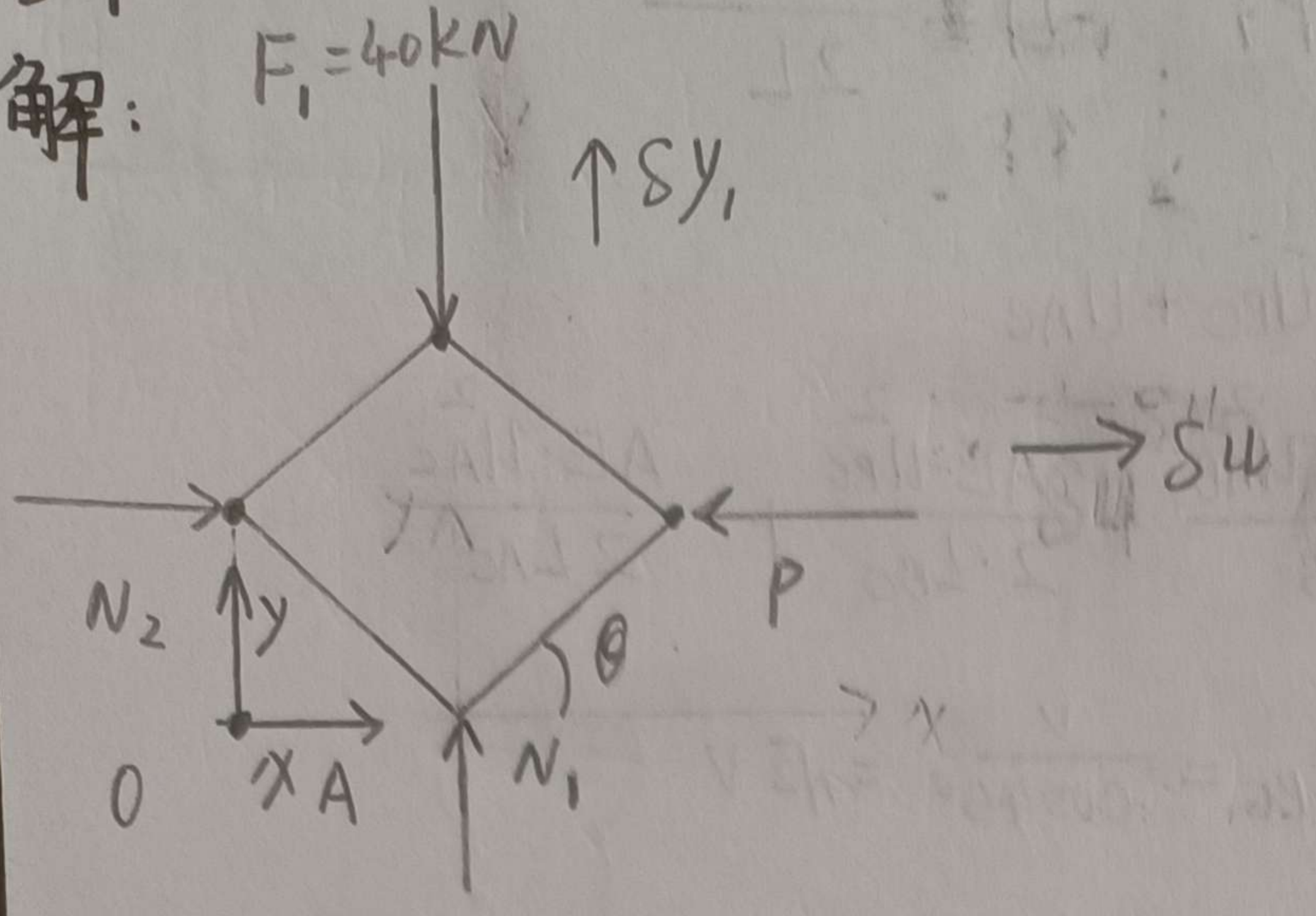
~~航~~空结构强度 HW3

3.1

解:



FBD

Degree of Free = 1, 是为 θ.

$$\delta W = -P\cdot\delta u - F_1\cdot\delta y_1$$

$$u = 2L\cos\theta \Rightarrow \delta u = -L\sin\theta\cdot\delta\theta\cdot 2$$

$$y_1 = L\sin\theta\cdot 2 \Rightarrow \delta y_1 = 2L\cos\theta\cdot\delta\theta$$

$$\therefore \delta W = +2P\cdot L\sin\theta\,\delta\theta - F_1\cdot 2L\cos\theta\,\delta\theta$$

