

## 2 Assignment 2

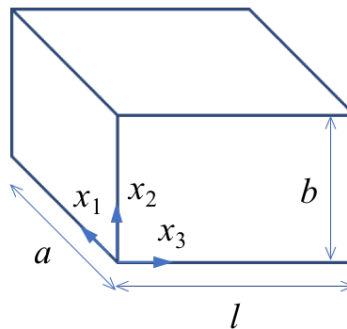
Question No.	1	2	3	4	5	6	Total
Score	12%	16%	17%	16%	18%	21%	100%

**Q 2.1.** Show that a closed box with sides of length  $a$ ,  $b$  and  $l$  resonates at frequencies

$$\omega = c\pi \left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 + \left( \frac{p}{l} \right)^2 \right]^{1/2}$$

where  $m$ ,  $n$  and  $p$  are integers. Hence determine the three lowest resonant frequencies of a rigid walled box with sides of length 1 m by 0.2 m by 0.3 m.

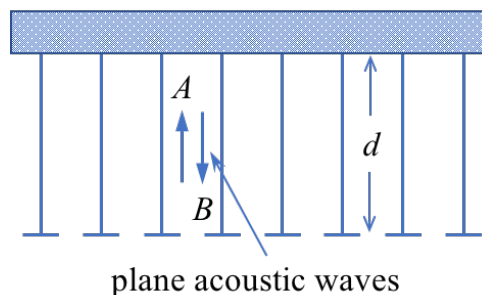
Total: [12%]



**Q 2.2.** For an acoustic liner of the form shown in the figure, the porosity (i.e. open area ratio) of the perforated sheet is  $\alpha$ , the pressure loss across a hole is  $0.1\rho_0 c u_h$ , where  $u_h$  is the acoustic velocity in the hole (i.e.  $u_h = u_n/\alpha$ ), and the honeycomb depth is  $d$ . Consider plane acoustic waves propagating in the honeycomb.

(i) Determine the surface impedance  $p'_s/u_n$ . [9%]

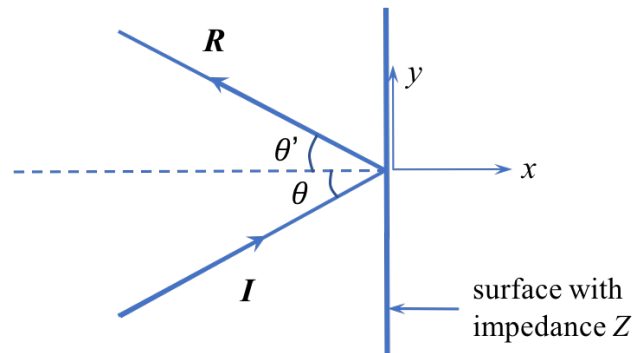
(ii) For this liner geometry, what honeycomb depth would you choose to optimally absorb sound of 1.45 kHz at an ambient temperature of 600 K? [7%]



结构? holes+honeycomb?  
p\_s?

Total: [16%]

**Q 2.3.** A plane wave of amplitude  $I$  travelling at an angle  $\theta$  to the positive  $x$  axis, where  $0 \leq \theta \leq \pi/2$ , is incident on an infinite plane wall at  $x = 0$ . The complex impedance of the wall is  $Z$ .



- (i) Show that the complex amplitude of the reflected wave,  $R$ , is given by

$$\frac{R}{I} = \frac{Z \cos \theta - \rho_0 c}{Z \cos \theta + \rho_0 c}$$

where  $\rho_0$  and  $c$  are the density and sound speed of the air for  $x < 0$ . [7%]

- (ii) In the case when  $Z$  is a purely real and positive constant, determine the minimum and maximum possible values of  $|R/I|$ , being careful to distinguish between the cases  $Z > \rho_0 c$  and  $Z < \rho_0 c$ . [8%]

- (iii) What happens to  $|R/I|$  when  $Z$  is purely imaginary? [2%]

Total: [17%]

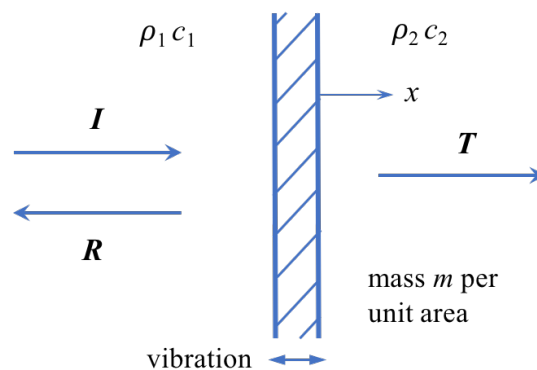
**Q 2.4.** A thin wall of mass per unit area  $m$  is located in the plane  $x = 0$  and is free to make small oscillations in the  $x$ -direction. The wall separates two acoustic media, of sound speed and density  $c_1$  and  $\rho_1$  in  $x < 0$  and  $c_2$  and  $\rho_2$  in  $x > 0$ . A plane sound wave is normally incident on the wall from  $x < 0$ .

**motion eq and velocity continuity\***

- (i) Determine an expression for the ratio of the magnitudes of the reflected and incident pressures,  $|R/I|$ , where  $R$  and  $I$  are the complex amplitudes of the reflected and incident waves, respectively. [12%]

- (ii) Identify two dimensionless numbers which determine the size of this quantity, and explain their physical significance. [4%]

Total: [16%]



**Q 2.5.** A ray travels through  $x \geq 0$  in a medium with spatially-varying sound speed

$$c(x) = c_0 e^{\alpha x}$$

where  $c_0$  and  $\alpha$  are constants. The ray passes through the origin, where it makes an angle  $\beta > 0$  to the positive  $x$  axis.

(i) Determine the equation of the ray path. **constant** [8%]

(ii) Sketch the shape of the ray path in the cases  $\alpha > 0$  and  $\alpha < 0$ , indicating the important values on the axes.  **$x = \frac{1}{\alpha} \ln(\sin \beta)$ ,  $\sin^{-1}(\sin \beta) - \pi$**  [10%]

You may assume without proof that

$$\int \frac{e^{\alpha x}}{\sqrt{1 - \sin^2 \beta e^{2\alpha x}}} dx = \frac{\sin^{-1}(\sin \beta e^{\alpha x})}{\alpha \sin \beta} + C$$

where  $C$  is a constant to be determined.

Total: [18%]

**Q 2.6.** Near a fjord where cold water enters the sea, the speed of sound  $c$  has a maximum at a depth of  $x = 100$  m. The speed of sound increases linearly with depth from  $1450 \text{ m s}^{-1}$  at the sea surface to  $1500 \text{ m s}^{-1}$  at the depth of  $x = 100$  m. At depths greater than 100 m,  $c$  decreases with increasing depth.

(i) Explain how the angle  $\theta$  between a ray and the  $x$ -axis varies with depth. Sketch the ray pattern produced by a source on the surface. [8%]

Consider the ray that forms the upper boundary of the shadow zone produced by a source on the sea surface.

(ii) At what angle  $\theta_0$  does this ray leave the source? [3%]

(iii) At what horizontal distance from the source does this ray again meet the surface?

**圆方程中的符号可变**

[10%]

Total: [21%]