a) ~~$u(t) = k_I \int_0$~~ An integral control will eliminate the steady-state error for a constant input.

$z = 1 + sT \quad s = \frac{z-1}{T}$

b) $C(s) = \frac{U(s)}{E(s)} = k_P + \frac{k_I}{s} + k_D \cdot s$ ~~$C(z) = k_P + k_I \cdot \frac{z}{z-1} + k_D \cdot \frac{z-1}{T}$~~

assuming fast sampling. $s = \frac{z-1}{T}$

$$\therefore C(z) = k_P + k_I \cdot \frac{T}{z-1} + k_D \cdot \frac{z-1}{T}$$

c) i) $P(z) = \mathcal{Z}\left[\frac{1-e^{-sT}}{s} \cdot \frac{s+10}{s}\right] = (1 - z^{-1})\mathcal{Z}\left[\frac{P(s)}{s}\right]$

$\frac{P(s)}{s} = \frac{s+10}{s^2} = \frac{1}{s} + \frac{10}{s^2}$ ~~$\mathcal{L}^{-1}\left\{\frac{P(s)}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{10}{s^2}\right\}$~~

$$(1-z^{-1})\mathcal{Z}\left\{\frac{1}{s} + \frac{10}{s^2}\right\} = (1-z^{-1})\left\{\frac{z}{z-1} + 10\frac{z}{(z-1)^2}\right\}$$

$$= \frac{z-1}{z} \cdot \frac{z}{z-1} + \frac{z-1}{z} \cdot 10\frac{z}{(z-1)^2}$$

$$= 1 + \frac{10}{z-1} = \frac{z+9}{z-1}$$

ii) ~~$\frac{Y(z)}{R(z)} = G(\cdot$~~

$H(z) = 1$

$$\frac{Y(z)}{R(z)} = \frac{C(z) \cdot P(z)}{1 + C(z) \cdot P(z)} \Rightarrow C(z) = \mathcal{Z}\{k\} = k.$$

$$= \frac{k \cdot \frac{z+9}{z-1}}{1 + k\frac{z+9}{z-1}} = k \cdot \frac{z+9}{z-1} \cdot \frac{z-1}{z-1+k(z+9)}$$

$$= \frac{k(z+9)}{z-1+k(z+9)} = \frac{z+9}{\frac{1}{k}(z-1)+(z+9)}$$

iii) ~~$y_{final} = \lim_{z\to1}(z-1)Y(z).$~~

~~$= \lim_{z\to1}(z-1)\frac{k(z+9)}{z-1+k(z+9)}$~~

~~characteristic~~

characteristic equation.

$\frac{1}{k}(z-1) + (z-9) = 0.$

① $z_1 = -1 \quad z_2 = -9.$

$\frac{1}{k}z - \frac{1}{k} + z - 9 = 0.$

$z(\frac{1}{k}+1) = (\frac{1}{k}+9)$

$z = \frac{1+9k}{k} \cdot \frac{k}{1+k} = \frac{1+9k}{1+k}.$

$|z| \le 1$

$\left|\frac{1+9k}{1+k}\right| \le 1$

$\therefore |1+9k| \le |1+k|$

$(1+9k)^2 \le (1+k)^2.$

$81k^2 + 18k + 1 \le k^2 + 2k + 1$

$80k^2 + 16k \le 0.$

$k(80k+16) \le 0.$

$k = 0$

$k = -\frac{16}{80} = -0.2$

$\therefore -0.2 \le k \le 0.$

~~$\therefore 1+9$~~

a) i) $G(z) = \frac{Y(z)}{U(z)} = \frac{3(z-0.4)}{(z+0.2)(z+0.5)(z+0.9)}$

$\frac{G(z)}{z} = \frac{3(z-0.4)z}{(z+0.2)(z+0.5)(z+0.9)} = \frac{A}{z+0.2} + \frac{B}{z+0.5} + \frac{C}{z+0.9}$

