Bayes Project

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Stochastic Volatility Analysis in R

To conduct the analysis, we will use the "stochvol" package developed by Darjus Hosszejni and Gregor Kastner.

```
library(stochvol)
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.4     v readr     2.1.5
## v forcats   1.0.0     v stringr   1.5.1
## v ggplot2   3.5.0     v tibble    3.2.1
## v lubridate 1.9.3     v tidyr     1.3.1
## v purrr     1.0.2
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

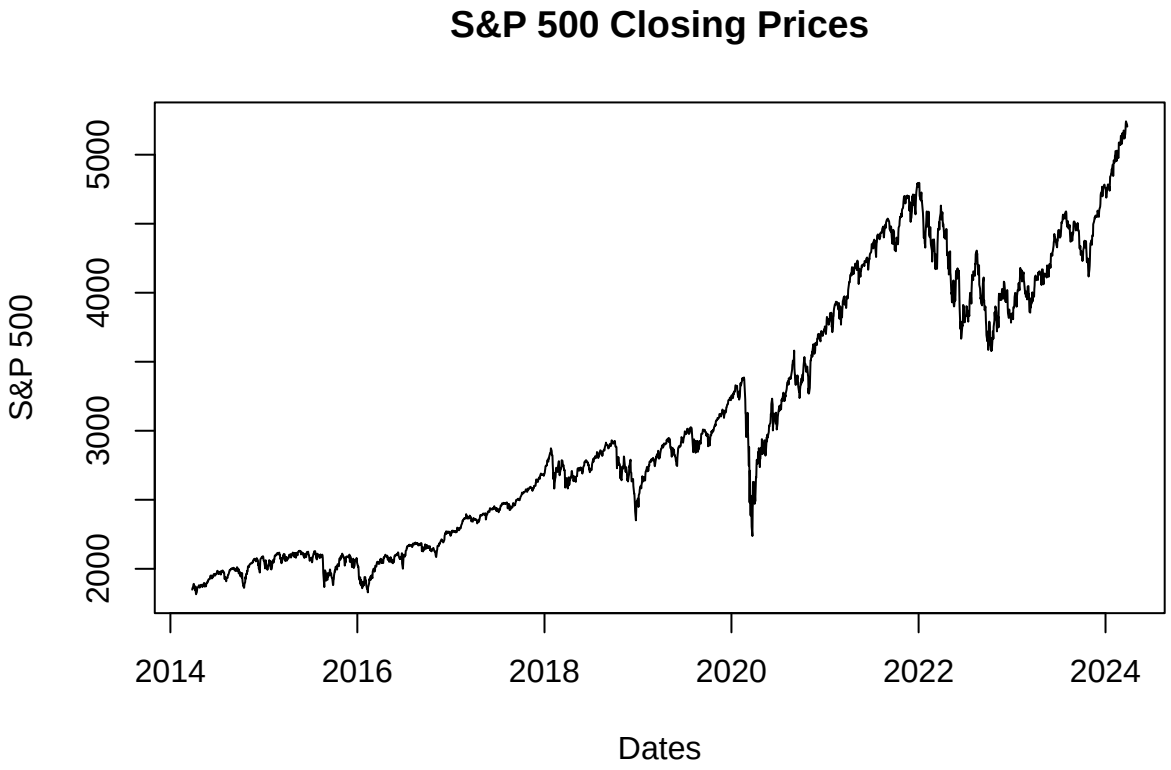
Data Import

To demonstrate an application of the model, we have chosen to model the closing prices of the S&P 500. We first get our data frame.

