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EC303. 2020/21.

a) i) $y(k) = \{1, 0, 0, -0.1, 1, 0, 0, \dots, 0\}$.

$X(z) = \sum_n x(n) z^{-n}$.　$Y(z) = 1 + 0\cdot z^{-1} + 0\cdot z^{-2} - 0.1 z^{-3} + z^{-4} + 0$

$= 1 - 0.1 z^{-3} + z^{-4}$.　ROC: the entire z plane except $z=0$ & $z=\infty$

ii) the minimum sampling frequency must be the twice of bandwidth of the signal. Nyquist-shannon sampling theorem. $f_s \geq 2 \times 200\,Hz = 400\,Hz > 250\,Hz$.

~~200Hz~~ Therefore, sampling frequency of 250 Hz can't be chosen because it will cause aliasing problem (overlapping of adjacent frequency components).

iii) $F(z) = \dfrac{z+1}{z^2 + 0.3z + 0.02}$

Sol: $F(z) = \dfrac{z+1}{(z+0.1)(z+0.2)} \Rightarrow \dfrac{F(z)}{z} = \dfrac{z+1}{z(z+0.1)(z+0.2)}$

let $\dfrac{F(z)}{z} = \dfrac{A}{z} + \dfrac{B}{z+0.1} + \dfrac{C}{z+0.2}$

$\Rightarrow z+1 = A(z+0.1)(z+0.2) + B\cdot z(z+0.2) + C\cdot z(z+0.1)$

① $z=0 \Rightarrow 1 = A\cdot 0.1 \cdot 0.2 + 0 + 0$.　$\therefore A = \dfrac{1}{0.02} = 50$.

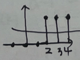
② $z=-0.1 \Rightarrow 0.9 = 0 + B\cdot(-0.1)(0.1) + 0$　$\therefore B = \dfrac{0.9}{-0.01} = -90$.

③ $z=-0.2 \Rightarrow 0.8 = 0 + 0 + C\cdot(-0.2)(-0.1)$　$\therefore C = \dfrac{0.8}{0.02} = 40$.

$\therefore \dfrac{F(z)}{z} = \dfrac{50}{z} + \dfrac{-90}{z+0.1} + \dfrac{40}{z+0.2}$　$\therefore F(z) = 50 - 90\dfrac{z}{z+0.1} + 40\dfrac{z}{z+0.2}$.

$\therefore f(k) = 50\delta(k) - 90(-0.1)^k + 40(0.2)^k$ ✗　$f(k) = 50\delta(k) - 90(-0.1)^k u(k)$ ~~$+40(0.2)u(k)$~~ $+40(0.2)^k u(k)$

b) $f(k) = \begin{cases} 4 & k = 2, 3, \dots \\ 0 & \text{otherwise.} \end{cases}$



~~Finite. Signal causal~~

~~$f(k) = \sum_{k=2}^{\infty}$~~　~~$X(z) = \sum_n x(n) z^{-1}$.~~　~~$F(z) = \sum f(k) z^{-k}$~~　$F(z) = \sum_{k=2}^{\infty} f(k) z^{-k}$

~~$f(k-2) = \begin{cases} 4 & k = 0, 1, 2, \dots \\ 0 & \text{otherwise.} \end{cases}$~~　~~$F(z) = \sum_{k=2} \sum_{k=0}^{\infty}$~~　~~let $f(k-2) = f(m)$~~　$m = k-2$.

~~$F_m(z) = \sum_{m=0}^{\infty} f(m) z^{-m} = \sum_{m=0}^{\infty} 4 z^{-m}$~~　$\because \sum_{k=0}^{\infty} ar^k = \dfrac{a}{1-r}$.　$a = 4$, $r = z^{-1}$, $k = m$.

