



Systems Thinking

Modularization-1:

Combinational Circuits and Sequential Circuits

组合电路与时序电路

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Outline

- What is systems thinking?
- Three objectives of systems thinking
- Abstraction
- Modularization 模块化 抽象一点也不“抽象”
 - Modularization and modules
 - Combinational circuits 组合电路
 - Logic gates and combinational circuits
 - The information hiding principle
 - Adders
 - An adder-subtractor controlled by multiplexers
 - Sequential circuits 时序电路
 - Types of memory cells and the D flip-flop
 - General organization of sequential circuit
 - A serial adder example
 - Instruction Set and Instruction Pipeline 指令集与指令流水线
 - Software Stack 软件栈
- Seamless transition

These slides acknowledge sources for additional data not cited in the textbook

4.1 Modularization and modules

模块化与模块

- Modularization is a systems thinking method, similar to divide-and-conquer in algorithmic thinking 模块化像算法思维的分治方法
 - Divide a system into multiple subsystems called modules 将系统划分成模块
 - Compose modules into a system (higher-level abstraction) 将模块组合成系统
 - 该系统成为一个更高级的抽象
- In a system with modularization
 - Two modules may be **interconnected**, but often **do not overlap**
 - 很多情况下，系统中的两个模块可以连接，但不重叠
- Modularization is a special form of abstraction where the information hiding principle is followed 模块是采用了信息隐藏原理的抽象
- Modularization, i.e., how to divide and compose a system, is an art, needing human imagination and creativity 模块化是艺术，需要创造力
- Understand how modularization works via a design journey
 - of higher and higher hardware subsystem abstractions
 - from designing **gates** to designing an **instruction pipeline**
 - 从逻辑门到指令流水线的旅行，学习模块化概念（模块如何组合成为系统）

4.2.1 Logic gates and combinational circuit

- Logic gates: electronic circuits realizing Boolean operators

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1



AND

与门

$$Z = X \cdot Y$$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

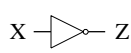


OR

或门

$$Z = X + Y$$

X	Z
0	1
1	0



NOT

非门

$$Z = \bar{X}$$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

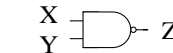


XOR

异或门

$$Z = X \oplus Y$$

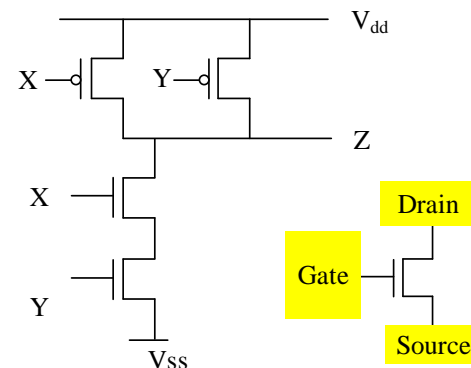
X	Y	Z
0	0	1
0	1	1
1	0	1
1	1	0



NAND

与非门

$$Z = \overline{X \cdot Y}$$



CMOS circuit for NAND

与非门的**CMOS**电路

晶体管行为

Gate 为1（高电平）：导通

Gate为0（低电平）：断开

Vss: ground (0)

