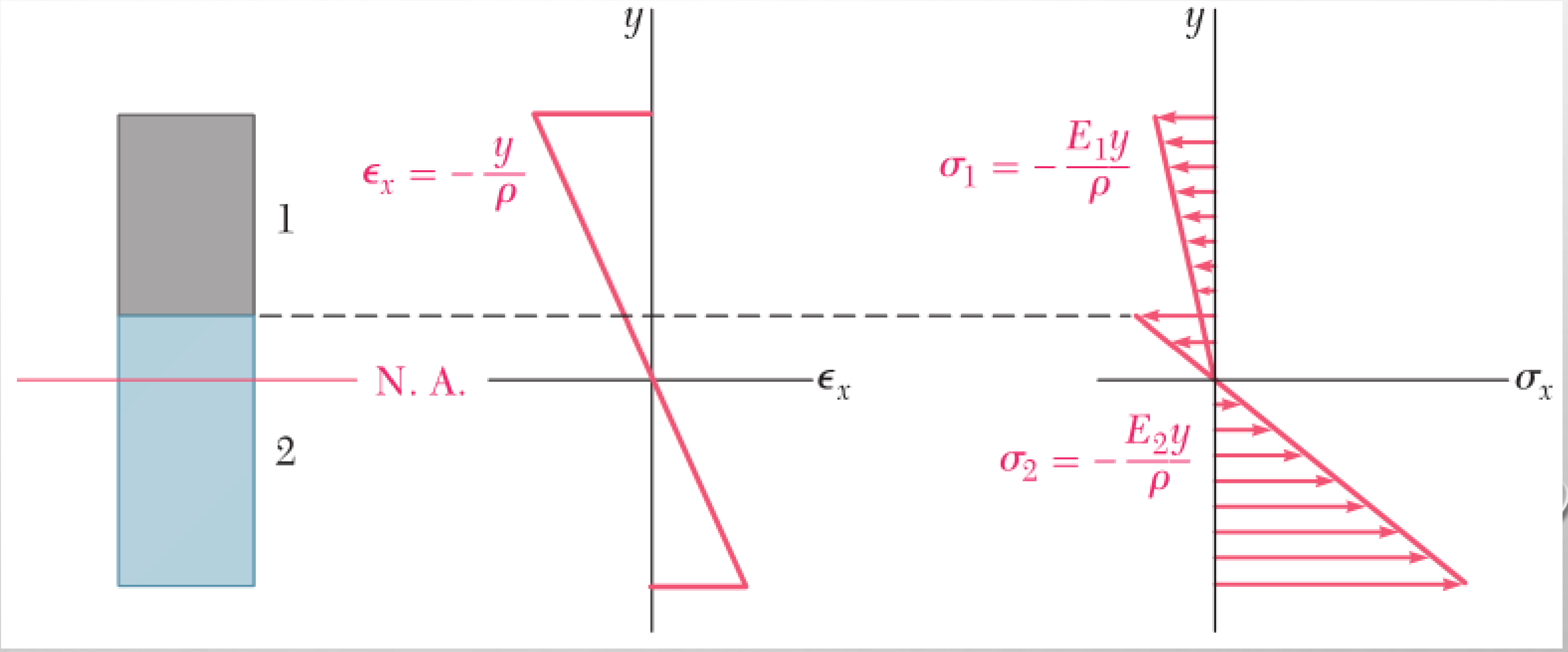
$$\text{Anticlastic curvature} = \frac{1}{\rho'} = \frac{\nu}{\rho}$$



§ 5.7 Bending member of several materials

- A bar consisting of two portions of different materials bonded together





We cannot assume that the neutral axis passes through the centroid of the composite section.

