常微分方程B　Week 2　邹佳驹 12012127

2.1节

29.解：

$$\frac{dy}{dt}-2y=t^2e^{2t} \quad ①$$

$$\frac{dy}{dt}-2y=0 \Rightarrow y=Ce^{\int 2dt}=C\cdot e^{2t}$$

$y=v(t)\cdot e^{2t}$ 代入①

$$v'(t)\cdot \cancel{e^{2t}}+v(t)\cdot 2\cancel{e^{2t}}-2v(t)\cancel{e^{2t}}=t^2\cancel{e^{2t}}$$

$$v'(t)=t^2$$

$$v(t)=\frac{1}{3}t^3+C_1$$

$$\therefore y=(\frac{1}{3}t^3+C_1)e^{2t}$$

$$=\frac{1}{3}t^3\cdot e^{2t}+C_1\cdot e^{2t}$$

30.解：$\frac{dy}{dt}+\frac{1}{t}y=\cos 2t$，$t>0$　①

$$\frac{dy}{dt}+\frac{1}{t}y=0 \Rightarrow y=C\cdot e^{\int -\frac{1}{t}dt}=C_1\cdot\frac{1}{t}$$

$y=v(t)\cdot\frac{1}{t}$ 代入①

$$v'(t)\cdot\frac{1}{t}+\cancel{v(t)\cdot(-1)\cdot\frac{1}{t^2}}+\cancel{\frac{1}{t^2}v(t)}=\cos 2t$$

$$v'(t)=t\cos 2t$$

$$v(t)=\int t\cos 2t\,dt=\frac{1}{4}\int m\cos m\,dm,\ m=2t$$

$$=\frac{1}{4}(2t\cdot\sin 2t+\cos 2t+C_2)$$

$$\therefore y=(\frac{t}{2}\sin 2t+\frac{1}{4}\cos 2t+\frac{C_2}{4})\cdot\frac{1}{t}$$

$$=\frac{1}{2}\sin 2t+\frac{1}{4t}\cos 2t+\frac{C_2}{4t}$$

2.2节

23.解：

$$\frac{dy}{dx}=\frac{ay+b}{cy+d}=\frac{(cy+d)\cdot\frac{a}{c}+b-\frac{ad}{c}}{cy+d}=\frac{a}{c}+\frac{b-\frac{ad}{c}}{cy+d}$$

$$\frac{dy}{dx}=\frac{ay+b}{cy+d},\quad cy+d\neq 0$$

(一)，$a\neq 0$时，

① $ay+b=0$

$$y=-\frac{b}{a}$$

② $ay+b\neq 0$

$$\frac{cy+d}{ay+b}dy=dx$$

$$\int\frac{(ay+b)\cdot\frac{c}{a}+d-\frac{bc}{a}}{ay+b}dy=\int dx$$

$$\int(\frac{c}{a}+\frac{d-\frac{bc}{a}}{ay+b})dy=x+C$$

$$\frac{c}{a}y+\frac{ad-bc}{a^2}\cdot\ln|ay+b|=x+C_1$$

$$\Rightarrow x=\frac{c}{a}y+\frac{ad-bc}{a^2}\cdot\ln|ay+b|+C_2.$$

(二)，$a=0$时

$$\frac{dy}{dx}=\frac{b}{cy+d} \Rightarrow x=\frac{c}{2b}y^2+\frac{d}{b}y-\frac{C_1}{b}$$

$(b\neq 0)$

2.3节

1.解：$Q(t)$: At time t, the concentration of dye

$$Q(0)=200\times 1=200g$$

$$\frac{dQ(t)}{dt}=+0-\frac{Q(t)}{200}$$

$$\Rightarrow Q(t)=200e^{-\frac{t}{200}}$$

$$Q(t)=\frac{1}{100}\times 200$$

$$200\cdot e^{-\frac{t}{200}}=2$$

$t=200\ln 100$ mins

≈ 921.0 mins

7. 解: $S(t)$: At time t, the loan

$$\frac{dS(t)}{dt} = 10\% S(t) - k, \quad S(0) = 8000 \text{ 刀}$$

$$S(3) = 0 \text{ 刀}$$

$$\Rightarrow S(t) = C \cdot e^{0.1t} + 10k$$

$$\begin{cases} 8000 = 10k + C \\ 0 = C \cdot e^{0.3} + 10k \end{cases}$$

$$\Rightarrow k = \frac{800 e^{0.3}}{e^{0.3} - 1} \approx 3087 \text{ 刀}$$

more $= 3087 \times 3 - 8000 = 1261$ 刀

9. 解:

a. $$\frac{dQ(t)}{dt} = -rQ(t)$$

$$\frac{dQ(t)}{dt} + rQ(t) = 0$$

$$Q(t) = C \cdot e^{-rt}$$

$$\frac{C \cdot e^{-rt}}{C \cdot e^{-r(t+5730)}} = \frac{2}{1}$$

$$r = \frac{\ln 2}{5730} \approx 1.210 \times 10^{-4}$$

b. $Q(0) = Q_0$

$C = Q_0$

$$\therefore Q(t) = Q_0 \cdot e^{-\frac{\ln 2}{5730} t}$$

c. $$\frac{1}{5} = \frac{Q_0 \cdot e^{\frac{-\ln 2}{5730} t}}{Q_0}$$

$$t = 5730 \log_2 5 \approx 13305 \text{ years}$$

12. 解:

$$\frac{dT(t)}{dt} = k \cdot (T(t) - T_0)$$

$T(0) = 200°F$

$T(1) = 190°F$

$T_0 = 70°F$

$$\Rightarrow T(t) = e^{kt}\left[\int e^{-kt}(-kT_0)\,dt + C\right]$$

$$= C \cdot e^{kt} + T_0$$

$$\begin{cases} 200 = 70 + C \\ 190 = C \cdot e^k + 70 \end{cases} \Rightarrow \begin{cases} C = 130 \\ k = \ln \frac{12}{13} \end{cases}$$

$$130 \times \left(\frac{12}{13}\right)^t + 70 = 150$$

$$t = \log_{\frac{12}{13}} \frac{8}{13} = \frac{\ln \frac{8}{13}}{\ln \frac{12}{13}} = \frac{\ln 8 - \ln 13}{\ln 12 - \ln 13}$$

≈ 6.066 mins