力学与航空航天工程系
DEPARTMENT OF MECHANICS AND AEROSPACE ENGINEERING

MECHANICS OF MATERIALS

YAHUI XUE (薛亚辉)

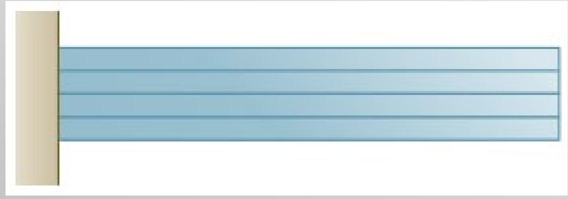
SPRING, 2022

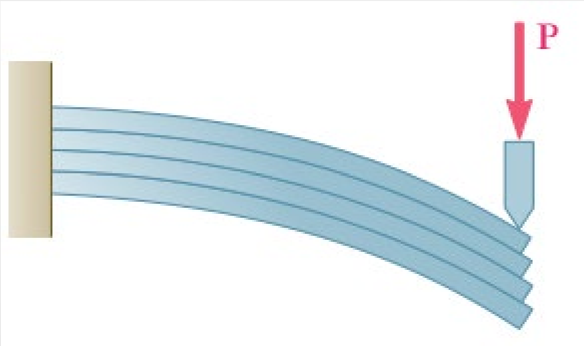
Lesson 8： Shearing stresses in beams and thin-walled Members

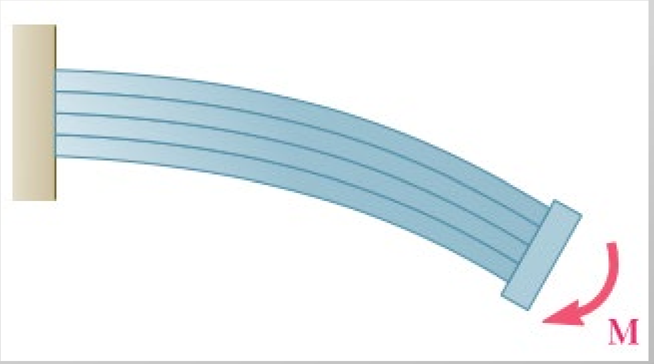
- Shear on the Horizontal Face of a Beam Element
- Determination of the Shearing Stresses in a Beam
- Distribution of Stresses in a Narrow Rectangular Beam
- Shearing Stresses in Thin-Walled Members
- Shear Center

§ 8.1 Introduction

- Consider a cantilever beam made of separate planks clamped together at one end. When a transverse load P is applied to the free end of this composite beam, the planks are observed to slide with respect to each other.
- In contrast, if a couple M is applied to the free end of the same composite beam, the various planks will bend into concentric arcs of circle and will not slide with respect to each other.





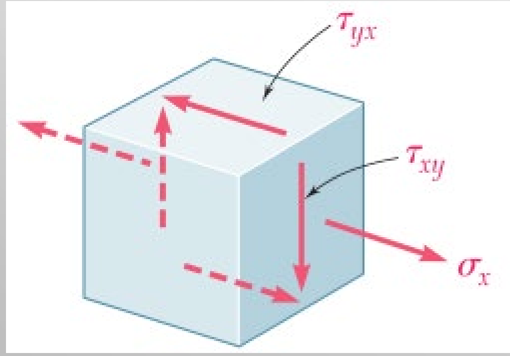


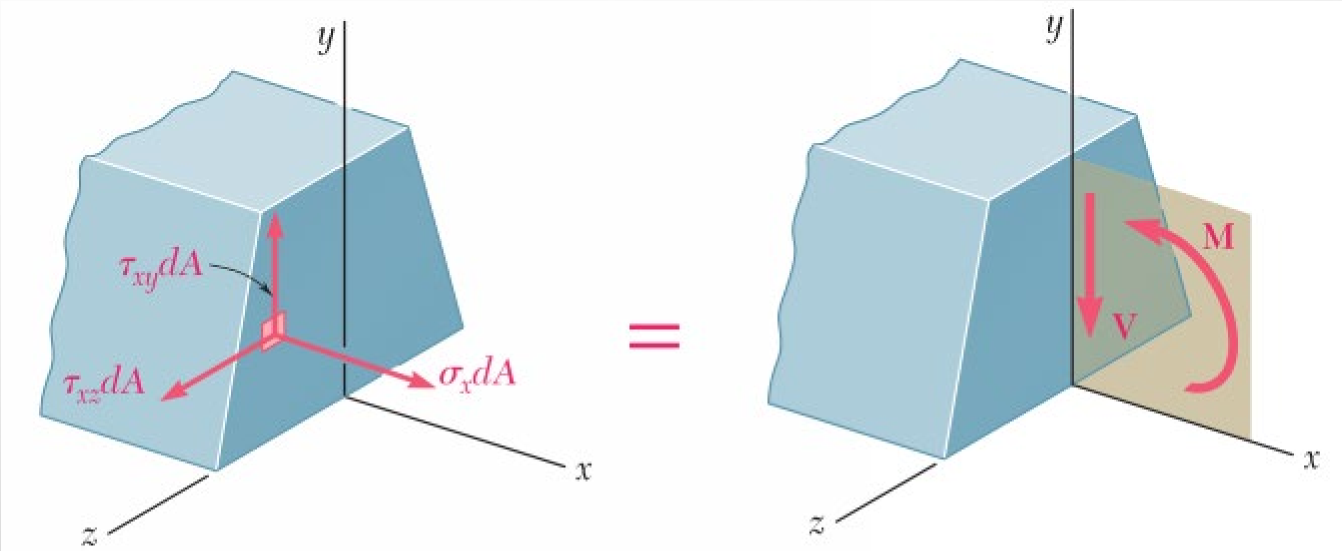
§ 8.1 Introduction

- Shearing stresses can be important, particularly in the design of short, stubby beams.

$$\int \tau_{xy} dA = -V$$

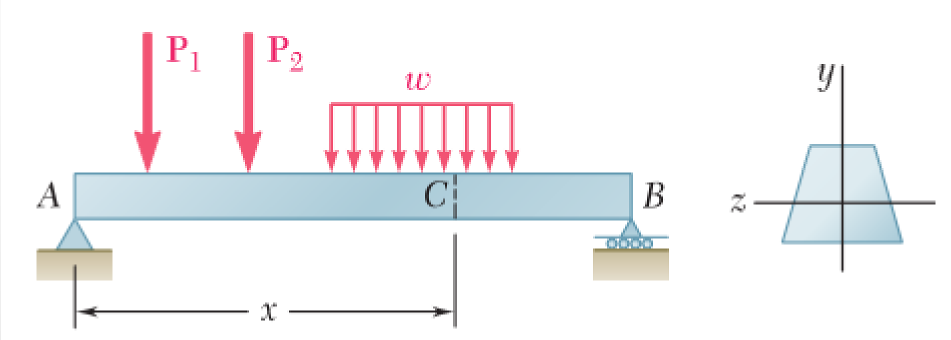
$$\int \tau_{xz} dA = 0$$

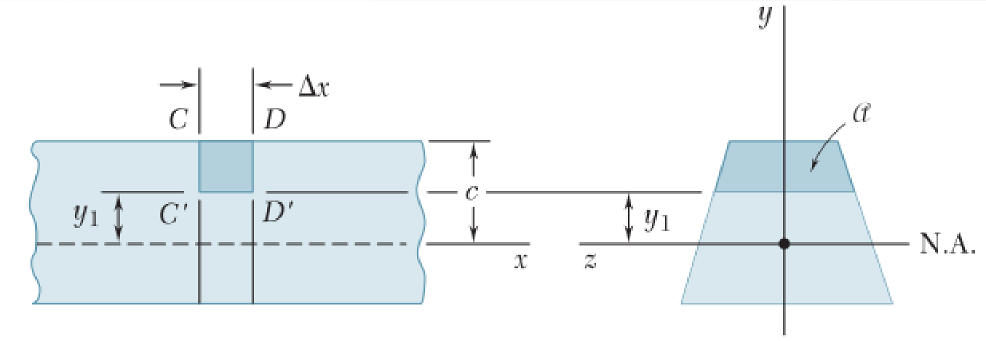




§ 8.2 Shear on the horizontal face

- Consider a prismatic beam AB with a vertical plane of symmetry that supports various concentrated and distributed loads





