Reference Answers of Homework 9

1. Consider an ideal Brayton cycle with air as the working fluid and the pressure ratio is 6. The minimum and maximum temperatures are 300 K and 1300 K, respectively. Now the pressure ratio is doubled without changing the minimum and maximum temperatures. Use constant specific heat at room temperature to determine:
(a) the change in the net work output per unit mass;
(b) the change in the thermal efficiency as a result of this modification.

ANS: Processes 1-2 and 3-4 are isentropic. Therefore, For $r_p = 6$,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{ K})(6)^{0.4/1.4} = 500.6\text{ K}$$

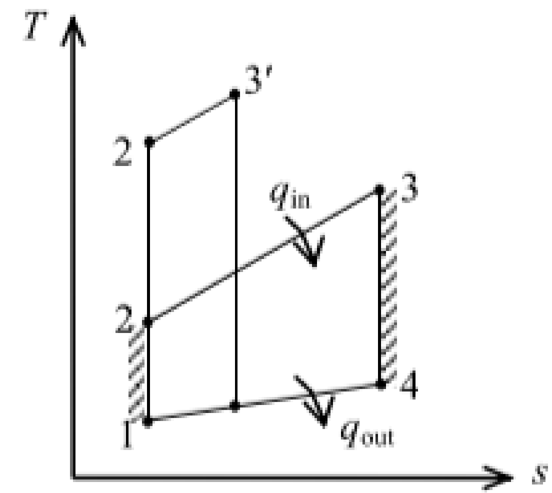
$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1300\text{ K})\left(\frac{1}{6}\right)^{0.4/1.4} = 779.1\text{ K}$$

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2) = (1.005\text{ kJ/kg}\cdot\text{K})(1300 - 500.6)\text{K} = 803.4\text{ kJ/kg}$$

$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1) = (1.005\text{ kJ/kg}\cdot\text{K})(779.1 - 300)\text{K} = 481.5\text{ kJ/kg}$$

$$w_{net} = q_{in} - q_{out} = 803.4 - 481.5 = 321.9\text{ kJ/kg}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{321.9\text{ kJ/kg}}{803.4\text{ kJ/kg}} = 40.1\%$$



For $r_p = 12$,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{ K})(12)^{0.4/1.4} = 610.2\text{ K}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1300\text{ K})\left(\frac{1}{12}\right)^{0.4/1.4} = 639.2\text{ K}$$

$$\begin{aligned} q_{\text{in}} &= h_3 - h_2 = c_p(T_3 - T_2) \\ &= (1.005\text{ kJ/kg}\cdot\text{K})(1300 - 610.2)\text{K} = 693.2\text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} q_{\text{out}} &= h_4 - h_1 = c_p(T_4 - T_1) \\ &= (1.005\text{ kJ/kg}\cdot\text{K})(639.2 - 300)\text{K} = 340.9\text{ kJ/kg} \end{aligned}$$

$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 693.2 - 340.9 = 352.3\text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{352.3\text{ kJ/kg}}{693.2\text{ kJ/kg}} = 50.8\%$$

Thus,

(*a*) $\Delta w_{\text{net}} = 352.3 - 321.9 = \mathbf{30.4\ kJ/kg}$ (increase)

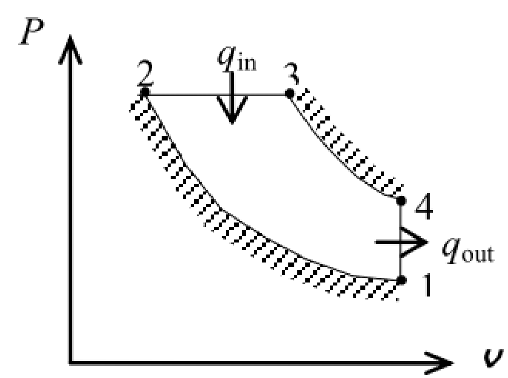
(*b*) $\Delta\eta_{\text{th}} = 50.8\% - 40.1\% = \mathbf{10.7\%}$ (increase)

2. An ideal Diesel cycle has a compression ratio of 16 and a cutoff ratio of 2. At the beginning of the compression process (state 1 shown in the figure), air is at 100 kPa and 25°C. Taking the specific heat c_p = 1.005 kJ/(kg·K) and the adiabatic constant k = 1.4 for air, determine:

(a) the temperature after the heat-addition process;

(b) the thermal efficiency of this cycle, and compare it with that of a Carnot cycle operating between the same temperature limits;

(c) the exergy destruction for each process in this cycle, assuming a source temperature of 2000 K and a sink temperature of 300 K.



