1+27 28

Problem 1

9.5 and 9.6 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.

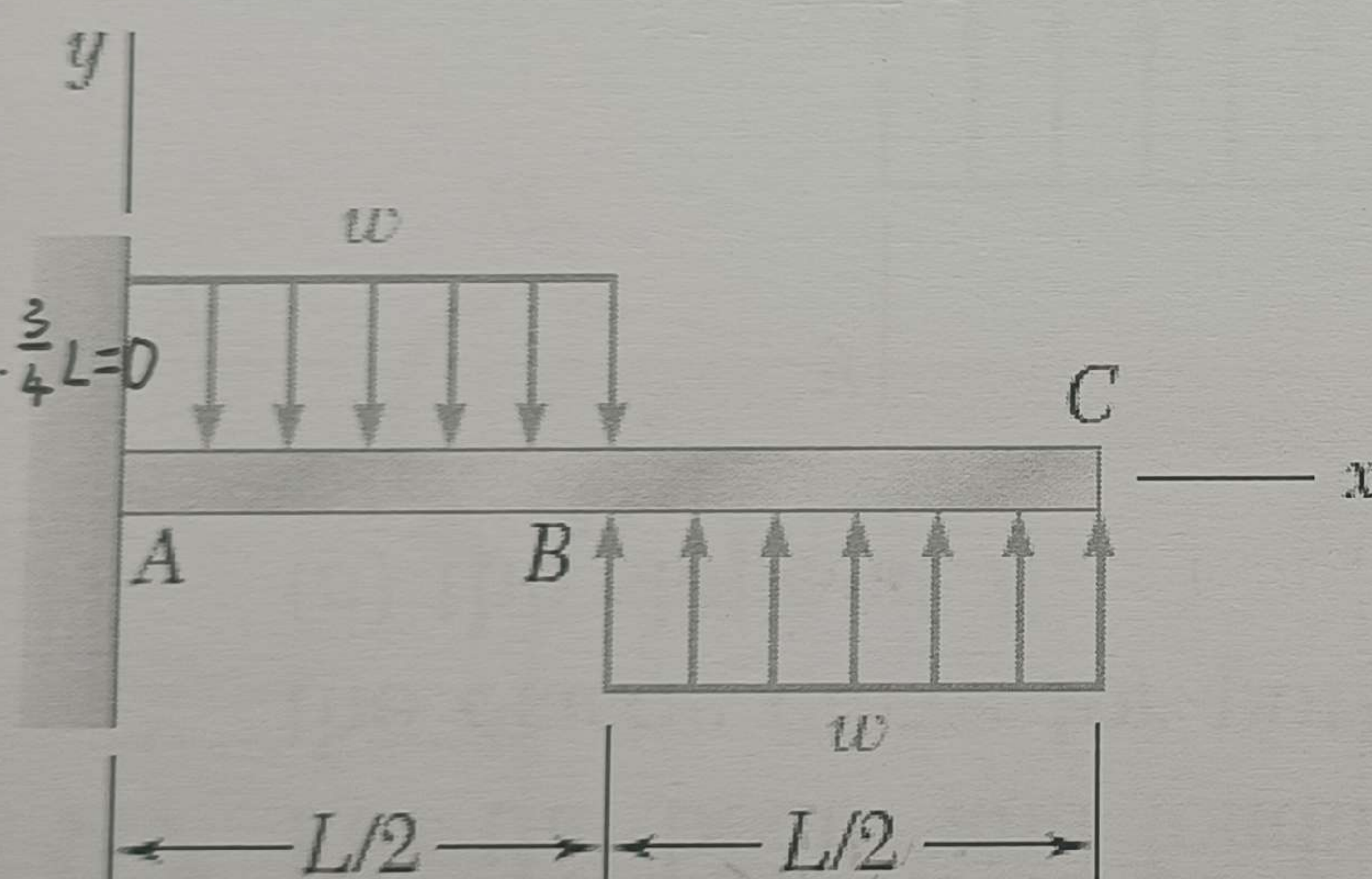


Fig. P9.5

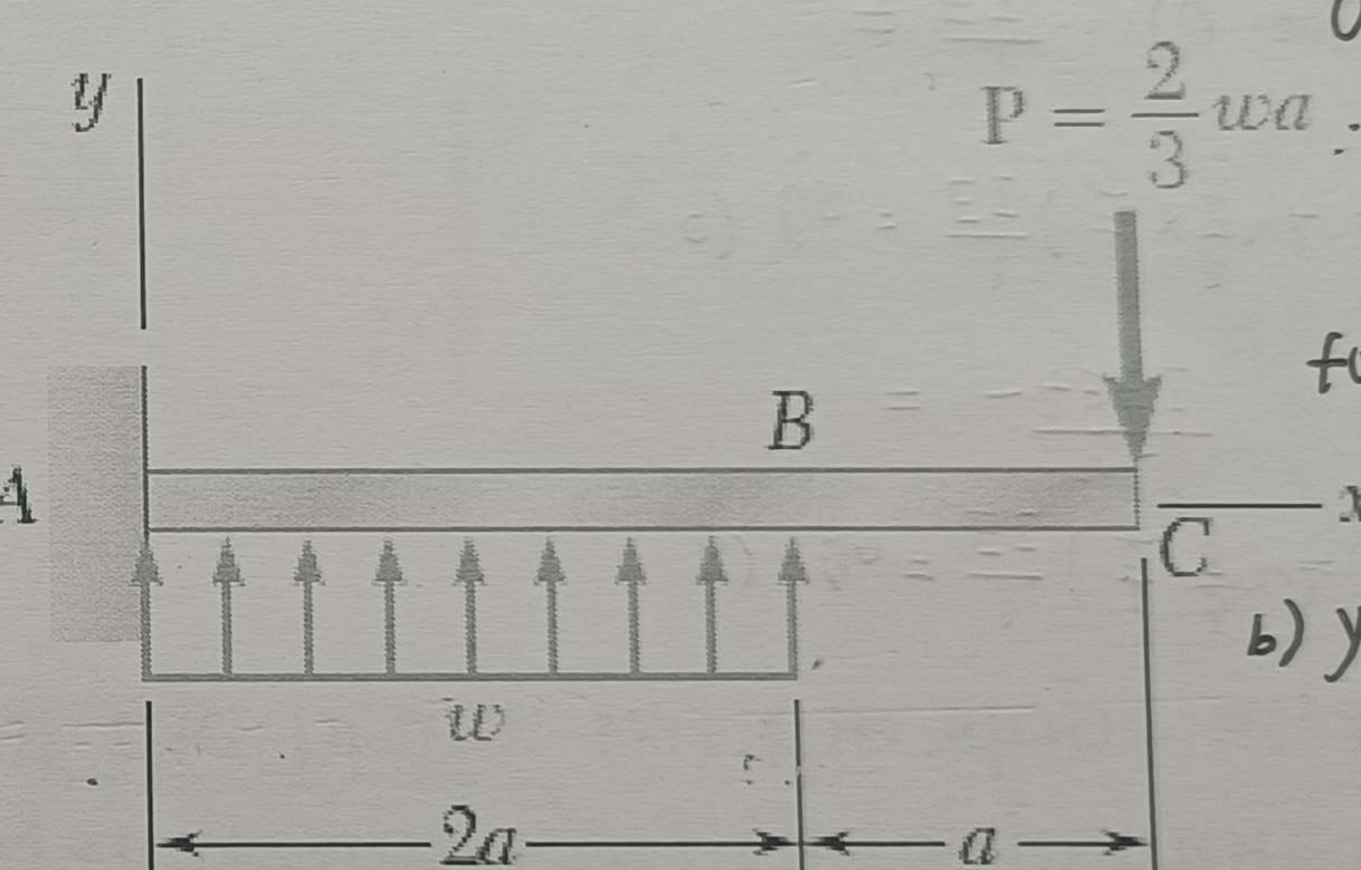
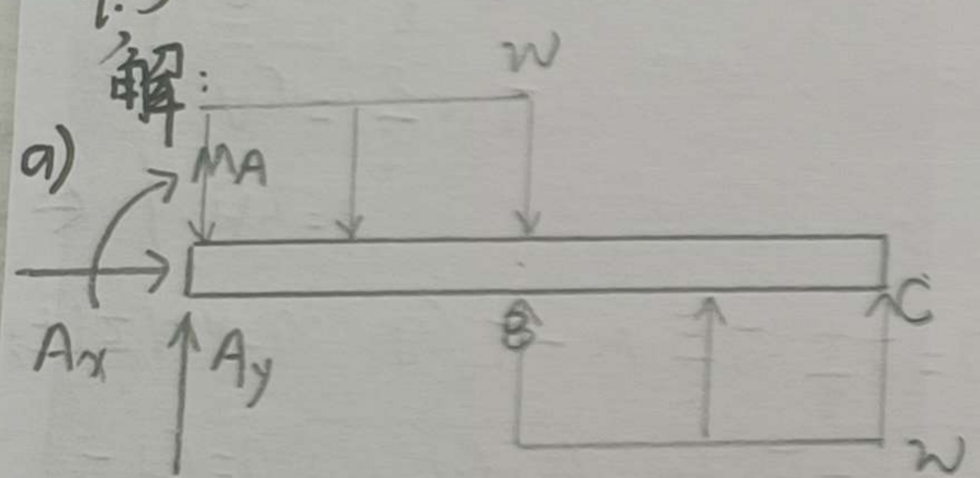


