Homework problems 39-42
Due in class, Friday, 4 December 2020

39. Determine the stress components acting on the inclined plane *AB* by using (a) the method of equilibrium, and (b) the method of stress transformation equations.

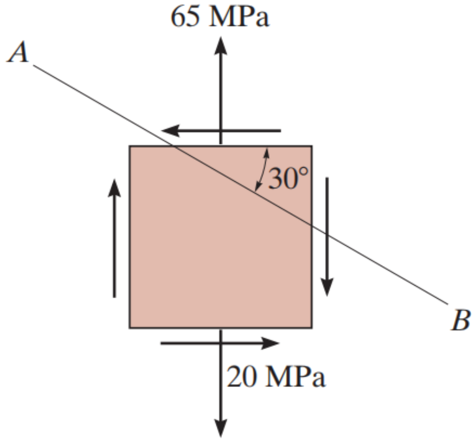


Figure 39

SOLUTION

$\swarrow + \Sigma F_{x'} = 0; \quad \sigma_{x'}\Delta A + 20\Delta A \sin 30^\circ \cos 30^\circ + 20\Delta A \cos 30^\circ \cos 60^\circ$

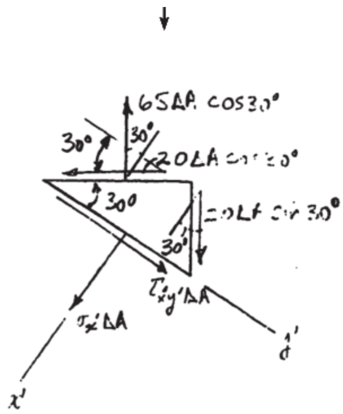
$- 65\Delta A \cos 30^\circ \cos 30^\circ = 0$

$\sigma_{x'} = 31.4 \text{ MPa}$ **Ans.**

$\searrow + \Sigma F_{y'} = 0; \quad \tau_{x'y'}\Delta A + 20\Delta A \sin 30^\circ \sin 30^\circ - 20\Delta A \cos 30^\circ \sin 60^\circ$

$- 65\Delta A \cos 30^\circ \sin 30^\circ = 0$

$\tau_{x'y'} = 38.1 \text{ MPa}$ **Ans.**



40. The wood beam is subjected to a load of 12 kN. If grains of wood in the beam at point *A* make an angle of 25° with the horizontal as shown, determine the normal and shear stress that act perpendicular and parallel to the grains due to the loading at *A*.

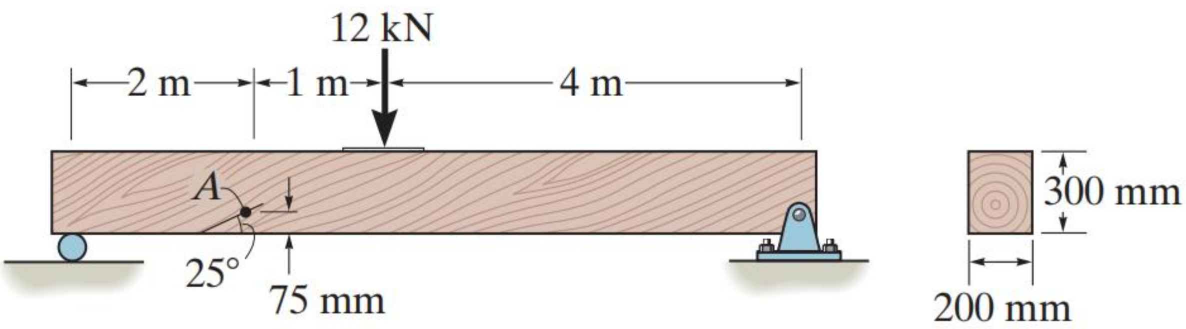


Figure 40

$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45(10^{-3})\ \text{m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My_A}{I} = \frac{13.714(10^3)(0.075)}{0.45(10^{-3})} = 2.2857\ \text{MPa}\ \ \text{(T)}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{6.875(10^3)(1.6875)(10^{-3})}{0.45(10^{-3})(0.2)} = 0.1286\ \text{MPa}$$

$$\sigma_x = 2.2857\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -0.1286\ \text{MPa} \qquad \theta = 115°$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

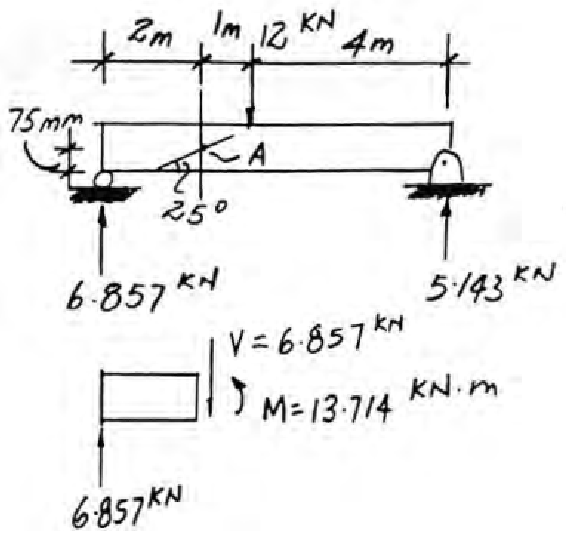
$$\sigma_{x'} = \frac{2.2857 + 0}{2} + \frac{2.2857 - 0}{2}\cos 230° + (-0.1286)\sin 230°$$

