



# Systems Thinking

Modularization-1:  
**Combinational Circuits and Sequential Circuits**  
组合电路与时序电路

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# Outline

- What is systems thinking?
- Three objectives of systems thinking
- Abstraction
- Modularization 模块化 抽象一点也不“抽象”
  - Modularization and modules
  - Combinational circuits 组合电路
    - Logic gates and combinational circuits
    - The information hiding principle
    - Adders
    - An adder-subtractor controlled by multiplexers
  - Sequential circuits 时序电路
    - Types of memory cells and the D flip-flop
    - General organization of sequential circuit
    - A serial adder example
  - Instruction Set and Instruction Pipeline 指令集与指令流水线
  - Software Stack 软件栈
- Seamless transition

*These slides acknowledge sources for additional data not cited in the textbook*

# 4.1 Modularization and modules

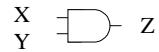
## 模块化与模块

- Modularization is a systems thinking method, similar to divide-and-conquer in algorithmic thinking 模块化像算法思维的分治方法
  - Divide a system into multiple subsystems called modules 将系统划分成模块
  - Compose modules into a system (higher-level abstraction) 将模块组合成系统
    - 该系统成为一个更高级的抽象
- In a system with modularization
  - Two modules may be interconnected, but often do not overlap
  - 很多情况下， 系统中的两个模块可以连接，但不重叠
- Modularization is a special form of abstraction where the information hiding principle is followed 模块是采用了信息隐藏原理的抽象
- Modularization, i.e., how to divide and compose a system, is an art, needing human imagination and creativity 模块化是艺术，需要创造力
- Understand how modularization works via a design journey
  - of higher and higher hardware subsystem abstractions
    - from designing gates to designing an instruction pipeline
    - 从逻辑门到指令流水线的旅行，学习模块化概念（模块如何组合成为系统）

## 4.2.1 Logic gates and combinational circuit

- Logic gates: electronic circuits realizing Boolean operators

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1



AND

与门

$$Z = X \cdot Y$$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

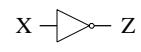


OR

或门

$$Z = X + Y$$

X	Z
0	1
1	0



NOT

非门

$$Z = \bar{X}$$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

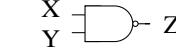


XOR

异或门

$$Z = X \oplus Y$$

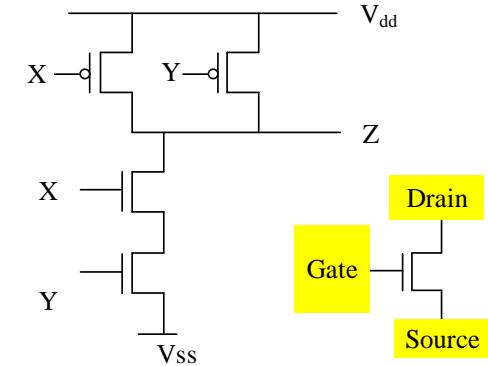
X	Y	Z
0	0	1
0	1	1
1	0	1
1	1	0



NAND

与非门

$$Z = \overline{X \cdot Y}$$



CMOS circuit for NAND

与非门的CMOS电路

晶体管行为

Gate 为1（高电平）：导通

Gate为0（低电平）：断开

Vss: ground (0)

Vdd: high voltage (1)

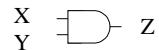
每个逻辑门都有半导体电路实现  
只需知道与非门的CMOS电路实现  
作为例子

X,Y都是高电平（1）时，下两个晶体管联通，上两个晶体管断开，Z连接到地（0）

## 4.2.1 Logic gates and combinational circuit

- Logic gates: electronic circuits realizing Boolean operators

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1



AND

$$Z = X \cdot Y$$

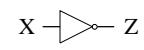
X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1



OR

$$Z = X + Y$$

X	Z
0	1
1	0



NOT

$$Z = \overline{X}$$

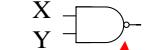
X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0



XOR

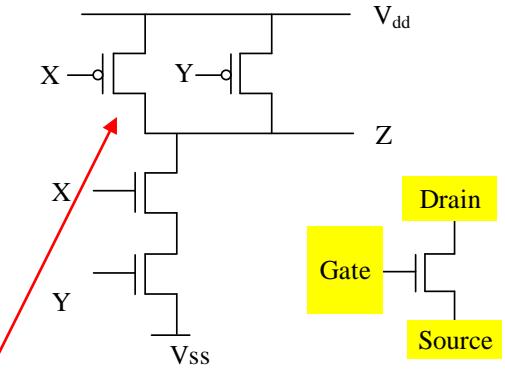
$$Z = X \oplus Y$$

X	Y	Z
0	0	1
0	1	1
1	0	1
1	1	0



NAND

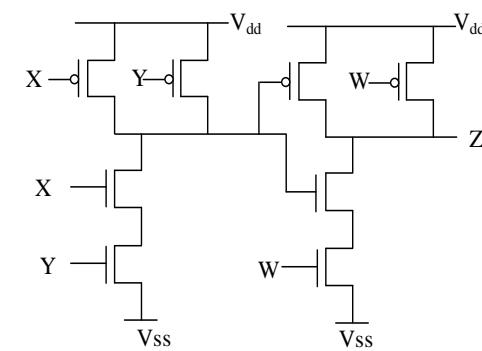
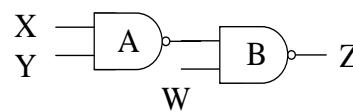
$$Z = \overline{X \cdot Y}$$



小圈表示NOT (非)

circle means NOT 高电平变成低电平，低电平变成高电平

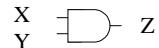
$$Z = \overline{(\overline{X} \cdot \overline{Y}) \cdot W}$$



## 4.2.1 Logic gates and combinational circuit

- Logic gates: electronic circuits realizing Boolean operators

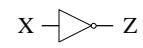
X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

 AND

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

 OR

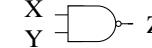
X	Z
0	1
1	0

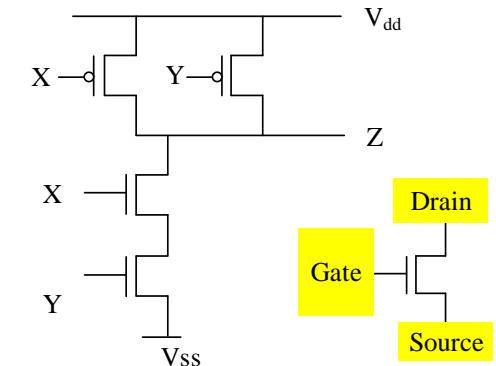
 NOT

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

 XOR

X	Y	Z
0	0	1
0	1	1
1	0	1
1	1	0

 NAND



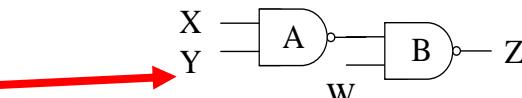
- A combinational circuit is comprised of interconnected gates without feedback wires   组合电路由逻辑门连接而成，没有反馈连线

● 教科书：组合电路实现命题逻辑表达式

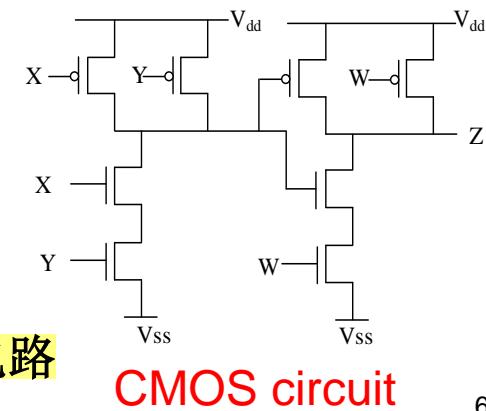
● Any combinational circuit has a corresponding Boolean expression  $\rightarrow Z = \overline{(X \cdot Y)} \cdot W$

● Any Boolean expression has a corresponding combinational circuit

● A combinational circuit can be shown as a logic diagram  $\rightarrow$



使用逻辑线路图表示组合电路



## 4.2.1 Logic gates and combinational circuit

- Logic gates: electronic circuits realizing Boolean operators

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

$\begin{array}{c} X \\ Y \end{array}$   z

AND

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

$\begin{array}{c} X \\ Y \end{array}$   z

OR

X	Z
0	1
1	0

$x \rightarrowtail z$

NOT

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

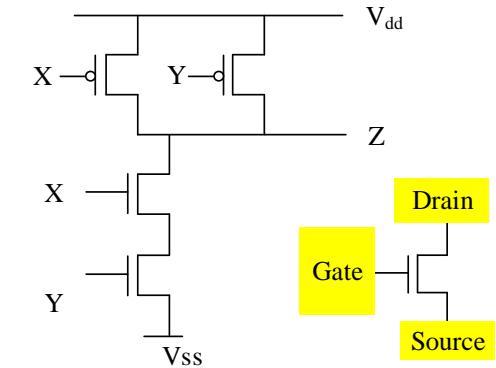
$\begin{array}{c} X \\ Y \end{array}$   z