Fig. P9.6

9.5 解:

a)

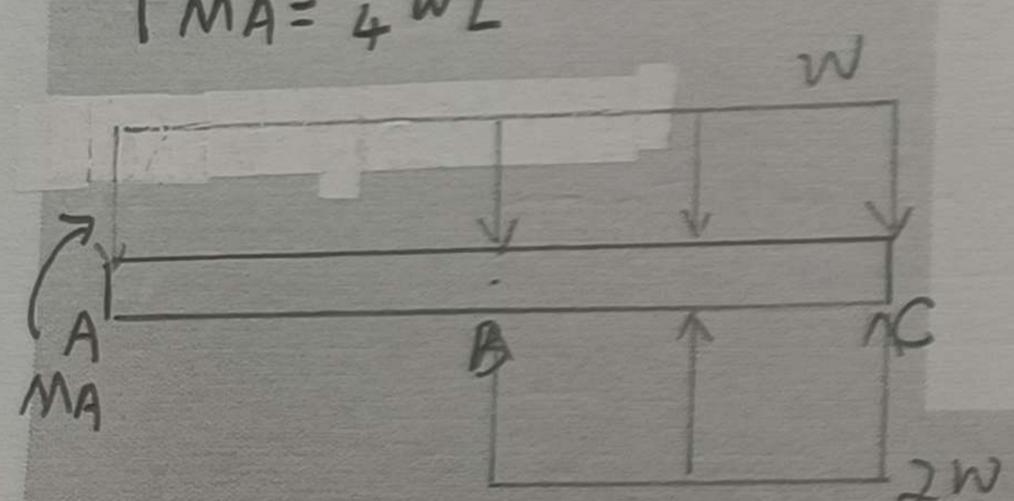


$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y - \frac{1}{2}wL + \frac{1}{2}wL = 0$

$\circlearrowleft \Sigma M_A = 0: \quad -M_A - \frac{1}{2}wL \cdot \frac{1}{4}L + \frac{1}{2}wL \cdot \frac{3}{4}L = 0$

$\Rightarrow \begin{cases} A_x = 0 \\ A_y = 0 \\ M_A = \frac{1}{4}wL^2 \end{cases}$



$V(x) = -w\langle x-0\rangle^1 + 2w\langle x-\frac{L}{2}\rangle^1$

$M(x) = +M_A - \frac{1}{2}w\langle x-0\rangle^2 + w\langle x-\frac{L}{2}\rangle^2$

$EI \cdot \frac{d^2y}{dx^2} = M(x)$

$EI \cdot \frac{dy}{dx} = EI\theta = \frac{1}{4}wL^2 \cdot x - \frac{1}{6}w\langle x-0\rangle^3 + \frac{1}{3}w\langle x-\frac{L}{2}\rangle^3 + C_1$

$EI \cdot y = \frac{1}{8}wL^2 \cdot x^2 - \frac{1}{24}w\langle x-0\rangle^4 + \frac{1}{12}w\langle x-\frac{L}{2}\rangle^4 + C_1x + C_2$

$\begin{cases} x=0,\ y_A=0,\ \theta_A=0 \\ x=L,\ M_C=0,\ V_C=0 \end{cases}$

$\Rightarrow 0 = 0-0+0+C_1,\ C_1=0$

$0 = C_2$

$\therefore y = \frac{1}{EI}\left(\frac{1}{8}wL^2x^2 - \frac{1}{24}w\langle x-0\rangle^4 + \frac{1}{12}w\langle x-\frac{L}{2}\rangle^4\right)$

for AB: $y = \frac{1}{EI}\left(\frac{1}{8}wL^2x^2 - \frac{1}{24}wx^4\right)$ $\boxed{\text{ANS}}$

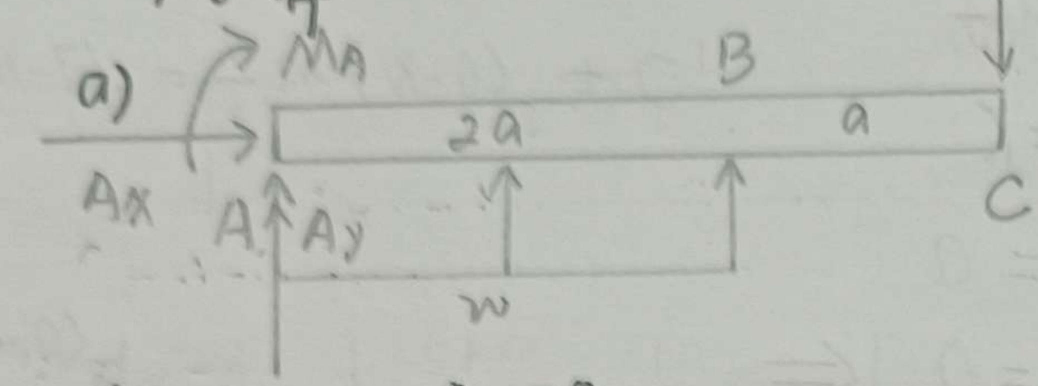
b) $y_B = \frac{1}{EI}\left(\frac{1}{8}wL^2 \cdot \frac{1}{4}L^2 - \frac{1}{24}w \cdot \frac{L^4}{16}\right)$

$= \frac{1}{EI} \cdot \frac{11}{384}wL^4$ $\boxed{\text{ANS}}$

c) $\theta_B = \frac{1}{EI} \cdot \left[\frac{1}{4}wL^2 \cdot \frac{L}{2} - \frac{1}{6}w \cdot \left(\frac{L}{2}\right)^3\right] = \frac{1}{EI} \cdot \frac{5}{48}wL^3$ $\boxed{\text{ANS}}$

9.6 解:

a)

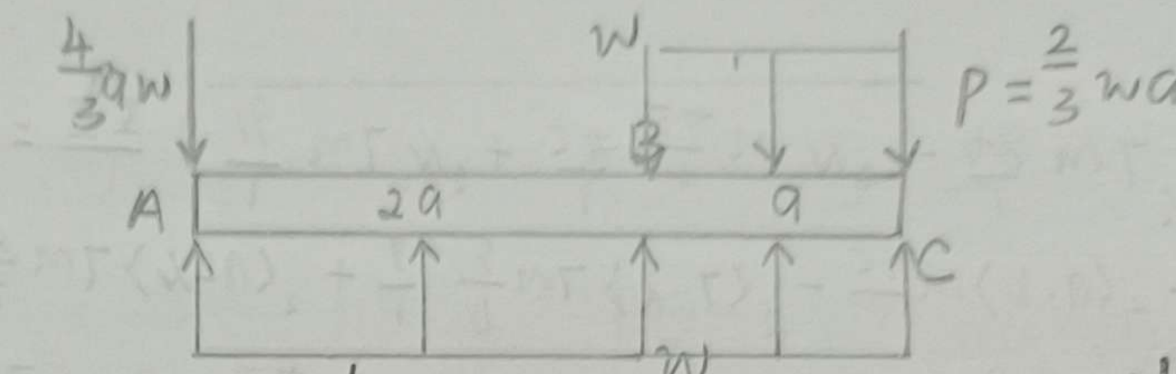


$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad +A_y + 2aw - \frac{2}{3}aw = 0$

