Derivation of Chaotic Attractor Equation and Chaotic Evolution Equation of High Order Discrete HNN Based on OGY Linear Control

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Considering N binary neurons, each of which has two states (e.g., $s_i = \pm 1$), they form a generalized Hopfield neural network by the first and two order connections. According to the Hebb's learning algorithm, the connection weight w_{ij} from the neuron j to the neuron i and the connection right of the neurons to the k,l to the neuron Ii w_{ijk} are respectively:

$$w_{ij} = \frac{C_{ij}}{N}\sum_{\mu=1}^{p} s_i^\mu s_j^\mu, \quad w_{ikl} = \frac{C_{ikl}}{N}\sum_{\mu=1}^{p} s_i^\mu s_k^\mu s_l^\mu \tag{1}$$

$\mathbf{s}^\mu = (s_1^\mu, ..., s_N^\mu)$ is the μ pattern, p is the number of patterns stored in network. C_{ij} and C_{ikl} is independent random variables, they obey these distributions respectively:

$$\rho(C_{ij}) = \frac{C}{N}\delta(C_{ij}-1) + (1-\frac{C}{N})\delta(C_{ij}) \tag{2}$$

$$\rho(C_{ikl}) = \frac{2C}{N^2}\delta(C_{ikl}-1) + (1-\frac{2C}{N^2})\delta(C_{ikl}) \tag{3}$$

C is the parameter representing the sparse degree of the network.

Consider neuron i, let $j_1, j_2, ..., j_{K_1}$ is K_1 neurons connected to j of i, and let $k_1 l_1, ..., k_{K_2} l_{K_2}$ are K_2 neuron pairs that satisfy $w_{ikl} \neq 0$. According to 2 and 3, the mean value of K_1 and K_2 are both C. Suppose the total input of neuron i is:

$$h_i(t) = \gamma_1 \sum_{r=1}^{K_1} w_{ij_r} s_{j_r}(t) + \gamma_2 \sum_{r=1}^{K_2} w_{ik_r} l_r s_{k_r}(t) s_{l_r}(t) + I_i(t) + \eta \tag{4}$$

$s_i(t)$ represents the state of neuron j at time t, γ_1 and γ_2 represent one-order and two-order weight respectively. For every neuron, there are variance σ_o and background gaussian noise η_i. Further, we introduce a external signal $I_i(t)$ to control the dynamic behaviors of the network.

Here we consider the parallel evolution formula (e.g., the state of all neuron changed at the same time). The state evolution equation of neuron i is:

$$s_i(t) = sgn[h_i(t)] \tag{5}$$

Suppose the initial state of the network is neighbored with pattern $\mathbf{s_1}$, which means:

$$m^1(0) = max\ m^\mu(0)|\mu = 1, 2, ..., p, \quad m^\mu(t) = \frac{1}{N}\mathbf{s}^\mu\mathbf{s}(t) \tag{6}$$

The latter equation in 6 is the similarity measurement with pattern μ at time t. In general, the state of every unit update every time step. We expect to associate the pattern $\mathbf{s}_1$. In oder to simplify the problem, we only consider the evolution of $m^1(t)$. We can get the evolution equation of $m^1(t)$:

$$m^1(t) = \frac{1}{N}\sum_{i=1}^{N} s_i^1 sgn\{[\gamma_1\frac{C}{N}m^1(t) + \gamma_2\frac{C}{N}(m^1(t))^2]s_i^1 + I_i(t) + \eta'\} \tag{7}$$

η' is the fusion of internal noise η_i and $\mathbf{s}^\mu$. The mean average of it is 0, and the total variance is σ_t. Thus, we can get the parallel evolution equation:

$$m(t) = 1 - 2\psi\{\gamma_1 m(t) + \gamma_2[m(t)]^2 + I(t)\} = F[m(t), I(t), \sigma] \tag{8}$$

Here,

$$\psi(y) = \frac{1}{\sqrt{2\pi}}\int_{\frac{y}{\sigma}}^{\infty} e^{-\frac{x^2}{2}}dx \tag{9}$$

and the total variance is:

$$\sigma = \sqrt{(\gamma_1^2 + \gamma_2^2)(\frac{(p-1)}{C}) + (\frac{\sigma_0 N}{C})^2} \tag{10}$$

1 Perspectives

Cognition-based deep learning has become one hot research topic, and some of the most important functions of our human brains like memory and attention associated with knowledge extracted from experience and the universe, like memory and attention, have been widely used in the design of a more human-like deep learning system. Meanwhile, the brain does not learn through a unified undifferentiated neural network. The brain is composed of multiple modular subsystems, with an unique and complicated way interacting among. Although deep neural network can process structural data well, it can't deal with dynamic clouds of data. What's more, data is really scarce in some fields. Deep learning systems can get a lot of inspirations from cognitive science, to alleviate and even eliminate those problems.

In this section, we will discuss the essential trend to apply more elements of cognitive science to build more dynamic, robust and intelligent deep learning systems. We are going to give a general framework of cognition-based learning firstly. Then we will discuss the key problems of fusing deep neural network with cognitive mechanisms and essential solutions.

