<u>ECO 82800 Panel Econometrics</u>

Homework 1
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Dataset: Baltagi and Griffin (1983), 18 OECD countries over 19 years, 1960-1978.
Variables:

(1) CO = Country.
(2) YR = Year.
(3) LN(Gas/Car): The logarithm of motor gasoline consumption per auto.
(4) LN(Y/N): The logarithm of real per-capita income.
(5) LN(Pmg/Pgdp): The logarithm of real motor gasoline price.
(6) LN(Car/N): The logarithm of the stock of cars per-capita.

Question 1: Replicate Table 2.5 on Baltagi's book

Table 2.5 Gasoline Demand Data. One-way Error Component Results

	β_1	β_2	β_3	ρ	σ_μ	σ_ν
OLS	0.890	−0.892	−0.763			
	(0.036)*	(0.030)*	(0.019)*			
Between	0.968	−0.964	−0.795			
	(0.156)	(0.133)	(0.082)			
Within	0.662	−0.322	−0.640			
	(0.073)	(0.044)	(0.030)			
WALHUS	0.545	−0.447	−0.605	0.75	0.197	0.113
	(0.066)	(0.046)	(0.029)			
AMEMIYA	0.602	−0.366	−0.621	0.93	0.344	0.092
	(0.066)	(0.042)	(0.027)			
SWAR	0.555	−0.402	−0.607	0.82	0.196	0.092
	(0.059)	(0.042)	(0.026)			
IMLE	0.588	−0.378	−0.616	0.91	0.292	0.092
	(0.066)	(0.046)	(0.029)			

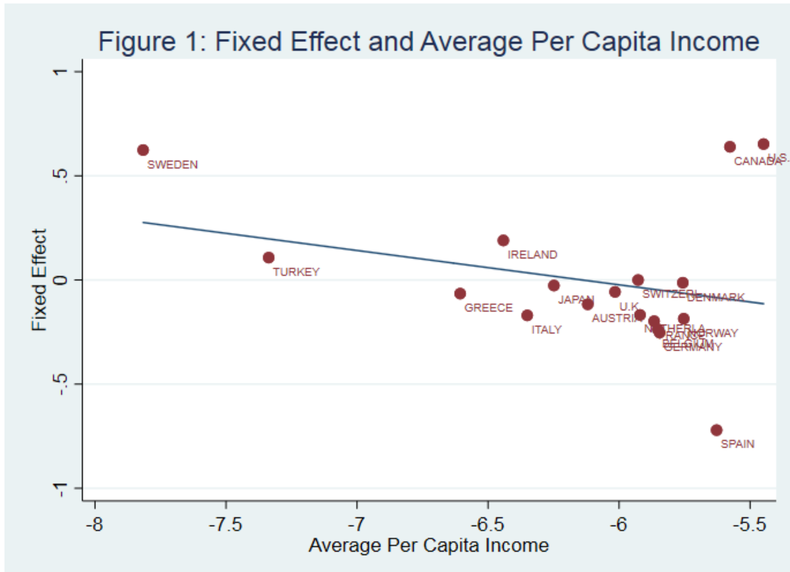
* These are biased standard errors when the true model has error component disturbances (see Moulton, 1986).
Source: Baltagi and Griffin (1983). Reproduced by permission of Elsevier Science Publishers B.V. (North-Holland).

Replicating Table 2.5

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	OLS	Between	Within	WALHUS	AMEMIYA	SWAR	IMLE
LINCOMEP	0.890***	0.968***	0.662***	0.543***	0.583***	0.759***	0.588
	(0.0358)	(0.156)	(0.0734)	(0.0558)	(0.0499)	(0.0423)	(0.694)
LRPMG	-0.892***	-0.964***	-0.322***	-0.471***	-0.567***	-0.767***	-0.378
	(0.0303)	(0.133)	(0.0441)	(0.0400)	(0.0386)	(0.0351)	(0.445)
LCARPCAP	-0.763***	-0.795***	-0.640***	-0.606***	-0.628***	-0.708***	-0.616*
	(0.0186)	(0.0825)	(0.0297)	(0.0249)	(0.0235)	(0.0213)	(0.291)
Standard errors in parentheses							
="* p<0.05	** p<0.01	*** p<0.001"					

Question 2

(b) Negative correlation between fixed effect and average per capita income.