$\circlearrowleft \Sigma M_A = 0: \quad -M_A + 2aw \cdot a - \frac{2}{3}wa \cdot 3a = 0$

$\Rightarrow \begin{cases} A_x = 0 \\ A_y = -\frac{4}{3}aw \\ M_A = 0 \end{cases}$



$V(x) = -\frac{4}{3}aw + w\langle x-0\rangle^1 - w\langle x-2a\rangle^1$

$M(x) = -\frac{4}{3}awx + \frac{1}{2}w\langle x-0\rangle^2 - \frac{1}{2}w\langle x-2a\rangle^2$

$EI\frac{d^2y}{dx^2} = M(x)$

$EI\frac{dy}{dx} = EI\theta = -\frac{2}{3}awx^2 + \frac{1}{6}w\langle x-0\rangle^3 - \frac{1}{6}w\langle x-2a\rangle^3 + C_1$

$EI\,y = -\frac{2}{9}awx^3 + \frac{1}{24}w\langle x-0\rangle^4 - \frac{1}{24}w\langle x-2a\rangle^4 + C_1x + C_2$

$\begin{cases} x=0,\ y=0,\ \theta=0 \\ x=3a,\ V=0,\ M=0 \end{cases}$

$\Rightarrow 0 = 0+0+0+C_1,\ C_1=0$

$0 = 0+0+0+0+C_2,\ C_2=0$

$\therefore y = \frac{1}{EI}\left(-\frac{2}{9}awx^3 + \frac{w}{24}\langle x-0\rangle^4 - \frac{w}{24}\langle x-2a\rangle^4\right)$

for AB

$y = \frac{1}{EI}\left(-\frac{2}{9}awx^3 + \frac{w}{24}x^4\right)$

b) $y_B = \frac{1}{EI}\left[-\frac{2}{9}aw(2a)^3 + \frac{w}{24}(2a)^4\right]$

$= \frac{1}{EI} \cdot \left(-\frac{10}{9}\right)w \cdot a^4$

c) $\theta_B = \frac{1}{EI}\left(-\frac{2}{3}aw \cdot 4a^2 + \frac{1}{6}w \cdot 8a^3\right)$

$= \frac{1}{EI} \cdot \left(-\frac{4}{3}a^3w\right)$

1+27 28

Problem 2

9.7 For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the slope at A, (*c*) the slope at *B*.

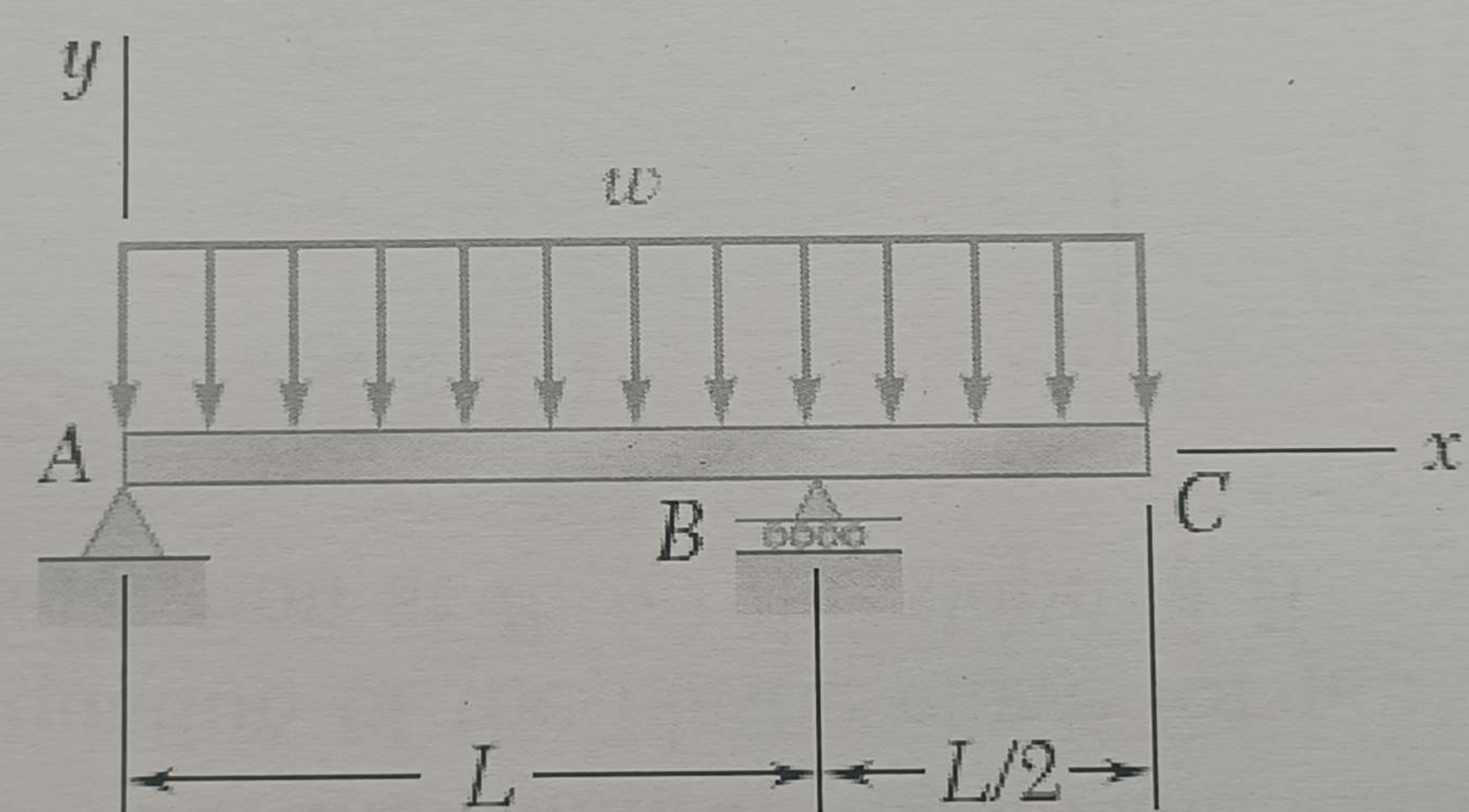
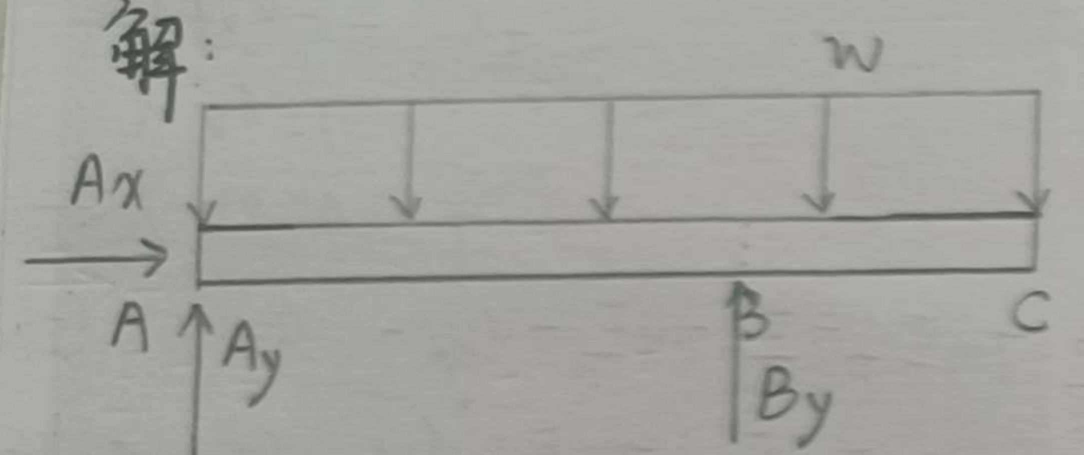


