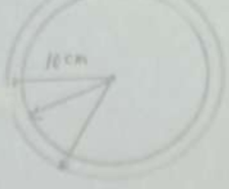
航空结构强度 HW 3

4.1

解: $r_m = 10cm$

$t = 2mm = 0.2cm$




exact value of J

$$J = \frac{\pi}{2}(c_o^4 - c_i^4)$$

$$= \frac{\pi}{2}\left[(10+0.1)^4 - (10-0.1)^4\right]$$

$$= \frac{10001}{25}\pi \quad cm^4$$

$$\tau_{max} = \frac{Tc_o}{J} = \frac{T\cdot(10+0.1)\times10^{-2}}{\frac{10001\pi}{25}\times(10^{-2})^4}$$

$$= 8036.5\ T$$

$$\alpha = \frac{\theta}{L} = \frac{T}{GJ} = \frac{T}{G\cdot\frac{10001\pi}{25}\times10^{-8}}$$

$$= 79569.5\frac{T}{G}$$

thin-wall approximation

$$J = 2\pi r^3 t = 2\pi\times(0.1)^3\times(2\times10^{-3})$$

$$= 1.2566\times10^{-5} \quad m^4$$

$$\tau = \frac{T}{2A_m t} = \frac{T}{2\times(\pi\times0.1^2)\times2\times10^{-3}} = 7957.7\ T$$

$$\alpha = \frac{T}{GJ} = 79579.8\frac{T}{G}$$

$$error_\tau = \frac{8036.5-7957.7}{8036.5} = 0.981\%$$

$$error_\alpha = \frac{79569.5-79579.8}{79569.5} = -0.0129\%$$

4.2

解:


$2\pi r_m = a$

$r_m = \frac{a}{2\pi}$

$r_o = \frac{a}{2\pi} + \frac{t}{2}$

$r_i = \frac{a}{2\pi} - \frac{t}{2}$

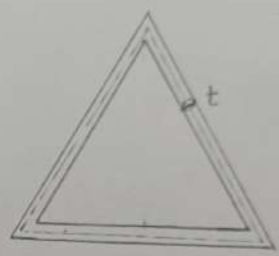
$$\tau = \frac{T}{2\pi r_m^2 t} = \frac{T}{2\pi\cdot\frac{a^2}{4\pi^2}\cdot t} = \frac{2\pi}{a^2 t}\cdot T$$

Torsional stiffness (TS) $= G\cdot J$

$$= G\cdot2\pi\cdot r_m^3\cdot t$$

$$= G\cdot2\pi\cdot\left(\frac{a}{2\pi}\right)^3\cdot t$$

$$= G\cdot\frac{a^3 t}{4\pi^2}$$




$3a_m = a$

$a_m = \frac{a}{3}$

$A_m = \frac{\sqrt{3}}{4}a_m^2 = \frac{\sqrt{3}a^2}{36}$

$$\tau = \frac{T}{2A_m t} = \frac{T}{2\cdot(\frac{\sqrt{3}}{4}a_m^2)\cdot t} = \frac{T}{\frac{\sqrt{3}}{18}a^2\cdot t} = \frac{6\sqrt{3}\,T}{a^2 t}$$

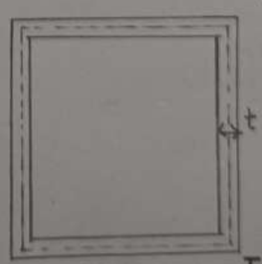
$TS = G\cdot J$

$$\alpha = \frac{\theta}{L} = \frac{T}{GJ} = \frac{T}{4GA_m^2}\oint\frac{ds}{t}$$

$$= \frac{T}{4G\cdot\frac{3a^4}{36^2}}\cdot\frac{a}{t} = \frac{108T}{Gta^3}$$

$$\therefore J = \frac{ta^3}{108}$$

$$\therefore TS = G\cdot\frac{ta^3}{108}$$




$4a_m = a$

$a_m = \frac{a}{4}$

$A_m = a_m^2 = \frac{a^2}{16}$

$$\tau = \frac{T}{2A_m t} = \frac{T}{2\times\frac{a^2}{16}\times t} = \frac{8T}{a^2 t}$$

$$\alpha = \frac{T}{GJ} = \frac{T}{4GA_m^2}\oint\frac{ds}{t} = \frac{T}{4G\cdot\frac{a^4}{16^2}}\cdot\frac{a}{t} = \frac{64T}{Gta^3}$$

$$\therefore J = \frac{ta^3}{64}$$

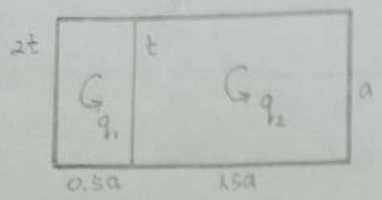
$$\therefore TS = G\cdot\frac{ta^3}{64}$$

