力学与航空航天工程系
DEPARTMENT OF MECHANICS AND AEROSPACE ENGINEERING

MECHANICS OF MATERIALS

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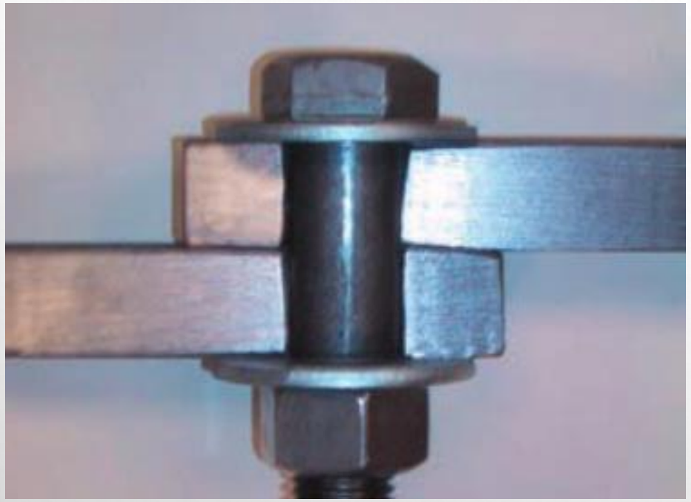
SPRING, 2022

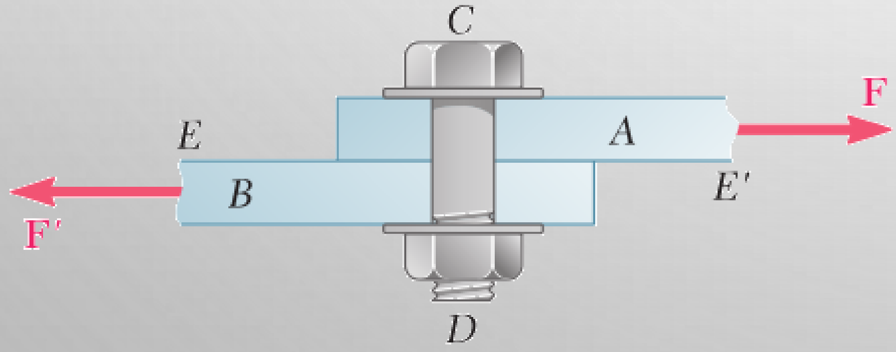
Lesson 3： Shear and bearing, strength calculation of connector

- Mechanical models of shear and bearing
- Stress and deformation characterization
- Stress calculation and design of connectors.

