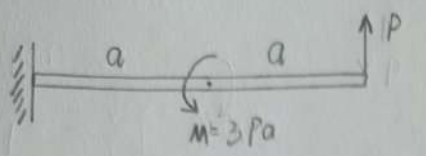
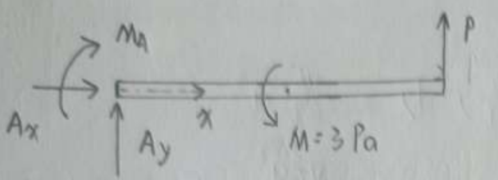
航空结构强度 HW4

5.2 (参考图5.29)

解:



FBD:



$\xrightarrow{+} \Sigma F_x: \ A_x = 0$

$+\uparrow \Sigma F_y: \ A_y + P = 0 \Rightarrow A_y = -P$

$\circlearrowleft \Sigma M_A: \ -M_A + 3P\cdot a + P\cdot 2a = 0 \Rightarrow M_A = 5Pa$

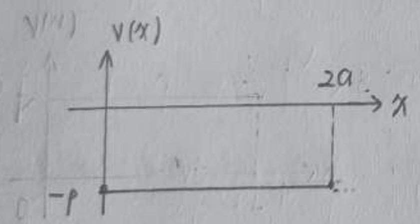
Singular Function:

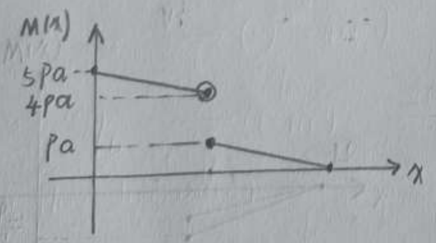
$V(x) = +A_y\langle x-0\rangle^0 = -P \qquad 0 \le x \le 2a$

$M(x) = A_y\langle x-0\rangle^1 + M_A\langle x-0\rangle^0 - M\langle x-a\rangle^0$

$$= \begin{cases} -Px + 5P\cdot a & 0 \le x < a \\ -Px + 2Pa & a \le x < 2a \end{cases}$$

$\therefore$



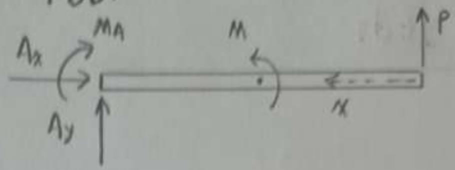


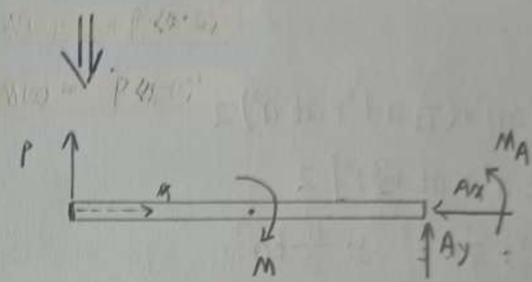
5.4

解: Complementary energy method:

$$v = \frac{\partial U^*}{\partial P}$$

FBD:





$V(x) = P\langle x-0\rangle^0$

$M(x) = P\langle x-0\rangle^1 + M\langle x-a\rangle^0$

$$= \begin{cases} Px & 0 \le x < a \\ Px + M & a \le x < 2a \end{cases}$$

$$U^* = \frac{1}{2}\int_0^{2a} \frac{M^2(x)}{EI} dx$$

$$= \frac{1}{2}\int_0^{a} \frac{M_1^2(x)}{EI} dx + \frac{1}{2}\int_a^{2a} \frac{M_2^2(x)}{EI} dx$$

$$\therefore v = \frac{\partial U^*}{\partial P} = \int_0^a \frac{M_1(x)}{EI}\frac{\partial M_1(x)}{\partial P} dx + \int_a^{2a} \frac{M_2(x)}{EI}\cdot\frac{\partial M_2(x)}{\partial P} dx$$

$$= \int_0^a \frac{1}{EI}\cdot Px^2 dx + \int_a^{2a} \frac{1}{EI}\cdot(Px+M)\cdot x \, dx$$

