Problem 1

Determine the [modulus of resilience] for each of the following metals:

(a) Stainless steel
AISI 302 (annealed): $E = 190$ GPa $\sigma_Y = 260$ MPa

(b) Stainless steel 2014-T6
AISI 302 (cold-rolled): $E = 190$ GPa $\sigma_Y = 520$ MPa

(c) Malleable cast iron: $E = 165$ GPa $\sigma_Y = 230$ MPa

解: a) $MR = \frac{1}{2}\sigma_Y \varepsilon_Y = \frac{1}{2}\sigma_Y^2 \cdot \frac{1}{E} = \frac{\frac{1}{2}\times(260\times10^6)^2}{190\times10^9} = 0.1779\ MPa = 177.9\ kPa$

b) $MR = \frac{1}{2}\frac{\sigma_Y^2}{E} = \frac{(520\times10^6)^2}{2\times190\times10^9} = 0.7116\ MPa = 711.6\ kPa$

c) $MR = \frac{1}{2}\frac{\sigma_Y^2}{E} = \frac{(230\times10^6)^2}{2\times165\times10^9} = 0.1603\ MPa = 160.3\ kPa$

Problem 2

Rod AB is made of a steel for which the yield strength is $\sigma_Y = 450$ MPa and $E = 200$ GPa; rod BC is made of an aluminum alloy for which $\sigma_Y = 280$ MPa and $E = 73$ GPa. Determine the maximum strain energy that can be acquired by the composite rod ABC without causing any permanent deformations.

解: for rod AB

$P_{AB} = \sigma_{Y_1} \cdot A_{AB} = 450\times10^6 \times \pi \times (5\times10^{-3})^2 = 11250\pi$ N