$\therefore 3(z-0.4) = A(z+0.5)(z+0.9) + B(z+0.2)(z+0.9) + C(z+0.2)(z+0.5)$

① $z=-0.2$. $-0.6 \cdot (-0.6) = A \cdot 0.3 \cdot 0.7 + 0 + 0 \Rightarrow$ ~~$A = -\frac{60}{7} = -8.5714$~~ $A = \frac{12}{7} \approx 1.7143$

② $z=-0.5$. $-1.5 \cdot (-0.9) = 0 + B(-0.3) \cdot 0.4 + 0 \Rightarrow$ ~~$B = \frac{45}{2} = 22.5$~~ $B = -\frac{45}{4} = -11.25$

③ $z=-0.9$. $-2.7(-1.3) = 0 + 0 + C(-0.7)(-0.4) \Rightarrow C = \frac{351}{28} \approx 12.5357$

$\therefore \frac{G(z)}{z} = \frac{12}{7} \frac{1}{z+0.2} - \frac{45}{4} \frac{1}{z+0.5} + \frac{351}{28} \cdot \frac{1}{z+0.9}$

$\therefore G(z) = \frac{12}{7} \frac{z}{z+0.2} - \frac{45}{4} \frac{z}{z+0.5} + \frac{351}{28} \cdot \frac{z}{z+0.9}$

$\therefore g(k) = \frac{12}{7}(-0.2)^k u(k) - \frac{45}{4}(-0.5)^k u(k) + \frac{351}{28} \cdot (-0.9)^k u(k)$

~~$y(k) = g(k) \cdot u(k)$~~

ii) $u(kT) = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$ $y_0(0) = 0$.

3.a) i) $G(z) = \frac{Y(z)}{U(z)} = \frac{3(z-0.4)}{z^3+1.62z^2+0.73z+0.09} = \frac{3(z-0.4)}{(z+0.2)(z+0.5)(z+0.9)}$

$\Rightarrow (z^3+1.62z^2+0.73z+0.09)Y(z) = 3(z-0.4)U(z)$.

$z^3 Y(z) + 1.62z^2 Y(z) + 0.73 z Y(z) + 0.09 Y(z) = 3z U(z) - 1.2U(z)$.

Transform it into difference equation in time domain:

$y(k+3) + 1.62y(k+2) + 0.73y(k+1) + 0.09y(k) = 3u(k+1) - 1.2u(k)$.

ii) ~~let $k=0$: $y(3)+1.62y(2)+0.73y(1)+0.09y(0) = 3u(1) - 1.2u(0)$~~

~~$y(3)+1.62y(2)+0.73y(1) = 3 - 1.2 = 1.8$.~~

~~$0.73y(1) = 1.8 - y(3) - 1.62y(2)$.~~

$Y(z) = \frac{3(z-0.4)}{(z+0.2)(z+0.5)(z+0.9)} \cdot U(z) = \frac{3(z-0.4)}{(z+0.2)(z+0.5)(z+0.9)} \cdot \frac{z}{z-1}$

$\frac{Y(z)}{z} = \frac{3(z-0.4)}{(z+0.2)(z+0.5)(z+0.9)(z-1)} = \frac{A}{z+0.2} + \frac{B}{z+0.5} + \frac{C}{z+0.9} + \frac{D}{z-1}$

① $z=-0.2$: $3 \cdot (-0.6) = A(0.3) \cdot 0.7 \cdot (-1.2)$ $A = \frac{50}{7}$.

② $z=-0.5$: $3 \cdot (-0.9) = B(-0.3) \cdot 0.4 (-1.5)$ $B = -15$.

③ $z=-0.9$: $3 \cdot (-1.3) = C(0.7)\cdot(-0.4)(-1.9) = -\frac{975}{133}$

④ $z=1$ $3 \cdot 0.6 = D(1.2 \times 1.5 \times 1.9) = \frac{10}{19}$.

