常微分方程B　邹佳驹 12012127　Week 1 HW

1.1节

7.解: direction field 见附图1

As $t\to\infty$, if $y>0$, $y\to 4$

if $y<0$, $y\to-\infty$

if $y=0$, $y=0$

14.解:

由 $y=-2$ 进行匹配: b, g,

由 $y=0$ ……: b.

$\therefore$ the direction field matchs: $y'=2+y$

15.解:

$y=3$, $y=0$ $\Rightarrow$: e, h.

$y=5$ $\Rightarrow$: h

$\therefore$ it matches: $y'=y(3-y)$

21.解:

a. $F=av^2$

$\frac{dv}{dt}=mg-F$

$\frac{dv}{dt}=mg-av^2$

b. $\cancel{\int\frac{dv}{mg-av^2}=\int dt}$ for a long time

$F=mg$

$av^2=mg$

$v=\sqrt{\frac{mg}{a}}$

c. $g=9.8m/s^2$

$49^2a=10\times9.8$

$a=\frac{98}{49^2}=\frac{2}{49}$

d. direction field 见附图2, $\frac{dv}{dt}=98-\frac{2}{49}v^2$

1.2节

1.解:

a. $\frac{dy}{dt}=-y+5$

$\frac{dy}{-y+5}=dt$

$\int\frac{-1}{-y+5}d(-y+5)=\int dt$

$-\ln|-y+5|=t+C_1$

$y=\pm a_1e^{-t}+5=5+A_1e^{-t}$

由 $y_0=5+A_1$ $\Rightarrow$ $y=5+(y_0-5)e^{-t}$

b. $\frac{dy}{dt}=-2y+5$

$\frac{dy}{-2y+5}=dt$

$\frac{1}{-2}\int\frac{1}{-2y+5}d(-2y+5)=\int dt$

$\frac{1}{-2}\ln|-2y+5|=t+C_2$

$y=\pm a_2e^{-2t}+\frac{5}{2}=\frac{5}{2}+A_2e^{-2t}$

$y_0=\frac{5}{2}+A_2$ $\Rightarrow$ $y=\frac{5}{2}+(y_0-\frac{5}{2})e^{-2t}$

c. $\frac{dy}{dt}=-2y+10$

$\frac{dy}{-2y+10}=dt$

$\frac{1}{-2}\int\frac{1}{-2y+10}d(-2y+10)=\int dt$

$\frac{1}{-2}\ln|-2y+10|=t+C_3$

$y=\pm a_3e^{-2t}+5=5+A_3e^{-2t}$

$y_0=5+A_3$ $\Rightarrow$ $y=5+(y_0-5)e^{-2t}$

resemble and differ:

a.c: 平衡解相同

b.c 趋于平衡解的速率快于a

10.解:

a. $\frac{dQ}{dt}=-rQ$

$\int \frac{dQ}{Q}=\int -r\,dt$

$\ln|Q|=-rt+C$

$Q=ae^{-rt}$

$\frac{Q_1}{Q_2}=\frac{e^{-rt_1}}{e^{-rt_2}}=e^{-rt_1+rt_2}$

$\frac{100}{82.04}=e^{r(t_2-t_1)}=e^{r}$

$r=\ln\frac{100}{82.04}$

≈ 0.19796

t: /week | ⇒ t: / day

Q: /mg | Q: /mg

$r\approx 0.02828$

b. $Q=ae^{-rt}$

$=a\left(\frac{1}{e^{r}}\right)^{t}$

由 $Q(0)=100$

$\Rightarrow Q(t)=100e^{-0.19796t}$ $\left(\begin{matrix}t:/\text{week}\\ Q:/\text{mg}\end{matrix}\right)$

或 $Q(t)=100e^{-0.02828t}$ $\left(\begin{matrix}t:/\text{day}\\ Q:/\text{mg}\end{matrix}\right)$

c. $Q=\frac{1}{2}Q_0$

$\left(\frac{1}{e^{r}}\right)^{t}=\frac{1}{2}$

$0.8204^{t}=\frac{1}{2}$

$t=\log_{0.8204}\frac{1}{2}\approx 3.501$ weeks

≈ 24.50 days

12.解:

a. $\frac{du}{dt}=-k(u-T)$

$\frac{du}{u-T}=-k\,dt$

$\int \frac{1}{u-T}\,d(u-T)=\int -k\,dt$

$\ln|u-T|=-kt+C$

$u(t)=a\cdot e^{-kt}+T$

$u(0)=a+T=u_0 \Rightarrow a=u_0-T$

$u(t)=(u_0-T)e^{-kt}+T$

b. $u(\tau)=(u_0-T)\frac{1}{2}+T=\frac{1}{2}u_0+\frac{1}{2}T=(u_0-T)e^{-k\tau}+T$

$(u_0-T)e^{-k\tau}=\frac{1}{2}u_0-\frac{1}{2}T$

$e^{-k\tau}=\frac{\frac{1}{2}u_0-\frac{1}{2}T}{u_0-T}$

$-k\tau=\ln\frac{\frac{1}{2}u_0-\frac{1}{2}T}{u_0-T}$

$k\tau=\ln 2$

1.3节

1.解: order: 2

linear

2.解: order: 2

nonlinear

6.解:

$y_1(t)=e^{-3t}$

$y_1'(t)=-3e^{-3t}$

$y_1''(t)=-3\cdot e^{-3t}\cdot(-3)=9e^{-3t}$

$9e^{-3t}+(-6)e^{-3t}-3e^{-3t}=0$

$y_1''(t)+2y_1'(t)-3y_1(t)=0$ check

$y_2(t)=e^{t}$

$y_2'(t)=e^{t}$

$y_2''(t)=e^{t}$

$e^{t}+2e^{t}-3e^{t}=0$

$y_2''(t)+2y_2'(t)-3y_2(t)=0$ check

2.1节 $p(t)\frac{dy}{dt}+Q(t)y=G(t)$

2.解: $y'-2y=t^2e^{2t}$

a. direction field 见附图3

b. the solution in infinity for large t

c. $\frac{dy}{dt}-2y=t^2e^{2t}$

$u(t)=e^{-2t}$

$\frac{d(e^{-2t}\cdot y)}{dt}=t^2e^{2t}\cdot e^{-2t}=t^2$

$y=\frac{1}{3}t^3\cdot e^{2t}+c\cdot e^{2t}$, $t\to+\infty$, $y\to+\infty$

4.解:

a. 见附图4

b. the solution is asymptotic to $\frac{3}{2}\sin 2t$ for large t

c. $\frac{dy}{dt}+\frac{1}{t}y=3\cos 2t$, $t>0$

$t\cdot\frac{dy}{dt}+y=3t\cdot\cos 2t$

$u(t)=t$, $\frac{d(ty)}{dt}=3t\cdot\cos 2t$, $y=\frac{3}{2}\sin 2t+\frac{3}{4t}\cos 2t+\frac{C}{t}$