Fig. P9.7

解:

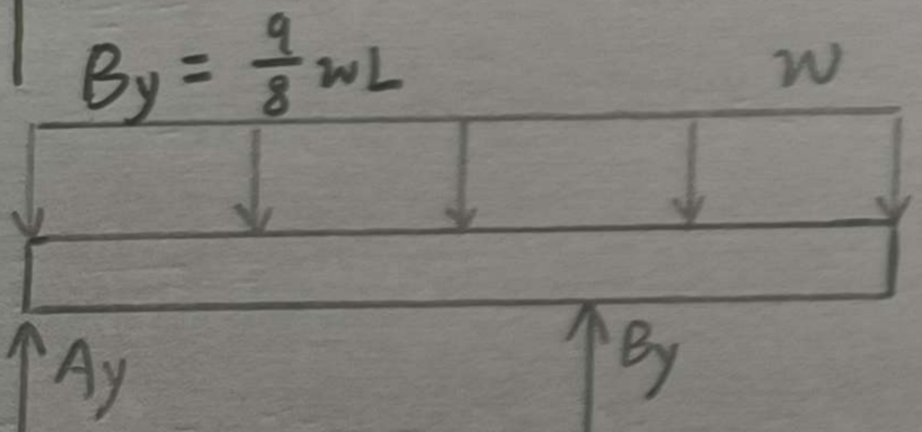


$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$\uparrow \Sigma F_y = 0: \quad +A_y + B_y - w \cdot \frac{3}{2}L = 0$

$\circlearrowleft \Sigma M_A = 0: \quad B_y L - w \cdot \frac{3}{2}L \cdot \frac{3}{4}L = 0$

$\Rightarrow \begin{cases} A_x = 0 \\ A_y = \frac{3}{8}wL \\ B_y = \frac{9}{8}wL \end{cases}$



$V(x) = +A_y\langle x-0\rangle^0 + B_y\langle x-L\rangle^0 - w\langle x-0\rangle^1$

$M(x) = A_y\langle x-0\rangle^1 + B_y\langle x-L\rangle^1 - w\frac{1}{2}\langle x-0\rangle^2$

$EI\frac{d^2y}{dx^2} = M(x)$

$EI\frac{dy}{dx} = EI\theta = \frac{1}{2}A_y\langle x-0\rangle^2 + \frac{1}{2}B_y\langle x-L\rangle^2 - \frac{1}{6}w\langle x-0\rangle^3 + C_1$

$EI\,y = \frac{1}{6}A_y\langle x-0\rangle^3 + \frac{1}{6}B_y\langle x-L\rangle^3 - \frac{1}{24}w\langle x-0\rangle^4 + C_1x + C_2$

A: $x=0,\ y=0,\ M=0$

B: $x=L,\ y=0,\ M=0$

C: $x=\frac{3}{2}L,\ y=0,\ M=0$

$\Rightarrow 0 = 0+0-0+C_2,\ C_2 = 0$

$0 = \frac{1}{6}\cdot\frac{3}{8}wL\cdot L^3 + 0 - \frac{1}{24}wL^4 + C_1L + 0,\quad C_1 = \frac{-1}{48}wL^3$

$\therefore y = \frac{1}{EI}\left(\frac{1}{6}\cdot\frac{3}{8}wL\langle x-0\rangle^3 + \frac{1}{6}\cdot\frac{9}{8}wL\langle x-L\rangle^3 - \frac{1}{24}w\langle x-0\rangle^4 - \frac{1}{48}wL^3\cdot x\right)$

for AB: $y = \frac{1}{EI}\left(\frac{1}{16}wLx^3 + 0 - \frac{1}{24}wx^4 - \frac{1}{48}wL^3 x\right)$ $\boxed{\text{ANS}}$

b) $\theta_A = \frac{1}{EI}\left(0+0+0-\frac{1}{48}wL^3\right) = -\frac{wL^3}{48EI}$

c) $\theta_B = \frac{1}{EI}\left(\frac{1}{2}\times\frac{3}{8}wL\cdot L^2 + 0 - \frac{1}{6}wL^3 - \frac{1}{48}wL^3\right) = \frac{1}{EI}\cdot(0\ wL^3) = 0$

Problem 3

9.31 and 9.32 Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.