~~$\therefore F_m(z) = \dfrac{4}{1-z^{-1}}$~~　~~$F_k(z)$ $f(k) = f(m+2)$~~

~~$\because x(n+1) \xleftrightarrow{Z} zX(z) - zx(0)$　$\therefore F(z) = z\cdot F_m(z) - z\cdot f(0)$~~



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$f(k)=\begin{cases}4 & k=2,3,\dots \\ 0 & \text{otherwise.}\end{cases}$

Sol: $\because X(z)=\sum_n x(n)z^{-n}$. $\therefore F(z)=\sum_k f(k)z^{-k}$.

$F(z)=\sum_{k=2}^{\infty} f(k)z^{-k}$.

~~F(z)=~~ $\because \boxed{x(n-k) \xleftrightarrow{\mathcal{Z}} z^{-k}\cdot X(z)}$

$\therefore f(k-2) \xleftrightarrow{\mathcal{Z}} z^{-2}F(z)=z^{-2}\cdot\sum_{k-2=0}^{\infty} f(k-2)z^{-(k-2)}$

$\therefore \mathcal{Z}\{f(k-2)\}=z^{-2}\cdot\sum_{k-2=0}^{\infty} 4\cdot z^{-k-2}$.

$\because \boxed{\sum_{k=0}^{\infty} ar^k=\frac{a}{1-r}}$

$\therefore \sum_{k-2=0}^{\infty} 4\cdot(z^{-1})^{k-2}=\frac{4}{1-z^{-1}}=\frac{4z}{z-1}$

$\therefore \mathcal{Z}\{f(k-2)\}=z^{-2}\cdot\frac{4z}{z-1}=4\frac{z^{-1}}{z-1}=4\frac{1}{z(z-1)}$

$\mathcal{Z}\{f(k)\}=z^2\cdot\mathcal{Z}\{f(k-2)\}=z^2\cdot\frac{4z}{z-1}=\frac{4z^3}{z-1}$

c) $x(k)-ax(k-1)=u(k)$ $-1<a<1$, $x(k)=0$ for $k<0$, $u(k)$ sampled step.

i) Take the z-transform of both sides: $X(z)-a\cdot z^{-1}\cdot X(z)=U(z)=\frac{z}{z-1}$

$\therefore (1-az^{-1})X(z)=\frac{z}{z-1}$ $\therefore X(z)=\frac{z}{z-1}\cdot\frac{1}{1-az^{-1}}=\frac{z}{z-1}\cdot\frac{z}{z-a}=\frac{z^2}{(z-1)(z-a)}$

ii) $\because X(z)=\frac{z^2}{(z-1)(z-a)} \Rightarrow \frac{X(z)}{z}=\frac{z^2}{z(z-1)(z-a)}=\frac{A}{z}+\frac{B}{(z-1)}+\frac{C}{(z-a)}$.

$\Rightarrow z^2=A(z-1)(z-a)+B\cdot z(z-a)+C\cdot z(z-1)$

① $z=0$: $0=A(-1)(-a)$ $A=0$.

② $z=1$: $1=B\cdot 1\cdot(1-a)$ $B=\frac{1}{1-a}$

③ $z=a$: $a^2=C\cdot a\cdot(a-1)$ $C=\frac{a^2}{a^2-a}=\frac{a}{a-1}$

$\therefore \frac{X(z)}{z}=\frac{1}{1-a}\cdot\frac{1}{z-1}+\frac{a}{a-1}\cdot\frac{1}{z-a}$

$\therefore X(z)=\frac{1}{1-a}\cdot\frac{z}{z-1}+\frac{a}{a-1}\cdot\frac{z}{z-a}$.

$\therefore x(k)=\frac{1}{1-a}\underbrace{(1)^k}_{u(k)}+\frac{a}{a-1}\cdot\underbrace{a^k}_{u(k)}=\frac{1}{1-a}\cdot u(k)+\frac{a}{a-1}\cdot a^k\cdot u(k)$

iii) final value theorem.