XOR

X	Y	Z
0	0	1
0	1	1
1	0	1
1	1	0

$\begin{array}{c} X \\ Y \end{array}$   z

NAND

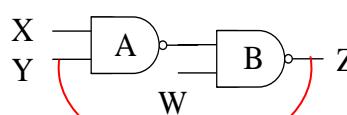


CMOS circuit for NAND

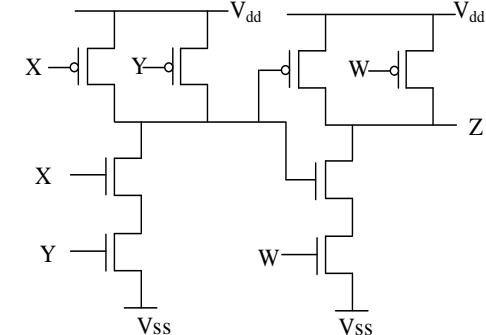
A combinational circuit is comprised of interconnected gates without feedback wires  
组合电路由门电路连接而成，没有反馈连线

- Any combinational circuit has a corresponding Boolean expression
- Any Boolean expression has a corresponding combinational circuit
- A combinational circuit can be shown as a logic diagram

$$Z = \overline{(X \cdot Y)} \cdot W$$



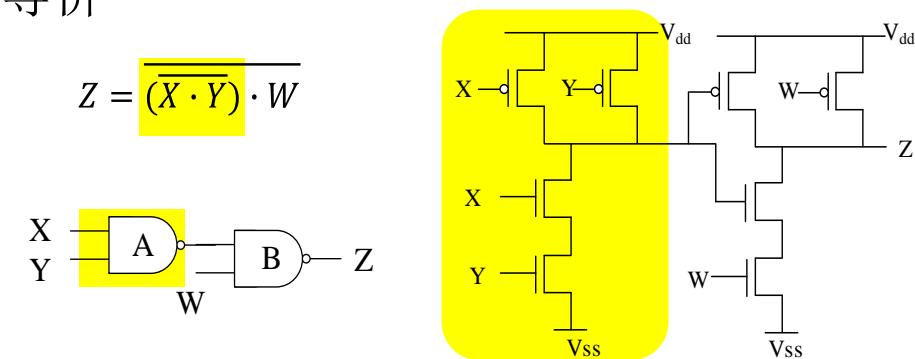
Such **feedback wire**  
is not allowed  
**不允许反馈线路**



CMOS circuit

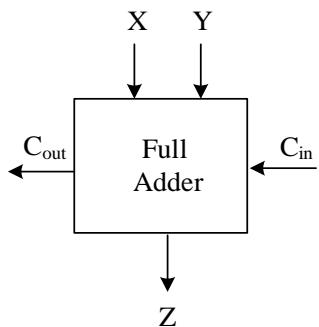
## 4.2.2 The information hiding principle 信息隐藏原理

- A module only exposes its interface and visible behaviors, but hides internal details and internal behavior
  - 模块仅暴露接口和可见行为，隐藏内部细节和内部行为
- Three types of abstractions are shown of the same combinational circuit  
一个组合电路的三种表示，它们逻辑等价
  - Boolean expression 布尔表达式
  - Logic diagram 逻辑线路图
  - CMOS circuit CMOS电路
  - 布尔表达式、逻辑线路图更简洁
- 三个黄色区域表示同样的  
**2-输入-1-输出与非门**
- The three yellow areas  
are different abstractions for the same thing
  - a 2-input-1-output NAND gate
  - The Boolean expression and the logic diagram abstractions hide internal details of the CMOS implementation  
**CMOS电路最复杂，暴露了内部**
  - The former two are simpler and allow different implementations



## 4.2.3 Adders 加法器

- Add two unsigned numbers X and Y to generate result sum Z
  - Consider the carry-in bit  $C_{in}$  and the carry-out bit  $C_{out}$
  - Use the same algorithm we use when doing addition by pen and paper
- For one bit, design a **full adder 全加器**
  - “全”：考虑进位输入 ( $C_{in}$ ) 与进位输出 ( $C_{out}$ )，而不只是本位输入 (X,Y) 与本位输出 (Z)
  - Here, “full” means the adder considers carry-in and carry-out bits
  - **1-minute quiz:** given X, Y and  $C_{in}$ , what are the expressions of Z and  $C_{out}$ ?

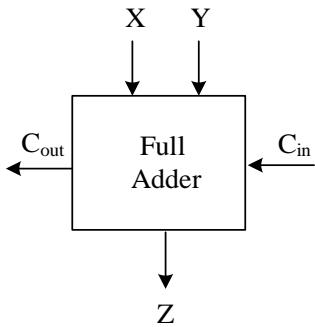


Full adder symbol

# Full adder 全加器

代表了设计方法1：直接连接逻辑门

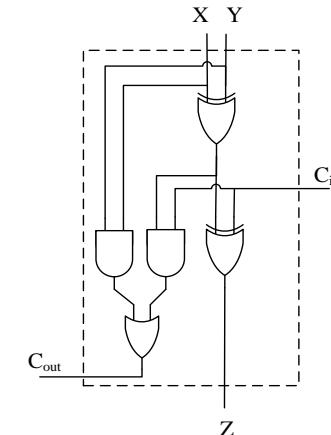
- Add two unsigned numbers X and Y to generate result sum Z
  - Consider the carry-in bit  $C_{in}$  and the carry-out bit  $C_{out}$
  - Use the same algorithm we use when doing addition by pen and paper
- For one bit, design a full adder
  - Here, “full” means the adder considers carry-in and carry-out bits
  - **1-minute quiz:** given X, Y and  $C_{in}$ , what are the expressions of Z and  $C_{out}$ ?
  - A: Derive the truth table from the manual addition method  
Then, derive the Boolean expressions for Z and  $C_{out}$
  - $Z = X \oplus Y \oplus C_{in}$
  - $C_{out} = (X \cdot Y) + (X \oplus Y) \cdot C_{in}$



Full adder symbol  
全加器是系统

Cin	X	Y	Z	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Truth table

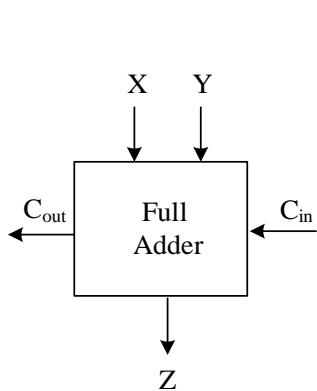


Implementation by gates  
逻辑门是模块

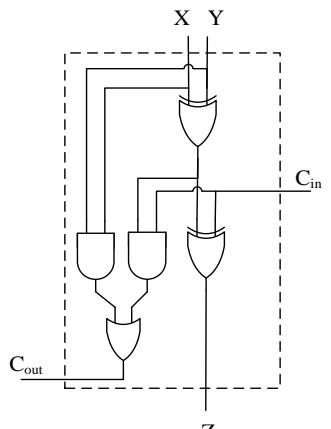
# Ripple-carry adder 波纹进位加法器

代表了设计方法2：串行连接多个模块

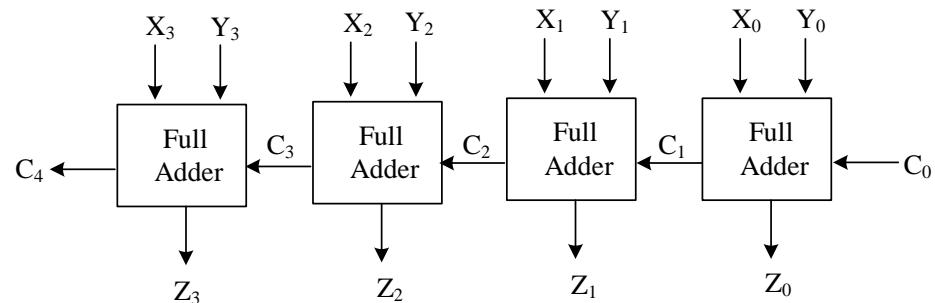
- Add two unsigned numbers X and Y to generate result sum Z
  - Consider the carry-in bit  $C_{in}$  and the carry-out bit  $C_{out}$
  - Use the same algorithm we use when doing addition by pen and paper
- Design an n-bit **ripple-carry adder** (assuming n=4 in example)  
串联4个全加器（模块），实现一个4位波纹进位加法器（系统）
  - Use  $X+Y=1011+1001 = 10100$  to verify that the 4-bit adder works correctly
  - 将模块组合成系统的抽象过程可能会丢失信息
  - **1-minute quiz:** How much time to do the addition? Use gate delay as the unit  
4位波纹进位加法器产生多少级门延迟？n位呢？