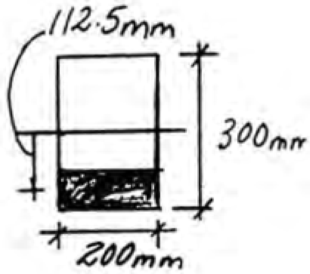
$$= 0.507\ \text{MPa}$$ **Ans.**

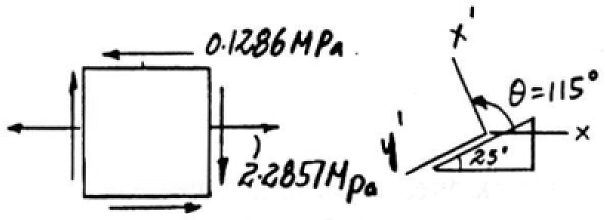
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{2.2857 - 0}{2}\right)\sin 230° + (-0.1286)\cos 230°$$

$$= 0.958\ \text{MPa}$$ **Ans.**







41. The state of stress at a point is shown on the element. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.

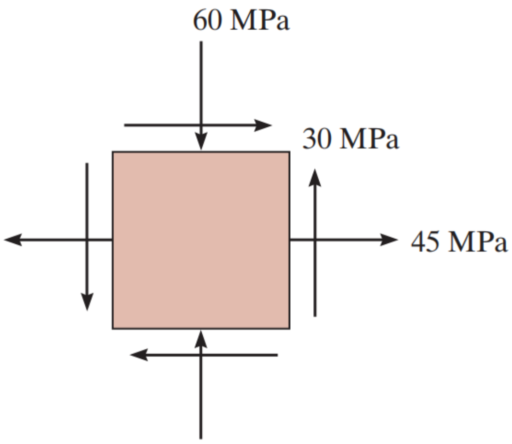


Figure 41

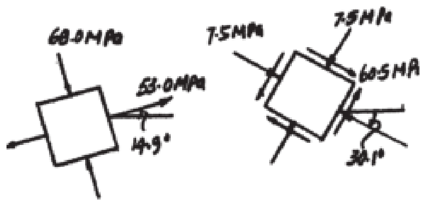
SOLUTION

$\sigma_x = 45$ MPa $\quad \sigma_y = -60$ MPa $\quad \tau_{xy} = 30$ MPa

a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + (30)^2}$$

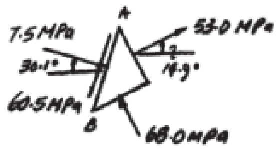
$\sigma_1 = 53.0$ MPa $\qquad \sigma_2 = -68.0$ MPa **Ans.**



Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{30}{(45 - (-60))/2} = 0.5714$$

$\theta_p = 14.87, \quad -75.13$



Use Eq. 9–1 to determine the principal plane of σ_1 and σ_2:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 14.87^\circ$$

$$= \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2}\cos 29.74^\circ + 30\sin 29.74^\circ = 53.0 \text{ MPa}$$

Therefore $\theta_{p1} = 14.9^\circ$ and $\theta_{p2} = -75.1^\circ$ **Ans.**

b)

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2}$$

$= 60.5$ MPa **Ans.**

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50 \text{ MPa}$$ **Ans.**

Orientation of maximum in-plane shear stress:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(45 - (-60))/2}{30} = -1.75$$

$\theta_s = -30.1^\circ$ and $\theta_s = 59.9^\circ$ **Ans.**

By observation, in order to preserve equilibrium along AB, τ_{max} has to act in the direction shown.

Ans:
$\sigma_1 = 53.0$ MPa,
$\sigma_2 = -68.0$ MPa,
$\theta_{p1} = 14.9^\circ$,
$\theta_{p2} = -75.1^\circ$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 60.5$ MPa,
$\sigma_{avg} = -7.50$ MPa,
$\theta_s = -30.1^\circ$, $\theta_s = 59.9^\circ$

42. Determine the principal stresses and the absolute maximum shear stress. Specify the corresponding orientations.

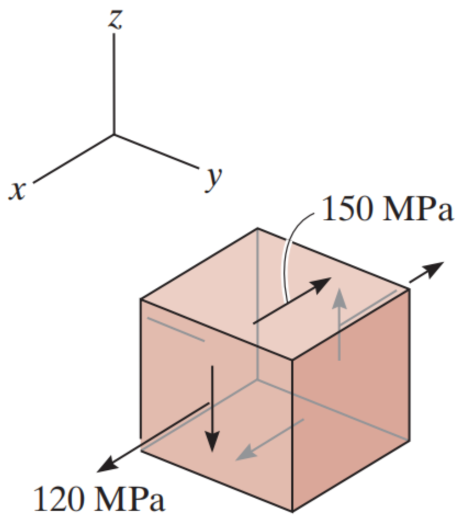


Figure 42

SOLUTION

For x - z plane:

$R = CA = \sqrt{(120 - 60)^2 + 150^2} = 161.55$

$\sigma_1 = 60 + 161.55 = 221.55 \text{ MPa}$

$\sigma_2 = 60 - 161.55 = -101.55 \text{ MPa}$

$\sigma_1 = 222 \text{ MPa} \qquad \sigma_2 = -102 \text{ MPa}$ **Ans.**

$\tau_{\substack{\text{abs}\\ \text{max}}} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2} = \dfrac{221.55 - (-101.55)}{2} = 162 \text{ MPa}$ **Ans.**