Quiz 11

Date: 2022-04-25 Name: SID:

Using the ***Laplace transform*** to solve the given initial value problems.

Q1. $y'' - 2y' + 4y = 0,\ y(0) = 2,\ y'(0) = 0.$

Q2. $y'' - 2y' + 2y = e^{-t},\ y(0) = 0,\ y'(0) = 1.$

11. Taking the Laplace transform of the differential equation we obtain $[s^2Y(s) - sy(0) - y'(0)] - 2[sY(s) - y(0)] + 4Y(s) = 0$. Using the initial conditions and solving for $Y(s)$ we obtain $Y(s) = (2s-4)/(s^2-2s+4)$. Completing the square in the denominator, we have

$$Y(s) = \frac{2s-4}{(s-1)^2+3} = \frac{2(s-1)}{(s-1)^2+3} - \frac{2}{(s-1)^2+3}$$

which (using line 14 in Table 6.2.1) gives $y(t) = 2e^t\cos(\sqrt{3}t) - (2\sqrt{3}/3)e^t\sin(\sqrt{3}t)$.

$\dfrac{\quad}{4-\omega^2}\cos 2t$.

16. Taking the Laplace transform of both sides of the ODE, we obtain

$$[s^2Y(s) - sy(0) - y'(0)] - 2[sY(s) - y(0)] + 2Y(s) = \frac{1}{s+1}.$$

Solving for $Y(s)$ we get

$$Y(s) = \frac{1}{s^2-2s+2} + \frac{1}{(s^2-2s+2)(s+1)}.$$

Using partial fractions on the second term, we have

$$\frac{1}{(s^2-2s+2)(s+1)} = \frac{1}{5}\frac{1}{s+1} + \frac{1}{5}\frac{3-s}{s^2-2s+2}.$$

Therefore, we can write

$$Y(s) = \frac{1}{5}\frac{1}{s+1} + \frac{1}{5}\frac{8-s}{s^2-2s+2}.$$

Completing the square in the denominator for the last term, we have

$$\frac{8-s}{s^2-2s+2} = -\frac{(s-1)-7}{(s-1)^2+1}.$$

Therefore,

$$Y(s) = \frac{1}{5}\frac{1}{s+1} - \frac{1}{5}\frac{(s-1)-7}{(s-1)^2+1},$$

which implies that $y = (e^{-t} - e^t\cos t + 7e^t\sin t)/5$.