Full adder symbol  
全加器是模块



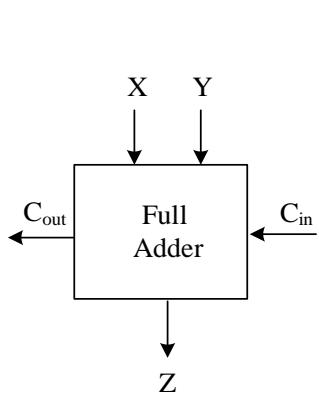
Its implementation by gates



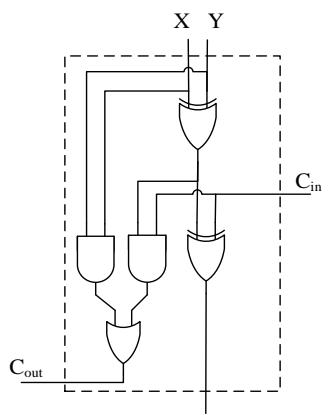
A 4-bit ripple-carry adder  
4位波纹进位加法器是系统

# Ripple-carry adder

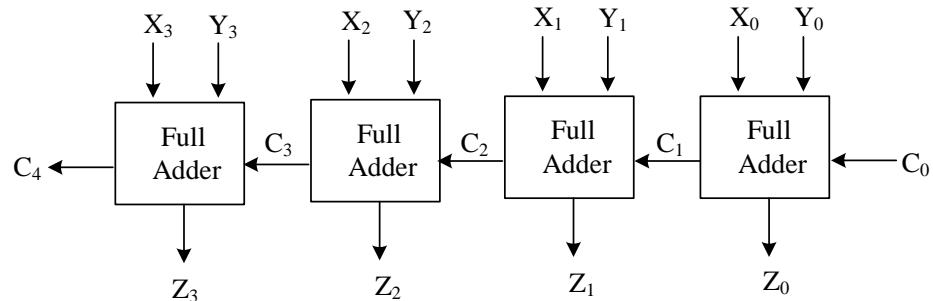
- For n-bit, design an n-bit ripple-carry adder (assuming n=4 in example)
- 将模块组合成系统的抽象过程可能会丢失信息
  - 1-minute quiz: How much time to do the addition? Use gate delay as the unit
  - 问题：4位波纹进位加法器产生多少级门延迟？n位呢？
  - Answer 1: total delay ~ **3n**; about 12 gate delays when n=4
    - Because each full adder needs 3 gate delays to generate carry-out
  - 答案：4位波纹进位加法器产生**4\*3=12**级门延迟，因为每个全加器产生3级门延迟
- Is this answer correct?



Full adder symbol



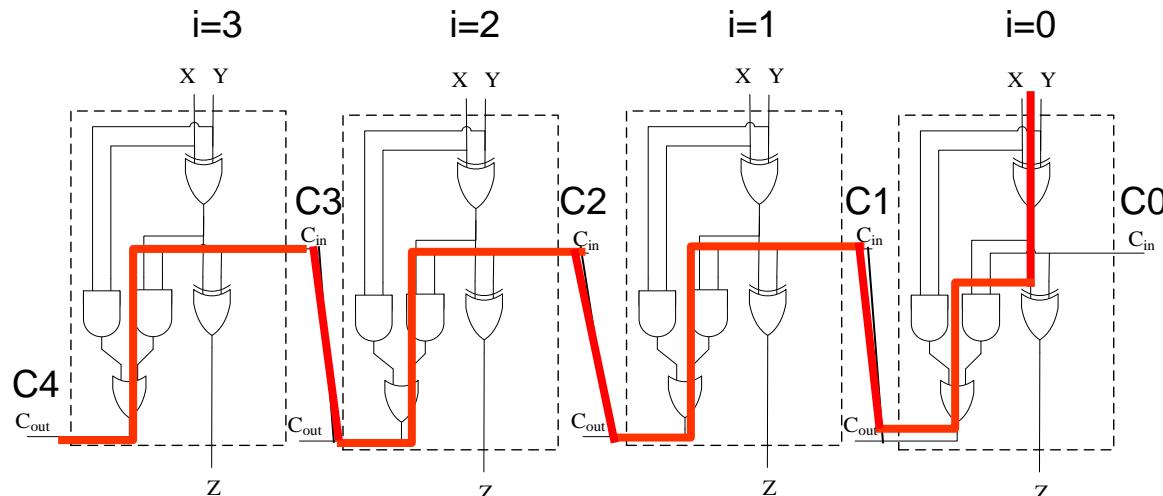
Its implementation by gates



A 4-bit ripple-carry adder

# Ripple-carry adder

- Design an n-bit ripple-carry adder (assuming n=4 in example)
  - **1-minute quiz:** How much time to do the addition? Use gate delay as the unit
  - Answer 2: **2n+1**. There are 9 gate delays for n=4  
正确答案：4位波纹进位加法器产生 $2 \times 4 + 1 = 9$ 级门延迟  
因为当算出 $C_1$ 时， $X_i \oplus Y_i$  也已经被算出来了， $1 \leq i < n$ 
    - Only  $C_1$  needs 3 gate delays, and another  $C_i$  needs only 2 additional gate delays.
    - Why? Because for  $1 \leq i < n$ ,  $X_i \oplus Y_i$  is already generated when  $C_1$  becomes available
      - Red line shows the longest path 红线展示了最长路径



Expand to show the gate-level details

# A much faster adder

代表了设计方法3: 并行连接,

并行进位的加法器  
使用中间模块Gi和Pi

- Instead of compute the carry bits one by one in  $n$  steps, we can compute all the  $n$  carry bits in parallel in one step

先算出全部进位比特值

- This step needs 3 gate delays
- Afterwards, the  $n$  sum bits are computed in parallel in one step

- This step needs 1 gate delay

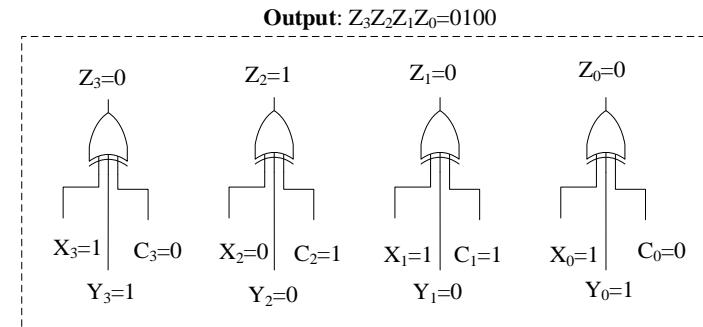
- 4 gate delays in total

只需4级门延迟 4 vs.  $2n-1$

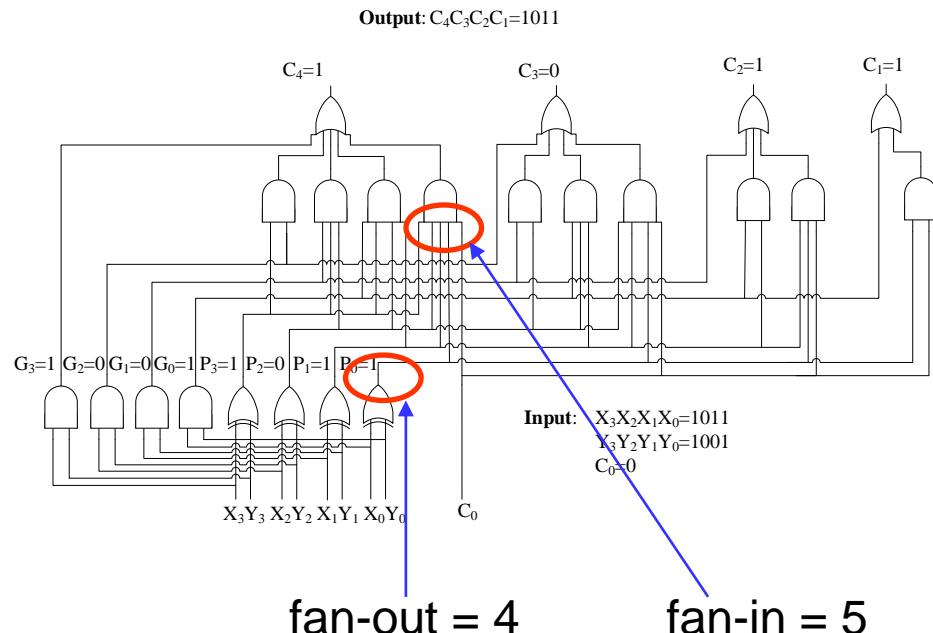
- Compared to  $2n-1$  gate delays for ripple-carry adder