```
data <- data.frame(read.table("sp500.csv", header = T, sep = ","))
data$Date <- as.Date(data$Date,"%m/%d/%Y")
data <- arrange(data, Date)
dates <- data$Date[2:nrow(data)]
sp500 <- data$Close.Last
```

We can plot the returns:

```
plot(data$Date, sp500, xlab = "Dates", ylab = "S&P 500", main = "S&P 500 Closing Prices", type = "l")
```

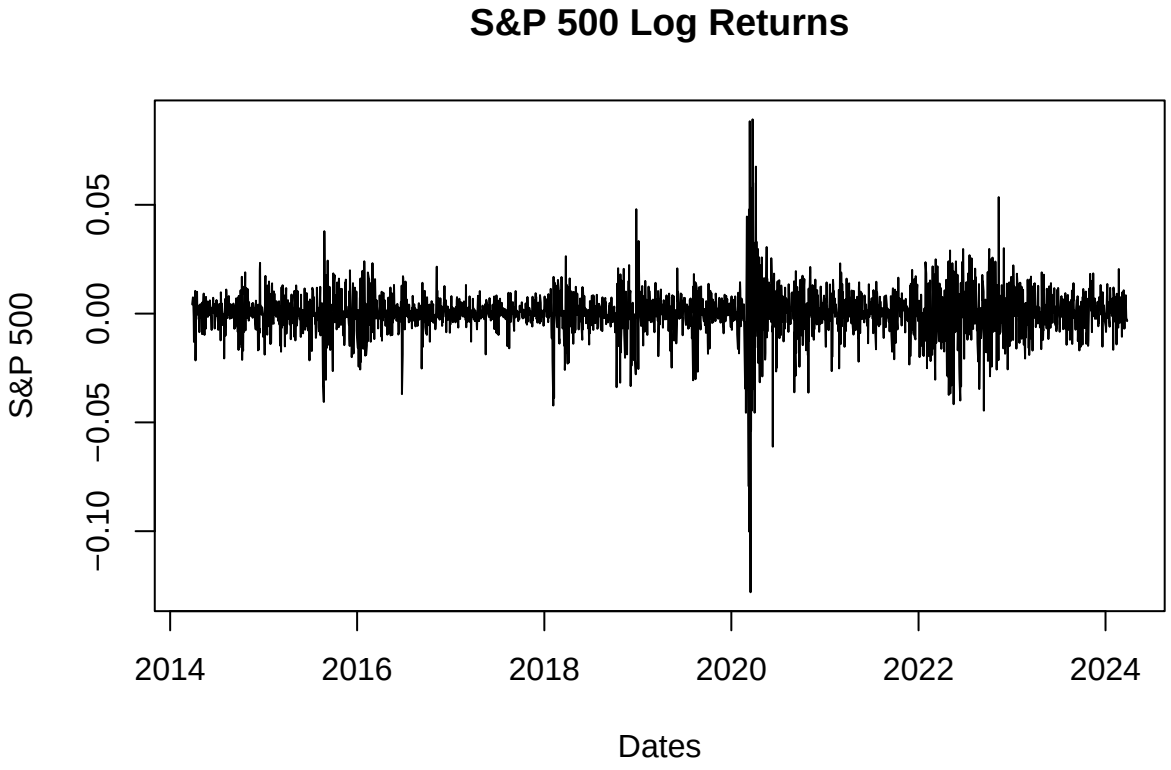


In order to better model our data, we convert the closing prices of the S&P 500 to log returns.

```
rets <- logret(sp500, demean = TRUE)
```

We can visualize the log returns in a plot:

```
plot(dates, rets, xlab = "Dates", ylab = "S&P 500", main = "S&P 500 Log Returns", type = "l")
```



Modeling

In the "stochvol" package, the modeling is implemented via the "svsample" function. To construct the mode, we have chosen "ar0" as the design matrix specification. This is because we want to demonstrate the constant mean model of stochastic volatility, where $h_{t+1} = \mu + \phi(h_t - \mu) + \eta_t$. We will draw for 10,000 iterations and have a burn-in period of 1000.

