OWN Sp 2015

ECO 82800
Panel Econometrics

Midterm Exam
26 March 2015, 9:30am-11:30pm

This exam is a closed-book, closed-notes exam. Calculators without matrix functions are allowed. The exam consists of seven questions, most of them with parts. Points per question are as indicated; parts are weighted equally. The total is 100 points. Budget your time. You may answer the questions in any order, but *label them clearly and keep parts of a question together.*

Several questions refer to Stata output that is attached at the end of this exam. The data used to generate this output refer to 18 OECD countries over the period 1960-1978 (19 years). The variables are `lgaspcar` (log of gasoline demand per car), `lincomep` (log of real income per capita), `lrpmg` (log of real gasoline price), and `lcarpcap` (log of the stock of cars per capita). In the Stata output, irrelevant output is omitted.

1. (21 points)
A one-factor model is often written as $y_{it} = \alpha + X'_{it}\beta + u_{it}$ where $u_{it} = \mu_i + \nu_{it}$ for $i = 1, \dots, N$ and $t = 1, \dots, T$. X'_{it} is a $1 \times K$ row vector of explanatory variables. ν_{it} is distributed IID$(0, \sigma_\nu^2)$.
   a. Anticipating that this model will be estimated as a fixed effects model (i.e., with $\alpha = 0$), set up this model in a matrix notation.
   b. In matrix form, derive the FE estimator of β.
   c. A variation of a one-factor model is $y_{it} = d'_t\alpha_i + X'_{it}\beta + \nu_{it}$, where d'_t is a $1 \times m$ vector of individual-invariant variables (possibly including one element equal to 1 for all t) and where ν_{it} is distributed IID$(0, \sigma_\nu^2)$. In matrix form, derive the FE estimator of β for this model.

$y_i = D\alpha_i + X_i\beta + \nu_i \rightarrow y = (I_N \otimes D)\alpha + X\beta + \nu$

2. (16 points)
Examine the attached Stata output, and answer the following questions.
   a. Comment on the sign and significance of the OLS regression results: are they plausible?
   b. Comment on the sign and significance of the fixed effects (FE) regression results: are they plausible?
   c. The magnitude of the OLS estimates differs substantially from that of the FE estimates. Give the proper econometric reason for this.
   d. How would you test whether the fixed effects are a necessary part of the regression model?

Z_{it} is omitted variable

3. (16 points)
Examine the attached Stata output, and answer the following questions.
   a. The random effects (RE) model is computed with the "sa" option, referring to Swamy-Arora. What does that mean?
   b. The Stata command `xttest0` implements the Breusch-Pagan LM test for random effects. What does the test here tell us?
   c. In general, why is the LM test flawed? How can the flaw be fixed?
   d. However, the flaw does not impact the outcome of the LM test here. Therefore, given the outcome of the LM test, what can you say about the OLS results that you discussed in Q2a?

4. (20 points)

With the help of the attached Stata output, answer the following questions.

a. In general, what hypothesis does the Hausman test examine?

b. What is the result of the Hausman test in this particular application?

c. Arellano (1993) offered an alternative approach to the Hausman test. For your information, relevant formulas are given below. Given the output, what does Arellano's test imply in this particular application?

$$\begin{pmatrix} y_i^+ \\ \bar{y}_i \end{pmatrix} = \begin{pmatrix} X_i^+ & 0 \\ \bar{X}_i & \bar{X}_i \end{pmatrix} \begin{pmatrix} \beta \\ \gamma \end{pmatrix} + \begin{pmatrix} u_i^+ \\ \bar{u}_i \end{pmatrix}$$

where $y_i^+ = \left(y_{i1}^+ \dots y_{i,T-1}^+\right)'$ and $y_{it}^+ = \left(\frac{T-t}{T-t+1}\right)^{1/2} \left(y_{it} - \frac{1}{T-t}\sum_{s=t+1}^{T} y_{is}\right)$.

d. In what way does the Stata output make use of the advantage that is inherent in Arellano's approach? Precisely, what is this advantage?

