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Panel Econometrics Homework 1

Question 1.
The question asked to replicate Table 1, "Gasoline Demand Data. One-way Error Component Results" in Stata. The do file used to replicate the results is attached at the end of the homework. My results are reported in Table 2. We can see that the results of Amemiya and Swamy-Arora estimators did not match exactly the results presented in Baltagi's book meanwhile the rest is similar.

Table 1: Table from Baltagi's book

Table 2.5 Gasoline Demand Data. One-way Error Component Results

	β_1	β_2	β_3	ρ	σ_μ	σ_ν
OLS	0.890	−0.892	−0.763			
	(0.036)*	(0.030)*	(0.019)*			
Between	0.968	−0.964	−0.795			
	(0.156)	(0.133)	(0.082)			
Within	0.662	−0.322	−0.640			
	(0.073)	(0.044)	(0.030)			
WALHUS	0.545	−0.447	−0.605	0.75	0.197	0.113
	(0.066)	(0.046)	(0.029)			
AMEMIYA	0.602	−0.366	−0.621	0.93	0.344	0.092
	(0.066)	(0.042)	(0.027)			
SWAR	0.555	−0.402	−0.607	0.82	0.196	0.092
	(0.059)	(0.042)	(0.026)			
IMLE	0.588	−0.378	−0.616	0.91	0.292	0.092
	(0.066)	(0.046)	(0.029)			

* These are biased standard errors when the true model has error component disturbances (see Moulton, 1986).
Source: Baltagi and Griffin (1983). Reproduced by permission of Elsevier Science Publishers B.V. (North-Holland).

Table 2: Gasoline Demand Data.

	(1) OLS	(2) Between	(3) Within	(4) WALHUS	(5) AMEMIYA	(6) SWAR	(7) IMLE
β_1	0.890***	0.968***	0.662***	0.543***	0.583***	0.759***	0.588
	(24.86)	(6.22)	(9.02)	(9.74)	(11.67)	(17.93)	(0.85)
β_2	-0.892***	-0.964***	-0.322***	-0.471***	-0.567***	-0.767***	-0.378
	(-29.42)	(-7.25)	(-7.29)	(-11.79)	(-14.70)	(-21.83)	(-0.85)
β_3	-0.763***	-0.795***	-0.640***	-0.606***	-0.628***	-0.708***	-0.616*
	(-41.02)	(-9.64)	(-21.58)	(-24.30)	(-26.68)	(-33.21)	(-2.12)
β_0	2.391***	2.542***	2.403***	1.906***	1.900***	2.159***	2.136
	(20.45)	(4.82)	(10.66)	(11.18)	(12.62)	(16.37)	(0.95)
Obs.	342	342	342	342	342	342	342

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Question 2.

The do file used to answer this question is attached at the end of the homework. Figure 1 shows a negative relationship between the fixed effects and per capita income of the country.

Figure 1: Fixed effects vs Income per capita

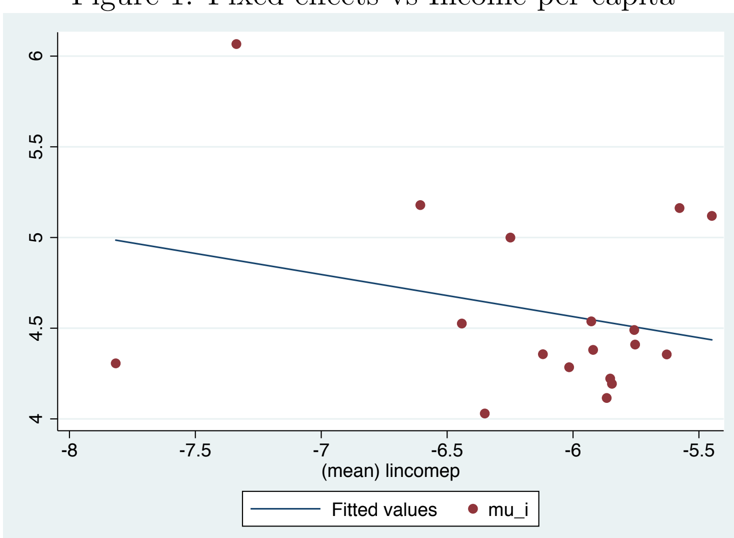

Table 3 report the cross correlations between the estimated country fixed effects and the mean by country of the other independent variables. There are no statistically significant correlations.