```
S <- 10000
b <- 1000
res <- svsample(rets, priormu = c(0,sqrt(10)), designmatrix = "ar0", draws = S, burnin = b)
```

```
## Done!
```

```
## Summarizing posterior draws...
```

We may view the results of the model below:

```
summary(res)
```

```
##
## Summary of 'svdraws' object
##
## Prior distributions:
## mu        ~ Normal(mean = 0, sd = 3.162278)
## (phi+1)/2 ~ Beta(a = 5, b = 1.5)
## sigma^2   ~ Gamma(shape = 0.5, rate = 0.5)
## nu        ~ Infinity
## rho       ~ Constant(value = 0)
## beta      ~ Normal(mean = 0, sd = 10000)
##
## Stored 10000 MCMC draws after a burn-in of 1000.
## No thinning.
```

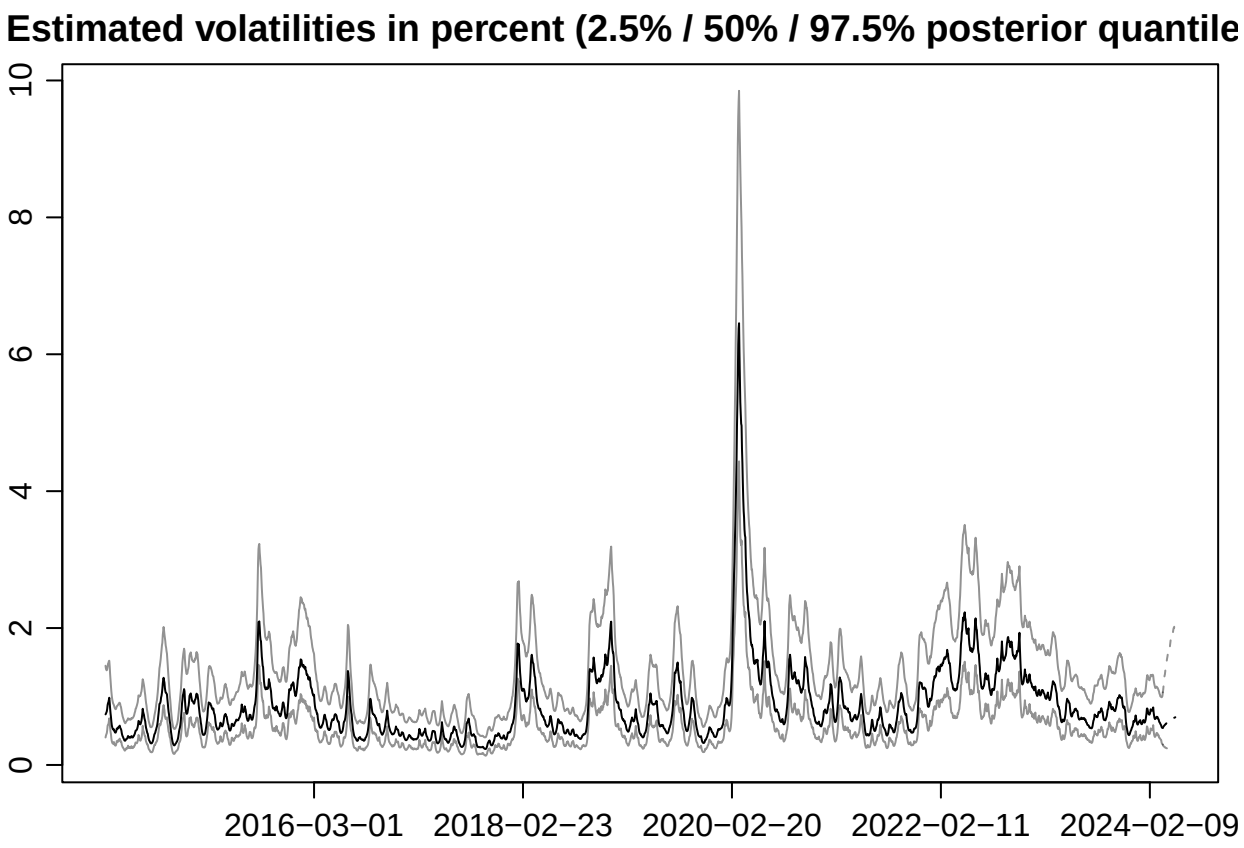
```
##
## Posterior draws of SV parameters (thinning = 1):
##               mean      sd       5%     50%     95%  ESS
## mu        -9.7383 0.16314 -10.0050 -9.7390 -9.4672 6315
## phi        0.9594 0.00826   0.9452  0.9599  0.9722  