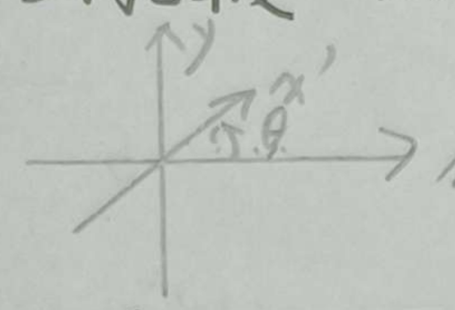
航空结构强度 HW2 12012127 邹佳驹

2.6

解:



counterclockwise 30°

$\theta = 30^\circ$

$$[\sigma'] = \begin{bmatrix} \cos30^\circ & \sin30^\circ \\ -\sin30^\circ & \cos30^\circ \end{bmatrix} \begin{bmatrix} 10 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} \cos30^\circ & -\sin30^\circ \\ \sin30^\circ & \cos30^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt3}{2} & \frac12 \\ -\frac12 & \frac{\sqrt3}{2} \end{bmatrix} \begin{bmatrix} 10 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} \frac{\sqrt3}{2} & -\frac12 \\ \frac12 & \frac{\sqrt3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 5\sqrt3+\frac32 & \frac32\sqrt3+1 \\ -5+\frac32\sqrt3 & -\frac32+\sqrt3 \end{bmatrix} \begin{bmatrix} \frac{\sqrt3}{2} & -\frac12 \\ \frac12 & \frac{\sqrt3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 10.598 & -1.964 \\ -1.964 & 1.4019 \end{bmatrix}$$

2.8

解:

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} x^2y & (a^2-y^2)x & 0 \\ (a^2-y^2)x & (y^3-3a^2y)/3 & 0 \\ 0 & 0 & 2az^2 \end{bmatrix}$$

$$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{yx}}{\partial y} + \frac{\partial\sigma_{zx}}{\partial z} + f_x = 0$$

$$\Rightarrow f_x = (-1)\,(2x\cdot y + (-2y)x + 0) = \underset{\sim}{0}$$

$$\frac{\partial\sigma_{xy}}{\partial x} + \frac{\sigma_{yy}}{\partial y} + \frac{\sigma_{zy}}{\partial z} + f_y = 0$$

$$\Rightarrow f_y = (-1)\left[(a^2-y^2) + \frac13(3y^2-3a^2)\right] = \underset{\sim}{0}$$

$$\frac{\partial\sigma_{xz}}{\partial x} + \frac{\partial\sigma_{yz}}{\partial y} + \frac{\partial\sigma_{zz}}{\partial z} + f_z = 0$$