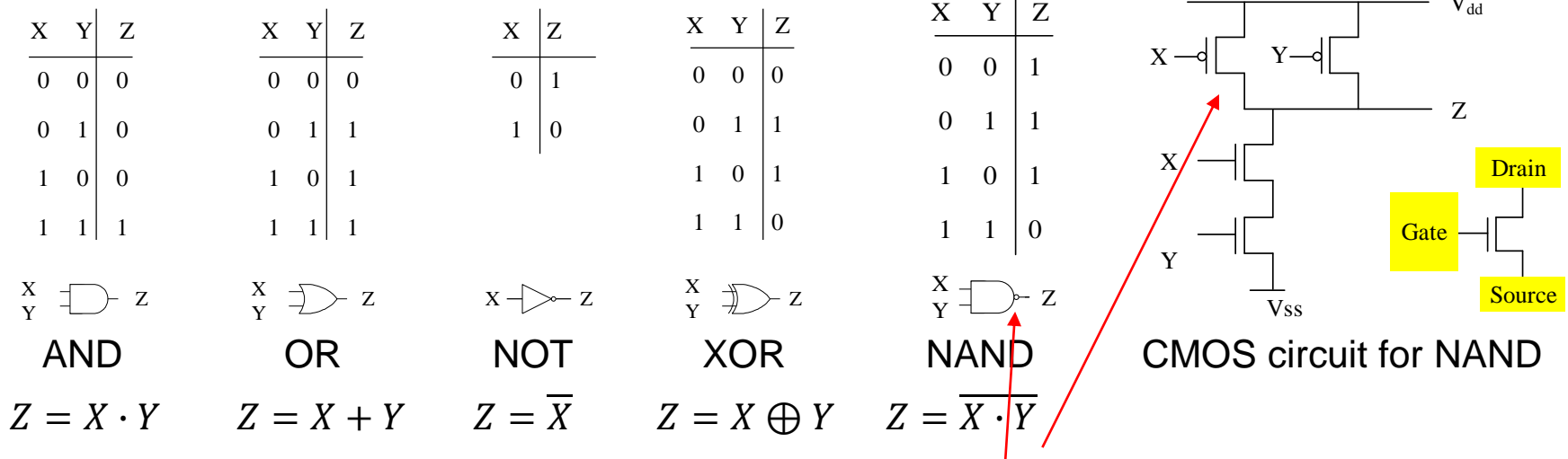
Vdd: high voltage (1)

每个逻辑门都有半导体电路实现
只需知道与非门的**CMOS**电路实现
作为例子

X,Y都是高电平（1）时，下两个晶体管联通，上两个晶体管断开，Z连接到地（0）

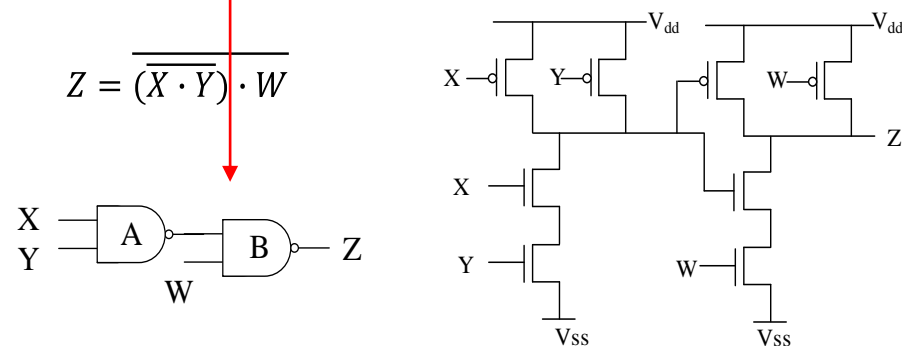
4.2.1 Logic gates and combinational circuit

- Logic gates: electronic circuits realizing Boolean operators



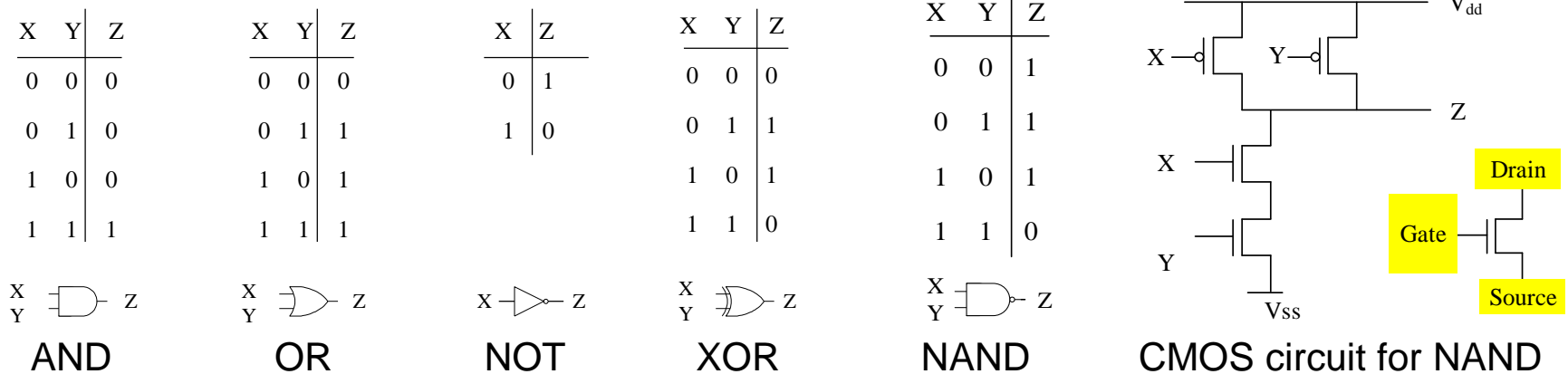
小圈表示NOT (非)