Table 3: Cross-correlation table

Variables	mu_i	(mean) lincomep	(mean) lrpmg	(mean) lcarpcap
mu_i	1.0000			
(mean) lincomep	-0.2747	1.0000		
	(0.2699)			
(mean) lrpmg	-0.0547	0.4636	1.0000	
	(0.8294)	(0.0526)		
(mean) lcarpcap	-0.6535	0.4801	-0.4039	1.0000
	(0.0033)	(0.0437)	(0.0964)	

Question 3.

Using the gasoline demand data of Baltagi and Griffin, I computed the forward orthogonal deviations estimator of Arellano in Matlab. Here I attached the code, the estimation gave me the following vector of estimated coefficients: $[-0.0302, 0.5197, -0.3247, -0.6236]$.

```
clear
load ps1.mat
% I construct the transforming factor
diag = diag([18/19 17/18 16/17 15/16 14/15 13/14 12/13 11/12 10/11 9/10 8/9 7/8 6/7 5/6 4/5 3/4 2/3 1/2]);
% I create the matrix that compute deviations from the forward mean
A_plus=triu(ones(18,19));
for i=1:18
    for j=2:19
        if j>i
    A_plus(i,j)=-1/(19-i);
        end
    end
end
A_plus_diag=diag.^(1/2)*A_plus;
A_plus_diag_stacked=kron(eye(18),A_plus_diag);
% Now I transform the variables of the model
Y_star=A_plus_diag_stacked*lgaspcar;
X1_star=A_plus_diag_stacked*lincomep;
X2_star=A_plus_diag_stacked*lrpmg;
X3_star=A_plus_diag_stacked*lcarpcap;
X_star=[ones(324,1) X1_star X2_star X3_star];
% Now I compute the OLS estimator over the transformed model
beta=inv(X_star'*X_star)*X_star'*Y_star
```

Question 4. We want to show that:
$\tilde{u}'(I_n \otimes J_T)\tilde{u} = \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}\tilde{u}_{it}\tilde{u}_{is} = \sum_{i=1}^{N}\sum_{t=1}^{T}\tilde{u}_{it}(\sum_{s=1}^{T}\tilde{u}_{is})$
where $\tilde{u}$ is a $NT*1$ vector, I_n is a identity matrix of dimension $N*N$, J_T is a defined as $\iota_T\iota_T'$.

First note that $(I_n \otimes J_T)$ is equal to a $NT*NT$ matrix of the form:

$$\begin{pmatrix} J_T & 0 & 0 & 0 & ... & 0 \\ 0 & J_T & 0 & 0 & ... & 0 \\ 0 & 0 & J_T & 0 & ... & 0 \\ .... \\ .... \\ 0 & 0 & 0 & 0 & ... & J_T \end{pmatrix},$$

Hence, we have that:
$\tilde{u}'(I_n \otimes J_T)\tilde{u}$ can be written as:
$[\tilde{u}_{11}\ \tilde{u}_{12}\ ...\ \tilde{u}_{1T}\ \tilde{u}_{21}\ ....\ \tilde{u}_{2T}\ ...\ \tilde{u}_{NT}](I_n \otimes J_T)[\tilde{u}_{11}\ \tilde{u}_{12}\ ...\ \tilde{u}_{1T}\ \tilde{u}_{21}\ ....\ \tilde{u}_{2T}\ ...\ \tilde{u}_{NT}]'$

Multiplying the first two matrices we get a $1*NT$ matrix of the form:
$[\sum_{s=1}^{T}\tilde{u}_{1t}\ ...\ \sum_{s=1}^{T}\tilde{u}_{1t}\ \sum_{s=1}^{T}\tilde{u}_{2t}\ ...\ \sum_{s=1}^{T}\tilde{u}_{Nt}]$