$$\Rightarrow f_z = (-1)\,(0 + 0 + 4az) = \underset{\sim}{-4az}$$

$$\therefore \vec f = 0\,\vec i + 0\,\vec j - 4az\,\vec k$$

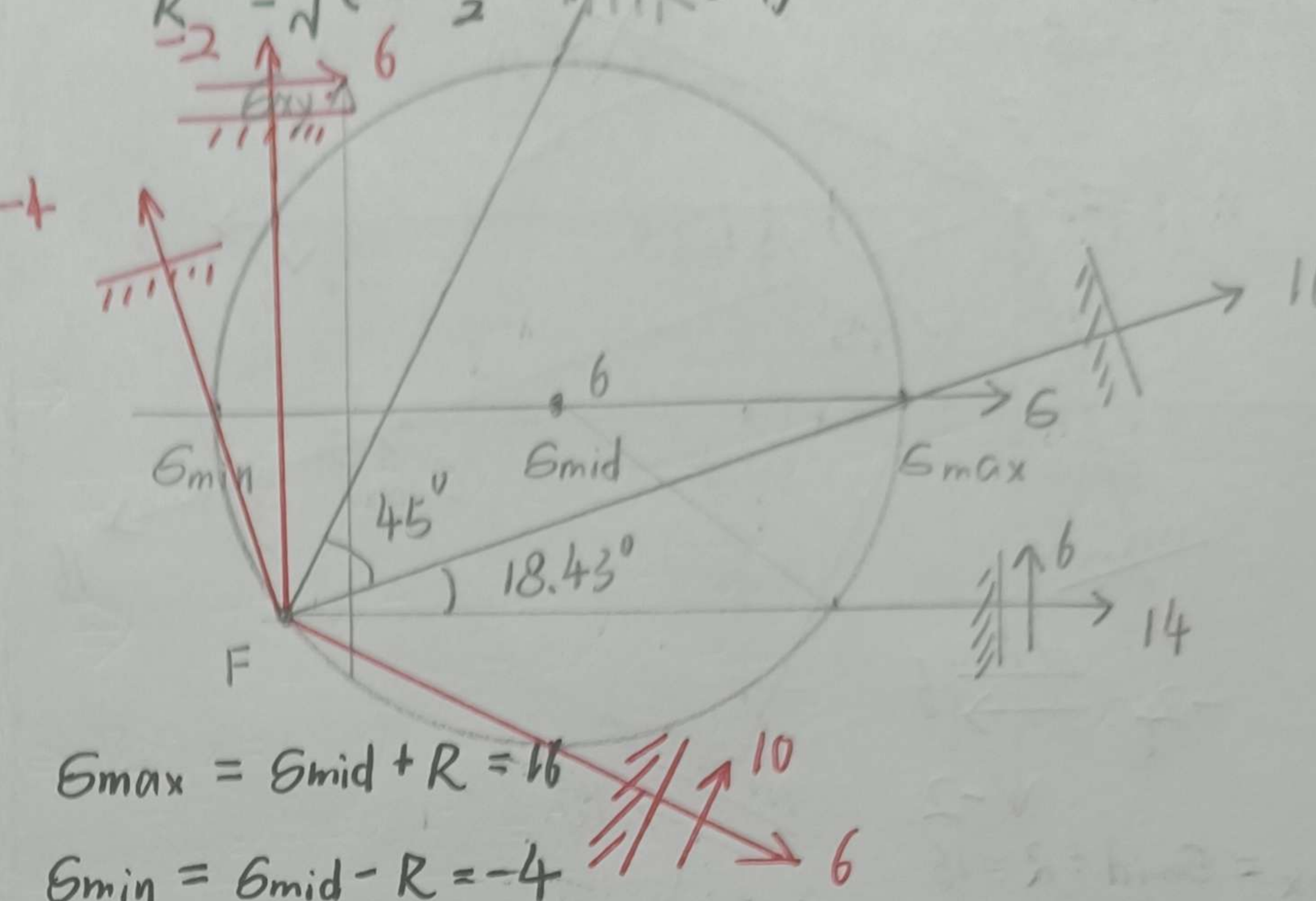
for the equilibrium.

2.10

解: 由图: $\sigma_{xx} = 14$ $\sigma_{yy} = -2$ $\sigma_{xy} = 6$

$$\sigma_{mid} = \frac{\sigma_{xx}+\sigma_{yy}}{2} = 6$$

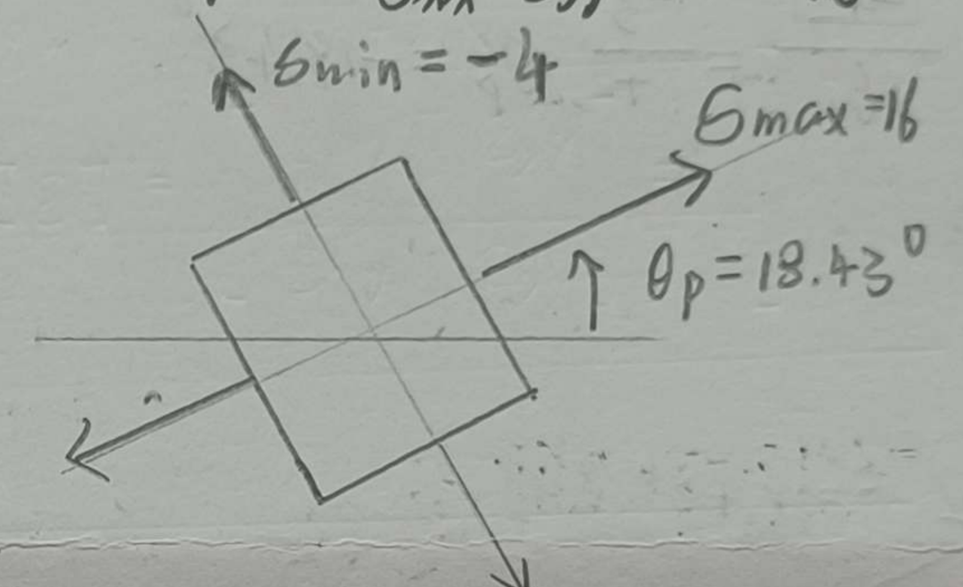
$$R = \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = \sqrt{8^2+6^2} = 10$$



