空气动力学 HW11

10.2

解: isentropically.

$P_0 = 1\text{ atm}$

$P_e = 0.3143\text{ atm}$

$$\frac{P_0}{P_e} = \left(\frac{T_0}{T_e}\right)^{\frac{\gamma}{\gamma-1}} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{\gamma-1}} = \frac{1}{0.3143}$$

$$\Rightarrow \quad M_e = 1.400$$

$$\frac{A_e}{A^*} = \left(\frac{1}{M_e^2}\cdot\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}\right)^{\frac{1}{2}}$$

$$= \frac{1}{M_e}\cdot\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{2\gamma-2}}$$

$$= \frac{1}{1.4}\times\left[\frac{2}{2.4}\times(1+0.2\times1.4^2)\right]^{\frac{2.4}{0.8}}$$

$$= 1.1149 \quad \boxed{\text{ANS}}$$

10.3

解: $P_e = 8.92\times10^4\ Pa = P_{0.2}$

$P_0 = 2.02\times10^5\ Pa = P_{0.1}$

$$\frac{P_e}{P_0} = \frac{P_{0.2}}{P_{0.1}} = \frac{8.92\times10^4}{2.02\times10^5} = 0.4416$$

From appendix B, $M_e = 2.65$

From appendix A, at $M_e=2.65$, $\frac{A_e}{A^*} = 3.036$

$\boxed{\text{ANS}}$

10.5

解:

$$\dot m = \rho^* A^* u^* = \rho^* A^* a^*$$

$$\frac{P_0}{P^*} = \left(\frac{\rho_0}{\rho^*}\right)^{\gamma} = \left(\frac{T_0}{T^*}\right)^{\frac{\gamma}{\gamma-1}} = \left(1+\frac{\gamma-1}{2}\cdot1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \quad \rho^* = \rho_0\left(\frac{\gamma+1}{2}\right)^{\frac{-1}{\gamma-1}} = \frac{P_0}{RT_0}\left(\frac{\gamma+1}{2}\right)^{\frac{-1}{\gamma-1}}$$

$$a^* = \sqrt{\gamma R T^*} = \sqrt{\gamma\cdot R\cdot\frac{2}{\gamma+1}T_0}$$

$$\therefore \quad \dot m = A^*\cdot\frac{P_0}{RT_0}\left(\frac{\gamma+1}{2}\right)^{\frac{-1}{\gamma-1}}\sqrt{\gamma R\frac{2}{\gamma+1}\cdot T_0}$$

$$= \frac{P_0A^*}{\sqrt{T_0}}\cdot\frac{1}{\sqrt R}\cdot\sqrt{\left(\frac{\gamma+1}{2}\right)^{\frac{-2}{\gamma-1}}\cdot\frac{2}{\gamma+1}\cdot\gamma}$$

$$= \frac{P_0A^*}{\sqrt{T_0}}\cdot\sqrt{\frac{\gamma}{R}}\cdot\sqrt{\left(\frac{2}{\gamma+1}\right)^{1+\frac{2}{\gamma-1}}}$$

$$= \frac{P_0\cdot A^*}{\sqrt{T_0}}\cdot\sqrt{\frac{\gamma}{R}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

10.7

解: $\frac{A_e}{A_t} = 1.616$

$P_e = 0.947\text{ atm}$, $P_0 = 1.0\text{ atm}$

isentropic flow

$$\frac{P_0}{P_e} = \left(\frac{T_0}{T_e}\right)^{\frac{\gamma}{\gamma-1}} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{\gamma-1}} = \frac{1.0}{0.947}$$

$$\Rightarrow \quad M_e = 0.280$$

$$\left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

$$= \frac{1}{0.28^2}\cdot\left[\frac{2}{2.4}\cdot(1+0.2\times0.28^2)\right]^{\frac{2.4}{0.4}} = 4.6896$$

$$\frac{A_e}{A^*} = 2.1656$$

$$\frac{A_t}{A^*} = \frac{A_t}{A_e}\cdot\frac{A_e}{A^*} = \frac{1}{1.616}\times2.1656 = 1.34$$

$$= \frac{1}{M_t^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_t^2\right)\right]^{\frac{\gamma+1}{\gamma-1}} \Rightarrow M_t = 0.5$$