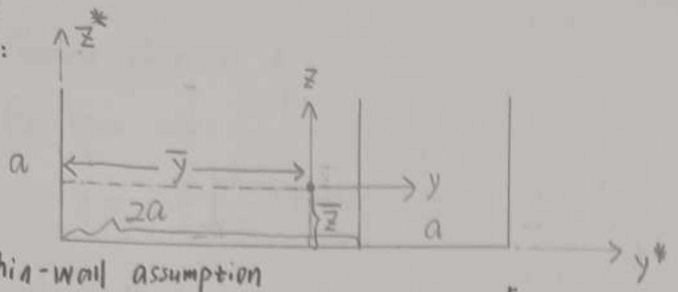
$$= \frac{1}{EI}\cdot P\cdot\frac{1}{3}a^3 + \frac{1}{EI}\cdot\left(\frac{1}{3}Px^3 + \frac{1}{2}M\cdot x^2\right)\Big|_a^{2a}$$

$$= \frac{43}{6}Pa^3\cdot\frac{1}{EI}$$

航空结构强度 HW4

5.6

解:



Thin-wall assumption

$$\bar{y}\cdot A = at\cdot 0+2at\cdot a+at\cdot 2a+at\cdot\tfrac{5}{2}a+at\cdot 3a$$

$$\bar{y} = \frac{2a^2t+2a^2t+\frac{5}{2}a^2t+3a^2t}{at\cdot 3+2at+at} = \frac{19}{12}a$$

$$\bar{z}\cdot A = at\cdot\frac{a}{2}\cdot 3+2at\cdot 0+at\cdot 0$$

$$\bar{z} = \frac{\frac{3}{2}a^2t}{6at} = \frac{1}{4}a$$

$$I_{yy} = \int_A z^2 dA = \left[\frac{1}{12}ta^3+\left(\frac{a}{2}-\frac{a}{4}\right)^2\cdot at\right]\times 3$$

$$+\left[\frac{1}{12}\cdot 3a\cdot t^3+\left(\frac{1}{4}a\right)^2\cdot 3a\cdot t\right]$$

$$= \frac{5}{8}ta^3+\frac{1}{4}at^3$$

$$I_{zz} = \int_A y^2 dA = \left[\frac{1}{12}at^3+\left(\frac{19}{12}a\right)^2\cdot at\right]+\left[\frac{1}{12}at^3+\left(\frac{5}{12}a\right)^2\cdot at\right]$$

$$+\left[\frac{1}{12}at^3+\left(\frac{17}{12}a\right)^2\cdot at\right]$$

$$+\left[\frac{1}{12}\cdot t\cdot(3a)^3+\left(\frac{1}{12}a\right)^2\cdot 3a\cdot t\right]$$

$$= \frac{1}{4}at^3+\frac{167}{24}a^3t$$

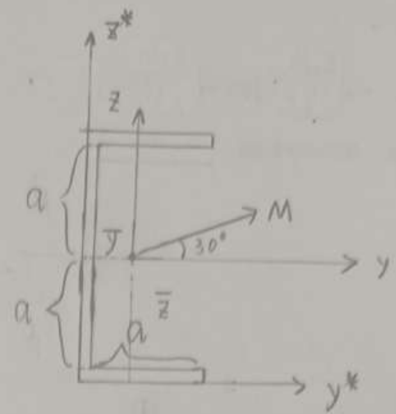
$$I_{yz} = \int_A yz\,dA = at\cdot\left(-\frac{19}{12}a\right)\cancel{(-2a)}\cdot\frac{1}{4}a+at\cdot\frac{5}{12}a\cdot\frac{1}{4}a$$

$$+at\cdot\frac{17}{12}a\cdot\frac{1}{4}a+3at\cdot\left(-\frac{1}{12}a\right)\cdot\left(-\frac{1}{4}a\right)$$

$$= \frac{1}{8}a^3t$$

5.7

解:



$a = 10cm = 0.1m$, $t = 5mm = 0.005m$

$$\bar{y}\cdot A = 2at\cdot\frac{t}{2}+(a+t)\cdot t\cdot\frac{a+t}{2}\cdot 2$$

$$\bar{y} = \frac{at^2+(a+t)^2\cdot t}{2at+(a+t)\cdot t\cdot 2} = \frac{461}{16400}\ m$$

$$\bar{z} = a+t = 0.105\ m$$

$$M_y = M\cdot\cos 30^\circ = 150\sqrt{3}\ N\cdot m$$

$$M_z = M\sin 30^\circ = 150\ N\cdot m$$

$$I_y = \frac{1}{12}\cdot t\cdot(2a)^3+\left[\frac{1}{12}\cdot(a+t)\cdot t^3+(a+t)t\left(a+\frac{t}{2}\right)^2\right]\cdot 2$$

$$= 1.43671\times 10^{-5}\ m$$

$$I_z = \frac{1}{12}\cdot 2a\cdot t^3+\left(\bar{y}-\frac{t}{2}\right)^2\cdot 2at$$

$$+\left[\frac{1}{12}\cdot t\cdot(a+t)^3+\left(\frac{a+t}{2}-\bar{y}\right)^2\cdot(a+t)\cdot t\right]$$

$$= 4.26343\times 10^{-6}\ m$$

$$\sigma = +\frac{M_y\cdot z}{I_y}-\frac{M_z\cdot y}{I_z}$$, Since $I_{yz}=0$

$$= (18.0835z-35.18294y)\ MPa$$

At $z=(a+t)$, $y=-\bar{y}$

$$\sigma_{\max\,T} = \frac{M_y}{I_y}(a+t)+\frac{M_z}{I_z}\cdot\bar{y} = 2.8878\ MPa$$

(Tension)

At $z=-(a+t)$, $y=a+t-\bar{y}$

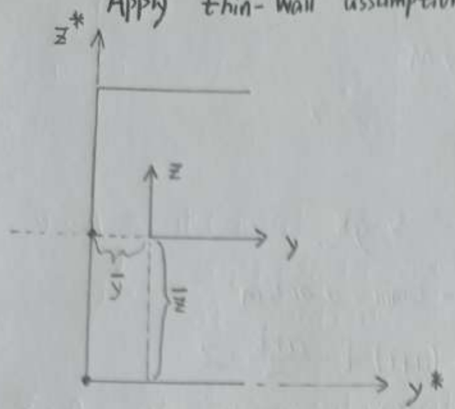
$$\sigma_{\max\,C} = \frac{-M_y}{I_y}(a+t)-\frac{M_z}{I_z}(a+t-\bar{y})$$

$$= -4.604\ MPa$$ (Compression)

5.7

解: Since $t = 5mm \ll a = 10cm$

Apply thin-wall assumption



$$\bar{z} \cdot A = 0 + 2a \cdot t \cdot a + at \cdot 2a$$

$$\bar{z} = \frac{4ta^2}{4ta} = a$$

$$\bar{y} \cdot A = at \cdot \frac{a}{2} \cdot 2 + 0$$

$$\bar{y} = \frac{a^2 t}{4at} = \frac{1}{4}a$$

$$\therefore I_y = \frac{1}{12} \cdot t \cdot (2a)^3 + \left(\frac{1}{12} \cdot a \cdot t^3 + at \cdot a^2\right) \cdot 2$$

$$= \frac{1}{6}at^3 + \frac{8}{3}ta^3 = 1.3335 \times 10^{-5}\ m^4$$

$$I_z = \left(\frac{1}{12} \cdot 2a \cdot t^3 + \bar{y}^2 \cdot 2at\right) + \left[\frac{1}{12} \cdot t \cdot a^3 + at\left(\frac{1}{4}a\right)^2\right] \cdot 2$$

$$= \frac{19}{24}ta^3 + \frac{1}{6}at^3 = 3.9604 \times 10^{-6}\ m^4$$

$$I_{yz} = 2at \cdot \left(-\frac{1}{4}a\right) \cdot 0 + at \cdot \frac{1}{4}a \cdot a + at \cdot \frac{1}{4}a(-a) = 0$$