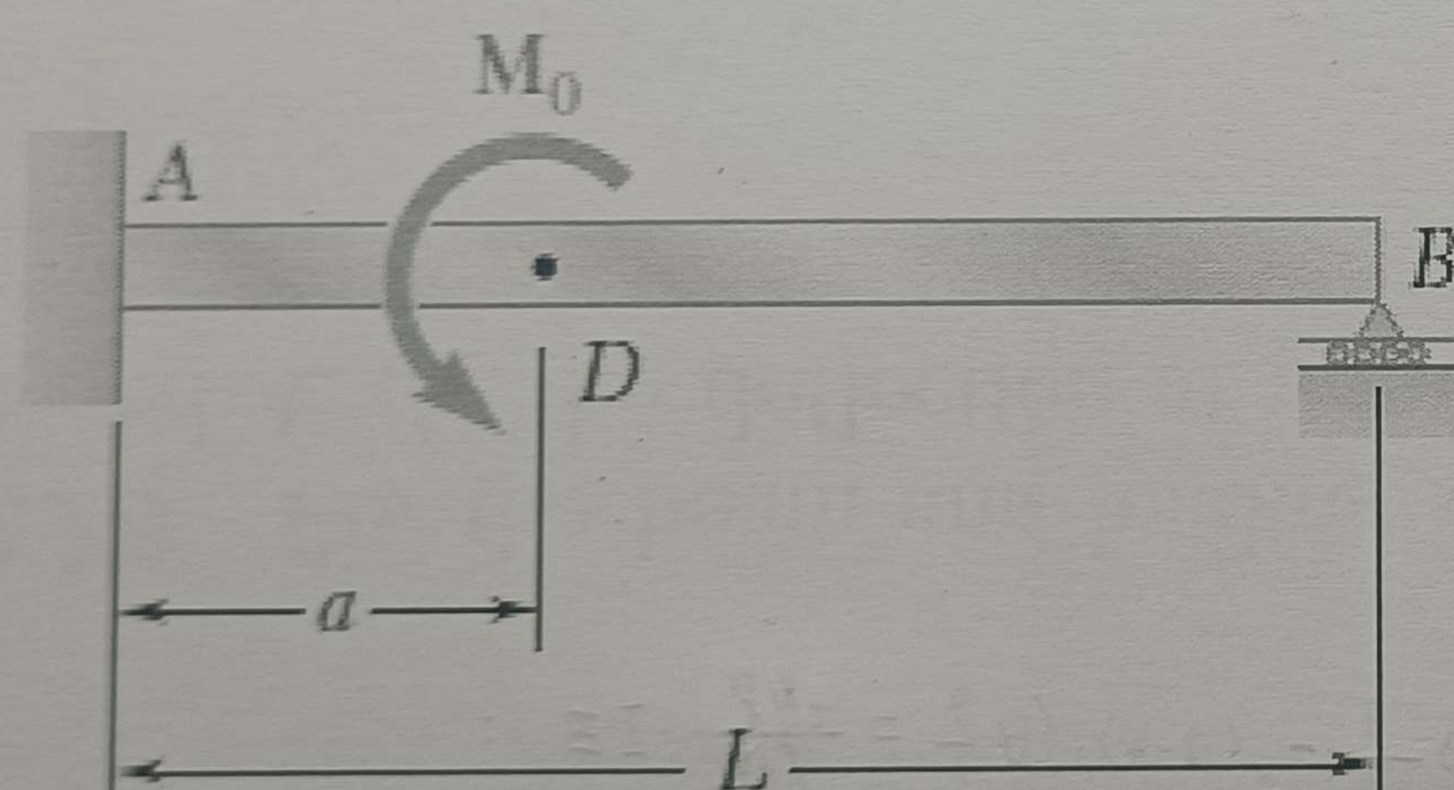


Fig. P9.31

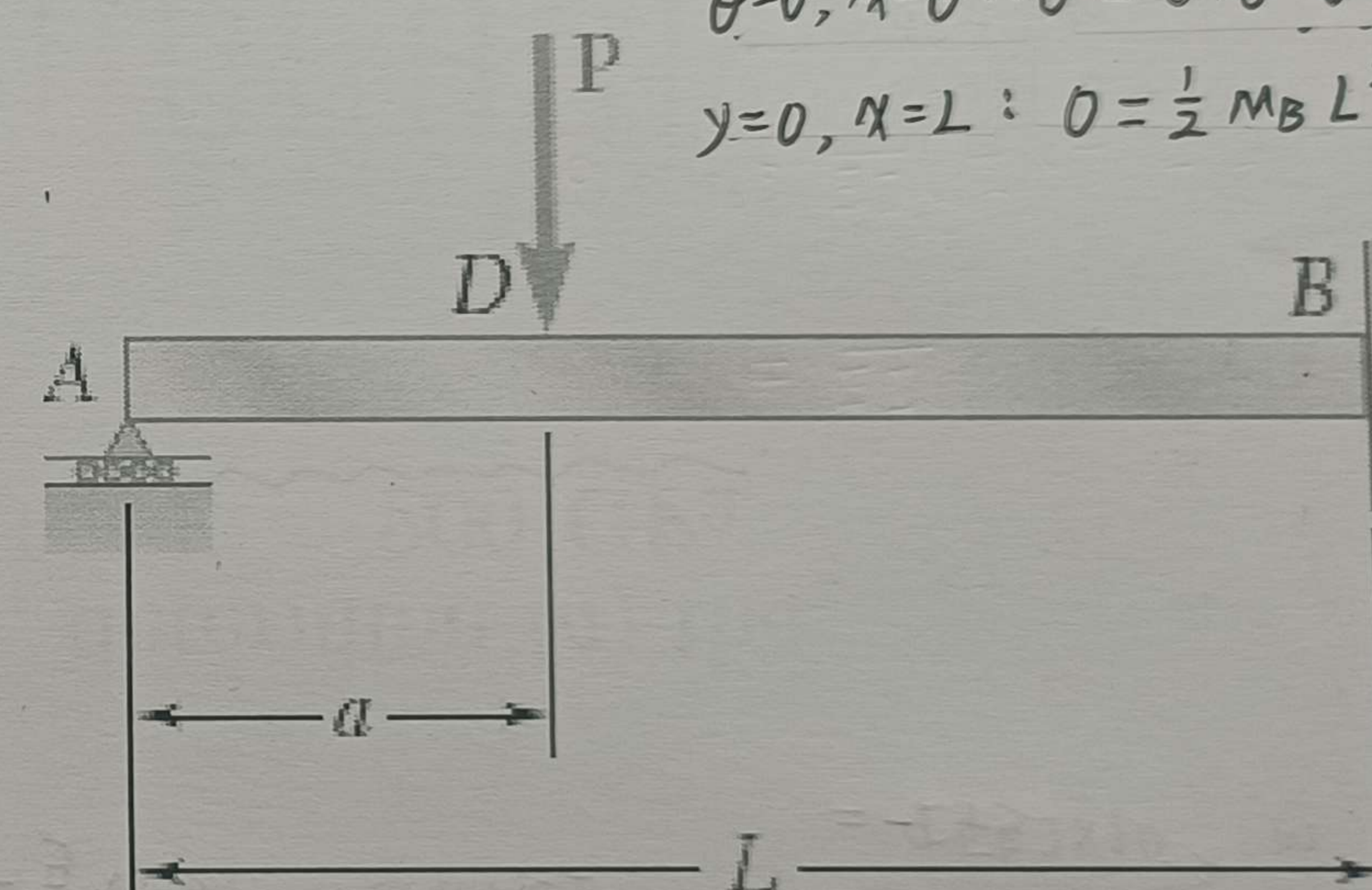
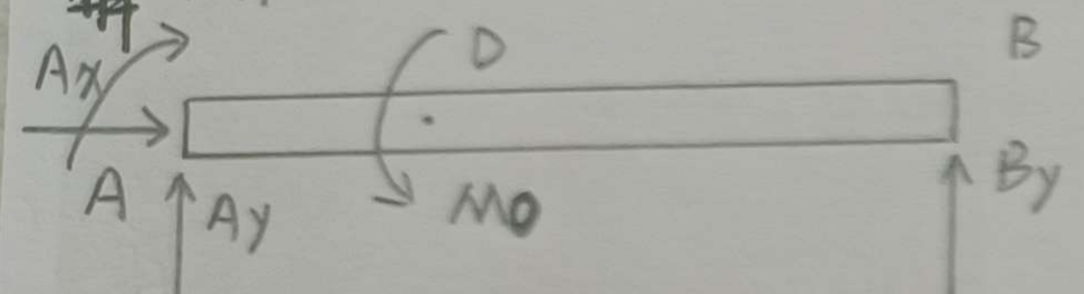


Fig. P9.32

9.31

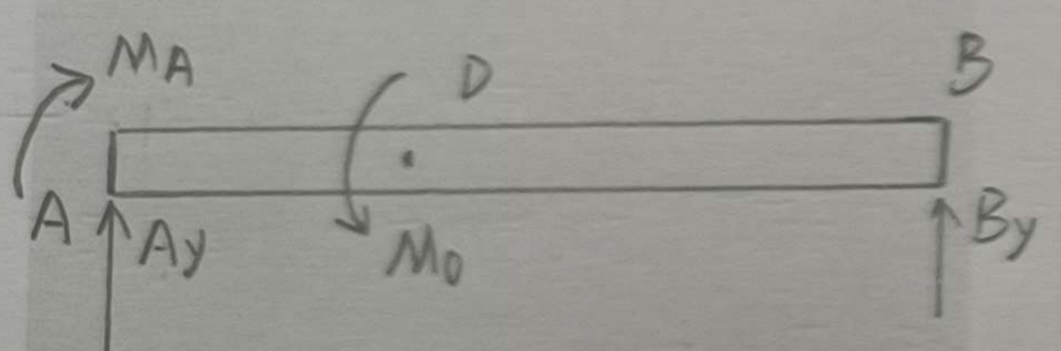
解: M_A



$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y + B_y = 0$

$\circlearrowleft \Sigma M_A = 0: \quad -M_A + M_0 + B_y \cdot L = 0$



$V(x) = +A_y \langle x-0 \rangle^0 + B_y \langle x-L \rangle^0$

$M(x) = +M_A \langle x-0 \rangle^0 - M_0 \langle x - \frac{L}{3} \rangle^0 + A_y \langle x-0 \rangle^1 + B_y \langle x-L \rangle^1$

$EI \cdot \frac{d^2y}{dx^2} = M(x)$

$EI \cdot \frac{dy}{dx} = EI \cdot \theta = M_A \langle x-0 \rangle^1 - M_0 \langle x - \frac{L}{3} \rangle^1 + A_y \cdot \frac{1}{2} \langle x-0 \rangle^2 + B_y \cdot \frac{1}{2} \langle x-L \rangle^2 + C_1$

$EI \cdot y = M_A \cdot \frac{1}{2} \langle x-0 \rangle^2 - M_0 \frac{1}{2} \langle x - \frac{L}{3} \rangle^2 + \frac{1}{6} A_y \langle x-0 \rangle^3 + \frac{1}{6} B_y \langle x-L \rangle^3 + C_1 x + C_2$

A: $x=0,\ y=0,\ \theta=0$

B: $x=L,\ y=0,\ M=0$

$\Rightarrow$ $y=0, x=0: \ 0 = 0-0+0+0+C_2, \quad C_2 = 0$

$\theta=0, x=0: \ 0 = 0-0+0+0+C_1, \quad C_1 = 0$

$y=0, x=L: \ 0 = \frac{1}{2} M_A \cdot L^2 - M_0 \cdot \frac{1}{2} \cdot \frac{4}{9} L^2 + \frac{1}{6} A_y \cdot L^3 + 0 + 0 + 0$