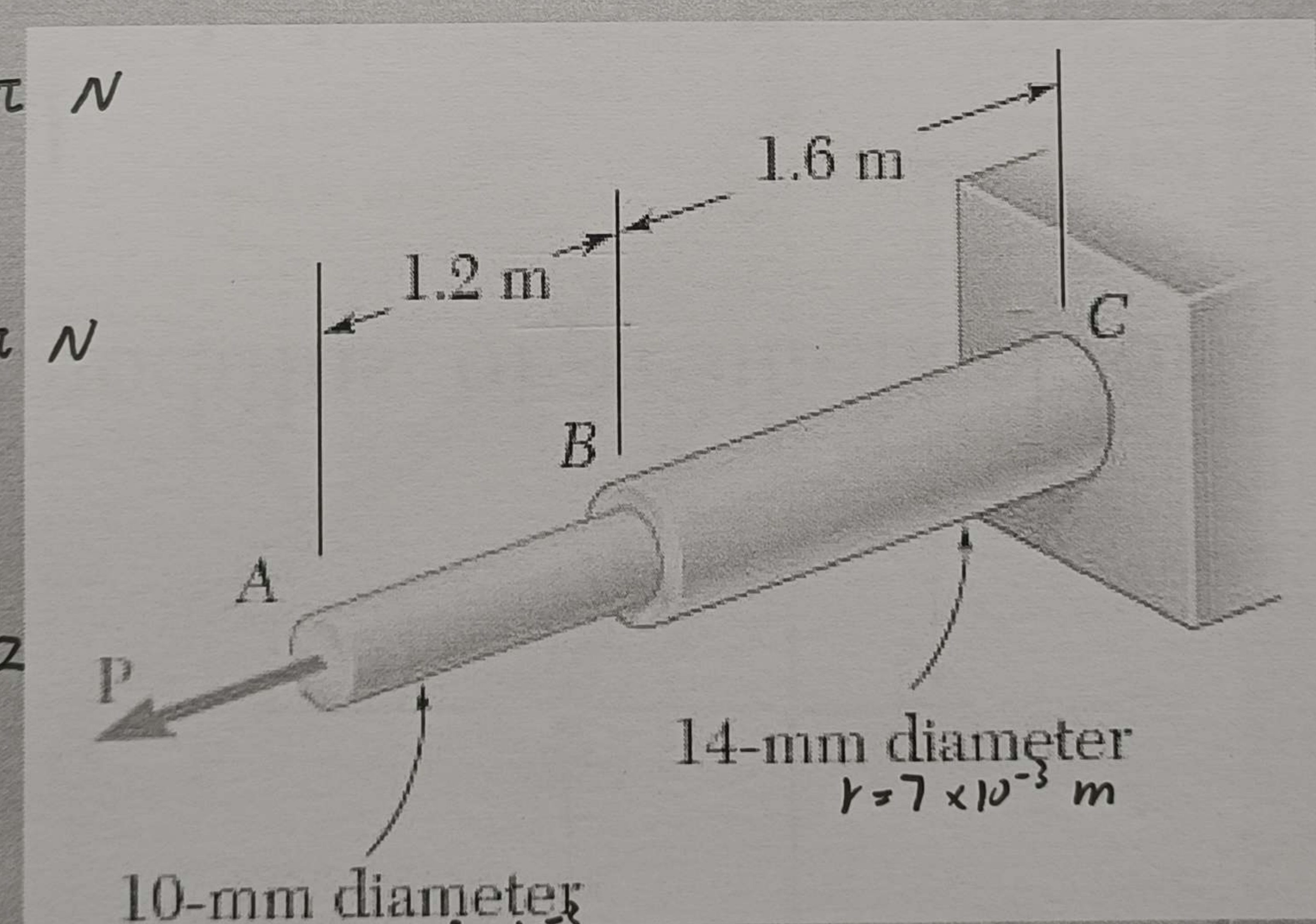
for rod BC

$P_{BC} = \sigma_{Y_2} \cdot A_{BC} = 280\times10^6 \times \pi \times (7\times10^{-3})^2 = 13720\pi$ N

So $P_{max} = 11250\pi$ N

$U_{AB} = \int \frac{\sigma_{Y_1}^2}{2E} dV = \frac{(450\times10^6)^2}{2\times200\times10^9} \times \pi \times (5\times10^{-3})^2 \times 1.2$

$= \frac{243}{16}\pi$ J



$\sigma_{BC} = \frac{P_{max}}{A_{BC}}$

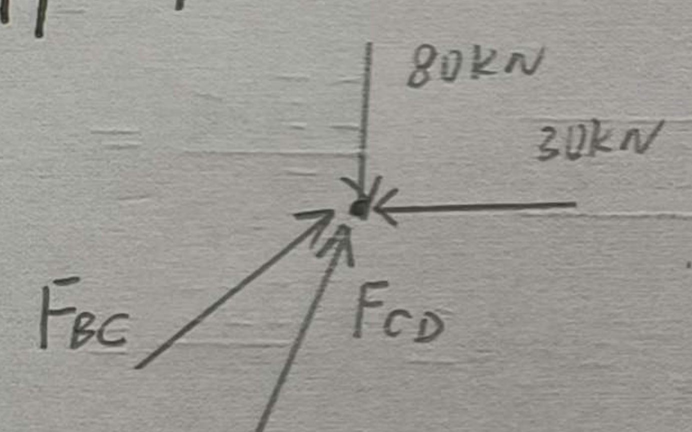
$U_{BC} = \int \frac{\sigma_{BC}^2}{2E} dV = \frac{P_{max}^2}{2E A_{BC}^2} \cdot A_{BC} \cdot L_{BC} = \frac{P_{max}^2}{2E A_{BC}} \cdot L_{BC} = \frac{(11250\pi)^2 \times 1.6}{2\times73\times10^9 \times \pi \times (7\times10^{-3})^2} = 88.925$ J