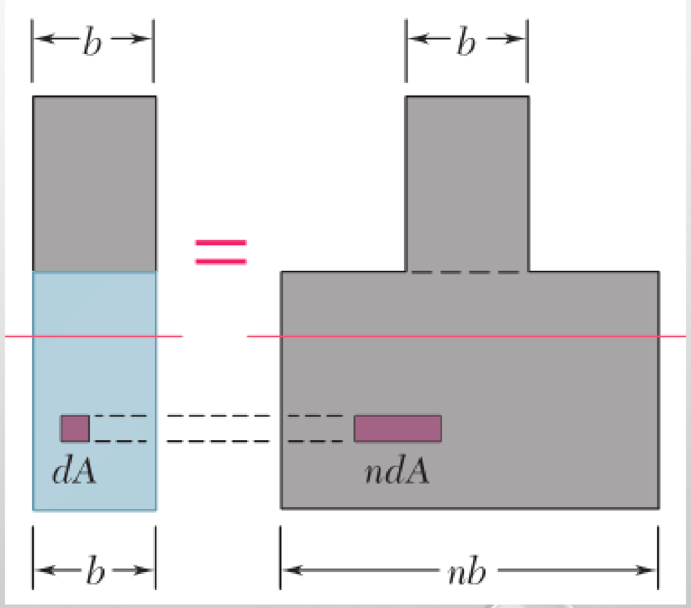
§ 5.8 Transformed section of the member

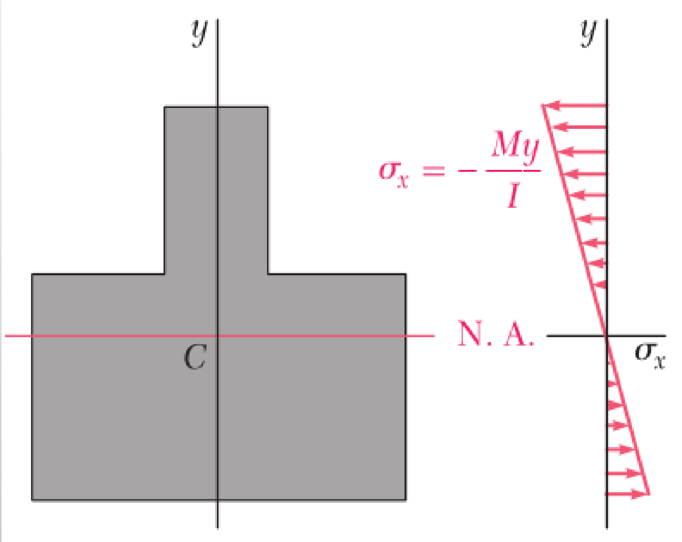
- The transformed section represents the cross section of a member made of a homogeneous material with a modulus of elasticity