(c) Yes.

Correlation Table				
	mu_i	lincomep	lrpmg	lcarpcap
mu_i	1			
lincomep	-0.2882	1		
lrpmg	-0.8133	0.4636	1	
lcarpcap	0.2706	0.4801	-0.4039	1

```
** Panel Econometrics        **
** Assignment 1              **
** Author: Chuxin Liu        **
** Last Updated: 03/06/2019 **

clear
set more off
capture: log close
cd "C:\Users\cliu\Documents\GitHub\PanelEconometrics\HW1"
use "gasoline.dta", clear

********************************************************************************
* Question 1
encode country, generate(ncountry)
* a) Gasoline Demand Data. One-way Error Component Results
tsset ncountry year /* declare panel data*/
matrix results = J(14,3,.)

eststo clear
* OLS
eststo OLS: reg lgaspcar lincomep lrpmg lcarpcap
* Between
eststo Between: xtreg lgaspcar lincomep lrpmg lcarpcap, be
* Within
eststo Within: xtreg lgaspcar lincomep lrpmg lcarpcap, fe
* WALHUS
/* ssc install spregxt */
/* check if spregxt is installed */
/* nc(#): Number of Cross Sections Units */
/* model(ols): Linear Panel Models (Non Spatial) */
/* run(xtwh): [NEW] Wallace-Hussain Random-Effects Panel Regression */
eststo Walhus: spregxt lgaspcar lincomep lrpmg lcarpcap, nc(18) model(ols) run(xtwh)
* AMEMIYA
/* run(xtam): [NEW] Amemiya Random-Effects Panel Regression */
eststo Amemiya: spregxt lgaspcar lincomep lrpmg lcarpcap, nc(18) model(ols) run(xtam)
* SWAR (Swamy-Arora)
/* run(xtsa): [NEW] Swamy-Arora Random-Effects Panel Regression */
eststo Swar: spregxt lgaspcar lincomep lrpmg lcarpcap, nc(18) model(ols) run(xtsa)
* IMLE
/* run(xtmlem): [NEW] Trevor Breusch MLE Random-Effects Panel Regression */
eststo IMLE: spregxt lgaspcar lincomep lrpmg lcarpcap, nc(18) model(ols) run(xtmlem)

esttab using Table1.csv, label se noobs nocons title(Replicating Table 2.5) //
    mtitles("OLS" "Between" "Within" "WALHUS" "AMEMIYA" "SWAR" "IMLE") replace

* Question 2
tsset ncountry year
xtreg lgaspcar lincomep lrpmg lcarpcap, fe
collapse lgaspcar lincomep lrpmg lcarpcap, by(ncountry)
*(a)
gen mu_i = lgaspcar-_b[lincomep]*lincomep-_b[lrpmg]*lrpmg-_b[lcarpcap]*lcarpcap-_b[_cons]
*(b)
twoway (lfit mu_i lincomep) (scatter mu_i lincomep, mlabel(ncountry) mlabsize(vsmall) //
    mlabposition(5)), ytitle(Fixed Effect) xtitle(Average Per Capita Income) //
    title(Figure 1: Fixed Effect and Average Per Capita Income) legend(off)

graph export Figure1.png, replace
*(c)
corr mu_i lincomep lrpmg lcarpcap
```

Question 4: Show that $\tilde{u}'(I_n \otimes J_T)\tilde{u} = \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} \tilde{u}_{it}\tilde{u}_{is} = \sum_{i=1}^{N}\sum_{i=1}^{T} \tilde{u}_{it}\left(\sum_{s=1}^{T}\hat{u}_{is}\right)$

Note that:

- $(I_n \otimes J_T)$ is a $NT * NT$ matrix: $\begin{pmatrix} J_T & 0 & 0 & \cdots & 0 \\ 0 & J_T & 0 & \cdots & 0 \\ 0 & 0 & J_T & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & J_T \end{pmatrix}$

- $\hat{u}$ is $NT \times 1$ vector

Hence $\tilde{u}'(I_n \otimes J_T)\tilde{u} = [\hat{u}_{11}\ \hat{u}_{12} \cdots \hat{u}_{1T}\ \hat{u}_{21} \cdots \hat{u}_{2T} \cdots \hat{u}_{NT}](I_n \otimes J_T)\tilde{u}$