$M_A = M_0 + B_y \cdot L$

$A_y = -B_y$

$\Rightarrow 0 = \frac{1}{2} L^2 (M_0 + B_y L) - M_0 \cdot \frac{2}{9} L^2 + \frac{1}{6} L^3 \cdot (-B_y)$

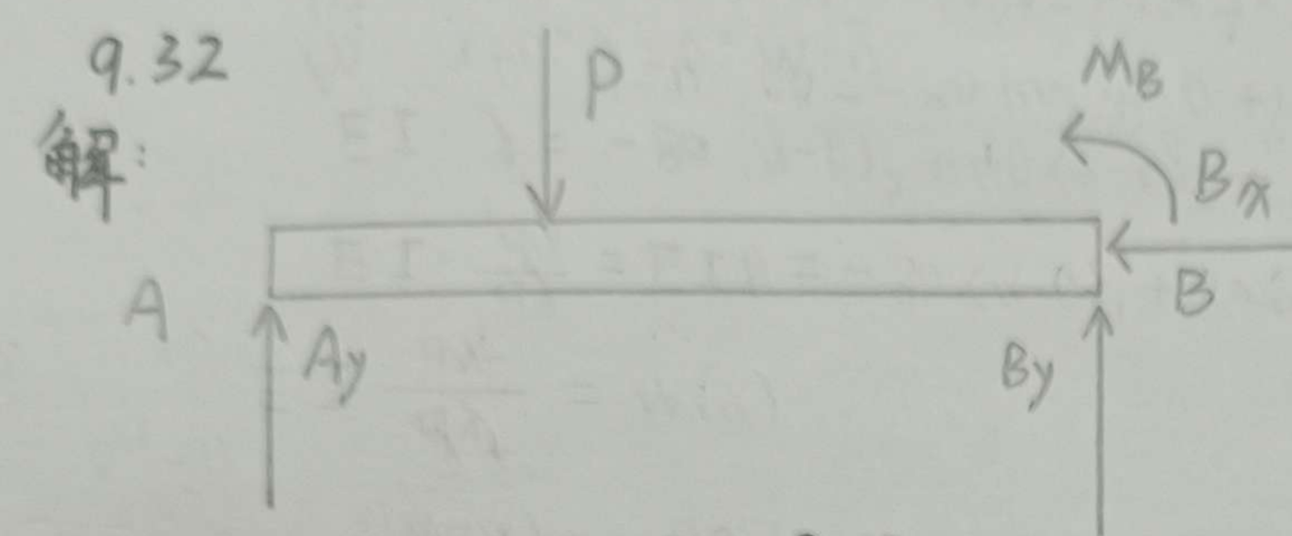
$\Rightarrow B_y = \frac{-5M_0}{6L}$ $\boxed{\text{ANS}}$

$y_D = \frac{1}{2} M_A \cdot \frac{1}{9} L^2 - 0 + \frac{1}{6} A_y \cdot \frac{L^3}{27} + 0 + 0 + 0$

$= \frac{7}{486} M_0 L^2$ $\boxed{\text{ANS}}$

9.32

解:

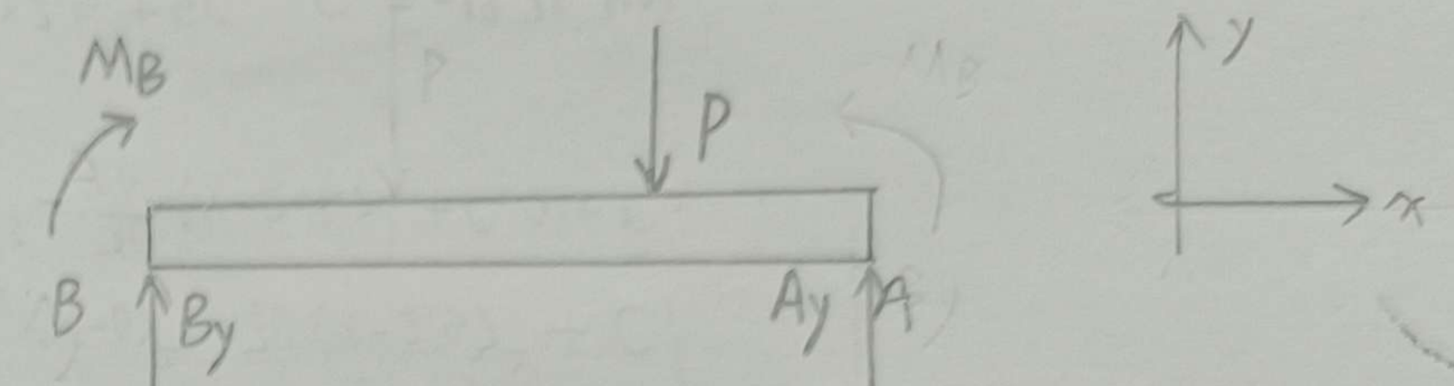


$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - P = 0$

$\xrightarrow{+} \Sigma F_x = 0: \quad -B_x = 0$

$\circlearrowleft \Sigma M_B = 0: \quad M_B + P \cdot \frac{2}{3} L - A_y \cdot L = 0$

$\Rightarrow \begin{cases} B_x = 0 \\ B_y = P - A_y \\ M_B = A_y L - \frac{2}{3} PL \end{cases}$



由里向外.

$V(x) = +B_y \langle x-0 \rangle^0 - P \langle x - \frac{2}{3} L \rangle^0$

$M(x) = +M_B \langle x-0 \rangle^0 + B_y \langle x-0 \rangle^1 - P \langle x - \frac{2}{3} L \rangle^1$

$EI \frac{d^2y}{dx^2} = M(x)$

$EI \frac{dy}{dx} = M_B \langle x-0 \rangle^1 + \frac{1}{2} B_y \langle x-0 \rangle^2 - \frac{1}{2} P \langle x - \frac{2}{3} L \rangle^2 + C_1 = EI \cdot \theta$

$EI \cdot y = \frac{1}{2} M_B \langle x-0 \rangle^2 + \frac{1}{6} B_y \langle x-0 \rangle^3 - \frac{1}{6} P \langle x - \frac{2}{3} L \rangle^3 + C_1 x + C_2$

A: $x=L,\ y=0,\ M=0$

B: $x=0,\ y=0,\ \theta=0$

$\Rightarrow$ $y=0, x=0: \ 0 = 0+0-0+C_2, \ C_2 = 0$

$\theta=0, x=0: \ 0 = 0+0-0+C_1, \ C_1 = 0$

$y=0, x=L: \ 0 = \frac{1}{2} M_B L^2 + \frac{1}{6} B_y \cdot L^3 - \frac{1}{6} P \cdot \frac{1}{27} L^3$

$\Rightarrow B_y = \frac{13}{27} P, \quad A_y = \frac{14}{27} P$ $\boxed{\text{ANS}}$

