Reference Answers of Homework 8

1. An ideal gas is contained in a piston-cylinder device and undergoes a power cycle as follows: 1-2 isentropic compression from an initial temperature T_1 = 20°C with a compression ratio r = 5; 2-3 constant pressure heat addition; 3-1 constant volume heat rejection. The gas has constant specific heats with c_v = 0.7 kJ/kg·K and R = 0.3 kJ/kg·K.

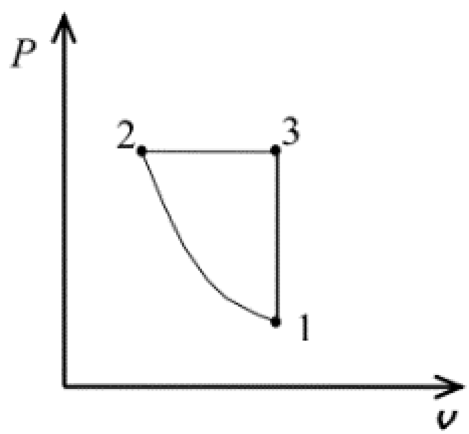
(a) Sketch the P-v and T-s diagrams for the cycle;

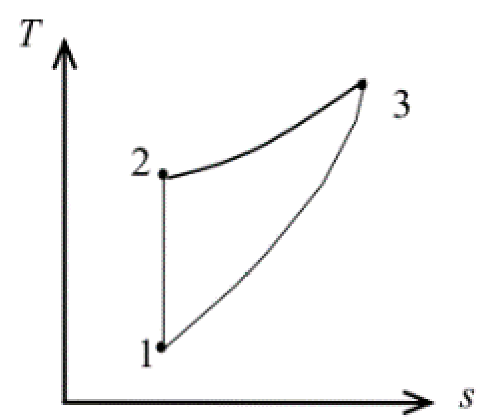
(b) Determine the heat and work interactions for each process, in kJ/kg;

(c) Determine the cycle thermal efficiency;

(d) Obtain thermal efficiency as a in terms of compression ratio r and adiabatic coefficient k.

(a) The P-v and T-s diagrams of the cycle are shown in the figures.





(b) Noting that $c_p = c_v + R = 0.7 + 0.3 = 1.0\ \text{kJ/kg}\cdot\text{K}$

$$k = \frac{c_p}{c_v} = \frac{1.0}{0.7} = 1.429$$

Process 1-2: Isentropic compression

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = T_1 r^{k-1} = (293\ \text{K})(5)^{0.429} = 584.4\ \text{K}$$

$$w_{1-2,\text{in}} = c_v(T_2 - T_1) = (0.7\ \text{kJ/kg}\cdot\text{K})(584.4 - 293)\ \text{K} = \mathbf{204.0 kJ/kg}$$

$$q_{1-2} = \mathbf{0}$$

From ideal gas relation, $\frac{T_3}{T_2} = \frac{v_3}{v_2} = \frac{v_1}{v_2} = r \longrightarrow T_3 = (584.4)(5) = 2922$

Process 2-3: Constant pressure heat addition

$$w_{2-3,\text{out}} = \int_2^3 P\,dv = P_2(v_3 - v_2) = R(T_3 - T_2)$$

$$= (0.3\ \text{kJ/kg}\cdot\text{K})(2922 - 584.4)\ \text{K} = \mathbf{701.3 kJ/kg}$$

$$q_{2-3,\text{in}} = w_{2-3,out} + \Delta u_{2-3} = \Delta h_{2-3}$$

$$= c_p(T_3 - T_2) = (1\ \text{kJ/kg}\cdot\text{K})(2922 - 584.4)\ \text{K} = \mathbf{2338 kJ/kg}$$