$$= (+2PL\sin\theta - F_1\cdot 2L\cos\theta)\cdot\delta\theta = 0$$

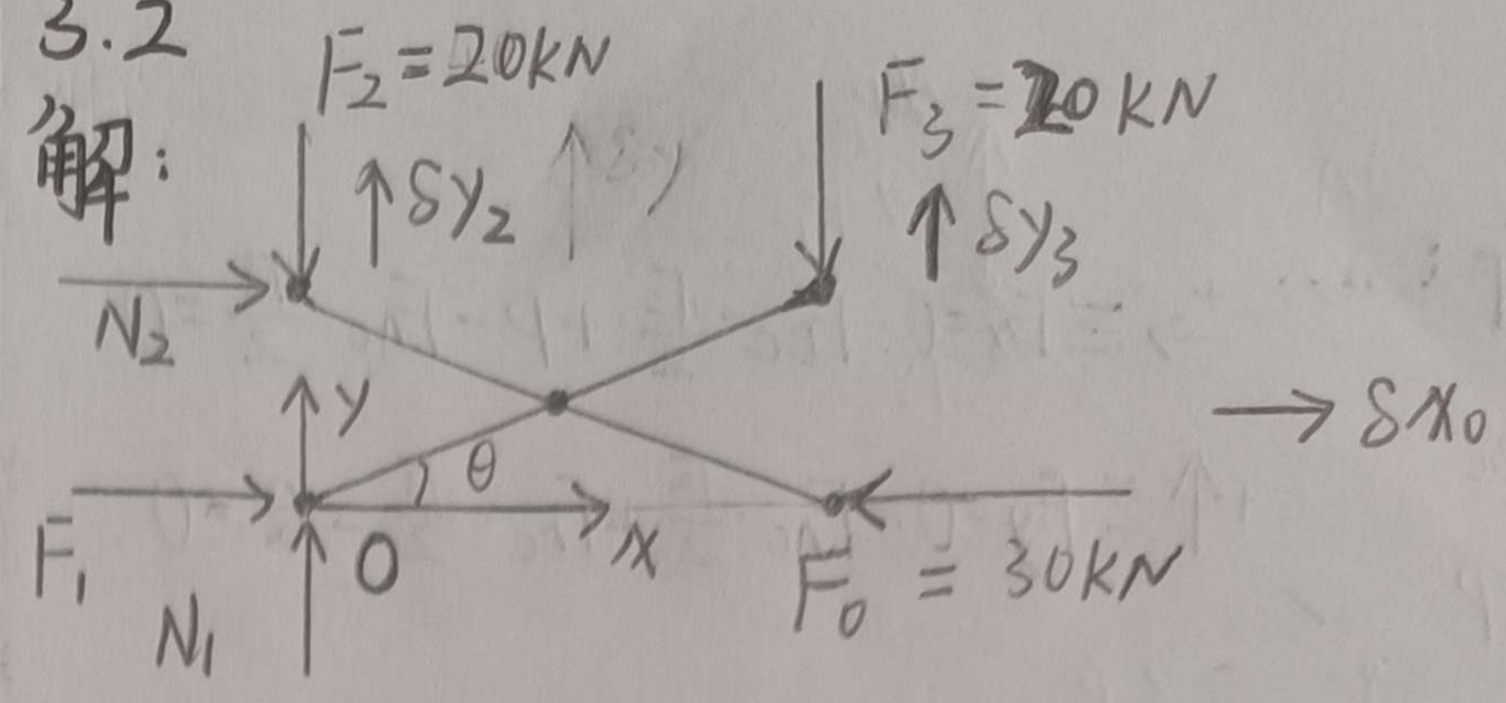
$$\therefore 2PL\sin\theta - F_1\cdot 2L\cos\theta = 0$$

$$P = \frac{2F_1\cdot\cos\theta}{2\sin\theta} = \frac{2\times 40\cos 45^\circ}{2\times\sin 45^\circ}$$

$$= 40\ kN \qquad \boxed{ANS}$$

3.2

解:



FBD

DOF = 1, that's θ.

$$\delta W = -F_0\,\delta x_0 - F_2\,\delta y_2 - F_3\,\delta y_3$$

$$x_0 = L\cos\theta \Rightarrow \delta x_0 = -L\sin\theta\,\delta\theta$$

$$y_2 = y_3 = L\sin\theta \Rightarrow \delta y_2 = \delta y_3 = L\cos\theta\,\delta\theta$$

$$\therefore \delta W = +F_0\cdot L\sin\theta\,\delta\theta - (F_2+F_3)L\cos\theta\cdot\delta\theta$$

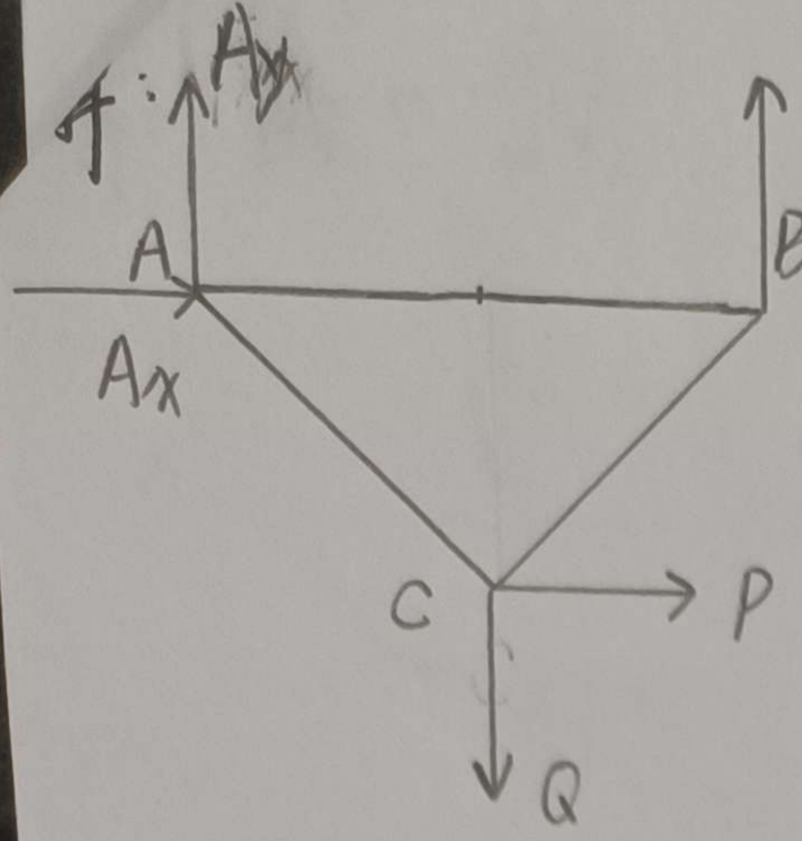
$$= [F_0\cdot L\sin\theta - (F_2+F_3)L\cos\theta]\cdot\delta\theta = 0$$

$$F_0\cdot L\sin\theta = (F_2+F_3)L\cos\theta$$

$$\therefore \tan\theta = \frac{40}{30} = \frac{4}{3}$$

$$\theta = 53.13^\circ$$

解:



$\xrightarrow{+} \sum F_x = 0: A_x + P = 0$

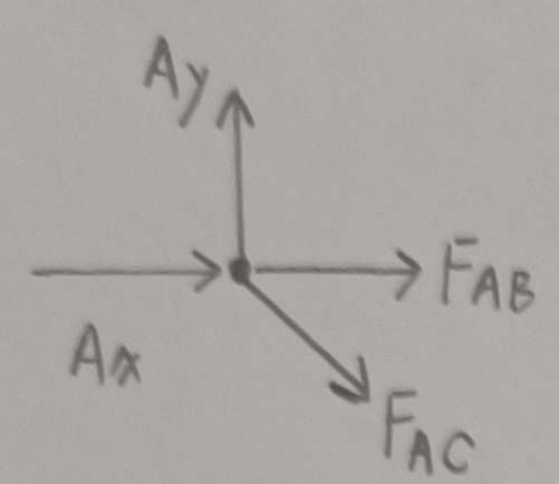
$+\uparrow \sum F_y = 0: A_y + B_y - Q = 0$