$f(\infty)=\lim_{z\to1}(z-1)F(z)=\lim_{z\to1}\left(\frac{z-1}{z}\right)F(z)=\lim_{z\to1}(1-z^{-1})F(z)$

$f(\infty)=\lim_{z\to1}(z-1)F(z)=\lim_{z\to1}(z-1)\frac{z^2}{(z-1)(z-a)}=\lim_{z\to1}\frac{z^2}{z-a}=\frac{1}{1-a}$



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a) $\theta(z) = \dfrac{k(z-0.5)}{z^2-z+0.25+k} V(z)$

Characteristic equation. : $z^2-z+0.25+k=0$.

$\because z = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

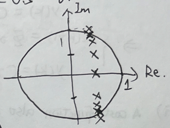
① $k=0$. $z^2-z+0.25=0$. $\Rightarrow z=\frac{1}{2}$

② $k=0.25$. $z^2-z+0.5=0$. $\Rightarrow z_1=0.5+0.5i \quad z_2=0.5-0.5i$.

③ $k=0.5$. $z^2-z+0.75=0$. $\Rightarrow z_1=0.5+0.7071i \quad z_2=0.5-0.7071i$.

④ $k=0.75$ $\quad z^2-z+1=0$. $\Rightarrow z_1=0.5+0.866i \quad z_2=0.5-0.866i$.

⑤ $k=1$ $\quad z^2-z+1.25=0 \Rightarrow z_1=0.5+i \quad z_2=0.5-i$.




The stability of the system is determined by the the location of the root. When the roots are outside the unit circle, the system is unstable.

when $k=0.75$. the two roots are on the unit circle. (circle. it's critical stability)

when $k=1$ the two roots are outside the ~~cir~~ unit circle.

Therefore ~~whe k>0.75 the system goes unstable.~~

~~between 0.75 k=0.75 and k=2.~~

the System goes unstable between $k=0.75$ and $k=1$.

b). $P(s) = \dfrac{1}{2s+1} e^{-2s}$.

Sol: $ZOH(s) = \dfrac{1-e^{-sT}}{s}$ $\quad P(z) = \mathcal{Z}\left[\dfrac{1-e^{-sT}}{s}\cdot P(s)\right] = (1-z^{-1})\mathcal{Z}\left[\dfrac{P(s)}{s}\right]$

$T=1 \therefore P(z) = \mathcal{Z}\left[\dfrac{1-e^{-s}}{s}\cdot\dfrac{e^{-2s}}{2s+1}\right]$ $\qquad \dfrac{P(s)}{s} = \dfrac{\frac{1}{2}}{s(s+\frac{1}{2})}\cdot e^{-2s}$.

$= \mathcal{Z}\left[\dfrac{e^{-2s}}{s(2s+1)}\right]\cdot(1-z^{-1})$ $\qquad \mathcal{L}^{-1}\left\{\dfrac{P(s)}{s}\right\} = (1-e^{-\frac{1}{2}(t-2)})\delta(t-2)$

$\mathcal{Z}\{(1-e^{-\frac{1}{2}(t-2)})\delta(t-2)\}$

$= (1-z^{-1})\,\mathcal{Z}\left[\dfrac{\frac{1}{2}e^{-2s}}{s(s+\frac{1}{2})}\right] = \mathcal{Z}\{1-e^{-\frac{1}{2}t}\}\cdot z^{-2} = \dfrac{z(1-e^{-\frac{1}{2}})}{(z-1)(z-e^{-\frac{1}{2}})}\cdot z^{-2}$

$\mathcal{Z}\left[\frac{1}{2}\left(\dfrac{1}{s+\frac{1}{2}}\right)e^{-2s}\right] = \frac{1}{2}\mathcal{Z}\left[\dfrac{1}{s+\frac{1}{2}}\cdot e^{-2s}\right]$ $\quad \cancel{\mathcal{L}^{-1}\left(\dfrac{1}{s+\frac{1}{2}}\cdot e^{-2s}\right) = e^{-\frac{1}{2}k}\cdot\delta(t-2)}$ ×