Process 3-1: Constant volume heat rejection

$$q_{3-1,\text{out}} = \Delta u_{1-3} = c_v(T_3 - T_1) = (0.7\ \text{kJ/kg}\cdot\text{K})(2922 - 293)\ \text{K} = \mathbf{1840.3 kJ/kg}$$

$$w_{3-1} = \mathbf{0}$$

(c) Net work is $w_{\text{net}} = w_{2-3,\text{out}} - w_{1-2,\text{in}} = 701.3 - 204.0 = 497.3\ \text{kJ/kg}\cdot\text{K}$

The thermal efficiency is then $\eta_{\text{th}} = \dfrac{w_{\text{net}}}{q_{\text{in}}} = \dfrac{497.3\ \text{kJ}}{2338\ \text{kJ}} = 0.213 = \mathbf{21.3\%}$

(d) The expression for the cycle thermal efficiency is obtained as follows:

$$\frac{R}{c_p} = \frac{c_p - c_v}{c_p} = 1 - \frac{c_v}{c_p} = 1 - \frac{1}{k}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{w_{2-3,\text{out}} - w_{1-2,\text{in}}}{q_{\text{in}}}$$

$$= \frac{R(T_3 - T_2) - c_v(T_2 - T_1)}{c_p(T_3 - T_2)}$$

$$= \frac{R}{c_p} - \frac{c_v(T_1 r^{k-1} - T_1)}{c_p(rT_1 r^{k-1} - T_1 r^{k-1})}$$

$$= \frac{R}{c_p} - \frac{c_v T_1 r^{k-1}\left(1 - \dfrac{T_1}{T_1 r^{k-1}}\right)}{c_p T_1 r^{k-1}(r-1)}$$

$$= \frac{R}{c_p} - \frac{1}{k(r-1)}\left(1 - \frac{T_1}{T_1 r^{k-1}}\right)$$

$$= \frac{R}{c_p} - \frac{1}{k(r-1)}\left(1 - \frac{1}{r^{k-1}}\right)$$

$$= \left(1 - \frac{1}{k}\right) - \frac{1}{k(r-1)}\left(1 - \frac{1}{r^{k-1}}\right)$$

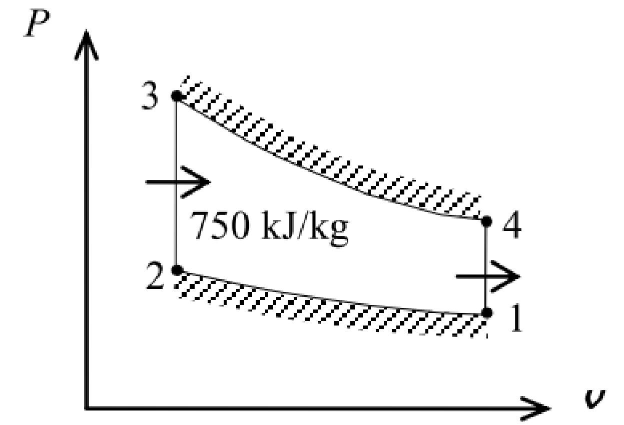
2. An ideal Otto cycle has a compression ratio of 8. At the beginning of the compression process, air is at 95 kPa and 27°C, and 750 kJ/kg of heat is transferred to air during the constant-volume heat-addition process. Using constant specific heats at room temperature, determine:

(a) the pressure and temperature at the end of the heat-addition process,

(b) the net work output,

(c) the thermal efficiency, and

(d) the mean effective pressure for the cycle.



(a) Process 1-2: isentropic compression.

