EXECUTIVE SUMMARY

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In this project we explore Kou jump diffusion model. The motivation of this project comes from several empirical investigations. (i) The leptokurtic feature that the return distribution of assets may have higher peak and two (asymmetric) heavier tails than those of the normal distribution; (ii) volatility smile phenomenon in option markets; (iii) asymmetric jump risk matters during hedging, while traditional delta hedging only captures diffusion. We introduce a tractable model, proposed by Steven Kou in 2002, which incorporates all these features. The contents of this project can be divided into four parts.

1. Simulation

Kou's model for asset price has two components, a diffusion part modeled by a Brownian motion as Black-Scholes, and a jump part with Poisson counting process as Merton, but with size of each jump controlled by a (asymmetric) double exponential distribution. We construct functions for simulating stocks paths under Kou's model. The key idea I learned from Glasserman's *Monte Carlo Methods in Financial Engineering* for this part is that we first draw the jump count from Poisson distribution, then draw a binomial split between upward and downward jumps, lastly sum them up using Gamma distributions. We output plots with multiple paths simulated simultaneously. We illustrate that even with small jump rate, paths generated under Kou and Black-Scholes can still be very different. But in general Kou path tends to coincide with Black-Scholes path when lambda goes to zero. We visualize this simple mathematical observation.

2. Pricing

In the original paper of Kou, Kou writes down the explicitly closed formula of Eurepean option pricing. It is expressed in terms of so call Hh_n functions. The expression involves several infinite series and cdf's of normal distributions, which makes the practical computation cumbersome, and even inaccurate when volatility and jump rate are large. Kou later together with his coauthors proposed to use Laplace transform (which can be written explicitly) and two-sided Euler inversion algorithm to compute the price. With this idea, we construct functions for pricing. Compare its accuracy with Monte-Carlo methods. Moreover, we illustrate how the call option price changes under varying strikes and different jump rates. As one may expect, when jump rate vanishes the curve will coincide with the one given by Black-Scholes pricing.

3. Hedging

Fortunately we don't have to compute the delta of call option price analytically. The Laplace transform of delta is again given explicitly and simple, hence we just need to do inversion. Given delta, we implement delta hedging. But as we mentioned at the beginning, delta hedging is not perfect as it is under Black-Scholes. The jump residuals remain under delta hedging, that's the drawback of the original delta hedging in jump diffusion model proposed by Merton (1976). We instead follow the idea of Grünewald, B. and Trautmann, S., apply the local risk-minimizing (LRM) hedging strategy. Roughly speaking, one can define the expected quadratic hedging error which measures the risk and we want to minimize it. Given that the only asset we are using is stock, we can't do it directly, but we can do it locally, namely during a small time interval. Grünewald, B. and Trautmann, S. give a closed formula for the minimizing problem by considering it as the orthogonal projection of the discounted payoff onto a closed subspace of a Hilbert space with L^2 norm. Such LRM strategy incorporate both diffusion and jumps partly (there is a weight for both components). We identify their formula in our double exponential jump diffusion case. We implement this strategy by plotting the distribution of profit of hedged portfolio with 100 simulations, and show that it does behave better then delta hedging in some cases (depending on parameters you are chosen). Lastly, we introduce a theoretically perfect hedging strategy, which is hedging with options. That is, one can hold two assets, one stock and one other European option. In this setting, Cont, R., Tankov, P., and Voltchkova, E. (2007) gives a closed solution for minimizing the global quadratic hedging error. We use their formula for hedge ratios and compare hedging performance with previous two strategies.

4. Calibration

We import two-year data of SPY and plot its one year stock path. We calibrate Kou's pricing model to SPY market price by gradient descent. We find that with the initial parameters, LRM hedging perform worse than delta hedging. But the situation turned opposite if we use calibrated parameters. We also test leptokurtosis for SPY daily log returns. It turns out that the distribution of SPY returns has notable excess kurtosis, and has right tail slightly heavier than left tail. With calibrated parameters, we also simulate 10000 stock paths under Kou's model. Overall, they have significant kurtosis which matches the leptokurtic feature but too large, and the distribution of simulated log returns exhibits an opposite skewness.

5. Future Directions (incomplete lists)

(i) Show Kou's model captures volatility smile
(ii) Explore better calibration method and fix the above discrepancy
(iii) Kou's model has a particular advantage over Merton's model in path-dependent options (barrier, lookback, Asian), explore the model in these contexts.
(iv) Combine stochastic volatility into Kou's model.