$$\sum F_x = 0 \quad \Delta H + \int (\sigma_C - \sigma_D)\, dA = 0$$

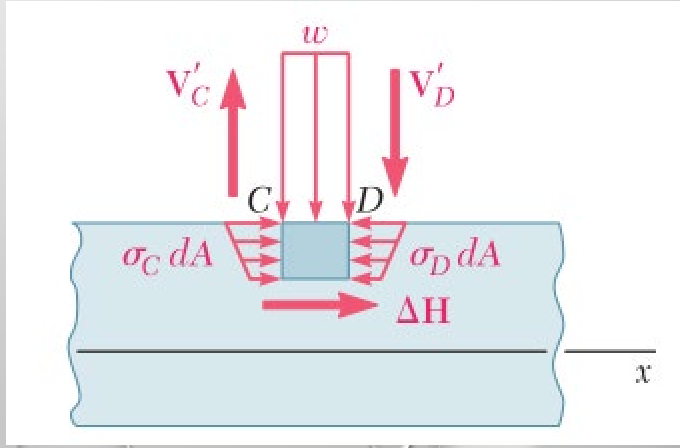
$$\Delta H = \frac{M_D - M_C}{I} \int_\alpha y\, dA = \frac{VQ}{I} \Delta x$$

$$\boxed{q = \frac{VQ}{I}}$$

shear per unit length

horizontal shearing force

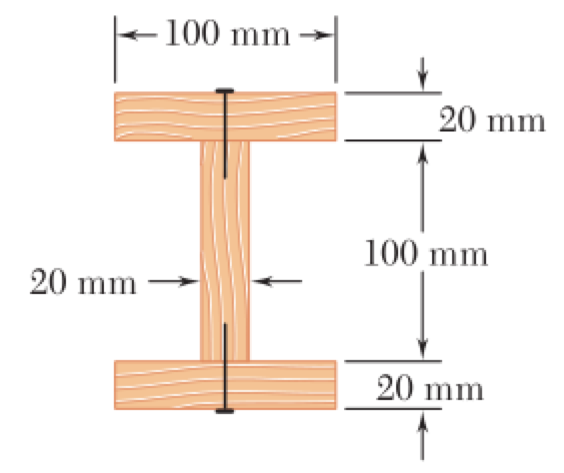
Q, first moment of portion α

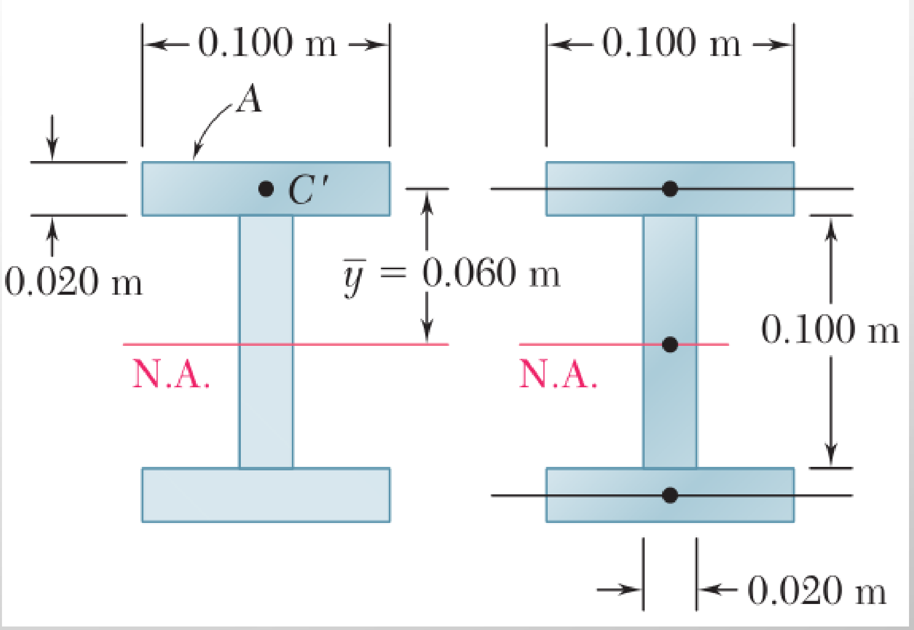


Example 8.1

(Beer, Page 386)

A beam is made of three planks, 20 ×100 mm in cross section, nailed together. Knowing that the spacing between nails is 25 mm and that the vertical shear in the beam is V =500 N, determine the shearing force in each nail.

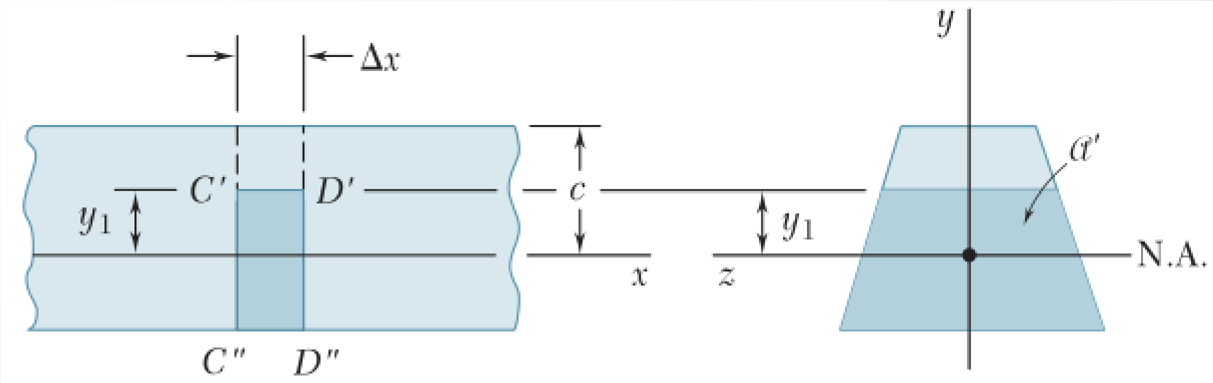




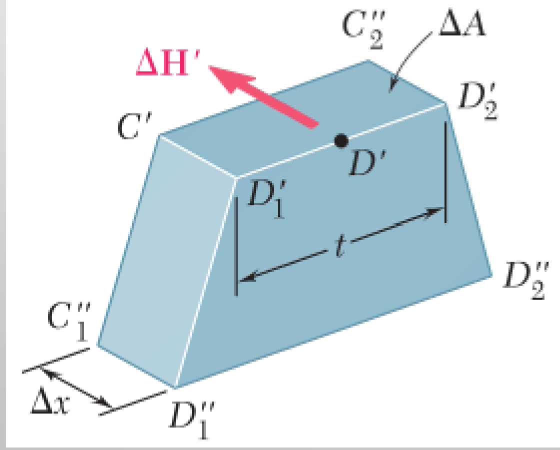
§ 8.3 Determination of the shear stresses in a beam

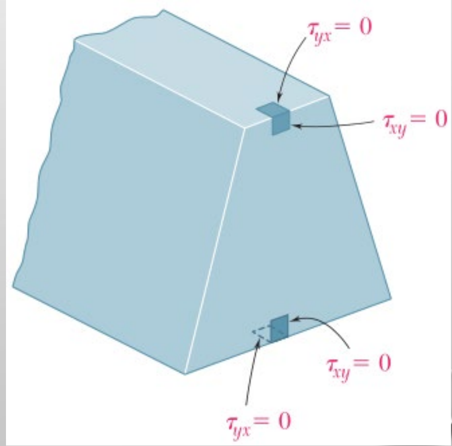
- Consider again a beam with a vertical plane of symmetry, subjected to various concentrated or distributed loads applied in that plane. The magnitude ΔH of the shearing force exerted on the horizontal face of the element can be obtained