$$n = \frac{E_2}{E_1} \qquad dF_1 = -\frac{E_1 y}{\rho} dA$$

$$dF_2 = -\frac{E_2 y}{\rho} dA = -\frac{E_1 y}{\rho}(ndA)$$

$$\text{centroid}: \int y dA = 0$$

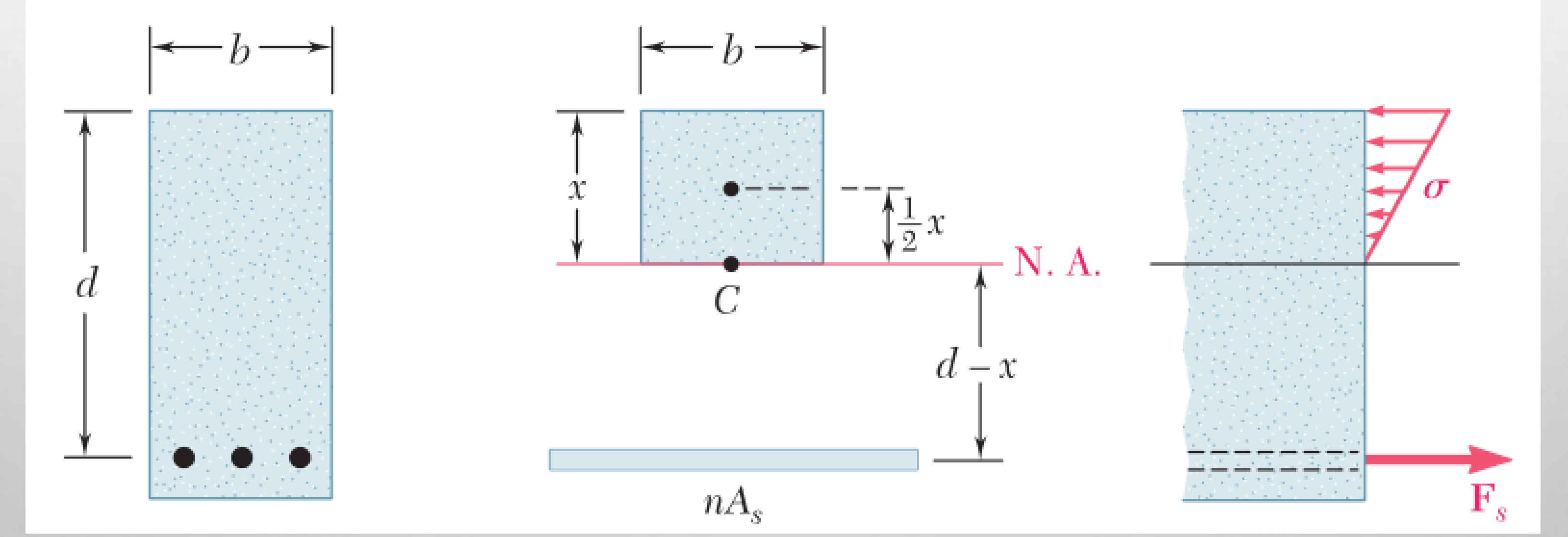




I, moment of inertia of the transformed section

§ 5.9 Reinforced concrete beams

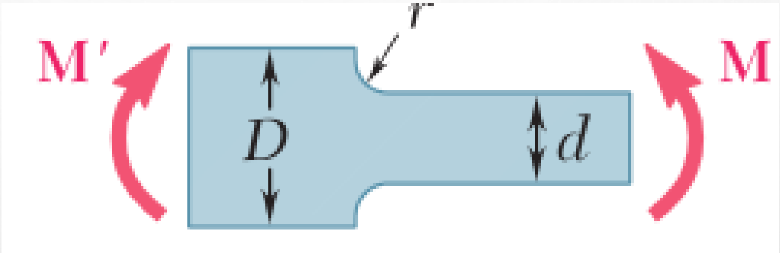
- These beams, when subjected to positive bending moments, are reinforced by steel rods placed a short distance above their lower face.



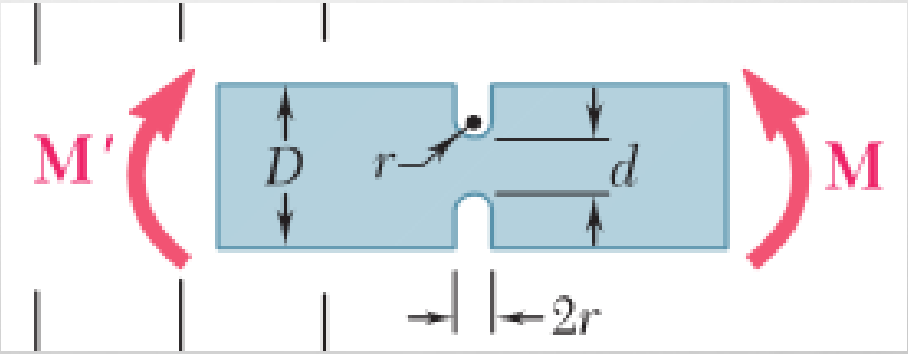
$$(bx)\frac{1}{2}x - nA_s(d-x) = 0$$

§ 5.10 Stress concentration

- Higher stresses will also occur if the cross section of the member undergoes a sudden change.