circle means NOT 高电平变成低电平，低电平变成高电平



4.2.1 Logic gates and combinational circuit

- Logic gates: electronic circuits realizing Boolean operators

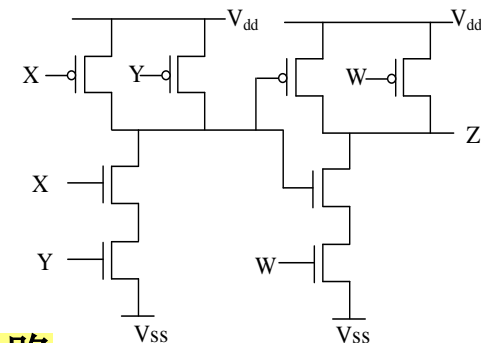
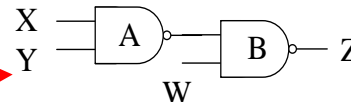


- A combinational circuit is comprised of interconnected gates without feedback wires 组合电路由逻辑门连接而成，没有反馈连线

- 教科书：组合电路实现命题逻辑表达式

- Any combinational circuit has a corresponding **Boolean expression** $\longrightarrow Z = \overline{(\overline{X \cdot Y}) \cdot W}$
- Any Boolean expression has a corresponding combinational circuit

- A combinational circuit can be shown as a **logic diagram** \longrightarrow



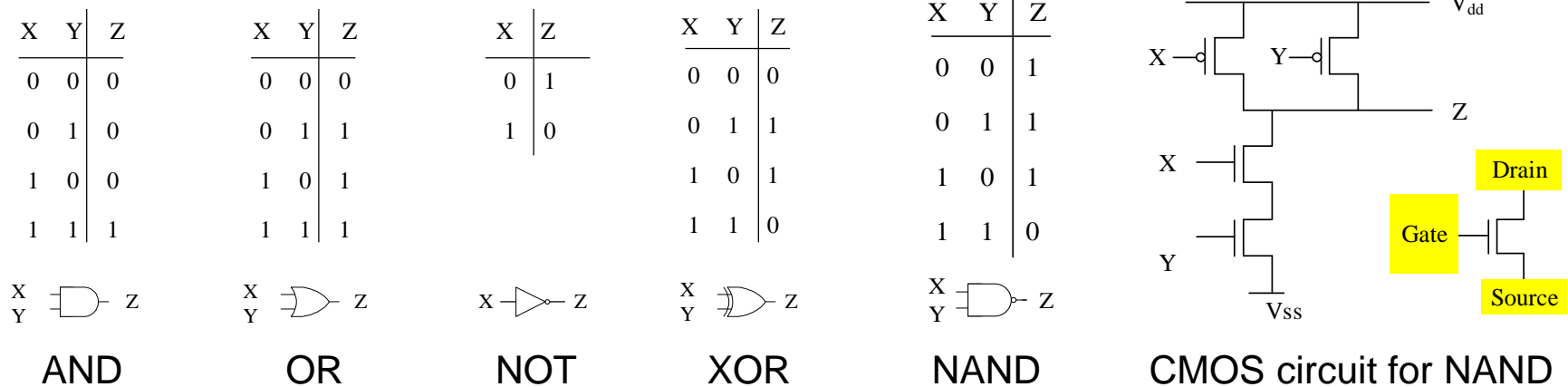
- 不用更加繁杂的CMOS电路

使用逻辑线路图表示组合电路

CMOS circuit

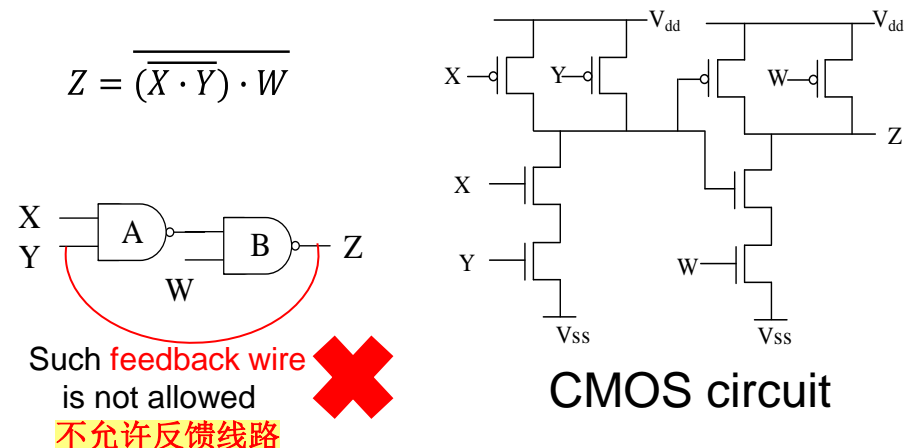
4.2.1 Logic gates and combinational circuit

- Logic gates: electronic circuits realizing Boolean operators



A combinational circuit is comprised of interconnected gates without feedback wires
 组合电路由门电路连接而成，没有反馈连线

- Any combinational circuit has a corresponding Boolean expression
- Any Boolean expression has a corresponding combinational circuit
- A combinational circuit can be shown as a logic diagram



4.2.2 The information hiding principle 信息隐藏原理