$\circlearrowleft \sum M_A = 0: B_y \cdot L + P \cdot \frac{L}{2} - Q \cdot \frac{L}{2} = 0$

$\Rightarrow A_x = -P$, $A_y = \frac{1}{2}(Q+P)$

$B_y = \frac{1}{2}(Q-P)$

node A:



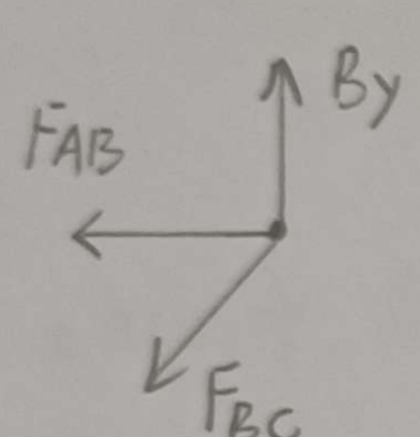
$\xrightarrow{+} \sum F_x = 0: F_{AB} + F_{AC} \cdot \frac{\sqrt{2}}{2} + A_x = 0$

$+\uparrow \sum F_y = 0: A_y - F_{AC} \cdot \frac{\sqrt{2}}{2} = 0$

$\Rightarrow F_{AC} = \frac{\sqrt{2}}{2}(Q+P)$

$F_{AB} = \frac{1}{2}(P-Q)$

node B:



$+\uparrow \sum F_y = 0: B_y - \frac{\sqrt{2}}{2} F_{BC} = 0$

$\Rightarrow F_{BC} = \frac{\sqrt{2}}{2}(Q-P)$

Since $u_i = \frac{\partial U^*}{\partial P_i}$, $U^* = \frac{LF^2}{2AE}$

$$U^*_{total} = U^*_{AB} + U^*_{AC} + U^*_{BC}$$

$$= \frac{L}{4A \cdot E} \cdot \frac{1}{4}(P-Q)^2 + \frac{\frac{\sqrt{2}}{2}L}{2AE} \cdot \frac{1}{2}(Q+P)^2 + \frac{\frac{\sqrt{2}}{2}L}{2AE} \cdot \frac{1}{2}(Q-P)^2$$

$$= \frac{L}{16AE} \cdot \left[(4\sqrt{2}+1)P^2 + (4\sqrt{2}+1)Q^2 - 2PQ\right]$$

$$u = \frac{\partial U^*_{total}}{\partial P} = \frac{L}{16AE} \cdot \left[(8\sqrt{2}+2)P - 2Q\right]$$

$$v = \frac{\partial U^*_{total}}{\partial Q} = \frac{L}{16AE} \cdot \left[(8\sqrt{2}+2)Q - 2P\right]$$

namely, $u = \frac{L}{8AE} \cdot \left[(4\sqrt{2}+1)P - Q\right]$

$v = \frac{L}{8AE} \cdot \left[(4\sqrt{2}+1)Q - P\right]$ $\boxed{\text{ANS}}$

3.4

解: 由3.3得:

$F_{AC} = \frac{\sqrt{2}}{2}(Q+P)$

$F_{AB} = \frac{1}{2}(P-Q)$

$F_{BC} = \frac{\sqrt{2}}{2}(Q-P)$

$\Rightarrow u_{AC} = \frac{\sqrt{2}}{2}(u+v)$

$u_{AB} = \frac{F_{AB} \cdot L}{2A \cdot E} = \frac{(P-Q) \cdot L}{4AE}$

$u_{BC} = \frac{\sqrt{2}}{2}(v-u) + \frac{\sqrt{2}}{2} \cdot \frac{(P-Q) \cdot L}{4AE}$

Since $U = \frac{1}{2}F\Delta = \frac{AE}{2L}\Delta^2$

$$U_{total} = \frac{AE}{\sqrt{2}L} \cdot \frac{1}{2}(u+v)^2 + \frac{2AE}{2L} \cdot \frac{(P-Q)^2 \cdot L^2}{16A^2E^2} + \frac{AE}{\sqrt{2}L} \cdot \left(\frac{\sqrt{2}}{2}(v-u) + \frac{\sqrt{2}}{2}\frac{(P-Q)L}{4AE}\right)^2$$

$$P = \frac{\partial U_{total}}{\partial u} = \frac{AE}{\sqrt{2}L}(u+v) + \frac{\sqrt{2}AE}{L} \cdot \left(\frac{\sqrt{2}}{2}(v-u) + \frac{\sqrt{2}}{2}\frac{(P-Q)L}{4AE}\right)\left(-\frac{\sqrt{2}}{2}\right)$$

$$= \frac{AE}{\sqrt{2}L}(u+v) + \frac{AE}{\sqrt{2}L}\left(u - v + \frac{Q-P}{4AE} \cdot L\right)$$

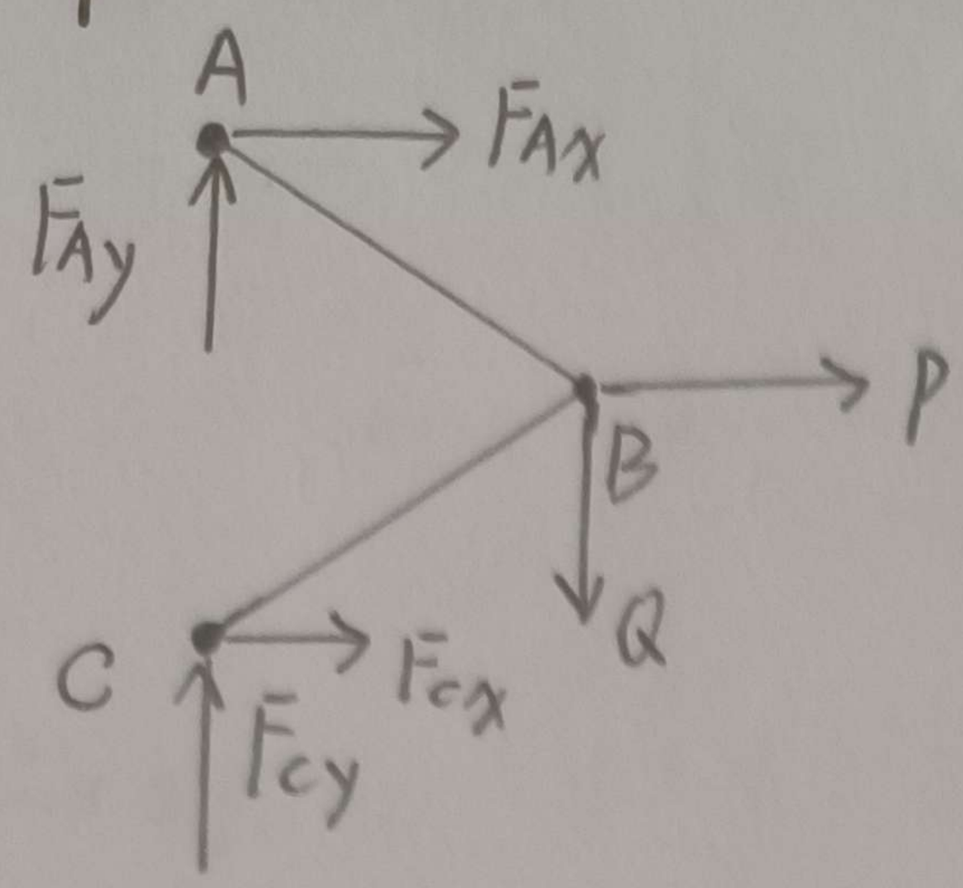
$$= \frac{AE}{L}\sqrt{2}\,u + \frac{Q-P}{4\sqrt{2}}$$