$$\sigma_{max} = K\frac{Mc}{I}$$



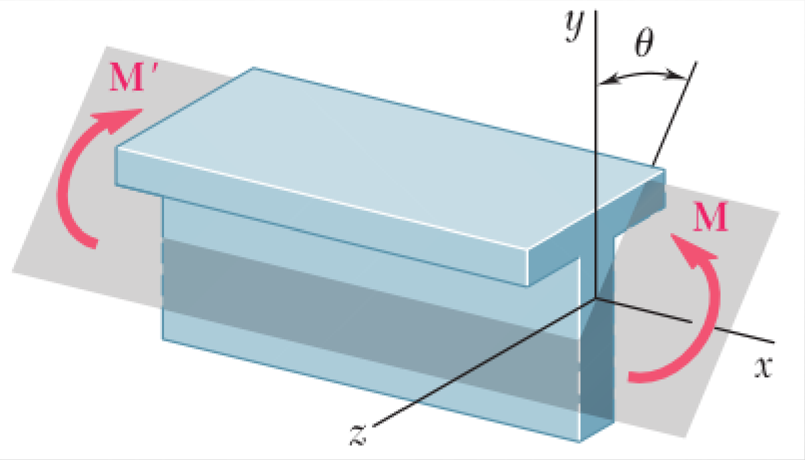
§ 5.11 Unsymmetric bending

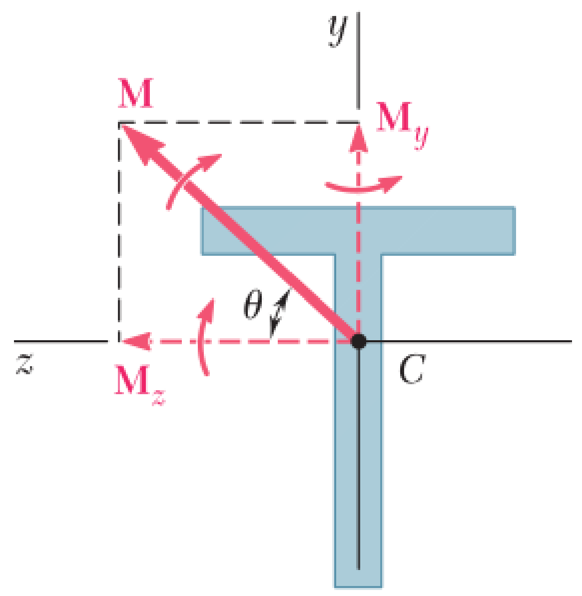
- The principle of superposition can be used to determine stresses in the most general case of unsymmetric bending.