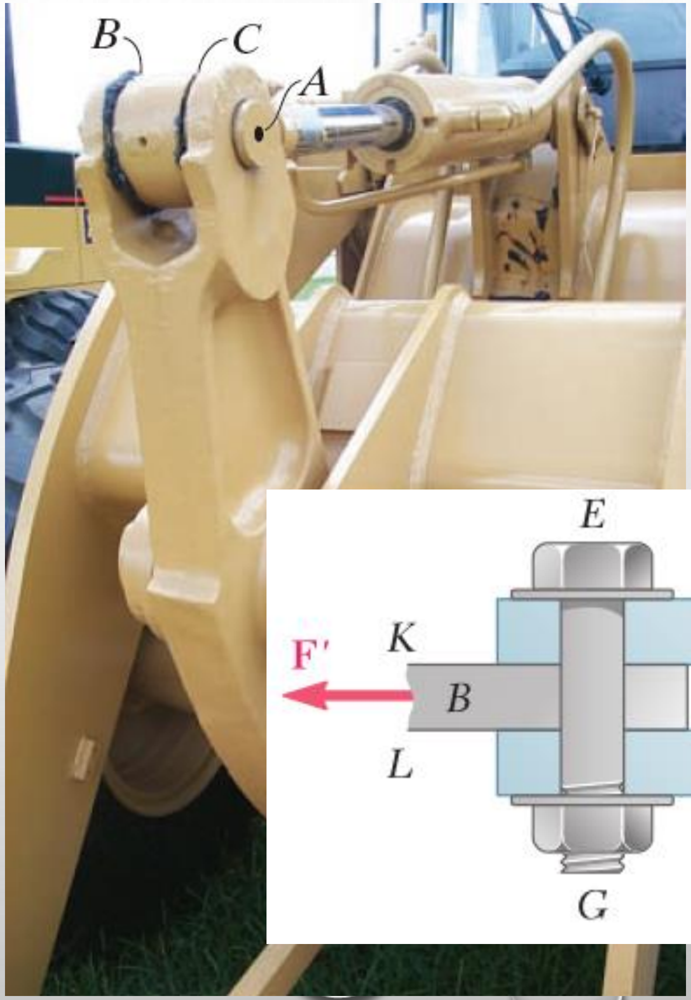
§ 3.1 Examples of shear and bearing

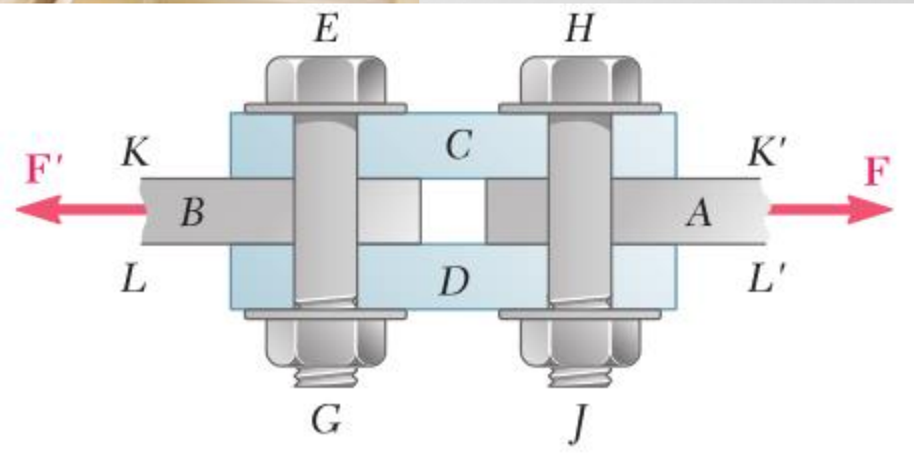
Bolt





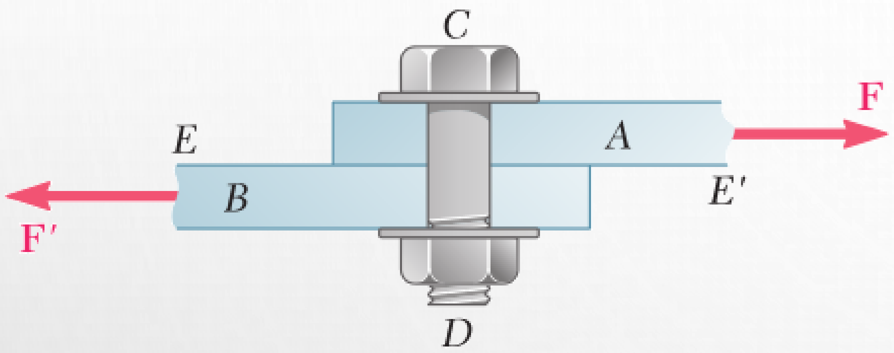
Single shear

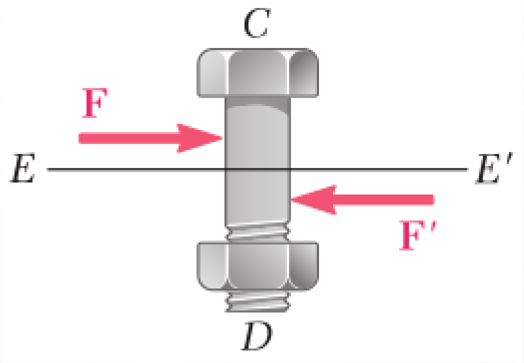


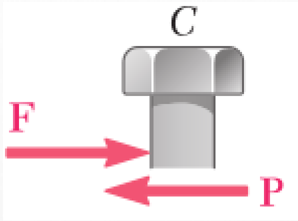


Double shear

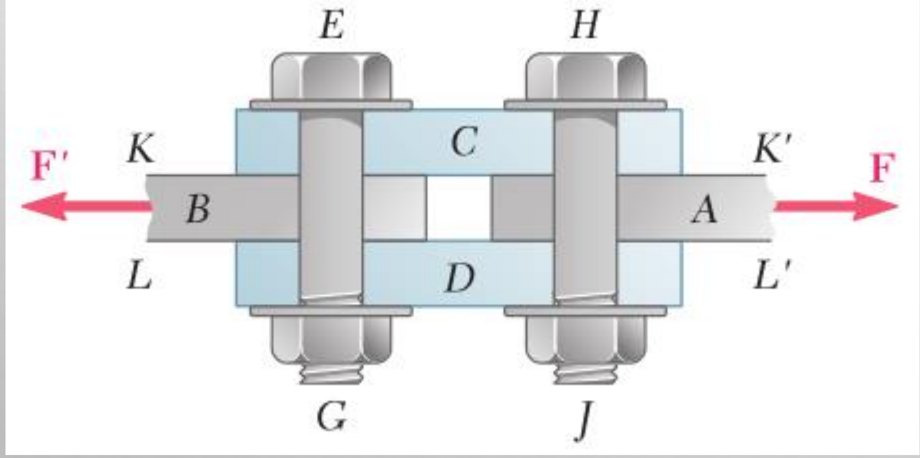
§ 3.2 Shear stress

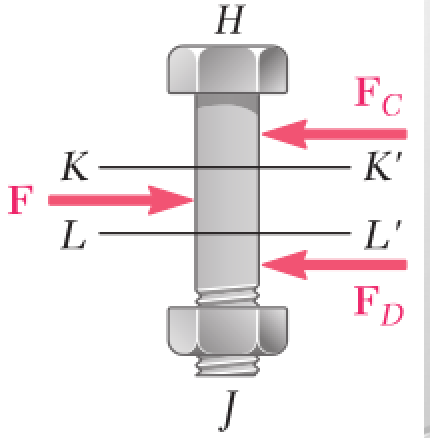


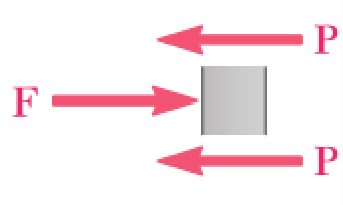




$$\tau_{\text{avg}} = \frac{F}{A}$$







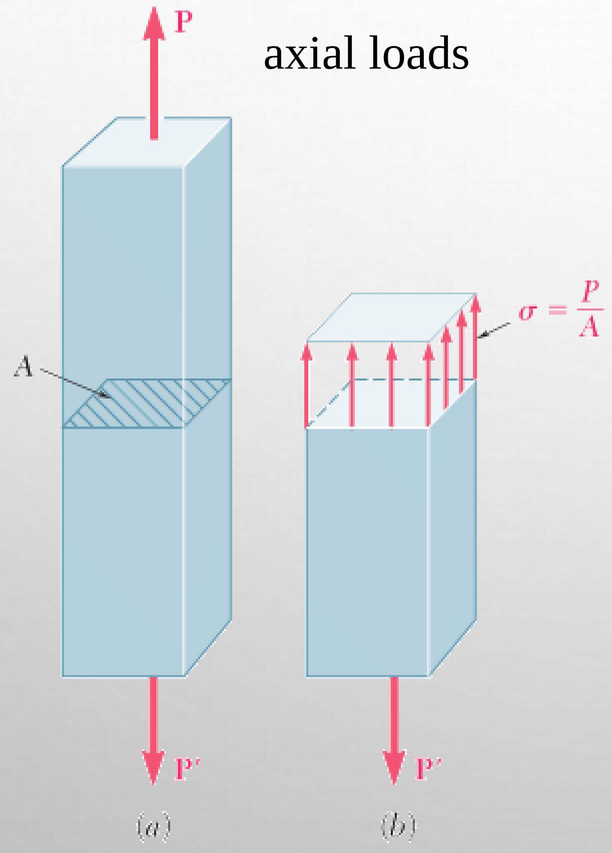
$$\tau_{\text{avg}} = \frac{F}{2A}$$

§ 3.2 Shear stress

- Comparison with normal stress