$\Rightarrow u = \frac{L}{8AE}\left[(4\sqrt{2}+1)P - Q\right]$

同理 $Q = \frac{\partial U_{total}}{\partial v} \Rightarrow v = \frac{L}{8AE}\left[(4\sqrt{2}+1)Q - P\right]$

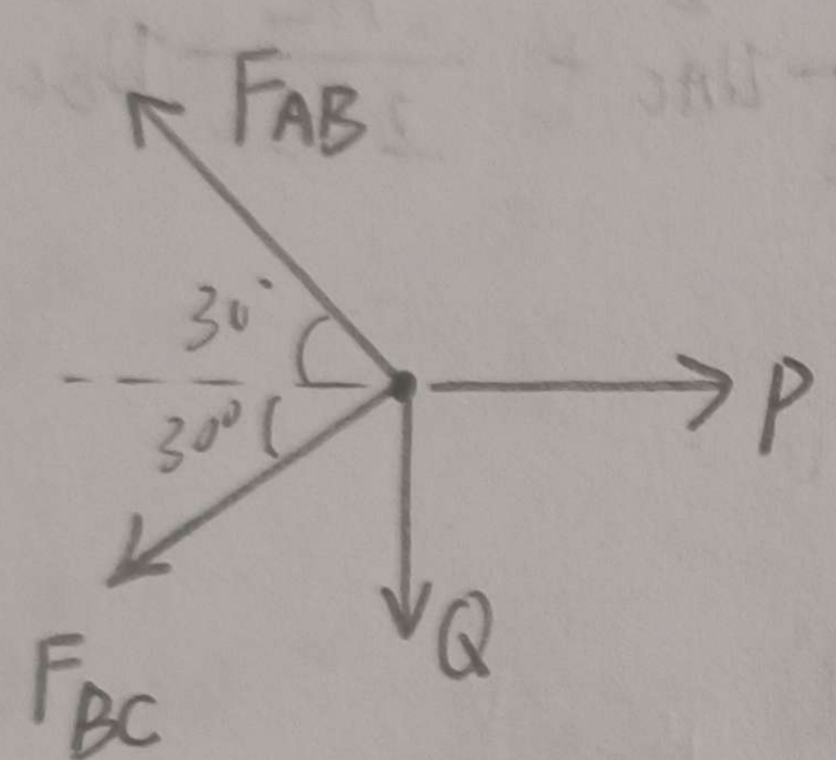
3.5

解：Assume 水平力 P 作用于 B 点



FBD

node B：



$\xrightarrow{+} \Sigma F_x = 0: P - F_{AB} \cdot \frac{\sqrt{3}}{2} - F_{BC} \cdot \frac{\sqrt{3}}{2} = 0$

$+\uparrow \Sigma F_y = 0: F_{AB} \cdot \frac{1}{2} - Q - F_{BC} \cdot \frac{1}{2} = 0$

