第 9 周习题
常微分方程 B

April 12, 2022

1. In each of the following exercises, use the method of variation of parameters to find a particular solution of the given differential equation.

(1) $y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0$

(2) $y'' - 2y' + y = e^t/(1 + t^2)$

2. Given the differential equation

$$ty'' - (t + 1)y' + y = t^2e^{2t}, \quad t > 0,$$

verify that $y_1(t) = t+1$ and $y_2(t) = e^t$ are solutions of the corresponding homogeneous equation; then find a particular solution of the nonhomogeneous equation.

3. A mass of 100 g stretches a spring 5 cm. If the mass is set in motion from its equilibrium position with a downward velocity of 10 cm/s, and if there is no damping, determine the position u of the mass at any time t. When does the mass first return to its equilibrium position? Determine the frequency, period, amplitude, and phase of the motion.

4. A spring is stretched 10 cm by a force of 3 N. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 N when the velocity of the mass is 5 m/s. If the mass is pulled down 5 cm below its equilibrium position and given an initial downward velocity of 10 cm/s, determine its position u at any time t. Find the quasi-frequency μ and the ratio of μ to the natural frequency of the corresponding undamped motion.

5. A series circuit (串联电路) has a capacitor of 10^{-5} F, a resistor of 3×10^2 Ω, and an inductor of 0.2 H. The initial charge on the capacitor is 10^{-6} C and there is no initial current. Find the charge Q on the capacitor at any time t.

6. Assume that the system described by the differential equation $mu'' + \gamma u' + ku = 0$ is either critically damped or overdamped. Show that the mass can pass through the equilibrium position at most once, regardless of the initial conditions. (*Hint*. Determine all possible values of t for which $u = 0$.)

7. Verify that e^t, e^{-t}, and e^{-2t} are solutions of the differential equation

$$y''' + 2y'' - y' - 2y = 0.$$

Determine their Wronskian. Do they form a fundamental set of solutions?

8. For each of the following differential equations, find its general solution.

(1) $y''' - 3y'' + 3y' - y = 0$.

(2) $y^{(4)} - 4y''' + 4y'' = 0$.

(3) $y^{(4)} + 2y'' + y = 3 + \cos 2t$.