

Devide and Conquer

- a. You can choose *three* from problems 1-6.
- b. For problems 1-6, you should do at least the following things:
 - (1) Describe your algorithm in natural language AND pseudo-code;
 - (2) Draw a “subproblem reduction graph”, where nodes represent subproblems, and edges describe the “reduction relationship” between them for every problem you choose in problems 1-6;
 - (3) Prove the correctness of your algorithm;
 - (4) Analyse the complexity of your algorithm

Question1

You are interested in analyzing some hard-to-obtain data from two separate databases. Each database contains n numerical values, so there are $2n$ values total and you may assume that no two values are the same. You'd like to determine the median of this set of $2n$ values, which we will define here to be the n^{th} smallest value.

However, the only way you can access these values is through queries to the databases. In a single query, you can specify a value k to one of the two databases, and the chosen database will return the k^{th} smallest value that it contains. Since queries are expensive, you would like to compute the median using as few queries as possible. Give an algorithm that finds the median value using at most $O(\log n)$ queries.

Question2

Consider an n -node complete binary tree T , where $n = 2^d - 1$ for some d . Each node v of T is labeled with a real number x_v . You may assume that the real numbers labeling the nodes are all distinct. A node v of T is a local minimum if the label x_v is less than the label x_w for all nodes w that are joined to v by an edge.

You are given such a complete binary tree T , but the labeling is only specified in the following implicit way: for each node v , you can determine the value x_v by probing the node v . Show how to find a local minimum of T using only $O(\log n)$ probes to the nodes of T .

Question3

There are N rooms in a prison, one for each prisoner, and there are M religions, and each prisoner will follow one of them. If the prisoners in the adjacent room are of the same religion, escape may occur. Please give an $O(n)$ algorithm to find out how many states escape can occur.

For example, there are 3 rooms and 2 kinds of religions, then 6 different states escape will occur.

Question4

Given the head of a linked list, return the list after sorting it in ascending order

Question5

Please implement QuickSort with a 3-way partition

Question6

There are n ropes with different length. If m ropes of the same length are cut from them, please find the longest length of each of these m ropes