5. (12 points)

Examine the attached Stata output, and answer the following questions.

a. After the OLS output, some Stata commands compute the LM1 statistic. What hypothesis does this statistic examine?

b. The value of LM1 equals 1484. How would you evaluate this value, and what does the outcome of the test imply?

c. Given the result of this test, what else would you want to test for?

6. (5 points)

Comment briefly on the following statement: "The only way to estimate a two-way random effects model is by GLS."

7. (10 points)

Hashem Pesaran designed an estimator for the following model:

$$y_{ij} = \alpha_i' d_t + \beta_i' x_{it} + e_{it}$$
$$e_{it} = \gamma_i' f_t + \epsilon_{it}$$
$$x_{it} = A_i' d_t + \Gamma_i' f_t + \nu_{it}$$

where d_t is a vector of observable individual-invariant factors, f_t is a vector of unobservable individual-invariant factors that may be correlated with d_t, and ϵ_{it} and ν_{it} are independent of each other and of f_t and d_t. β_i varies across i: $\beta_i \sim iid\left(\beta, \sigma_\beta^2\right)$. The objective is to estimate β. Pesaran offers two estimators, a CCEMG and a CCEP estimator. Explain the key insight that both estimators rely on.

```
. su lgaspcar lincomep lrpmg lcarpcap;

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
    lgaspcar |       342    4.296242    .5489071   3.380209   6.156644
    lincomep |       342   -6.139425    .6345925  -8.072523  -5.221232
       lrpmg |       342   -.5231032    .6782225  -2.896497   1.125311
    lcarpcap |       342   -9.041805    1.218896  -13.47518  -7.536176
-------------+--------------------------------------------------------
```

```
. xtreg lgaspcar lincomep lrpmg lcarpcap,fe

Fixed-effects (within) regression               Number of obs      =       342
Group variable: ctry                            Number of groups   =        18

R-sq:  within  = 0.8396                         Obs per group: min =        19
       between = 0.5755                                        avg =      19.0
       overall = 0.6150                                        max =        19

                                                F(3,321)           =    560.09
corr(u_i, Xb)  = -0.2468                        Prob > F           =    0.0000

------------------------------------------------------------------------------
    lgaspcar |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lincomep |   .6622498    .073386     9.02   0.000     .5178715    .8066282
       lrpmg |  -.3217025   .0440992    -7.29   0.000    -.4084626   -.2349424
    lcarpcap |  -.6404829   .0296788   -21.58   0.000    -.6988726   -.5820933
       _cons |    2.40267   .2253094    10.66   0.000     1.959401    2.84594
-------------+----------------------------------------------------------------
     sigma_u |  .34841289
     sigma_e |  .09233034
         rho |  .93438173   (fraction of variance due to u_i)
------------------------------------------------------------------------------
. estimates store fixed;


. xtreg lgaspcar lincomep lrpmg lcarpcap,sa

Random-effects GLS regression                   Number of obs      =       342
Group variable: ctry                            Number of groups   =        18

R-sq:  within  = 0.8363                         Obs per group: min =        19
       between = 0.7099                                        avg =      19.0
       overall = 0.7309                                        max =        19

                                                Wald chi2(3)       =   1642.20
corr(u_i, X)   = 0 (assumed)                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
    lgaspcar |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lincomep |   .5549858   .0591282     9.39   0.000     .4390967    .6708749
       lrpmg |  -.4203893   .0399781   -10.52   0.000    -.498745   -.3420336
    lcarpcap |  -.6068402    .025515   -23.78   0.000    -.6568487   -.5568316
       _cons |   1.996699    .184326    10.83   0.000     1.635427    2.357971
-------------+----------------------------------------------------------------
     sigma_u |  .19554468
     sigma_e |  .09233034
         rho |  .81769856   (fraction of variance due to u_i)
------------------------------------------------------------------------------
. estimates store random;
. xttest0;

Breusch and Pagan Lagrangian multiplier test for random effects

        lgaspcar[cid,t] = Xb + u[cid] + e[cid,t]

        Estimated results:
                     |       Var     sd = sqrt(Var)