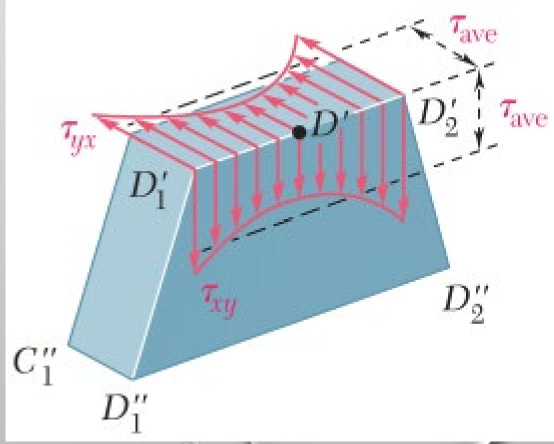
$$\tau_{\text{avg}} = \frac{\Delta H}{\Delta A} = \frac{VQ}{I}\frac{\Delta x}{t\Delta x} = \frac{VQ}{It}$$



average shearing stress

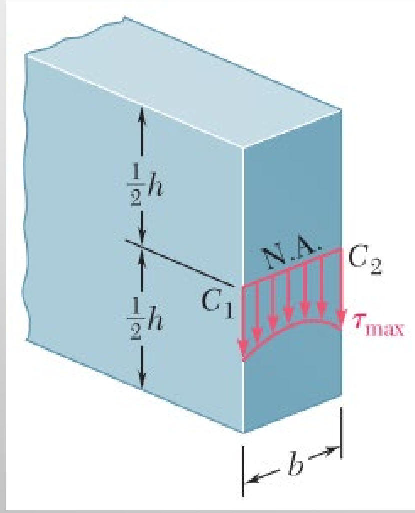






§ 8.4 Shearing stresses in a narrow regular beam

- For a beam of rectangular section of width *b* and depth *h* with *b* ≤1/4 *h*, the variation of the shearing stress τ_{xy} across the width of the beam is less than 0.8% of τ_{avg} .

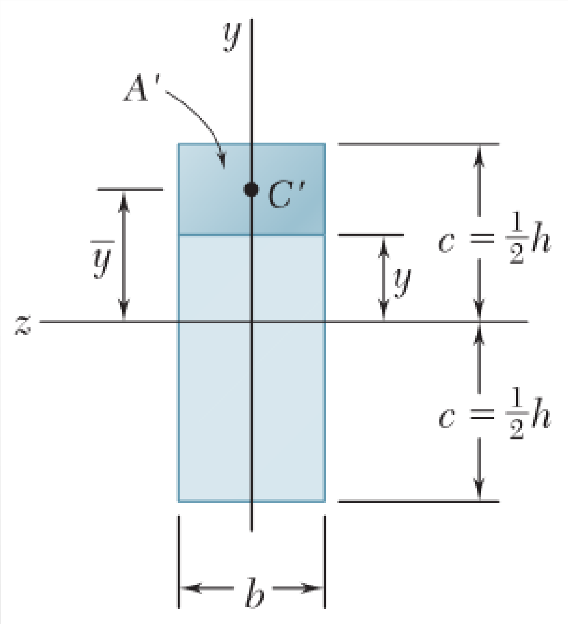


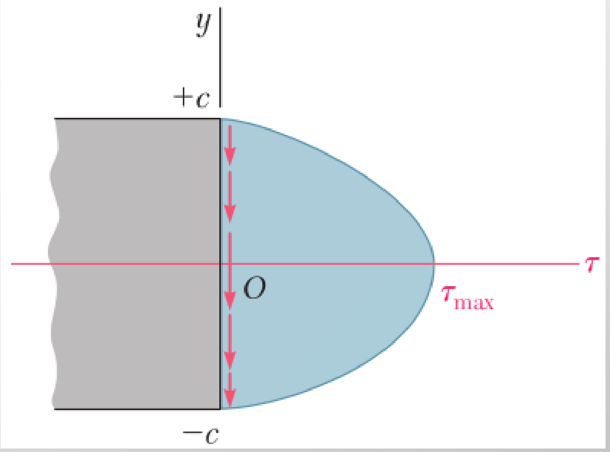
$$\tau_{xy} = \frac{VQ}{It}$$

$$Q = A'\bar{y} = \tfrac{1}{2}b(\tfrac{1}{4}h^2 - y^2)$$

$$I = \tfrac{1}{12}bh^3 = \tfrac{2}{3}bc^3$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A}\left(1 - \frac{y^2}{c^2}\right)$$



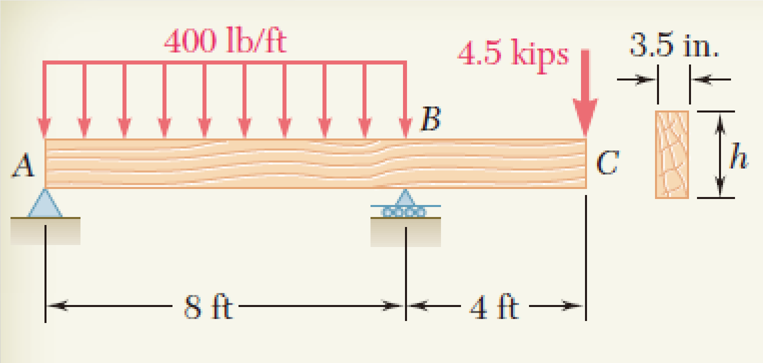


$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$

Example 8.2

(Beer, Page 389)

Knowing that the allowable shearing stress for the timber beam is $\tau_{all} = 0.250$ ksi, check that the design obtained in that sample problem is acceptable from the point of view of the shearing stresses. h=14.55 in.

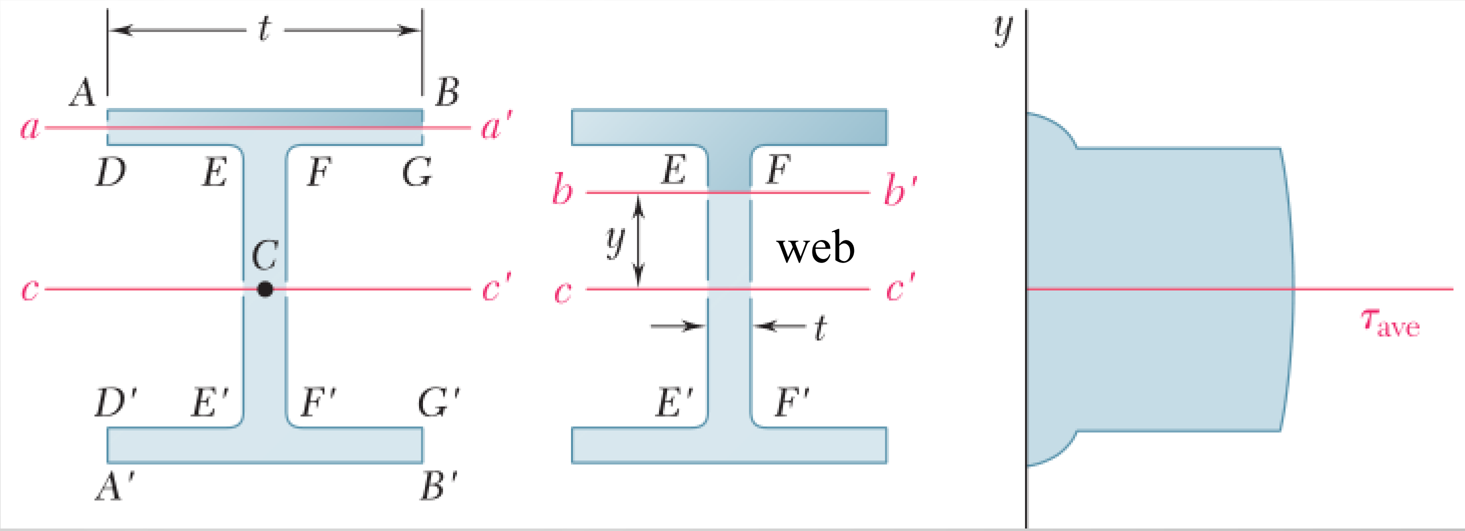


§ 8.5 Shearing stresses in an American standard beam (S-beam) or a wide-flange beam (W-beam)

- For a beam of rectangular section of width b and depth h with b $\leq$1/4 h, the variation of the shearing stress τ_{xy} across the width of the beam is less than 0.8% of τ_{avg} .