$\Rightarrow F_{AB} = Q + \frac{1}{\sqrt{3}} P$

$F_{BC} = \frac{1}{\sqrt{3}} P - Q$

$$u_i = \frac{\partial U^*}{\partial F_i}$$

$$U^*_{total} = U^*_{AB} + U^*_{BC}$$

$$= \frac{L}{2AE} (F_{AB}^2 + F_{BC}^2)$$

$$= \frac{L}{2AE} \cdot (2Q^2 + \frac{2}{3} P^2)$$

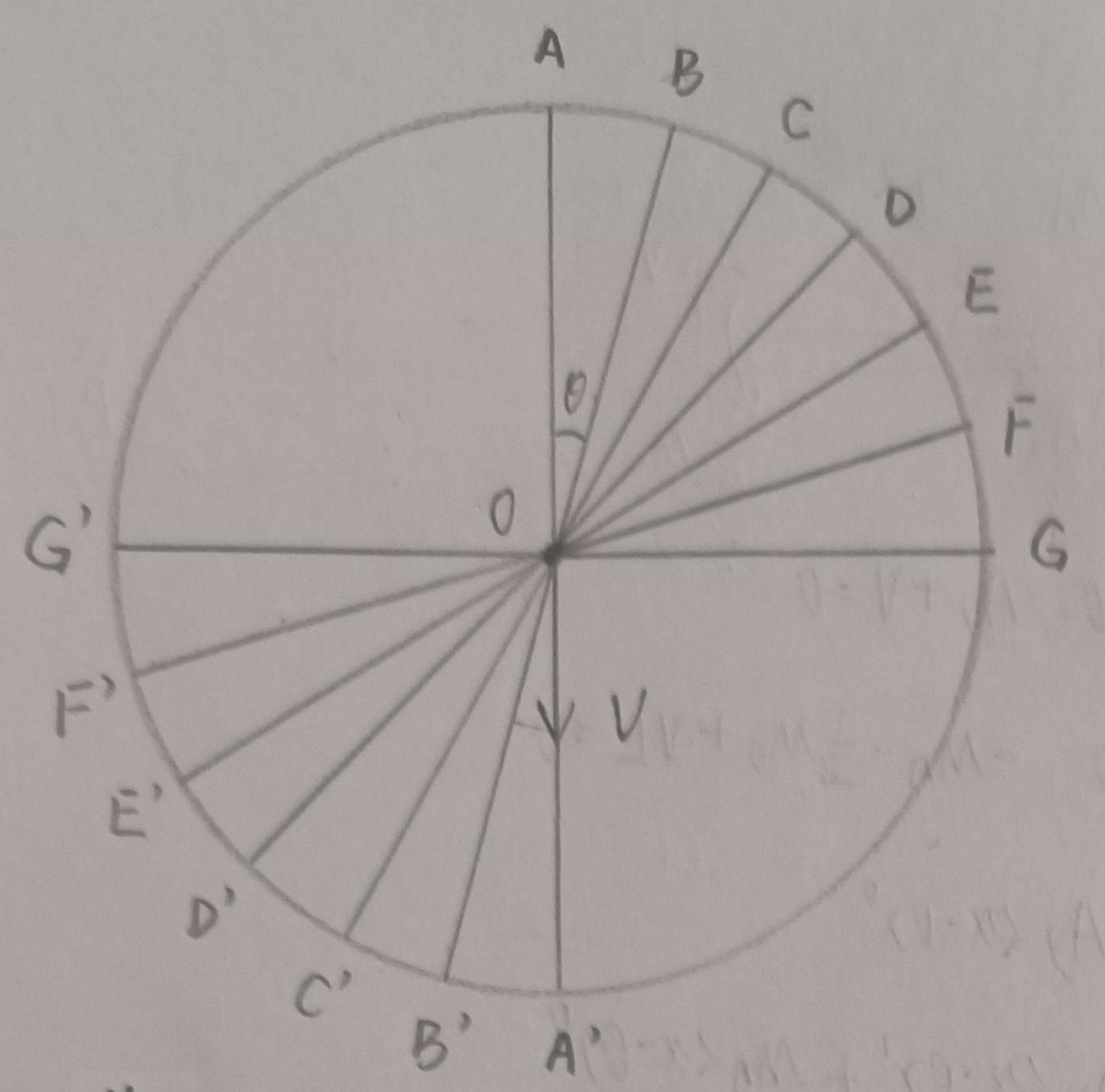
$$= \frac{L}{AE} (Q^2 + \frac{1}{3} P^2)$$

$u_B = \frac{\partial U^*}{\partial P} = \frac{L}{AE} \cdot \frac{2}{3} P = 0$, since $P = 0$

$v_B = \frac{\partial U^*}{\partial Q} = \frac{L}{AE} \cdot 2Q$ $\boxed{\text{ANS}}$

3.9

解:



$V_{OA} = -V_{OA'} = v$

$V_{OB} = -V_{OB'} = v\cos\theta$

$V_{OC} = -V_{OC'} = v\cos 2\theta$

$\vdots$

$V_{OF} = -V_{OF'} = v\cos 5\theta$

$U = \frac{1}{2}\Delta F = \frac{AE}{2L}\Delta^2$

$$U = \frac{AE}{\cancel{2}L}\cdot\cancel{2}(v^2 + 2v^2\cos^2\theta + 2v^2\cos^2 2\theta + 2v^2\cos^2 3\theta + 2v^2\cos^2 4\theta + 2v^2\cos^2 5\theta)$$

$$P = \frac{\partial U}{\partial v} = \frac{AE}{L}(2v + 4v\cos^2\theta + 4v\cos^2 2\theta + 4v\cos^2 3\theta + 4v\cos^2 4\theta + 4v\cos^2 5\theta)$$

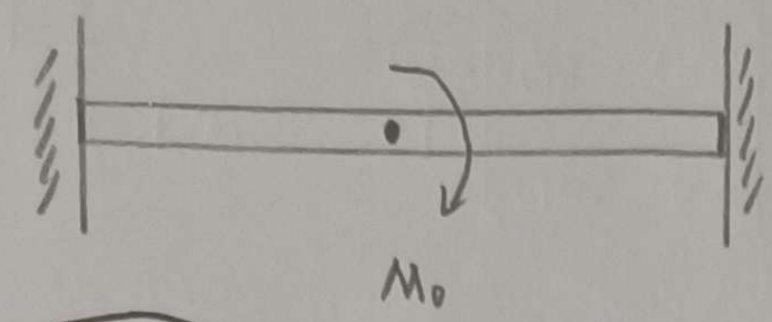
$$v = \frac{PL}{AE}\cdot\frac{1}{2+4(\cos^2\theta+\cos^2 2\theta+\cos^2 3\theta+\cos^2 4\theta+\cos^2 5\theta)}$$

$$= \frac{2000\times 0.4}{0.15\times 10^{-6}\times 200\times 10^{9}}\times\frac{1}{2+4\cdot\sum_{i=1}^{5}\cos^2(15^\circ i)}$$

$$= \frac{1}{450}\,\text{cm} = 2.22\ \text{mm}$$

3.12 2L, EI

解：



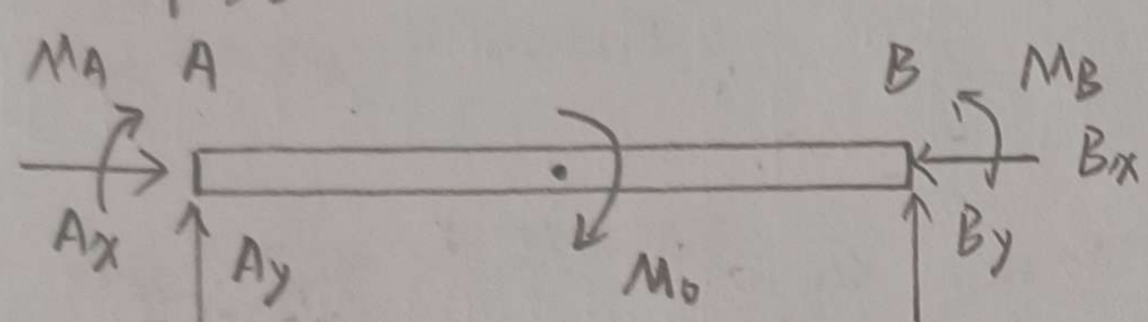
$$\theta = \frac{\partial U^*}{\partial M_0}$$