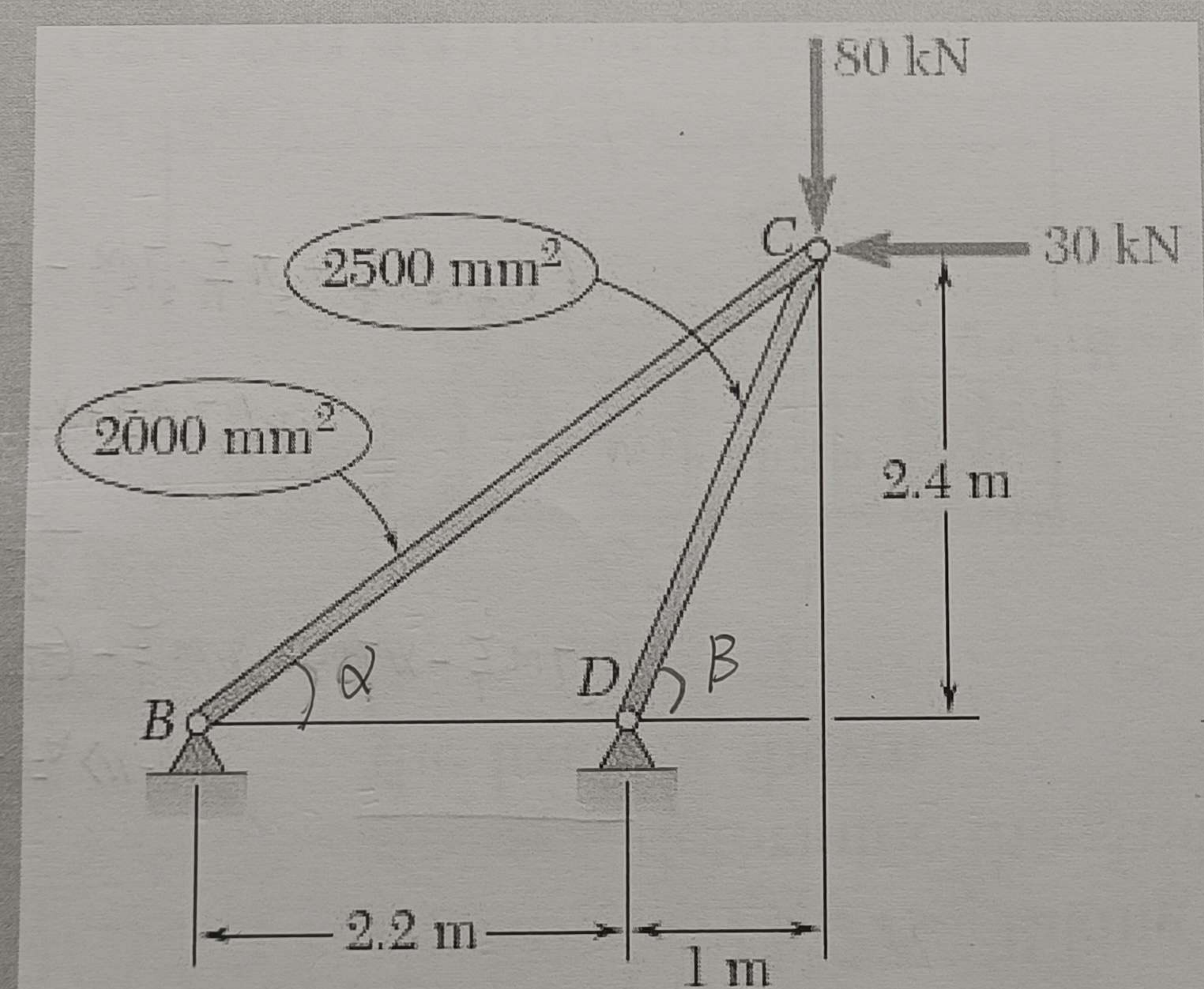
$\therefore U_{ABC} = U_{AB} + U_{BC} = 136.6$ J $\boxed{\text{ANS}}$

Problem 3

Each member of the truss shown is made of aluminum and has the cross-sectional area shown. Using $E = 72$ GPa, determine the strain energy of the truss for the loading shown.

解: for node C





$\xrightarrow{+} \sum F_x = 0: \ -30 + F_{BC} \cdot \cos\alpha + F_{CD} \cos\beta = 0$

$+\uparrow \sum F_y = 0: \ -80 + F_{BC} \cdot \sin\alpha + F_{CD} \sin\beta = 0$

$\cos\alpha = \dfrac{3.2}{\sqrt{3.2^2 + 2.4^2}} = 0.8, \ \sin\alpha = 0.6$

$\cos\beta = \dfrac{1}{\sqrt{1 + 2.4^2}} = \dfrac{5}{13}, \ \sin\beta = \dfrac{12}{13}$

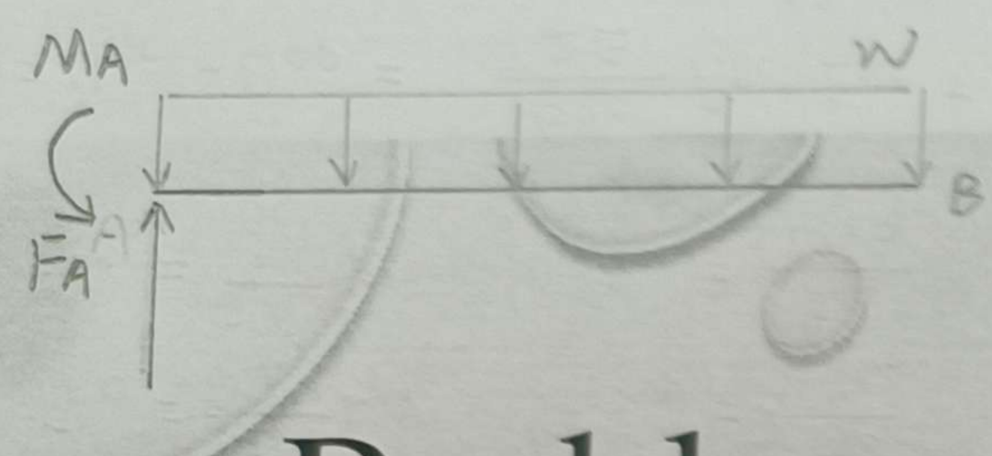
$\Rightarrow F_{BC} = -\dfrac{200}{33} kN, \quad \sigma_{BC} = \dfrac{F_{BC}}{A_{BC}}$