$$\tau_{avg} = \frac{VQ}{It}$$

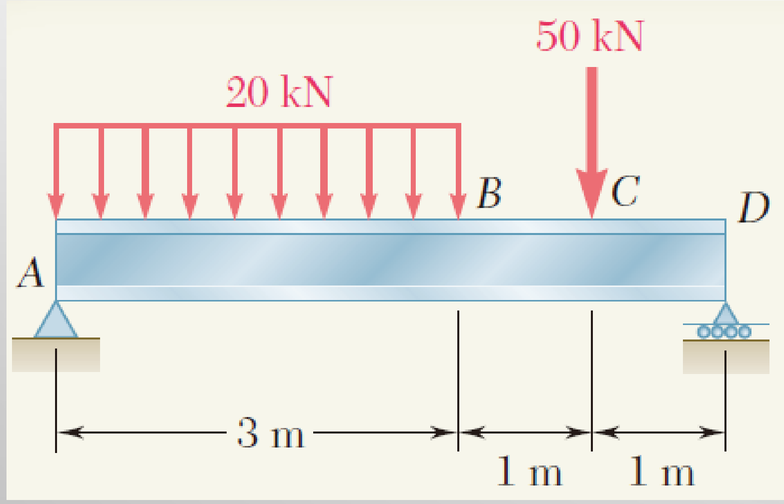
$$\tau_{max} = \frac{V}{A_{web}}$$

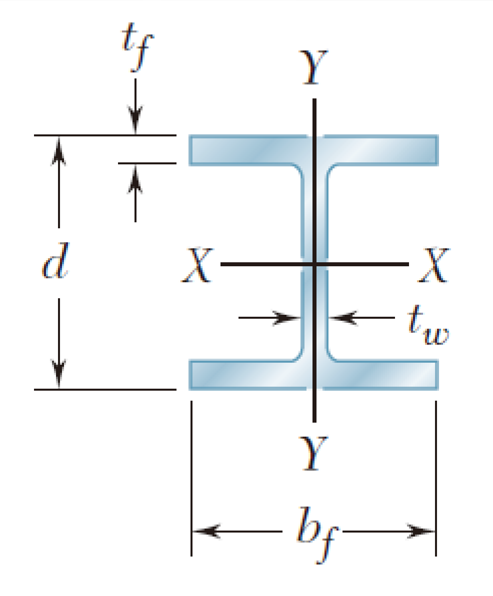


Example 8.3

(Beer, Page 389)

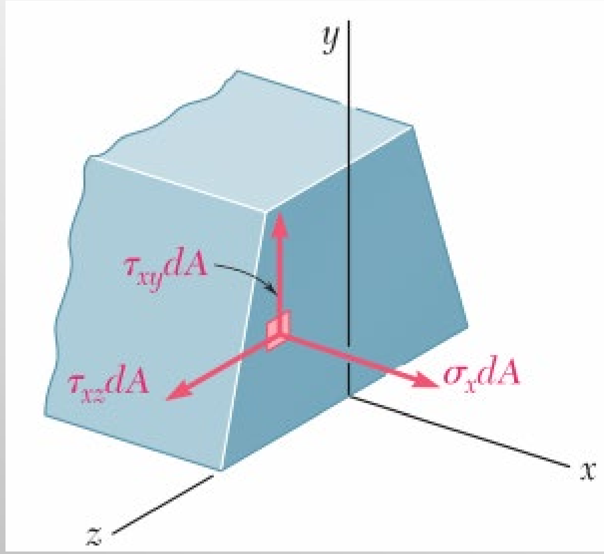
Knowing that the allowable shearing stress for the steel beam of Sample is $\tau_{all} = 90$ MPa, check that the W360 × 32.9 shape obtained in that sample problem is acceptable from the point of view of the shearing stresses. (d=349mm, t=5.8mm)

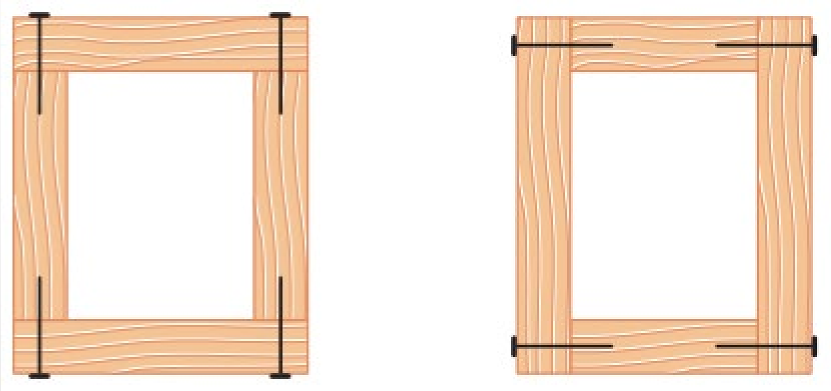




§ 8.6 Longitudinal shear on a beam of arbitrary shape

- Determine q if the planks had been joined along vertical surfaces. Could you determine q if the planks had been joined along vertical surfaces? What about the horizontal components τ_{xz} of the stresses in the flanges





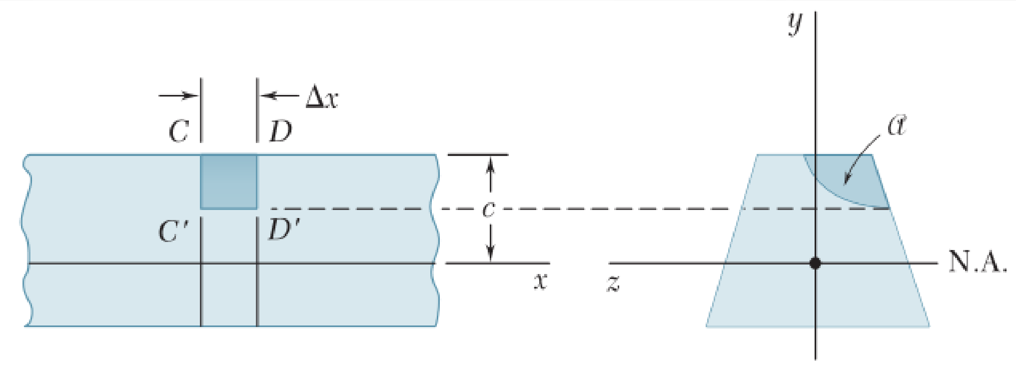
§ 8.6 Longitudinal shear on a beam of arbitrary shape

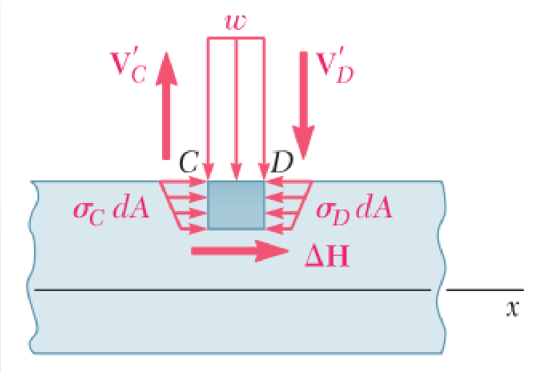
- Determine q if the planks had been joined along vertical surfaces. Could you determine q if the planks had been joined along vertical surfaces? What about the horizontal components τ_{xz} of the stresses in the flanges