$$U^* = \frac{1}{2}\int_0^{2L} EI\left(\frac{dy}{dx}\right)^2 dx$$

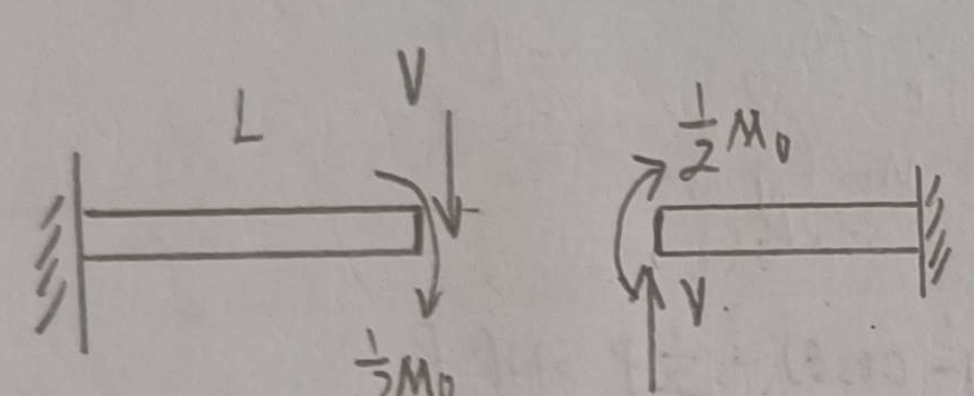
$$= \frac{1}{2}\int_0^{2L} \frac{M^2(x)}{EI} dx$$

$$\frac{\partial U^*}{\partial M_0} = \int_0^{2L} \frac{M(x)}{EI}\cdot\frac{\partial M(x)}{\partial M_0} dx$$

FBD

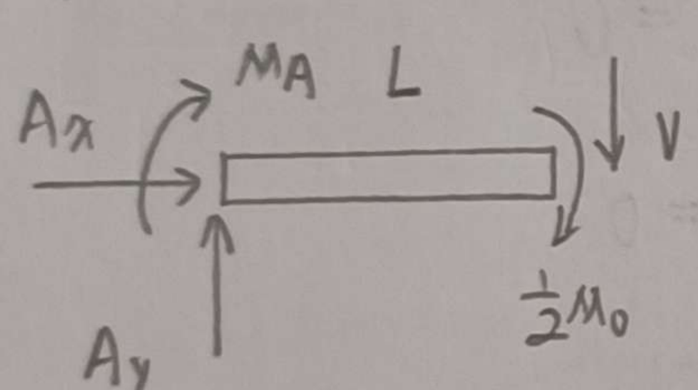


考虑对称性：



(反对称荷载
正对称 轴力为0).

FBD：



$+\uparrow \Sigma F_y = 0$: $A_y - V = 0$

$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

$\circlearrowleft \Sigma M_A = 0$: $-M_A - \frac{1}{2}M_0 - V\cdot L = 0$

$$V(x) = A_y\langle x-0\rangle^0$$

$$M(x) = A_y\langle x-0\rangle^1 + M_A\langle x-0\rangle^0$$

$$= A_y x + M_A$$

$$= Vx + (-\tfrac{1}{2}M_0) - VL$$

$$\frac{\partial U^*}{\partial V} = \int_0^L \frac{M(x)}{EI}\,\frac{\partial M(x)}{\partial V} dx = 0, \quad \frac{\partial M(x)}{\partial V} = x - L$$

$$= \frac{1}{EI}\int_0^L \left(Vx - \tfrac{1}{2}M_0 - VL\right)(x-L)\,dx = 0$$

$$\Rightarrow V = \frac{-3M_0}{4L}, \quad M(x) = \frac{1}{4}M_0 - \frac{3M_0}{4L}x$$

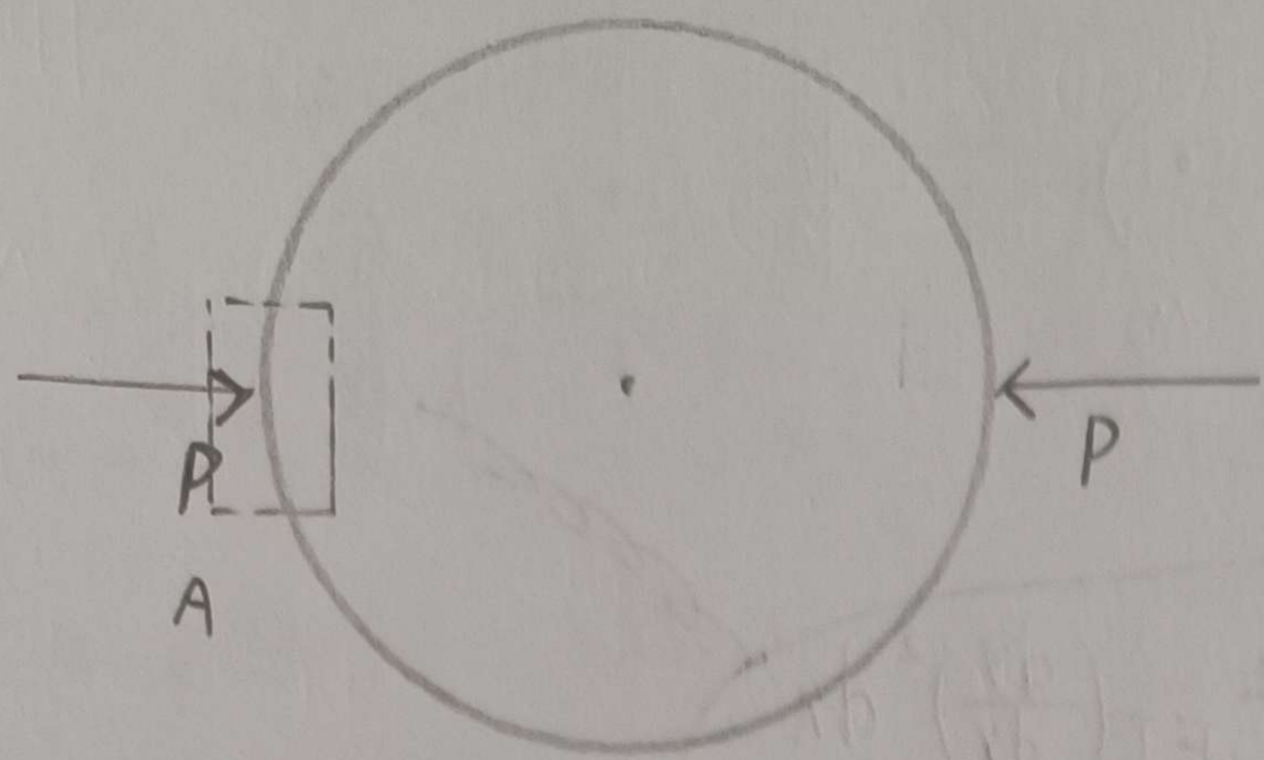
$$\frac{\partial U^*}{\partial(\frac{1}{2}M_0)} = \int_0^L \frac{M(x)}{EI}\,\frac{\partial M(x)}{\partial(\frac{1}{2}M_0)} dx = \theta.$$