$\therefore$ We can apply:

$$\sigma = + \frac{M_y \cdot z}{I_y} - \frac{M_z \cdot y}{I_z}$$

$$= \frac{150 z}{1.3335 \times 10^{-5}} - \frac{150 y}{3.9604 \times 10^{-6}}$$

$$= 19.4831 z - 37.8750 y \quad MPa$$

max tension: $\sigma_{mT} = 19.4831 \times 0.1 - 37.8750 \times \left(-\frac{1}{4} \times 0.1\right)$

$$= 2.3952\ MPa$$

max compression: $\sigma_{mc} = 19.4831 \times (-0.1) - 37.8750 \times \left(\frac{3}{4} \times 0.1\right)$

$$= -4.7889\ MPa$$

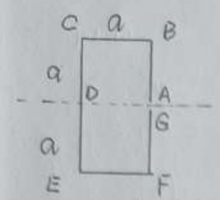
5.8

解:

Assume thickness t

for open cell

$$q = -\frac{V}{I}\int_{s=0}^{s} y t\, ds$$



$$I = \frac{1}{12}\cdot t\cdot(2a)^3 + \left(\frac{1}{12}\cdot a\cdot t^3 + at\cdot a^2\right)\cdot 2$$
$$+\left[\frac{1}{12} t\cdot a^3 + at\cdot\left(\frac{a}{2}\right)^2\right]\cdot 2$$
$$= \frac{10}{3} t\cdot a^3 + \frac{1}{6} a\cdot t^3 \approx \frac{10}{3} t a^3$$

AB段: $q = -\frac{V}{I}\cdot\int_0^{y_0} y\cdot t\, dy$ from $q_A = 0$

$$= -\frac{V}{I}\cdot t\cdot\frac{1}{2}y_0^2$$

$$\therefore q_B = -\frac{Vt}{I}\cdot\frac{1}{2}a^2$$

BC段: $q = q_B - \frac{V}{I}\int_0^{s} a t\, ds$

$$= -\frac{Vt}{2I}a^2 - \frac{V}{I}\cdot ta\cdot s$$

$$\therefore q_C = -\frac{Vt}{2I}a^2 - \frac{Vt}{I}a^2 = \frac{-3Vt}{2I}a^2$$

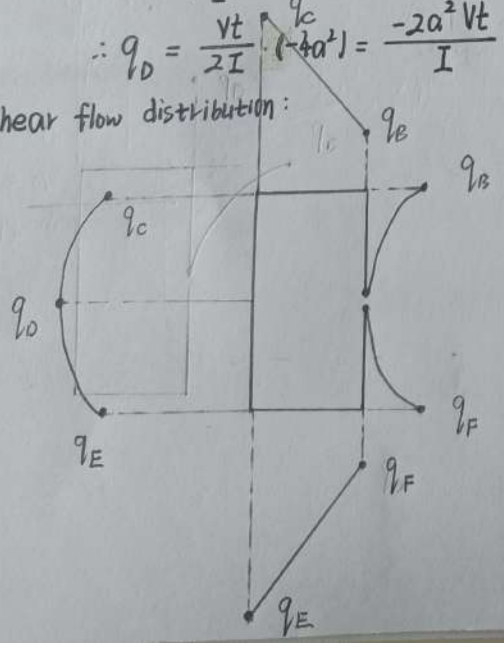
CD段: $q = q_C - \frac{V}{I}\int_a^{y} y t\,(-dy)$

$$= \frac{-3Vt}{2I}a^2 + \frac{Vt}{I}\cdot\frac{1}{2}(y^2 - a^2)$$

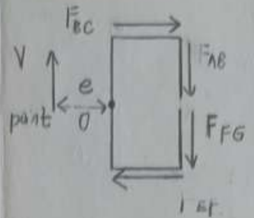
$$= \frac{Vt}{2I}(y^2 - 4a^2)$$

$$\therefore q_D = \frac{Vt}{2I}\cdot(-4a^2) = \frac{-2a^2 Vt}{I}$$

shear flow distribution:



To obtain shear center.



$$F_{AB} = \int_0^{a} \frac{Vt}{2I} y^2\, dy = \frac{Vt}{6I}a^3$$

$$F_{BC} = -\left(\frac{q_B + q_C}{2}\right)\cdot a = \frac{Vta^3}{I}$$

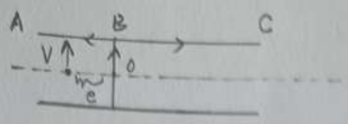
Torque $T = 2F_{AB}\cdot a + F_{BC}\cdot 2a$

$$= \frac{7Vta^4}{3I} = V\cdot e$$

$$\therefore e = \frac{7t}{3I}a^4 \approx \frac{7ta^4}{3\times\frac{10}{3}ta^3} = \frac{7}{10}a$$

5.10

解: wall thickness t



$$I = \frac{1}{12}t\cdot a^3 + \left[\frac{1}{12}\cdot 3a\cdot t^3 + 3at\left(\frac{a}{2}\right)^2\right]\cdot 2$$

$$= \frac{1}{2}at^3 + \frac{19}{12}ta^3 \approx \frac{19}{12}ta^3$$

AB段: $q_A = 0$

Since $q = -\frac{V}{I}\int_{s=0}^{s} yt\,ds$

$$q_{B_1} = -\frac{V}{I}\cdot\int_0^a \frac{a}{2}t\,ds = -\frac{Vt}{2I}a^2$$

$$q = -\frac{V}{I}\int_0^s \frac{a}{2}t\,ds = -\frac{Vt}{2I}as$$

BC段: $q_C = 0$

$$q_{B_2} = -\frac{V}{I}\int_0^{2a}\frac{a}{2}t\,ds = -\frac{Vt}{I}a^2$$

$$q = -\frac{V}{I}\int_0^s \frac{a}{2}t\,ds = -\frac{Vt}{2I}as$$

OB段: $q = q_B - \frac{V}{I}\int_0^y y\cdot t(-dy)$

$$= -\frac{Vt}{2I}a^2 + \frac{Vt}{I}\cdot\frac{1}{2}y^2$$

$$= \frac{Vt}{2I}(y^2 - a^2)$$

$$F_{AB} = \frac{1}{2}\times\left(0 + \frac{Vta^2}{2I}\right)\cdot a = \frac{Vt}{4I}a^3$$

$$F_{BC} = \frac{Vta^2}{I}\cdot\frac{1}{2}\cdot 2a = \frac{Vta^3}{I}$$

$$T = -F_{AB}\cdot a + F_{BC}\cdot a$$

$$= \frac{3Vta^4}{4I}$$

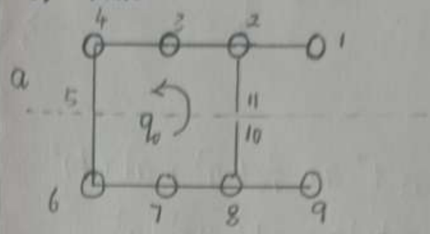
$$e = \frac{T}{V} = \frac{3ta^4}{4I} \approx \frac{3ta^4}{4\times\frac{19}{12}ta^3} = \frac{9}{19}a$$

5.14

解:

a) $I = 4Ba^2\times 2 = 8Ba^2$

b) Make a cut:



Starting from node 1:

$q^V_{12} = -\frac{VBa}{I}$, $q^V_{23} = q^V_{12} + q^V_{11\,2} - \frac{VBa}{I} = -\frac{2VBa}{I}$