- Constrained by fan-in and fan-out in practice 扇入扇出限制

- A gate can only safely receive a few inputs
- A gate output cannot drive too many wires



Input:  $X_3X_2X_1X_0=1011, Y_3Y_2Y_1Y_0=1001, C_3C_2C_1C_0=0110$



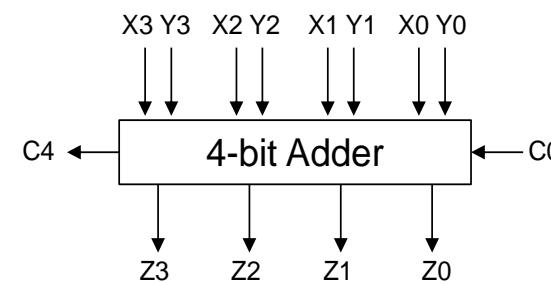
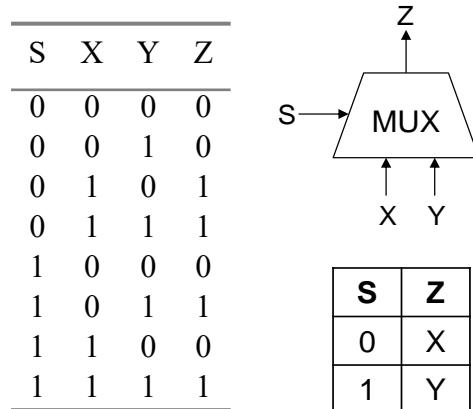
## 4.2.4 A combinational circuit can compute multiple functions via selection by control signals

代表了设计方法4：使用多路复用器和控制信号（本例中的选择输入S）

- An adder-subtractor controlled by a **multiplexer** (MUX in short)
  - Control signal S to select addition ( $S=0$ ) or subtraction ( $S=1$ )
  - Subtraction is adding negative, e.g.,  $5-5 = 5+(-5)$ 
    - The negative value of a number X is the two's complement of X
  - Let  $X = Y = 5$ , i.e.,  $X_3X_2X_1X_0 = Y_3Y_2Y_1Y_0 = 0101$ 
    - Then,  $X - Y = 5 + (-5) = 5 + \text{Two's complement of } 5$

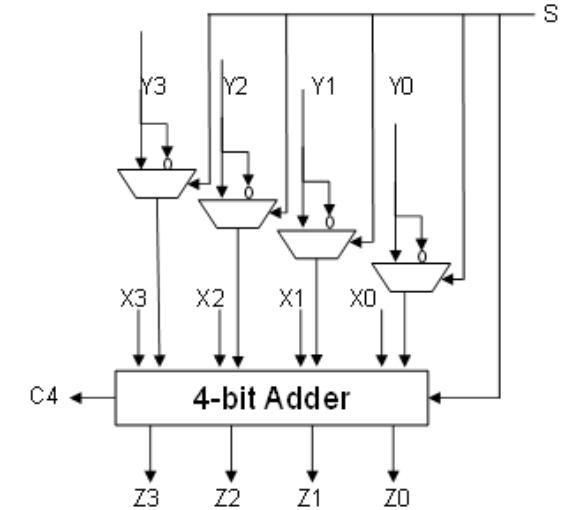
$$Z = \begin{cases} X & \text{if } S = 0 \\ Y & \text{if } S = 1 \end{cases}$$

$$\begin{aligned} &= 0101 + (\bar{0}\bar{1}\bar{0}\bar{1} + 0001) \\ &= 0101 + 1011 \\ &= 10000 = C_4 Z_3 Z_2 Z_1 Z_0 \end{aligned}$$



A multiplexer 多路复用器

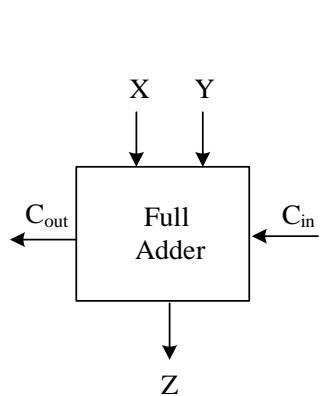
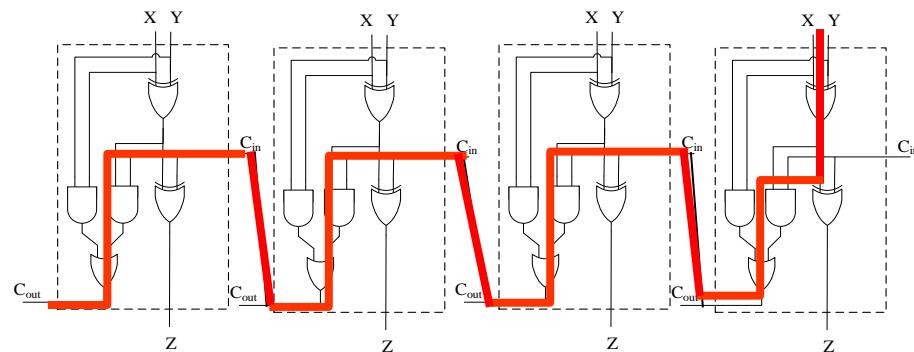
采用多路复用器的**加减器**，控制信号S选择做加法还是做减法



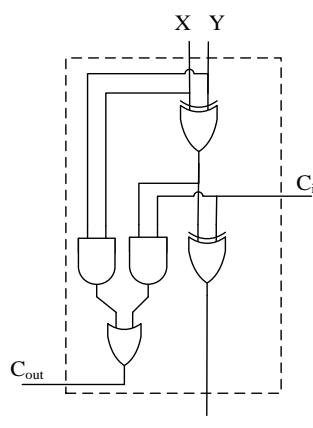
A two's complement 4-bit adder and an adder-subtractor

# 通过实例理解模块化的一般原理

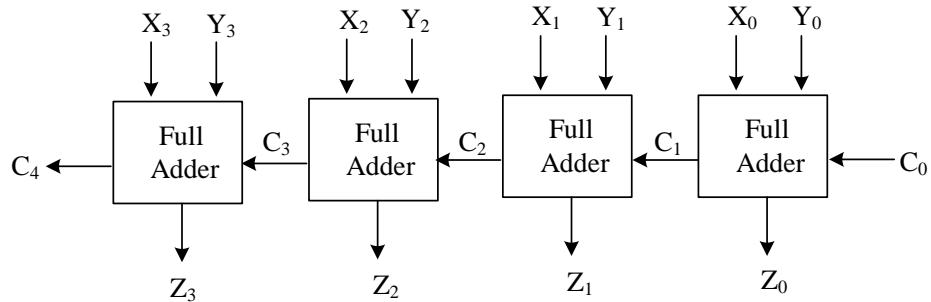
- 模块是采用信息隐藏原理的抽象
  - 全加器模块是一个组合电路抽象，隐藏了虚框内的逻辑电路细节
- 系统中的两个模块可以连接，但不重叠
  - 波纹进位加法器中的两个全加器可连接，但两个全加器的内部电路不重叠
- 模块化是艺术，需要人的创造力
  - 波纹进位加法器比较直接简单
  - 并行进位加法器则需要艺术：  
体现在中间表示P, G两种模块（留作练习）