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (300\text{K})(8)^{0.4} = 689\text{ K}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \longrightarrow P_2 = \frac{v_1}{v_2}\frac{T_2}{T_1}P_1 = (8)\left(\frac{689\text{ K}}{300\text{ K}}\right)(95\text{ kPa}) = 1745\text{ kPa}$$

Process 2-3:

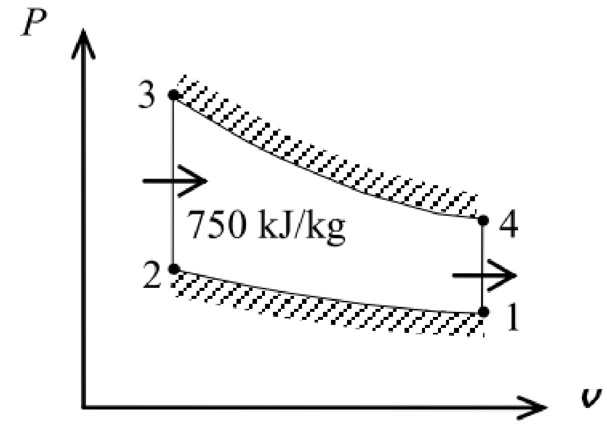
$$q_{23,\text{in}} = u_3 - u_2 = c_v(T_3 - T_2)$$

$$750\text{ kJ/kg} = (0.718\text{ kJ/kg}\cdot\text{K})(T_3 - 689)\text{K}$$

$$T_3 = \mathbf{1734\text{ K}}$$

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow P_3 = \frac{T_3}{T_2}P_2 = \left(\frac{1734\text{ K}}{689\text{ K}}\right)(1745\text{ kPa}) = \mathbf{4392\text{ kPa}}$$

(b) Process 3-4: isentropic expansion.



$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = (1734 \text{ K})\left(\frac{1}{8}\right)^{0.4} = 755 \text{ K}$$

Process 4-1: v = constant heat rejection.

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1) = (0.718 \text{ kJ/kg}\cdot\text{K})(755-300)\text{K} = 327 \text{ kJ/kg}$$

$$w_{net,out} = q_{in} - q_{out} = 750 - 327 = \mathbf{423 \text{ kJ/kg}}$$

(c) $$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{423\text{kJ/kg}}{750\text{kJ/kg}} = \mathbf{56.4\%}$$

(d) $$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300 \text{ K})}{95 \text{ kPa}} = 0.906 \text{ m}^3/\text{kg} = v_{max}$$

$$v_{min} = v_2 = \frac{v_{max}}{r}$$

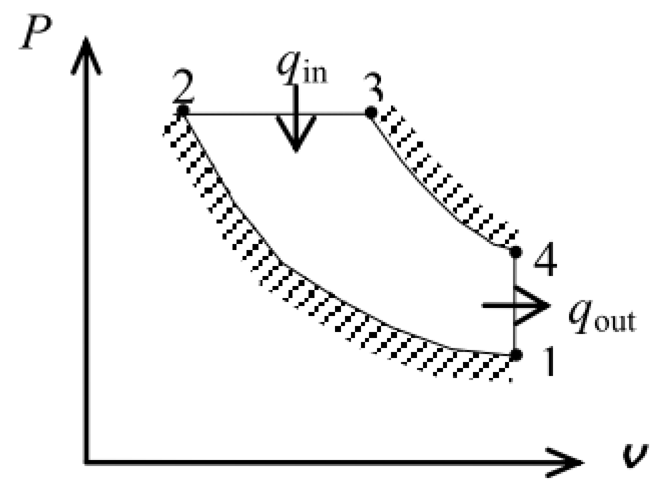
$$\text{MEP} = \frac{w_{net,out}}{v_1 - v_2} = \frac{w_{net,out}}{v_1(1-1/r)} = \frac{423 \text{ kJ/kg}}{(0.906 \text{ m}^3/\text{kg})(1-1/8)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{534 \text{ kPa}}$$

3. An air-standard Diesel cycle has a compression ratio of 16 and a cutoff ratio of 2. At the beginning of the compression process, air is at 95 kPa and 27°C. Using constant specific heats at room temperature, determine:

(a) the temperature after the heat-addition process,;

(b) the thermal efficiency;

(c) the mean effective pressure for the cycle.



(a)

Process 1-2: isentropic compression.

