Homework

① Derive the differential equations governing the damped two degree-of-freedom system shown in Figure using x_1 and x_2 as generalized coordinates.

② Determine the response of the system of Figure due to a force $F(t)=20\sin 20t$ N applied to the block whose displacement is x_2 using the method of undetermined coefficients. Use m=10 kg, k=90,000 N/m, and c=100 N·s/m.

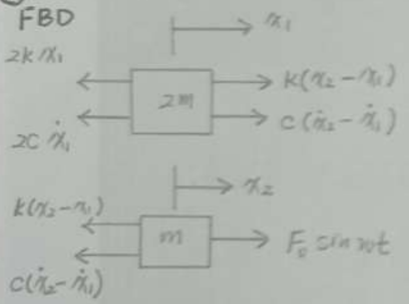
$x_1 = u_1 \sin(wt) + v_1 \cos(wt)$

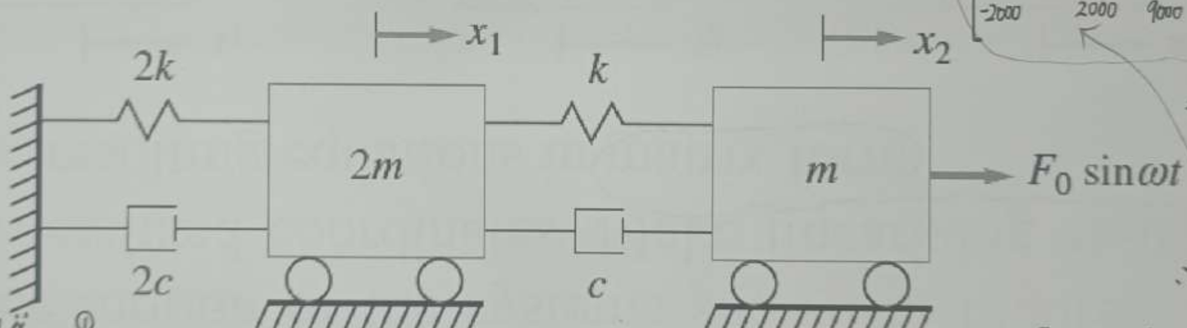
$x_2 = u_2 \sin(wt) + v_2 \cos(wt)$

$$\begin{bmatrix} 19000 & -9000 & -6000 & 2000 \\ -9000 & 3000 & 2000 & -2000 \\ 6000 & -2000 & 19000 & -9000 \\ -2000 & 2000 & 9000 & 3000 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 20 \\ 0 \\ 0 \end{bmatrix}$$

解:

① FBD





$\Rightarrow u_1 = 1.035\times10^{-2}$

$u_2 = 2.261\times10^{-2}$

$v_1 = -0.173\times10^{-2}$

$v_2 = -0.178\times10^{-2}$

$\therefore x_1(t) = 0.014\sin(20t+0.165)$

$x_2(t) = 0.023\sin(20t+0.078)$ $\boxed{\text{ANS}}$

$\Rightarrow \underbrace{k(x_2-x_1)-2kx_1}_{kx_2-3kx_1} + \underbrace{c(\dot{x}_2-\dot{x}_1)-2c\dot{x}_1}_{(c\dot{x}_2-3c\dot{x}_1)} = 2m\ddot{x}_1$ ①

$F_0\sin wt - k(x_2-x_1) - c(\dot{x}_2-\dot{x}_1) = m\ddot{x}_2$ ②

Arrange: $2m\ddot{x}_1 + c(3\dot{x}_1-\dot{x}_2) + k(3x_1-x_2) = 0$

$m\ddot{x}_2 + c(\dot{x}_2-\dot{x}_1) + k(x_2-x_1) = F_0\sin wt$

$$\Rightarrow \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 3c & -c \\ -c & c \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 3k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ F_0\sin wt \end{bmatrix} \quad \boxed{\text{ANS}}$$

② Assume

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\sin wt + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\cos wt$$

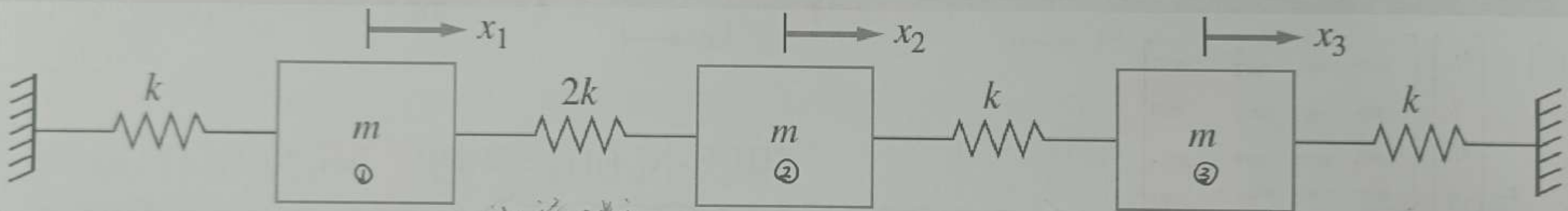
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = w\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\cos wt + (-w)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\sin wt$$

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = -w^2\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\sin wt - w^2\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\cos wt$$

代入回 $\boxed{\text{ANS}}$ 中可得

Homework

Use Lagrange's equations to derive the differential equations governing the motion of the systems shown in Figures. Use the indicated generalized coordinates. Make linearizing assumptions, and write the resulting equations in matrix form



解: Lagrange's equation

$2x_1^2 + x_2^2 - 2x_1x_2$

$$T = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 + \frac{1}{2} m \dot{x}_3^2$$

$$V = \frac{1}{2} k x_1^2 + \frac{1}{2} \times 2k (x_2 - x_1)^2 + \frac{1}{2} \times k (x_3 - x_2)^2 + \frac{1}{2} k x_3^2$$

$$L = T - V$$

mass ①: $\frac{\partial L}{\partial \dot{x}_1} = m \dot{x}_1$, $\frac{\partial L}{\partial x_1} = -[k x_1 + k \cdot 2 (x_2 - x_1)(-1)]$

$-2kx_2 + 2kx_1$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = m\ddot{x}_1 + kx_1 - 2k(x_2 - x_1)$$
$$= m\ddot{x}_1 + 3kx_1 - 2kx_2 = Q_1 = 0 \quad (1)$$

mass ②: $\frac{\partial L}{\partial \dot{x}_2} = m\dot{x}_2$, $\frac{\partial L}{\partial x_2} = 0 - [k \cdot 2 \cdot (x_2 - x_1) + k(x_3 - x_2)(-1)] = 2kx_1 - 3kx_2 + kx_3$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} = m\ddot{x}_2 - 2kx_1 + 3kx_2 - kx_3$$

mass ③: $\frac{\partial L}{\partial \dot{x}_3} = m\dot{x}_3$, $\frac{\partial L}{\partial x_3} = 0 - [k(x_3 - x_2) + kx_3] = kx_2 - 2kx_3$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) - \frac{\partial L}{\partial x_3} = m\ddot{x}_3 - kx_2 + 2kx_3 = Q_3 = 0 \quad (3)$$

from (1)(2)(3)

$$\Rightarrow \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 3k & -2k & 0 \\ -2k & 3k & -k \\ 0 & -k & 2k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{\text{ANS}}$