《Fundamentals of Electric Circuits》 homework 5

6.13 Find the voltage across the capacitors in the circuit of Fig. 6.49 under dc conditions. (10')

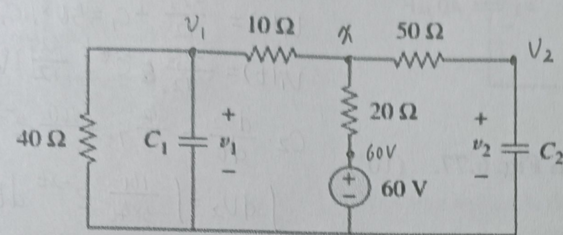


Figure 6.49

解:

$$\begin{cases} \frac{V_1-0}{40} = \frac{x-V_1}{10} \\ \frac{60-x}{20} = \frac{x-V_1}{10} + \frac{x-V_2}{50} \\ \frac{x-V_2}{50} = 0 \end{cases}$$

$\Rightarrow V_1 = \frac{240}{7} V = 34.29 V \quad V_2 = \frac{300}{7} V = 42.86 V$

6.19 Find the equivalent capacitance between terminals a and b in the circuit of Fig. 6.53. All capacitances are in μF. (10')

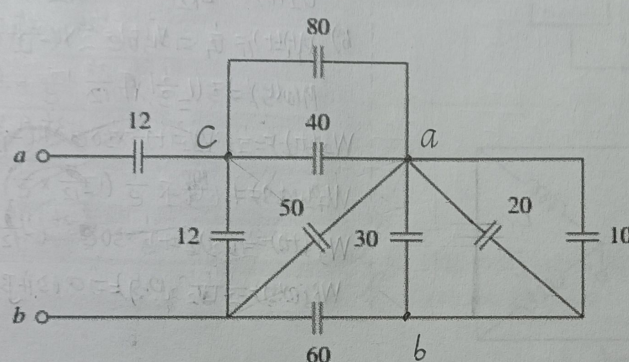
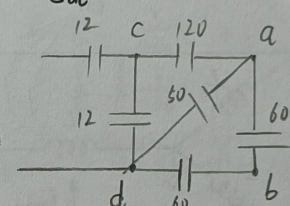


Figure 6.53

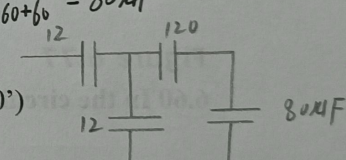
解:

$C_{ob} = 30+20+10 = 60 \mu F$

$C_{ac} = 80+40 = 120 \mu F$



$C_{adb} = 50 + \frac{60 \times 60}{60+60} = 80 \mu F$



$C_{eq} = \left(\frac{120 \times 80}{120+80} + 12\right) || 12$

$= \frac{12 \times 60}{12+60} = 10 \mu F$

6.22 Obtain the equivalent capacitance of the circuit in Fig. 6.56. (10')

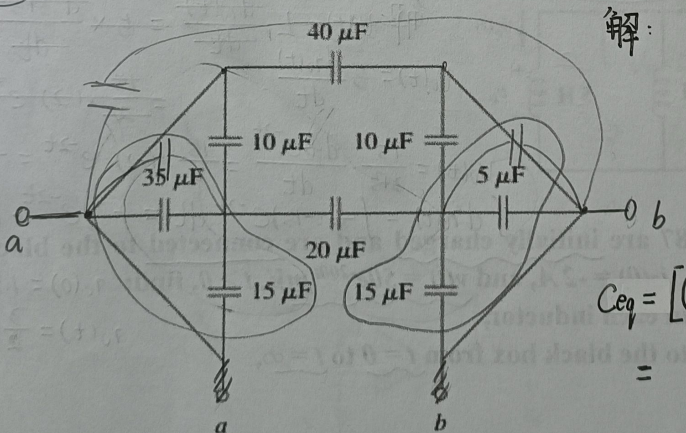


Figure 6.56

解:

$C_{eq} = [(10+35+15) || 20] || (10+5+15) + 40$

$= \frac{20 \times 60}{20+60} || 30 + 40$

$= \frac{15 \times 30}{15+30} + 40 = 50 \mu F$

6.32 In the circuit of Fig. 6.64, let $i_s = 50e^{-2t}$ mA and $v_1(0) = 50V$, $v_2(0) = 20V$. Determine: (a) $v_1(t)$ and $v_2(t)$, (b) the energy in each capacitor at $t = 0.5s$. (10')