Full adder symbol  
全加器是模块



Its implementation by gates



A 4-bit ripple-carry adder  
4位波纹进位加法器是系统

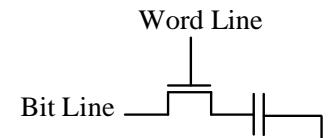
## 4.3 Sequential circuits

- **Sequential circuit = combinational circuit + state circuit**  
时序电路 = 组合电路 + 状态电路, 以实现多步计算过程
  - With states, a system can execute multi-step computational processes
  - Each step computes two types of values
    - The current output values                          当前输出
    - The next state values                          下一状态
  - Both are computed from
    - The current input values                          当前输入
    - The current state values                          当前状态
- Correspondences to logic thinking abstractions    实现逻辑思维
  - A combinational circuit is equivalent to a Boolean expression
  - A sequential circuit is equivalent to an automaton
  - 组合电路实现布尔表达式; 时序电路实现自动机
- States in hardware circuits are implemented by two types of basic circuits  
状态电路有两种实现方法: 存储单元, 触发器
  - Memory cells
  - flip-flops: logic circuits with feedback wires
    - We discuss only the D flip-flop

## 4.3.1 Types of memory technology 存储器类别

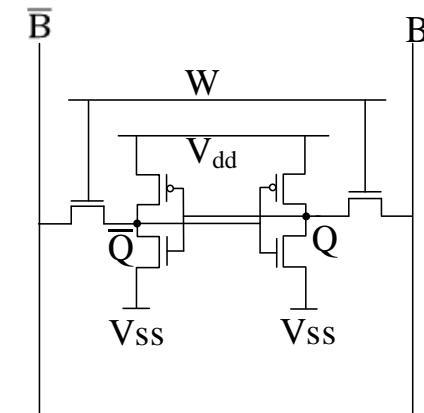
- Non-volatile memory (**NVM**): content is retained when power is off
  - 计算机下电后，非易失存储器中的内容不会丢失
  - ROM** (read-only memory) 只读存储器 例子：开机后读取数据与指令
  - Read-write NVM 可读可写的非易失存储器 例子：U盘
- Volatile memory: content is lost when power is off
  - 计算机下电后，易失存储器中的内容会丢失
  - DRAM** (dynamic random access memory) 动态随机访问存储器
    - Simple and inexpensive 成本低
    - Needs to constantly refresh its content (once every 7.8-128  $\mu$ s)
    - Because the capacitor leaks electricity after charging
  - SRAM** (static random access memory) 静态随机访问存储器
    - More complex but avoid refreshing overhead
    - Faster but more expensive than DRAM

Type	Latency	Price \$/GB
Register	100s ps	N/A
SRAM	100s ps ~ 10 ns	\$100's ~1000s /GB
DRAM	10s ~ 100 ns	\$2~4 /GB
NVM: Main Memory	100 ns ~ 10 $\mu$ s	\$6 / GB
NVM: SSD	10 $\mu$ s ~ 1 ms	\$0.1~0.2 / GB
Hard Disk: HDD	> A few ms	\$0.02 / GB



**DRAM cell**

1 transistor and  
1 capacitor  
只需一个晶体管



**SRAM cell**

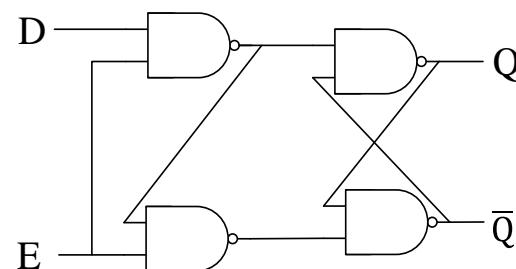
6 transistors  
需要6个晶体管

## 4.3.2 D flip-flop D触发器

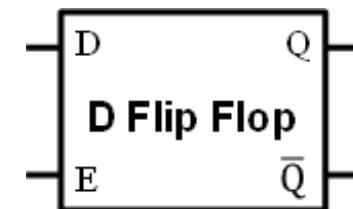
- We can also use gates to form state circuits
  - With feedback wires 触发器是包含反馈线路的逻辑门电路
- A common type is the **delay flip-flop**, or D flip-flop
  - 2-input-2-output, where the 2 states (outputs) are negation of each other
    - Input D is data input; Input E (enable signal) is often the clock signal CLK  
状态Q值是数据输入D一个时钟周期前的值，即D延迟一周期后变成Q
  - Functionality: when enabled, Q is D delayed one clock cycle
    - When E=0, Q remains the same; when E=1, the next Q will be the current D

E	D	Q	Q <sub>next</sub>
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

D flip-flop: Truth table



Internal logic diagram

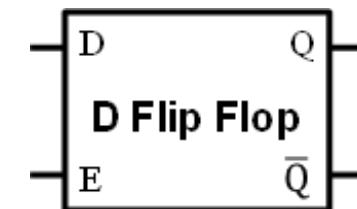
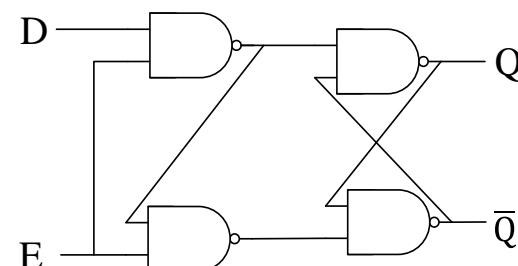
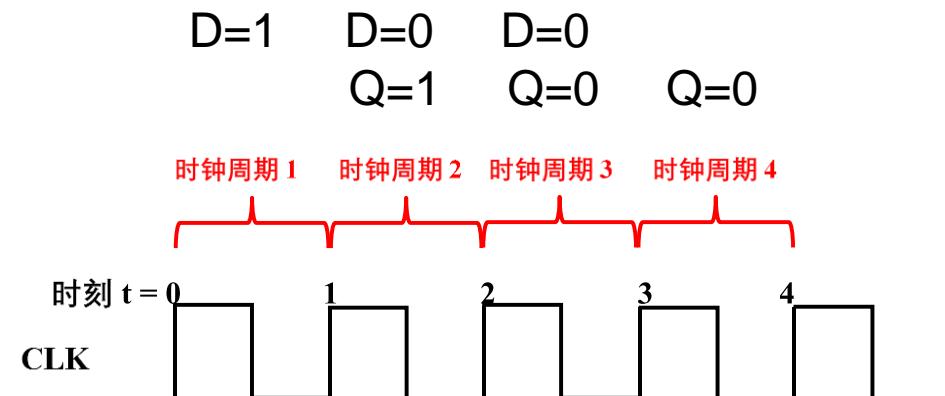


Symbol

## 4.3.2 D flip-flop D触发器

- Input D is data input; **Input E (enable signal) is often the clock signal CLK**
- D是数据输入，E常常是时钟信号CLK
- 状态Q值是数据输入D一个时钟周期前的值，即D延迟一周期后变成Q

