Reference Answers of Homework 11

1. In class, we have made a detailed analysis of the second law efficiency of a refrigerator that operates on the ideal vapor-compression refrigeration cycle. Please make a similar analysis for a heat pump.

ANS: The second-law efficiency of a heat pump operating on a vapor-compression refrigeration cycle is defined as

$$\eta_{\mathrm{II,HP}} = \frac{\dot{E}x_{\dot{Q}_H}}{\dot{W}} = \frac{\dot{W}_{\min}}{\dot{W}} = 1 - \frac{\dot{E}x_{\mathrm{dest,total}}}{\dot{W}}$$

Substituting

$$\dot{W} = \frac{\dot{Q}_H}{\mathrm{COP_{HP}}} \quad \text{and} \quad \dot{E}x_{\dot{Q}_H} = \dot{Q}_H\left(1 - \frac{T_0}{T_H}\right)$$

into the second-law efficiency equation

$$\eta_{\mathrm{II,HP}} = \frac{\dot{E}x_{\dot{Q}_H}}{\dot{W}} = \frac{\dot{Q}_H\left(1 - \frac{T_0}{T_H}\right)}{\frac{\dot{Q}_H}{\mathrm{COP_{HP}}}} = \dot{Q}_H\left(1 - \frac{T_0}{T_H}\right)\frac{\mathrm{COP_{HP}}}{\dot{Q}_H} = \frac{\mathrm{COP_{HP}}}{\frac{T_H}{T_H - T_L}} = \frac{\mathrm{COP_{HP}}}{\mathrm{COP_{Carnot}}}$$

since $T_0 .= T_L$.

2. A refrigerator operates on the ideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid. The refrigerant evaporates at -10°C and condenses at 57.9°C. The refrigerant absorbs heat from a space at 5°C and rejects heat to ambient air at 25°C. Determine

(a) the cooling capacity (in kJ/kg), and the COP;

(b) the exergy destruction in each component of the cycle and the total exergy destruction in the cycle;

(c) the second-law efficiency of the compressor, evaporator, and the cycle.

ANS:

(a) The properties of R-134a are (Tables A-11 through A-13)

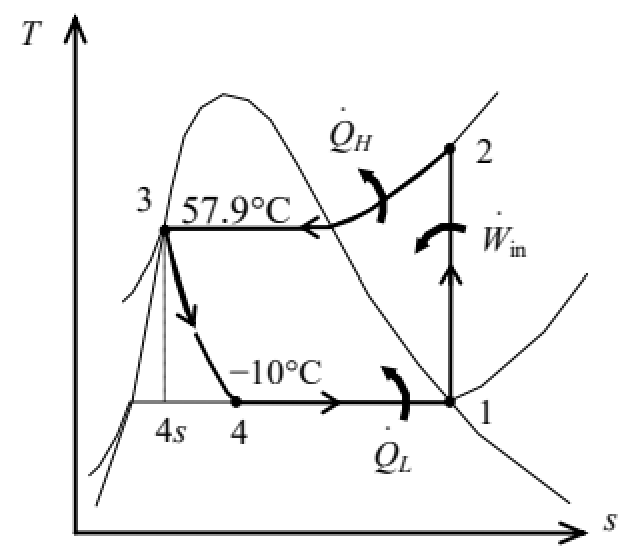
$$\left.\begin{array}{l} T_1 = -10°\text{C} \\ x_1 = 1 \end{array}\right\} \begin{array}{l} h_1 = 244.55 \text{ kJ/kg} \\ s_1 = 0.9378 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = P_{\text{sat@57.9°C}} = 1600 \text{ kPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 287.89 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 1600 \text{ kPa} \\ x_3 = 0 \end{array}\right\} \begin{array}{l} h_3 = 135.96 \text{ kJ/kg} \\ s_3 = 0.4792 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$h_4 = h_3 = 135.96 \text{ kJ/kg}$$

$$\left.\begin{array}{l} T_4 = -10°\text{C} \\ h_4 = 135.96 \text{ kJ/kg} \end{array}\right\} s_4 = 0.5252 \text{ kJ/kg}\cdot\text{K}$$



