

空气动力学 HW 8

8.1

解: $a = \sqrt{\gamma R T}$, $T = 230 K$

$$= \sqrt{1.4 \times 287 \times 230}$$

$$= 304.0 \text{ m/s}$$

8.3

解:

$$a = \sqrt{\gamma R T} = \sqrt{1.4 \times 287 \times 300} = 347.19 \text{ m/s}$$

$$M = \frac{V}{a} = \frac{250}{347.19} = 0.72$$

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

$$\Rightarrow T_0 = (1 + \frac{0.4}{2} \times 0.72^2) \times 300 = 331.1 \text{ K} \quad \boxed{\text{ANS}}$$

$$\frac{P_0}{P} = (\frac{T_0}{T})^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow P_0 = 1.2 \times (\frac{331.1}{300})^{\frac{1.4}{0.4}} = 1.695 \text{ atm} \quad \boxed{\text{ANS}}$$

$$(M^*)^2 = \frac{(\gamma+1)M^2}{2 + (\gamma-1)M^2}$$

$$\Rightarrow M^* = \sqrt{\frac{2.4 \times 0.72^2}{2 + 0.4 \times 0.72^2}} = 0.7508 \quad \boxed{\text{ANS}}$$

$$\text{At } M=1, \frac{T_0}{T^*} = \frac{\gamma+1}{2}$$

$$\Rightarrow T^* = \frac{2}{\gamma+1} T_0 = \frac{2}{2.4} \times 331.1 = 275.92 \text{ K} \quad \boxed{\text{ANS}}$$

$$\frac{P_0}{P^*} = (\frac{T_0}{T^*})^{\frac{\gamma}{\gamma-1}} = (\frac{\gamma+1}{2})^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow P^* = \frac{P_0}{(\frac{\gamma+1}{2})^{\frac{\gamma}{\gamma-1}}} = \frac{1.695}{1.2^{\frac{1.4}{0.4}}} = 0.8954 \text{ atm} \quad \boxed{\text{ANS}}$$

8.4

解: $R = 1716 \text{ ft} \cdot \text{lb} / (\text{slug} \cdot ^\circ \text{R})$

$$a = \sqrt{\gamma R T} = \sqrt{1.4 \times 1716 \times 700} = 1296.8 \text{ ft/s}$$

$$M = \frac{V}{a} = \frac{2983}{1296.8} = 2.3$$

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

$$\Rightarrow T_0 = (1 + \frac{0.4}{2} \times 2.3^2) \times 700 = 1440.6 ^\circ \text{R} \quad \boxed{\text{ANS}}$$

$$\frac{P_0}{P} = (\frac{T_0}{T})^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow P_0 = 1.6 \times (\frac{1440.6}{700})^{\frac{1.4}{0.4}} = 20.01 \text{ atm} \quad \boxed{\text{ANS}}$$

$$(M^*)^2 = \frac{(\gamma+1)M^2}{2 + (\gamma-1)M^2}$$

$$\Rightarrow M^* = \sqrt{\frac{2.4 \times 2.3^2}{2 + 0.4 \times 2.3^2}} = 1.756 \quad \boxed{\text{ANS}}$$

$$T^* = \frac{2}{2.4} \times 1440.6 = 1200.5 ^\circ \text{R} \quad \boxed{\text{ANS}}$$

$$P^* = \frac{20.01}{1.2^{\frac{1.4}{0.4}}} = 10.58 \text{ atm} \quad \boxed{\text{ANS}}$$

8.5

解: $\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$

$$\Rightarrow T_0 = (1 + \frac{0.4}{2} \times 4) \times 230 = 414 \text{ K} \quad \boxed{\text{ANS}}$$

$$\frac{P_0}{P} = \left(\frac{T_0}{T} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\begin{aligned} \Rightarrow P_0 &= P \cdot \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}} \\ &= 1 \times (1 + 0.8)^{\frac{1.4}{0.4}} = 7.824 \text{ atm} \quad \boxed{\text{ANS}} \end{aligned}$$

8.6

解: At 10000 ft,

$$\rho_\infty = 1.7556 \text{ slugs/ft}^3$$

$$P_\infty = 1455.6 \text{ lb/ft}^2$$

$$T_\infty = 483.04 \text{ } ^\circ\text{R}$$

$$\begin{aligned} \left. \begin{aligned} \frac{T_0}{T_\infty} &= 1 + \frac{\gamma-1}{2} M_\infty^2 \\ \frac{T_0}{T} &= 1 + \frac{\gamma-1}{2} M^2 \end{aligned} \right\} \Rightarrow T = T_\infty \cdot \frac{1 + \frac{\gamma-1}{2} M_\infty^2}{1 + \frac{\gamma-1}{2} M^2} \\ &= 483.04 \times \frac{1 + 0.2 \times 0.82^2}{1 + 0.2 \times 1^2} \\ &= 456.67 \text{ } ^\circ\text{R} \quad \boxed{\text{ANS}} \end{aligned}$$

isentropic

$$\frac{P}{P_\infty} = \left(\frac{T}{T_\infty} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\begin{aligned} \Rightarrow P &= \left(\frac{1 + \frac{\gamma-1}{2} M_\infty^2}{1 + \frac{\gamma-1}{2} M^2} \right)^{\frac{\gamma}{\gamma-1}} P_\infty \\ &= \left(\frac{1 + 0.2 \times 0.82^2}{1 + 0.2 \times 1^2} \right)^{\frac{1.4}{0.4}} \times 1455.6 \end{aligned}$$

$$= 1195.9 \text{ lb/ft}^2 \quad \boxed{\text{ANS}}$$