$$\theta = \frac{1}{EI}\int_0^L \left(\tfrac{1}{4}M_0 - \frac{3M_0}{4L}x\right)\left(\tfrac{1}{2} - \frac{3x}{2L}\right) dx$$

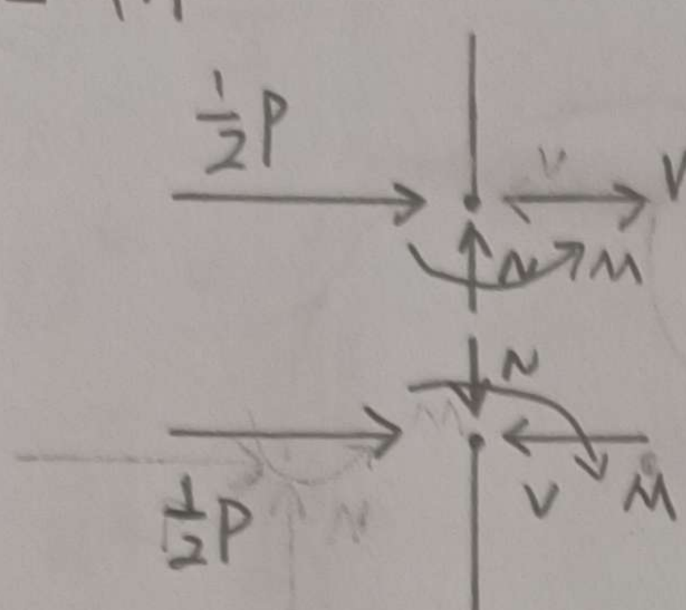
$$\theta = \frac{1}{EI}\cdot\frac{1}{8}M_0 L$$

3.13

解:



区域A:

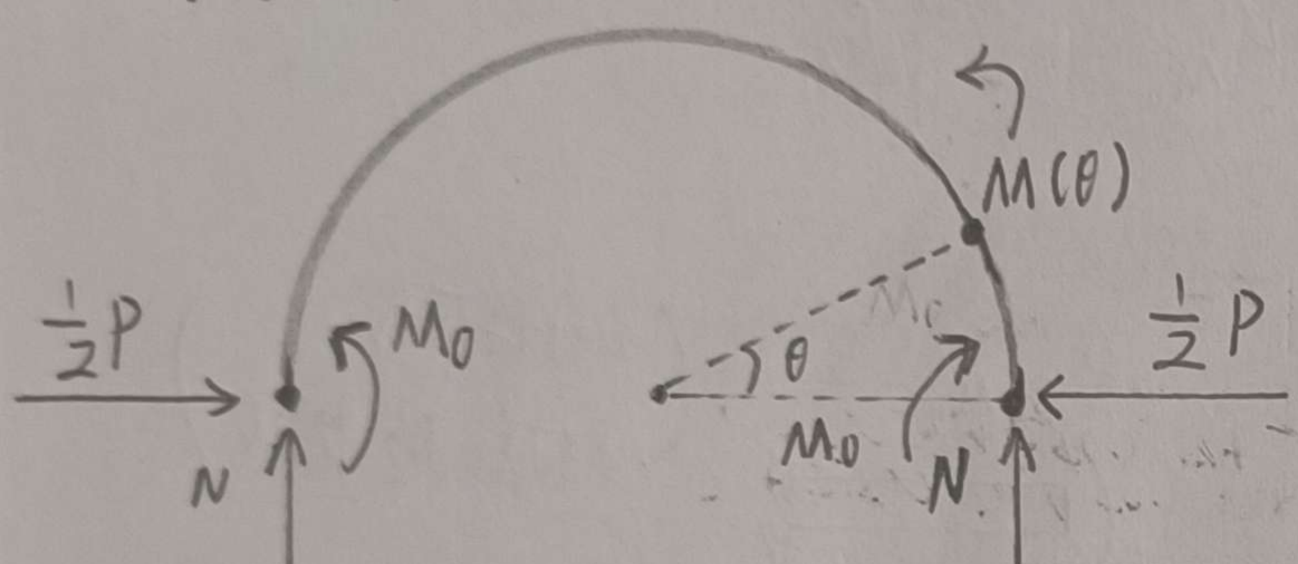


同向荷载

对称轴处反向剪力为0.

$V=0.$

对称性:



$$M(\theta) = -M_0 + N\cdot R(1-\cos\theta) - \frac{1}{2}P\sin\theta R$$

$$\frac{\partial U^*}{\partial N} = \int_0^{\pi} \frac{M(\theta)}{EI}\frac{\partial M(\theta)}{\partial N} R\,d\theta = 0$$

$$\frac{\partial U^*}{\partial M_0} = \int_0^{\pi} \frac{M(\theta)}{EI}\frac{\partial M(\theta)}{\partial M_0} R\,d\theta = 0$$

$$\Rightarrow M_0 = \frac{-PR}{\pi}$$

$$N = 0$$

$$M(\theta) = \frac{PR}{\pi} - \frac{1}{2}P\sin\theta R$$

$$\therefore 2u = \frac{\partial U^*}{\partial(\frac{1}{2}P)} = \int_0^{\pi}\frac{M(\theta)}{EI}\cdot\frac{\partial M(\theta)}{\partial(\frac{1}{2}P)} R\,d\theta$$

$$= \int_0^{\pi}\frac{(\frac{PR}{\pi} - \frac{1}{2}P\sin\theta R)}{EI}\cdot(\frac{R}{\pi} - \frac{1}{2}\sin\theta R)2R\,d\theta$$

