Homework problems 47-48
Due in class, Friday, 18 December 2020

47. The steel beam has an allowable bending stress σ_{allow} = 140 MPa and an allowable shear stress of τ_{allow} = 90 MPa. Determine the maximum load that can safely be supported.

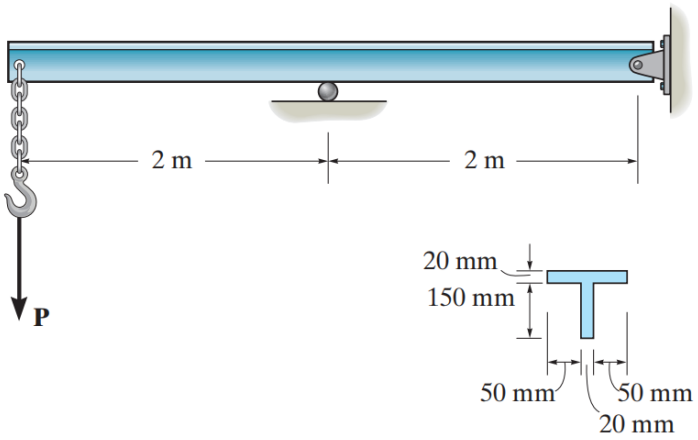
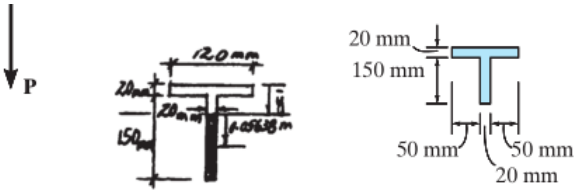


Figure 47



SOLUTION

Section Properties:

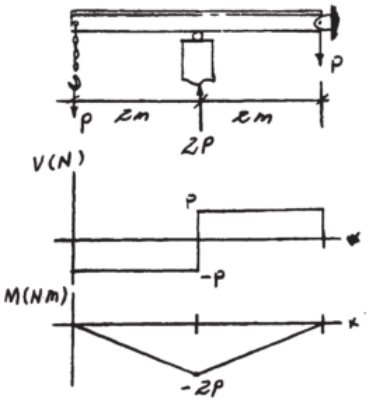
$$\bar{y} = \frac{(10)(120)(20) + (95)(150)(20)}{120(20) + 150(20)} = 57.22 \text{ mm}$$

$$Q_{max} = \bar{y}'A' = (0.05638)(0.02)(0.170 - 0.05722) = 0.127168(10^{-3}) \text{ m}^3$$

$$I = \frac{1}{12}(0.12)(0.02^3) + 0.12(0.02)(0.05722 - 0.01)^2 +$$

$$\frac{1}{12}(0.02)(0.15^3) + 0.15(0.02)(0.095 - 0.05722)^2 = 15.3383(10^{-6}) \text{ m}^4$$

$$S = \frac{I}{c} = \frac{15.3383(10^{-6})}{(0.170 - 0.05722)} = 0.136005(10^{-3}) \text{ m}^3$$



For Moment:

$$M = \sigma_{allow} S$$

$$2P = 140(10^6)(0.136005)(10^{-3})$$

$$P = 9520 \text{ N} = 9.52 \text{ kN} \quad \textbf{(Controls)} \qquad \textbf{Ans.}$$

For Shear:

$$V \overset{?}{=} \tau_{allow}\left(\frac{It}{Q_{max}}\right)$$

$$P = 90(10^6)\left(\frac{15.3383(10^{-6})(0.02)}{0.127168(10^{-3})}\right) = 217106 = 217 \text{ kN}$$

48. Determine the variation in the width b as a function of x for the cantilevered beam that supports a uniform distributed load along its centerline so that it has the same maximum bending stress σ_{allow} throughout its length. The beam has a constant depth t.

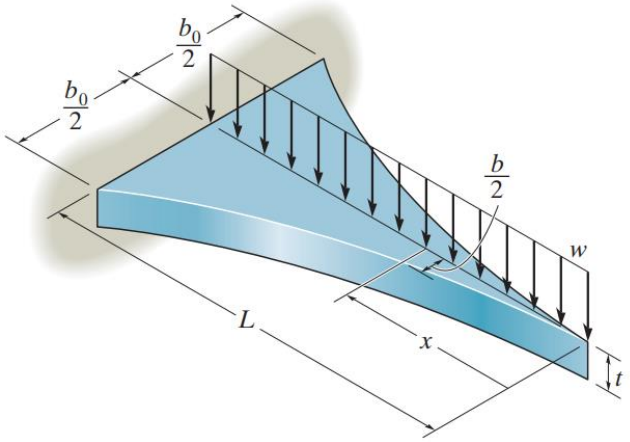


Figure 48

SOLUTION

Section Properties:

$$I = \frac{1}{12} b\, t^3 \qquad S = \frac{I}{c} = \frac{\frac{1}{12} b\, t^3}{\frac{t}{2}} = \frac{t^2}{6} b$$

Bending Stress:

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{\frac{wx^2}{2}}{\frac{t^2}{6}b} = \frac{3wx^2}{t^2 b} \qquad \textbf{(1)}$$

At $x = L, b = b_0$

$$\sigma_{\text{allow}} = \frac{3wL^2}{t^2 b_0} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) yields:

$$\frac{3wx^2}{t^2 b} = \frac{3wL^2}{t^2 b_0}$$

$$b = \frac{b_0}{L^2} x^2 \qquad \textbf{Ans.}$$