

Model of Investment and Input Choice and Productivity

1. Exogenous Capital

(a) The firm chooses M_{it} and L_{it} to maximize the profit:

$$\max_{L_{it}, M_{it}} \Pi_{it}(M_{it}, L_{it} | K_{it}, P^M, W, P) = P \times \Omega_{it} K_{it}^{\alpha_K} M_{it}^{\alpha_M} L_{it}^{\alpha_L} - P^M \times M_{it} - W \times L_{it}$$

First order condition:

$$\begin{aligned} \frac{\partial \Pi_{it}}{\partial M_{it}} &= \frac{\alpha_M \times P \times \Omega_{it} K_{it}^{\alpha_K} M_{it}^{\alpha_M} L_{it}^{\alpha_L}}{M_{it}} - P^M = 0 \\ \frac{\partial \Pi_{it}}{\partial L_{it}} &= \frac{\alpha_L \times P \times \Omega_{it} K_{it}^{\alpha_K} M_{it}^{\alpha_M} L_{it}^{\alpha_L}}{L_{it}} - W = 0 \end{aligned}$$

Input demand functions:

$$\begin{aligned} M_{it} &= \left(\frac{\alpha_M^{1-\alpha_L} \alpha_L^{\alpha_L} P \Omega_{it} K_{it}^{\alpha_K}}{W^{\alpha_L} (P^M)^{1-\alpha_L}} \right)^{\frac{1}{1-\alpha_M-\alpha_L}} \\ L_{it} &= \left(\frac{\alpha_L^{1-\alpha_M} \alpha_M^{\alpha_M} P \Omega_{it} K_{it}^{\alpha_K}}{W^{1-\alpha_M} (P^M)^{\alpha_M}} \right)^{\frac{1}{1-\alpha_M-\alpha_L}} \end{aligned}$$

Labor to material ratio:

$$\frac{L_{it}}{M_{it}} = \frac{\alpha_L P^M}{\alpha_M W}$$

(b)

$$\max_{L_{it}, M_{it}} \Pi_{it}(M_{it}, L_{it} | K_{it}, P^M, \epsilon, \eta) = (\Omega_{it} K_{it}^{\alpha_K} M_{it}^{\alpha_M} L_{it}^{\alpha_L})^{1-\frac{1}{\epsilon}} - P^M \times M_{it} - L_{it}^{1+\eta}$$

First order condition:

$$\begin{aligned} \frac{\partial \Pi_{it}}{\partial M_{it}} &= \left(\frac{\alpha'_M \times \Omega_{it}^{1-\frac{1}{\epsilon}} K_{it}^{\alpha'_K} M_{it}^{\alpha'_M} L_{it}^{\alpha'_L}}{M_{it}} \right) - P^M = 0 \\ \frac{\partial \Pi_{it}}{\partial L_{it}} &= \left(\frac{\alpha'_L \times \Omega_{it}^{1-\frac{1}{\epsilon}} K_{it}^{\alpha'_K} M_{it}^{\alpha'_M} L_{it}^{\alpha'_L}}{L_{it}} \right) - (1+\eta)L_{it}^\eta = 0 \end{aligned}$$

$$\alpha'_i = \alpha_i^{1-\frac{1}{\epsilon}}, i = K, M, L$$

Input demand functions:

$$L_{it} = \left(\frac{\alpha_L'^{1-\alpha_M'} \alpha_M'^{\alpha_M'} \Omega_{it}^{1-\frac{1}{\epsilon}} K_{it}^{\alpha_K'}}{(1+\eta)^{1-\alpha_M'} (P^M)^{\alpha_M'}} \right)^{\frac{1}{(1+\eta)(1-\alpha_M')-\alpha_L'}}$$

$$M_{it} = \left(\frac{\alpha_M'^{1-\frac{\alpha_L'}{1+\eta}} \alpha_L'^{\frac{\alpha_L'}{1+\eta}} \Omega_{it}^{1-\frac{1}{\epsilon}} K_{it}^{\alpha_K'}}{\frac{\alpha_L'}{(1+\eta)^{1+\eta}} (P^M)^{1-\frac{\alpha_L'}{1+\eta}}} \right)^{\frac{1}{1-\alpha_M'-\frac{\alpha_L'}{1+\eta}}}$$

(c)

$$\max_{L_{it}, M_{it}} \Pi_{it}(M_{it}, L_{it} | K_{it}, P^M, \epsilon, \eta) = Q_{it}^{1-\frac{1}{\epsilon}} - P^M \times M_{it} - L_{it}^{1+\eta}$$

$$Q_{it} = A_{it} \left(\alpha_k (K_{it})^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_k - \alpha_m) (B L_{it})^{\frac{\sigma-1}{\sigma}} + \alpha_m (M_{it})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

First order condition:

$$\frac{\partial \Pi_{it}}{\partial M_{it}} = \left(1 - \frac{1}{\epsilon} \right) Q_{it}^{-\frac{1}{\epsilon}} \times \frac{\partial Q_{it}}{\partial M_{it}} - P^M = 0$$

$$\frac{\partial \Pi_{it}}{\partial L_{it}} = \left(1 - \frac{1}{\epsilon} \right) Q_{it}^{-\frac{1}{\epsilon}} \times \frac{\partial Q_{it}}{\partial L_{it}} - (1 + \eta) L_{it}^\eta = 0$$

$$\frac{\partial Q_{it}}{\partial M_{it}} = \alpha_m Q_{it}^{\frac{1}{\sigma}} M_{it}^{-\frac{1}{\sigma}}$$

$$\frac{\partial Q_{it}}{\partial L_{it}} = \alpha_L Q_{it}^{\frac{1}{\sigma}} B^{\frac{\sigma-1}{\sigma}} L_{it}^{-\frac{1}{\sigma}}$$

$$\frac{L_{it}^{-\eta-\frac{1}{\sigma}}}{M_{it}^{-\frac{1}{\sigma}}} = \frac{\alpha_M (1 + \eta)}{\alpha_L P^M B^{\frac{\sigma-1}{\sigma}}}$$

(d) Please refer to the code *tauchen.m*

(e) Please refer to the code *simulationps1.m*

(f) Olley-Pakes Covariance: 0.8626; Weighted productivity: 0.8620; Unweighted productivity: -0.0063

Olley-Pakes Covariance:
0.8626

weighted_productivity:
0.8620

unweighted_productivity:
-0.0063

2. Endogenous Capital

(a) Bellman Equation:

$$\begin{aligned}
 V(\Omega_{it}, K_{it}) &= \max_{I_{it}} \pi(\Omega_{it}, K_{it}) - C(I_{it}, K_{it}) \\
 &+ \beta \int_{\Omega_{it+1}} V(\Omega_{it+1}, \delta K_{it} + I_{it}) \phi(\Omega_{it+1} | \Omega_{it}) d\Omega_{it+1} \\
 \pi(\Omega_{it}, K_{it}) &= P \times \Omega_{it} K_{it}^{\alpha_K} M_{it}^{\alpha_M} L_{it}^{\alpha_L} - P^M \times M_{it} - W \times L_{it} \\
 P &= P^M = W = 1 \\
 C(I_{it}, K_{it}) &= I_{it}^2
 \end{aligned}$$

(b) Please refer to the code *Dynamic_Investment.m*

Discrete the productivity with tauchen 1986 and simulate 50 states for capital.

$$I_t = K_t - \delta K_{t-1}$$