$$M_z = M\cos\theta$$

$$M_y = M\sin\theta$$

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

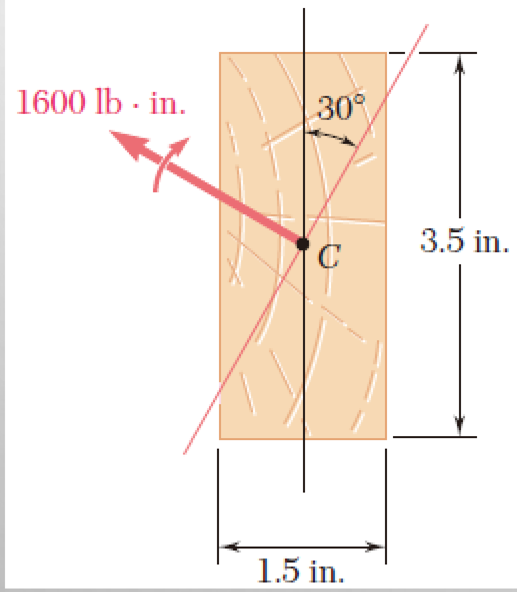


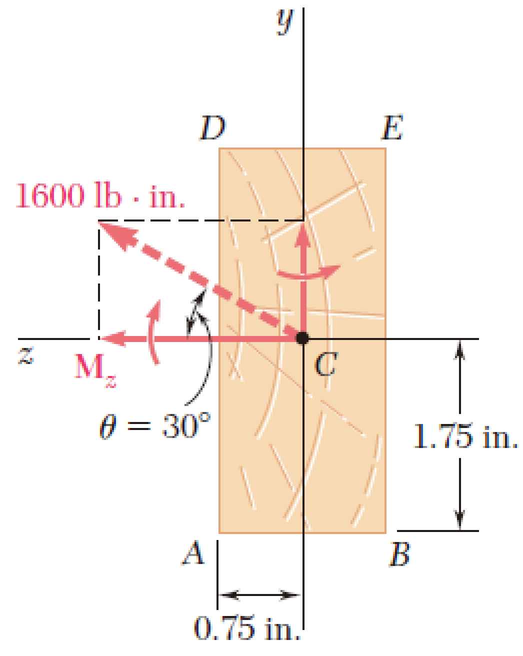


Example 5.5

(Beer, Page 285)

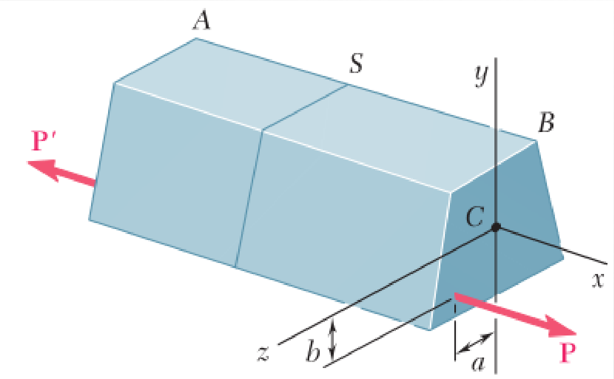
A 1600-lb · in. couple is applied to a wooden beam, of rectangular cross section 1.5 by 3.5 in., in a plane forming an angle of 308 with the vertical. Determine (a) the maximum stress in the beam, (b) the angle that the neutral surface forms with the horizontal plane.

