常微分方程B HW6

1.解

(1) linear

inhomogeneous

(2) $t^2 y'' + \sin t\, y' - y = -\cos t$

linear

inhomogeneous

(3) nonlinear

(4) $(1-t^2)y'' + 0y' + (-3)y = 0$

linear

homogeneous

2.解:

(1) $y'' + 3y' - 4y = 0$

$a=1,\ b=3,\ c=-4$

$r^2 + 3r - 4 = 0$

$r_1 = -4,\ r_2 = 1$

$y = C_1 e^{-4t} + C_2 e^{t}$

(2) $a=2,\ b=-1,\ c=-1$

$2r^2 - r - 1 = 0$

$r_1 = \frac{-1}{2},\ r_2 = 1$

$y = C_1 e^{-\frac{1}{2}t} + C_2 e^{t}$

3.解: $2r^2 + (-3)r + 1 = 0$

$r_1 = \frac{1}{2},\ r_2 = 1$

$y = C_1 e^{\frac{1}{2}t} + C_2 e^{t}$

$$\left.\begin{aligned} y(0)=2 &\Rightarrow 2 = C_1 + C_2 \\ y'(0)=\tfrac{1}{2} &\Rightarrow C_1\cdot\tfrac{1}{2} + C_2\cdot 1 = \tfrac{1}{2} \end{aligned}\right\} \Rightarrow \begin{aligned} C_1 &= 3 \\ C_2 &= -1 \end{aligned}$$

$\therefore y = 3e^{\frac{1}{2}t} - e^{t}$

4.解: $\begin{cases} y_1''(t) - y_1'(t) - 2y_1(t) = 0 \\ y_2''(t) - y_2'(t) - 2y_2(t) = 0 \end{cases}$

(1) $\frac{y_2(t)}{y_1(t)} = \frac{e^{2t}}{e^{-t}} = e^{3t} \neq \text{const}$

$y_1(t),\ y_2(t)$ linearly independent solutions

$W[y_1(t), y_2(t)] = \begin{vmatrix} e^{-t} & e^{2t} \\ -e^{-t} & 2e^{2t} \end{vmatrix} = 2e^{t} + e^{t} = 3e^{t} \neq 0$

(2) $\frac{y_2(t)}{y_1(t)} = \frac{t\cdot e^{-3t}}{e^{-3t}} = t \neq \text{const}$, $\begin{cases} y_1''(t) + 6y_1'(t) + 9y_1(t) = 0 \\ y_2''(t) + 6y_2'(t) + 9y_2(t) = 0 \end{cases}$

$y_1(t),\ y_2(t)$ linearly independent solutions

$W[y_1(t), y_2(t)] = \begin{vmatrix} e^{-3t} & te^{-3t} \\ -3e^{-3t} & (1-3t)e^{-3t} \end{vmatrix} = (1-3t)e^{-6t} + 3te^{-6t} \neq 0$

5.解:

$W[f,g] = \begin{vmatrix} f & g \\ f' & g' \end{vmatrix} = fg' - f'g$

$W[u,v] = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = (f+2g)\cdot(f'-g') - (f-g)(f'+2g')$

$= -3(fg' - gf')$

$= -3t\cos t - 3\sin t$

6.解: $ty'' + 2y' + t\cdot e^{t} y = 0$

$y'' + \frac{2}{t}y' + e^{t}y = 0$

$W[y_1, y_2](t) = e^{-\int \frac{2}{t}dt} = \frac{1}{t^2}\cdot C$

$W[y_1, y_2](1) = 2 \Rightarrow C = 2$

$W[y_1, y_2](5) = \frac{2}{25}$

7. 解:

if $y = \sin t^2$ is a solution

$$y' = \cos t^2 \cdot 2t$$

$$y'' = -\sin t^2 \cdot 2t \cdot 2t + \cos t^2 \cdot 2$$

$$-\sin t^2 \cdot 2t \cdot 2t + 2\cos t^2 + p(t) \cdot \cos t^2 \cdot 2t + q(t) \cdot \sin t^2 = 0$$

let $t=0$, $0+2+0+0 \neq 0$

so $y=\sin t^2$ is not a solution on an interval containing $t=0$