$B_x = 0$

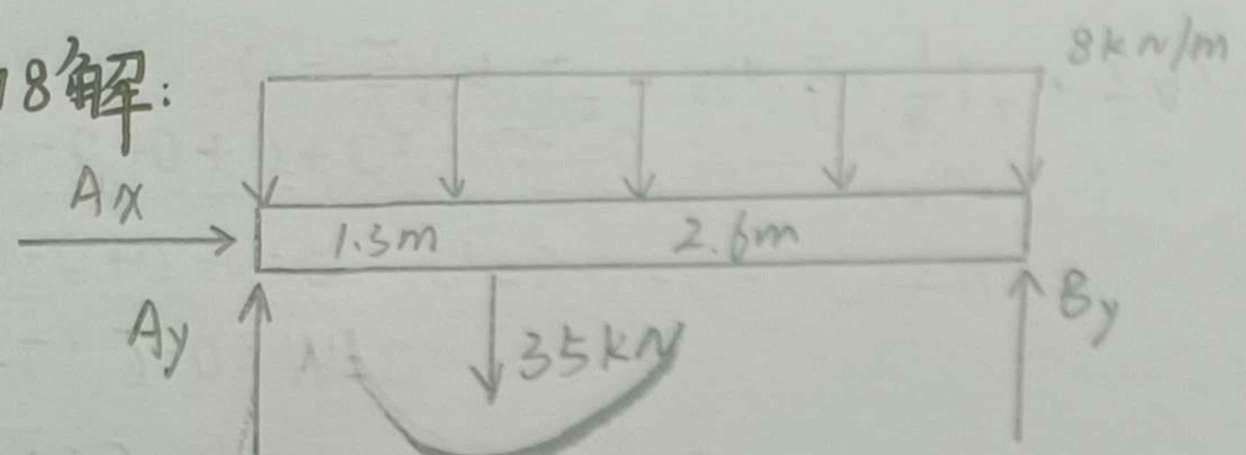
$M_B = \frac{4}{-27} PL$

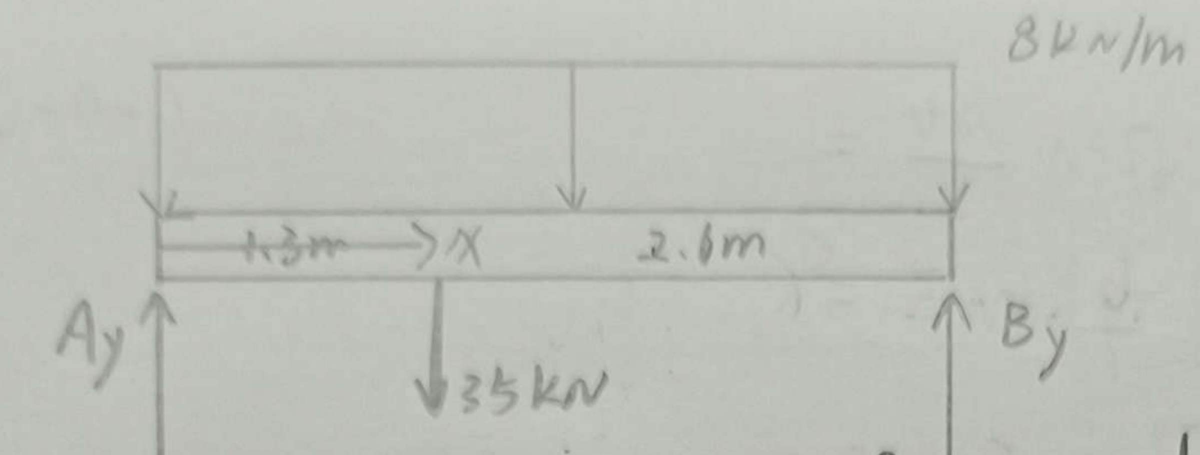
$y_D = \frac{1}{EI} \left(\frac{1}{2} M_B \cdot \frac{4}{9} L^2 + \frac{1}{6} B_y \frac{8}{27} L^3 \right)$

$= \frac{1}{EI} \cdot PL^3 \cdot \frac{-20}{2187}$

$\frac{\ }{54} = \frac{\ }{54}P$

9.78解:





$\xrightarrow{+} \Sigma F_x=0: A_x=0$

$+\uparrow \Sigma F_y=0: A_y+B_y-35-8\times3.9=0$

Problem 4

$\circlearrowleft + \Sigma M_A=0: -35\times1.3+B_y\times3.9-8\times3.9\times\frac{3.9}{2}=0$

$\Rightarrow A_x=0$, $A_y=38.93kN$, $B_y=27.27kN$

$V(x) = +A_y\langle x-0\rangle^0 - 35\langle x-1.3\rangle^0 - 8\langle x-0\rangle^1$

$M(x) = A_y\langle x-0\rangle^1 - 35\langle x-1.3\rangle^1 - 4\langle x-0\rangle^2$

$EI\frac{d^2y}{dx^2} = M(x)$

$EI\cdot\frac{dy}{dx} = \frac12 A_y\langle x-0\rangle^2 - \frac{35}{2}\langle x-1.3\rangle^2 - \frac43\langle x-0\rangle^3 + C_1 = EI\cdot\theta$

$EI\cdot y = \frac16 A_y\langle x-0\rangle^3 - \frac{35}{6}\langle x-1.3\rangle^3 - \frac13\langle x-0\rangle^4 + C_1x + C_2$

A: $x=0$, $y=0$, $M=0$

B: $x=3.9m$, $y=0$, $M=0$

$\Rightarrow$: $y=0$, $x=0$: $0=C_2$

$y=0$, $x=3.9m$: $0=\frac16 A_y\cdot3.9^3 - \frac{35}{6}\times2.6^3 - \frac13 3.9^4 + C_1\cdot3.9$

$\Rightarrow C_1 = -52.63$

$\theta_A = \frac{1}{EI}(0-0-0-52.63)$

$= \frac{-52.63\times10^3}{200\times10^9\times102\times10^{-6}} = -2.580\times10^{-3}$ rad

$y_C = \frac{1}{EI}[\frac16 A_y\times1.3^3 - 0 - \frac13\times1.3^4 + (-52.63)\times1.3]$

$=-2.702\times10^{-3}$ m

9.77 and 9.78 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

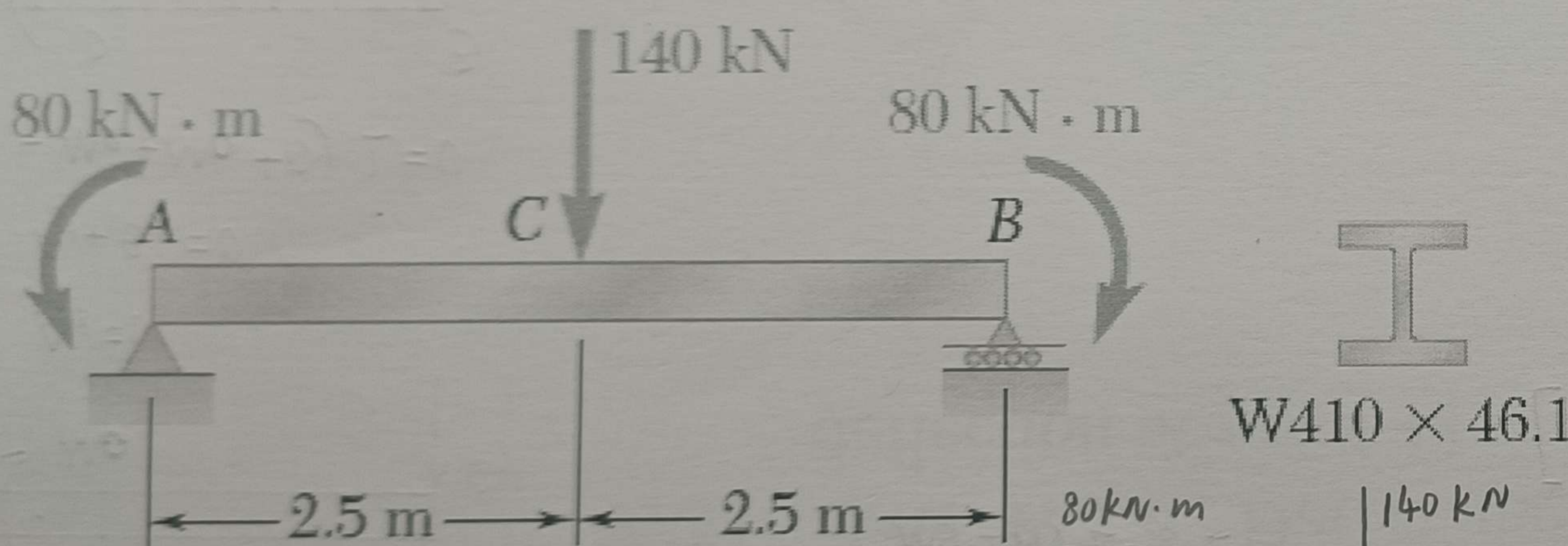
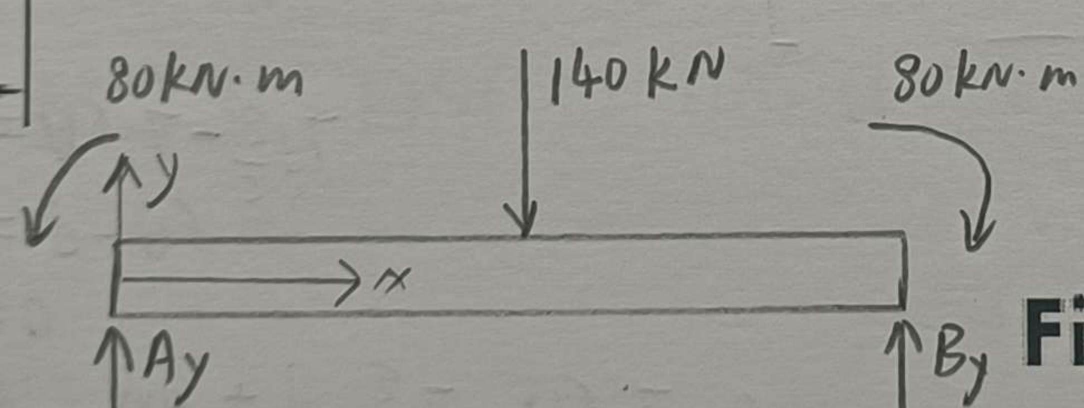