$F_{CD} = \dfrac{2990}{33} kN, \quad \sigma_{CD} = \dfrac{F_{CD}}{A_{CD}}$

$$U = U_{BC} + U_{CD} = \frac{\sigma_{BC}^2}{2E} A_{BC} \cdot L_{BC} + \frac{\sigma_{CD}^2}{2E} \cdot A_{CD} \cdot L_{CD} = \frac{F_{BC}^2}{2E \cdot A_{BC}} \cdot L_{BC} + \frac{F_{CD}^2}{2E \cdot A_{CD}} \cdot L_{CD}$$

$$= \frac{(\frac{200}{33} \times 10^3)^2 \times 4}{2 \times 72 \times 10^9 \times 2000 \times 10^{-6}} + \frac{(\frac{2990}{33} \times 10^3)^2 \times 2.6}{2 \times 72 \times 10^9 \times 2500 \times 10^{-6}} = 59.80 \text{ J} \quad \boxed{\text{ANS}}$$

11.24解:



$+\uparrow \Sigma F_y=0:\ F_A - wL = 0$

$\circlearrowleft \Sigma M_A=0:\ M_A - wL\cdot\frac{L}{2} = 0$

$\Rightarrow\ F_A = wL$

$M_A = \frac{1}{2}wL^2$

Problem 4

$w(x) = w\langle x-0\rangle^0 \qquad V(x) = -w\langle x-0\rangle^1 + F_A\langle x-0\rangle^0$

$M(x) = -\frac{1}{2}w\langle x-0\rangle^2 + F_A\langle x-0\rangle^1 - M_A\langle x-0\rangle^0 = -\frac{1}{2}wx^2 + wLx - \frac{1}{2}wL^2$

$U = \int_0^L \frac{M^2(x)}{2EI}dx$

$= \frac{1}{2EI}\int_0^L\left(\frac{1}{4}w^2x^4 - w^2Lx^3 + \frac{3}{2}w^2L^2x^2 - w^2L^3x + \frac{1}{4}w^2L^4\right)dx$

$= \frac{1}{2EI}\left(\frac{1}{4}w^2\times\frac{1}{5}L^5 - w^2L\cdot\frac{1}{4}L^4 + \frac{3}{2}w^2L^2\cdot\frac{1}{3}L^3 - w^2L^3\cdot\frac{1}{2}L^2 + \frac{1}{4}w^2L^4\cdot L\right)$

$= \frac{w^2L^5}{40EI}$ $\boxed{\text{ANS}}$

11.24 through 11.27 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.