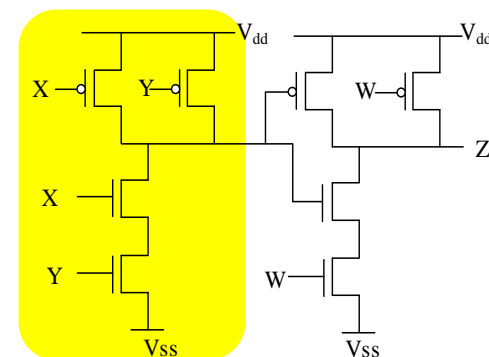
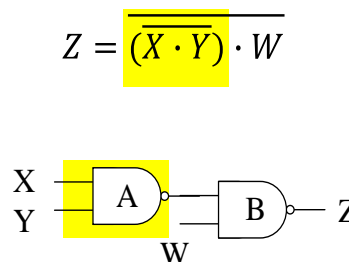
- A module only exposes its interface and visible behaviors, but hides internal details and internal behavior

- 模块仅暴露接口和可见行为，隐藏内部细节和内部行为

- Three types of abstractions are shown of the same combinational circuit
一个组合电路的三种表示，它们逻辑等价

- Boolean expression 布尔表达式
- Logic diagram 逻辑线路图
- CMOS circuit CMOS电路

- 布尔表达式、逻辑线路图更简洁



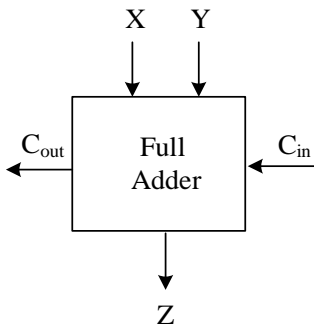
- 三个黄色区域表示同样的
2-输入-1-输出与非门

- The three yellow areas
are different abstractions for the same thing

- a 2-input-1-output NAND gate
- The Boolean expression and the logic diagram abstractions hide internal details of the CMOS implementation CMOS电路最复杂，暴露了内部
- The former two are simpler and allow different implementations

4.2.3 Adders 加法器

- Add two unsigned numbers X and Y to generate result sum Z
 - Consider the carry-in bit C_{in} and the carry-out bit C_{out}
 - Use the same algorithm we use when doing addition by pen and paper
- For one bit, design a **full adder 全加器**
 - “全”：考虑进位输入（ C_{in} ）与进位输出（ C_{out} ），而不只是本位输入（ X, Y ）与本位输出（ Z ）
 - Here, “full” means the adder considers carry-in and carry-out bits
 - **1-minute quiz**: given X , Y and C_{in} , what are the expressions of Z and C_{out} ?



