Homework problems 43-46
Due in class, Friday, 28 February 2020

43. The state of strain at the point on the support has components of $\varepsilon_x = 350(10^{-6})$, $\varepsilon_y = 400(10^{-6})$, $\gamma_{xy} = -675(10^{-6})$. Use (a) the strain-transformation equations, (b) Mohr's circle, to determine the in-plane principal strains, the maximum in-plane shear strain and the average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.

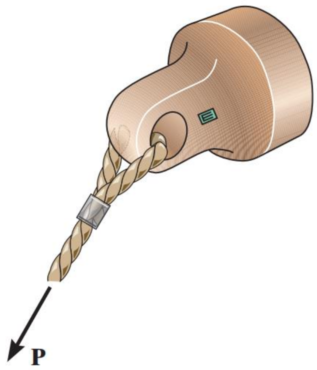
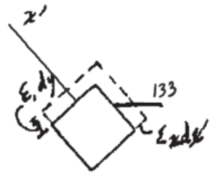


Figure 43

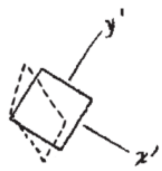
a)



$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{350 + 400}{2} \pm \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$\epsilon_1 = 713(10^{-6})$ **Ans.** $\epsilon_2 = 36.6(10^{-6})$ **Ans.**



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-675}{(350 - 400)}$$

$\theta_{p1} = 133°$ **Ans.**

b)

$$\frac{(\gamma_{x'y'})_{\max}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\frac{(\gamma_{x'y'})_{\max}}{2} = \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$(\gamma_{x'y'})_{\max} = 677(10^{-6})$ **Ans.**

$\epsilon_{avg} = \dfrac{\epsilon_x + \epsilon_y}{2} = \dfrac{350 + 400}{2} = 375(10^{-6})$ **Ans.**

$$\tan 2\theta_s = \frac{-(\epsilon_x - \epsilon_y)}{\gamma_{xy}} = \frac{350 - 400}{675}$$

$\theta_s = -2.12°$ **Ans.**

44. The 45° strain rosette is mounted on a steel shaft. The following readings are obtained from each gage: $\varepsilon_a = -200(10^{-6})$, $\varepsilon_b = 300(10^{-6})$, and $\varepsilon_c = 250(10^{-6})$. Determine the in-plane principal strains.

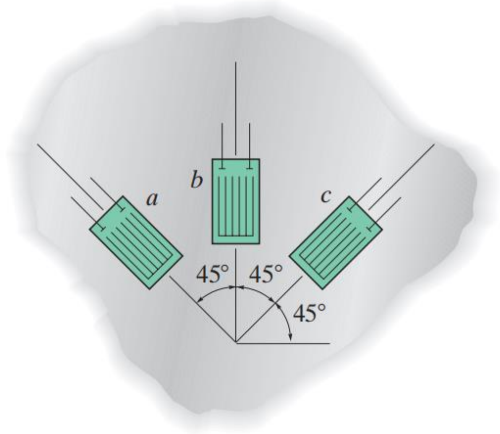


Figure 44

With $\theta_a = 45°, \theta_b = 90°$ and $\theta_c = 135°$,

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

$$300(10^{-6}) = \varepsilon_x \cos^2 45° + \varepsilon_y \sin^2 45° + \gamma_{xy}\sin 45°\cos 45°$$

$$\varepsilon_x + \varepsilon_y + \gamma_{xy} = 600(10^{-6}) \qquad \textbf{(1)}$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$$

$$-250(10^{-6}) = \varepsilon_x \cos^2 90° + \varepsilon_y \sin^2 90° + \gamma_{xy}\sin 90°\cos 90°$$

$$\varepsilon_y = -250(10^{-6})$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy}\sin\theta_c\cos\theta_c$$

$$-450(10^{-6}) = \varepsilon_x \cos^2 135° + \varepsilon_y \sin^2 135° + \gamma_{xy}\sin 135°\cos 135°$$

$$\varepsilon_x + \varepsilon_y - \gamma_{xy} = -900(10^{-6}) \qquad \textbf{(2)}$$

Substitute the result of ε_y into Eq. (1) and (2) and solve them

$$\varepsilon_x = 100(10^{-6}) \qquad \gamma_{xy} = 750(10^{-6})$$

In accordance to the established sign convention, $\varepsilon_x = 100(10^{-6})$, $\varepsilon_y = -250(10^{-6})$ and $\frac{\gamma_{xy}}{2} = 375(10^{-6})$. Thus,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{100 + (-250)}{2}\right](10^{-6}) = -75(10^{-6}) \qquad \textbf{Ans.}$$

Then, the coordinates of the reference point A and the center C of the circle are

$A(100, 375)(10^{-6}) \qquad C(-75, 0)(10^{-6})$

Thus, the radius of the circle is

$$R = CA = \left(\sqrt{\left[100 - (-75)\right]^2 + 375^2}\right)(10^{-6}) = 413.82(10^{-6})$$

Using these results, the circle is shown in Fig. a.