E	D	Q	$Q_{next}$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1



D flip-flop: Truth table

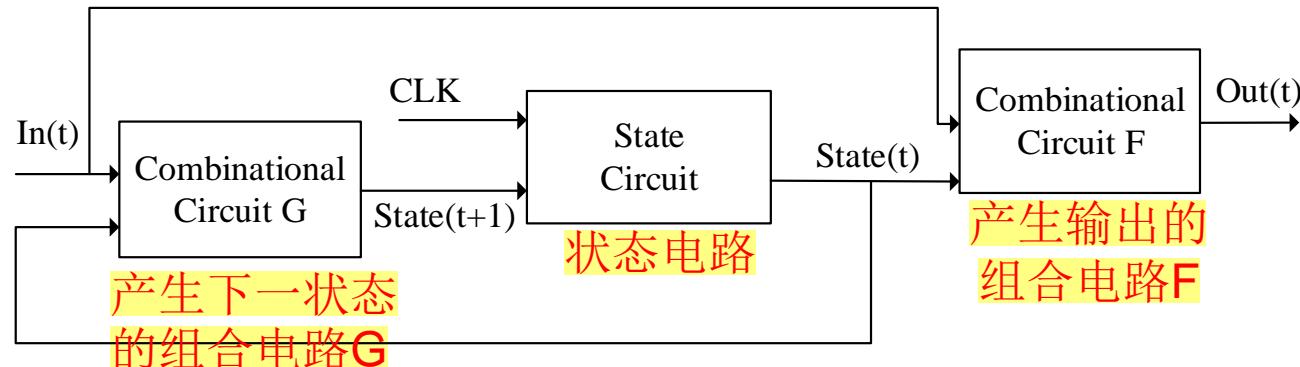
Internal logic diagram

Symbol

### 4.3.3 General organization of sequential circuits

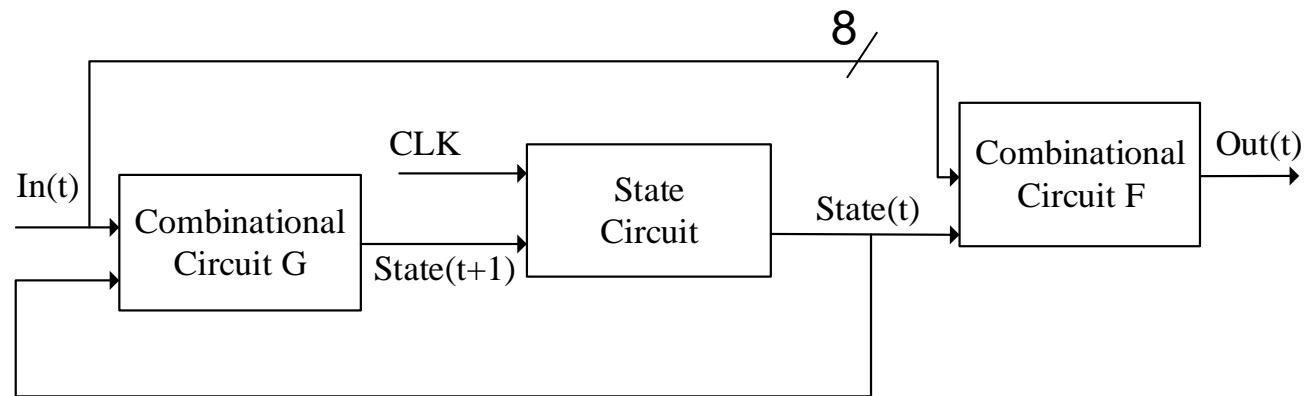
时序电路的一般组成：两个组合电路，一个状态电路；时钟信号驱动

- Any sequential circuit can be organized as shown below
  - Comprised of two combinational circuits and a state circuit
  - Driven by a clock signal CLK
  - The state circuit consists of one or more D flip-flops
    - The output circuit F:  $Out(t) = F(Out(t), State(t))$  输出电路
    - The next-state circuit G:  $State(t+1) = G(Out(t), State(t))$  下一状态电路
- Logic diagram of a sequential circuit 时序电路的逻辑线路图
  - Also called synchronous sequential circuit
    - Because the sequential circuit is synchronized by the clock signal



### 4.3.3 General organization of sequential circuits

- Any sequential circuit can be organized as shown below
  - Comprised of two combinational circuits and a state circuit
  - Driven by a clock signal CLK
  - The state circuit consists of one or more D flip-flops
  - The output circuit F:  $\text{Out}(t) = F(\text{In}(t), \text{State}(t))$
  - The next-state circuit G:  $\text{State}(t+1) = G(\text{In}(t), \text{State}(t))$
- An input or output may have multiple bits (此处输入In是8条线)

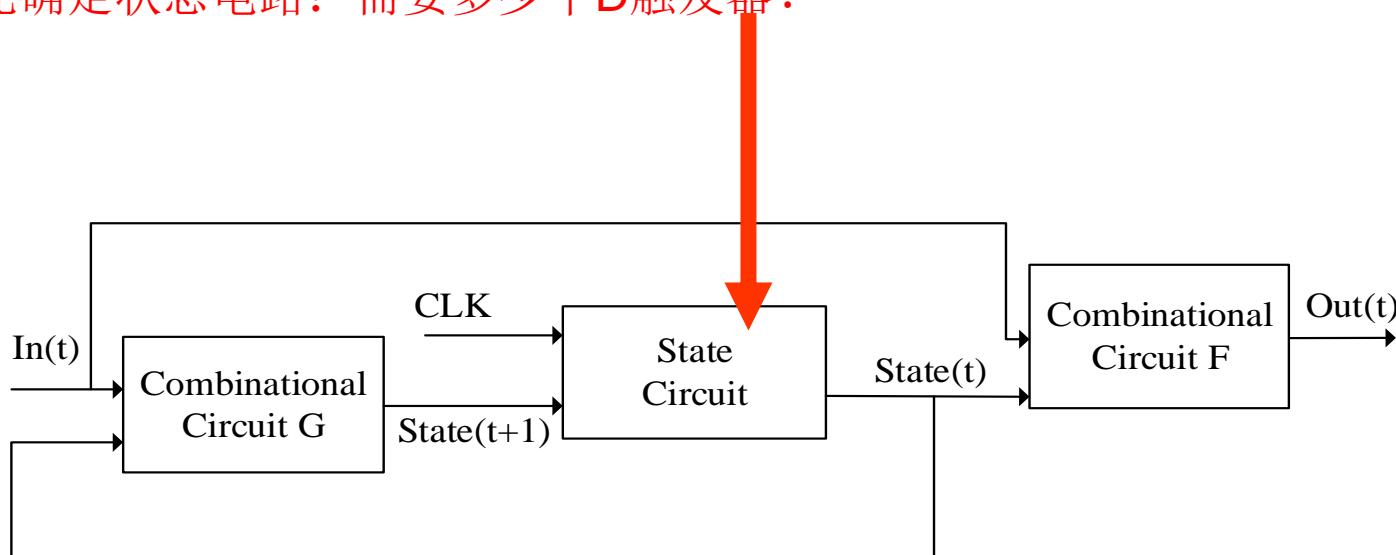


We will go through the process of designing a 4-bit serial adder

## 4.3.4 Design a 4-bit serial adder

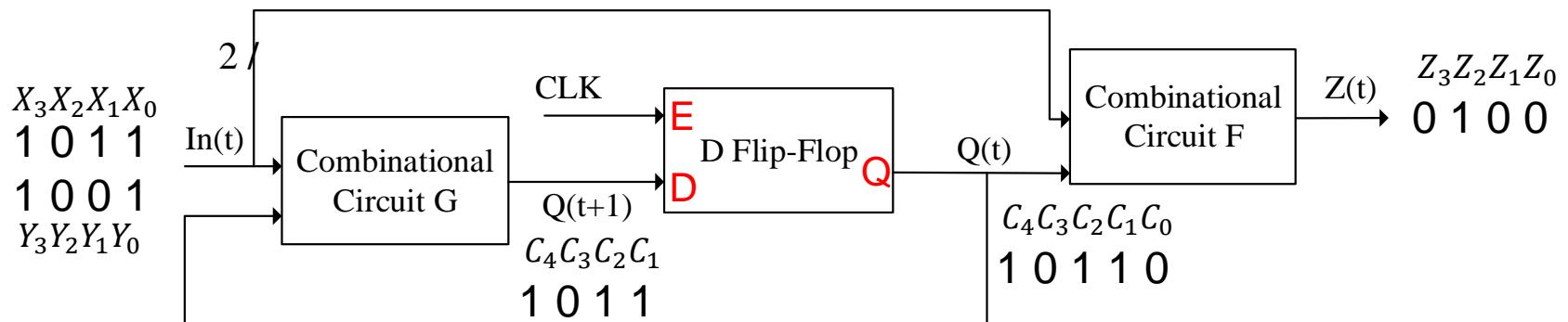
### 设计4位串行加法器

- Perform  $Z_3Z_2Z_1Z_0 = X_3X_2X_1X_0 + Y_3Y_2Y_1Y_0$  in 4 steps  
需要四步（四个时钟周期）
  - Each step performs a full addition 每一步执行一个全加操作
- Verify:  $11_{10} + 9_{10} = 1011_2 + 1001_2 = 10100_2 = 20_{10} = 4_{10}$  and overflow  
用一个实例验证： 输入X=1011, Y=1001； 则输出Z=0100且产生溢出
- Design process 设计过程
  - Q: how many bits (or D flip-flops) are needed for the state circuit?
  - 首先确定状态电路：需要多少个D触发器？