1.1 General Framework of Cognition-based Deep Learning

We suggest the general framework of designing cognition-based deep learning systems. These framework use cognitive mechanisms in a particular way. It can help build more dynamic, robust and intelligent systems. More accurately speaking, it can process unconstructed data as constructed one with the help of our memory with concepts, especially associative memory. And it can reasoning, infer based on the knowledge by gaining structural feature map with hierarchical knowledge sets in the top-down manner. As feedback is also very essential in our human brains, we can monitor this mechanism by designing two feedback loops. One is knowledge feedback loop, to update our knowledge based on attention select network, which is aimed at deciding what we need to see. Another is memory feedback loop, to update our memory (especially the experience), and gains high-level concepts after measuring actions/decisions the system make. The general framework is shown in Fig1.

1.2 Key Problems and Potential Solutions

This part will discuss the future directions of cognition-based deep learning. It is organized by current problems and essential solutions.

Associative Memory Human brains can associate patterns similar to the input patterns when being stimulated. Associative memory model was once a hit in 1980s and 1990s, accompanied with the booming of Hopfield Neural Network [1], a typical network that can store patterns and realize associative memory. Due to the potential chaos state of network evolution, HNNs alone is difficult to handle natural real-world problems well. However, it has the potential as it is an important kind of brain-like neural network. Besides, synesthesia is a typical

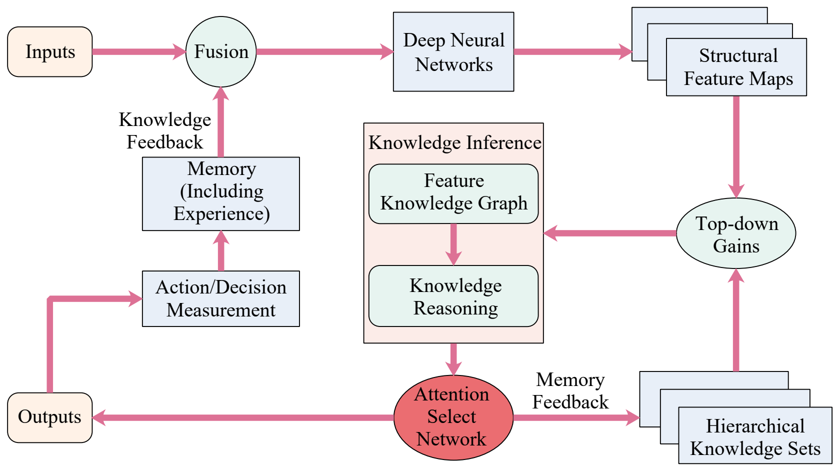

Fig. 1: General framework of cognition-based deep learning

perceptual phenomenon in cognitive science. That is, a person can activate a sensory when stimulated by another sensory (e.g., grapheme-color synesthesia means a person can directly associate a colorful image when listening to music). The proposition of an effective associative memory model by combining human-like neural network and synesthesia with deep learning is a promising direction. A recent successful attempt was Dense Associative Memory [2], which combined associative memory with deep learning and achieved a good result on MNIST dataset.

Interpretable Network with Cognitive Mechanisms For we human, it's difficult to understand how deep neural networks work and how they react towards a task. However, interpretable systems in many applications are of vital importance. For example, suppose that there is a person who may be in the early stage of cancer, the system based on deep neural networks needs to infer whether he is suffering from cancer. We can gather all features of the person as the input of DNNs, such as age, history of disease. The question is why we can trust the output of this system as we can not check the correctness. What if the process of inference can be understood or monitored (e.g., the decision tree) by an expert? Interpretability is important in these fields.

[3] proposed a tree regularization to interpret the neural network in the perspective of decision tree. This method can not train towards the typical back-propagation learning rule as the tree is undifferentiated. They suggested replacing trees with multi-layer perceptrons in the training phase, but this solution is not very elegent and does not create a really interpretable network indeed. According to psychological experiments [4], humans tend to assign the same name to similarly shaped items rather than to items with similar color, texture or size. [5] proposed shape Matching Networks (MNs) with inception network, which has the state-of-the-art one-shot learning performance on ImageNet. And [6] found

that this kind of networks that exhibits a similar shape bias to that observed in humans. Cognitive mechanisms like shape bias, decision and inference can help design more interpretable neural networks.

Cognition-based Deep Reinforcement Learning Deep reinforcement learning has raised a lot of interests nowadays. However, due to the uncertainty of the state space and the complexity of the reward function, it is difficult for the traditional trial-and-error strategies to associate continuous actions with reward. Imagination is utilized to make use of the knowledge embedded in the model. However, deep reinforcement learning is still in its early stage.

As decision making and feedback mechanism are very similar to that of humans, there is a trend to apply cognitive mechanisms to reinforcement learning. As for attention mechanism, [7] proposed the Deep Attention Recurrent Q-Network (DARQN), which largely outperformed the traditional Deep Q-Network (DQN) on Atari 2600 games by incorporating what they called 'soft' and 'hard' attention mechanisms. [8] further improved DARQN by implementing a multi-focus attention network where the agent is capable of attending to multiple important elements. Further, as for memory mechanism, [9] extended the typical LSTM-based memory network to choose more sophisticated addressing schemes over the past k frames. [10] used a spatially structured 2D memory image to learn to store arbitrary information about the environment over long time lags. As for our human knowledge mechanism, in which field we call usually transfer learning. [11] proposed a policy distillation (i.e., knowledge-based reinforcement learning policy) architecture for deep reinforcement learning by using task-specific high-level convolutional features as the inputs to the multi-task policy network. However, how to hierarchically reconstruct the knowledge and uncover the hidden characteristics, how to abstract our knowledge and experience for the feasibility to deal with unstructured data by fusion, how to design a generalized attention selection network, may remain issues that lead the future research direction in this field.

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