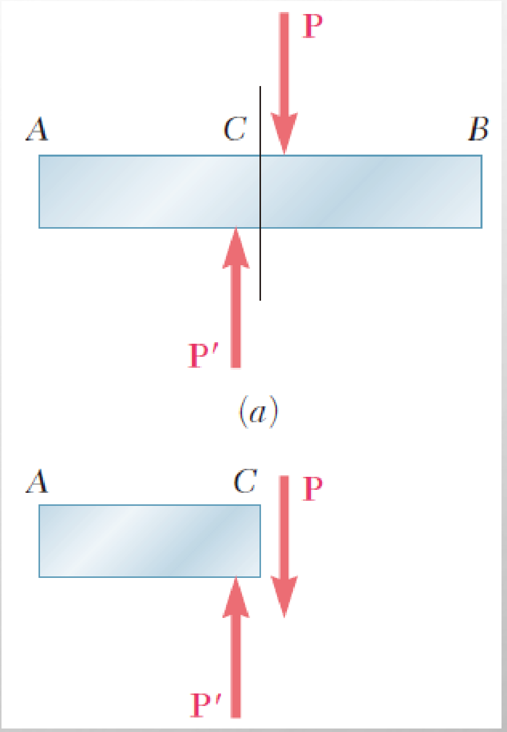
axial loads

$$\sigma_{avg} = \frac{P}{A}$$

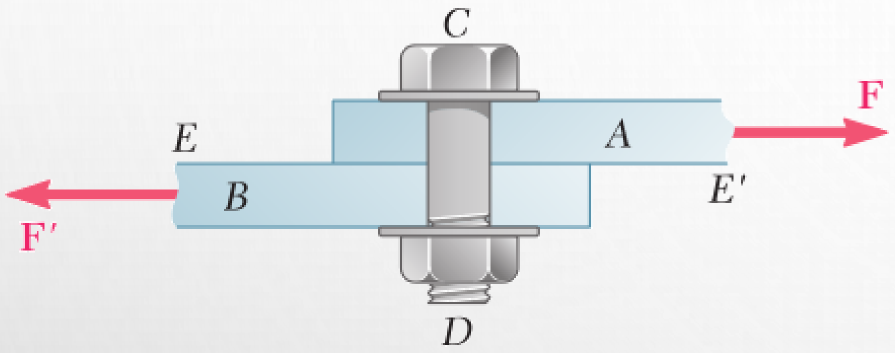


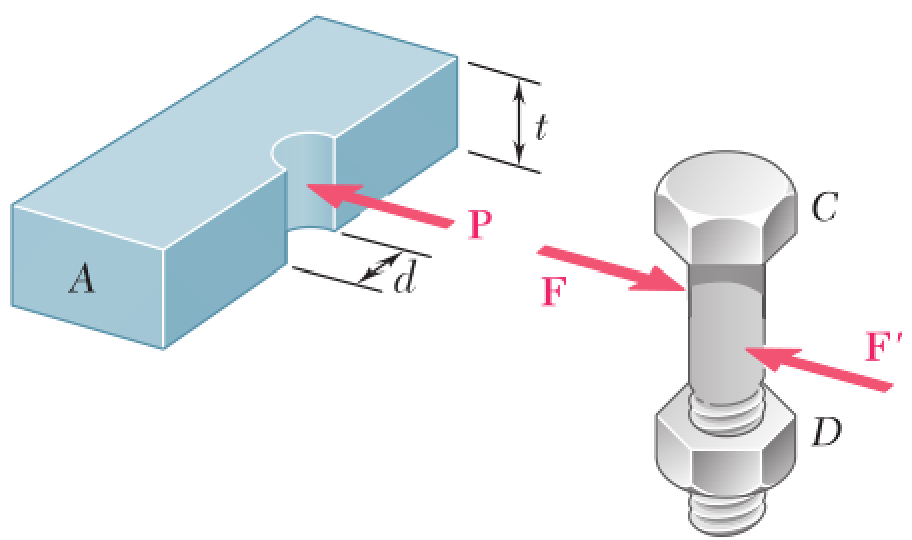
transverse loads

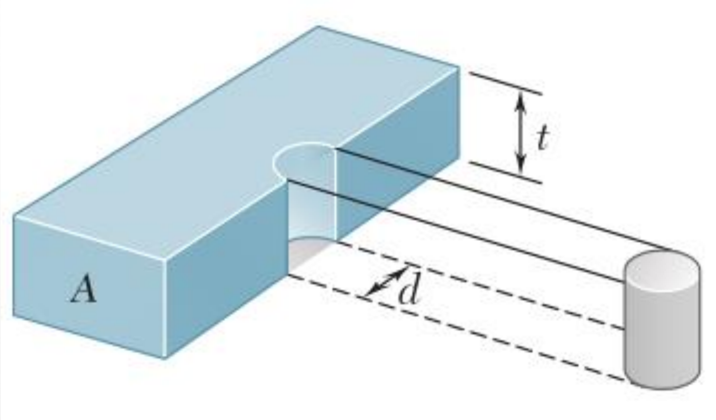
$$\tau_{avg} = \frac{P}{A}$$



§ 3.3 Bearing stress



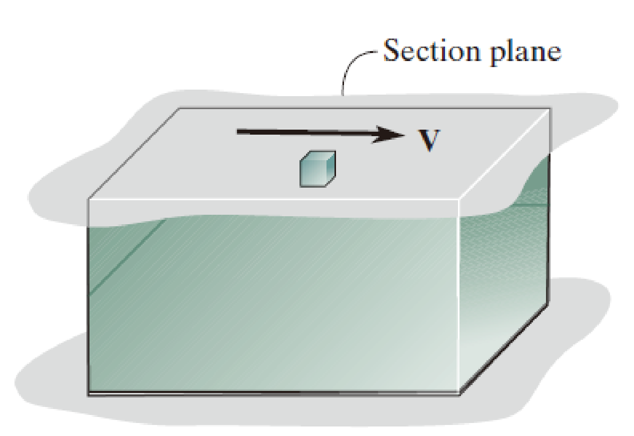


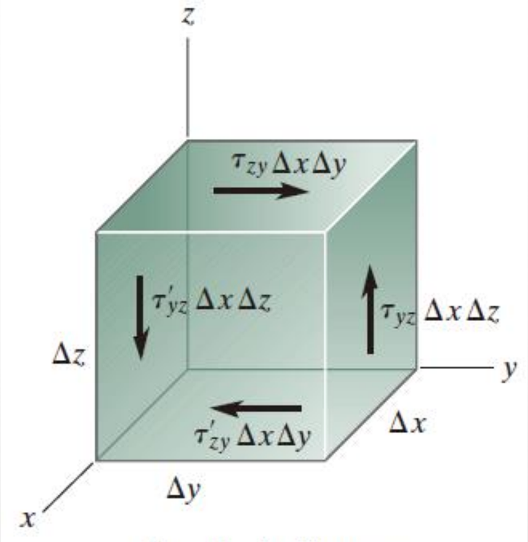


$$\sigma_{\mathrm{b}} = \frac{P}{A} = \frac{P}{td}$$

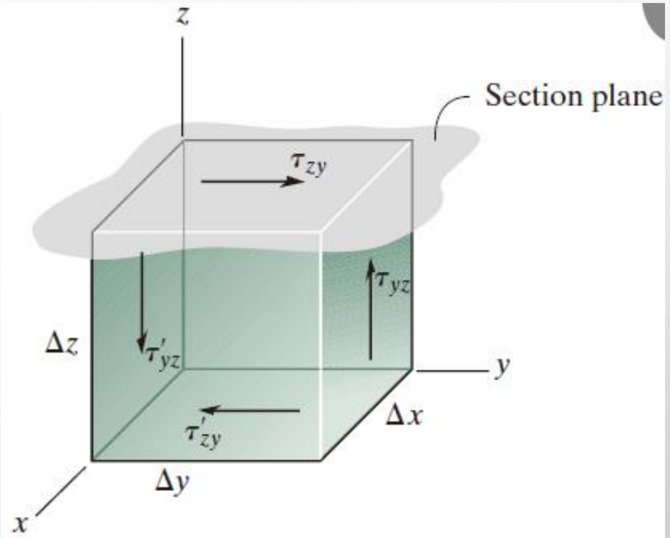
§ 3.4 Shear stress equilibrium

- The block has been sectioned and is subjected to the internal shear force V.