$y(k) = \frac{50}{7}(-0.2)^k u(k) - 15(-0.5)^k u(k) - \frac{975}{133}(-0.9)^k u(k) + \frac{10}{19} 1^k u(k)$

$y(1) =$

b) i) $T(z) = \frac{G(z)D(z)}{1+G(z)D(z)} = \frac{3(z-0.4)}{(z+0.2)(z+0.5)(z+0.9)} \cdot \frac{0.15(z+0.9)}{z-0.4} \cdot \frac{1}{1+G(z)D(z)}$

$= \frac{0.45}{(z+0.2)(z+0.5)} \cdot \frac{(z+0.2)(z+0.5)}{(z+0.2)(z+0.5)+0.45}$

$= \frac{0.45}{(z+0.2)(z+0.5)+0.45} = \frac{0.45}{z^2+0.7z+0.55}$

ii) characteristic equation: $z^2+0.7z+0.55=0$.

$x_1 = -\frac{7}{20} + j\frac{3\sqrt{19}}{20} \doteq -0.35+j0.6538$

$x_2 = -\frac{7}{20} - j\frac{3\sqrt{19}}{20} \doteq -0.35-j0.6538$.

$\sqrt{(\frac{7}{20})^2+(\frac{3\sqrt{19}}{20})^2} = \frac{\sqrt{55}}{10} \doteq 0.7416 < 1$. $\therefore$ stable.

iii) for a unit step input $R(z) = \frac{z}{z-1}$

~~the steady-state error can be found by using the final value theorem~~

~~in z domain:~~ $\cancel{e_{ss} = \lim_{z\to 1}(z-1)R(z)}$.

~~The error signal $E(z) = R(z) - Y(z)$. $Y(z) = T(z) \cdot R(z)$.~~

~~Determine the steady-state error for a unit step input:~~

$\cancel{e_{ss} = \frac{1}{1+K_p}}$

Final Value ~~Theorth~~ Theorem: $y_{final} = \lim_{z\to 1}(z-1)Y(z)$.

steady-state error: $e[n] = r[n] - y[n]$.

$E(z) = R(z) - Y(z)$.

unit step: $R(z) = \frac{z}{z-1}$. $Y(z) = T(z) \cdot R(z)$.

$E(z) = R(z)(1-T(z))$

$= \frac{z}{z-1}\left(1 - \frac{0.45}{z^2+0.7z+0.55}\right)$

$= \frac{z}{z-1} \cdot \frac{z^2+0.7z+0.1}{z^2+0.7z+0.55}$

$e_{ss} = \lim_{z\to 1}(z-1)E(z) \Rightarrow e_{ss} = \lim_{z\to 1} z \cdot \frac{z^2+0.7z+0.1}{z^2+0.7z+0.55}$.

$= \frac{1+0.7+0.1}{1+0.7+0.55} = \frac{4}{5} = 0.8$.

c). ① Accuracy and Precision.

Digital computers can handle complex ~~comp~~ mathmatical computations with high precision and accuracy. It can process feedback and adjust outputs with minimal error.

②. Flexibility and Programmability.

~~It easy modifi~~ Digital computers are highly ~~flexibit~~ flexible and programmable allowing for easy modifications and updates to control ~~system~~ algorithms. adjusted or optimized as system requirements change or as new information becomes available.

③. Integration and Scalability.

Digital computers can seamlessly integrate with other digital systems and components. Digital computer systems are ~~scalability~~ scalable, they can be expanded or modified to accommodate larger or more complex systems, without significant changes to the underlying hardware or software architecture.

4. a) i) necessary: $f'(x)=0$. sufficient $f''(x)>0 \Rightarrow x^*$ minimum.

$f''(x)<0 \Rightarrow x^*$ maximum.

ii). $f(x)=C^Tx+\frac{1}{2}x^TQx$.

$f'(x)=Qx+C$.

$f''(x)=Q$.

$x^{k+1}=x^k-[\nabla^2 f'(x^k)]^{-1}\nabla f(x^k)$

$x^1=x^0-Q^{-1}(Qx^0+C)=-Q^{-1}C$

$x^2=x^1-Q^{-1}(Qx^1+C)=-Q^{-1}C$

$x^*=-Q^{-1}C$.

iii) $\cancel{\nabla f(x^{k+1})\ \nabla f(x^k)=}$ ~~For~~

~~For a quadratic function~~ $\cancel{f(x)=\frac{1}{2}x^TQx.}$ ~~let~~ $\cancel{p^k=\nabla f(x^k) \Rightarrow \nabla f(x^k)=Qx^k}$.

