空气动力学 HW9

8.7

解:

$$\frac{p_2}{p_1} = 1+\frac{2\gamma}{\gamma+1}(M_1^2-1)$$

$$\Rightarrow p_2 = \left[1+\frac{2.8}{2.4}(2.6^2-1)\right]\times 1 = 7.72 \text{ atm} \quad \boxed{\text{ANS}}$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\cdot\frac{\rho_1}{\rho_2} = \left[1+\frac{2\gamma}{\gamma+1}(M_1^2-1)\right]\cdot\left[\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right]$$

$$\Rightarrow T_2 = \left[1+\frac{2.8}{2.4}\times(2.6^2-1)\right]\cdot\left(\frac{2+0.4\times 2.6^2}{2.4\times 2.6^2}\right)\times 288$$

$$= 644.64 \text{ K} \quad \boxed{\text{ANS}}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$

$$p_1 = \rho_1 R T_1$$

$$\Rightarrow \rho_2 = \frac{p_1}{RT_1}\cdot\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2} = \frac{1.01\times10^5}{287\times288}\times\frac{2.4\times2.6^2}{2+0.4\times2.6^2}$$

$$= 4.214 \text{ kg/m}^3 \quad \boxed{\text{ANS}}$$

$$M_2 = \sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2-\frac{\gamma-1}{2}}} = \sqrt{\frac{1+\frac{0.4}{2}\times2.6^2}{1.4\times2.6^2-\frac{0.4}{2}}} = 0.5039 \quad \boxed{\text{ANS}}$$

$$\frac{T_{0.2}}{T_2} = 1+\frac{\gamma-1}{2}M_2^2$$

$$\Rightarrow T_{0.2} = \left(1+\frac{0.4}{2}\times0.5039^2\right)\times644.64 = 677.37 \text{ K} \quad \boxed{\text{ANS}}$$

$$\frac{P_{0.2}}{P_2} = \left(\frac{T_{0.2}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow P_{0.2} = 7.72\times\left(1+\frac{0.4}{2}\times0.5039^2\right)^{\frac{1.4}{0.4}} = 9.181 \text{ atm} \quad \boxed{\text{ANS}}$$

$$s_2-s_1 = C_p\ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

$$= \frac{287\times1.4}{1.4-1}\times\ln\frac{644.64}{288} - 287\times\ln\frac{7.72}{1}$$

$$= 222.78 \ (\text{J/kg}\cdot\text{K}) \quad \boxed{\text{ANS}}$$

8.9

解:

$$s_2-s_1 = C_p\ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \quad ①$$

$$\frac{T_2}{T_1} = \left[1+\frac{2\gamma}{\gamma+1}(M_1^2-1)\right]\cdot\left[\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right]$$

$$= \left[1+\frac{2.8}{2.4}(M_1^2-1)\right]\cdot\left(\frac{2+0.4M_1^2}{2.4M_1^2}\right)$$

$$= \left(\frac{7}{6}M_1^2-\frac{1}{6}\right)\cdot\left(\frac{5+M_1^2}{6M_1^2}\right)$$

$$= \frac{17}{18}+\frac{7}{36}M_1^2-\frac{5}{36M_1^2}$$

$$\frac{p_2}{p_1} = 1+\frac{2\gamma}{\gamma+1}(M_1^2-1)$$

$$= 1+\frac{2.8}{2.4}(M_1^2-1) = \frac{7}{6}M_1^2-\frac{1}{6}$$

$$\therefore C_p\ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

$$= \ln\left(\frac{T_2}{T_1}\right)^{C_p} - \ln\left(\frac{p_2}{p_1}\right)^R$$

$$= \ln\frac{\left(\frac{p_2}{p_1}\cdot\frac{\rho_1}{\rho_2}\right)^{\frac{\gamma R}{\gamma+1}}}{\left(\frac{p_2}{p_1}\right)^R} = \ln\left(\frac{\rho_1}{\rho_2}\right)^{\frac{\gamma R}{\gamma+1}}\cdot\left(\frac{p_2}{p_1}\right)^{\frac{-R}{\gamma+1}}$$

$$= \ln\left[\left(\frac{2+0.4M_1^2}{2.4M_1^2}\right)^{\frac{1.4\times287}{2.4}}\cdot\left(\frac{7}{6}M_1^2-\frac{1}{6}\right)^{\frac{-287}{2.4}}\right]$$