$q^V_{11\,2} = 0$, $q^V_{34} = q^V_{23} - \frac{VBa}{I} = -\frac{3VBa}{I}$

$q^V_{45} = q^V_{34} - \frac{VBa}{I} = -\frac{4VBa}{I}$

c) for cell one

$$8a\cdot q_0 = \left(0 + \frac{2VBa^2}{I} + \frac{3VBa^2}{I} + \frac{4VBa^2}{I}\right)\cdot 2$$

$$\Rightarrow q_0 = \frac{9VBa}{4I}$$

$$\therefore q_{11\,2} = \frac{9VBa}{4I} - 0 = \frac{9VBa}{4I}$$

d) $q_{23} = \frac{9VBa}{4I} - \frac{2VBa}{I} = \frac{VBa}{4I}$

$$q_{34} = \frac{9VBa}{4I} - \frac{3VBa}{I} = \frac{-3VBa}{4I}$$

$$q_{45} = \frac{9VBa}{4I} - \frac{4VBa}{I} = \frac{-7VBa}{4I}$$

Compute torque about node 5:

$$T = 2A_1q_0 + \left(q^V_{12}\cdot a\cdot 2a + q^V_{23}\cdot a\cdot 2a + q^V_{34}\cdot a\cdot 2a\right)$$

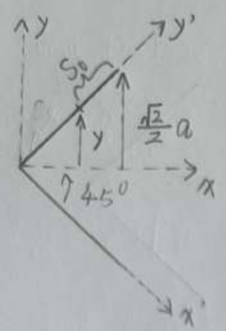
$$= 2\cdot 4a^2\cdot\frac{9VBa}{4I} - 2a^2\cdot\left(\frac{VBa}{I} + \frac{2VBa}{I} + \frac{3VBa}{I}\right)$$

$$= \frac{6VBa^3}{I} = \frac{6VBa^3}{8Ba^2} = \frac{3}{4}a\cdot V$$

$\therefore e = \frac{3}{4}a$ on the right of node 5

5.16

解:



$Ox'y'$ 坐标下: $I_{x'x'} = \frac{1}{12}t^3 \cdot a + \frac{1}{12}t \cdot a^3 + at \cdot (\frac{a}{2})^2$

$= \frac{1}{3}ta^3 + \frac{1}{12}at^3 \approx \frac{1}{3}ta^3$

$I_{y'y'} = \frac{1}{12}t^3 a + \frac{1}{12} \cdot t \cdot a^3 + at(\frac{a}{2})^2$

$= \frac{1}{3}ta^3 + \frac{1}{12}at^3 \approx \frac{1}{3}ta^3$

$I_{x'y'} = 0$

$\theta = 45^\circ$,

$$I_{xx} = \frac{I_{x'x'} + I_{y'y'}}{2} + \frac{I_{x'x'} - I_{y'y'}}{2}\cos 2\theta - I_{x'y'} \cdot \sin 2\theta$$

$= \frac{1}{3}ta^3 + \frac{1}{12}at^3 \approx \frac{1}{3}ta^3$

for open cell : $q = -\frac{V}{I}\int_{s=0}^{s} yt\,ds$

$\therefore q = -\frac{Vt}{I_{xx}} \cdot \int_0^{s_0} y\,ds$

$\frac{(a-s_0)}{\sqrt{2}} = y$

ds dy

$\Rightarrow q = -\frac{Vt}{I_{xx}}\int_0^{s}(\frac{a-s}{\sqrt{2}})ds$

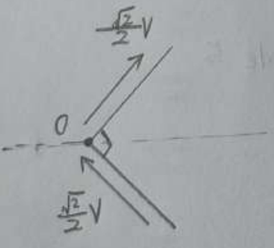
$= \frac{-Vt}{I_{xx}}\left(\frac{a}{\sqrt{2}}s - \frac{1}{\sqrt{2} \cdot 2}s^2\right)$

$\sqrt{2}y = a - s$

$s = a - \sqrt{2}y$

$= \frac{+Vt}{\sqrt{2}I_{xx}} \cdot (y^2 - \frac{1}{2}a^2)$

$\therefore q_{max} = \frac{-\sqrt{2}Vt}{4I_{xx}} \cdot a^2$ at $y = 0$



$T_0 = 0 = e \cdot V$

$\therefore e = 0$

$\therefore$ shear center is point O.