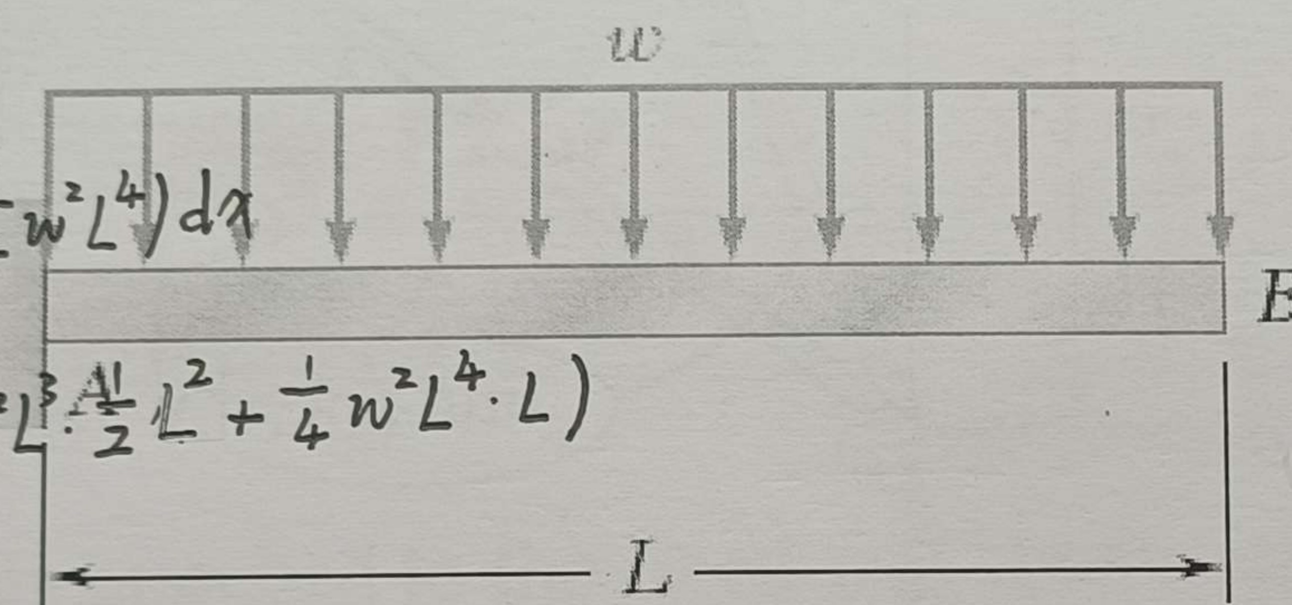


Fig. P11.24