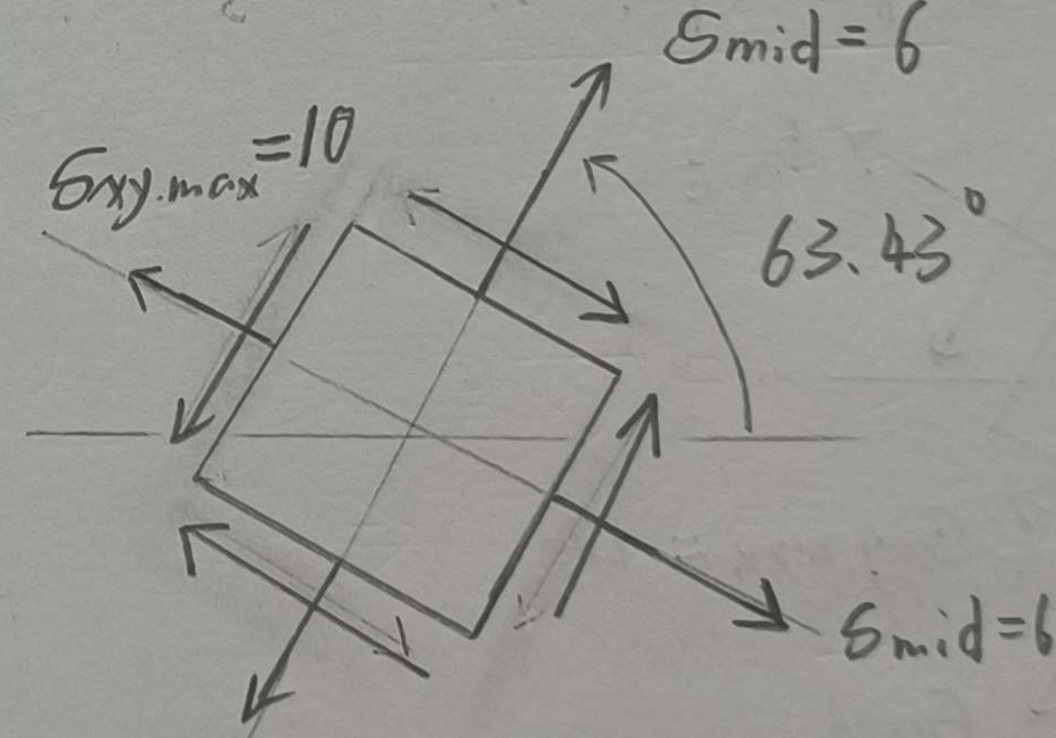
$\sigma_{max} = \sigma_{mid} + R = 16$

$\sigma_{min} = \sigma_{mid} - R = -4$

$$\tan 2\theta_p = \frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}} = \frac{12}{16} = \frac34,\quad \theta_p = 18.43^\circ$$