$\cancel{\mathcal{L}^{-1}\{P(s)\} = p(t)}$ $\quad \cancel{\mathcal{L}^{-1}\left\{\dfrac{1}{2s+1}e^{-2s}\right\} = \mathcal{L}^{-1}\left\{\dfrac{1}{2s+1}\right\}}$ $\quad \cancel{\mathcal{L}\{x(t-t_0)\} = X(s)e^{-st_0}}$

$\cancel{\therefore \mathcal{L}\{p(t-2)\} = \dfrac{1}{2s+1}\cdot e}$ $\quad \cancel{\therefore p(t) = \frac{1}{2}\mathcal{L}^{-1}\left\{\left(\dfrac{1}{s+\frac{1}{2}}\right)e^{-2s}\right\} = \frac{1}{2}e^{-\frac{1}{2}(t-2)}\delta(t-2)}$

$\therefore P(z) = (1-z^{-1})\cdot\dfrac{z(1-e^{-\frac{1}{2}})}{(z-1)(z-e^{-\frac{1}{2}})}\cdot z^{-2} = \dfrac{z-1}{z}\cdot\dfrac{z(1-e^{-\frac{1}{2}})}{(z-1)(z-e^{-\frac{1}{2}})}\cdot z^{-2} = \dfrac{1-e^{-\frac{1}{2}}}{z-e^{-\frac{1}{2}}}\cdot z^{-2}$



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input voltage $u$. output current $y$.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-\frac{1}{R_1C} & 0\\ 0 & -\frac{R_2}{L}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{R_1C}\\ \frac{1}{L}\end{bmatrix}u.$$

$\dot{x} = A\vec{x} + B\vec{u}$.
$\vec{y} = C\vec{x} + D\vec{u}$.

$$y = \begin{bmatrix}-\frac{1}{R_1} & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \frac{1}{R_1}u.$$

uncontrollable matrix given by: $|Q_c| = |[B \;\; AB]| = 0$.

where $A = \begin{bmatrix}-\frac{1}{R_1C} & 0\\ 0 & -\frac{R_2}{L}\end{bmatrix}$ $B = \begin{bmatrix}\frac{1}{R_1C}\\ \frac{1}{L}\end{bmatrix}$.

$$AB = \begin{bmatrix}-\frac{1}{R_1C}\cdot\frac{1}{R_1C}+0\\ 0-\frac{R_2}{L}\cdot\frac{1}{L}\end{bmatrix} = \begin{bmatrix}-\frac{1}{(R_1C)^2}\\ -\frac{R_2}{L^2}\end{bmatrix}$$

$$\therefore |[B \;\; AB]| = 0. \Rightarrow \left|\begin{bmatrix}\frac{1}{R_1C} & -\frac{1}{(R_1C)^2}\\ \frac{1}{L} & -\frac{R_2}{L^2}\end{bmatrix}\right| = \frac{1}{R_1C}\cdot\left(-\frac{R_2}{L^2}\right) + \frac{1}{(R_1C)^2}\cdot\frac{1}{L}$$

$$= -\frac{R_2}{R_1CL^2} + \frac{1}{(R_1C)^2L} = 0.$$

$$\frac{R_2}{R_1CL^2} = \frac{1}{R_1^2C^2L} \Rightarrow R_1^2R_2C^2L = R_1CL^2.$$

$$R_1R_2C = L \Rightarrow R_1\cdot R_2 = \frac{L}{C}$$

d) $f(x,y,z) = \frac{1}{2}(x^2+y^2+z^2)$

s.t. $g_1(x,y,z) = x-y=0$.
$g_2(x,y,z) = x+y+z-1=0$.