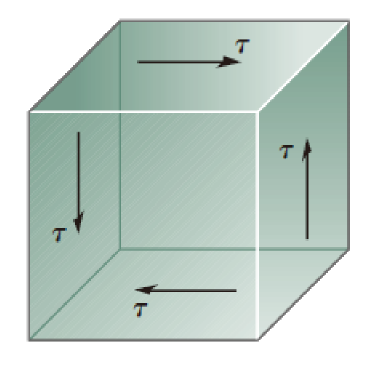
Free-body diagram



$$\Sigma F = 0, \Sigma M = 0 : \tau_{zy} = \tau_{yz} = \tau'_{zy} = \tau'_{yz} = \tau$$

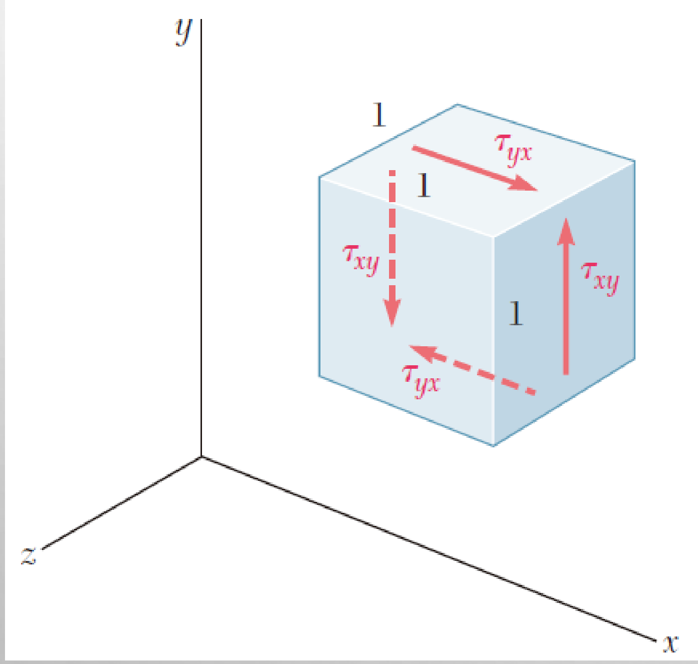
complementary property of shear

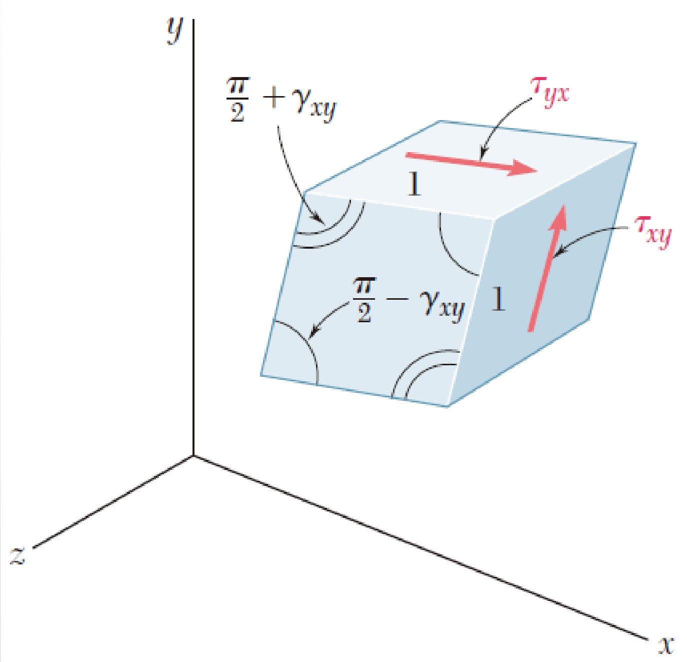
Pure shear

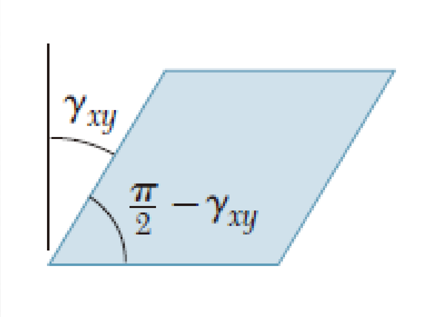


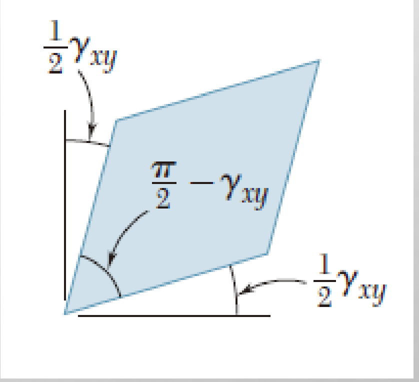
§ 3.5 Shear strain

- Deformation of cubic element due to shearing stresses





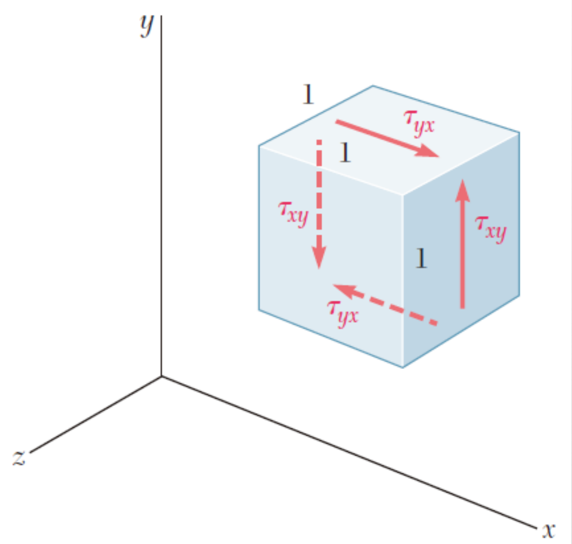


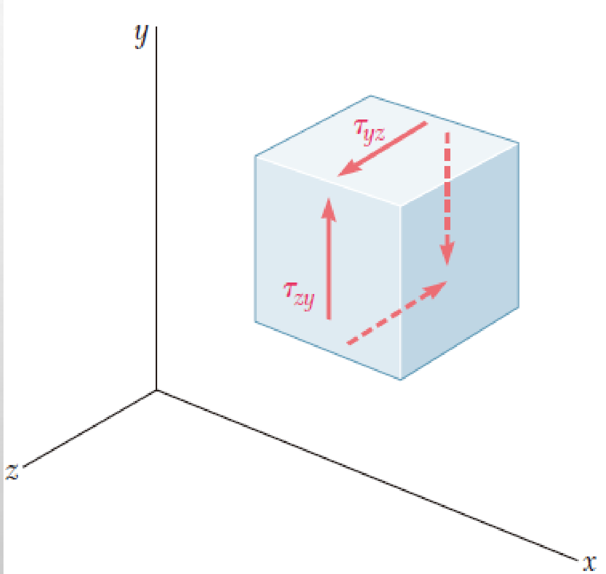


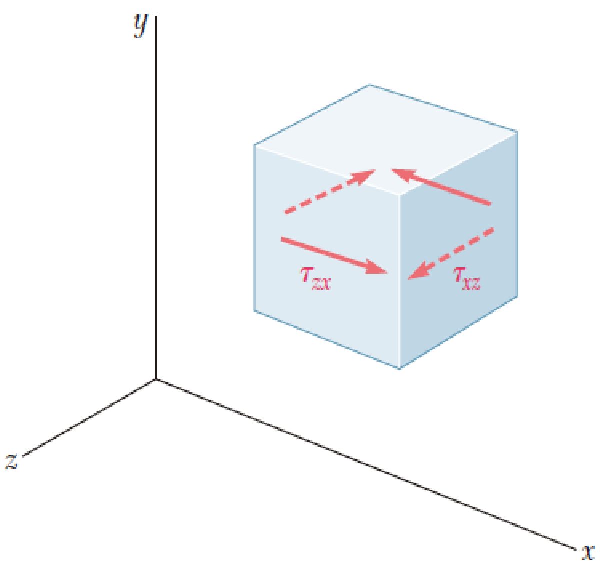
§ 3.6 Hooke's law of shear

- For any homogeneous isotropic material

$$\tau_{xy} = G\gamma_{xy} \qquad \tau_{yz} = G\gamma_{yz} \qquad \tau_{zx} = G\gamma_{zx}$$







G, shear modulus of elasticity or rigidity

§ 3.7 Generalized Hooke’s law