航空结构强度

4.3

解:

Torsional stiffness : GJ




$$T = 2A_1 q_1 + 2A_2 q_2$$

$$2GA_1\alpha = q_1 \cdot \frac{2a}{2t} + (q_1 - q_2)\cdot\frac{a}{t}$$

$$2GA_2\alpha = q_2 \cdot \frac{4a}{2t} + (q_2 - q_1)\cdot\frac{a}{t}$$

$$A_1 = 0.5a^2$$

$$A_2 = 1.5a^2$$

$$\Rightarrow \quad \alpha = \frac{5T}{27Gta^3}$$

$$q_1 = \frac{2T}{9a^2} = \frac{6}{5}Gat\alpha$$

$$q_2 = \frac{7}{27}\frac{T}{a^2} = \frac{7}{5}Gat\alpha$$

$$\therefore GJ = \frac{T}{\alpha} = \frac{27}{5}Gta^3$$

4.4

解:

$$\tau = \frac{q}{t}$$

$$\tau_1 = \frac{q_1}{2t} = \frac{3}{5}Ga\alpha$$

$$\tau_2 = \frac{q_2}{t} = \frac{7}{5}Ga\alpha$$

$$\tau_3 = \frac{q_1 - q_2}{t} = -\frac{1}{5}Ga\alpha$$

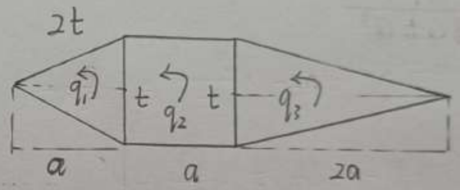
$$\tau_{max} = \tau_2 = \frac{7}{5}Ga\alpha$$

$$= \frac{7}{5}\times 20\times 10^9 \times 0.1 \times \frac{5}{180}\times\pi$$

$$= 244.34\ MPa$$

4.6

解:




$$T = 2A_1q_1 + 2A_2q_2 + 2A_3q_3$$

$$2GA_1\alpha = q_1\cdot\frac{2\times\frac{\sqrt{5}}{2}a}{2t} + (q_1 - q_2)\cdot\frac{a}{t}$$

$$2GA_2\alpha = q_2\cdot\frac{2a}{2t} + (q_2 - q_1)\cdot\frac{a}{t} + (q_2 - q_3)\cdot\frac{a}{t}$$

$$2GA_3\alpha = q_3\cdot\frac{2\times\frac{\sqrt{17}}{2}a}{2t} + (q_3 - q_2)\cdot\frac{a}{t}$$

$$A_1 = \frac{1}{2}a^2, \quad A_2 = a^2, \quad A_3 = a^2$$

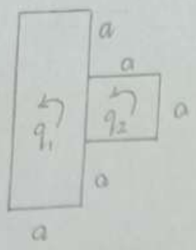
$$\Rightarrow \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \begin{Bmatrix} 1.1424 \\ 1.4198 \\ 1.1170 \end{Bmatrix} taG\alpha$$

$$\alpha = \frac{T}{6.216\ ta^3G}, \quad \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \begin{Bmatrix} 0.1838 \\ 0.2284 \\ 0.1797 \end{Bmatrix}\frac{T}{a^2}$$

$$GJ = \frac{T}{\alpha} = 6.216\ ta^3G$$

4.7

解：




$$T = 2A_1 q_1 + 2A_2 q_2$$

$$2GA_1\alpha = q_1 \cdot \frac{7a}{t} + (q_1 - q_2)\cdot\frac{a}{t}$$

$$2GA_2\alpha = q_2 \cdot \frac{3a}{t} + (q_2 - q_1)\cdot\frac{a}{t}$$

$$A_1 = 3a^2,\quad A_2 = a^2$$

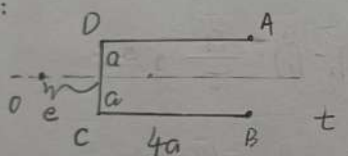
$$\Rightarrow\quad q_1 = \frac{26}{31}aG\alpha t = \frac{13T}{48a^2}$$

$$q_2 = \frac{22}{31}aG\alpha t = \frac{11T}{48a^2}$$

$$\alpha = \frac{31T}{96Gta^3}$$

4.9

解：




$$w_A - w_B = -2\alpha A_S.$$

$$A_S = \int_0^s \frac{p\,ds}{2}$$

$$A_{AD} = \int_0^{4a}\frac{a\,ds}{2} = \frac{a}{2}\cdot 4a = 2a^2 = A_{S_{BC}}$$