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (300\text{K})(16)^{0.4} = 909.4\text{K}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow T_3 = \frac{v_3}{v_2} T_2 = 2T_2 = (2)(909.4\text{K}) = \mathbf{1818.8K}$$

(b) $q_{in} = h_3 - h_2 = c_p(T_3 - T_2) = (1.005\text{ kJ/kg}\cdot\text{K})(1818.8 - 909.4)\text{K} = 913.9\text{ kJ/kg}$

Process 3-4: isentropic expansion.

$$T_4 = T_3\left(\frac{\boldsymbol{v}_3}{\boldsymbol{v}_4}\right)^{k-1} = T_3\left(\frac{2\boldsymbol{v}_2}{\boldsymbol{v}_4}\right)^{k-1} = (1818.8\text{K})\left(\frac{2}{16}\right)^{0.4} = 791.7\text{K}$$

Process 4-1: $\boldsymbol{v}$= constant heat rejection.

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1) = (0.718\text{kJ/kg}\cdot\text{K})(791.7 - 300)\text{K} = 353\text{kJ/kg}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{353\text{ kJ/kg}}{913.9\text{ kJ/kg}} = \mathbf{61.4\%}$$

(c) $w_{net.out} = q_{in} - q_{out} = 913.9 - 353 = 560.9\text{kJ/kg}$

$$\boldsymbol{v}_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{ K})}{95\text{ kPa}} = 0.906\text{ m}^3/\text{kg} = \boldsymbol{v}_{max}$$

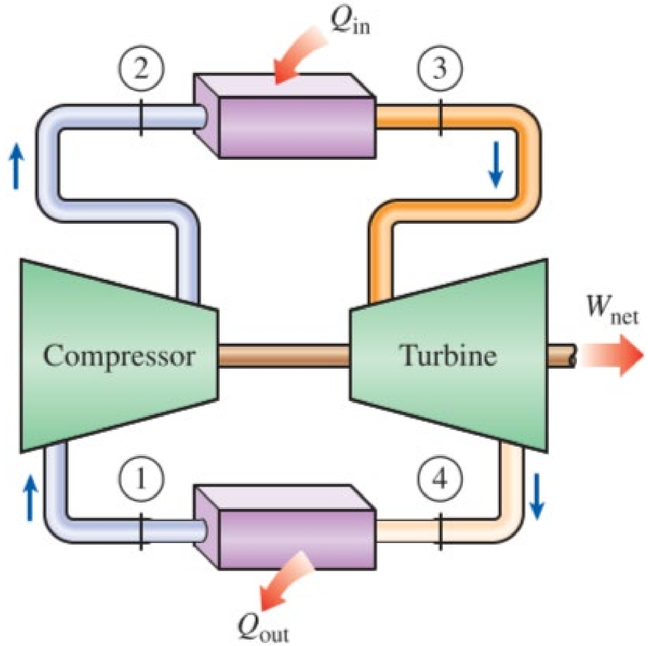
$$\boldsymbol{v}_{min} = \boldsymbol{v}_2 = \frac{\boldsymbol{v}_{max}}{r}$$

$$\text{MEP} = \frac{w_{net,out}}{\boldsymbol{v}_1 - \boldsymbol{v}_2} = \frac{w_{net,out}}{\boldsymbol{v}_1(1 - 1/r)} = \frac{560.9\text{ kJ/kg}}{(0.906\text{ m}^3/\text{kg})(1 - 1/16)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{660.4\text{ kPa}}$$

4. Consider a simple Brayton cycle using air as the working fluid. It has a pressure ratio of 12 and a maximum cycle temperature of 600 °C. The compressor is operated with 100 kPa and 15 °C at the inlet. Please use constant specific heats at room temperature to determine which process below will has a lager impact on the back-work ratio.

(a) a compressor isentropic efficiency of 80 percent;

(b) a turbine isentropic efficiency of 80 percent.



