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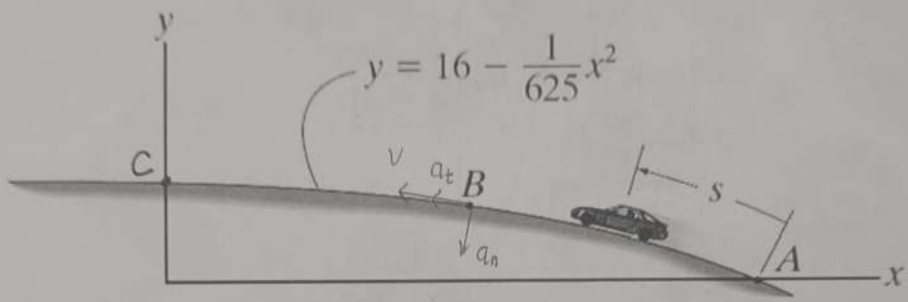
The car passes point A with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s. Determine the magnitude of the car's acceleration when it reaches point B, where $s = 51.5$ m and $x = 50$ m.

If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5\ \text{m/s}^2$, determine the magnitude of the car's acceleration when $s = 101.68$ m and $x = 0$.



1. 解: Consider n-t system

At point B, $V = 25 - 0.15 \times 51.5 = 17.275$ m/s

$$\frac{dy}{dx} = -\frac{2}{625}x$$

$$\frac{d^2y}{dx^2} = -\frac{2}{625}$$

$$\Rightarrow \text{At point B, } \rho = \frac{\left[1 + \left(\frac{-2}{625} \times 50\right)^2\right]^{\frac{3}{2}}}{\left|-\frac{2}{625}\right|} = +324.58\ \text{m}$$

$$\therefore a_n = \frac{v^2}{\rho} = \frac{17.275^2}{324.58} = 0.9194\ \text{m/s}^2$$

$$\because a_t = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = \frac{dv}{ds} \cdot v = -0.15(25 - 0.15S) = -0.15 \times (25 - 0.15 \times 51.5) = -2.5913\ \text{m/s}^2$$

$$\therefore a = \sqrt{a_n^2 + a_t^2} = 2.7496\ \text{m/s}^2$$

2. 解: Consider n-t system.

$$a_t = \frac{dv}{dt} \Rightarrow dv = a_t \cdot dt$$

$$v = \frac{ds}{dt} \Rightarrow ds = v\,dt$$

At point C: $V = V_A + a_t \cdot t = 20 + 0.5t$

$$S = 0 + V_A \cdot t + \frac{1}{2} a_t \cdot t^2 \Rightarrow 101.68 = 20t + 0.25t^2$$

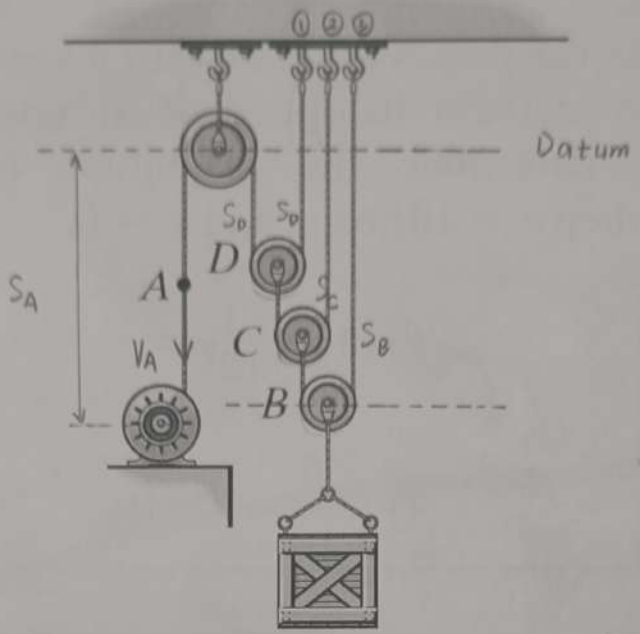
$$\Rightarrow t = 4.796\text{s} \qquad V = 22.40\ \text{m/s}$$

At point C: $$\rho = \frac{(1+0)^{\frac{3}{2}}}{\left|-\frac{2}{625}\right|} = 312.5\ \text{m}$$

$$a_n = \frac{v^2}{\rho} = \frac{22.4^2}{312.5} = 1.606\ \text{m/s}^2$$

$$\therefore a = \sqrt{a_n^2 + a_t^2} = 1.682\ \text{m/s}^2$$

Starting from rest, the cable can be wound onto the drum of the motor at a rate of $v_A = (3t^2)$ m/s, where t is in seconds. Determine the time needed to lift the load 7 m.