Full adder symbol

Full adder 全加器

代表了设计方法1：直接连接逻辑门

- Add two unsigned numbers X and Y to generate result sum Z

- Consider the carry-in bit C_{in} and the carry-out bit C_{out}
- Use the same algorithm we use when doing addition by pen and paper

- For one bit, design a full adder

- Here, “full” means the adder considers carry-in and carry-out bits
- **1-minute quiz:** given X, Y and C_{in} , what are the expressions of Z and C_{out} ?

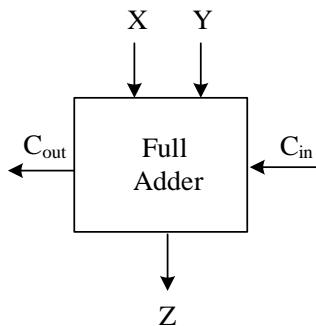
- A: Derive the truth table from the manual addition method

Then, derive the Boolean expressions for Z and C_{out}

采用二进制加法原理导出真值表
继而导出输出Z和 C_{out} 的布尔表达式

- $Z = X \oplus Y \oplus C_{in}$

- $C_{out} = (X \cdot Y) + (X \oplus Y) \cdot C_{in}$

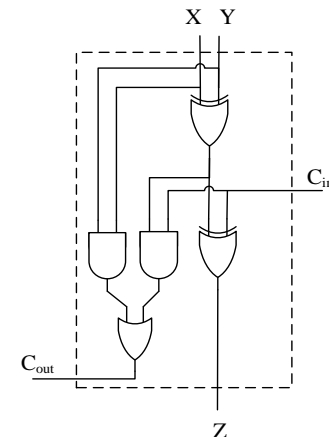


Full adder symbol

全加器是系统

C_{in}	X	Y	Z	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Truth table



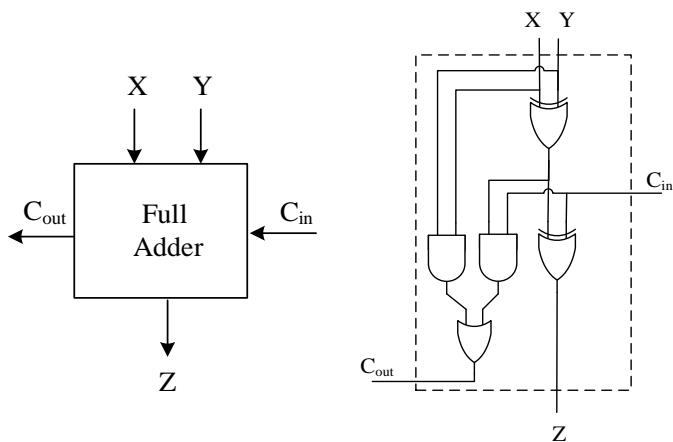
Implementation by gates

逻辑门是模块

Ripple-carry adder 波纹进位加法器

代表了设计方法2: 串行连接多个模块

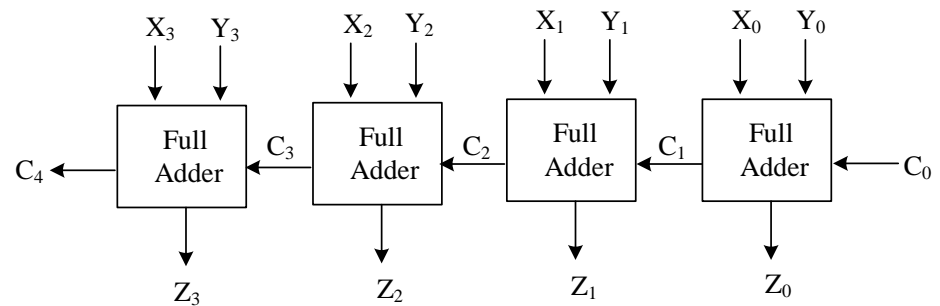
- Add two unsigned numbers X and Y to generate result sum Z
 - Consider the carry-in bit C_{in} and the carry-out bit C_{out}
 - Use the same algorithm we use when doing addition by pen and paper
- Design an n -bit ripple-carry adder (assuming $n=4$ in example)
串联4个全加器（模块），实现一个4位波纹进位加法器（系统）
 - Use $X+Y=1011+1001=10100$ to verify that the 4-bit adder works correctly
- 将模块组合成系统的抽象过程可能会丢失信息
- **1-minute quiz:** How much time to do the addition? Use gate delay as the unit
4位波纹进位加法器产生多少级门延迟？ n 位呢？



