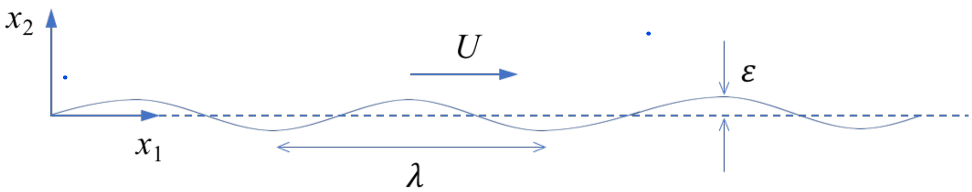
1 Assignment 1

Question No.	1	2	3	4	5	Total
Score	26%	18%	18%	24%	14%	100%

Q 1.1. A sinusoid wavy wall (in two-dimensions) of amplitude ε ($\ll 1$) is moving at a speed U in the x_1-direction.



(i) Write down the boundary conditions satisfied by the velocity potential. [14%]

(ii) Show that the wall generates a travelling wave in the x_2 direction only if it moves at supersonic speeds. [12%]

Total: [26%]

Q 1.2. The radius of an isolated spherical bubble formed by boiling water expands uniformly from nothing to 10^{-3} m in 10^{-3} s. The velocity potential outside the bubble can be written in the form $\phi(r,t) = f(t - r/c)/r$. Show that

$$\begin{cases} f(t) = 0 & \text{for} \quad t < 0 \\ f(t) = -u_0^3 t^2 & \text{for} \quad 0 < t < t_0 \end{cases}$$

where $t_0 = 10^{-3}$ s and $u_0 = 1$ m/s. [10%]

Hence, determine the pressure field at a distance r from the centre of the bubble, throughout the period $t < t_0 + r/c$. [8%]

Hint: On the bubble surface $|\partial f/\partial t|/(rc) \ll |f|/r^2$

Total: [18%]

Q 1.3. Consider a sphere of radius $a = 0.3$ m undergoing a small-amplitude pulsation with an angular frequency ω. The radial displacement on the surface of the sphere is given by $\varepsilon e^{i\omega t}$ with the amplitude $\varepsilon = 1$ mm.

(i) Show that the acoustic pressure radiated by the sphere, at a distance r from the center of the sphere, is given by

$$p'(r,t) = -\frac{a}{r}\frac{ka}{1+ika}\rho_0 c\omega\varepsilon e^{i\omega t}e^{-ik(r-a)}$$

where $k = \omega/c$, c is the speed of sound and ρ_0 is the ambient density. [10%]

(ii) Calculate the radiated acoustic power at a frequency of $f = 100$ Hz.

Take air density $\rho_0 = 1.2$ kg m^{-3} and speed of sound $c = 340$ m s^{-1}. [8%]

Total: [18%]

Q 1.4. By determining the energy flow to infinity, or otherwise, calculate the sound power radiated into air with mean density ρ_0 and speed of sound c by

(i) a sphere of mean radius a pulsating with normal surface velocity $Ue^{i\omega t}$, with $ka \ll 1$; [11%]

(ii) a hard sphere of radius a oscillating with velocity $Ue^{i\omega t}$, with $ka \ll 1$. [11%]

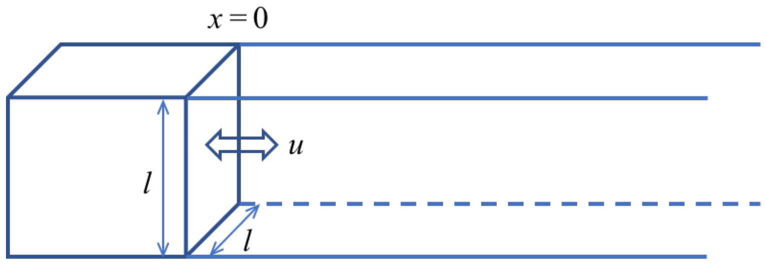
(iii) Compare the radiated sound power between the two cases. [2%]

Total: [24%]

Q 1.5. A square piston forming one side of an otherwise rigid, fixed cube of side $l = 0.25$ m vibrates with a small amplitude $\varepsilon = 1$ mm, and is positioned in an infinitely long tube with square cross-section of side l. The cube is oriented so that the vibration of the position is parallel to the walls of the tube.

(i) Find the 'cut-off' frequency f_c in Hz below which only the plane wave propagates within the tube. [6%]

(ii) At a frequency $f = 150$ Hz, determine the radiated acoustic pressure and sound power at a distance $r = 1$ m from the cube. [8%]



Take air density $\rho_0 = 1.2$ kg/m^3 and speed of sound $c = 340$ m/s.

Hint: The pressure perturbations in the tube have the form

$$p'(\boldsymbol{x}, t) = \sum_{m,n} \cos\left(\frac{m\pi y}{l}\right) \cos\left(\frac{n\pi z}{l}\right) A_{mn} e^{i(\omega t - k_{mn} x)}$$

$$\text{with} \qquad k_{mn} = \sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{l^2}(m^2 + n^2)}$$

Total: [14%]