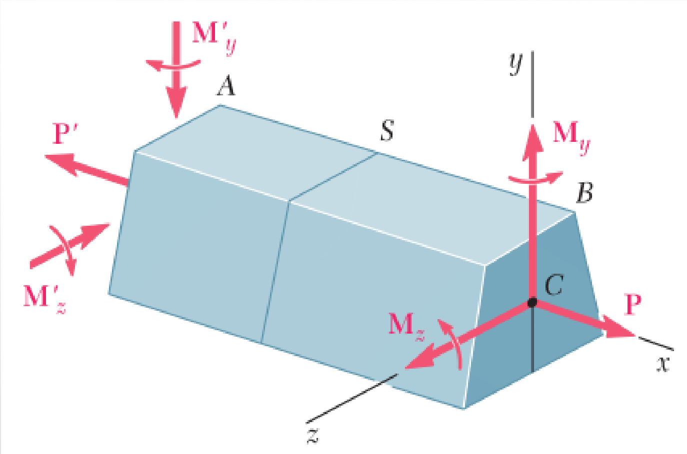




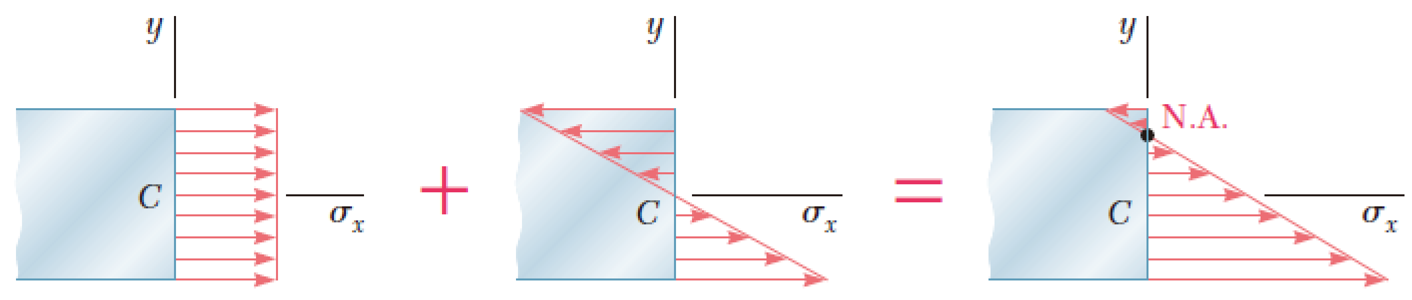
§ 5.12 Eccentric axial loading

- The distribution of stresses when the line of action of the loads does not pass through the centroid of the cross section, i.e., when the loading is eccentric.





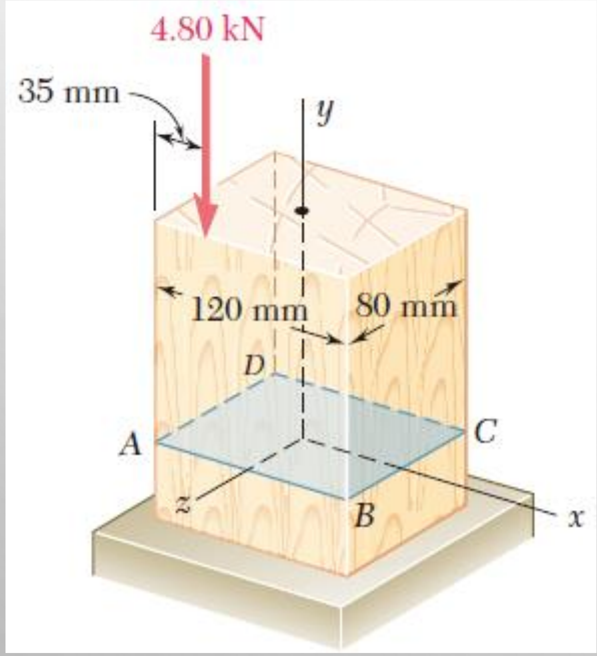
$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

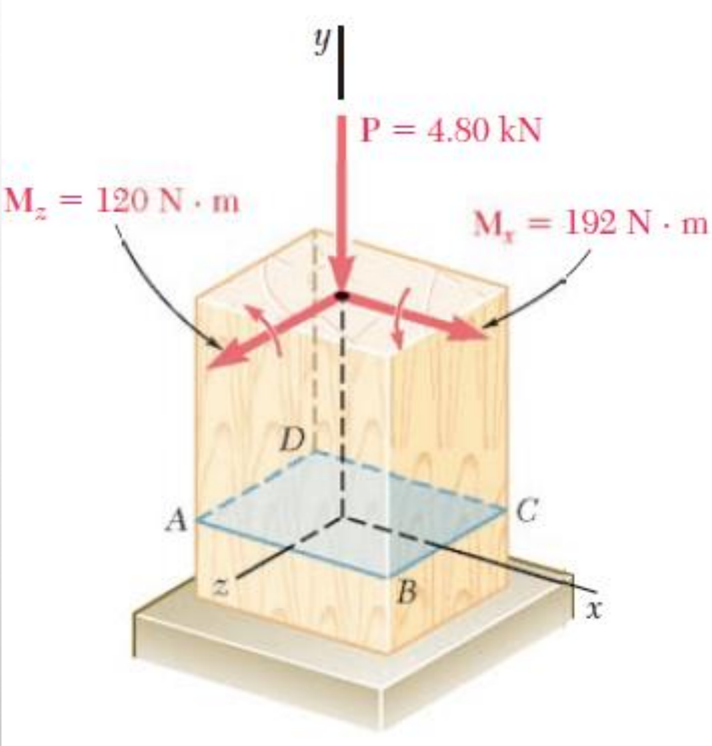


Example 5.6

(Beer, Page 285)