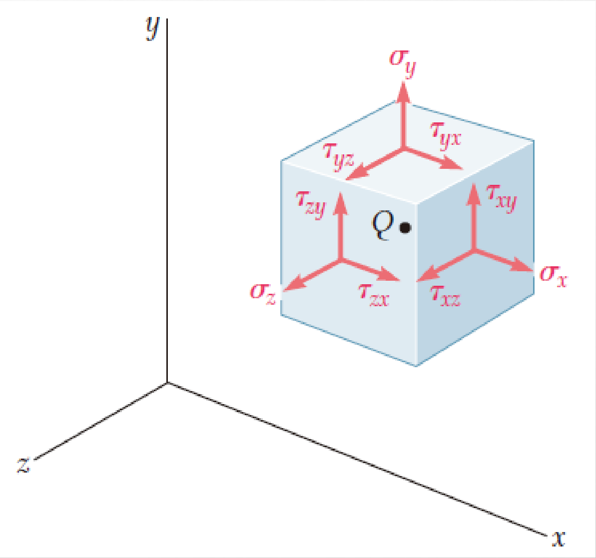
- For any homogeneous isotropic material under the most general stress condition,

$$\varepsilon_{\mathrm{x}} = +\frac{\sigma_{\mathrm{x}}}{E} - \frac{\nu\sigma_{\mathrm{y}}}{E} - \frac{\nu\sigma_{\mathrm{z}}}{E}$$

$$\varepsilon_{\mathrm{y}} = -\frac{\nu\sigma_{\mathrm{x}}}{E} + \frac{\sigma_{\mathrm{y}}}{E} - \frac{\nu\sigma_{\mathrm{z}}}{E}$$

$$\varepsilon_{\mathrm{y}} = -\frac{\nu\sigma_{\mathrm{x}}}{E} - \frac{\nu\sigma_{\mathrm{y}}}{E} + \frac{\sigma_{\mathrm{z}}}{E}$$

$$\tau_{xy} = \mathrm{G}\gamma_{xy},\ \tau_{yz} = \mathrm{G}\gamma_{yz},\ \tau_{zx} = \mathrm{G}\gamma_{zx}$$

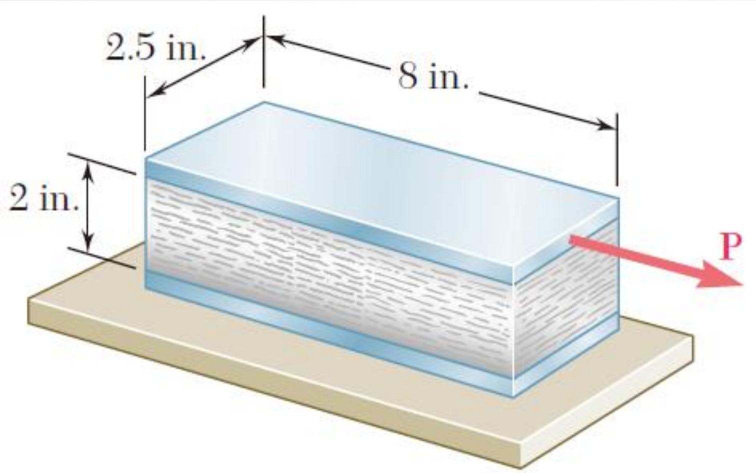


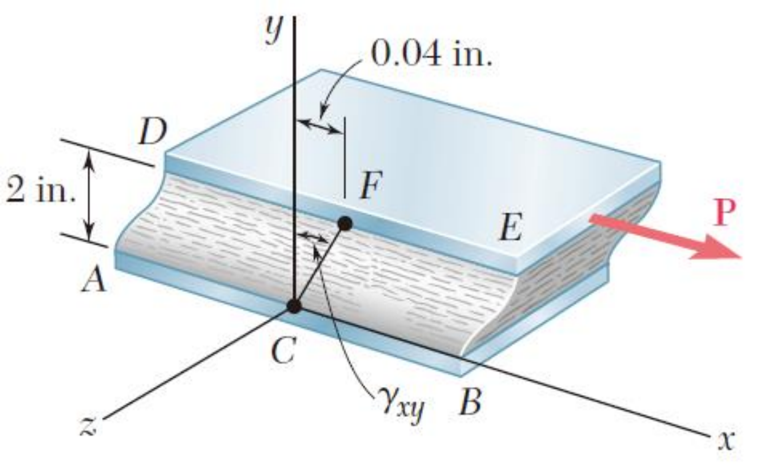
General state of stress

Example 3.1

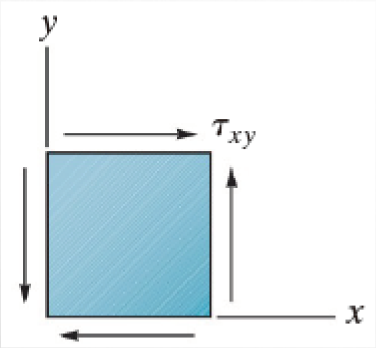
(Beer, Page 101)

