Quiz 2

Date: 2022-02-21 Name: SID:

Q1. Answer **True** or **False**. (3 pts)

(a) $\frac{d^2y}{(dx)^2} = y\frac{dy}{dx} + x$ is an ODE.

(b) $y_1 = -2$ and $y_2 = e^{\frac{1}{2}(t+1)} - 2$ are both solutions of $\frac{dy}{dx} = \frac{y}{2} + 1$.

(c) Consider the linear differential equation $x' = 2x + t^2e^t$ and assume that the function ϕ is a solution. Then any other solution to the equation is of the form $C\phi$ where C is a real constant.

Q2. Consider the initial value problem $y' - \frac{3}{2}y = 3t + 2e^t, y(0) = y_0$. Find the value of y_0 that separates solutions that grow positively as $t \to \infty$ from those that grow negatively. (4 pts)

Q3. Solve the differential equation $\frac{dy}{dx} = \frac{y-4x}{x-y}$.(3 pts)

Solution

Q1. T T F.

Q2. $y' - \frac{3}{2}y = 3t + 2e^t$ $\quad u(t) = e^{-\frac{3}{2}t}$

$$y = -2t - \frac{4}{3} - 4e^t + C \cdot e^{\frac{3}{2}t}$$

$$y_0 = -\frac{4}{3} - 4 + C \qquad C = y_0 + \frac{16}{3}$$

as $t \to +\infty$. $\quad y_0 = -\frac{16}{3}$.

Q3. $\frac{dy}{dx} = \frac{\frac{y}{x} - 4}{1 - \frac{y}{x}}$

$v = \frac{y}{x} \quad y = vx$

$v + x\frac{dv}{dx} = \frac{v-4}{1-v}$

$x\frac{dv}{dx} = \frac{v^2-4}{1-v}$

- $v = \pm 2 \quad y = \pm 2x$

- $v \neq \pm 2$ 时

$\frac{1-v}{v^2-4}dv = \frac{1}{x}dx$

$-\frac{3}{4}\frac{dv}{v+2} - \frac{1}{4}\frac{dv}{v-2} = \frac{1}{x}dx$

$|y+2x|^3 \cdot |y-2x| = C$