$$\sum F_x = 0$$

$$\Delta H + \int_{\alpha} (\sigma_C - \sigma_D) dA = 0$$

$$\Delta H = \frac{M_D - M_C}{I} \int_{\alpha} y dA = \frac{VQ}{I} \Delta x$$

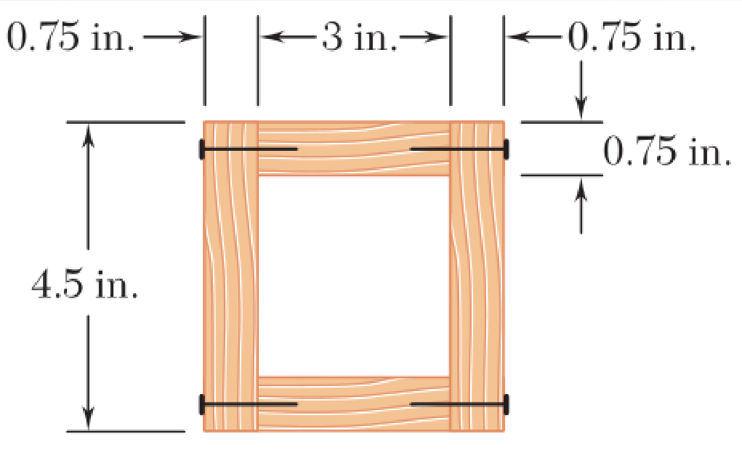


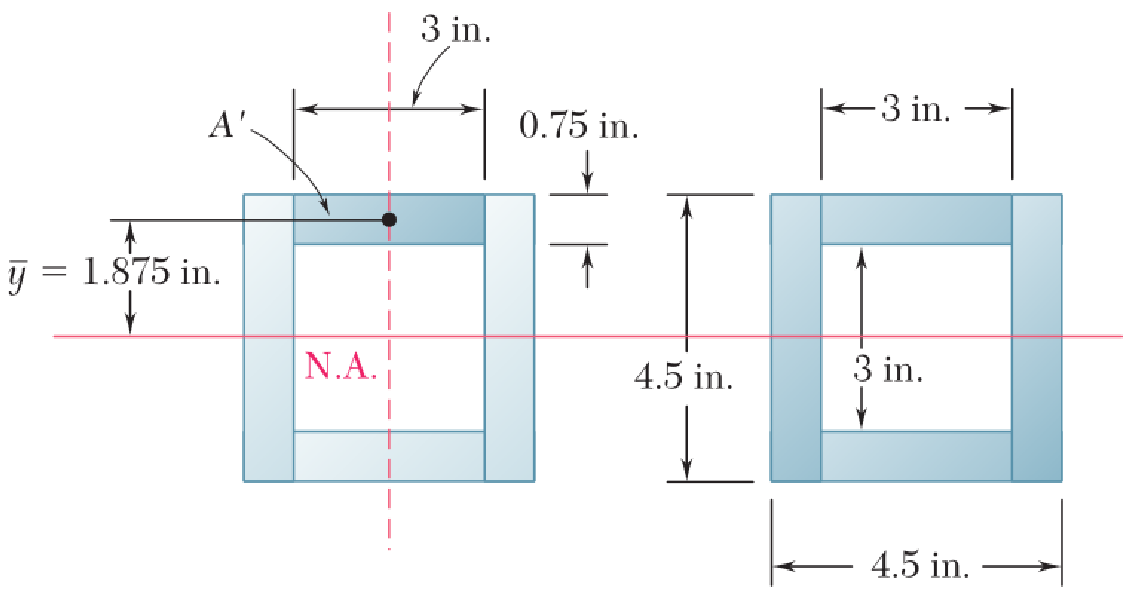


Example 8.4

(Beer, Page 401)

A square box beam is made of two 0.75 × 3-in. planks and two 0.75 × 4.5-in. planks, nailed together. Knowing that the spacing between nails is 1.75 in. and that the beam is subjected to a vertical shear of magnitude V = 600 lb, determine the shearing force in each nail.



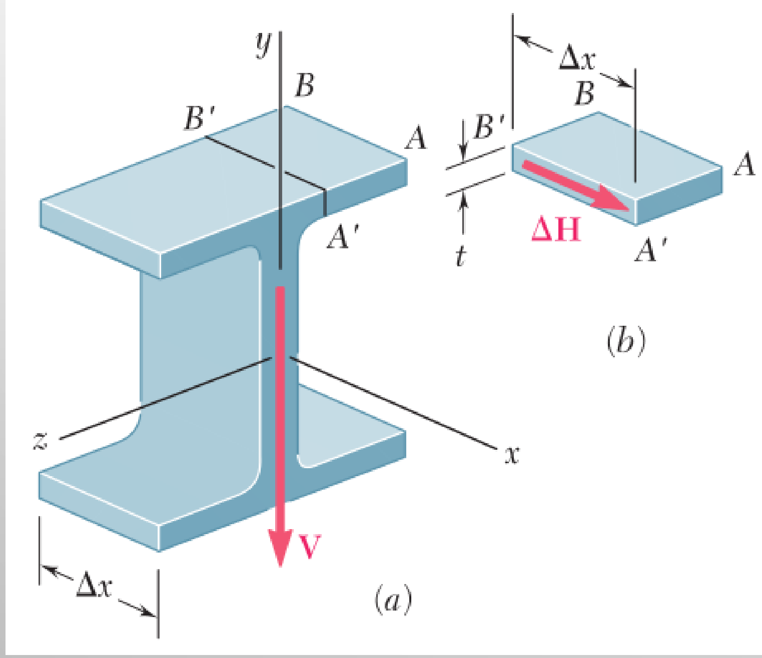


§ 8.7 Shear stress in shin-walled members

- Calculate both the shear flow and the average shearing stress in thin-walled members such as the flanges of wide-flange beams and box beams, or the walls of structural tubes.

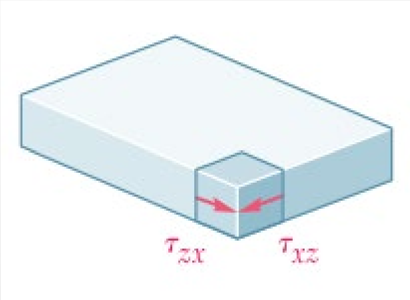
§ 8.7 Shear stress in shin-walled members

- Consider a segment of length Δx of a wide-flange beam and let V be the vertical shear in the transverse section shown. The longitudinal shear ΔH exerted on the element of A′ABB′ can be obtained.