$$A_{CD} = \int_0^{2a}\frac{e\,ds}{2} = a\cdot e$$

$$\therefore\ w_A - w_B = -2\alpha(2a^2 + 2a^2 + ae)$$

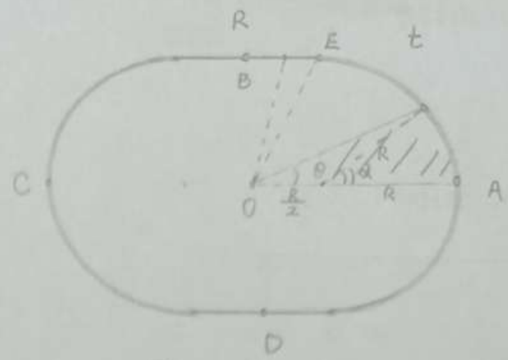
$$= -2\alpha(4a^2 + ae)$$

$$\alpha = \frac{T}{GJ} = \frac{T}{G\cdot\frac{10at^3}{3}} = \frac{3T}{10Gat^3}$$

$$\therefore\ w_A - w_B = \frac{-3T}{5Gt^3}(4a+e)$$

4.12

解：




$$w(s) - w_0 = \frac{T}{2A^2G}(A\delta_s - A_s\delta)$$

$$A = \pi R^2 + 2R^2$$

$$\delta_s = \int_0^s \frac{ds}{t},\quad A_s = \int_0^s \frac{p\,ds}{2}$$

from A to E:

$$\delta_s = \frac{R\cdot\alpha}{t}$$

$$A_s = \frac{1}{2}\alpha R^2 + \frac{R}{2}\cdot\frac{1}{2}\cdot R\sin\alpha$$

where: $\sin(\alpha-\theta) = \frac{1}{2}\sin\theta$.

$$A\delta_s - A_s\delta = \frac{AR}{t}\alpha - \left(\frac{1}{2}\alpha R^2 + \frac{R^2}{4}\sin\alpha\right)\cdot\delta$$

$$= \frac{R\alpha}{t}\cdot(\pi R^2 + 2R^2) - \left(\frac{1}{2}\alpha R^2 + \frac{R^2}{4}\sin\alpha\right)\cdot\frac{2\pi R + 2R}{t}$$

$$= \frac{R^3}{t}\left[\alpha - \frac{1}{2}\sin\alpha(\pi+1)\right]$$

$$f(\alpha) = \alpha - \frac{1}{2}\sin\alpha(\pi+1)$$

$$f'(\alpha) = 1 - \frac{1}{2}\cos\alpha(\pi+1) = 0;\quad \cos\alpha = \frac{2}{\pi+1}$$

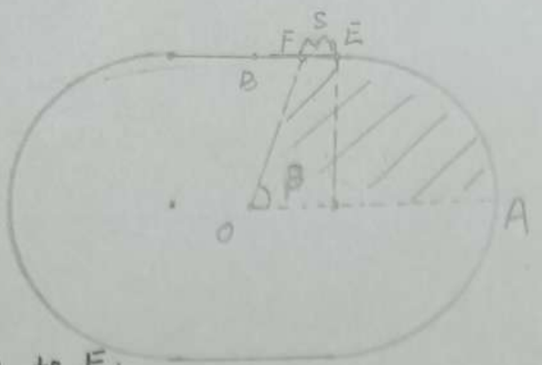
$$\alpha = 1.0668\ \text{rad}$$

$$= 61.12^\circ$$

$$A\delta_s - A_s\delta = \frac{R^3}{t}\cdot(-0.7465)$$

$$w_1 - w_0 = \frac{T}{2A^2G}\cdot\frac{R^3}{t}(-0.7465)$$

$$= -0.37325\,\frac{TR^3}{A^2Gt}$$




from A to F:

$$\delta_s = \frac{\frac{\pi R}{2} + s}{t}$$

$$A_s = \frac{\pi R^2}{4} + \left(s + \frac{R}{2}\right) \cdot R \cdot \frac{1}{2}$$

$$A\delta_s - A_s\delta = (\pi R^2 + 2R^2) \cdot \frac{\frac{\pi R}{2} + s}{t}$$

$$- \left[\frac{\pi R^2}{4} + \frac{R}{2}\left(s + \frac{R}{2}\right)\right] \cdot \frac{2\pi R + 2R}{t}$$

$$= \frac{\pi R^3}{2t} + \frac{s}{t} R^2 - \frac{R^3}{2t}(\pi + 1)$$

if $s = \frac{R}{2}$, $A\delta_s - A_s\delta = 0$

if $s = 0$, $A\delta_s - A_s\delta = \frac{-R^3}{2t}$

$$w(E) - w_0 = \frac{T}{2A^2G} \cdot \frac{-R^3}{2t} < (w_1 - w_0)$$

∴ 在 $\widehat{AE}$ 上，$\alpha = 61.12°$ 时

maximum warp $= -0.37325 \frac{TR^3}{A^2Gt}$