ANS: (a) Process 1-2: isentropic compression.

$$T_2 = T_1(\frac{v_1}{v_2})^{k-1} = 298\text{ K} \times 16^{0.4} = 903.4\text{K}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \rightarrow T_3 = \frac{v_3}{v_2} T_2 = 2T_2 = 2 \times 903.367\text{ K}{=}1806.7\text{ K}$$

(b) $q_{in} = h_3 - h_2 = c_p(T_3 - T_2) = 1.005\text{ kJ/kg}\cdot\text{K}(1806.7 - 903.4)\text{K} = 907.9\text{ kJ/kg}$

Process 3-4: isentropic expansion.

$$T_4 = T_3(\frac{v_3}{v_4})^{k-1} = T_3(\frac{2v_2}{v_4})^{k-1} {=} 1806.734\text{ K} \times (\frac{2}{16})^{0.4} = 786.4\text{K}$$

Process 4-1: v = constant heat rejection.

$$q_{\text{out}} = u_4 - u_1 = c_v(T_4 - T_1) = (0.717857 \text{ kJ/kg}\cdot\text{K})(786.4 - 298)\text{K}=350.6 \text{ kJ/kg}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{350.621 \text{ kJ/kg}}{907.884 \text{ kJ/kg}} = 61.4\%$$

For the thermal efficiency of a Carnot cycle operating between the same temperature limits,

$$\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - \frac{298 \text{ K}}{1806.734 \text{ K}} = 83.5\%$$

(c)

$$\chi_{\text{des},1-2} = 0 \qquad \chi_{\text{des},3-4} = 0$$

$$R = c_p - c_v = c_p - c_p/k = c_p(1 - 1/k) = 1.005 \text{ kJ/kg}\cdot\text{K(1-1/1.4)=0.287143 kJ/kg}\cdot\text{K}$$

$$s_3 - s_2 = C_V \ln\frac{T_3}{T_2} + R\ln\frac{v_3}{v_2} = 0.717857 \text{ kJ/kg}\cdot\text{K}\cdot\ln\left(\frac{1806.734 \text{ K}}{903.367 \text{ K}}\right) + 0.287143 \text{ kJ/kg}\cdot\text{K}\cdot\ln 2 = 0.695315 \text{ kJ/kg}\cdot\text{K}$$

$$s_1 - s_4 = C_V \ln\frac{T_1}{T_4} + R\ln\frac{v_1}{v_4} = 0.717857 \text{ kJ/kg}\cdot\text{K}\cdot\ln\left(\frac{298 \text{ K}}{786.427 \text{ K}}\right) + 0.287143 \text{ kJ/kg}\cdot\text{K}\cdot\ln 1 = -0.696613\text{kJ/kg}\cdot\text{K}$$

$$\chi_{\text{des},2-3} = T_0\left[(s_3 - s_2) - \frac{q_{\text{in}}}{T_{\text{source}}}\right] = 300 \text{ K}\left[0.695315 \text{ kJ/kg}\cdot\text{K - }\frac{907.884 \text{ kJ/kg}}{2000 \text{ K}}\right] = 72.4119 \text{ kJ/kg}$$

$$\chi_{\text{des},4-1} = T_0\left[(s_1 - s_4) - \frac{q_{\text{out}}}{T_{\text{sink}}}\right] = 300 \text{ K}\left[-0.696613 \text{ kJ/kg}\cdot\text{K} + \frac{350.621 \text{ kJ/kg}}{300 \text{ K}}\right] = 141.637 \text{ kJ/kg}$$

3. A Brayton cycle with regeneration using air as the working fluid has a pressure ratio of 7. The minimum and maximum temperatures in the cycle are 310 and 1150 K. Assuming an isentropic efficiency of 75 percent for the compressor, 80 percent for the turbine and an effectiveness of 70 percent for the regenerator, and given that c_p = 1.04 kJ/kg·K and the adiabatic coefficient k is 1.4, determine:

(a) the air temperature at the turbine exit;

(b) the energy of the exhaust gases at the exit of the regenerator;

(c) the net work output;

(d) the thermal efficiency;

(e) the total exergy destruction, assuming a source temperature of 1500 K and a sink temperature of 290 K.