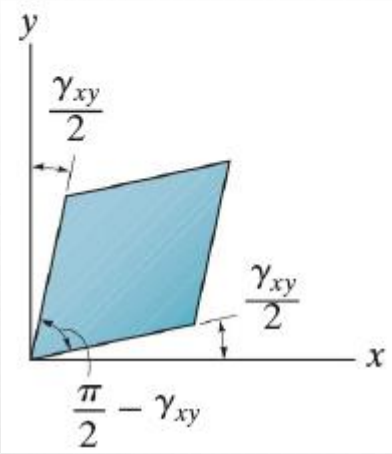
A rectangular block of a material with a modulus of rigidity G = 90 ksi is bonded to two rigid horizontal plates. The lower plate is fixed, while the upper plate is subjected to a horizontal force P. Knowing that the upper plate moves through 0.04 in. under the action of the force, determine (a) the average shearing strain in the material, (b) the force P exerted on the upper plate.





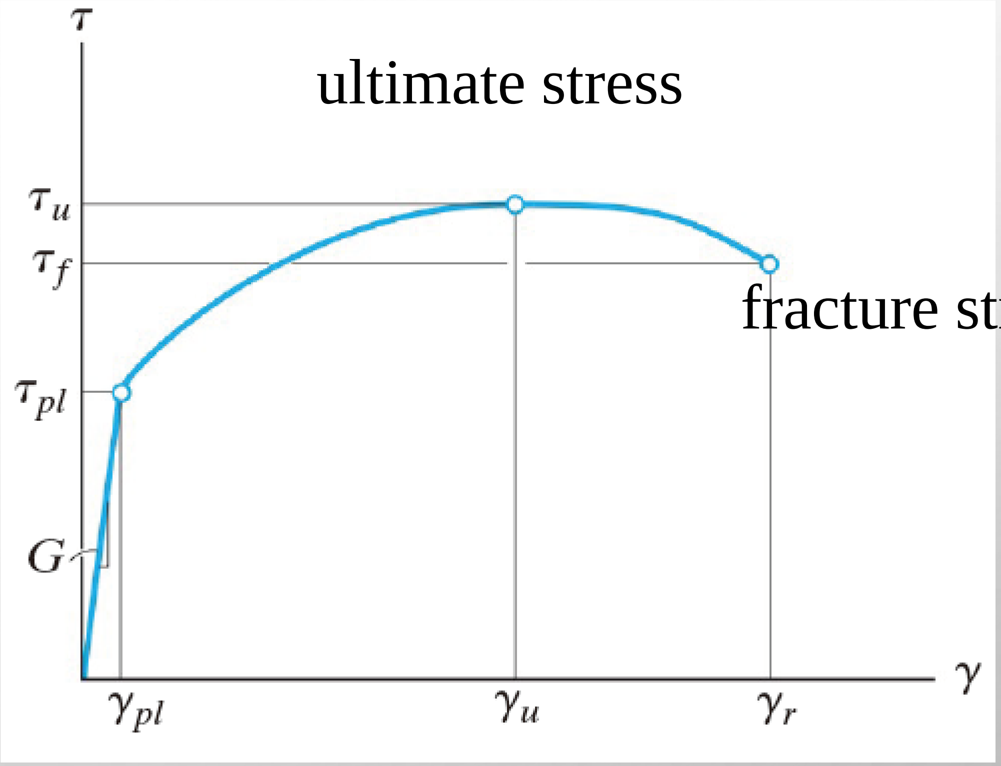
§ 3.8 The shear-stress diagram





$$\tau_{xy} = G\gamma_{xy}$$

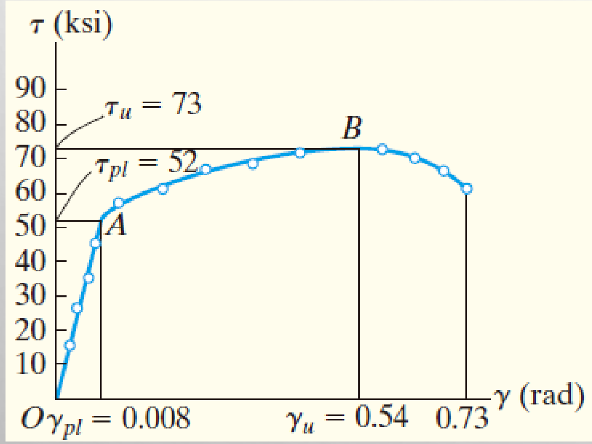
Hooke's law of shear

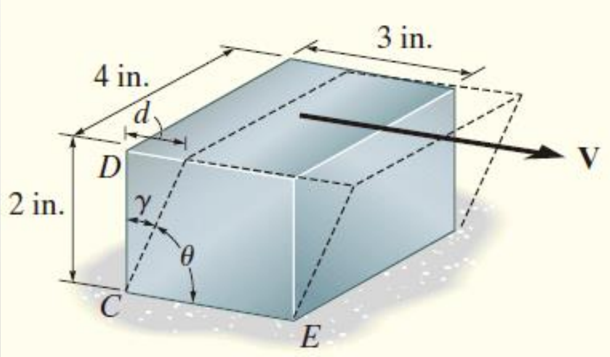


Example 3.2

(Hibbeler, Page 109)

A specimen of titanium alloy is tested in torsion and the shear stress–strain diagram is as shown. Determine the shear modulus G, the proportional limit, and the ultimate shear stress. Also, determine the maximum distance *d* that the top of a block of this material, shown in the right figure, could be displaced horizontally if the material behaves elastically when acted upon by a shear force *V*. What is the magnitude of *V* necessary to cause this displacement?



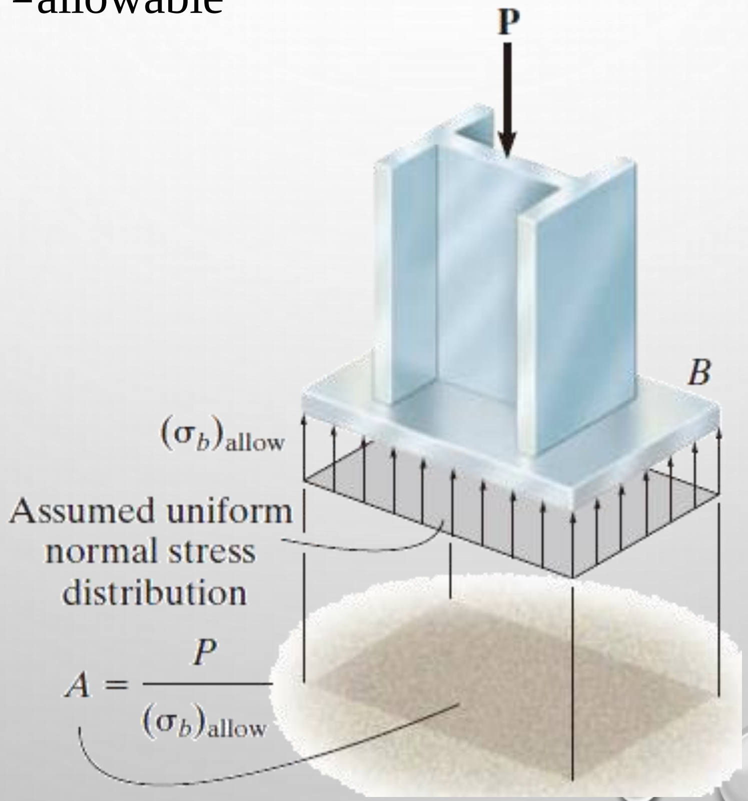


§ 3.9 Allowable stress design

$$\text{Factor of safety} = \frac{[F]_{\text{ultimate}}}{[F]_{\text{allowable}}}$$