             ---------+-----------------------------
             lgaspcar |    .301299       .5489071
                    e |   .0085249       .0923303
                    u |   .0382377       .1955447

        Test:   Var(u) = 0
                            chibar2(01) =  1465.55
                          Prob > chibar2 =   0.0000

. hausman fixed random;

                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |     fixed        random       Difference          S.E.
-------------+----------------------------------------------------------------
    lincomep |   .6622498     .5549858         .107264        .0434669
       lrpmg |  -.3217025    -.4203893        .0986868        .0186143
    lcarpcap |  -.6404829    -.6068402       -.0336428        .0151597
------------------------------------------------------------------------------
                           b = consistent under Ho and Ha; obtained from xtreg
            B = inconsistent under Ha, efficient under Ho; obtained from xtreg
```

```
    Test:  Ho:  difference in coefficients not systematic

                  chi2(3) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =       302.80
                Prob>chi2 =       0.0000
                (V_b-V_B is not positive definite)


. reg lgaspcar lincomep lrpmg lcarpcap;

      Source |       SS       df       MS              Number of obs =     342
-------------+------------------------------           F(  3,   338) =  664.00
       Model |  87.8386024     3  29.2795341            Prob > F      =  0.0000
    Residual |  14.9043581   338  .044095734           R-squared     =  0.8549
-------------+------------------------------           Adj R-squared =  0.8536
       Total |  102.742961   341  .301299005           Root MSE      =  .20999

------------------------------------------------------------------------------
    lgaspcar |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lincomep |   .8899616   .0358058    24.86   0.000     .8195313    .9603919
       lrpmg |  -.8917979   .0303147   -29.42   0.000    -.9514272   -.8321685
    lcarpcap |  -.7633727   .0186083   -41.02   0.000    -.7999754   -.7267701
       _cons |   2.391326   .1169343    20.45   0.000     2.161315    2.621336
------------------------------------------------------------------------------

. predict u,resid;
. g uu = u^2;
. quietly su uu;
. scalar den = r(sum);
. egen uc = total(u),by(cid);
. quietly g ucuc = uc^2 if year == tmin;
. scalar numA = r(sum);
. g uul = u*u[_n-1] if year ~= tmin;
(18 missing values generated)
. quietly su uul;
. scalar numB = r(sum);
. scalar A = (numA/den) - 1;
. scalar B = numB/den;
. scalar LM1 = (N*T*T)/(2*(T-1)*(T-2)) * (A*A - 4*A*B + 2*T*B*B);
. display "LM1 = " LM1 ";
LM1 = 1484.571

*** Variables with a last letter of "d" denote forward orthogonal differences.
*** Variables with a last letter of "d" denote individual-specific means.
*** x0=1 for all (i,t)
. reg lgaspcard lincomepd lrpmgd lcarpcapd lincomepm lrpmgm lcarpcapm x0m,noconst vce(hc2);

Linear regression                                       Number of obs =     342
                                                        F(  7,   335) = 2426.94
                                                        Prob > F      =  0.0000
                                                        R-squared     =  0.9907
                                                        Root MSE      =  .09892

------------------------------------------------------------------------------
             |               Robust HC2
   lgaspcard |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   lincomepd |     .66225    .078864     8.40   0.000      .507119     .817381
      lrpmgd |  -.3217025   .0479153    -6.71   0.000    -.4159553   -.2274497
   lcarpcapd |   -.640483   .0439033   -14.59   0.000    -.7268438   -.5541222
   lincomepm |   .3053265    .242636     1.26   0.209    -.1719556    .7826087
      lrpmgm |  -.6418482   .1813578    -3.54   0.000    -.9985919   -.2851046
   lcarpcapm |  -.1548162   .0988906    -1.57   0.118     -.349341    .0397086
         x0m |    2.54163   .6091093     4.17   0.000     1.343469    3.739791
------------------------------------------------------------------------------

.         matrix b=e(b)';

.         matrix V=e(V);

.         matrix Wald = b[4..6,1]'*invsym(V[4..6,4..6])*b[4..6,1];

.         matrix list Wald;

symmetric Wald[1,1]
            y1
y1  15.446815
```

OLS { income +, price −, car stock −

y is gasoline / car

x_3 is car / pop — division bias?

And large elasticity