(a) The energy interactions in the components and the COP are

$$q_L = h_1 - h_4 = 244.55 - 135.96 = \mathbf{108.6 kJ/kg}$$

$$q_H = h_2 - h_3 = 287.89 - 135.96 = 151.9\,\text{kJ/kg}$$

$$w_{\text{in}} = h_2 - h_1 = 287.89 - 244.55 = 43.34\,\text{kJ/kg}$$

$$\text{COP} = \frac{q_L}{w_{\text{in}}} = \frac{108.6\,\text{kJ/kg}}{43.34\,\text{kJ/kg}} = \mathbf{2.506}$$

(b) The exergy destruction in each component of the cycle is determined as follows

Compressor:

$$s_{\text{gen},1\text{-}2} = s_2 - s_1 = 0$$

$$Ex_{\text{dest},1\text{-}2} = T_0 s_{\text{gen},1\text{-}2} = \mathbf{0}$$

Condenser:

$$s_{\text{gen},2\text{-}3} = s_3 - s_2 + \frac{q_H}{T_H} = (0.4792 - 0.9378)\,\text{kJ/kg}\cdot\text{K} + \frac{151.9\,\text{kJ/kg}}{298\,\text{K}} = 0.05125\,\text{kJ/kg}\cdot\text{K}$$

$$Ex_{\text{dest},2\text{-}3} = T_0 s_{\text{gen},2\text{-}3} = (298\,\text{K})(0.05125\,\text{kJ/kg}\cdot\text{K}) = \mathbf{15.27 kJ/kg}$$

Expansion valve:

$$s_{\text{gen},3\text{-}4} = s_4 - s_3 = 0.5252 - 0.4792 = 0.04595\ \text{kJ/kg}\cdot\text{K}$$

$$Ex_{\text{dest},3\text{-}4} = T_0 s_{\text{gen},3\text{-}4} = (298\ \text{K})(0.04595\ \text{kJ/kg}\cdot\text{K}) = \mathbf{13.69 kJ/kg}$$

Evaporator:

$$s_{\text{gen},4\text{-}1} = s_1 - s_4 - \frac{q_L}{T_L} = (0.9378 - 0.5252)\ \text{kJ/kg}\cdot\text{K} - \frac{108.6\ \text{kJ/kg}}{278\ \text{K}} = 0.02202\ \text{kJ/kg}\cdot\text{K}$$

$$Ex_{\text{dest},4\text{-}1} = T_0 s_{\text{gen},4\text{-}1} = (298\ \text{K})(0.02202\ \text{kJ/kg}\cdot\text{K}) = \mathbf{6.56 kJ/kg}$$

The **total exergy destruction** can be determined by adding exergy destructions in each component:

$$\begin{aligned}\dot{E}x_{\text{dest,total}} &= \dot{E}x_{\text{dest},1\text{-}2} + \dot{E}x_{\text{dest},2\text{-}3} + \dot{E}x_{\text{dest},3\text{-}4} + \dot{E}x_{\text{dest},4\text{-}1} \\ &= 0 + 15.27 + 13.69 + 6.56 \\ &= \mathbf{35.52 kJ/kg}\end{aligned}$$

(c) The exergy of the heat transferred from the low-temperature medium is

$$Ex_{q_L} = -q_L\left(1 - \frac{T_0}{T_L}\right) = -(108.6\ \text{kJ/kg})\left(1 - \frac{298}{278}\right) = 7.813\ \text{kJ/kg}$$

The second-law efficiency of the cycle is

$$\eta_{\text{II}} = \frac{Ex_{q_L}}{w_{\text{in}}} = \frac{7.813}{43.34} = 0.180 = \mathbf{18.0\%}$$

The total exergy destruction in the cycle can also be determined from

$$Ex_{\text{dest,total}} = w_{\text{in}} - Ex_{q_L} = 43.34 - 7.813 = 35.53\,\text{kJ/kg}$$

The result is practically identical as expected.