Then we multiply this matrix to $[\tilde{u}_{11}\ \tilde{u}_{12}\ ...\ \tilde{u}_{1T}\ \tilde{u}_{21}\ ....\ \tilde{u}_{2T}\ ...\ \tilde{u}_{NT}]'$ to get the following scalar:
$\tilde{u}_{11}\sum_{s=1}^{T}\tilde{u}_{1s}+\tilde{u}_{12}\sum_{s=1}^{T}\tilde{u}_{1s} +...+ \tilde{u}_{1T}\sum_{s=1}^{T}\tilde{u}_{1s} +...+ \tilde{u}_{N1}\sum_{s=1}^{T}\tilde{u}_{Ns} +...+ \tilde{u}_{NT}\sum_{s=1}^{T}\tilde{u}_{Ns}$

Then we can do a summation to get:
$\sum_{i=1}^{N}(\tilde{u}_{i1}\sum_{s=1}^{T}\tilde{u}_{is} + \tilde{u}_{i2}\sum_{s=1}^{T}\tilde{u}_{is} + \tilde{u}_{i3}\sum_{s=1}^{T}\tilde{u}_{is} +...+ \tilde{u}_{iT}\sum_{s=1}^{T}\tilde{u}_{is})$

Finally, doing another summation of T elements we get the desired result:
$\sum_{i=1}^{N}\sum_{t=1}^{T}\tilde{u}_{it}(\sum_{s=1}^{T}\tilde{u}_{is})$

which by properties of summation can also be written as:
$\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}\tilde{u}_{it}\tilde{u}_{is}$

Question 5.

Consider the model:

$y_{it} = x_{it}'\beta + \alpha_i + \eta_{it}$

Stack the observations for t=1,...,T giving:

$Y_i = X_i'\beta + \alpha_i 1_T + \eta_i$

Define a matrix that create deviations from the group mean:

$Q_T = I_T - 1_T(1_T'1_T)^{-1}1_T'$

$Q_T = I_T - P_T$

where

$P_T = 1_T(1_T'1_T)^{-1}1_T' = T^{-1}1_T 1_T'$

Note that:

$P_T 1_T = 1_T$, $Q_T 1_T = 0$

$P_T Y_i = 1_T(1_T'1_T)^{-1}1_T' y_i = 1_T \bar{Y}_i$

$Q_T Y_i = (I_T - P_T)Y_i = Y_i - 1_T\bar{Y}_i = \tilde{Y}_i$

The transformed error components model is then:

$Q_T Y_i = Q_T X_i \beta + \alpha_i Q_T 1_T + Q_T \eta_i$

which can also be written as:

$\tilde{Y}_i = \tilde{X}_i\beta + \tilde{\eta}_i$

or stacked by observation is:

$\tilde{Y} = \tilde{X}\beta + \tilde{\eta}$

The FE estimator is an OLS on this transformed system:

$\hat{\beta}_{FE} = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{Y} = (\sum_{i=1}^{n}\tilde{X}_i'\tilde{X}_i')^{-1}\sum_{i=1}^{n}\tilde{X}_i'\tilde{y}_i$

In the case of the LSDV estimator, let's begin with the stacked model on observations over t:

$Y_i = X_i'\beta + \alpha_i 1_T + \eta_i$

which can be stacked to get:

$Y = X\beta + D\alpha + \eta = X\beta + (I_n \otimes 1_T)\alpha + \eta$

Recall that when we consider a general partitioned regression equation:

$y = X_1\beta_1 + X_2\beta_2 + \epsilon$

the OLS estimators for β_1 and β_2 can be expressed as:

$\hat{\beta}_1 = (X_1'Q_2X_1)^{-1}X_1'Q_2y$, $Q_2 = I_n - P_{X_2}$

$\hat{\beta}_2 = (X_2'Q_1X_2)^{-1}X_2'Q_1y$, $Q_1 = I_n - P_{X_1}$

where

$P_{X_1} = X_1(X_1'X_1)^{-1}X_1'$, $P_{X_2} = X_2(X_2'X_2)^{-1}X_2'$

Hence, using these formulas we have that:

$\hat{\beta_F E} = (X'Q_DX)^{-1}X'Q_Dy$

where

$Q_D = I_{T_n} - P_D$, $P_D = D(D'D)^{-1}D'$, $D = I_n \otimes 1_T$

Now,

$P_D = (I_n \otimes 1_T)[(I_n \otimes 1_T)'(I_n \otimes 1_T)]^{-1}(I_n \otimes 1_T)'$

$P_D = (I_n \otimes 1_T)[I_n \otimes 1_T' 1_T]^{-1}(I_n \otimes 1_T)'$
$P_D = (I_n \otimes 1_T)[I_n \otimes (1_T' 1_T)^{-1}](I_n \otimes 1_T')' = I_n \otimes P_T$
Therefore,
$Q_D = I_{Tn} - P_D = I_{Tn} - (I_n \otimes P_T) = I_n \otimes Q_T$
Hence, we have that:
$\hat{\beta}_{LSDV} = (X'Q_D X)^{-1} X' Q_D y$
$\hat{\beta}_{LSDV} = (X'(I_n \otimes Q_T)X)^{-1} X'(I_n \otimes Q_T)y$
$\hat{\beta}_{LSDV} = (\sum_{i=1}^{n} \tilde{X}_i' \tilde{X}_i')^{-1} \sum_{i=1}^{n} \tilde{X}_i' \tilde{y}_i$
which is equal to what we got using the transformed model for the within estimator.

```
capture: log close
cd "/Users/ercio/Box Sync/CUNY/Panel Econometrics/homework/"
set more off
log using "hw1_log", text replace
**********************************************
** Homework 1 - Ercio Munoz
**********************************************
use "ps1.dta", clear
sum
describe
encode country, generate(ncountry)
* a) Gasoline Demand Data. One-way Error Component Results
tsset ncountry year
matrix results = J(14,3,.)
* OLS
eststo clear
eststo OLS: quietly reg lgaspcar lincomep lrpmg lcarpcap
* Between
eststo Between: quietly xtreg lgaspcar lincomep lrpmg lcarpcap, be
* Within
eststo Within: quietly xtreg lgaspcar lincomep lrpmg lcarpcap, fe
* WALHUS
*xtregwhm lgaspcar lincomep, id(ncountry) it(year)
eststo Walhus: quietly spregxt lgaspcar lincomep lrpmg lcarpcap, nc(18) model(ols) run(xtwh)
* AMEMIYA
* xtregam lgaspcar lincomep lrpmg lcarpcap, id(ncountry) it(year)
eststo Amemiya: quietly spregxt lgaspcar lincomep lrpmg lcarpcap, nc(18) model(ols) run(xtam)
* SWAR (Swamy-Arora)
eststo Swar: quietly spregxt lgaspcar lincomep lrpmg lcarpcap, nc(18) model(ols) run(xtsa)
* IMLE
eststo IMLE: quietly spregxt lgaspcar lincomep lrpmg lcarpcap, nc(18) model(ols) run(xtmlem)

esttab using table1.tex, label title(Gasoline Demand Data. table\label{tab1}) mtitles("OLS" "Between" "Within" "WALHUS"
```

```
"AMEMIYA" "SWAR" "IMLE") replace

* Question 2)
preserve
tsset ncountry year
reg lgaspcar lincomep lrpmg lcarpcap i.ncountry
collapse lgaspcar lincomep lrpmg lcarpcap, by(ncountry)
gen mu_i = lgaspcar-_b[lincomep]-_b[lrpmg]-_b[lcarpcap]
graph twoway (lfit mu_i lincomep) (scatter mu_i lincomep)
graph export scatter_graph.pdf, replace
corrtex mu_i lincomep lrpmg lcarpcap, file(correlation) replace case sig dig(4) key(table2)
restore
matrix fe = e(b)'
matrix fixedeffects = fe[4..21,1]
matrix cons = J(18,1,_b[_cons])
matrix ffs = fixedeffects + cons
```