$\boxed{\text{ANS}}$

$$\frac{P_0}{P_t} = \left(1+\frac{\gamma-1}{2}M_t^2\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow P_t = 0.843\text{ atm}$$

$\boxed{\text{ANS}}$

10.8

解: $T_0 = 288K$

$A_t = 0.3\ m^2$

Since $M_t = 0.5 < 1$,

We choose $\dot{m} = \rho_e u_e A_e$ to calculate

$$\frac{P_0}{P_e} = \frac{1}{0.947} = 1.056$$

from Appendix A: $M_e = 0.28$

$$\frac{T_0}{T_e} = 1.016 \Rightarrow T_e = \frac{T_0}{1.016} = 283.46\ K$$

$\because P_e = \rho_e R T_e$

$$\Rightarrow \rho_e = \frac{P_e}{RT_e} = \frac{0.947\times10^5\times1.01}{287\times283.46} = 1.1757\ kg/m^3$$

$$u_e = M_e a_e = M_e\cdot\sqrt{\gamma R T_e}$$

$$= 0.28\times\sqrt{287\times1.4\times283.46} = 94.50\ m/s$$

$\therefore \dot{m} = 53.86\ kg/s$

10.9

解: $\frac{A_e}{A_t} = 1.53$

$P_0 = 1\ atm$

a) $P_e = 0.94\ atm$

$$\frac{P_0}{P_e} = \frac{1}{0.94} = 1.064$$

From Appendix A: $\frac{A_e}{A^*} = 2.035 > \frac{A_e}{A_t}$, $A_t > A^*$

$\therefore$ the flow is subsonic at throat

From Appendix A: $M_e = 0.3$

b) $P_e = 0.886$

$$\frac{P_0}{P_e} = \frac{1}{0.886} = 1.129$$

From Appendix A: $\frac{A_e}{A^*} = 1.529 \approx \frac{A_e}{A_t}$, $A_t \approx A^*$

$\therefore$ the flow is sonic at throat

From Appendix A: $M_e = 0.42$

c) $P_e = 0.75\ atm$

$\frac{P_0}{P_e} = 1.333$,

From Appendix A: $\frac{A_e}{A^*} = 1.127 < \frac{A_e}{A_t}$, $A_t < A^*$

it means there is shock wave inside the nozzle.

d) $P_e = 0.154\ atm$

$\frac{P_0}{P_e} = 6.494$, based on c)

there is shock wave inside the nozzle

10.11

解: $\frac{A_e}{A_t} = 6.79$

$P_e = 1.448\ atm = P_{0.2}$

$$\left(\frac{A_e}{A^*}\right)^2 = \left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{\gamma-1}} = 6.79^2$$

$\Rightarrow M_e = 3.5$

From Appendix B: $\frac{P_{0.2}}{P_{0.1}} = 0.2129$

$\therefore P_{0.1} = 6.8013\ atm$

10.13

解: $\dot{m} = 287.2\ kg/s$

$T_0 = 3600\ k$

$A_t = 0.2\ m^2$

$R_g = \frac{R}{M} = \frac{8.314472}{16\times10^{-3}} = 519.65\ J/(kg\cdot k)$

$\dot{m} = \rho_t \cdot A_t \cdot u_t = \rho^* \cdot A^* \cdot a^*$

$\frac{\rho_0}{\rho^*} = \left(\frac{T_0}{T^*}\right)^{\frac{1}{\gamma-1}} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}}$

$\Rightarrow\ T^* = \frac{2}{\gamma+1}\cdot T_0 = \frac{2}{2.2}\times 3600 = 3273\ k$

$a^* = \sqrt{\gamma R_g T^*} = \sqrt{1.2\times519.65\times3273}$

$= 1428.57\ m/s$

$\rho^* = \frac{\dot{m}}{A^*\cdot a^*} = 1.0052\ kg/m^3$

$\rho_0 = 1.6189\ kg/m^3$

$P_0 = \rho_0 R_g T_0 = 3.0285\times10^6\ Pa$ $\boxed{\text{ANS}}$