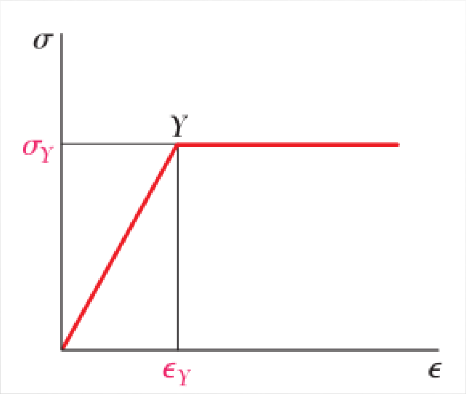
A vertical 4.80-kN load is applied as shown on a wooden post of rectangular cross section, 80 by 120 mm (Fig. 4.65). (a) Determine the stress at points A, B, C, and D. (b) Locate the neutral axis of the cross section.





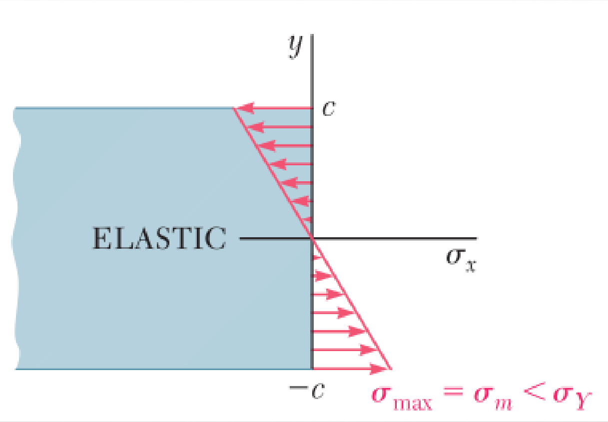
§ 5.13 Members made of an elastoplastic material

- Members of circular cross section subjected to twisting couples, or **torques**, **T** and **T′**