The second-law efficiency of the compressor is determined from

$$\eta_{\text{II,Comp}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\dot{W}_{\text{rev}}}{\dot{W}_{\text{act, in}}} = \frac{\dot{m}[h_2 - h_1 - T_0(s_2 - s_1)]}{\dot{m}(h_2 - h_1)}$$

Since the compression through the compressor is isentropic ($s_2 = s_1$), the second-law efficiency is 100%.

The second-law efficiency of the evaporator is determined from

$$\eta_{\text{II, Evap}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\dot{Q}_L(T_0 - T_L)/T_L}{\dot{m}[h_4 - h_1 - T_0(s_4 - s_1)]} = 1 - \frac{\dot{X}_{\text{dest,4-1}}}{\dot{X}_4 - \dot{X}_1} = 1 - \frac{x_{\text{dest,4-1}}}{x_4 - x_1} = 1 - \frac{6.56\,\text{kJ/kg}}{14.37\,\text{kJ/kg}} = \mathbf{54.4\%}$$

where

$$\begin{aligned} x_4 - x_1 &= h_4 - h_1 - T_0(s_4 - s_1) \\ &= (135.96 - 244.55)\,\text{kJ/kg} - (298\,\text{K})(0.5252 - 0.9378)\,\text{kJ/kg}\cdot\text{K} \\ &= 14.37\,\text{kJ/kg} \end{aligned}$$

3. A heat pump that operates on the ideal vapor-compression cycle with refrigerant-134a is used to heat water from 15 to 45°C at a rate of 0.12 kg/s. The condenser and evaporator pressures are 1.4 and 0.32 MPa, respectively. Determine the power input to the heat pump.

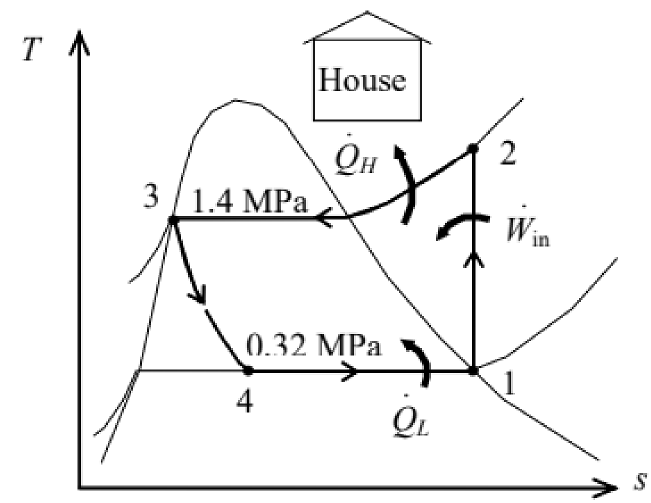
ANS: In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

$$\left.\begin{array}{l} P_1 = 320 \text{ kPa} \\ \text{sat. vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g\,@320\,\text{kPa}} = 251.93 \text{ kJ/kg} \\ s_1 = s_{g\,@\,320\,\text{kPa}} = 0.93026 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 1.4 \text{ MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 282.60 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 1.4 \text{ MPa} \\ \text{sat. liquid} \end{array}\right\} h_3 = h_{f\,@\,1.4\,\text{MPa}} = 127.25 \text{ kJ/kg}$$

$$h_4 \cong h_3 = 127.25 \text{ kJ/kg} \quad (\text{throttling})$$



The heating load of this heat pump is determined from

$$\dot{Q}_H = [\dot{m}c(T_2 - T_1)]_{\text{water}} = (0.12 \text{ kg/s})(4.18 \text{ kJ/kg}\cdot{}^\circ\text{C})(45-15)^\circ\text{C} = 15.05 \text{ kW}$$

and $$\dot{m}_R = \frac{\dot{Q}_H}{q_H} = \frac{\dot{Q}_H}{h_2 - h_3} = \frac{15.05 \text{ kJ/s}}{(282.60 - 127.25) \text{ kJ/kg}} = 0.09686 \text{ kg/s}$$

Then, $$\dot{W}_{\text{in}} = \dot{m}_R(h_2 - h_1) = (0.09686 \text{ kg/s})(282.60 - 251.93) \text{ kJ/kg} = \mathbf{2.97\ kW}$$