# Design process

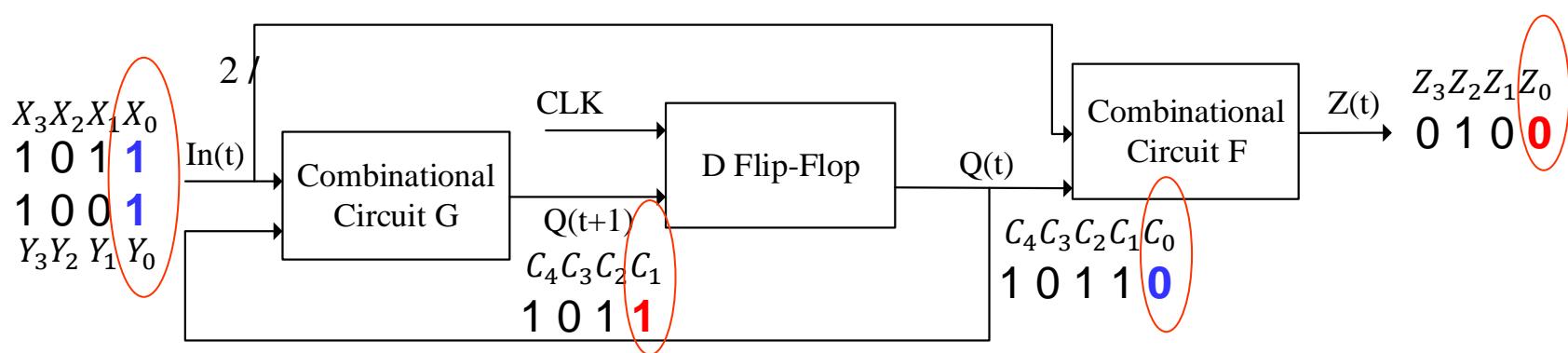
- Design process
  - Q: how many bits (or D flip-flops) are needed for the state circuit?
    - A: 1 bit to hold the carry bit. **Only one D flip-flop is needed.** Denote the state as Q.  
**Clock signal as the Enable signal**  
只需要一个 D触发器，其状态Q表示当前进位，其Enable端连接时钟信号CLK
  - Draw the logic diagram for the sequential circuit  
确定状态电路后，只需要画出组合电路G、F（确定它们的布尔表达式）
    - How?  
使用加法原理理解串行加法器的时序电路
    - 某些同学更喜欢使用实例来理解



# Design process

- Draw the logic diagram for the sequential circuit
  - 确定组合电路G、F的布尔表达式
  - How?  
使用实例和加法原理理解串行加法器的时序电路
    - Step 1: inputs in blue, outputs in red. Note that  $C_0 = 0$

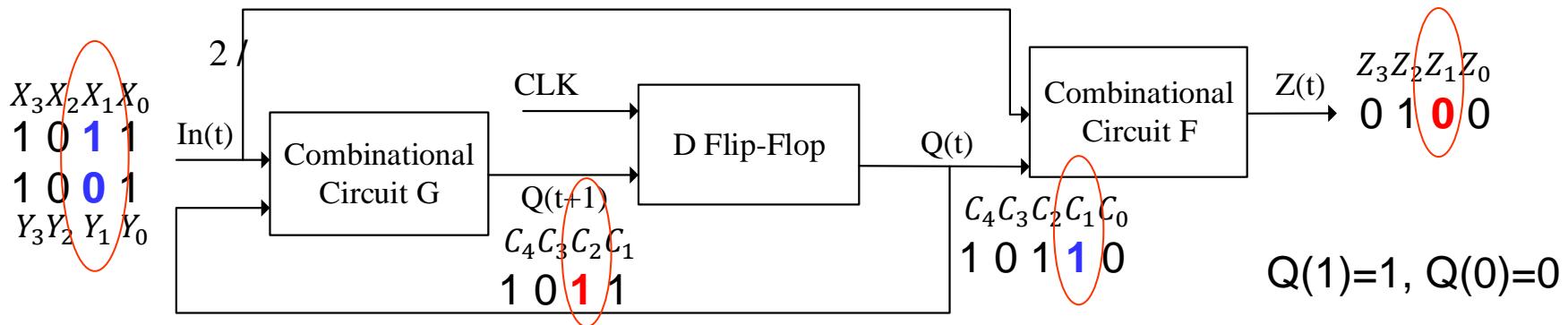
$$\begin{array}{r} 1011 = X \\ 1001 = Y \\ \hline 101\textcolor{red}{1}0 = C \\ 010\textcolor{red}{0} = Z \end{array}$$



# Design process

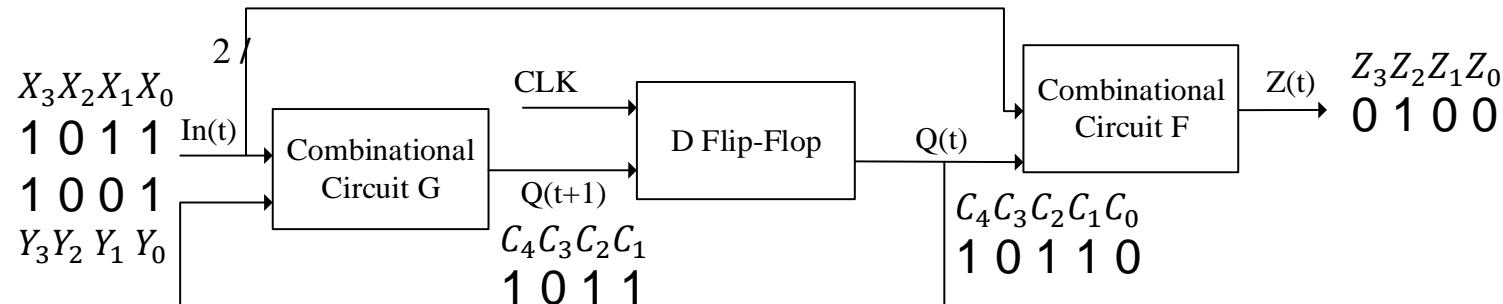
- Draw the logic diagram for the sequential circuit
  - 确定组合电路G、F的布尔表达式
  - How?
    - 使用实例和加法原理理解串行加法器的时序电路
  - Step 2: inputs in blue, outputs in red.

$$\begin{array}{r} 1011 = X \\ 1001 = Y \\ \hline 10110 = C \\ 0100 = Z \end{array}$$

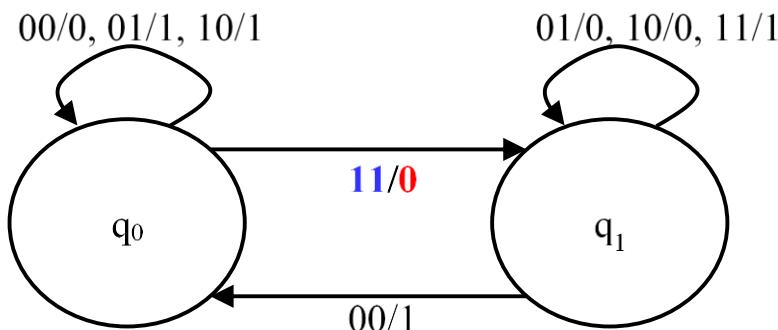


# Design process

- Draw the logic diagram for the sequential circuit
  - Derive the state-transition diagram 画出时序电路的状态转换图



XY/Z



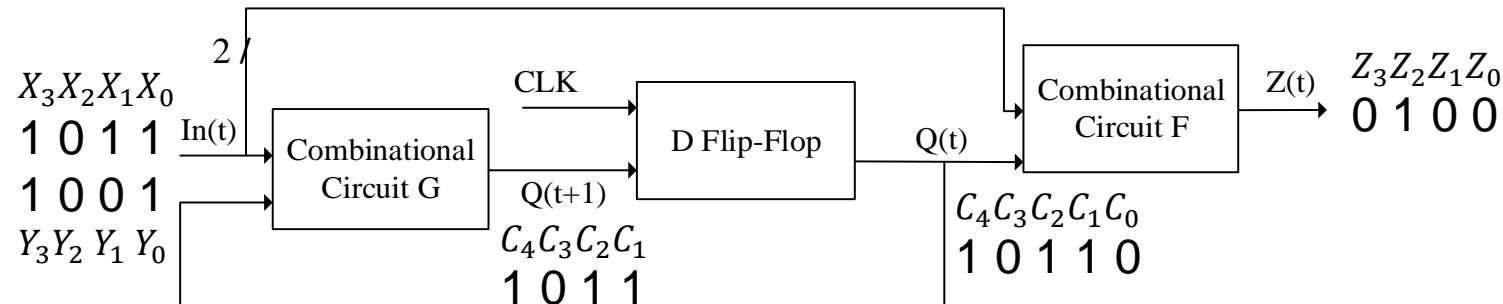
画出时序电路的状态转换图

$q_0$  denotes  $C=0$   
 $q_1$  denotes  $C=1$

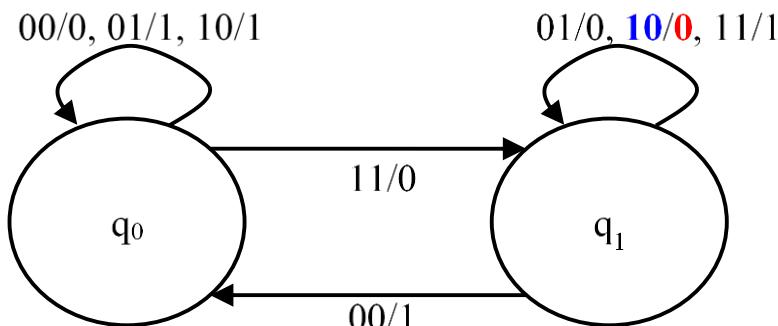
$$\begin{array}{r} 1011 = X \\ 1001 = Y \\ + \\ 10110 = C \\ 0100 = Z \end{array}$$