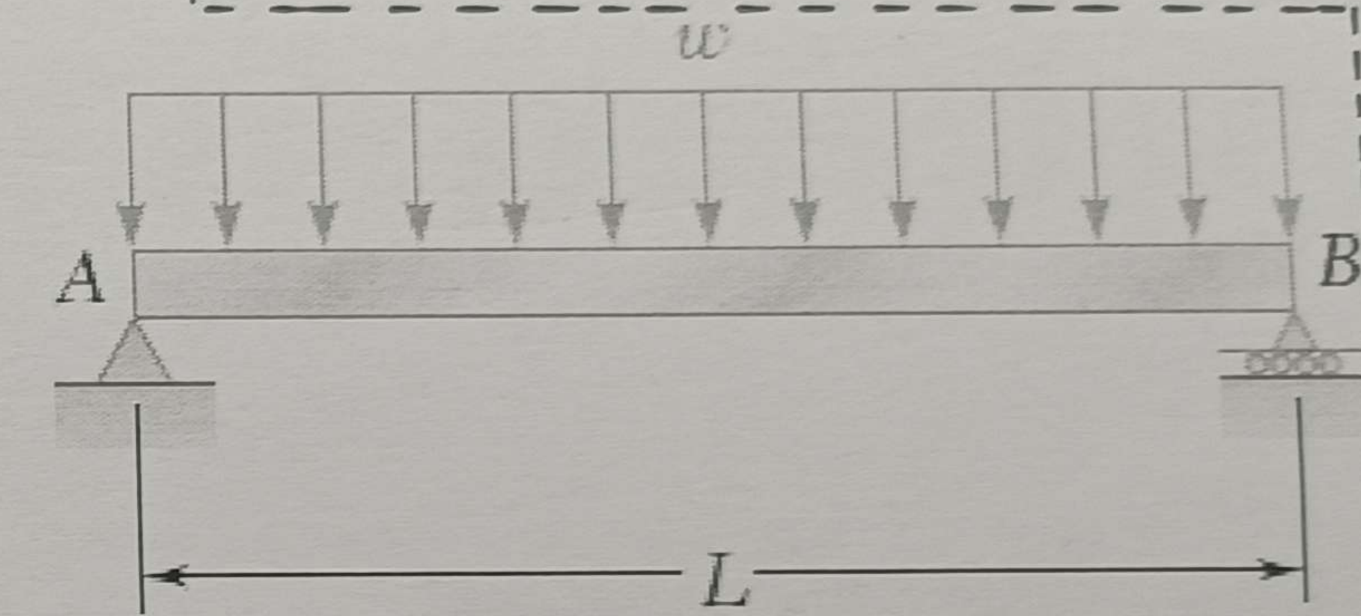
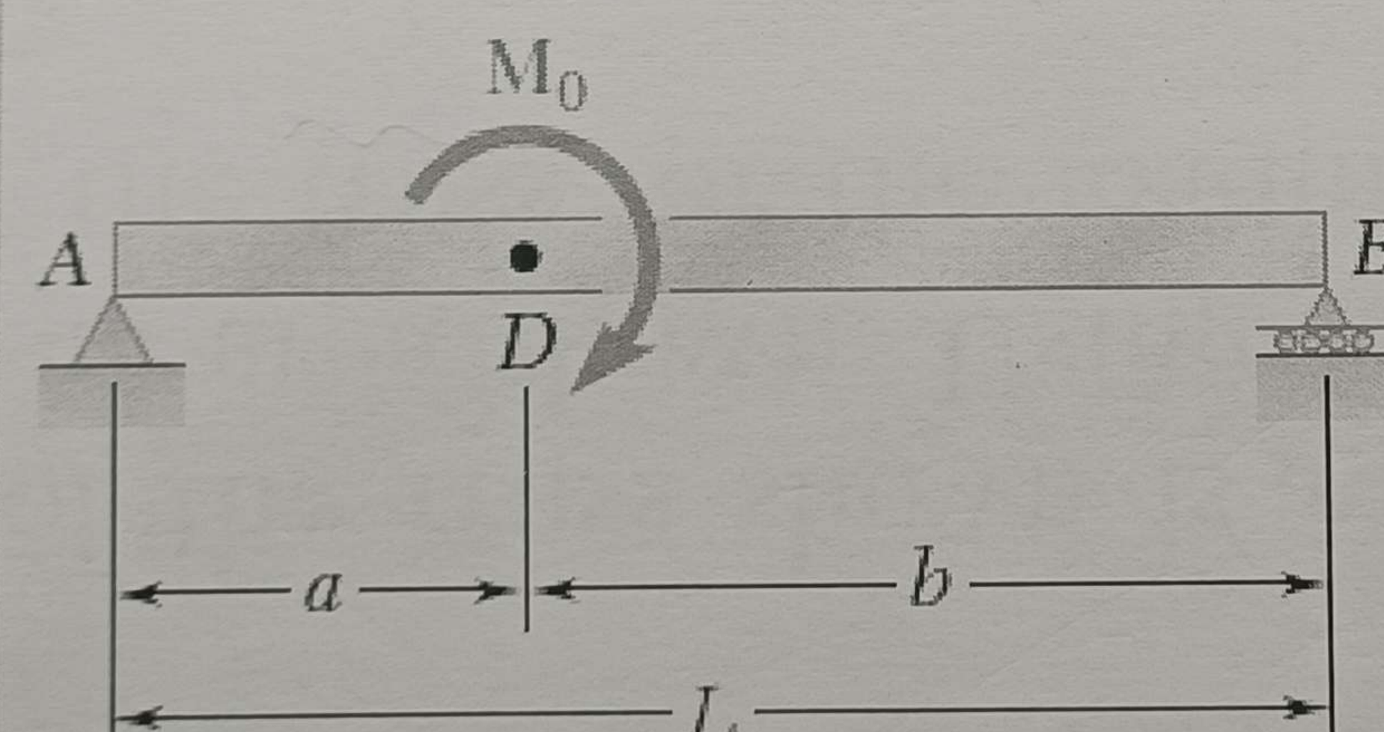