ANS:

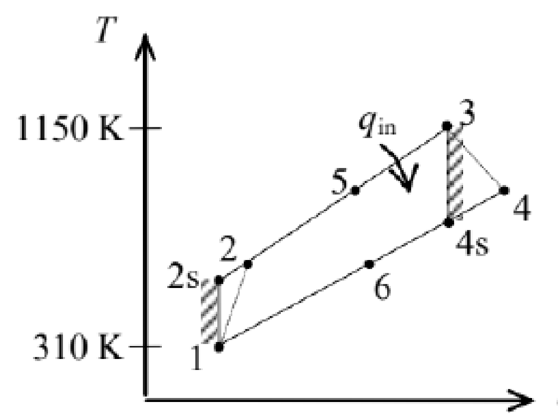
(a) $\frac{P_2}{P_1} = \frac{P_3}{P_4} = 7, \quad T_1 = 310\text{K}$

$$T_{2s} = T_1 r_p^{\frac{k-1}{k}} = 310\text{K} \times (7)^{\frac{0.4}{1.4}} = 540.53\text{K}$$

$$\eta_{Comp} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{T_{2s} - T_1}{T_2 - T_1} = 0.75 \Rightarrow T_2 = 617.37\text{K}$$

$$T_{4s} = T_3 \cdot \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}} = 1150\text{K} \times \left(\frac{1}{7}\right)^{\frac{0.4}{1.4}} = 659.54\text{K}$$

$$\eta_{Turb} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{T_3 - T_4}{T_3 - T_{4s}} = 0.80 \Rightarrow T_4 = 757.63\text{K}$$



(b) $\varepsilon = \dfrac{h_5 - h_2}{h_4 - h_2} = \dfrac{h_4 - h_6}{h_4 - h_2} = \dfrac{T_4 - T_6}{T_4 - T_2} = 0.70 \Rightarrow T_6 = 659.45\text{K}$

$$h_6 = Cp \cdot T_6 = 1.04\text{kJ} / (\text{kg} \cdot \text{K}) \times 659.45\text{K} = 685.83\text{kJ/kg}$$

(c) $w_{net} = h_3 - h_4 - h_2 + h_1 = 1.04\text{kJ} / (\text{kg} \cdot \text{K}) \times (1150\text{K} - 757.63\text{K} - 617.37\text{K} + 310\text{K}) = 88.4\text{kJ/kg}$

(d) $q_{in} = h_3 - h_5 = Cp(T_3 - T_5)$， $\eta = \dfrac{w_{net}}{q_{in}} = 19.56\%$

(e) $X_{des,12} = T_0 \cdot S_{gen,12} = T_0[Cp\ln(\frac{T_2}{T_1}) - R\ln(\frac{P_2}{P_1})] = 32.13\text{kJ/kg}$

$$X_{des,34} = T_0 \cdot S_{gen,34} = T_0[Cp\ln(\frac{T_4}{T_3}) - R\ln(\frac{P_4}{P_3})] = 41.79\text{kJ/kg}$$

$$X_{des,53} = T_0(S_3 - S_5 - \frac{q_{in}}{T_H}) = T_0[Cp\ln(\frac{T_3}{T_5}) - R\ln(\frac{P_3}{P_5}) - \frac{q_{in}}{T_H}] = 55.74\text{kJ/kg}$$

$$X_{des,61} = T_0(S_1 - S_6 - \frac{q_{out}}{T_L}) = T_0[Cp\ln(\frac{T_1}{T_6}) - R\ln(\frac{P_1}{P_6}) + \frac{q_{out}}{T_L}] = 135.61\text{kJ/kg}$$

$$X_{des,regen} = T_0[(S_5 - S_2) - (S_6 - S_4)] = T_0[Cp\ln(\frac{T_5}{T_2}) - R\ln(\frac{P_5}{P_2}) - Cp\ln(\frac{T_6}{T_4}) + R\ln(\frac{P_6}{P_4})]$$
$$= 86.37\text{kJ/kg}$$

$$X_{total} = \sum X_i = 351.64\text{kJ/kg}$$