$t\to\infty$, y is asymptotic to $\frac{3}{2}\sin 2t$

2.2节

2.解: $\frac{dy}{dx}+y^2\sin x=0$

$$\frac{dy}{dx}=-y^2\sin x$$

①$y\neq 0$, $\frac{1}{+y^2}dy=-\sin x\,dx$　　② $y=0$

$$\int\frac{1}{y^2}dy=\int-\sin x\,dx$$

$$\frac{-1}{y}=\cos x+C$$

$$y=\frac{-1}{\cos x+C}$$

$\therefore y=\begin{cases}0\\ \frac{-1}{\cos x+c}\end{cases}$

6.解: $\frac{dy}{dx}=\frac{x^2}{1+y^2}$

$$\int(1+y^2)\,dy=\int x^2\,dx$$

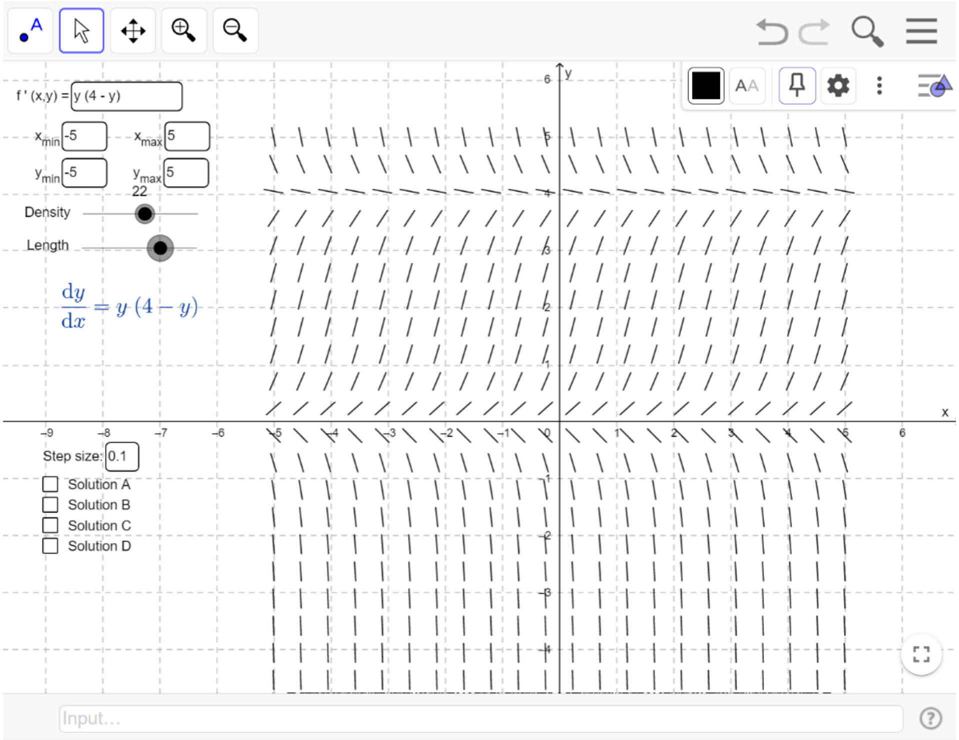
$$y+\frac{1}{3}y^3=\frac{1}{3}x^3+C'$$

$$x^3=3y+y^3+C$$

$$x=\sqrt[3]{3y+y^3+C}$$

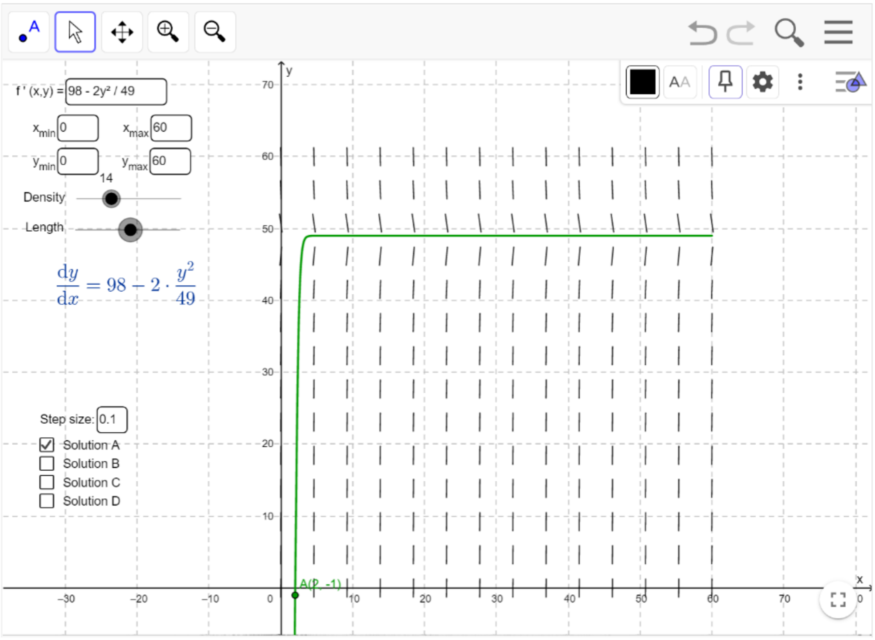
附图 1

1.1 节 7 题



附图 2

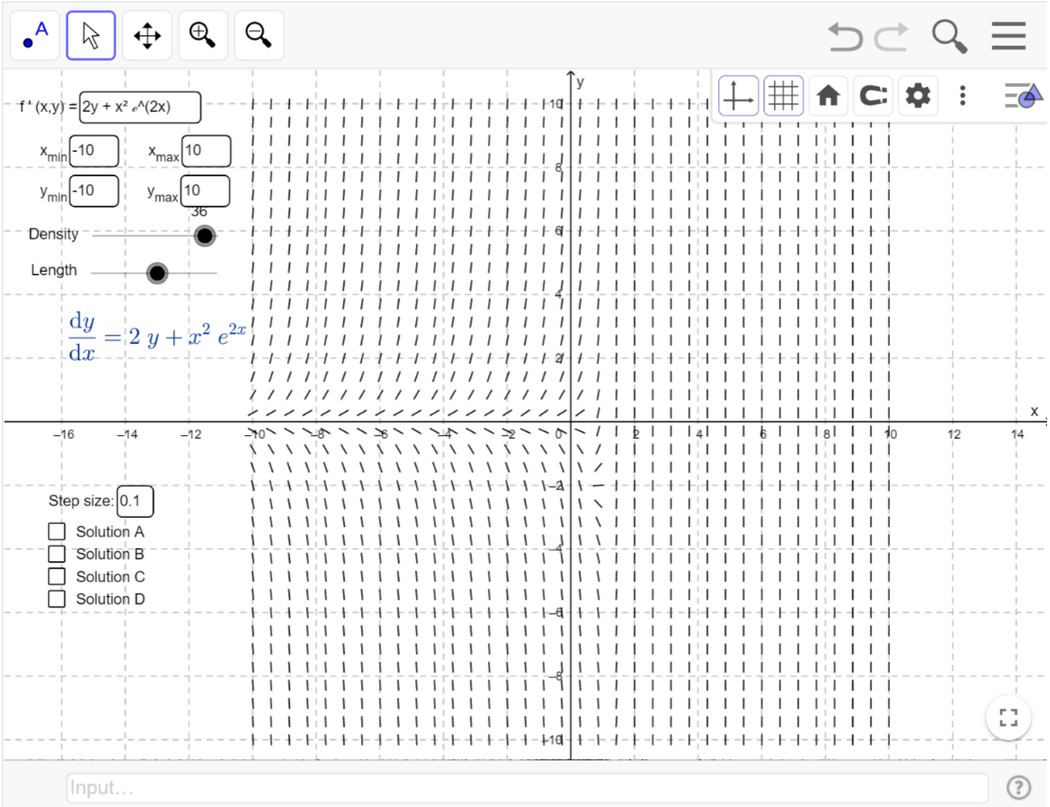
1.1节 21 题



和图 1.1.3 比较，有相同的平衡解

附图 3

2.1 节 2 题



附图 4

2.1 节 4 题