Q1.

A pair of standard spur external gears have parameters as follow: $m = 4\ mm, a = 20°, h_a^* = 1$, reference centre distance $a = 90\ mm$, gear ratio $i_{12} = 1.5$.

(1) Determine the tooth number of two gears: z_1, z_2

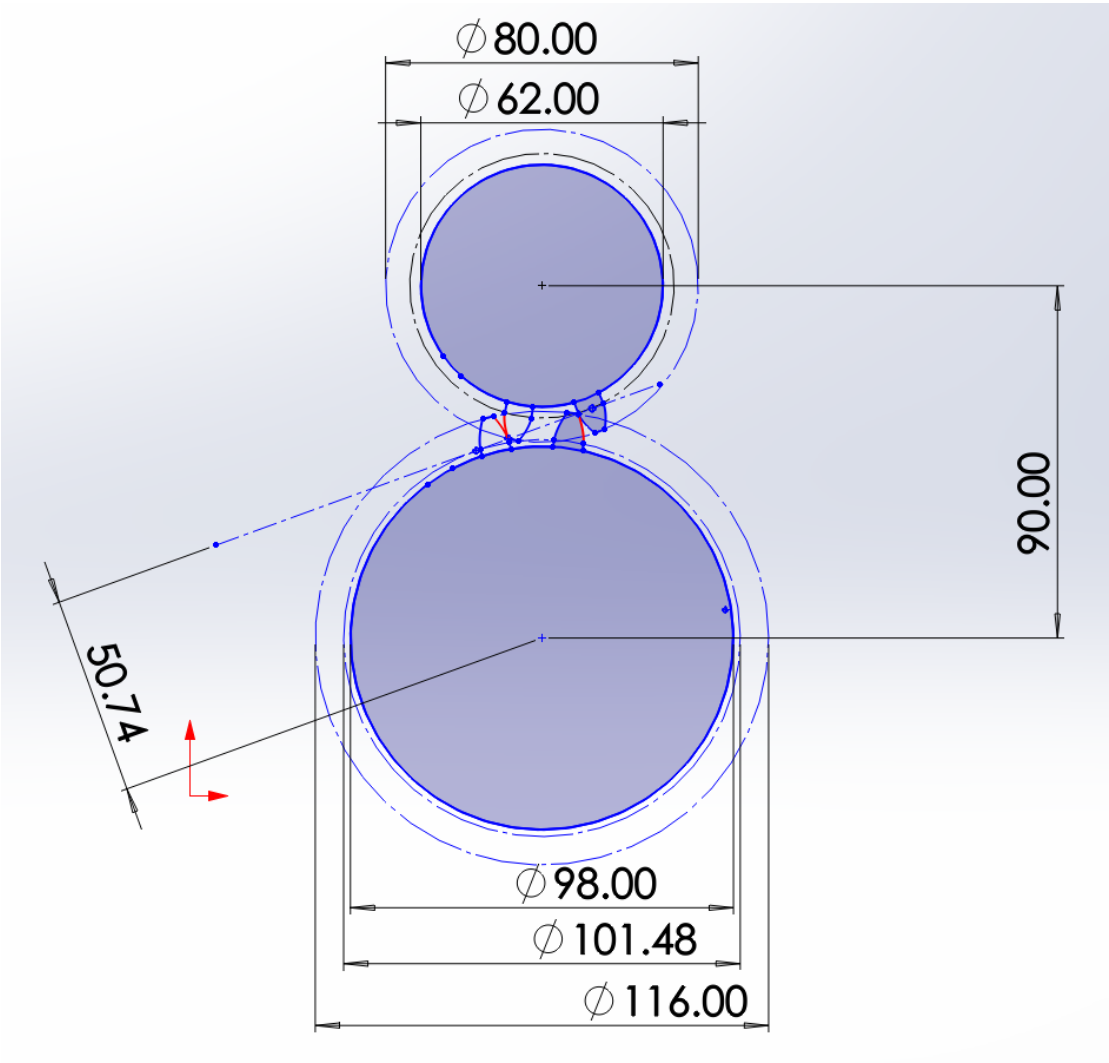
$$i_{12} = \frac{z_2}{z_1} = 1.5$$
$$a = \frac{m}{2}(z_1 + z_2) = 90\ \text{mm} \quad \text{where } m = 4\ \text{mm}$$
$$\Rightarrow z_1 = 18 \quad z_2 = 27$$

(2) Determine the radius of the reference circle and the radius of the addendum circle of two gears: $r_1, r_2;\ r_{a1}, r_{a2}$

$$i_{12} = \frac{r_2}{r_1} = 1.5$$
$$a = r_1 + r_2 = 90\ \text{mm}$$
$$\Rightarrow \quad r_1 = 36\ \text{mm} \quad r_2 = 54\ \text{mm}$$
$$r_{a_1} = r_1 + h_a = r_1 + h_a^* \cdot \text{m} = 36 + 1 \times 4 = 40\ \text{mm}$$
$$r_{a_2} = r_2 + h_a^* \cdot m = 54 + 1 \times 4 = 58\ \text{mm}$$