Full adder symbol

全加器是模块

Its implementation by gates

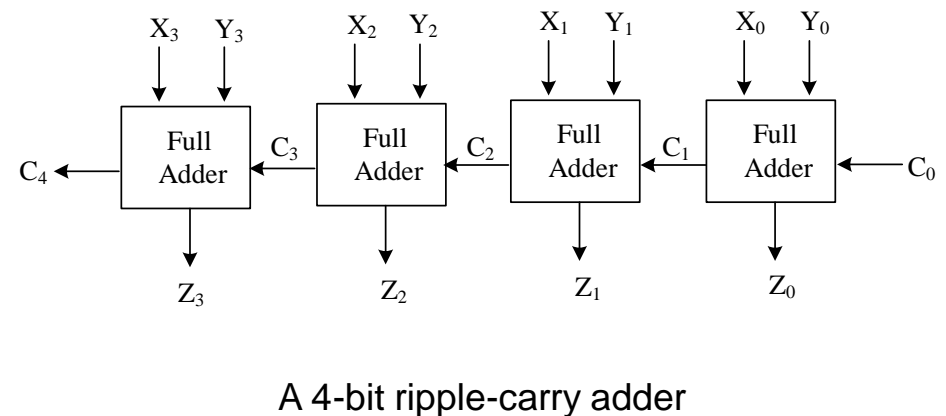
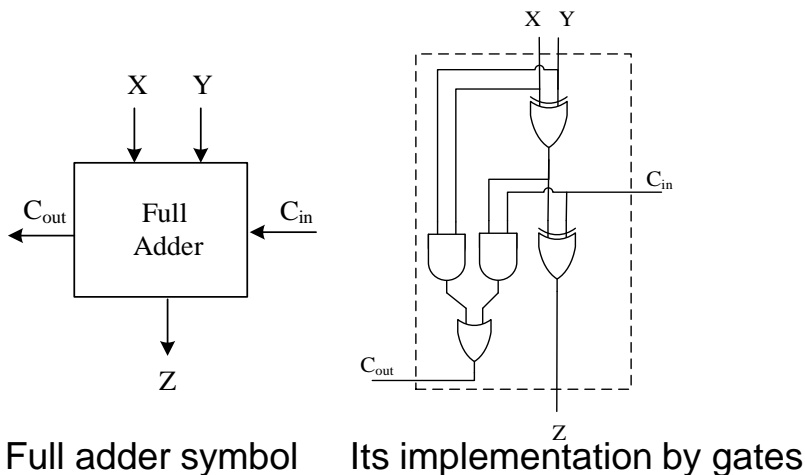