解: Three cords

cord ①: $S_A + 2S_D + l_1 = L_1$

cord ②: $S_C + (S_C - S_D) + l_2 = L_2$

cord ③: $S_B + (S_B - S_C) + l_3 = L_3$

time derivative

$$V_A + 2V_D = 0 \qquad \text{where } V_A = -3t^2$$

$$2V_C - V_D = 0$$

$$2V_B - V_C = 0$$

$$\Rightarrow V_B = \tfrac{1}{2}V_C = \tfrac{1}{4}V_D = -\tfrac{1}{8}V_A = +\tfrac{3}{8}t^2$$

$$\frac{ds}{dt} = V_B$$

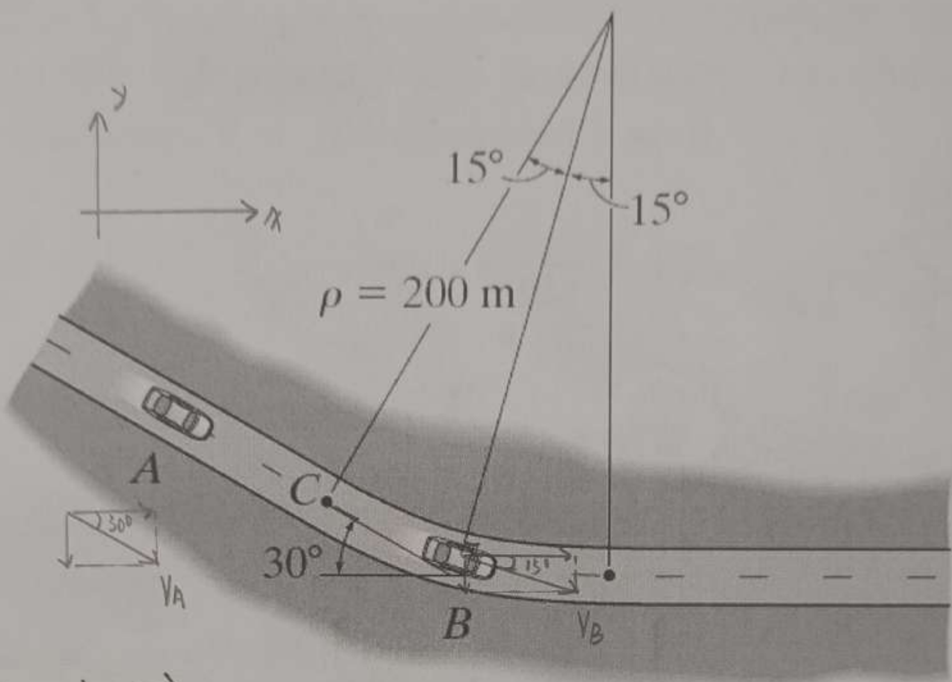
$$ds = V_B \cdot dt$$

$$s = \int_0^t \tfrac{3}{8}t^2\,dt = \tfrac{1}{8}t^3 = 7 \text{ m}$$

$$t = 2\sqrt[3]{7}\text{ s} = 3.826 \text{ s}$$

Jet

At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A.



解:

$$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}}$$

$$\vec{V_{B/A}} = \vec{V_B} - \vec{V_A}$$

$$\vec{V_A} = V_A \cos 30^\circ \vec{i} - V_A \sin 30^\circ \vec{j}$$ where $V_A = 25\ m/s$

$$\vec{V_B} = V_B \cos 15^\circ \vec{i} - V_B \sin 15^\circ \vec{j}$$ where $V_B = 15\ m/s$

$$\therefore \vec{V_{B/A}} = (V_B \cos 15^\circ - V_A \cos 30^\circ)\vec{i} + (V_A \sin 30^\circ - V_B \sin 15^\circ)\vec{j}$$

$$= -7.162\,\vec{i} + 8.618\,\vec{j}$$