$\theta_p + 45^\circ = 63.43^\circ$ to get $\sigma_{xy\,max}$ at σ_{mid}

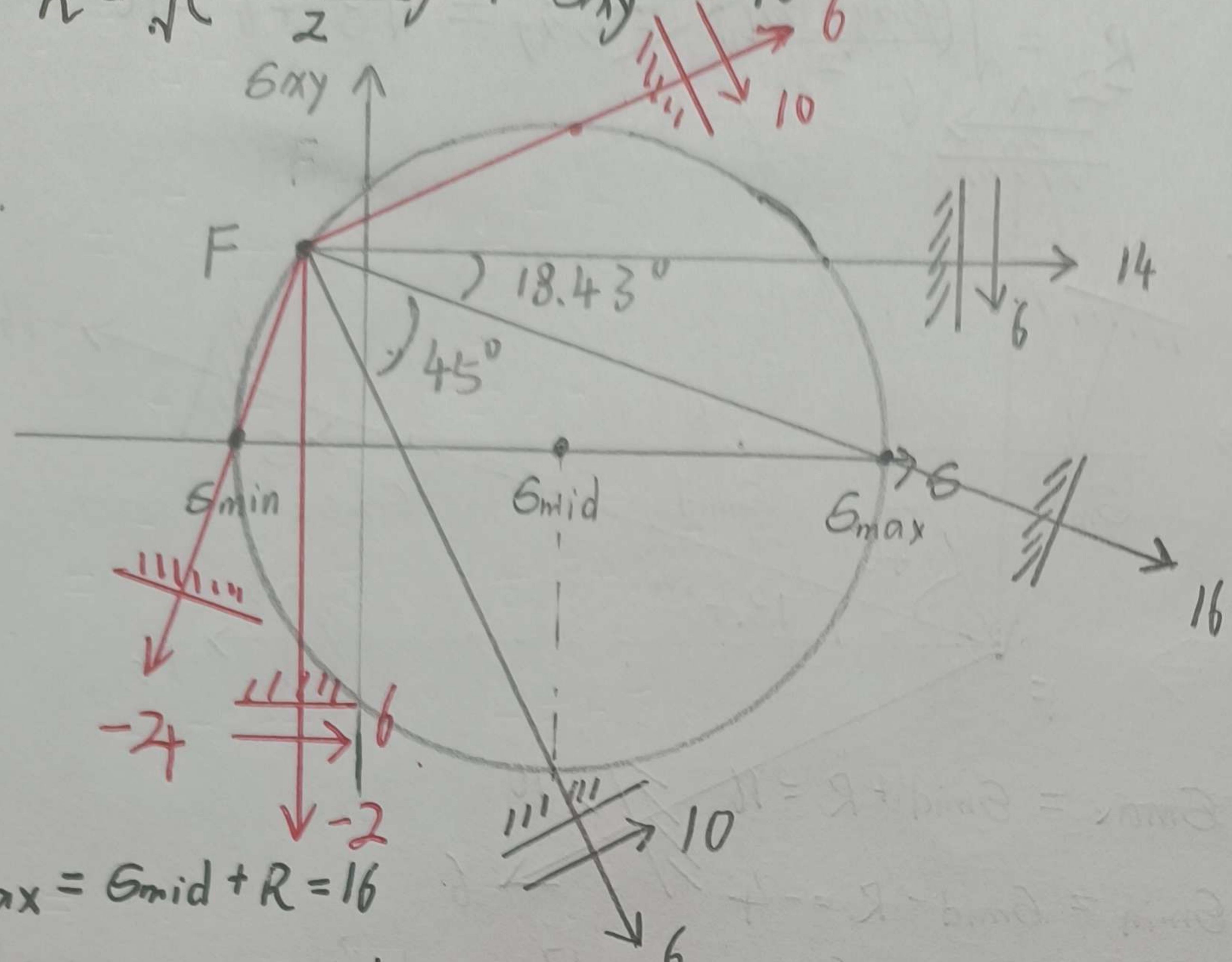


2.11

解：由图：$\varepsilon_{xx}=14$，$\varepsilon_{yy}=-2$，$\varepsilon_{xy}=-6$

$$\varepsilon_{mid}=\frac{\varepsilon_{xx}+\varepsilon_{yy}}{2}=6$$

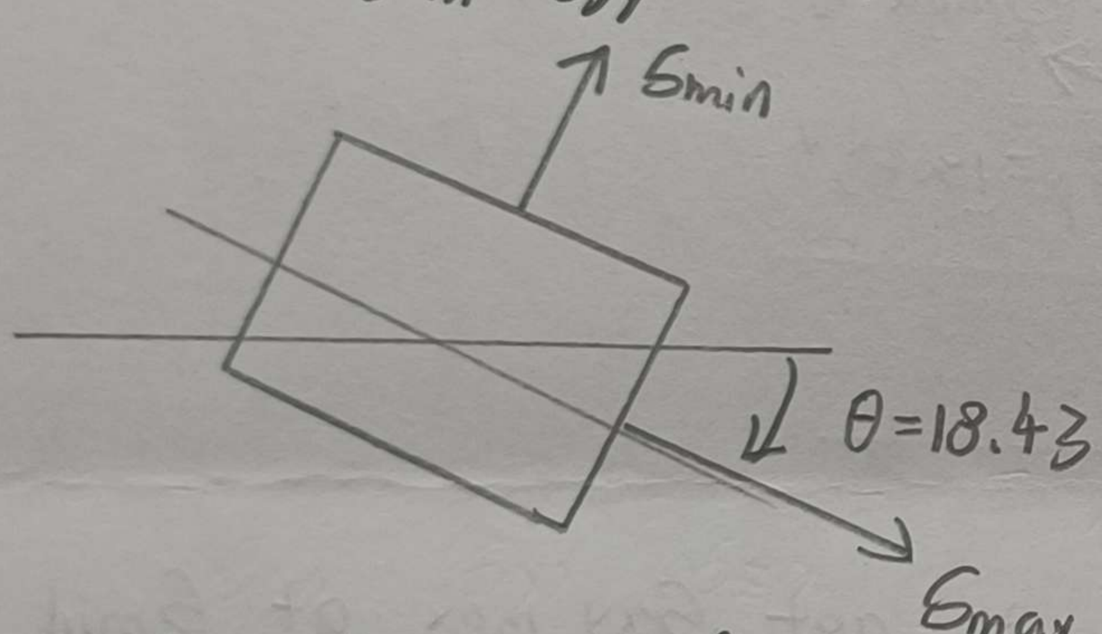
$$R=\sqrt{\left(\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\right)^2+\varepsilon_{xy}^2}=10$$