Fig. P9.77



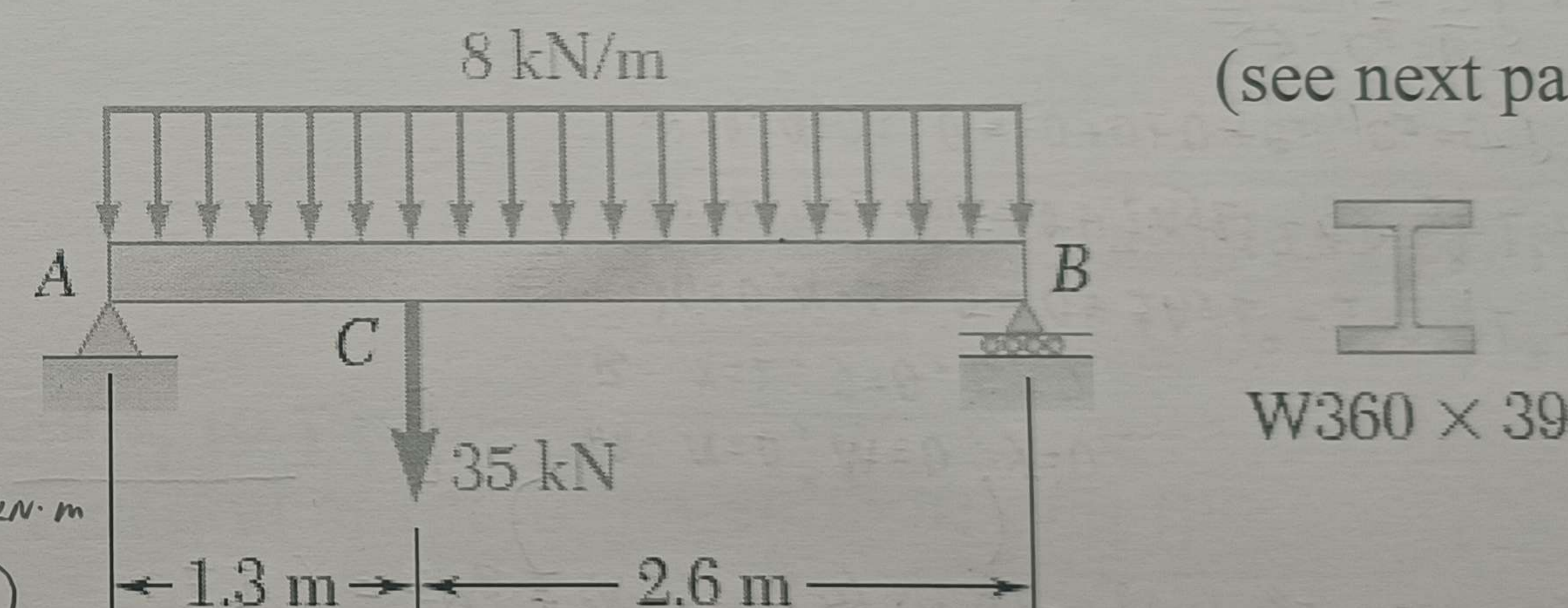


Fig. P9.78

(see next page)

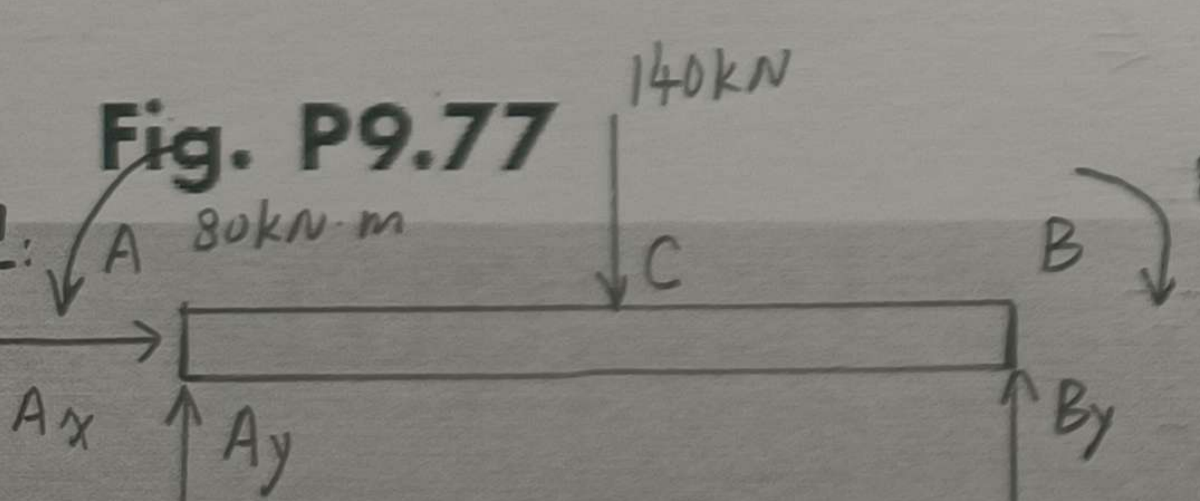
$\theta_A = \frac{1}{EI}(0+0+0-0-18.75) = \frac{-18.75\times10^3}{200\times10^9\times156\times10^6\times10^{-12}}$

$=-6.010\times10^{-4}$ rad

$y_C = \frac{1}{EI}(-40\times2.5^2+0+\frac16\times70\times2.5^3-0-18.75\times2.5)$

$=-3.673\times10^{-3}$ m

9.77解:



$\xrightarrow{+} \Sigma F_x=0: A_x=0$

$+\uparrow \Sigma F_y=0: +A_y+B_y-140=0$

$\circlearrowleft + \Sigma M_A=0: +80-140\times2.5-80+B_y\cdot5=0$

$\Rightarrow A_x=0$, $B_y=70kN$

$A_y=70kN$

$V(x)=+A_y\langle x-0\rangle^0-140\langle x-2.5\rangle^0$

$M(x)=-80\langle x-0\rangle^0+80\langle x-5\rangle^0+A_y\langle x-0\rangle^1-140\langle x-2.5\rangle^1$

$EI\frac{d^2y}{dx^2}=M(x)$

$EI\cdot\frac{dy}{dx}=EI\theta=-80\langle x-0\rangle^1+80\langle x-5\rangle^1+\frac12A_y\langle x-0\rangle^2-70\langle x-2.5\rangle^2+C_1$

$EI\cdot y=-40\langle x-0\rangle^2+40\langle x-5\rangle^2+\frac16A_y\langle x-0\rangle^3-\frac{70}{3}\langle x-2.5\rangle^3+C_1x+C_2$

A: $x=0$, $y=0$, $\therefore$ $0=0+0+0-0+C_2$, $C_2=0$

B: $x=5m$, $y=0$, $\therefore$ $0=-40\times25+0+\frac16\times70\times125-\frac{70}{3}\times2.5^3+5C_1$, $C_1=-18.75$