Model of Investment and Input Choice and Productivity

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1. Exogenous Capital

(a) The firm chooses M_{it} and L_{it} to maximize the profit:

$$\max_{L_{it},M_{it}} \Pi_{it}(M_{it},L_{it}|K_{it},P^M,W,P) = P \times \Omega_{it}K_{it}^{\alpha_K}M_{it}^{\alpha_M}L_{it}^{\alpha_L} - P^M \times M_{it} - W \times L_{it}$$

First order condition:

$$\frac{\partial \Pi_{it}}{\partial M_{it}} = \frac{\alpha_M \times P \times \Omega_{it}K_{it}^{\alpha_K}M_{it}^{\alpha_M}L_{it}^{\alpha_L}}{M_{it}} - P^M = 0$$

$$\frac{\partial \Pi_{it}}{\partial L_{it}} = \frac{\alpha_L \times P \times \Omega_{it}K_{it}^{\alpha_K}M_{it}^{\alpha_M}L_{it}^{\alpha_L}}{L_{it}} - W = 0$$

Input demand functions:

$$M_{it} = \left(\frac{\alpha_M^{1-\alpha_L}\alpha_L^{\alpha_L}P\Omega_{it}K_{it}^{\alpha_K}}{W^{\alpha_L}(P^M)^{1-\alpha_L}}\right)^{\frac{1}{1-\alpha_M-\alpha_L}}$$

$$L_{it} = \left(\frac{\alpha_L^{1-\alpha_M}\alpha_M^{\alpha_M}P\Omega_{it}K_{it}^{\alpha_K}}{W^{1-\alpha_M}(P^M)^{\alpha_M}}\right)^{\frac{1}{1-\alpha_M-\alpha_L}}$$

Labor to material ratio:

$$\frac{L_{it}}{M_{it}} = \frac{\alpha_L P^M}{\alpha_M W}$$

(b)

$$\max_{L_{it},M_{it}} \Pi_{it}(M_{it},L_{it}|K_{it},P^M,\epsilon,\eta) = \left(\Omega_{it}K_{it}^{\alpha_K}M_{it}^{\alpha_M}L_{it}^{\alpha_L}\right)^{1-\frac{1}{\epsilon}} - P^M \times M_{it} - L_{it}^{1+\eta}$$

First order condition:

$$\frac{\partial \Pi_{it}}{\partial M_{it}} = \left(\frac{\alpha_M' \times \Omega_{it}^{1-\frac{1}{\epsilon}}K_{it}^{\alpha_K'}M_{it}^{\alpha_M'}L_{it}^{\alpha_L'}}{M_{it}}\right) - P^M = 0$$

$$\frac{\partial \Pi_{it}}{\partial L_{it}} = \left(\frac{\alpha_L' \times \Omega_{it}^{1-\frac{1}{\epsilon}}K_{it}^{\alpha_K'}M_{it}^{\alpha_M'}L_{it}^{\alpha_L'}}{L_{it}}\right) - (1+\eta)L_{it}^{\eta} = 0$$

$$\alpha_i' = \alpha_i^{1-\frac{1}{\epsilon}}, i = K, M, L$$

Input demand functions:

$$L_{it} = \left(\frac{{\alpha'}_L^{1-\alpha'_M} {\alpha'}_M^{\alpha'_M} \Omega_{it}^{1-\frac{1}{\epsilon}} K_{it}^{\alpha'_K}}{(1+\eta)^{1-\alpha'_M} (P^M)^{\alpha'_M}} \right)^{\frac{1}{(1+\eta)(1-\alpha'_M)-\alpha'_L}}$$

$$M_{it} = \left(\frac{{\alpha'_M}^{1-\frac{\alpha'_L}{1+\eta}} {\alpha'_L}^{\frac{\alpha'_L}{1+\eta}} \Omega_{it}^{1-\frac{1}{\epsilon}} K_{it}^{\alpha'_K}}{(1+\eta)^{\frac{\alpha'_L}{1+\eta}} (P^M)^{1-\frac{\alpha'_L}{1+\eta}}} \right)^{\frac{1}{1-\alpha'_M-\frac{\alpha'_L}{1+\eta}}}$$

(c)

$$\max_{L_{it}, M_{it}} \Pi_{it}(M_{it}, L_{it} | K_{it}, P^M, \epsilon, \eta) = Q_{it}^{1-\frac{1}{\epsilon}} - P^M \times M_{it} - L_{it}^{1+\eta}$$

$$Q_{it} = A_{it} \left(\alpha_k (K_{it})^{\frac{\sigma-1}{\sigma}} + (1-\alpha_k-\alpha_m)(BL_{it})^{\frac{\sigma-1}{\sigma}} + \alpha_m (M_{it})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

First order condition:

$$\frac{\partial \Pi_{it}}{\partial M_{it}} = \left(1-\frac{1}{\epsilon}\right) Q_{it}^{-\frac{1}{\epsilon}} \times \frac{\partial Q_{it}}{\partial M_{it}} - P^M = 0$$

$$\frac{\partial \Pi_{it}}{\partial L_{it}} = \left(1-\frac{1}{\epsilon}\right) Q_{it}^{-\frac{1}{\epsilon}} \times \frac{\partial Q_{it}}{\partial L_{it}} - (1+\eta) L_{it}^{\eta} = 0$$

$$\frac{\partial Q_{it}}{\partial M_{it}} = \alpha_m Q_{it}^{\frac{1}{\sigma}} M_{it}^{-\frac{1}{\sigma}}$$

$$\frac{\partial Q_{it}}{\partial L_{it}} = \alpha_L Q_{it}^{\frac{1}{\sigma}} B^{\frac{\sigma-1}{\sigma}} L_{it}^{-\frac{1}{\sigma}}$$

$$\frac{L_{it}^{-\eta-\frac{1}{\sigma}}}{M_{it}^{-\frac{1}{\sigma}}} = \frac{\alpha_M (1+\eta)}{\alpha_L P^M B^{\frac{\sigma-1}{\sigma}}}$$

(d) Please refer to the code *tauchen.m*

(e) Please refer to the code *simulationps1.m*

(f) Olley-Pakes Covariance: 0.8626; Weighted productivity: 0.8620; Unweighted productivity: -0.0063

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Olley-Pakes Covariance:
    0.8626

weighted_productivity:
    0.8620

unweighted_productivity:
   -0.0063
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2. Endogenous Capital

(a) Bellman Equation:

$$V(\Omega_{it}, K_{it}) = \max_{I_{it}} \pi(\Omega_{it}, K_{it}) - C(I_{it}, K_{it}) + \beta \int_{\Omega_{it+1}} V(\Omega_{it+1}, \delta K_{it} + I_{it}) \phi(\Omega_{it+1} \mid \Omega_{it}) d\Omega_{it+1}$$

$$\pi(\Omega_{it}, K_{it}) = P \times \Omega_{it} K_{it}^{\alpha_K} M_{it}^{\alpha_M} L_{it}^{\alpha_L} - P^M \times M_{it} - W \times L_{it}$$

$$P = P^M = W = 1$$

$$C(I_{it}, K_{it}) = I_{it}^2$$

(b) Please refer to the code *Dynamic_Investment.m*

Discrete the productivity with tauchen 1986 and simulate 50 states for capital.

$$I_t = K_t - \delta K_{t-1}$$