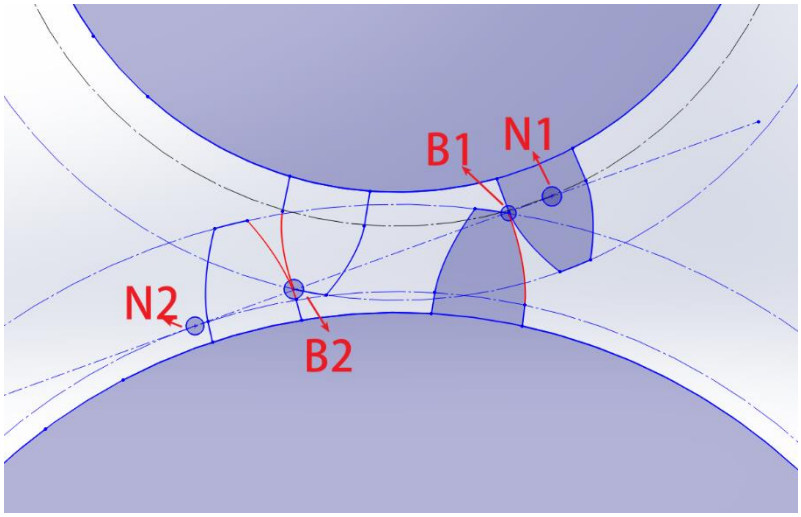
(3) Draw the addendum circle, actual line of action $\overline{B_1B_2}$, and theoretical line of action $\overline{N_1N_2}$of this pair of gears.

Please see the following pictures in the next page.



The addendum circles are drawn in this picture, whose diameters are 80mm and 116mm respectively.

Zoom in this picture to find actual line of action $\overline{B_1B_2}$, and theoretical line of action $\overline{N_1N_2}$of this pair of gears as follows.



Q2.

What are the important properties of involutes? What are the advantages of involute gear transmission?

Properties:

1)$\overline{BK} = \widehat{AB}$

2) The generating line(BK), the normal to the involute(red line), the tangent line to the base circle.

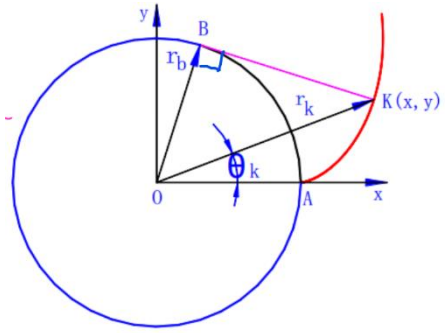
3)$\rho_K = \overline{BK} = \sqrt{r_K^2 - r_b^2}$ ， $\rho_A = 0$

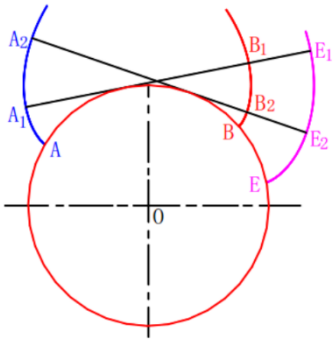
4)The normal distance between two involutes of the same base circle remains the same.

$$\overline{A_1B_1} = \overline{A_2B_2} = \widehat{AB} \text{ and } \overline{B_1E_1} = \overline{B_2E_2} = \widehat{BE}$$

5) The shape of an involute depends only on the radius of its base circle. As the radius rb of the base circle approaches infinity, the involute becomes a straight line.

6) No involute exists inside its base circle.





Advantages:

1) Involute gears have separability of the centre distance.
   (即使两轮中心距稍有改变，其角速比仍保持不变，仍具有良好传动性能，可以用于设计变位齿轮)
2) The position of the action line of the force keeps unchanged. Therefore, the magnitude and the direction of the force keeps unchanged. The frame will not vibrate.
3) Easy to manufacture.

Q3.

What is the contact ratio? What is the relationship between the magnitude of the contact ratio and the tooth number z, module m, pressure angle α, coefficient of addendum h_a^*, coefficient of bottom clearance c^*, and centre distance a?

Contact ratio: the ratio of actual line of action and base pitch(实际啮合弧B_1B_2与基节p_b之比)

$$\varepsilon = \frac{B_1B_2}{p_b}$$

The value of the contact ratio indicates the average number of tooth pairs in contact during a cycle to share the load.

$$r_{a_1} = r_1 + h_a^* m$$
$$r_{a_2} = r_2 + h_a^* m$$
$$\alpha_{a_1} = \cos^{-1}\left(\frac{r_{b_1}}{r_{a_1}}\right)$$
$$\alpha_{a_2} = \cos^{-1}\left(\frac{r_{b_2}}{r_{a_2}}\right)$$
$$r_{b_1} = r_1 \cos\ \alpha$$
$$r_{b_2} = r_2 \cos\ \alpha$$
$$\alpha_{a_1} = \cos^{-1}\left(\frac{r_1 \cos\ \alpha}{r_1 + h_a^* m}\right)$$
$$\alpha_{a_2} = \cos^{-1}\left(\frac{r_2 \cos\ \alpha}{r_2 + h_a^* m}\right)$$
$$a = \frac{m}{2}(Z_1 + Z_2) = r_1 + r_2$$

Combine the above euqtions and we can get:

$$\varepsilon_\alpha = \frac{1}{2\pi}\left[z_1\left(\tan\alpha_{a_1} - \tan\alpha\right) + z_2\left(\tan\alpha_{a_2} - \tan\alpha\right)\right]$$
$$= \frac{1}{2\pi}\left[z_1\left(\tan\cos^{-1}\left(\frac{r_1\cos\ \alpha}{r_1 + h_a^* m}\right) - \tan\alpha\right)\right.$$
$$\left. + z_2\left(\tan\cos^{-1}\left(\frac{r_2\cos\ \alpha}{r_2 + h_a^* m}\right) - \tan\alpha\right)\right]$$

Q4.

What are the characteristics of standard gear transmissions with reference center distances?

reference center distance $a = r_1 + r_2 = \frac{m}{2}(z_1 + z_2)$

1) the reference circles coincide with their pitch circles.
2) The two reference circles are tangential to each other.
3) bottom clearance c=c*m=$h_f - h_a$
4) the backlash b=0