$V_1(t) = -2083.33e^{-2t} + 2133.33$

$V_2(t) = -416.667e^{-2t} + 436.667$

$P_1 = 11.21$

0.803

1.606

$C \cdot V = q$

$C \cdot \frac{dV}{dt} = i$

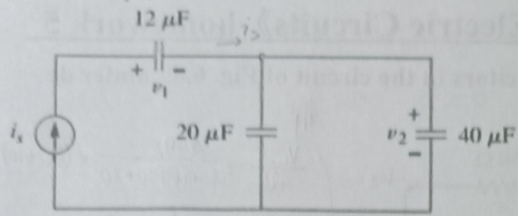


Figure 6.64

解:

a) $C_1 \cdot \frac{dV_1}{dt} = i_s = 50e^{-2t}$ mA

$\int dV_1 = \int \frac{50e^{-2t}}{C_1} dt$

$V_1 = \frac{50}{12} \cdot \frac{1}{-2} \cdot e^{-2t} + C_1$ kV

$V_1(0) = \frac{-25}{12} + C_1 = 50\times10^{-3}$ kV, $C_1 = 2133$ V

$V_1(t) = -2083e^{-2t} + 2133$ V

$C_2 \cdot \frac{dU_2}{dt} = \frac{2}{3} i_s = \frac{100}{3} e^{-2t}$ mA

$\int dV_2 = \int \frac{100}{3\times40} e^{-2t} dt$

$V_2 = \frac{5}{6} \times \frac{1}{-2} e^{-2t} + C_2$ kV

$U_2(0) = \frac{5}{-12} + C_2 = 20\times10^{-3}$ kV, $C_2 = 436.7$ V

$U_2(t) = -416.7e^{-2t} + 436.7$ V

b) $W_1(t) = \frac{1}{2} C_1 V_1^2(t) = \frac{1}{2} \times 12\times10^{-6} \times (-2083e^{-2t} + 2133)^2$

$W_1(0.5) = 11.21$ J

$W_2(t) = \frac{1}{2} C_2 V_2^2(t) = \frac{1}{2} \times 40\times10^{-6} \times (-416.7e^{-2t} + 436.7)^2$

$W_2(0.5) = 1.606$ J

$W_3(t) = \frac{1}{2} C_3 V_2^2(t)$

$W_3(0.5) = \frac{1}{2} W_2(0.5) = 0.8031$ J

6.55 Find L_{eq} in each of the circuits in Fig. 6.77. (10')

解:

a) $L_{eq} = L + 2L \parallel (L \parallel L)$

$= L + \frac{2L \cdot \frac{1}{2}L}{2L + \frac{1}{2}L}$

$= 1.4L$

b) $L_{eq} = L \parallel 2(L \parallel 2)$

$= \frac{L^2}{L+L} = \frac{L}{2}$

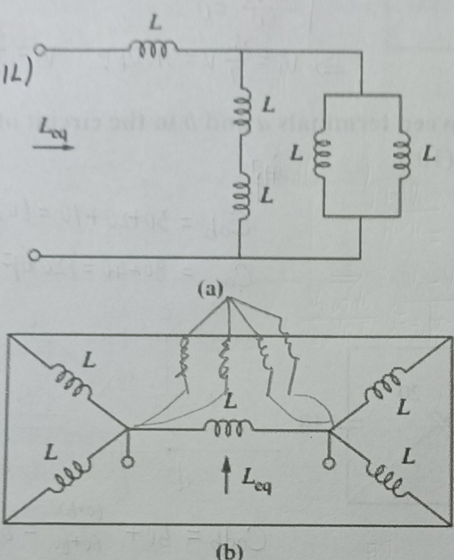


Figure 6.77

6.60 In the circuit of Fig.6.82, $i_o(0) = 2$ A. Determine $i_o(t)$ and $v_o(t)$ for $t > 0$. (10')

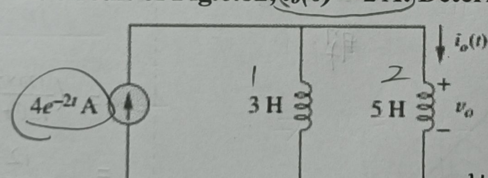


Figure 6.82

解: ~~$V_0(t) = L_1 \frac{di_0(t)}{dt} = 5 \times \frac{d \frac{3}{3+5} \times 4e^{-2t}}{dt}$~~

$V_0(t) = 5 \frac{di_0(t)}{dt}$

~~$= \frac{15}{2} \times (-2) e^{-2t} = -15e^{-2t}$ V~~

$V_0(t) = \frac{15}{3+5} \cdot \frac{d4e^{-2t}}{dt} = \frac{15}{2} \cdot (-2) \cdot e^{-2t} = -15e^{-2t}$ V

$\int di_0(t) = \int \frac{1}{5}(-15)e^{-2t} dt = -3 \cdot e^{-2t} \cdot \frac{1}{(-2)} + C$

$i_0(0) = 1.5 + C = 2$, $C = 0.5$

$i_0(t) = \frac{3}{2} \cdot e^{-2t} + 0.5$ A

6.65 The inductors in Fig. 6.87 are initially charged and are connected to the black box at $t = 0$. If $i_1(0) = 4$A, $i_2(0) = -2$ A, and $v(t) = 50e^{-200t}$ mV, $t \geq 0$, find:

(a) the energy initially stored in each inductor,

(b) the total energy delivered to the black box from $t = 0$ to $t = \infty$,

(c) $i_1(t)$ and $i_2(t)$, $t \geq 0$,

(d) $i(t)$, $t \geq 0$. (15')