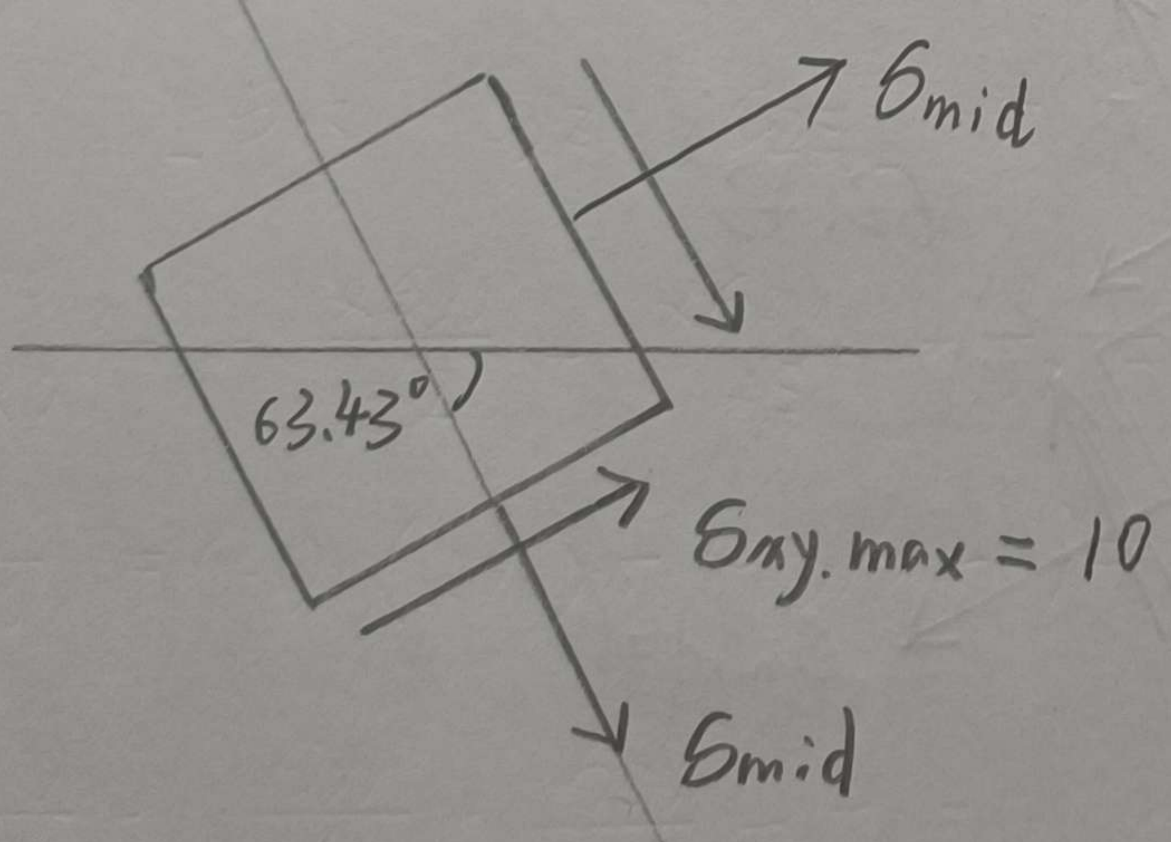
$\varepsilon_{max}=\varepsilon_{mid}+R=16$