A 4-bit ripple-carry adder

4位波纹进位加法器是系统

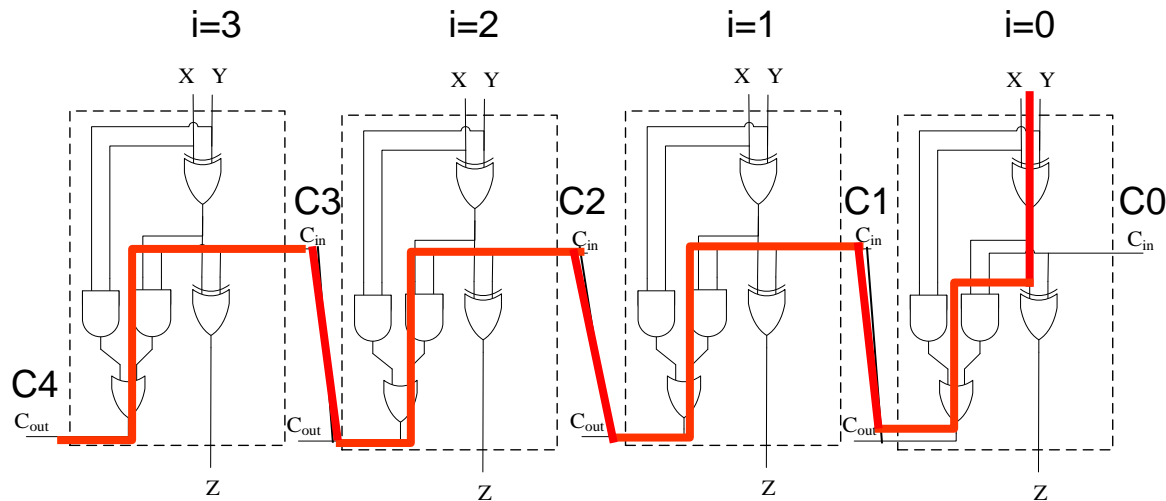
Ripple-carry adder

- For n-bit, design an n-bit ripple-carry adder (assuming $n=4$ in example)
- 将模块组合成系统的抽象过程可能会丢失信息
 - 1-minute quiz:** How much time to do the addition? Use gate delay as the unit
 - 问题: 4位波纹进位加法器产生多少级门延迟? n 位呢?
 - Answer 1: total delay $\sim 3n$; about 12 gate delays when $n=4$
 - Because each full adder needs 3 gate delays to generate carry-out
 - 答案: 4位波纹进位加法器产生 $4 \times 3 = 12$ 级门延迟, 因为每个全加器产生3级门延迟
 - Is this answer correct?



Ripple-carry adder

- Design an n-bit ripple-carry adder (assuming $n=4$ in example)
 - **1-minute quiz:** How much time to do the addition? Use gate delay as the unit
 - Answer 2: **$2n+1$** . There are 9 gate delays for $n=4$
正确答案: 4位波纹进位加法器产生 $2*4+1=9$ 级门延迟
因为当算出 C_1 时, $X_i \oplus Y_i$ 也已经被算出来了, $1 \leq i < n$
 - Only C_1 needs 3 gate delays, and another C_i needs only 2 additional gate delays.
 - Why? Because for $1 \leq i < n$, $X_i \oplus Y_i$ is already generated when C_1 becomes available
 - Red line shows the longest path 红线展示了最长路径



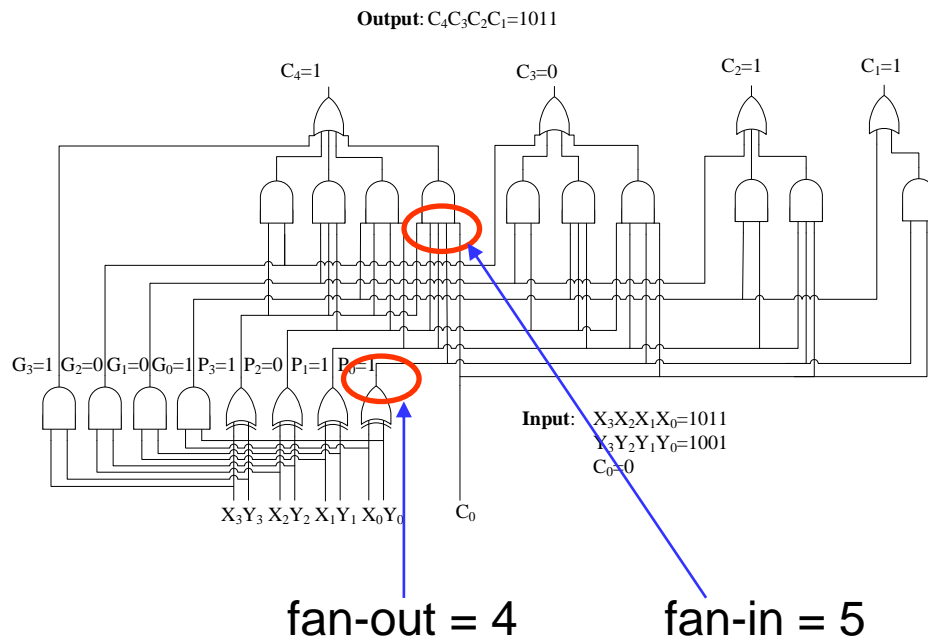
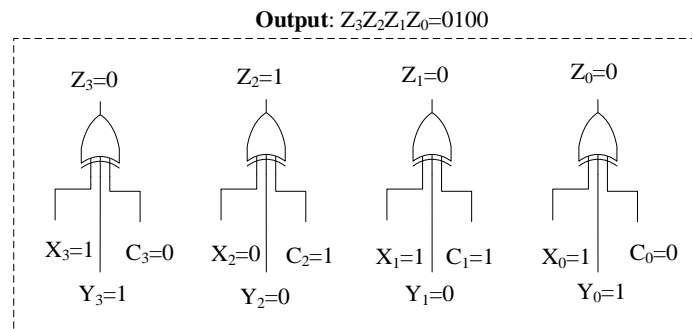
Expand to show the gate-level details