解: a) $W_1 = \frac{1}{2} L_1 i_1^2 = \frac{1}{2} \times 5 \times 16 = 40$ J

$W_2 = \frac{1}{2} L_2 i_2^2 = \frac{1}{2} \times 20 \times 4 = 40$ J

b) $E = W_1 + W_2 = 80$ J

c) $V(t) = L_1 \frac{di_1(t)}{dt}$

$\int di_1(t) = \int \frac{V(t)}{L_1} dt = \int \frac{-50e^{-200t}}{5} dt = \frac{10}{\mp 200} \cdot e^{-200t} + C_1$ mA

$i_1(0) = 4000 = +0.05 + C_1$, $C_1 = 3999.95$ mA

$i_1(t) = \frac{1}{20} e^{-200t} + 3999.95$ mA

$$v(t) = L_2 \frac{di_2(t)}{dt}$$

$$\int di_2(t) = \int \frac{1}{L_2} v(t)dt = \frac{1}{20}\int 50e^{-200t}dt$$

$$i_2(t) = \frac{5}{2}\cdot\frac{1}{-200}\cdot e^{-200t} + C_2$$

$$i_2(0) = \frac{5}{-400} + C_2 = -2000,\ C_2 = -2000.0125 \text{ mA}$$

$$i_2(t) = \frac{1}{-80}\cdot e^{-200t} - 2000.0125 \text{ mA}$$

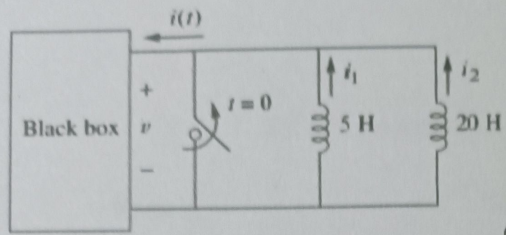


Figure 6.87

d) $i(t) = i_1(t) + i_2(t)$

$$= \frac{1}{16}e^{-200t} + 1999.9375 \text{ mA}$$

6.72 At $t = 1.5$ ms calculate v_o due to the cascaded integrators in Fig. 6.89. Assume that the integrators are reset to 0 V at $t = 0$. (15')

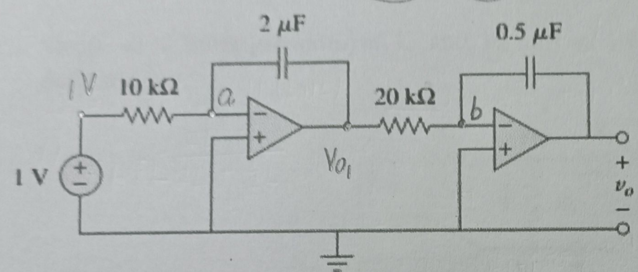


Figure 6.89

解:

$$\begin{cases} \dfrac{1-V_a}{10\times10^3} = 2\times10^{-6}\times\dfrac{d(V_a-V_{o1})}{dt\times10^{-3}} \\ \dfrac{V_{o1}-V_b}{20\times10^3} = 0.5\times10^{-6}\times\dfrac{d(V_b-V_o)}{dt\times10^{-3}} \\ V_a = V_b = 0 \end{cases}$$

$$\Rightarrow V_o = \frac{1}{400}t^2 + \left(\frac{C_1}{-10}\right)t + C_3 \text{ V (ms)}$$

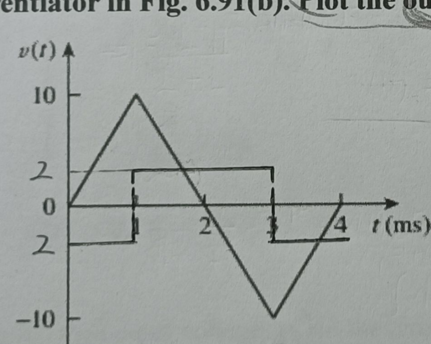
$$V_{o1} = \frac{1}{-20}t + C_1 \text{ V (ms)}$$

As $V_{o1}(0) = 0 = V_o(0) \Rightarrow C_1 = C_3 = 0$, $V_o(t) = \frac{t^2}{400}$

$$V_o(1.5) = \frac{1.5^2}{400} = 5.625\times10^{-3} \text{ V}$$

$$= 5.625 \text{ mV}$$

6.74 The triangular waveform in Fig. 6.91(a) is applied to the input of the op amp differentiator in Fig. 6.91(b). Plot the output. (10')

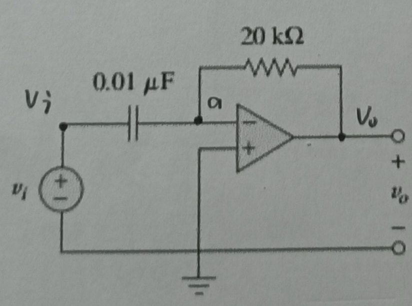


(a)

解:

$$\begin{cases} \dfrac{d(V_i-V_a)}{dt}\times0.01\times\cancel{10^{-3}} = \dfrac{V_a-V_o}{20}\cancel{\times10^{-3}} \\ V_a = 0 \end{cases}$$

$$\Rightarrow V_o = -0.2\frac{dV_i}{dt}$$



(b)

Figure 6.91