* $\tilde{u}'$ by $(I_n \otimes J_T)$ $= \left[\sum_{s=1}^{T}\hat{u}_{1s}\ \ \sum_{s=1}^{T}\tilde{u}_{2s} \cdots \sum_{s=1}^{T}\hat{u}_{Ts}\right] \cdot \tilde{u}$

* by $\tilde{u}$ $= \left[\sum_{s=1}^{T}\hat{u}_{1s}\ \ \sum_{s=1}^{T}\tilde{u}_{2s} \cdots \sum_{s=1}^{T}\hat{u}_{Ts}\right][\hat{u}_{11}\ \hat{u}_{12} \cdots \tilde{u}_{NT}]'$

$= \hat{u}_{11}\sum_{s=1}^{T}\hat{u}_{1s} + \hat{u}_{12}\sum_{s=1}^{T}\hat{u}_{2s} + \cdots + \tilde{u}_{NT}\cdot\sum_{s=1}^{T}\hat{u}_{Ts}$

* summation on N $= \sum_{i=1}^{N}\left(\hat{u}_{i1}\sum_{s=1}^{T}\hat{u}_{is} + \hat{u}_{i2}\sum_{s=1}^{T}\tilde{u}_{is} + \cdots + \hat{u}_{iT}\sum_{s=1}^{T}\hat{u}_{is}\right)$

* summation on T $= \sum_{i=1}^{N}\sum_{t=1}^{T}\hat{u}_{it}\left(\sum_{s=1}^{T}\tilde{u}_{is}\right)$

* properties of summation $= \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}\hat{u}_{it}\hat{u}_{is}$

Question 5 : Answer question 2.1

Prove that $\tilde{\beta}$ given (2.7): $\tilde{\beta} = (X'QX)^{-1}X'Qy$ can be obtained from OLS on (2.5): $y = \alpha \iota_{NT} + X\beta + Z_\mu \mu + \nu = Z\delta + Z_\mu \mu + \nu$, using results on partitioned inverse. This can easily be obtained using the Frisch – Waugh – Lovell theorem of Davidson and MacKinnon.

Hint: This theorem states that the OLS estimate of β from (2.5) will be identical to the OLS estimate of β from (2.6): $Qy = QX\beta + Q\nu$. Also, the least squares residuals will be the same.

- Define $Q = I - P$

$Qy = \tilde{y}$, $Qx = \tilde{x}$, the transformed error component model $Qy = QX\beta + Q\nu$ can be written as: $\tilde{Y}_i = \tilde{X}_i \beta + \tilde{\eta}_i$

The FE estimator is therefore an OLS estimator of $\tilde{Y} = \tilde{X}\beta + \tilde{\eta}$

$$\hat{\beta}_{FE} = (\tilde{X}'\tilde{X})^{-1}\tilde{X}\tilde{Y} = (X'QX)^{-1}X'Qy = \tilde{\beta}$$

- LSDV, within estimator

$$Y_i = X_i'\beta + \alpha_i 1_T + \eta_i \text{ , stack} \to Y = X\beta + D\alpha + \eta = X\beta + (I_n \otimes 1_T)\alpha + \eta$$

$$\beta = (X'Q_D X)^{-1}X'Q_D y \text{ where } Q_D = I - P_D,\ P_D = D(D'D)^{-1}D',\ D = I_n \otimes 1_T$$

To obtain Q_D:
$$\begin{aligned}
Q_D &= I - P_D \\
&= I - D(D'D)^{-1}D' \\
&= I - (I_n \otimes 1_T)\left[(I_n \otimes 1_T)'(I_n \otimes 1_T)\right]^{-1}(I_n \otimes 1_T)' \\
&= I - I_n \otimes P_T \\
&= I_n \otimes Q_T
\end{aligned}$$

$$\begin{aligned}
\beta_{LSDV} &= (X'Q_D X)^{-1}X'Q_D y = \left[X'(I_n \otimes Q_T)X\right]^{-1}X'(I_n \otimes Q_T)y \\
&= (X'QX)^{-1}X'Qy
\end{aligned}$$