$\cancel{\nabla f(x^{k+1})=Q(x^k+\lambda p^k)}$ $\nabla f(x^{k+1})=Qx^{k+1}$

$\cancel{=Q(x^k+\lambda Qx^k)}$ ~~Gradient vectors:~~ $\cancel{S=\frac{\nabla f(x)}{\|\nabla f(x)\|}=\frac{P}{\|\nabla f(x)\|}}$

$\cancel{=Qx^k+\lambda Q^2x^k}$ $s^k \perp s^{k+1}$.

$\cancel{\lambda^*=\frac{(p^k)^T\cdot p^k}{(p^k)^T A\cdot p^k}=\frac{(x^k)^TQ^2x^k}{(x^k)^TQ^3x^k}}$

4.a) iii) Proof: For quadratic function $f(x)=\frac{1}{2}x^TQx$. $\nabla f = Qx$.

gradient vectors direction $S_k = \frac{\nabla f(x^k)}{\|\nabla f(x^k)\|} = \frac{Qx^k}{\|Qx^k\|}$

$x^{k+1} = x^k + \lambda \nabla f(x^k)$

$p^k = \nabla f(x^k) = Qx^k$

$\lambda = \frac{(p^k)^T p^k}{(p^k)^T A p^k}$

$A = \nabla^2 f(x^k)$ Hesse matrix

$\lambda = \frac{(x^k)^T Q^2 x^k}{(x^k)^T Q^3 x^k}$

$\nabla f(x^{k+1}) = Q(x^k + \lambda Q x^k)$

$\nabla f(x^k) \cdot \nabla f(x^{k+1})^T$

$= \left[Qx^k \pm Q \cdot \frac{(x^k)^T Q^2 x^k}{(x^k)^T Q^3 x^k} Q x^k\right]^T \cdot Qx^k$

$= (x^k)^T Q^2 x^k \pm \frac{(x^k)^T Q^2 x^k}{(x^k)^T Q^3 x^k}(x^k)^T Q^3 x^k$

$= (x^k)^T Q^2 x^k \pm (x^k)^T Q^2 x^k = 0.$

$\nabla f(x^{k+1}) = Qx^{k+1} = Q(x^k + \alpha s^k) = Q(x^k + \frac{\alpha Q x^k}{\|Qx^k\|})$.

$s^k \perp s^{k+1}$ requires $(s^k, s^{k+1}) = 0$ or $[\nabla f(x^k)^T]\nabla f(x^{k+1}) = 0$.

$\alpha^k = \frac{(x^k)^T Q^2 x^k}{(x^k)^T Q^3 x^k}\|Qx^k\|$

$\nabla f(x^k)^T \cdot \nabla f(x^{k+1})$

$= (x^k)^T Q \left[Qx^k - \frac{(x^k)^T Q^2 x^k}{(x^k)^T Q^3 x^k} Q^2 x^k\right]$

$= (x^k)^T Q^2 x^k - (x^k)^T Q \frac{(x^k)^T Q^2 x^k}{(x^k)^T Q^3 x^k} Q^2 x^k$

$= (x^k)^T Q^2 x^k - (x^k)^T Q^2 x^k = 0.$

---

b) new tower (x,y).

$(10,45)$ $(15,25)$ $(20,10)$ $(55,20)$

$f(x,y) = \sqrt{(x-10)^2+(y-45)^2} + \sqrt{(x-15)^2+(y-25)^2} + \sqrt{(x-20)^2+(y-10)^2} + \sqrt{(x-55)^2+(y-20)^2}$

min $f(x,y)$

s.t. ~~$g(x,y) \le k$~~ $g_1(x,y) = \sqrt{(x-10)^2+(y-45)^2} \le k$.

$g_2(x,y) = \sqrt{(x-15)^2+(y-25)^2} \le k$.

$g_3(x,y) = \sqrt{(x-20)^2+(y-10)^2} \le k$.

$g_4(x,y) = \sqrt{(x-55)^2+(y-20)^2} \le k$.