Before shock wave: $0 = C_p\frac{T_{0.1}}{T_1} - R\ln\frac{P_{0.1}}{P_1}$ ②

After : $0 = C_p\frac{T_{0.2}}{T_2} - R\ln\frac{P_{0.2}}{P_2}$ ③

Combine ①②③: $s_2-s_1 = C_p\ln\frac{T_{0.2}}{T_{0.1}} - R\ln\frac{P_{0.2}}{P_{0.1}}$

$$= -R\ln\frac{P_{0.2}}{P_{0.1}}$$

$$\Rightarrow \frac{P_{0.2}}{P_{0.1}} = e^{-\frac{(s_2-s_1)}{R}} = e^{\frac{-199.5}{287}}$$

$$= 0.499$$

From Appendix B : $M_1 = 2.5$ at $\frac{P_{0.2}}{P_{0.1}} = 0.499$

8.11

解: $\frac{p_{0.1}}{p_1} = \frac{1.555}{1} = 1.555 < 1.893$

$\therefore \frac{p_{0.1}}{p_1} = \left(\frac{T_{0.1}}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}$

$\Rightarrow \left(1+\frac{0.4}{2}M_1^2\right)^{\frac{1.4}{0.4}} = 1.555$

$M_1 = 0.8199$

$V_1 = a_1 \cdot M_1 = \sqrt{\gamma R T_1}\cdot M_1$

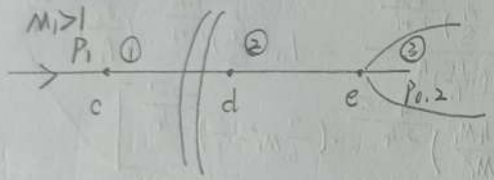
$= \sqrt{1.4\times 287\times 288}\times 0.8199$

$= 278.9$ m/s

8.14

解:

for supersonic flow



由①→②: $\frac{p_2}{p_1} = 1+\frac{2\gamma}{\gamma+1}(M_1^2-1)$

由②→③: $\frac{p_{0.2}}{p_2} = \left(\frac{T_{0.2}}{T_2}\right)^{\frac{\gamma}{\gamma-1}} = \left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}}$

$\therefore \frac{p_{0.2}}{p_1} = \frac{p_{0.2}}{p_2}\cdot\frac{p_2}{p_1}$

$= \left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}}\cdot\left[1+\frac{2\gamma}{\gamma+1}(M_1^2-1)\right]$

$M_2^2 = \frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2-\frac{\gamma-1}{2}}$

$\therefore \frac{p_{0.2}}{p_1} = \left(1+\frac{\gamma-1}{2}\cdot\frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2-\frac{\gamma-1}{2}}\right)^{\frac{\gamma}{\gamma-1}}\cdot\left[1+\frac{2\gamma}{\gamma+1}(M_1^2-1)\right]$

$= \left[\frac{(\gamma-1)[2+(\gamma-1)M_1^2]+4\gamma M_1^2-2\gamma+2}{4\gamma M_1^2-2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}}\cdot\left(\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right)$

$= \left[\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2-2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}}\cdot\left(\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right)$

8.16

解: $\frac{p_{0.2}}{p_1} = \frac{1.13}{0.1} = 11.3 > 1.893$

$\frac{p_{0.2}}{p_1} = 11.3 = \left[\frac{(1.4+1)^2 M_1^2}{5.6M_1^2-0.8}\right]^{\frac{1.4}{0.4}}\times\left(\frac{2.8M_1^2-0.4}{2.4}\right)$

$\approx \left(\frac{2.4^2 M_1^2}{5.6M_1^2}\right)^{\frac{1.4}{0.4}}\times\left(\frac{2.8M_1^2-0.4}{2.4}\right)$

$\Rightarrow M_1 = 3.0$ (近似)

And from Appendix B,

with $\frac{p_{0.2}}{p_1} = 11.3$, we get $M_1 = 2.9$