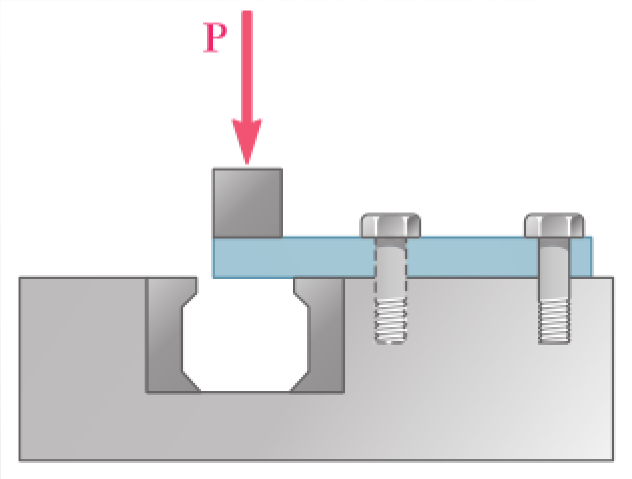
$$\text{F.S.} = \frac{[\sigma]_{\text{ultimate}}}{[\sigma]_{\text{allowable}}}$$

$$= \frac{[\tau]_{\text{ultimate}}}{[\tau]_{\text{allowable}}}$$



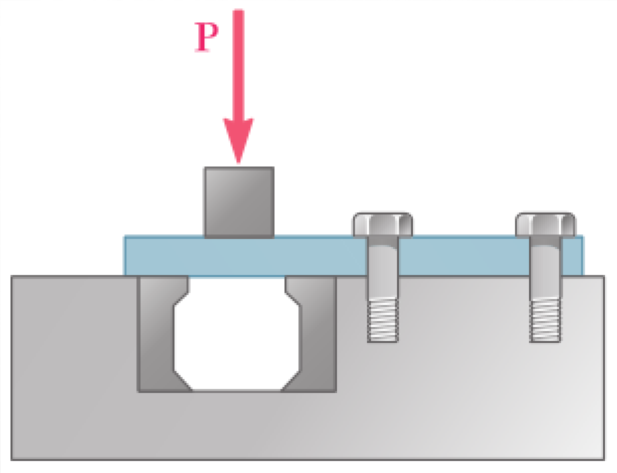
Cranes are often supported using bearing pads to give them stability.