# Design process

- Draw the logic diagram for the sequential circuit
  - Derive the state-transition diagram



XY/Z



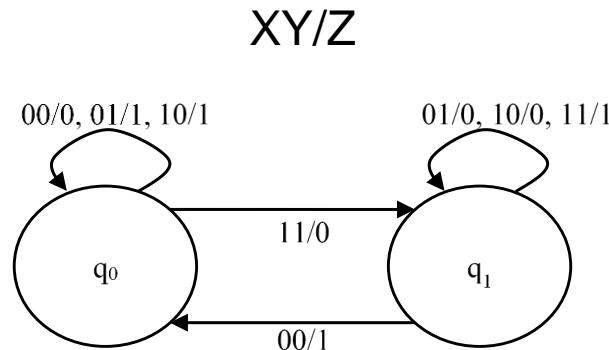
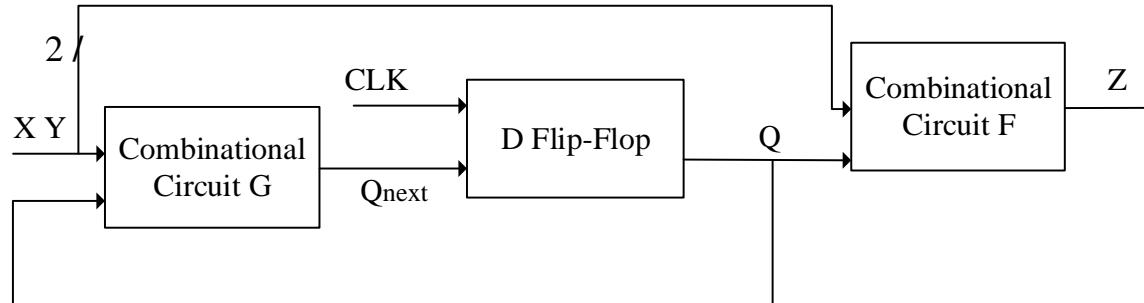
画出时序电路的状态转换图

$q_0$  denotes  $C=0$   
 $q_1$  denotes  $C=1$

$$\begin{array}{r} 1011 = X \\ 1001 = Y \\ + \\ 10110 = C \\ 0100 = Z \end{array}$$

# Design process

- Derive the state-transition diagram (remember that Q denotes carry C)
- Derive the truth table and Boolean expressions for F, G



$q_0$  denotes C=0  
 $q_1$  denotes C=1

Q	X	Y	Z	Q <sub>next</sub>
$q_0$	0	0	0	$q_0$
$q_0$	0	1	1	$q_0$
$q_0$	1	0	1	$q_0$
$q_0$	1	1	0	$q_1$
$q_1$	0	0	1	$q_0$
$q_1$	0	1	0	$q_1$
$q_1$	1	0	0	$q_1$
$q_1$	1	1	1	$q_1$

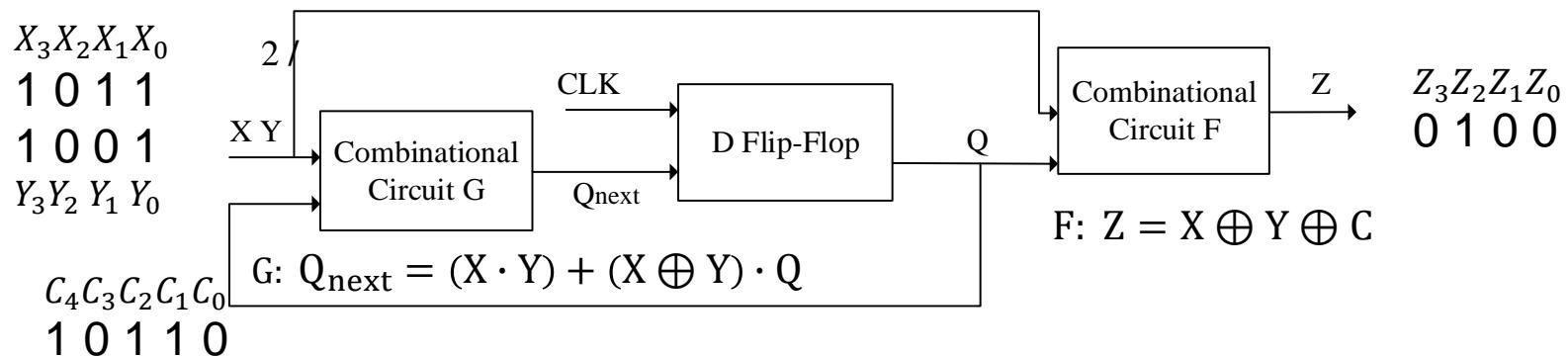
进而导出真值表  
 进而求出组合电路F和G的布尔逻辑表达式

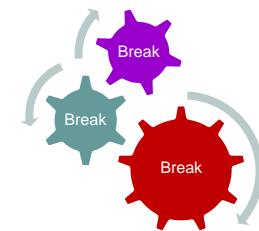
$$Z = F(X, Y, Q) = X \oplus Y \oplus C$$

$$Q_{\text{next}} = G(X, Y, Q) = (X \cdot Y) + (X \oplus Y) \cdot Q$$

# Design process

- Derive the state-transition diagram (remember that Q denotes carry C)
- Derive the truth table and Boolean expressions for F, G
- Verify the resulting sequential circuit,  
given  $X_3X_2X_1X_0 = 1011$ ,  $Y_3Y_2Y_1Y_0 = 1001$ ,  $C_0 = 0$ 
  - Step 1:  $X_0 = 1, Y_0 = 1, C_0 = \textcolor{red}{0}$ .  $Z_0 = \textcolor{red}{0}, C_1 = \textcolor{red}{1}$  最后，用实例验证
  - Step 2:  $X_1 = 1, Y_1 = 0, C_1 = \textcolor{red}{1}$ .  $Z_1 = \textcolor{red}{0}, C_2 = \textcolor{red}{1}$
  - Step 3:  $X_2 = 0, Y_2 = 0, C_2 = \textcolor{red}{1}$ .  $Z_2 = \textcolor{red}{1}, C_3 = \textcolor{red}{0}$
  - Step 4:  $X_3 = 1, Y_3 = 1, C_3 = \textcolor{red}{0}$ .  $Z_3 = \textcolor{red}{0}, C_4 = \textcolor{red}{1}$
- The final result is  $Z_3Z_2Z_1Z_0 = 0100$ , with  $C_4 = 1$  indicating overflow





# Take-Home Messages

- Modularization
  - Divide a system into multiple subsystems (modules), and compose modules into a system (higher-level abstraction), using information hiding
  - Understand modularization via a design journey
- Combinational circuits are circuits of gates without feedback wires
  - Equivalent to Boolean expression
    - How many combinational circuits are there, of n-input-1-output?
  - Boolean expressions and logic diagrams are simpler than CMOS circuits
  - Examples: adders, adder-subtractor with multiplexer
- Sequential circuits = state circuits + combinational circuits
  - State can be represented by a memory cell or a flip-flop
  - There are different types of memory, with different latencies and costs
    - Non-volatile memory (NVM): ROM (read-only memory), Read-write NVM
    - Volatile memory: DRAM, SRAM
  - A flip-flop, such as D flip-flop, is a circuit of gates with feedback wires
  - **Design a serial subtractor** to see how a sequential circuit generates the output and the next state from the inputs and the current state