The Coordinates of points B and D represent ε_1 and ε_2, respectively. Thus,

$$\varepsilon_1 = \left(-75 + 413.82\right)(10^{-6}) = 339(10^{-6}) \qquad \textbf{Ans.}$$

$$\varepsilon_2 = \left(-75 - 413.82\right)(10^{-6}) = -489(10^{-6}) \qquad \textbf{Ans.}$$

Referring to the geometry of the circle

$$\tan 2(\theta_P)_1 = \frac{375}{100 + 75} = 2.1429$$

$$(\theta_P)_1 = 32.5^\circ \quad (Counter\ Clockwise) \qquad \textbf{Ans.}$$

The deformed element for the state of principal strains is shown in Fig. b.

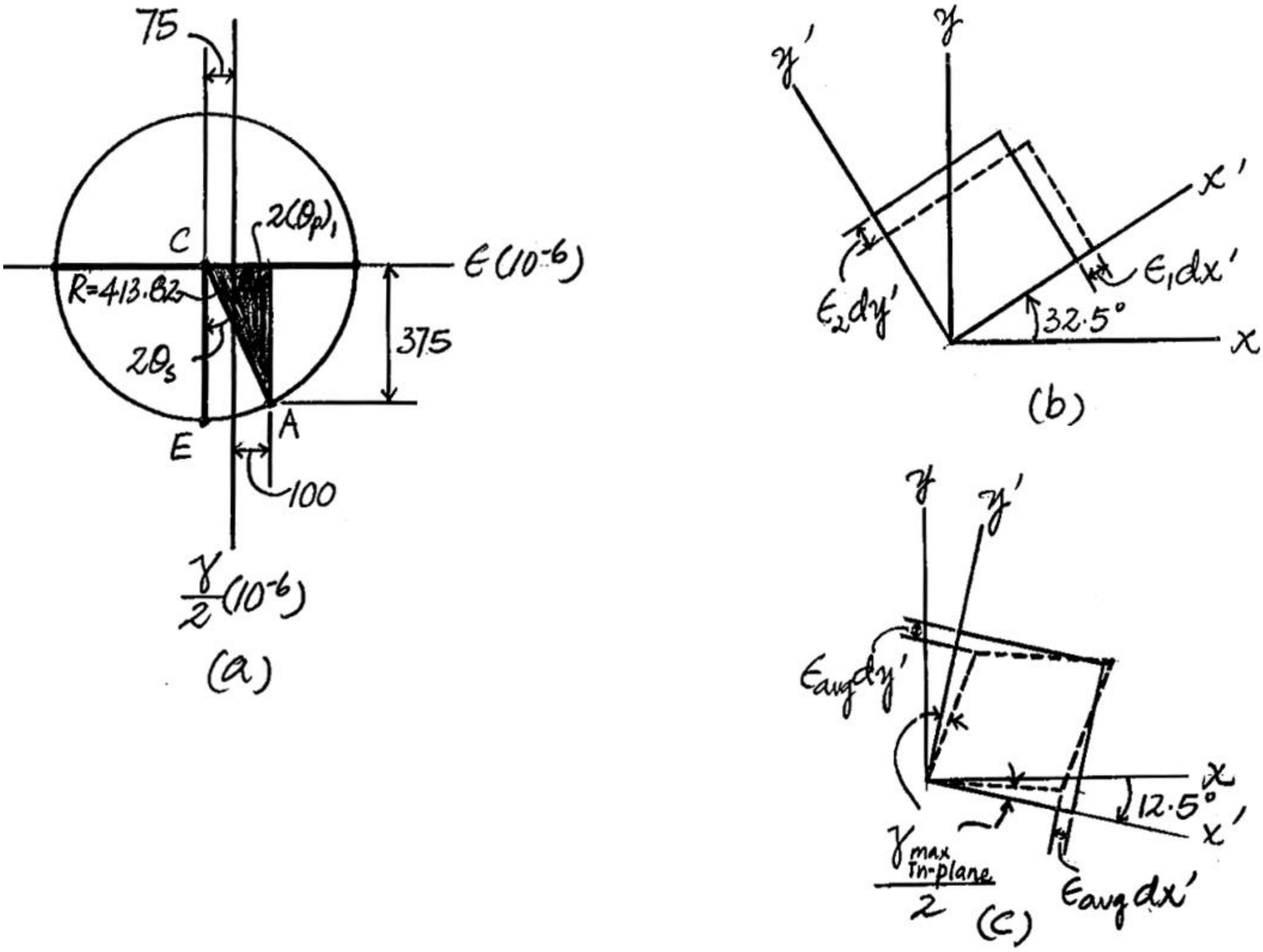
The coordinates of point E represent ε_{avg} and $\dfrac{\gamma_{\substack{\text{max}\\\text{in-plane}}}}{2}$. Thus

$$\frac{\gamma_{\substack{\text{max}\\\text{in-plane}}}}{2} = R = 413.82(10^{6}) \qquad \gamma_{\substack{\text{max}\\\text{in-plane}}} = 828(10^{-6}) \qquad \textbf{Ans.}$$

