MAE407 Jet Propulsion

Spring Semester, 2022–2023

Chapter 5
Solutions to Questions

Dr Yu Liu

liuy@sustech.edu.cn

http://faculty.sustech.edu.cn/liuy

南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Solution to Exercise 5.1 (discussed)

5.1 Temperature at inlet to engine $T_{02} = 259{\cdot}5$ K. Fan pressure rise 1·6, efficiency 90%

Temperature rise in fan $$\Delta T_{0f} = 259\cdot 5\left(\frac{1\cdot 6^{\gamma-1/\gamma} - 1}{0\cdot 9}\right) = \underline{41\cdot 44}\text{ K}$$

Temperature into core compressor $$T_{023} = 259{\cdot}5 + 41{\cdot}44 = \underline{300{\cdot}9}\text{ K}$$

Temperature rise in core compressor $$T_{03} - T_{023} = 300\cdot 9\left(\frac{25^{\gamma-1/\gamma} - 1}{0\cdot 9}\right)$$

$$= 504{\cdot}3\text{ K} \qquad = \text{Temp drop in core turbine}$$

Temperature out of core compressor $T_{03} = 300{\cdot}9 + 504{\cdot}3 = \underline{805{\cdot}2}$ K

Temperature into core turbine $T_{04} = 1450$ K

Temperature out of core turbine $T_{045} = 1450 - 504{\cdot}3 = \underline{945{\cdot}7}$ K

= temperature into LP turbine

Solution to Exercise 5.1 (cont.)

Core turbine $\Delta T_0 = 504{\cdot}3 = \eta_t 1450\left(1-(1/r_t)^{\gamma-1/\gamma}\right)$

$\therefore$ Core turbine pressure ratio $r_t = 5.52$

$\therefore$ Pressure at HP turbine exit = Pressure at LP turbine inlet $= \dfrac{40.p_{02}}{5\cdot 52} = 7\cdot 25 p_{02}$

But stagnation pressure at compressor inlet $p_{02} = 0{\cdot}460.10^5$

$\therefore$ Pressure at LP turbine inlet $p_{045} = 7{\cdot}25 \times 0{\cdot}46.10^5 = \underline{333}$ kPa

The pressure ratio across turbine is lower than that across compressor (for same enthalpy change) because the temperature change is proportional to the inlet temperature. By raising the inlet temperature to the turbine more work is produced for the same pressure ratio, or the same work for a smaller pressure ratio. Reducing the compressor inlet temperature leads to less work being required for same pressure ratio.

Solution to Exercise 5.2

Assume bypass ratio $bpr = 6$ and pressure ratio across fan (core and bypass) = 1.6

From Ex 5.1, temp rise across fan ΔT_{0f} = 41·44 K. Assume this temperature rise is the same for core and bypass streams.

If ΔT_{0LPT} = temp drop in LP turbine, and taking c_p equal for air and combustion products, the power balance for LP shaft is

$$(bpr+1)\dot{m}_{core} c_p \Delta T_{0f} = \dot{m}_{core} c_p \Delta T_{0LPT}$$

$$\therefore \qquad (6+1)41\cdot 44 = \Delta T_{0LPT}$$

i.e. LP turbine temperature drop $\Delta T_{0LPT} = \underline{290{\cdot}0}$ K

Solution to Exercise 5.2 (cont.)

Temp and pressure into LP turbine (from Ex 5·1) $T_{045} = 945{\cdot}7$ K, $p_{045} = 333$ kPa

Temperature out of LP turbine $T_{05} = T_{045} - \Delta T_{0LPT} = 945\cdot 7 - 290 = \underline{655{\cdot}7}$ K

$$\Delta T_{0LPT} = 290\cdot 0 = 945\cdot 7\,\eta_{LPT}\left(1 - \left(\frac{1}{r_{LPT}}\right)^{\gamma-1/\gamma}\right)$$

Taking $\eta_{LPT} = 0\cdot 9$, obtain pressure ratio across LP Turbine $r_{LPT} = \underline{4{\cdot}30}$

From Ex 5.1 $p_{045} = 333$ kPa

$\therefore$ Pressure out of LP turbine $p_{05} = \dfrac{p_{045}}{4\cdot 30} = \dfrac{333}{4\cdot 30} = \underline{77{\cdot}5}$ kPa

Solution to Exercise 5.3

5.3 $T_{02} = 288\ \text{K}$

$$\frac{T_{03}}{T_{02}} = 1 + \frac{r^{(\gamma-1)/\gamma} - 1}{\eta_{comp}}$$

For $\eta_c = 0 \cdot 90$ cooling air temperature $T_c = T_{03} = T_{02} + T_{02}\dfrac{40^{\frac{\gamma-1}{\gamma}} - 1}{0 \cdot 9} = 886\ \text{K}$

Metal temperature $T_m = T_4 - \varepsilon(T_4 - T_c) = 1600 - 0 \cdot 65(1600 - 886)$

$= \underline{1136\ \text{K}}$

For $\eta_c = 0 \cdot 85$ Cooling air Temperautre $T_c = T_{03} = T_{02} + T_{02}\dfrac{40^{\frac{\gamma-1}{\gamma}} - 1}{0 \cdot 85} = 922\ \text{K}$

Hence $T_m = \underline{1159\ \text{K}}$

Metal temperature increased by ΔT_m = 1159 - 1136 = 23 K

If creep life is halved by 10 K rise in T_m, creep life reduced by fall in compressor

efficiency to about $0 \cdot 5^{\frac{23}{10}} = \underline{0 \cdot 2}$ of former value

Solution to Exercise 5.3 (cont.)

If turbine metal temperature metal temperature held at $T_m = 1136\,\mathrm{K}$

while for lower compressor efficiency cooling air temperature $T_c = 922\,\mathrm{K}$

As before, $\varepsilon = 0\cdot 65 = (T_g - T_m)/(T_g - T_c)$

$\therefore$ $T_g = (1136 - 0\cdot 65 \times 922)/0\cdot 35 = 1533\,\mathrm{K}$

i.e. Turbine inlet temperature falls by 67K, as a result of a rise in compressor outlet temperature of 36K

Note: Assuming the effectiveness stays constant, the cooling air temperature has a greater effect on metal temperature than the temperature of the gas from the combustor into the turbine. For an engine a decrease in compressor efficiency would reduce net power from the core and reducing the turbine entry temperature to restore the turbine metal temperature would reduce the power further.