

Algorithm Design and Analysis - Assignment 4

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Problem1

Analysis: Assume $x_{ij} = 1$ means j-th job is assigned to i-th worker. Vice versa. Considering workers can work at the same time(same time used, multiple jobs done). 也就是说考虑可以并行工作。

then we have:

$$\min \max_i \sum_{j=1}^n C_{ij} x_{ij}$$

$$s. t. \sum_{i=1}^m x_{ij} = 1 \text{ for all } j$$

$$x_{ij} = 0/1 \text{ for all } i \text{ and } j$$

Problem2

Analysis: Assume $x_i = 1$ means i-th food is bought. Vice versa. At the same time, we say the nutrition is satisfied when it is greater than 0.

then we have:

$$\min \sum_{i=1}^m C_i x_i$$

$$s. t. \sum_{i=1}^m x_i W_{ij} > 0 \text{ for all } j$$

$$x_i = 0/1 \text{ for all } i$$

Problem3

Analysis: Assume x_{ij} is the amount of A of i-th place that is delivered to j-th market. Assume that the balance is achieved if and only if all A delivered is below the sum-output and all A accepted is below the sum-demand.

then we have:

$$\min \sum_{i=1}^n \sum_{j=1}^m C_{ij} x_{ij}$$

$$s. t. \sum_{j=1}^m x_{ij} \leq a_i \text{ for all } i$$

$$\sum_{i=1}^n x_{ij} \leq b_j \text{ for all } j$$

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j$$

Problem4

Analysis: Assume $x_{ij} = 1$ means i -th stuff is packed into j -th box. Vice versa.

then we have:

$$\min \sum_{j=1}^n \bigvee_{i=1}^m x_{ij}$$

$$s. t. \sum_{j=1}^n x_{ij} \leq 1 \text{ for all } i$$

$$\sum_{i=1}^n x_{ij} C_i \leq S_j \text{ for all } j$$

$$x_{ij} = 0/1 \text{ for all } i \text{ and } j$$

Problem6

Analysis: Assume $x_i = 1$ means i -th project is fund. Vice versa.

then we have:

$$\max \sum_{i=1}^n (c_i - b_i) x_i$$

$$s. t. \sum_{i=1}^n x_i b_i \leq B$$

$$x_i = 0/1 \text{ for all } i$$

Problem 7

Analysis: Assume we build x A-dorms and y B-dorms. Assume that profit is sum of c_i .

then we have:

$$\max xc_i + yc_j$$

$$s. t. x \leq n_i$$

$$y \leq n_j$$

$$xs_i + ys_j \leq s$$

$$x, y \geq 0$$

Problem 8

Analysis: Assume $x_i = 1$ means place A_i is chosen. Vice versa.

then we have:

$$\max \sum_{i=1}^7 (c_i - b_i)x_i$$

$$s. t. \sum_{i=1}^7 x_i b_i \leq B$$

$$x_1 + x_2 + x_3 \leq 2$$

$$x_4 + x_5 \geq 1$$

$$x_6 + x_7 \leq 1$$

$$x_i = 0/1 \text{ for all } i$$

Problem 9

Analysis: Assume $x_i = 1$ means i-th item is packed. Vice versa. Assume there are n items in total.

then we have:

$$\max \sum_{i=1}^n c_i x_i$$

$$s. t. \sum_{i=1}^n x_i a_i \leq b$$

$$x_i = 0/1 \text{ for all } i$$

Problem 10

Analysis: Assume $x_{ij} = 1$ means j-th job is assigned to i-th worker. Vice versa. Assume that num of jobs is less than num of workers, and there are m workers and n jobs. So $m \leq n$. Considering every job is assigned, but not every worker has job to do.

then we have:

$$\max \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

$$s. t. \sum_{i=1}^m x_{ij} = 1 \text{ for all } j$$

$$\sum_{j=1}^n x_{ij} \leq 1 \text{ for all } i$$

$$x_{ij} = 0/1 \text{ for all } i \text{ and } j$$