Fig. P11.25



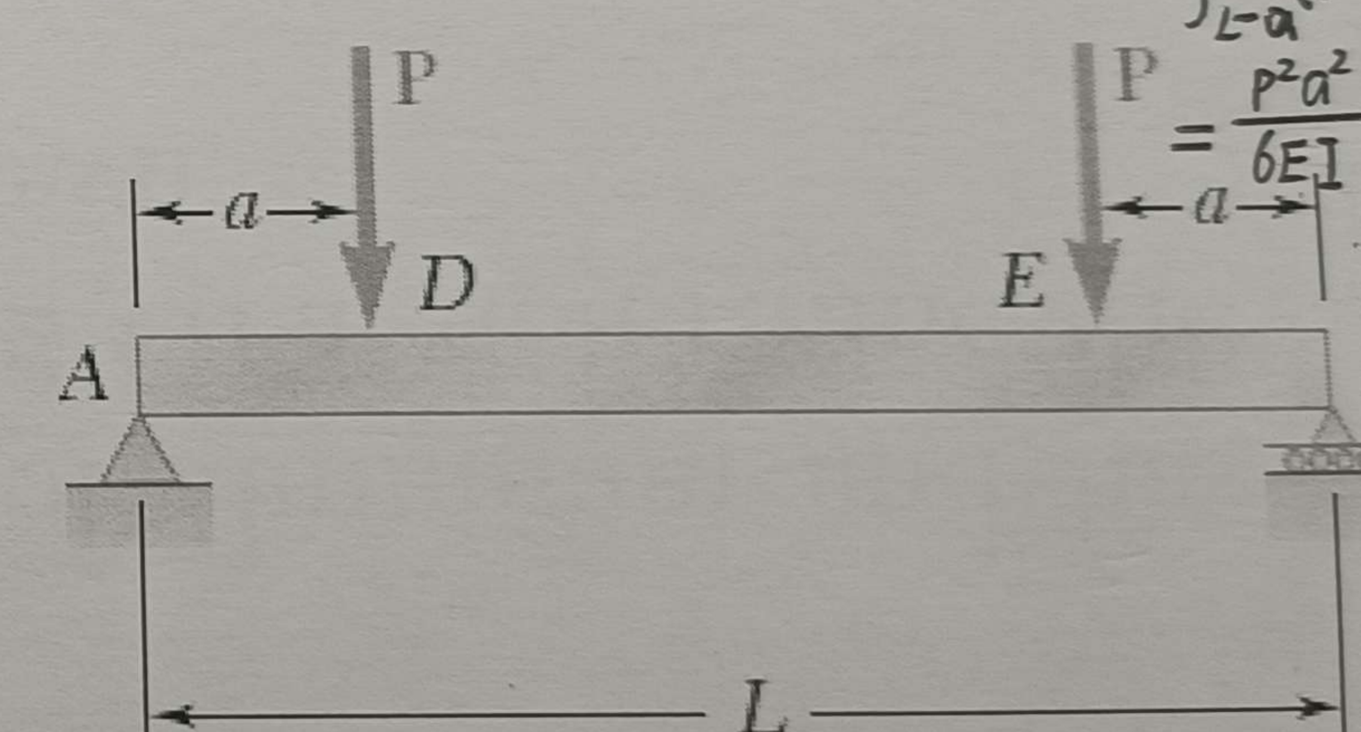
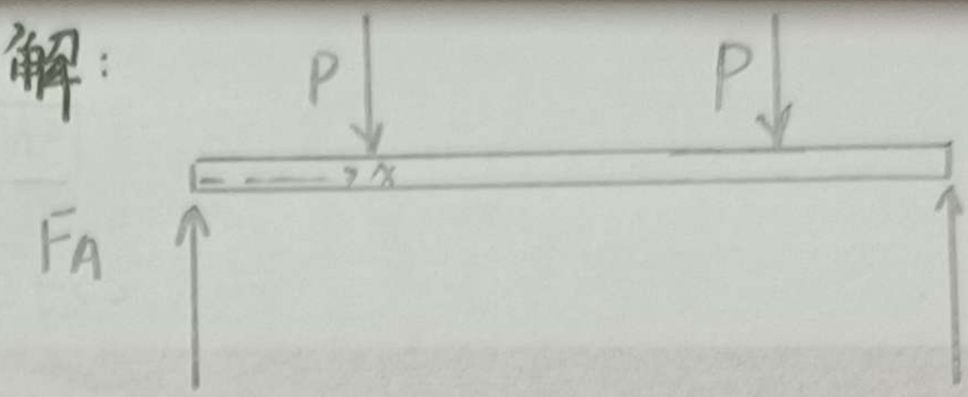


Fig. P11.27

11.27解:



$+\uparrow \Sigma F_y=0:\ F_A + F_B = 2P$

$\circlearrowleft \Sigma M_A=0:\ -P\cdot a - P(L-a) + F_B\cdot L = 0$

$\Rightarrow\ F_A = P \qquad F_B = P$

$V(x) = F_A\langle x-0\rangle^0 - P\langle x-a\rangle^0 - P\langle x-L+a\rangle^0$

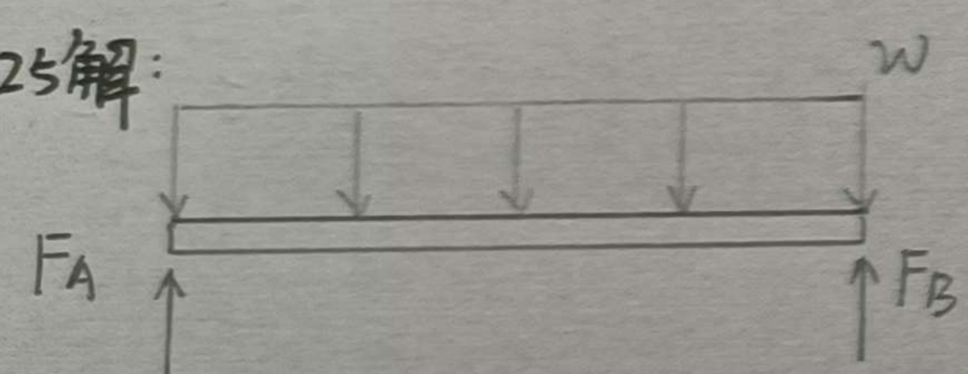
$M(x) = F_A\langle x-0\rangle^1 - P\langle x-a\rangle^1 - P\langle x-L+a\rangle^1$

$M(x) = \begin{cases} F_A x \\ F_A x - P(x-a) \\ F_A x - P(x-a) - P(x-L+a) \end{cases} = \begin{cases} Px & 0<x<a \\ aP & a<x<L-a \\ PL-Px & L-x<x<L \end{cases}$