$\varepsilon_{min}=\varepsilon_{mid}-R=-4$

$$\tan 2\theta_p=\frac{2\varepsilon_{xy}}{\varepsilon_{xx}-\varepsilon_{yy}}=\frac{-12}{16}=\frac{-3}{4},\ \theta_p=-18.43^\circ$$



$-18.43^\circ-45^\circ=-63.43^\circ$



2.12

解：

$$\varepsilon_{xx}'=c^2\varepsilon_{xx}+s^2\varepsilon_{yy}+2cs\,\varepsilon_{xy}$$

$\theta_1=0^\circ$，$\theta_2=120^\circ$，$\theta_3=240^\circ$ 依次代入计算

$$\varepsilon_1'=\cos^2 0^\circ\varepsilon_{xx}+0+0=\varepsilon_{xx}=3\times10^{-6}$$

$$\varepsilon_2'=\cos^2 120^\circ\varepsilon_{xx}+\sin^2 120^\circ\varepsilon_{yy}+2\cos120^\circ\cdot\sin120^\circ\varepsilon_{xy}=$$

$$\varepsilon_3'=\cos^2 240^\circ\varepsilon_{xx}+\sin^2 240^\circ\varepsilon_{yy}+2\cos240^\circ\sin240^\circ\varepsilon_{xy}=8\times10$$

$$\Rightarrow\quad \varepsilon_{xx}=3\times10^{-6}$$

$$\frac{1}{4}\varepsilon_{xx}+\frac{3}{4}\varepsilon_{yy}+(-1)\frac{\sqrt{3}}{2}\varepsilon_{xy}=5\times10^{-6}$$

$$\frac{1}{4}\varepsilon_{xx}+\frac{3}{4}\varepsilon_{yy}+\frac{\sqrt{3}}{2}\varepsilon_{xy}=8\times10^{-6}$$

$$\Rightarrow\quad \varepsilon_{xx}=3\times10^{-6}$$

$$\varepsilon_{yy}=\frac{23}{3}\times10^{-6}=7.67\times10^{-6}$$

$$\varepsilon_{xy}=\sqrt{3}\times10^{-6}=1.73\times10^{-6}$$

strain matrix

$$[\varepsilon]=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{yx}\\ \varepsilon_{xy} & \varepsilon_{yy}\end{bmatrix}=\begin{bmatrix}3\times10^{-6} & 1.73\times10^{-6}\\ 1.73\times10^{-6} & 7.67\times10^{-6}\end{bmatrix}$$

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2.13

解:

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2}+\frac{\partial^2 \varepsilon_{yy}}{\partial x^2}=\frac{\partial^2}{\partial x \partial y}\gamma_{xy}$$

$$A\cdot 2 + A\cdot 2 = C$$

$$\therefore C = 4A \quad \boxed{\text{ANS}}$$

$$\left.\begin{array}{l}\varepsilon_{xx}=\frac{\partial u}{\partial x}=Ay^2\\ \varepsilon_{yy}=\frac{\partial v}{\partial y}=Ax^2\end{array}\right\}\Rightarrow\left\{\begin{array}{l}u=Ay^2\cdot x+C_1(y)\\ v=Ax^2\cdot y+C_2(x)\end{array}\right. \quad (*).$$

$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=Cxy$$

$$A\cdot x\cdot 2y+C_1'(y)+Ay\cdot 2x+C_2'(x)=Cxy$$
$$=4Axy$$

$$C_1'(y)+C_2'(x)=0$$

$$\therefore\left\{\begin{array}{l}u=Axy^2+C_1(y)\\ v=Ayx^2+C_2(x)\end{array}\right. \quad \boxed{\text{ANS}}$$

其中 $C_1'(y)+C_2'(x)=0$

$C_1(y)=my+n$ (m,n,p为 const).