Referring to the geometry of the circle

$$\tan 2\theta_s = \frac{-100 + 75}{375} = 0.4667$$

$$\theta_s = 12.5^\circ \quad (Clockwise) \qquad \textbf{Ans.}$$


75
C
2(θp)1
ε(10^-6)
R=413.82
2θs
375
E
A
100
γ/2 (10^-6)
(a)
y'
y
x'
ε1dx'
ε2dy'
32.5°
x
(b)
y
y'
εavg dy'
x
12.5°
x'
γmax in-plane / 2
εavg dx'
(c)

45. A thin-walled spherical pressure vessel having an inner radius r and thickness t is subjected to an internal pressure p. Show that the increase in the volume within the vessel is $\Delta V = (2p\pi r^4/Et)(1 - v)$. Use a small-strain analysis.

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$\sigma_3 = 0$$

$$\varepsilon_1 = \varepsilon_2 = \frac{1}{E}(\sigma_1 - v\sigma_2)$$

$$\varepsilon_1 = \varepsilon_2 = \frac{pr}{2t\,E}(1 - v)$$

$$\varepsilon_3 = \frac{1}{E}(-v(\sigma_1 + \sigma_2))$$

$$\varepsilon_3 = -\frac{v\,pr}{t\,E}$$

$$V = \frac{4\pi r^3}{3}$$

$$V + \Delta V = \frac{4\pi}{3}(r + \Delta r)^3 = \frac{4\pi r^3}{3}(1 + \frac{\Delta r}{r})^3$$

where $\Delta V \ll V, \Delta r \ll r$

$$V + \Delta V - \frac{4\pi r^3}{3}\left(1 + 3\frac{\Delta r}{r}\right)$$

$$\varepsilon_{\mathrm{Vol}} = \frac{\Delta V}{V} = 3\left(\frac{\Delta r}{\mathrm{r}}\right)$$

Since $\varepsilon_1 = \varepsilon_2 = \dfrac{2\pi(r + \Delta r) - 2\pi r}{2\pi r} = \dfrac{\Delta r}{r}$

$$\varepsilon_{\mathrm{Vol}} = 3\varepsilon_1 = \frac{3pr}{2t\,E}(1 - v)$$

$$\Delta V = V e_{\mathrm{Vol}} = \frac{2p\pi r^4}{E\,t}(1 - v) \qquad \textbf{QED}$$

46. What is the equivalent bending moment M_e that, if applied alone to a solid bar with a circular cross section, would cause the same energy of distortion as the combination of an applied bending moment M and torque T.

Principal stresses:

$$\sigma_1 = \frac{M_e\, c}{I}; \qquad \sigma_2 = 0$$

$$u_d = \frac{1 + \nu}{3\,E}\left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)$$

$$(u_d)_1 = \frac{1 + \nu}{3\,E}\left(\frac{M_e^2 c^2}{I^2}\right) \tag{1}$$

Principal stress:

$$\sigma_{1,2} = \frac{\sigma + 0}{2} \pm \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + \tau^3}$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}; \qquad \sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

Distortion Energy:

Let $a = \frac{\sigma}{2}, b = \sqrt{\frac{\sigma^2}{4} + \tau^2}$

$$\sigma_1^2 = a^2 + b^2 + 2\,a\,b$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

$$\sigma_2^2 = a^2 + b^2 - 2\,a\,b$$

$$\sigma_2^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3\,b^2 + a^2$$

Apply $\sigma = \frac{M\,c}{I}; \qquad \tau = \frac{T\,c}{J}$

$$(u_d)_2 = \frac{1 + \nu}{3\,E}\left(3\,b^2 + a^2\right) = \frac{1 + \nu}{3\,E}\left(\frac{\sigma^2}{4} + \frac{3\sigma^2}{4} + 3\,\tau^2\right)$$

$$= \frac{1 + \nu}{3\,E}\left(\sigma^2 + 3\,\tau^2\right) = \frac{1 + \nu}{3\,E}\left(\frac{M^2 c^2}{I^2} + \frac{3\,T^2 c^2}{J^2}\right) \tag{2}$$

Equating Eq. (1) and (2) yields:

$$\frac{(1+\nu)}{3E}\left(\frac{M_e c^2}{I^2}\right) = \frac{1+\nu}{3E}\left(\frac{M^2 c^2}{I^2} + \frac{3T^2 c^2}{J^2}\right)$$

$$\frac{M_e^2}{I^2} = \frac{M^1}{I^2} + \frac{3T^2}{J^2}$$

$$M_e^2 = M^1 + 3T^2\left(\frac{I}{J}\right)^2$$

For circular shaft

$$\frac{I}{J} = \frac{\frac{\pi}{4}c^4}{\frac{\pi}{2}c^4} = \frac{1}{2}$$

Hence, $M_e^2 = M^2 + 3T^2\left(\frac{1}{2}\right)^2$

$$M_e = \sqrt{M^2 + \frac{3}{4}T^2}$$ **Ans.**