One Dimensional Conduction

- Steady Problem without Heat Source
  - Conduction through a pipe wall

    Consider a pipe wall with the inner and outer radii of r_1 and r_2, respectively, and the thermal conductivity λ. The temperatures at the inner and outer walls are T_1 and T_2, respectively. Determine the temperature profile through a plane wall.

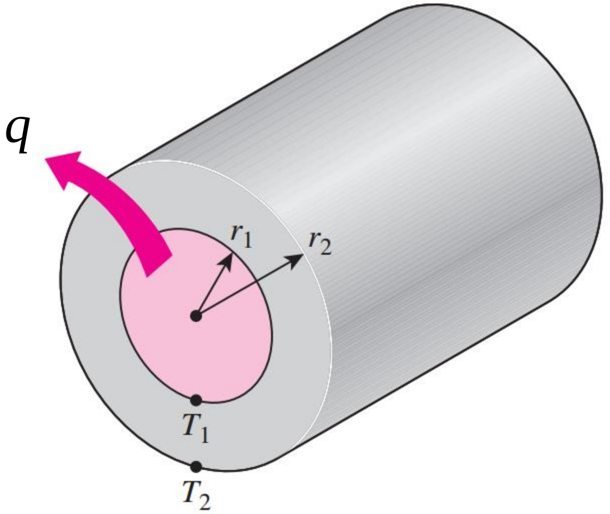
Solution:

Governing equation:

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=0$$

Boundary Condition:

$$r = r_1,\ T = T_1;\ r = r_2,\ T = T_2$$



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Solution:

Integrate the governing equation twice:

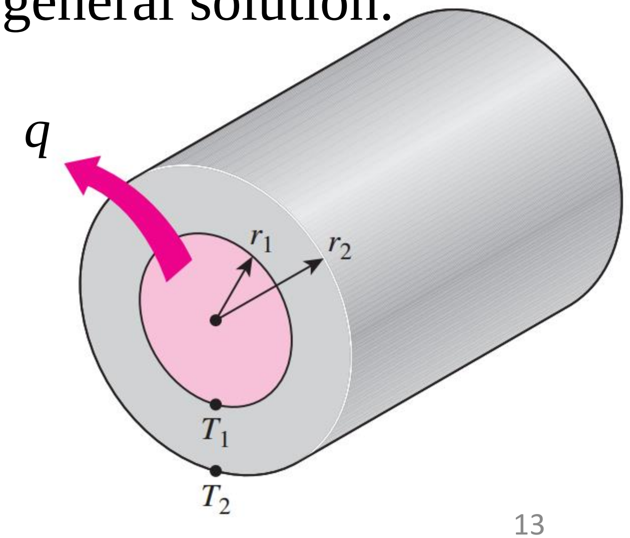
$$T = C_1 \ln r + C_2$$

Substitute the boundary conditions to the general solution:

$$C_1 = \frac{T_2 - T_1}{\ln\left(r_2/r_1\right)}$$

$$C_2 = T_1 - \ln r_1 \frac{T_2 - T_1}{\ln\left(r_2/r_1\right)}$$

$$T = T_1 + \frac{T_2 - T_1}{\ln\left(r_2/r_1\right)} \ln\left(r/r_1\right)$$



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<u>*Solution:*</u>

Heat flux q:

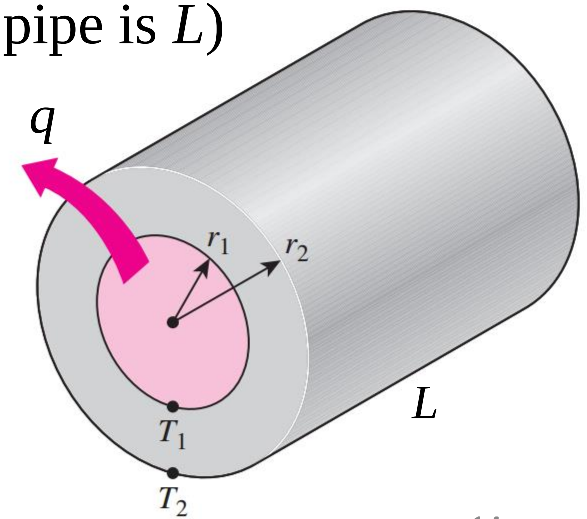
$$q = -\lambda \frac{dT}{dr} = \frac{\lambda}{r} \frac{T_1 - T_2}{\ln(r_2/r_1)}$$

Heat transfer rate Φ (Assume the length of pipe is L)

$$\Phi = qA = 2\pi r L \frac{\lambda}{r} \frac{T_1 - T_2}{\ln(r_2/r_1)} = \frac{2\pi L \lambda (T_1 - T_2)}{\ln(r_2/r_1)}$$

Thermal Resistance

$$R = \frac{T_1 - T_2}{\Phi} = \frac{\ln(r_2/r_1)}{2\pi L \lambda}$$



One Dimensional Conduction

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    The length of the wall is L

    Solution:

    The total thermal resistance:

$$R_t = R_1 + R_2 + R_3$$

$$= \frac{\ln(r_2/r_1)}{2\pi L\lambda_1} + \frac{\ln(r_3/r_2)}{2\pi L\lambda_2} + \frac{\ln(r_4/r_3)}{2\pi L\lambda_3}$$

$$\Phi = \frac{2\pi L(T_1 - T_4)}{\frac{\ln(r_2/r_1)}{\lambda_1} + \frac{\ln(r_3/r_2)}{\lambda_2} + \frac{\ln(r_4/r_3)}{\lambda_3}}$$