$$\Delta H = \frac{VQ}{I}\Delta x$$

$$\tau_{\text{avg}} = \frac{VQ}{It}$$

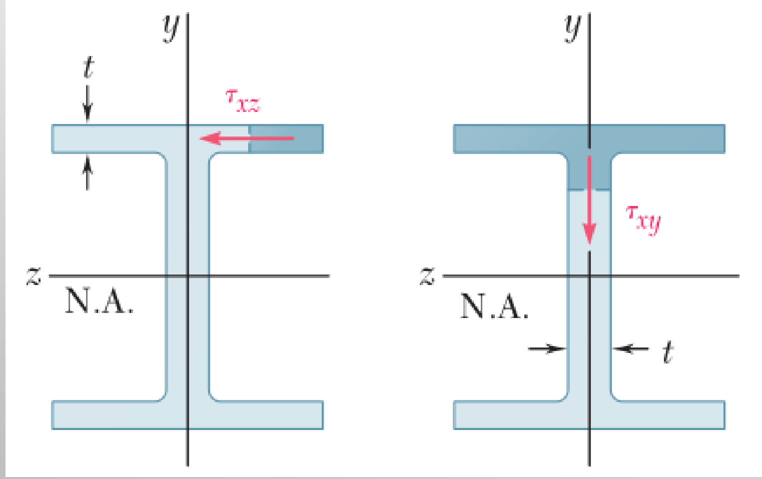


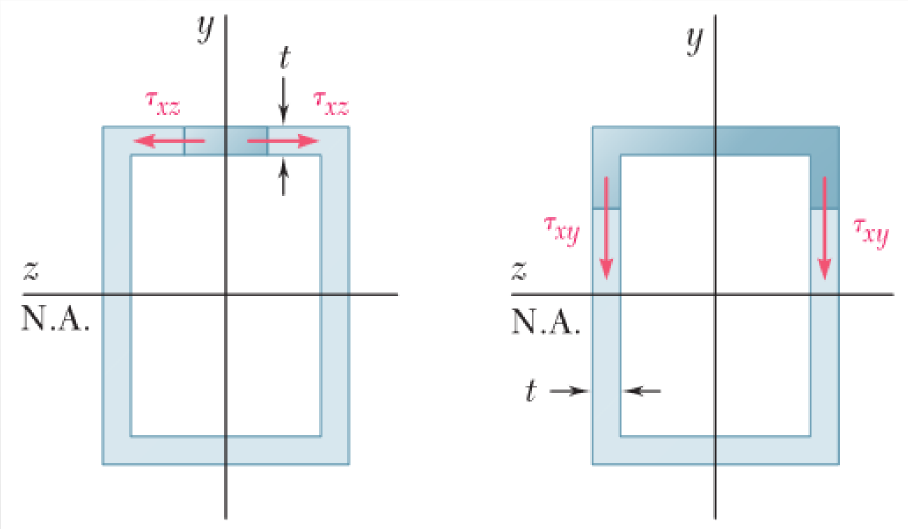
$$\tau_{\text{xz}} = \tau_{\text{zx}}$$

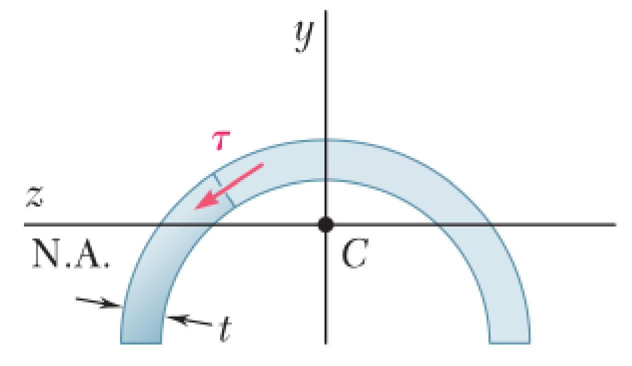
§ 8.7 Shear stress in shin-walled members

- Equation can be used to determine shearing stresses in box beams, half pipes, and other thin-walled members, as long as the loads are applied in a plane of symmetry of the member.

$$\tau_{\text{avg}} = \frac{VQ}{It}$$

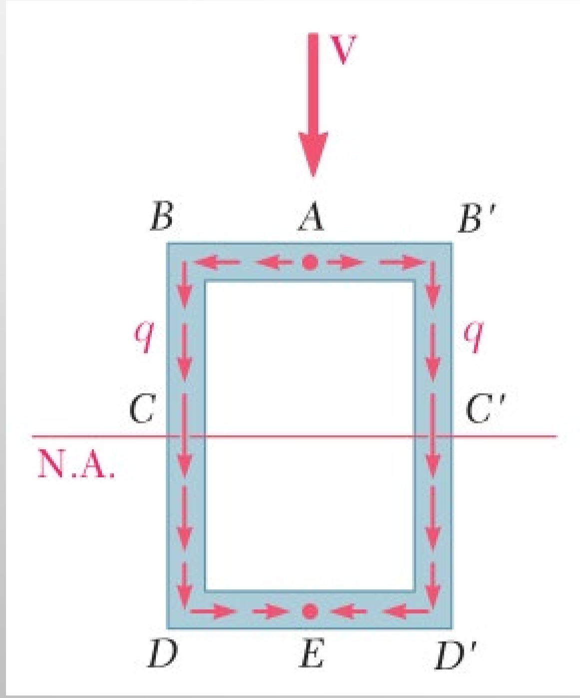


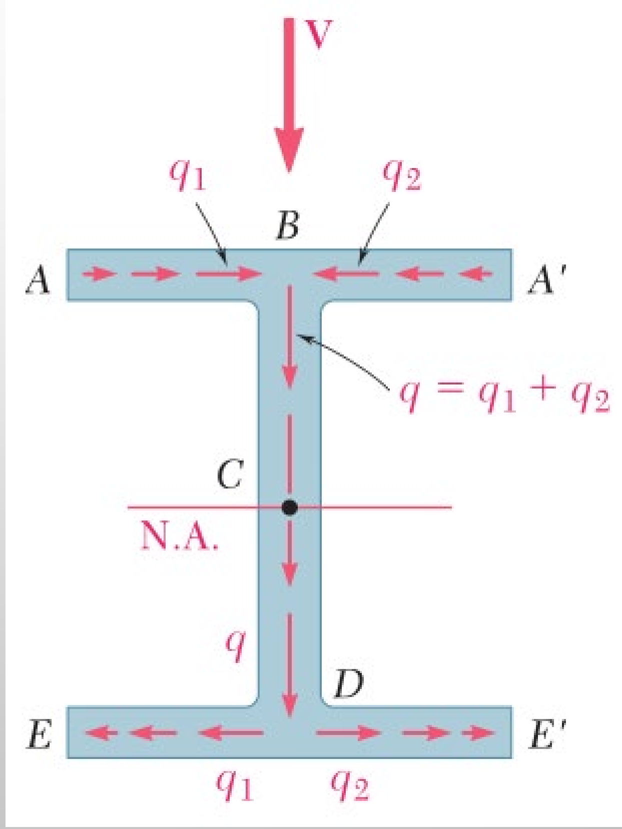




§ 8.7 Shear stress in shin-walled members

- The name of shear flow commonly used to refer to the shear per unit length, q, reflects the similarity between the properties of q that we have just described and some of the characteristics of a fluid flow through an open channel or pipe.

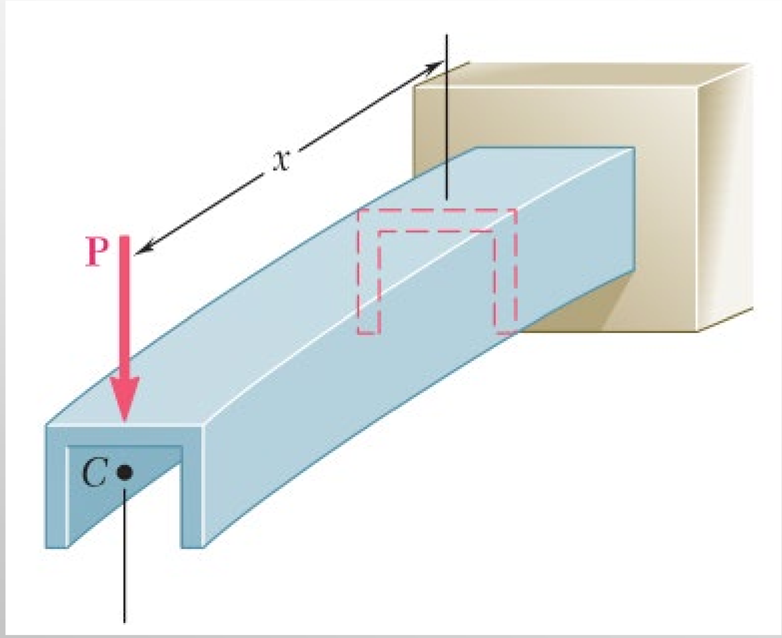




$$\tau_{avg} = \frac{VQ}{It}$$

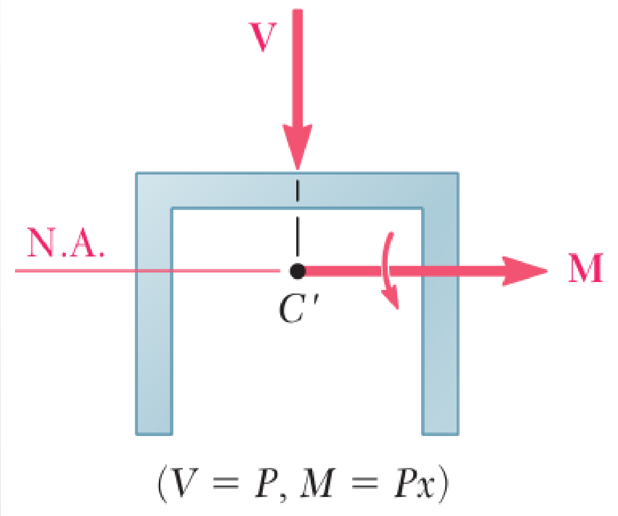
§ 8.8 Asymmetric loading of thin-walled members

- For members possessing a vertical plane of symmetry and loads applied in that plane