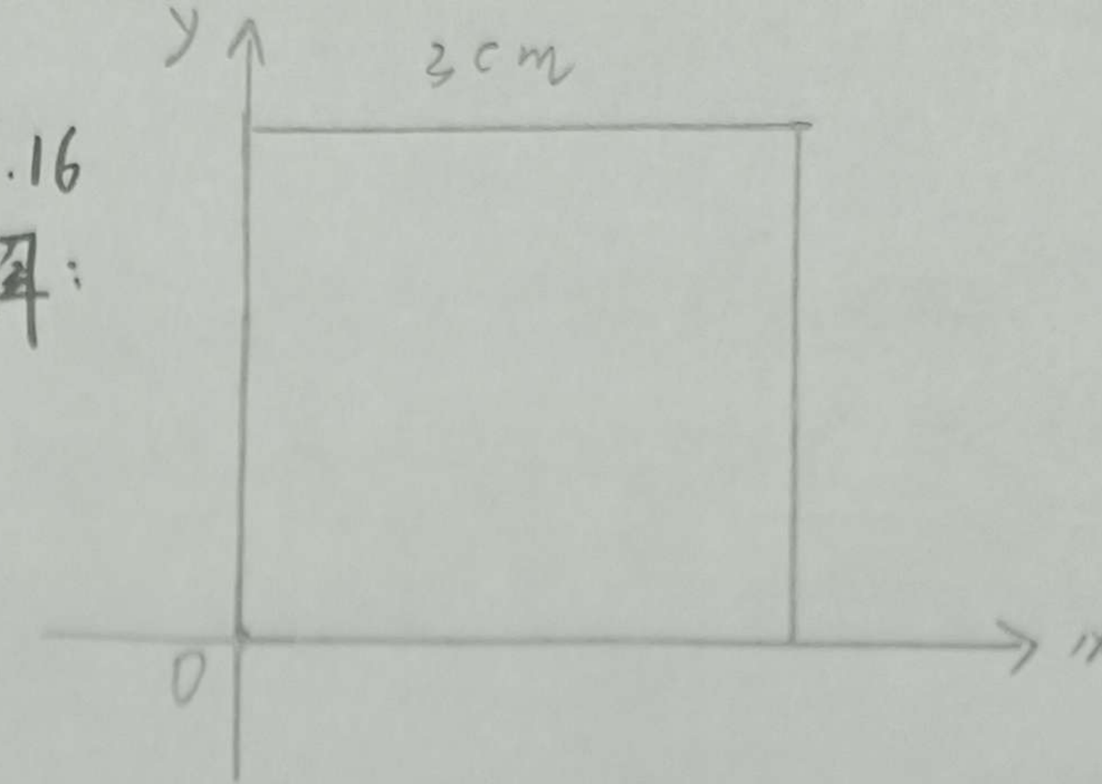
$C_2(x)=-mx+p$

particular solution: $u=Axy^2+my+n$

$v=Ayx^2+p-mx$

2.16

解:



$$\varepsilon_{xx}=\frac{\partial u}{\partial x}=3\times10^{-6},\ u=3\times10^{-6}x+C_1(y) \quad \text{cm}$$

$$\varepsilon_{yy}=\frac{\partial v}{\partial y}=5\times10^{-6},\ v=5\times10^{-6}y+C_2(x) \quad \text{cm}$$

$$\varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=-4\times10^{-6}$$

$$\Rightarrow C_1'(y)+C_2'(x)=-8\times10^{-6}$$

且 $C_1(0)=C_2(0)=0$ Since $u(0)=v(0)=0$

~~set $C_1(y)=Ay$~~

~~$C_2(x)=Bx$~~

~~$A+B=-8\times10^{-6}$~~

~~set $A=B=-4\times10^{-6}$~~

~~$u=3\times10^{-6}x-4\times10^{-6}y$ cm~~

~~$v=5\times10^{-6}y-4\times10^{-6}x$ cm~~

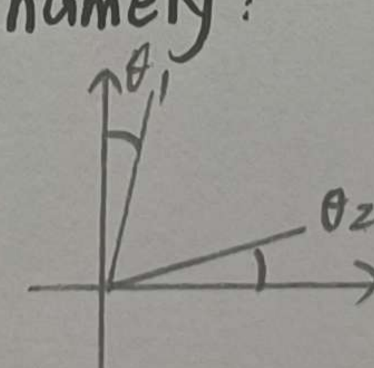
~~At (2, 1), $u=2\times10^{-6}$ cm~~ $\boxed{\text{ANS}}$

~~$v=-3\times10^{-6}$ cm~~

改:

the edge $x=0$ has no rotation

namely:



$$\theta_1=\frac{\partial u}{\partial y}=0$$

$$\Rightarrow C_1'(y)=0,\ C_1(y)=0$$

$$C_2'(x)=-8\times10^{-6}$$

$$C_2(x)=-8\times10^{-6}x$$

$$\therefore u=3\times10^{-6}x \quad \text{cm}$$

$$v=5\times10^{-6}y-8\times10^{-6}x \quad \text{cm}$$

At (2, 1), $u=6\times10^{-6}$ cm

$v=-11\times10^{-6}$ cm