MAE407 Jet Propulsion

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Chapter 1
Solutions to Questions

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Solution to Exercise 1.1

In x-y-z co-ordinates about centre of earth, position vector of (1) is given by

$$\mathbf{r}_1 = R_e(\cos\theta_1 \cos\phi_1, \cos\theta_1 \sin\phi_1, \sin\theta_1)$$

where $R_e =$ radius of earth, θ_1 is latitude and ϕ_1 is longitude

Likewise for (2), $\mathbf{r}_2 = R_e(\cos\theta_2 \cos\phi_2, \cos\theta_2 \sin\phi_2, \sin\theta_2)$

Around equator 1nm = $R_e\delta$, $\delta = 1$ minute of arc

$$\therefore \quad R_e = \frac{60\times 360}{2\pi} \text{ nm} = \underline{3438} \text{ nm}$$

Dot product between position vectors for (1) and (2) gives $R_e^2 \cos A$ where A is a subtended angle.

$$\therefore \quad \cos A = \cos\theta_1 \cos\phi_1 \cos\theta_2 \cos\phi_2 + \cos\theta_1 \sin\phi_1 \cos\theta_2 \sin\phi_2 + \sin\theta_1 \sin\theta_2$$

$$= \cos\theta_1 \cos\theta_2 \cos(\phi_1 - \phi_2) + \sin\theta_1 \sin\theta_2$$

London: $\theta_1 = 51\cdot5°\text{ N}$ $\quad \phi_1 = 0$

Sydney: $\theta_2 = 33\cdot9°\text{ S}$ $\quad \phi_2 = 151\cdot3°\text{E}$

$$\therefore \quad \cos A = 0\cdot6225\times0.8300\times(-0\cdot8772)+0\cdot7826\times(-0\cdot5578) = -0\cdot8892$$

$$\therefore \quad A = 152\cdot8° = 2\cdot667 \text{ rad}$$

$$\therefore \quad \text{Distance apart} = R_e A = 3438\times2\cdot667 = \underline{9168} \text{ nm} = \underline{16769} \text{ km}$$

Solution to Exercise 1.2

1.2 8000 Nautical Miles range $= 8000 \times 1\cdot829 = \underline{14632}$ km

31000 ft altitude $= 31.10^3 \times 0\cdot3048$ m $= \underline{9448}$ m

635·6 tonne $= 635\cdot6.10^3$ kg $= \dfrac{635\cdot6.10^3}{0\cdot4536}$ lbs $= \underline{1\cdot40.10^6}$ lbs

Estimate time of flight for maximum range at $M = 0\cdot85$ at 31000 ft altitude

Temperature = 226·7 K

$$a = \sqrt{\gamma RT} = \sqrt{1\cdot4 \times 287 \times 226\cdot73} = 301\cdot8 \text{ m/s}$$

$\therefore$ Flight speed $= 0\cdot85 \times 301\cdot8 = 256\cdot6$ m/s

Time of 14632 km flight $= \dfrac{14632}{256\cdot6 \times 3600} = \underline{15\cdot8 \text{ h}}$

Solution to Exercise 1.3

At 31000 ft, $M = 0{\cdot}85$

Speed $= 0{\cdot}85 \times 301{\cdot}8 = \underline{256{\cdot}5}$ m/s $= \underline{923{\cdot}5}$ km/h (see 1.2 above)

At 41000 ft $M = 0{\cdot}85$

Temp $= 216{\cdot}65$ K Speed of sound $= 295{\cdot}0$ m/s

Speed $= 0{\cdot}85 \times 295{\cdot}0 = \underline{250{\cdot}8}$ m/s $= \underline{902{\cdot}8}$ km/h

Solution to Exercise 1.4a) (discussed)

a) $dp = -\rho g dh$, $\rho = p/RT$, $T = T_{sl} - kh$ where T_{sl} is the sea-level temperature.

$$dp = \frac{-p}{R(T_{sl} - kh)} g dh \qquad \text{giving} \qquad [\ell n p]_{sl}^{H} = \frac{+g}{Rk}[\ell n(T_{sl} - kh)]_{sl}^{H}$$

$$\ell n(p/p_{sl}) = \frac{g}{Rk}\ell n\left(\frac{T_{sl} - kH}{T_{sl}}\right),$$ where p_{sl} = sea-level pressure and p is pressure at altitude H.

$$\therefore \quad p = p_{sl}\{1 - (k/T_{sl})H\}^{g/Rk} = p_{sl}(T/T_{sl})^{g/Rk}$$

$$p_{sl}\left\{1 - \frac{6 \cdot 5.10^{-3}}{288 \cdot 15}H\right\}^{\frac{9{\cdot}81}{287\times 6{\cdot}5.10^{-3}}} = p_{sl}\{1 - 2 \cdot 26.10^{-5}\}^{5{\cdot}26}$$

Take 35000 ft altitude, ($H = 10668$ m) $p/p_{sl} = 0 \cdot 235$

Compare with value in Table 1.2 $p/p_{sl} = 0 \cdot 238$

Discrepancy much smaller than error of approximation in the idealisation.

Above Tropopause $\quad dp = -\rho g dh = -\dfrac{p}{RT_T} g dh \qquad dp/p = -(g/RT_T)dh$

$$p/p_T = \exp\{-(g/RT_T)(H - 11.10^3)\} = \exp\{-1 \cdot 58.10^{-4}(H - 11.10^3)\}$$

where p_T is pressure at tropopause, 11.10^3 m.

Solution to Exercise 1.4b) (discussed)

b) $dp = -\rho g dh$. If p/ρ^{γ} = constant, $p^{1/\gamma} = K\rho$ and $p_{sl}^{1/\gamma} = K\rho_{sl}$,

where p_{sl} is sea-level pressure and K is a constant.

Hence $dp = -\dfrac{p^{1/\gamma}}{K} g dh$, $\displaystyle\int_0^H \frac{dp}{p^{1/\gamma}} = -\int_0^H \frac{g}{K} dh$

$$\left[p^{\gamma-1/\gamma}\right]_{p_{sl}}^{p_H} = -\frac{\gamma-1}{\gamma}\frac{gH}{K} \rightarrow \left(\frac{p_H}{p_{sl}}\right)^{\gamma-1/\gamma} = 1 - \frac{\gamma-1}{\gamma}\frac{gH}{Kp_{sl}^{\gamma-1/\gamma}}$$

Then eliminating the constant $K = p_{sl}^{/\gamma}/\rho_{sl}$ and using $p_{sl}/\rho_{sl} = RT_{sl}$ gives

$$\left(p/p_{sl}\right)^{\gamma-1/\gamma} = 1 - \frac{\gamma-1}{\gamma}\frac{gH}{RT_{sl}} \quad \text{or} \quad p/p_{sl} = \left[1 - \frac{\gamma-1}{\gamma}\frac{gH}{RT_{sl}}\right]^{\gamma/\gamma-1}$$

e.g. at 35000 ft $H = 10668$ m $p/p_{sl} = 0\cdot 2079$

$\rho/\rho_{sl} = (p/p_{sl})^{1/\gamma}$ $\therefore \rho_H/\rho_{sl} = 0\cdot 3257$, $T/T_{sl} = (p/p_{sl})^{\gamma-1/\gamma}$, $T_H/T_{sl} = 0\cdot 6384$