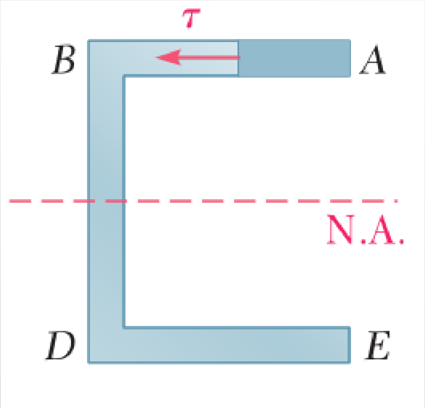
$$\sigma_x = -\frac{My}{I}$$

$$\tau_{avg} = \frac{VQ}{It}$$

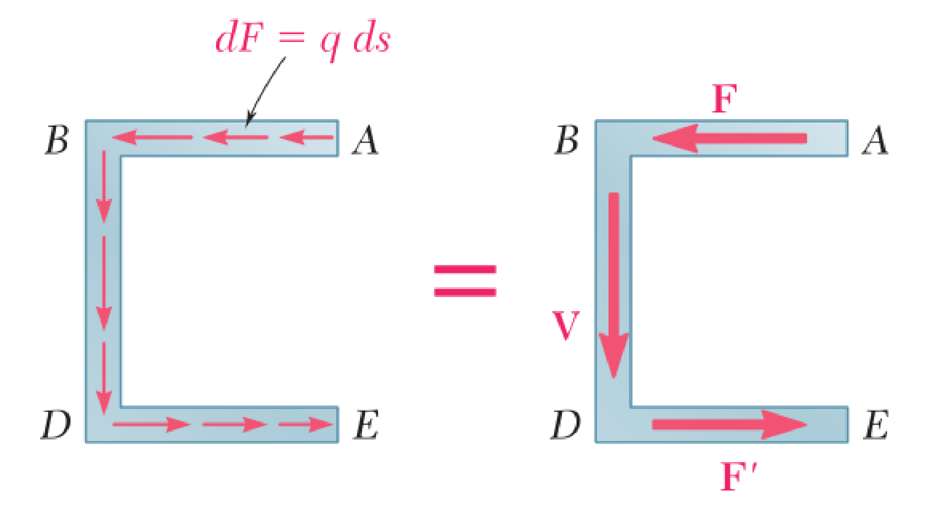


§ 8.8 Asymmetric loading of thin-walled members

- Is it possible to apply the vertical load P in such a way that the channel member will bend without twisting?



(a) Shear stress



(a) Shear flow q

(b) Resultant forces on elements

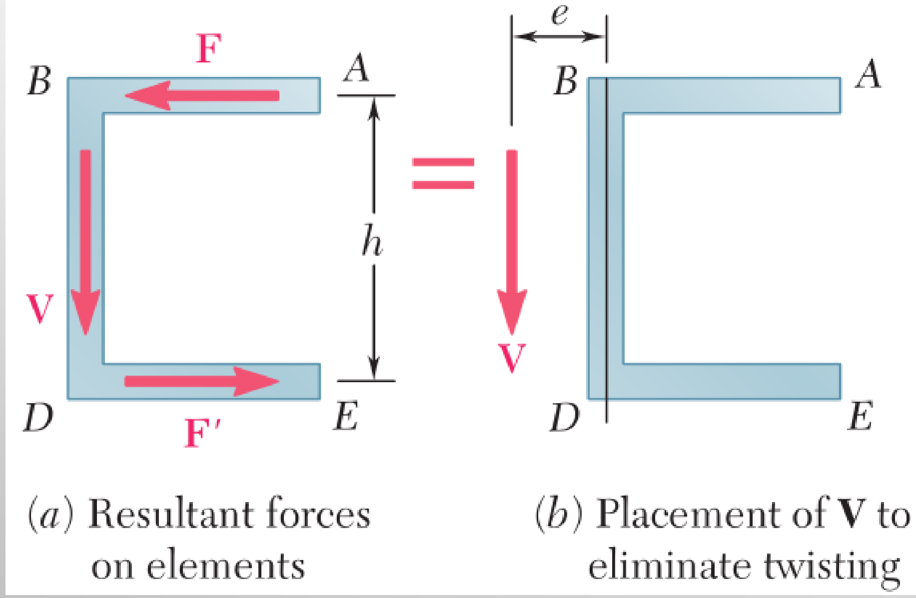
$$q = \tau t = \frac{VQ}{I}$$

$$F = \int_A^B q ds$$

$$V = \int_B^D q ds$$

§ 8.8 Asymmetric loading of thin-walled members

- Is it possible to apply the vertical load P in such a way that the channel member will bend without twisting?

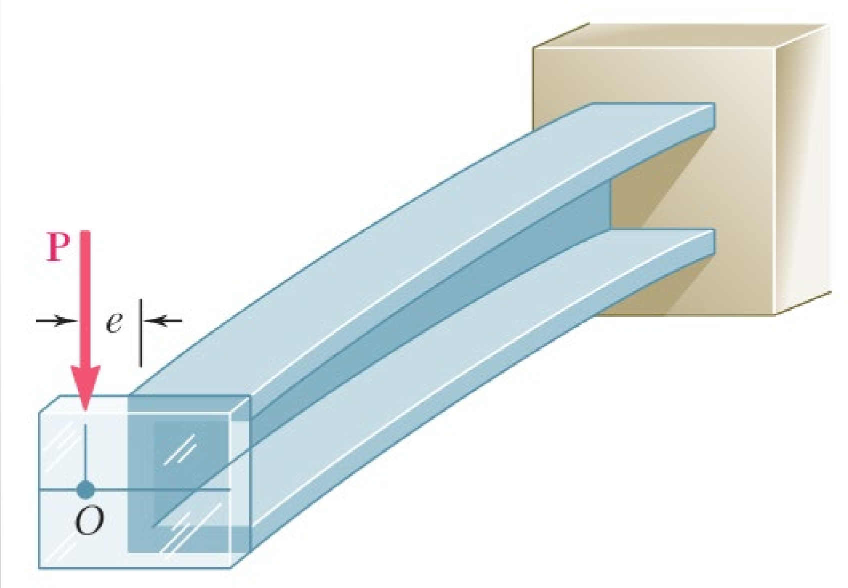


$$\Sigma M = 0$$

$$Fh = Ve$$

$$e = \frac{Fh}{V}$$

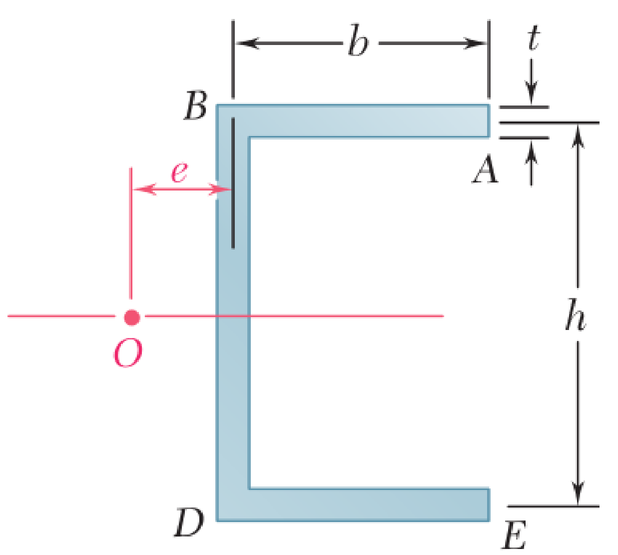
Shear center

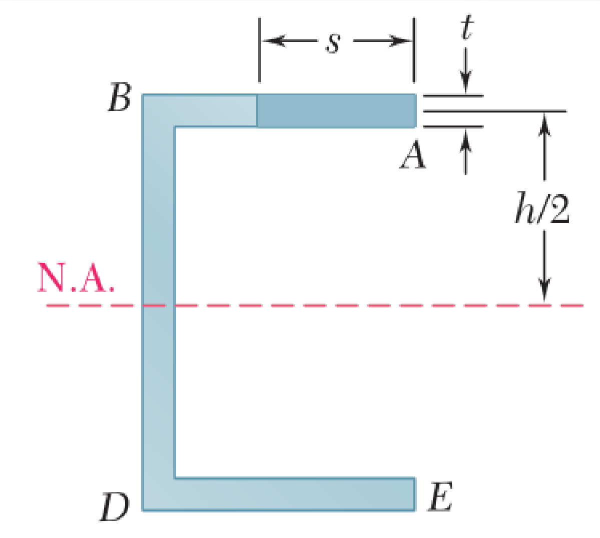


Example 8.5

(Beer, Page 417)

Determine the shear center O of a channel section of uniform thickness, knowing that $b = 4$ in., $h = 6$ in., and $t = 0.15$ in.