$$\mathcal{L}(x,y,z,\lambda_1,\lambda_2) = f(x,y,z) + \lambda_1 g_1(x,y,z) + \lambda_2 g_2(x,y,z)$$
$$= \frac{1}{2}(x^2+y^2+z^2) + \lambda_1(x-y) + \lambda_2(x+y+z-1)$$

$\frac{\partial\mathcal{L}}{\partial x} = x+\lambda_1+\lambda_2 = 0$. ①

$\frac{\partial\mathcal{L}}{\partial y} = y+(-\lambda_1)+\lambda_2 = 0$. ②

$\frac{\partial\mathcal{L}}{\partial z} = z+\lambda_2 = 0$. ③ $\Rightarrow z = -\lambda_2$.

$\frac{\partial\mathcal{L}}{\partial \lambda_1} = x-y=0$. ④ $\Rightarrow x=y$

$\frac{\partial\mathcal{L}}{\partial \lambda_2} = x+y+z-1=0$. ⑤.

From ①,②,④ $\Rightarrow$ $2\lambda_1 = 0$ $\therefore \lambda_1 = 0$.
~~From ①,②~~ $x+\lambda_2 = 0$.
$\because z+\lambda_2 = 0$.
$\therefore x-z=0$.
$\therefore x=y=z$.
$\therefore$ in ⑤. $3x-1=0$
$\therefore x=y=z=\frac{1}{3}$.
$\therefore \lambda_2 = -\frac{1}{3}$

Solution: $x^* = \frac{1}{3}$ $y^* = \frac{1}{3}$ $z^* = \frac{1}{3}$ $\lambda_1^* = 0$ $\lambda_2^* = -\frac{1}{3}$.

$$\therefore X^* = [x,y,z]^T = \left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right]^T$$



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3. a) $L\frac{di(t)}{dt} = v_L(t)$ $\quad v_{(R)}(t) = Ri(t)$ $\quad C\frac{dv_c(t)}{dt} = i(t).$

$$v_c(t) + v_L(t) + v_R(t) = v(t).$$

i) output $y(t) = i(t)$. input $u(t) = v(t)$.

state-space. $\dot{x}\cancel{(t)} = A\vec{x} + B\vec{u}.$

$$\vec{y} = C\vec{x} + D\vec{u}.$$

state variables $\quad x(t) = \begin{bmatrix} i(t) \\ v_c(t) \end{bmatrix} \quad \therefore \dot{x} = \frac{d}{dt}\begin{pmatrix} i(t) \\ v(t) \end{pmatrix}.$

$\therefore \frac{di(t)}{dt} = \frac{1}{L}v_L(t) \qquad \frac{dv_c(t)}{dt} = \frac{1}{C}i(t) \qquad v_L(t) = v(t) - \cancel{v_c(t)} - v_R(t)$

$$= v(t) - v_c(t) - Ri(t).$$

$$\therefore \frac{di(t)}{dt} = \frac{-R}{L}\cdot i(t) - \frac{1}{L}v_c(t) + \frac{1}{L}v(t).$$

$$\therefore \dot{x} = \frac{d}{dt}\begin{pmatrix} i(t) \\ v(t) \end{pmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u(t).$$

$$\dot{x} = A \cdot \vec{x} + B \cdot \vec{u}.$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

$$y = C \cdot \vec{x}$$

ii) $\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \xrightarrow{\text{Laplace.}} \begin{cases} sX(s) = AX(s) + BU(s) \\ Y(s) = CX(s) \end{cases} \Rightarrow \begin{cases} (sI_n - A)X(s) = BU(s) \\ Y(s) = CX(s) \end{cases}$

so that $X(s) = \cancel{(sI)}(sI_n - A)^{-1}U(s) \quad \therefore Y(s) = C\cdot(sI_n - A)^{-1}U(s)$