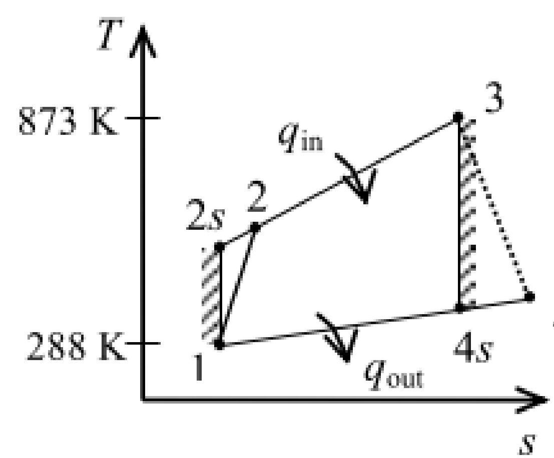
For the compression process,

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (288\text{ K})(12)^{0.4/1.4} = 585.8\text{ K}$$

$$\eta_C = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_p(T_{2s}-T_1)}{c_p(T_2-T_1)} \longrightarrow T_2 = T_1 + \frac{T_{2s}-T_1}{\eta_C}$$

$$= 288 + \frac{585.8-288}{0.80}$$

$$= 660.2\text{ K}$$



For the expansion process,

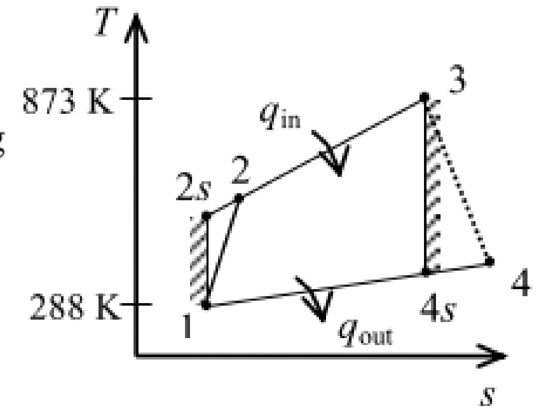
$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (873\text{ K})\left(\frac{1}{12}\right)^{0.4/1.4} = 429.2\text{ K}$$

$$\eta_T = \frac{h_3-h_4}{h_3-h_{4s}} = \frac{c_p(T_3-T_4)}{c_p(T_3-T_{4s})} \longrightarrow T_4 = T_3 - \eta_T(T_3-T_{4s})$$

$$= 873-(0.80)(873-429.2)$$

$$= 518.0\text{ K}$$

The isentropic and actual work of compressor and turbine are



$$W_{\text{Comp},s} = c_p(T_{2s} - T_1) = (1.005\,\text{kJ/kg}\cdot\text{K})(585.8 - 288)\text{K} = 299.3\,\text{kJ/kg}$$

$$W_{\text{Comp}} = c_p(T_2 - T_1) = (1.005\,\text{kJ/kg}\cdot\text{K})(660.2 - 288)\text{K} = 374.1\,\text{kJ/kg}$$

$$W_{\text{Turb},s} = c_p(T_3 - T_{4s}) = (1.005\,\text{kJ/kg}\cdot\text{K})(873 - 429.2)\text{K} = 446.0\,\text{kJ/kg}$$

$$W_{\text{Turb}} = c_p(T_3 - T_4) = (1.005\,\text{kJ/kg}\cdot\text{K})(873 - 518.0)\text{K} = 356.8\,\text{kJ/kg}$$

The back work ratio for 80% efficient compressor and isentropic turbine case is

$$r_{\text{bw}} = \frac{W_{\text{Comp}}}{W_{\text{Turb},s}} = \frac{374.1\,\text{kJ/kg}}{446.0\,\text{kJ/kg}} = \mathbf{0.8387}$$

The back work ratio for 80% efficient turbine and isentropic compressor case is

$$r_{\text{bw}} = \frac{W_{\text{Comp},s}}{W_{\text{Turb}}} = \frac{299.3\,\text{kJ/kg}}{356.8\,\text{kJ/kg}} = \mathbf{0.8387}$$

The two results are identical.