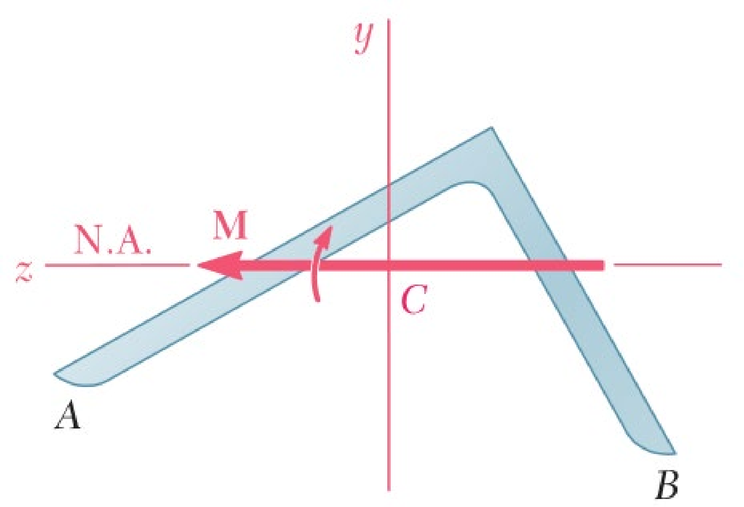


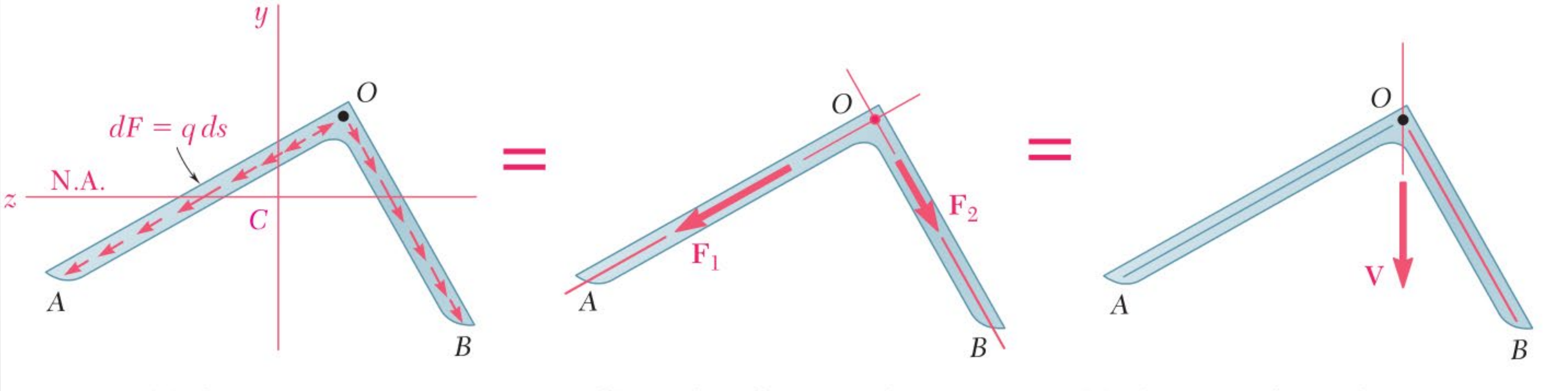


§ 8.8 Asymmetric loading of thin-walled members

- Turning our attention to thin-walled members possessing no plane of symmetry

Shear center

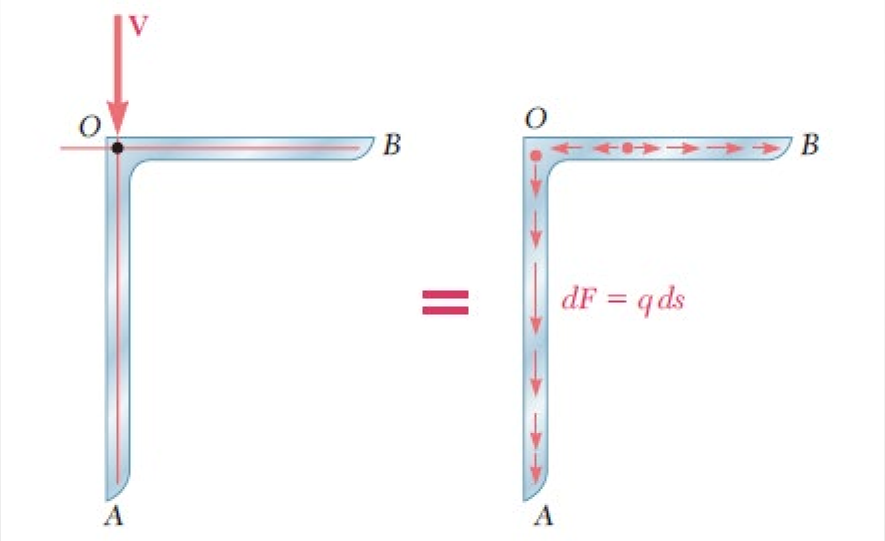




(a) Shear stresses (b) Resultant forces on elements (c) Placement of **V** to eliminate twisting

§ 8.8 Asymmetric loading of thin-walled members

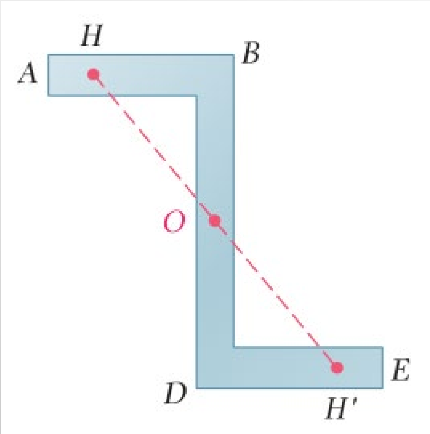
- Turning our attention to thin-walled members possessing no plane of symmetry

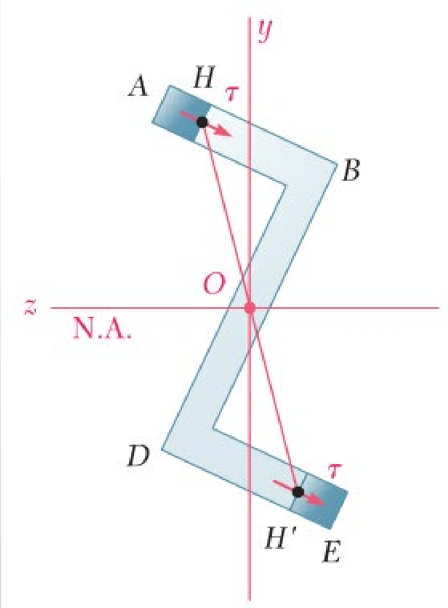


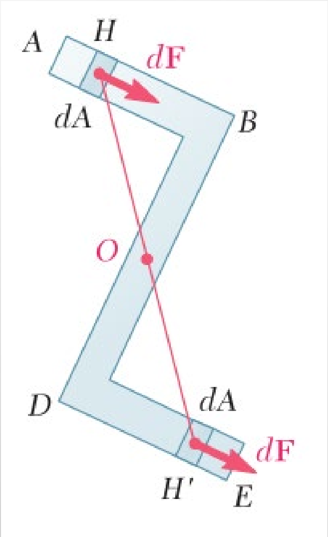
SAMPLE PROBLEM 6.6
at Beer, Page 422

§ 8.8 Asymmetric loading of thin-walled members

- Z section: the center of symmetry O coincides with the centroid of the cross section, and point O is also the shear center of the cross section.







§ 8.9 Summary

- **Stresses on a beam element**
- **Horizontal shear in a beam**
- **Shear flow**
- **Shearing stresses in a beam**
- **Longitudinal shear on curved surface**
- **Shearing stresses in thin-walled members**
- **Unsymmetric loading and shear center**