$\frac{Y(s)}{U(s)} = C\cdot(sI_n - A)^{-1} \quad \therefore$ open-loop zero is: $\begin{cases} (sI_n - A)X(s) - BU(s) = 0. \\ CX(s) = 0. \end{cases}$

$$\therefore \begin{bmatrix} sI_n - A & -B \\ C & 0 \end{bmatrix}\begin{bmatrix} X(s) \\ U(s) \end{bmatrix} = 0. \Rightarrow \begin{vmatrix} sI_n - A & -B \\ C & 0 \end{vmatrix} = 0.$$

iii) in i) $A = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \quad B = \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} \quad \begin{vmatrix} sI - A & -B \\ C & 0 \end{vmatrix} = 0 \cancel{= B C = 0}.$

$C = (1 \quad 0)$

$\cancel{\therefore B \cdot C}$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} = \begin{bmatrix} s + \frac{R}{L} & \frac{1}{L} \\ -\frac{1}{C} & s \end{bmatrix} \qquad s = 0.$$

$\cancel{s_1 = -(\frac{R}{L} + \frac{1}{L})}$

$$\begin{vmatrix} sI - A & -B \\ C & 0 \end{vmatrix} = \begin{vmatrix} s + \frac{R}{L} & \frac{1}{L} & \frac{1}{L} \\ -\frac{1}{C} & s & 0 \\ 1 & 0 & 0 \end{vmatrix} = 0 = (s + \frac{R}{L})(0) - \frac{1}{L}\cdot(0) + \frac{1}{L}(s) = \cancel{s^2 + (\frac{R}{L} + \frac{1}{L})s = 0.}$$

$\cancel{\therefore s_1 = 0.}$



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b) ? $s \to \frac{z-1}{T}$　　$\dot{x}(t) = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u(t).$

$\dot{x}(t) = Ax(t) + Bu(t)$

$y(t) = Cx(t)$　　$y(t) = [1 \quad 0]\, x(t).$

Continuous → discrete

$z = e^{sT}$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$z = 1 + sT + \frac{(sT)^2}{2!} + \cdots$

$T^2 \to 0.$

$z = 1 + sT$

$s = \frac{z-1}{T}$

Laplace $\Rightarrow \begin{cases} sX(s) = AX(s) + BU(s) \\ Y(s) = CX(s) \end{cases} \Rightarrow \begin{cases} \frac{z-1}{T} X(z) = AX(z) + BU(z) \\ Y(z) = CX(z). \end{cases}$

$\Rightarrow zX(z) - X(z) = ATX(z) + BTU(z)$

$zX(z) = (I_n + TA)X(z) + BTU(z).$

by taking inverse $z$ transform:

$x(k+1) = (I_n + TA)\cancel{x}\, x(k) + TBu(t).$

$y(k) = Cx(k)$

$\Rightarrow \begin{cases} x(k+1) = \Phi x(k) + \cancel{\Phi}\, \Gamma u(t) \\ y(k) = Cx(k). \end{cases}$　　where $\Phi = I_n + TA$

$\Gamma = TB.$

c) i) A cost function also known as an objective function or loss function. ~~It produ~~ It provides a scalar value that quantifies the performance of a solution or a model. It guides the optimisation ~~problem~~ algorithm towards the optimal solution. By ~~maximum or minimum~~ minimizing / maximizing the cost function, the algorithm iteratively improves the solution.

ii) An isopleth is a line on a diagram that represents a constant value of a particular variable. In the context of optimization, an isopleth typically connects points where the function takes the same value.

iii) ① necessary: $f'(x) = 0$.　② sufficient $f''(x) > 0$　$x^* \to$ minimum.

$f''(x) < 0$　$x^* \to$ maximum.

iv) $\max f(x) \Longleftrightarrow \min -f(x).$

Proof: Assume: ~~$x^*$ is the max~~ $\max f(x)$ is $f(x^*)$

$\Rightarrow f(x^*) \geq f(x)$

For the minimization of $-f(x)$.

$-f(x^*) \leq f(x).$

$x^*$ which maximizes $f(x)$ also minimizes $-f(x)$.



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