EC 303. 2021-2022.

a) i) $y(k) = \{1, 0, -0.5, 1, \dots\}$.

$\because X(z) = \sum_n x(n) \cdot z^{-n}$.

$\therefore Y(z) = 1 + 0 \cdot z^{-1} - 0.5 \cdot z^{-2} + z^{-3}, \dots$

~~$= 1 + 0 \cdot z^{-1}$~~

$\therefore Y(z) = 1 - 0.5 z^{-2} + z^{-3} + \dots$

ii) according Nyquist-Shannon sampling theorem:

the sample frequency $f_s \geq 2 \times f_{max}$

$\therefore f_s \geq 2 \times 100\,Hz = 200\,Hz$.

$150\,Hz < 200\,Hz$.

Hence, choosing 150 Hz ~~man~~ would lead to aliasing.

iii) Yes, a system can have more than two state space representations. Different choices of state variables can lead to different state-space models that describe the same system.

iv) $F(z) = \dfrac{z + 0.2}{(z-0.5)(z-0.6)} = \dfrac{z+0.2}{z^2 - 1.1z + 0.3}$

$$\begin{array}{r|l} & 0 \quad 1 \cdot z^{-1} \quad \frac{0.3}{1.12}\cancel{0.3} z^{-2} \\ z^2 - 1.1z + 0.3 & z + 0.2 \\ & -\ z - 1.1 + 0.3z^{-1} \\ \hline & +1.3 - 0.3z^{-1} \\ & \cancel{0.3 - 0.336z^{-1} + 0.09z^{-2}} \\ \hline & \cancel{1 + 0.036z^{-1}} \\ & \frac{0.3}{1.12} - 0.3z^{-1} + \frac{0.09}{1.12} z^{-2} \end{array}$$

Long-division: ~~wh~~ at $k=0$. $f(kT) = 0$.

$k=1$ $f(kT) = 1$.

$k=2$ $f(kT) = \frac{0.3}{1.12} = \frac{15}{56} \doteq 0.2679$

b) i) a cost function also known as an objective function. It guild the optimisation algorthm towards the ~~optimual~~ optimal solution, By maximing or minimizing the cost function, the algorthm iteratively improves the solution.

ii) An isopleth is a line on a diagram that represents a constant value of a particular variable. In the context of optimization, an isopleth typically connects points where the function takes the same value.

c) $\mathcal{L}(x, y, z, \lambda_1, \lambda_2) = f(x,y,z) + \lambda_1 g_1(x,y,z) + \lambda_2 g_2(x,y,z)$

$= \frac{1}{2}(x^2 + y^2 + z^2) + \lambda_1(x - y) + \lambda_2(x + y + z - 1)$.

$\frac{\partial f}{\partial x} = x + \lambda_1 + \lambda_2 = 0$. ①

$\frac{\partial f}{\partial y} = y - \lambda_1 + \lambda_2 = 0$ ②

$\frac{\partial f}{\partial z} = z + \lambda_2 = 0$ ③ $z = -\lambda_2$

$\frac{\partial f}{\partial \lambda_1} = x - y = 0$. ④ $x = y$

$\frac{\partial f}{\partial \lambda_2} = x + y + z - 1 = 0$ ⑤

$\lambda_1 = 0$.

From ①,② $\Rightarrow$ $x - y + 2\lambda_1 = 0 \overset{④}{\Rightarrow} \cancel{2x +} 2\lambda_1 = 0 \therefore \cancel{x = -\lambda_1 \Rightarrow}$

$x + y + 2\lambda_2 = 0 \overset{④}{\Rightarrow} 2x + \lambda_2 = 0 \therefore x = -\lambda_2 \overset{③}{\Rightarrow} x = z$.

$\therefore x = y = z$. in ⑤ $3x - 1 = 0 \Rightarrow x = \frac{1}{3} = y = z$.

~~$\therefore \lambda_1 = \frac{1}{3}$~~ $\lambda_2 = -\frac{1}{3}$

Solution: $x = y = z = \frac{1}{3}$ $\lambda_1 = 0$ $\lambda_2 = -\frac{1}{3}$