# Derivation of Chaotic Attractor Equation and Chaotic Evolution Equation of High Order Discrete HNN Based on OGY Linear Control

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Considering N binary neurons, each of which has two states (e.g., $s_i = \pm 1$), they form a generalized Hopfield neural network by the first and two order connections. According to the Hebb's learning algorithm, the connection weight $w_{ij}$ from the neuron j to the neuron i and the connection right of the neurons to the k,l to the neuron Ii $w_{ijk}$ are respectively:

$$w_{ij} = \frac{C_{ij}}{N}\sum_{\mu=1}^{p} s_i^{\mu} s_j^{\mu}, \quad w_{ikl} = \frac{C_{ikl}}{N}\sum_{\mu=1}^{p} s_i^{\mu} s_k^{\mu} s_l^{\mu} \tag{1}$$

$\mathbf{s}^{\mu} = (s_1^{\mu}, ..., s_N^{\mu})$ is the $\mu$ pattern, p is the number of patterns stored in network. $C_{ij}$ and $C_{ikl}$ is independent random variables, they obey these distributions respectively:

$$\rho(C_{ij}) = \frac{C}{N}\delta(C_{ij}-1) + (1-\frac{C}{N})\delta(C_{ij}) \tag{2}$$

$$\rho(C_{ikl}) = \frac{2C}{N^2}\delta(C_{ikl}-1) + (1-\frac{2C}{N^2})\delta(C_{ikl}) \tag{3}$$

C is the parameter representing the sparse degree of the network.

Consider neuron i, let $j_1, j_2, ..., j_{K_1}$ is $K_1$ neurons connected to j of i, and let $k_1 l_1, ..., k_{K_2} l_{K_2}$ are $K_2$ neuron pairs that satisfy $w_{ikl} \neq 0$. According to 2 and 3, the mean value of $K_1$ and $K_2$ are both C. Suppose the total input of neuron i is:

$$h_i(t) = \gamma_1 \sum_{r=1}^{K_1} w_{ij_r} s_{j_r}(t) + \gamma_2 \sum_{r=1}^{K_2} w_{ik_r} l_r s_{k_r}(t) s_{l_r}(t) + I_i(t) + \eta \tag{4}$$

$s_i(t)$ represents the state of neuron j at time t, $\gamma_1$ and $\gamma_2$ represent one-order and two-order weight respectively. For every neuron, there are variance $\sigma_o$ and background gaussian noise $\eta_i$. Further, we introduce a external signal $I_i(t)$ to control the dynamic behaviors of the network.

Here we consider the parallel evolution formula (e.g., the state of all neuron changed at the same time). The state evolution equation of neuron i is:

$$s_i(t) = sgn[h_i(t)] \tag{5}$$

Suppose the initial state of the network is neighbored with pattern $\mathbf{s_1}$, which means:

$$m^1(0) = max\ m^\mu(0)|\mu = 1, 2, ..., p, \quad m^\mu(t) = \frac{1}{N}\mathbf{s}^\mu\mathbf{s}(t) \tag{6}$$

The latter equation in 6 is the similarity measurement with pattern $\mu$ at time t. In general, the state of every unit update every time step. We expect to associate the pattern $\mathbf{s}_1$. In oder to simplify the problem, we only consider the evolution of $m^1(t)$. We can get the evolution equation of $m^1(t)$:

$$m^1(t) = \frac{1}{N}\sum_{i=1}^{N} s_i^1 sgn\{[\gamma_1\frac{C}{N}m^1(t) + \gamma_2\frac{C}{N}(m^1(t))^2]s_i^1 + I_i(t) + \eta'\} \tag{7}$$

$\eta'$ is the fusion of internal noise $\eta_i$ and $\mathbf{s}^\mu$. The mean average of it is 0, and the total variance is $\sigma_t$. Thus, we can get the parallel evolution equation:

$$m(t) = 1 - 2\psi\{\gamma_1 m(t) + \gamma_2[m(t)]^2 + I(t)\} = F[m(t), I(t), \sigma] \tag{8}$$

Here,

$$\psi(y) = \frac{1}{\sqrt{2\pi}}\int_{\frac{y}{\sigma}}^{\infty} e^{-\frac{x^2}{2}}dx \tag{9}$$

and the total variance is:

$$\sigma = \sqrt{(\gamma_1^2 + \gamma_2^2)(\frac{(p-1)}{C}) + (\frac{\sigma_0 N}{C})^2} \tag{10}$$