§ 3.10 Ultimate shearing strength



Single shear test

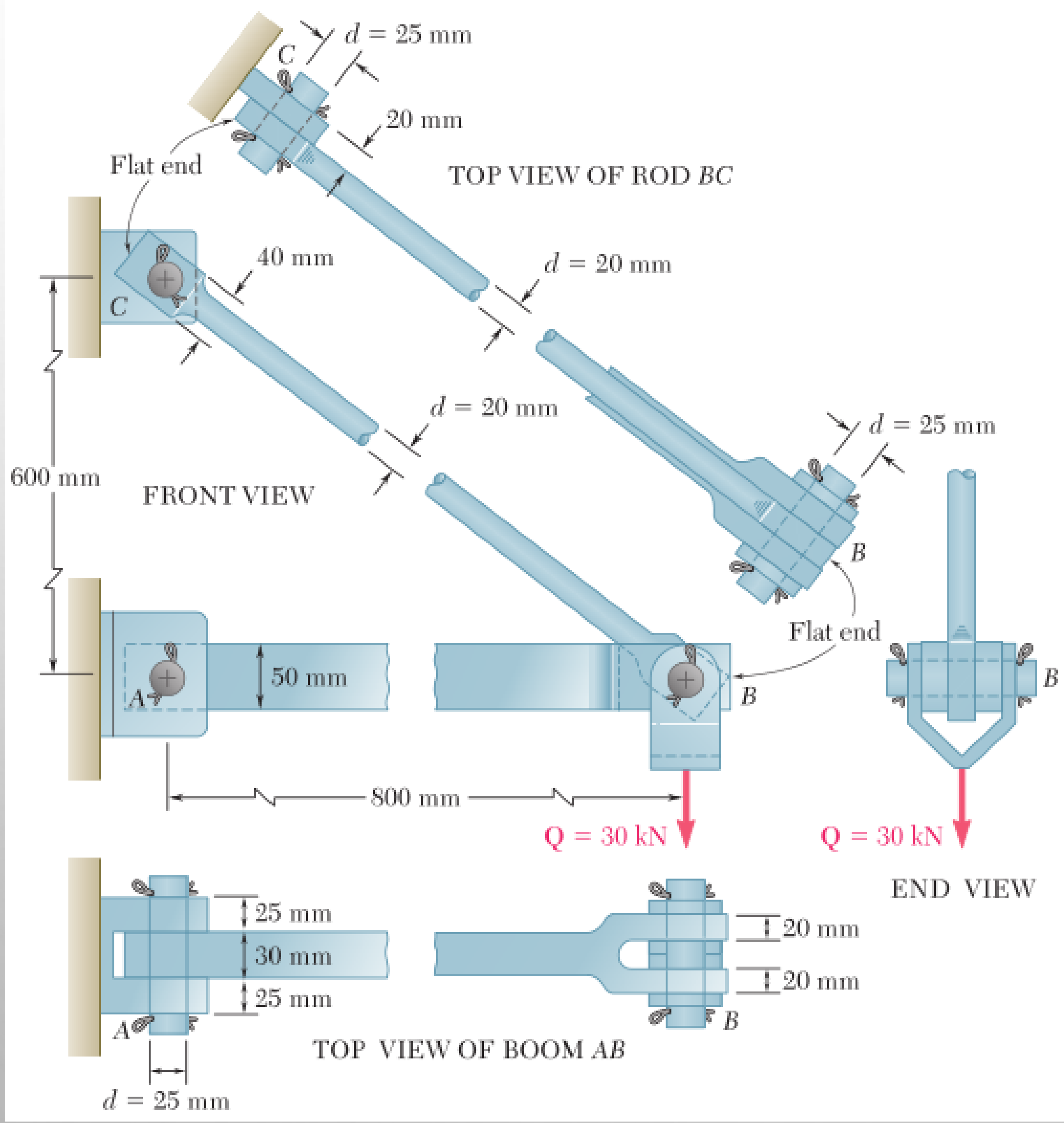
$$\tau_U = \frac{P_U}{A}$$



Double shear test

$$\tau_U = \frac{P_U}{2A}$$

§ 3.11 Analysis and design of simple structures

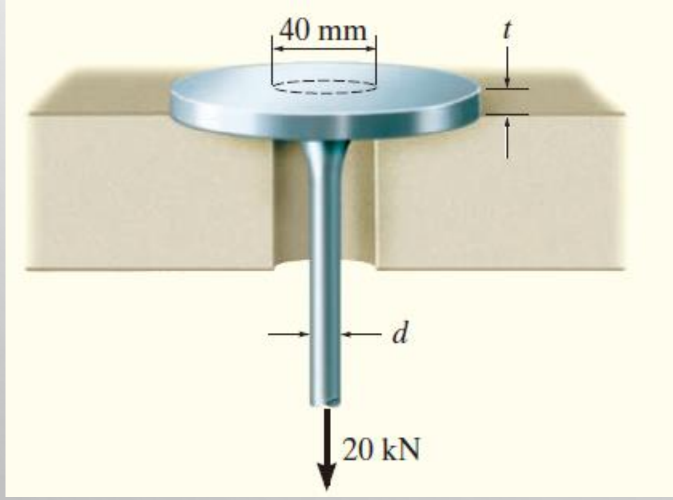


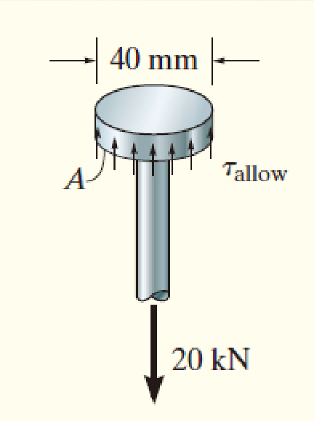
The 20-mm-diameter rod BC has flat ends of 20 × 40-mm rectangular cross section, while boom AB has a 30 × 50-mm rectangular cross section and is fitted with a clevis at end B. Both members are connected at B by a pin from which the 30-kN load is suspended by means of a U-shaped bracket. Boom AB is supported at A by a pin fitted into a double bracket, while rod BC is connected at C to a single bracket. All pins are 25 mm in diameter.

Example 3.3

(Hibbeler, Page 51)

- The suspender rod is supported at its end by a fixed-connected circular disk as shown. If the rod passes through a 40-mm-diameter hole, determine the minimum required diameter of the rod and the minimum thickness of the disk needed to support the 20-kN load. The allowable normal stress for the rod is sallow = 60 MPa, and the allowable shear stress for the disk is tallow = 35 MPa.

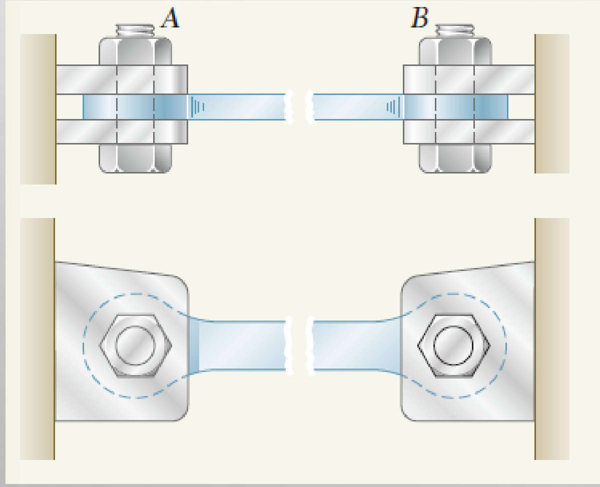


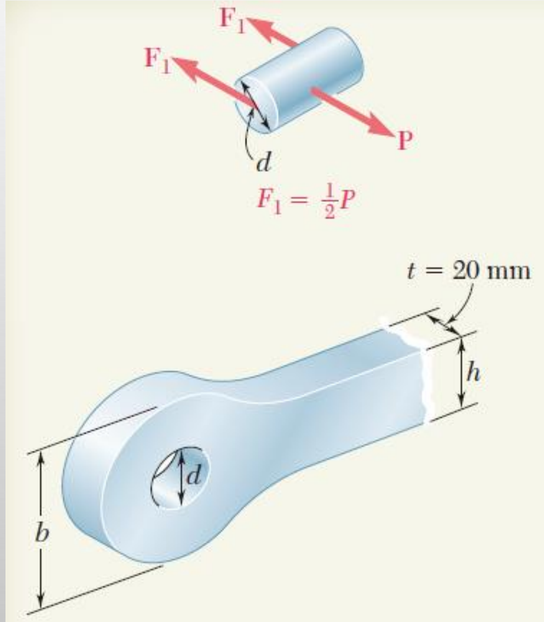


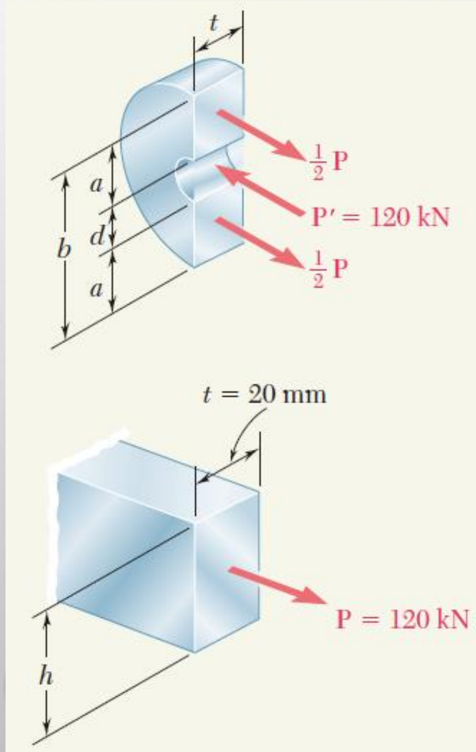
Example 3.4

(Beer, Page 19)

- The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120$ kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are: $\sigma = 175$ MPa, $\tau = 100$ MPa, $\sigma_b = 350$ MPa. Design the tie bar by determining the required values of (a) the diameter d of the bolt, (b) the dimension b at each end of the bar, (c) the dimension h of the bar.







§ 3.11 Summary

- Examples of shear and bearing
- **Single shear and double shear**
- **Shear stress and bearing stress**
- **Shear stress-strain diagram**
- **Factor of safety and allowable stress**