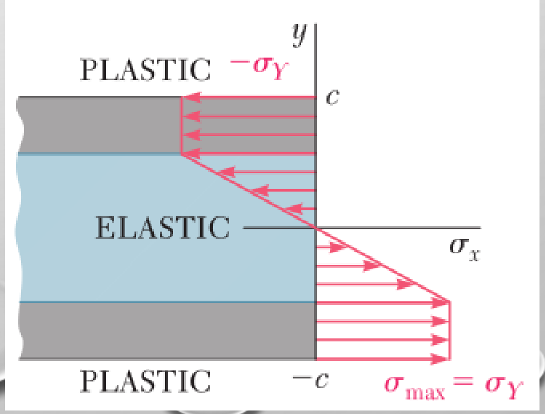


$$\sigma_m = \frac{Mc}{I}$$

$$\sigma_Y = \frac{M_Y c}{I}$$

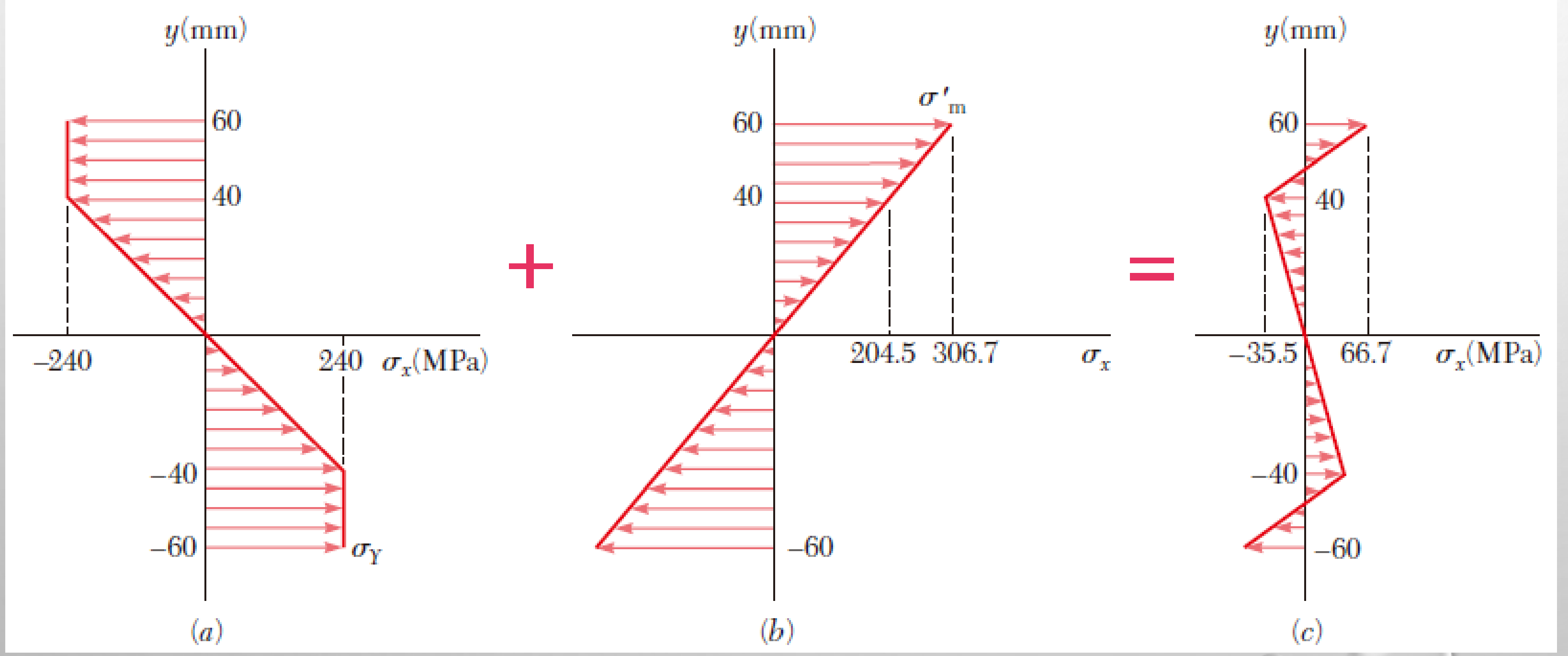


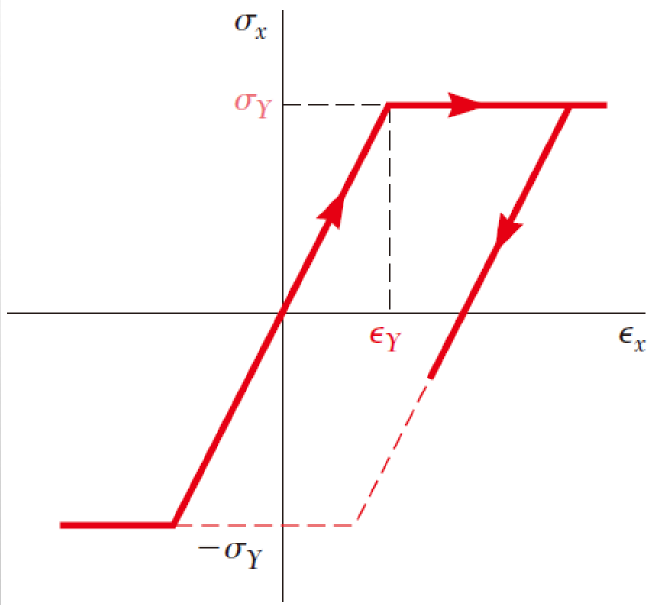
$$M = -2b\int_0^{y_Y} y\left(-\frac{\sigma_Y}{y_Y}y\right)dy - 2b\int_{y_Y}^{c} y(-\sigma_Y)dy$$



§ 5.14 Residual stress

- The residual stresses are obtained by applying the principle of superposition in a manner similar to that described for an axial centric loading and used again for torsion.





§ 5.14 Summary

- **Pure bending**
- **Stresses and deformation in members**
- **Elastic section modulus**
- **Curvature of member**
- **Members made of several materials**
- **Unsymmetric bending**
- **General eccentric axial loading**