中国科学院大学

试 题 专 用 纸

课程编号：B0912009Y
课程名称：算法设计与分析
任课教师：卜东波

注意事项：
1.考试时间为 150 分钟，考试方式 闭 卷；
2.全部答案写在答题本（纸）上；
3.考试结束后，请将本试卷和答题本（纸）、草稿纸一并交回。

Problem 1 Russian Doll Envelopes .
You are given a 2D array of integers envelopes where envelopes[i] = [w_i,h_i] represents the width and the height of an envelope.
One envelope can fit into another if and only if both the width and height of one envelope are greater than the other envelope 's width and height.
Return the maximum number of envelopes you can Russian doll (i.e., put one inside the other).
Note : You cannot rotate an envelope.
(1)Write an expression for this problem.
(2)Write the pseudocode algorithm for this problem.
(3)Calculate the time and space complexity of the algorithm.

Problem 2. **crossing a river**
A pony is crossing a river. The river is divided into some number of units, and at each unit, there may or may not exist a stone. The pony can jump on a stone, but it cannot jump into the water.
Given a list of stones' positions (in units) in sorted ascending order, determine if the pony can cross the river by landing on the last stone. Initially, the pony is on the first stone and assumes the first jump must be 1 unit. If the pony's last jump was k units, its next jump must be either k - 1, k, or k + 1 units. The pony can only jump in the forward direction.
Example:
Input: stones = [0,1,3,5,6,8,12,17]
Output: true
Explanation: The pony can jump to the last stone by jumping 1 unit to the 2nd stone, then 2 units to the 3rd stone, then 2 units to the 4th stone, then 3 units to the 6th stone, 4 units to the 7th stone, and 5 units to the 8th stone.
(1)Write an expression for the dynamic programming.
(2)Write the pseudocode algorithm for this problem.
(3)Calculate the time and space complexity of the algorithm.

Problem 3. **XOR tree**
A tree with weighted edges has n nodes, and the index of nodes start from 1 to n. There is a unique path between any two nodes, and the XOR sum of all edges on this path is called the specificity score. Please design an algorithm to find the highest specificity score.
(**Notion:** XOR: It is used in logical operations. The mathematical symbol for this is "⊕" and the computer symbol is "xor". The algorithm is as follows: a ⊕ b = (¬a ∧ b) ∨ (a ∧ ¬b))
Input format :
The first row is an integer n, representing the number of points.
And then the n -1 line gives u,v, and w, which respectively means that point u and point v on the tree have an edge, and the weight of the edge is w.
Output format :
An integer represents the answer
Input:
4
1 2 3
2 3 4
2 4 6
Output:
7

(1)Write an expression for this problem.
(2)Write the pseudocode algorithm for this problem.
(3)Calculate the time and space complexity of the algorithm.

Problem 4. Car Manufacturers

With the rapid development of social economy, the number of family cars has entered rapid growth. Car manufacturers need to deploy car carriers to transport cars to all parts of the country. The common double-deck car carrier has upper and lower layers each loaded with 1 row of cars, which is recorded as type 1-1; The lower and upper layers are respectively loaded with 1 and 2 rows of cars, which are recorded as type 1-2.

There are two types of car in the factory, of which the number of 1-2 type car is not more than $\boldsymbol{\alpha}$ times of the number of 1-1 type car; There are three types of cars to be transported, which are respectively referred to as cars I, II and III. Due to the height limit, a car carrier III can only be installed in the lower level of the 1-1 sedan car; The length of car carrier I is the shortest among the three kinds of car carriers; Each car, the lower loading area, can be equivalent to a rectangle; The lower level strives to be full, and the upper two columns strive to be symmetrical to ensure the smooth running of the sedan; All car carriers were placed longitudinally, and the longitudinal and transverse safety distance between adjacent car carriers was at least $\boldsymbol{\beta}$ m.

Taking the total car operating cost as the optimization goal, this paper tries to establish an integer programming model for this problem, and gives a simple explanation of the parameters used in the modeling process

(1)Establish the objective function.
(2)Make a comprehensive list of constraints.
(3)You do not need to solve this integer programming problem

Problem 5. **TSP**

There are n cities, V_1, V_2... V_n, D_{ij} is used to represent the distance between V_i and V_j. A salesman starts from one city and goes to other cities once and only once, and then returns to the original city. Find the shortest route around the journey.

(1)Establish the objective function.
(2)Make a comprehensive list of constraints.
(3)You do not need to solve this integer programming problem

Problem 6. Using Minimum Cost Maximum Flow to solve mathematical problem.

For a network, there is one source and one sink. Every edge e_{ij} is directed and has two value c_{ij} and a_{ij}. c_{ij} means the maximum flow of the edge.a_{ij} is a positive coefficient number which means that if the flow of the edge is x, the cost is ax^2.

You need to design a network flow algorithm for this problem , no need for solving it.

Problem 7. **Guarding the village**

There are n villages and m bidirectional roads between them. Each village has some guards. Specifically, the $i-th$ village has a_i guards. Now the king urge each guard either to stay in his village or to go to one of the neighboring villages.

Please design an algorithm to check if it is possible that after the decree the $i-th$ village has a_i village has exact b_i guards, no need for solving it.