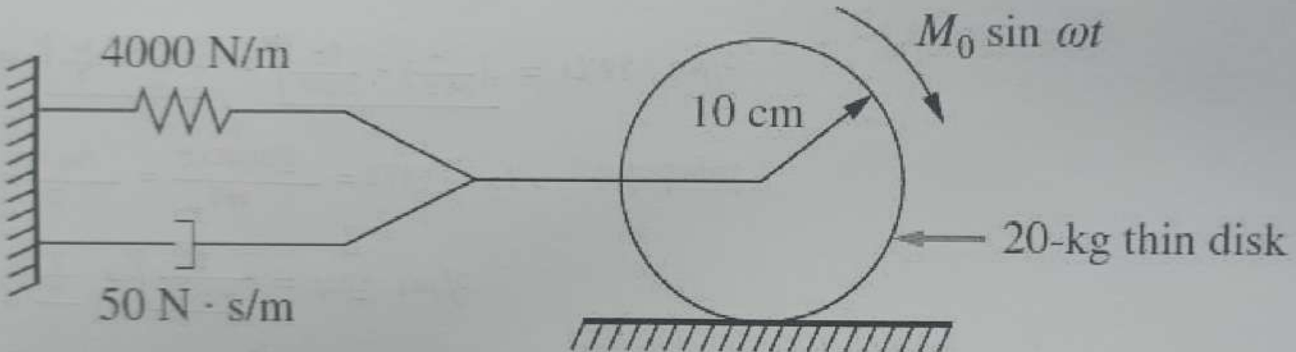
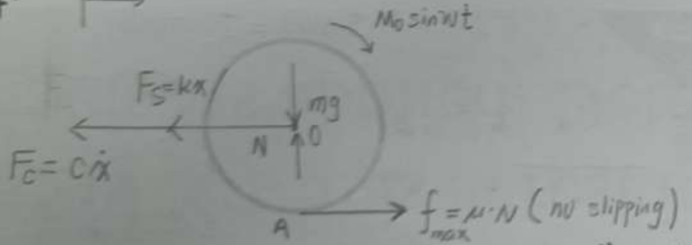
Homework

If $\omega = 16.5$ rad/s, what is the maximum value of M_0 such that the disk of Figure P4.17 rolls without slip?

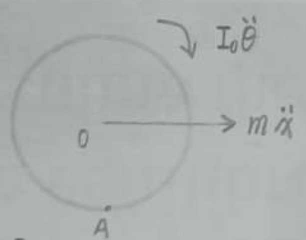


$\mu = 0.12$

解: $\curvearrowright \theta$ $\quad \vdash\!\rightarrow x$ FBD



KD



$+\curvearrowright \Sigma M_A = M_0 \sin wt - kx \cdot R - c\dot{x} \cdot R = I_0 \ddot{\theta} + m\ddot{x} R$

$\xrightarrow{+} \Sigma F_x = f - c\dot{x} - kx = m\ddot{x}$

$\Rightarrow (I_0 + mR^2)\ddot{\theta} + cR^2\dot{\theta} + kR^2\theta = M_0 \sin wt$

$W_n = \sqrt{\frac{kR^2}{I_0 + mR^2}} = \sqrt{\frac{2k}{3m}} = 11.55$ rad/s , $r = \frac{w}{w_n} = \frac{16.5}{11.55} = 1.429$

$\zeta = \frac{cR^2}{2\sqrt{(I_0 + mR^2) \cdot kR^2}} = \frac{c}{2\sqrt{1.5 km}} = 0.072$

initial, $t = 0$, $\dot{x} = 0$

$k \cdot X = f_{max} = \mu N \Rightarrow X = \frac{\mu N}{k} = 5.886 \times 10^{-3}$ m

is the max displacement

Magnification factor $M = \frac{I_A W_n^2 X}{M_0} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

$\Rightarrow M_0 = I_A W_n^2 X \cdot \sqrt{(1-r^2)^2 + (2\zeta r)^2}$, $I_A = \frac{3}{2} m R^2$

$= 0.25$ N·m $\boxed{\text{ANS}}$

Homework

A SDOF system with m=20 kg, k=10,000 N/m, and c=540 N·s/m is at rest in equilibrium when a 50 N·s impulse is applied. Determine the response of the system.

解: impulse $I = 50N \cdot S$

$$\ddot{x} + 2\xi w_n \dot{x} + w_n^2 x = \frac{I}{m}\delta(t)$$

$$w_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10000}{20}} = 10\sqrt{5} \text{ rad/s}$$

$$\xi = \frac{c}{2m w_n} = \frac{540}{2 \times 20 \times 10\sqrt{5}} = 0.6037 < 1$$, Underdamped

$$w_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = \sqrt{\frac{10000}{20} - \left(\frac{540}{40}\right)^2} = 17.83 \text{ rad/s}$$

$$x(t) = \frac{I}{m w_d} e^{-\xi w_n t} \sin w_d t = 0.14 e^{-13.50t} \sin(17.83t) \quad \boxed{\text{ANS}}$$