Quiz 7

Date: 2022-03-28 Name: SID:

Q1. Give the general solutions to the following equations and solve the initial value problems. (6pts)

(1) $y'' - y' - 6y = 0, y(0) = 0, y'(0) = 5$;

(2) $y'' - 9y = 0, y(1) = y'(1) = 1$.

Q2. Calculate the Wronskian of $y_1(t) = cos3t$ and $y_2(t) = sin3t$ for equation $y'' + 9y = 0$. Then decide whether y_1 and y_2 are linearly independent and whether they form a fundamental set of solutions. (4pts)

Q1 (1) $y'' - y' - 6y = 0$ $\lambda^2 - \lambda - 6 = 0$

general solution $y = c_1 e^{3t} + c_2 e^{-2t}$

$$\begin{cases} c_1 + c_2 = 0 \\ 3c_1 - 2c_2 = 5 \end{cases} \Rightarrow \begin{cases} c_1 = 1 \\ c_2 = -1 \end{cases}$$

$$y = e^{3t} - e^{-2t}$$

(2) $y'' - 9y = 0$ $\lambda = \pm 3$ $y = c_1 e^{3t} + c_2 e^{-3t}$

$$\begin{cases} c_1 e^3 + c_2 e^{-3} = 1 \\ 3c_1 e^3 - 3c_2 e^{-3} = 1 \end{cases} \Rightarrow \begin{cases} c_1 = \frac{2}{3} e^{-3} \\ c_2 = \frac{1}{3} e^3 \end{cases}$$

$$y = \frac{2}{3} e^{3t-3} + \frac{1}{3} e^{3-3t}$$

Q2. $W[\cos 3t, \sin 3t] = \begin{vmatrix} \cos 3t & \sin 3t \\ -3\sin 3t & 3\cos 3t \end{vmatrix} = 3 \neq 0$

independent.

form a fundamental set of solutions.