$U = \int\frac{M^2(x)}{2EI} = \frac{1}{2EI}\left(\int_0^a P^2x^2dx + \int_a^{L-a}a^2P^2dx + \int_{L-a}^{L}(PL-Px)^2dx\right)$

$= \frac{P^2a^2}{6EI}(3L-4a)$ $\boxed{\text{ANS}}$

11.25解:



$+\uparrow \Sigma F_y=0:\ F_A + F_B - wL = 0 \quad\Rightarrow\quad F_A = \frac{1}{2}wL$

$\circlearrowleft \Sigma M_A=0:\ -wL\cdot\frac{L}{2} + F_B\cdot L = 0 \qquad F_B = \frac{1}{2}wL$

$w(x) = w\langle x-0\rangle^0$

$V(x) = -w\langle x-0\rangle^1 + F_A\langle x-0\rangle^0$

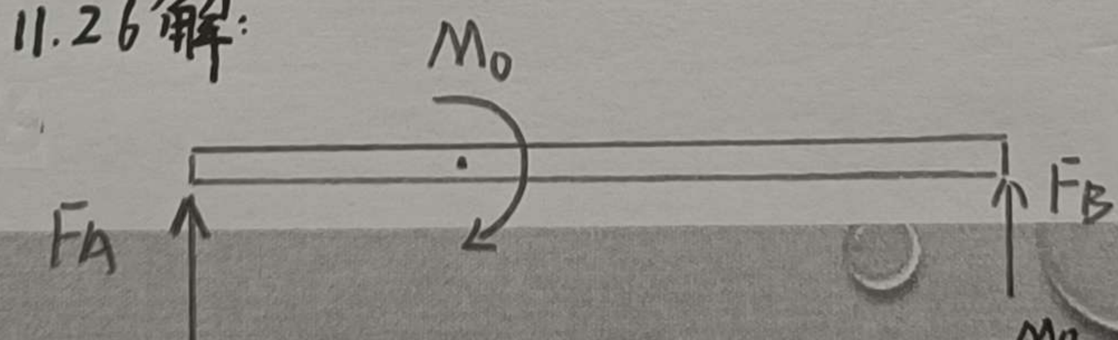
$M(x) = -\frac{1}{2}w\langle x-0\rangle^2 + F_A\langle x-0\rangle^1 = -\frac{1}{2}wx^2 + \frac{1}{2}wLx$

$U = \int_0^L\frac{M^2(x)}{2EI}dx = \frac{1}{2EI}\int_0^L\left(\frac{1}{4}w^2x^4 + \frac{1}{4}w^2L^2x^2 - \frac{1}{2}w^2Lx^3\right)dx$

$= \frac{1}{2EI}\left(\frac{1}{4}w^2\cdot\frac{1}{5}L^5 + \frac{1}{4}w^2L^2\cdot\frac{1}{3}L^3 - \frac{1}{2}w^2L\cdot\frac{1}{4}L^4\right)$

$= \frac{w^2L^5}{240EI}$ $\boxed{\text{ANS}}$

11.26解:



$+\uparrow \Sigma F_y=0:\ F_A + F_B = 0 \quad\Rightarrow\quad F_A = \frac{-M_0}{L}$

$\circlearrowleft \Sigma M_A=0:\ -M_0 + F_B\cdot L = 0 \qquad F_B = \frac{M_0}{L}$

$V(x) = F_A\langle x-0\rangle^0$

$M(x) = F_A\langle x-0\rangle^1 + M_0\langle x-a\rangle^0 = \begin{cases} \frac{-M_0}{L}x & 0<x<a \\ -\frac{M_0x}{L} + M_0 & a<x<L \end{cases}$

$U = \int_0^a\frac{M^2(x)}{2EI}dx + \int_a^{a+b}\frac{M^2(x)}{2EI}dx$

$= \frac{1}{2EI}\int_0^a\frac{M_0^2x^2}{L^2}dx + \frac{1}{2EI}\int_a^{a+b}\left(M_0^2 + \frac{M_0^2x^2}{L^2} - \frac{2xM_0^2}{L}\right)dx$

$= \frac{1}{2EI}\left(\frac{M_0^2}{L^2}\cdot\frac{1}{3}a^3 + M_0^2\cdot b + \frac{M_0^2}{L^2}\cdot\frac{1}{3}[(a+b)^3-a^3] - \frac{M_0^2}{L}[(a+b)^2-a^2]\right)$

$= \frac{M_0^2}{2EI}\left(\frac{a^3+b^3}{3L^2}\right) = \frac{M_0^2(a^3+b^3)}{6EIL^2}$ $\boxed{\text{ANS}}$,