Homework problems 53-55
Due in class, Friday, 25 December 2020

53. Determine the critical buckling load for the column. The material can be assumed rigid.

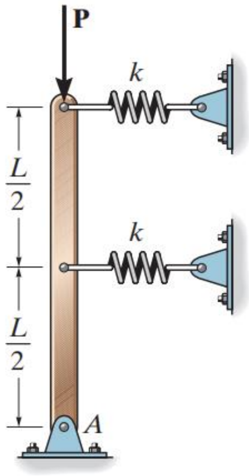


Figure 53

SOLUTION

$F_1 = k(L\,\theta); \qquad F_2 = k\left(\frac{L}{2}\theta\right)$

$\zeta + \Sigma M_A = 0; \qquad P(\theta)(L) - (F_1 L) - F_2\left(\frac{L}{2}\right) = 0$

$P(\theta)(L) - kL^2\,\theta - k\left(\frac{L}{2}\right)^2\theta = 0$

Require:

$P_{cr} = kL + \frac{kL}{4} = \frac{5kL}{4}$ **Ans.**

54. Determine the maximum load P the frame can support without buckling member AB. Assume that AB is made of steel and is pinned at its ends for y–y axis buckling and fixed at its ends for x–x axis buckling. E_{st} = 200 GPa, σ_Y = 360 MPa.

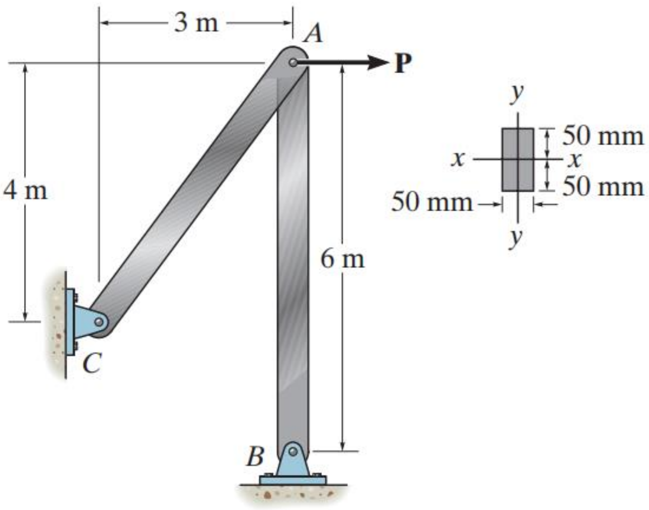
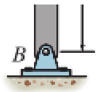


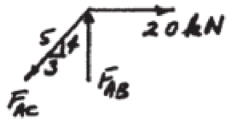
Figure 54

SOLUTION



$$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{AC}\left(\frac{3}{5}\right) + P = 0$$

$$F_{AC} = \frac{5}{3}P$$



$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - \frac{5}{3}P\left(\frac{4}{5}\right) = 0$$

$$F_{AB} = \frac{4}{3}P$$

$$I_y = \frac{1}{12}(0.10)(0.05)^3 = 1.04167(10^{-6})\text{m}^4$$

$$I_x = \frac{1}{12}(0.05)(0.10)^3 = 4.16667(10^{-6})\text{m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

x–x axis buckling:

$$P_{cr} = \frac{\pi^2(200)(10^9)(4.16667)(10^{-6})}{(0.5(6))^2} = 914 \text{ kN}$$

y–y axis buckling:

$$P_{cr} = \frac{\pi^2(200)(10^9)(1.04167)(10^{-6})}{(1(6))^2} = 57.12 \text{ kN}$$

y–y axis buckling controls

$$\frac{4}{3}P = 57.12$$

$$P = 42.8 \text{ kN} \qquad \textbf{Ans.}$$

Check:

$$\sigma_{cr} = \frac{P}{A} = \frac{57.12(10^3)}{(0.1)(0.05)} = 11.4 \text{ MPa} < \sigma_Y \qquad \textbf{OK}$$

55 (optional). The ideal column is subjected to the force F at its midpoint and the axial load P. Determine the maximum moment in the column at midspan. EI is constant.
Hint: Establish the differential equation for deflection. The general solution is $v = C_1 \sin kx + C_2 \cos kx - c^2x/k^2$, where $c^2 = F/2EI$, $k^2 = P/EI$.

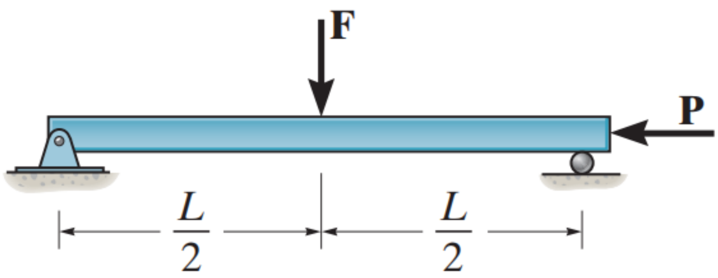
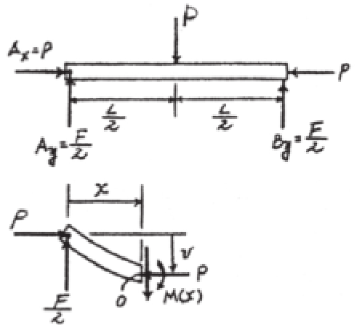


Figure 55

SOLUTION



Moment Functions: FBD(b).

$$\circlearrowleft +\Sigma M_o = 0; \qquad M(x) + \frac{F}{2}x + P(v) = 0$$

$$M(x) = -\frac{F}{2}x - Pv \qquad \textbf{(1)}$$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -\frac{F}{2}x - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = -\frac{F}{2EI}x$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right) - \frac{F}{2P}x \qquad \textbf{(2)}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P} \qquad \textbf{(3)}$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0$, $v = 0$. From Eq. (2), $C_2 = 0$

At $x = \frac{L}{2}$, $\frac{dv}{dx} = 0$. From Eq. (3),

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{F}{2P}$$

$$C_1 = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

Elastic Curve:

$$v = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P}x$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - x\right]$$

SOLUTION

However, $v = v_{\max}$ at $x = \frac{L}{2}$. Then,

$$v_{\max} = \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

Maximum Moment: The maximum moment occurs at $x = \frac{L}{2}$. From Eq. (1),

$$M_{\max} = -\frac{F}{2}\left(\frac{L}{2}\right) - Pv_{\max}$$

$$= -\frac{FL}{4} - P\left\{\frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]\right\}$$

$$= -\frac{F}{2}\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) \qquad \textbf{Ans.}$$