$$= \frac{PR^3}{4\pi EI}(\pi^2 - 8)$$

3.16

解:

proof 1:

$$U = \frac{1}{2}\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22}\end{bmatrix}\begin{bmatrix} x_1 \\ x_2\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} x_1a_{11}+x_2a_{12} & x_1a_{12}+x_2a_{22}\end{bmatrix}\begin{bmatrix} x_1 \\ x_2\end{bmatrix}$$

$$= \frac{1}{2}(x_1^2a_{11} + x_1x_2a_{12} + x_1x_2a_{12} + a_{22}x_2^2)$$

$$= \frac{1}{2}(a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2)$$

proof 2:

$$\{y\} = \begin{bmatrix} y_1 \\ y_2\end{bmatrix} = \begin{bmatrix}\frac{\partial U}{\partial x_1} \\ \frac{\partial U}{\partial x_2}\end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{12}x_2 + a_{22}x_2\end{bmatrix}$$

$$[A]\{x\} = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22}\end{bmatrix}\begin{bmatrix} x_1 \\ x_2\end{bmatrix}$$

$$= \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{12}x_1 + a_{22}x_2\end{bmatrix}$$

$$\therefore \{y\} = [A]\{x\}$$

$$\begin{bmatrix}\frac{\partial U}{\partial x_1} \\ \frac{\partial U}{\partial x_2}\end{bmatrix} = \frac{\partial U}{\partial X} = \{y\} = [A]\{x\}$$

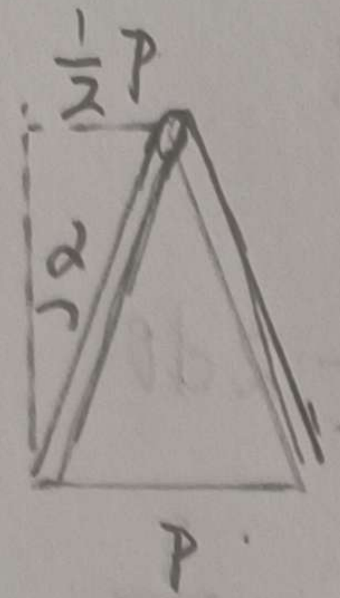
3.20

解：

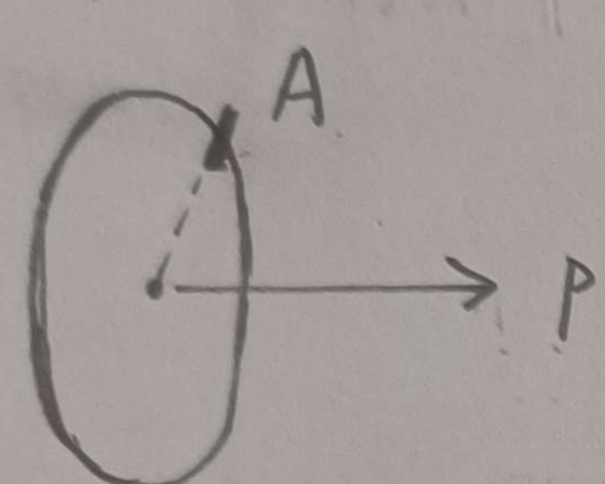
由 pitch，弹簧的夹角

$$\tan\alpha = \frac{\frac{1}{2}P}{2R} = \frac{\pi r}{2R}$$

如下图：



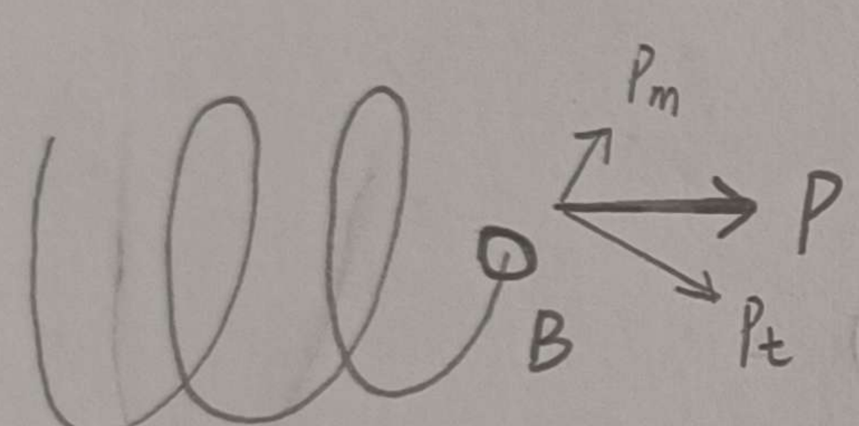
对于一个圆环而言



截面A处仅有 $T = PR$

Since 截面A平行于力P作用线.

对于螺旋线



截面B垂直于螺旋线

∴ B与P间存在夹角 α.

$$\begin{aligned} P_m &= P\sin\alpha \\ P_t &= P\cos\alpha \end{aligned} \Rightarrow \begin{aligned} M &= P\sin\alpha\cdot R \\ T &= P\cos\alpha\cdot R \end{aligned}$$

$$U^* = \int_0^{2\pi n}\frac{T^2}{2GJ}R\,d\theta + \int_0^{2\pi n}\frac{M^2}{2EI}R\,d\theta$$

$$\frac{\partial U^*}{\partial P} = \int_0^{2\pi n}\frac{T}{GJ}\frac{\partial T}{\partial P}R\,d\theta + \int_0^{2\pi n}\frac{M}{EI}\frac{\partial M}{\partial P}R\,d\theta$$

$$= \frac{P\cos^2\alpha R^3}{GJ}\cdot 2\pi n + \frac{P\sin^2\alpha R^3}{EI}\cdot 2\pi n$$

$$= 2\pi n P R^3\left(\frac{\cos^2\alpha}{GJ} + \frac{\sin^2\alpha}{EI}\right)$$

$$k = \frac{1}{2\pi n R^3\left(\frac{\cos^2\alpha}{GJ} + \frac{\sin^2\alpha}{EI}\right)}$$

$$\alpha \to 0,\ k = \frac{GJ}{2\pi n R^3} = \frac{G\cdot\frac{\pi}{2}r^4}{2\pi n R^3}$$

$$= \frac{G r^4}{4 n R^3}$$