433
## sigma      0.3180 0.02783   0.2747  0.3167  0.3662  222
## exp(mu/2)  0.0077 0.00063   0.0067  0.0077  0.0088 6315
## sigma^2    0.1019 0.01795   0.0754  0.1003  0.1341  222
##
## Posterior draws of regression coefficients (thinning = 1):
##           mean      sd      5%     50%   95%  ESS
## beta_0 0.00049 0.00013 0.00029 0.00049 7e-04 5923
```

Plotting and Forecasting

We want to plot the volatility according to out model. We construct a plot of the volatilities implied by our model, and also forecast the volatility for the upcoming month.

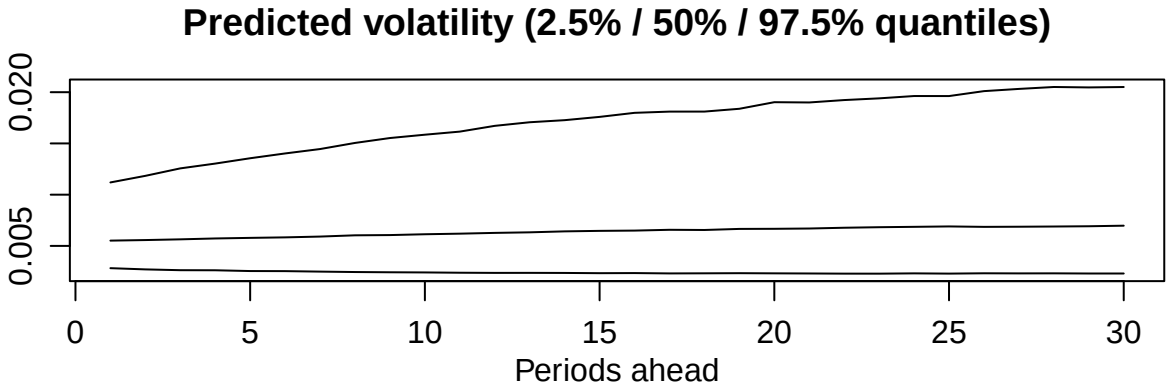
The plot shows both the volatility and the 95% confidence intervals.

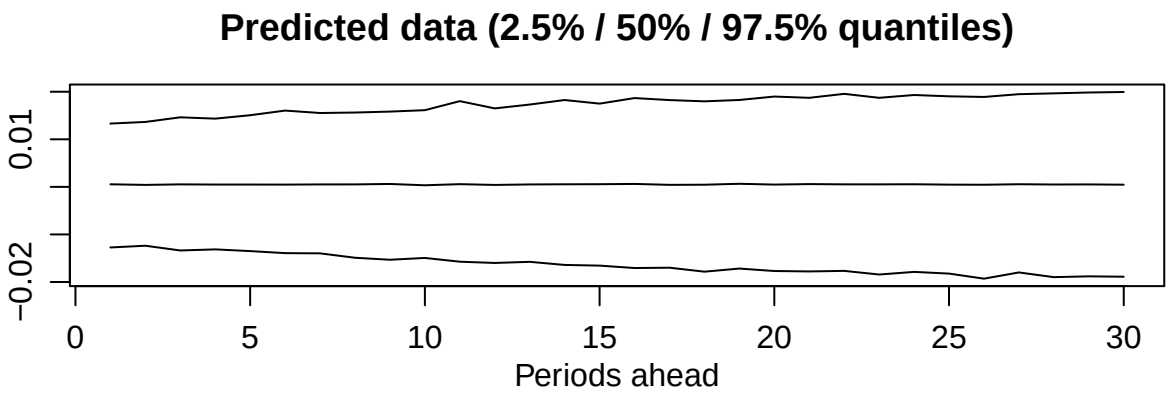
```
res <- updatesummary(res, quantiles = c(0.025,0.5,0.975))
volplot(res, dates = dates, forecast = 30)
```



We can construct forecasts of the upcoming 30 days using our model:

```
preds <- predict(res, 30)
plot(preds, quantiles = c(0.025,0.5,0.975))
```

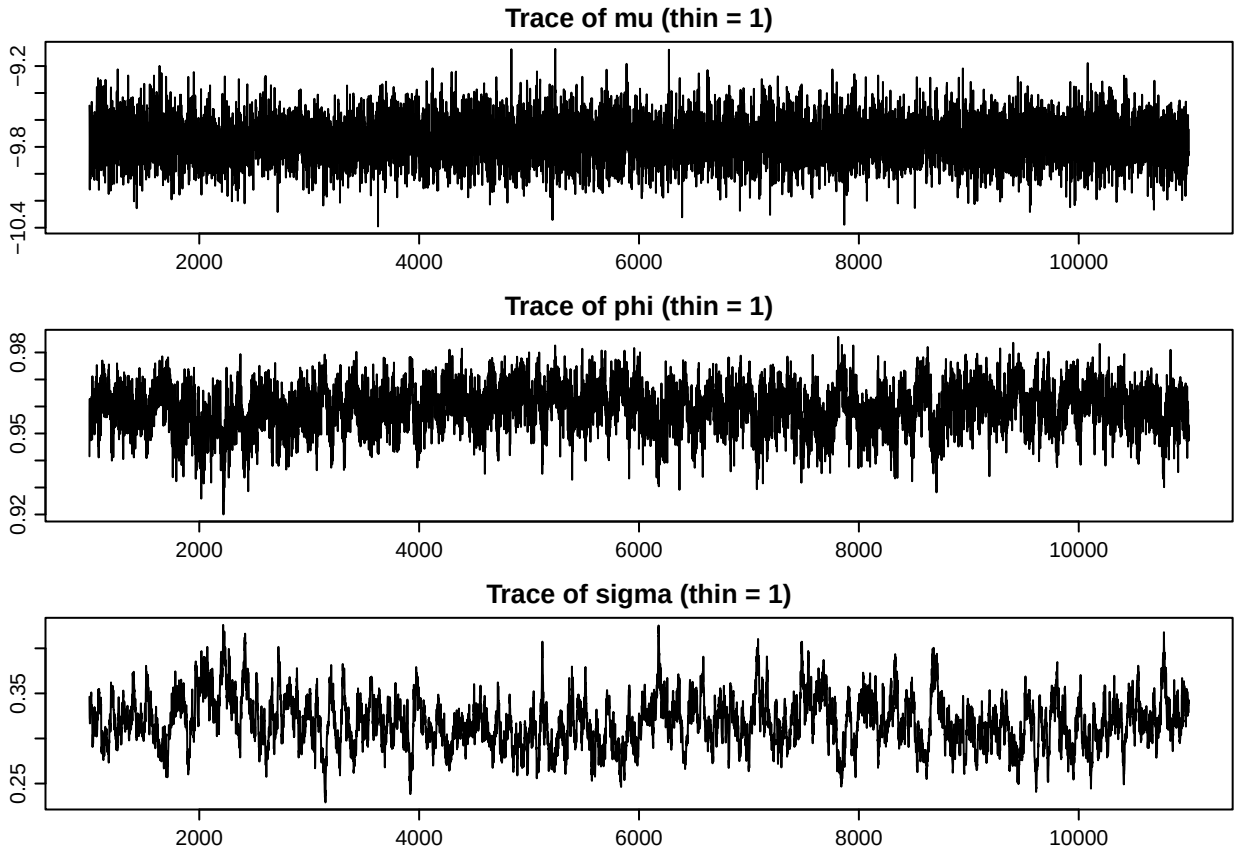




Analysis of Variables

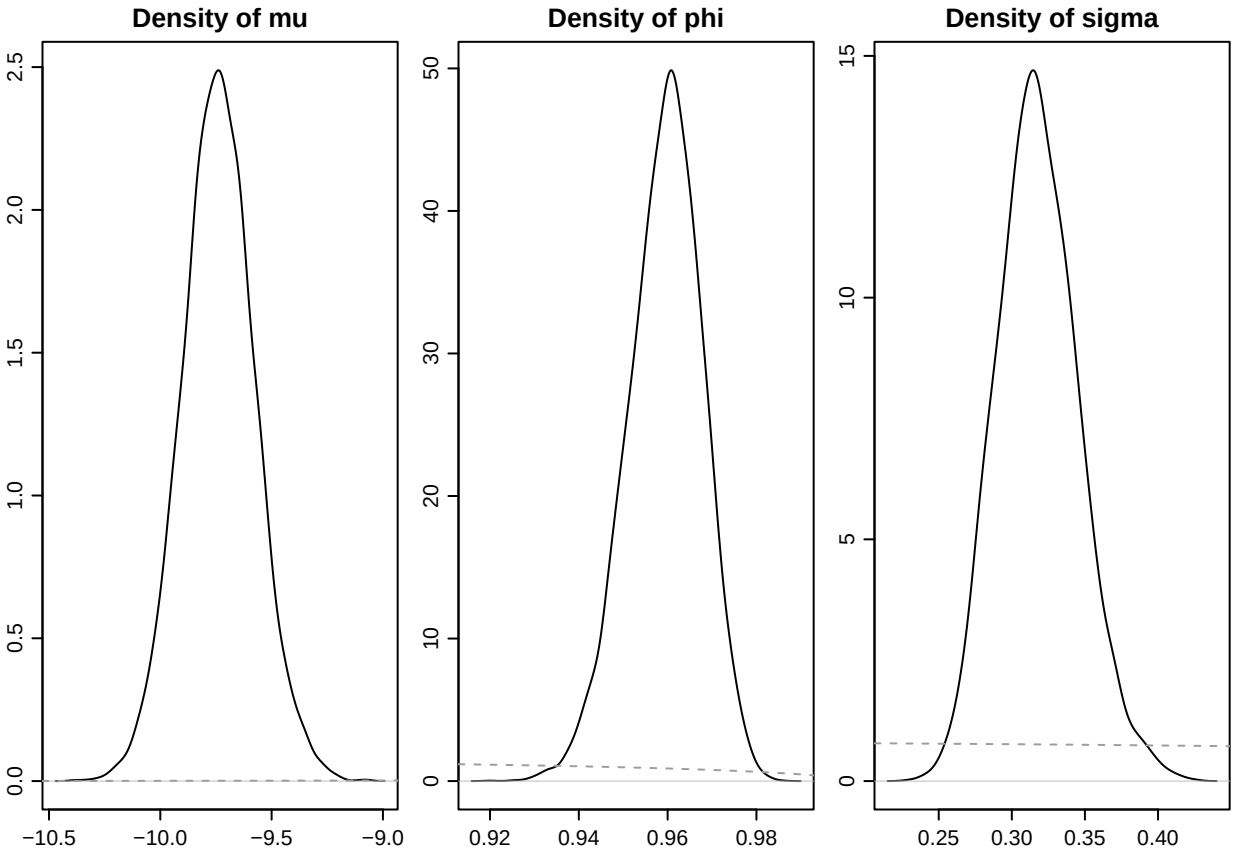
We can also construct the plots of our variables in the different stages of the draw:

```
par(mfrow = c(3, 1))
paratraceplot(res)
```



The densities of the variables are given below:

```
par(mfrow = c(1, 3))
paradensplot(res, showobs = FALSE)
```


Density of mu
Density of phi
Density of sigma
0.0
0.5
1.0
1.5
2.0
2.5
−10.5
−10.0
−9.5
−9.0
0
10
20
30
40
50
0.92
0.94
0.96
0.98
0
5
10
15
0.25
0.30
0.35
0.40