A much faster adder

代表了设计方法3: 并行连接, 使用中间模块Gi和Pi

- Instead of compute the carry bits one by one in n steps, we can compute all the n carry bits in parallel in one step
先算出全部进位比特值
 - This step needs 3 gate delays
- Afterwards, the n sum bits are computed in parallel in one step
 - This step needs 1 gate delay
- 4 gate delays in total
只需4级门延迟 4 vs. $2n-1$
 - Compared to $2n-1$ gate delays for ripple-carry adder
- Constrained by fan-in and fan-out in practice 扇入扇出限制
 - A gate can only safely receive a few inputs
 - A gate output cannot drive too many wires

并行进位的加法器



4.2.4 A combinational circuit can compute multiple functions via selection by control signals

代表了设计方法4：使用多路复用器和控制信号（本例中的选择输入S）

- An adder-subtractor controlled by a **multiplexer** (MUX in short)

- Control signal S to select addition (S=0) or subtraction (S=1)

- Subtraction is adding negative, e.g., $5-5 = 5+(-5)$

- The negative value of a number X is the two's complement of X

- Let $X = Y = 5$, i.e., $X_3X_2X_1X_0 = Y_3Y_2Y_1Y_0 = 0101$

- Then, $X - Y = 5 + (-5) = 5 + \text{Two's complement of } 5$

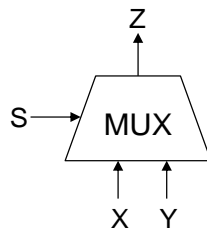
$$\rightarrow 0101 + (\overline{0101} + 0001)$$

$$= 0101 + 1011$$

$$= 10000 = C_4 Z_3 Z_2 Z_1 Z_0$$

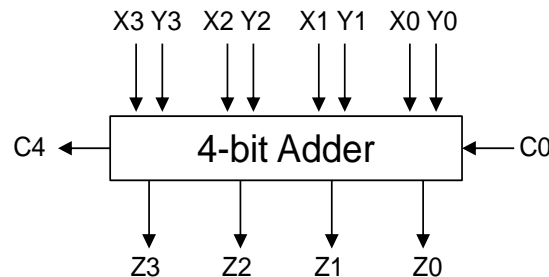
$$Z = \begin{cases} X & \text{if } S = 0 \\ Y & \text{if } S = 1 \end{cases}$$

S	X	Y	Z
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

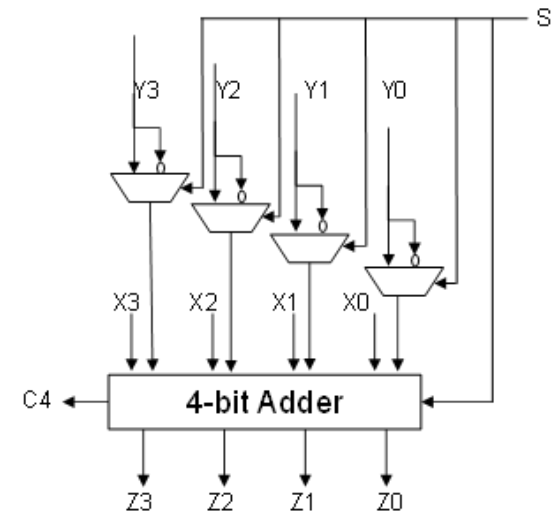


S	Z
0	X
1	Y

A multiplexer 多路复用器



A two's complement 4-bit adder

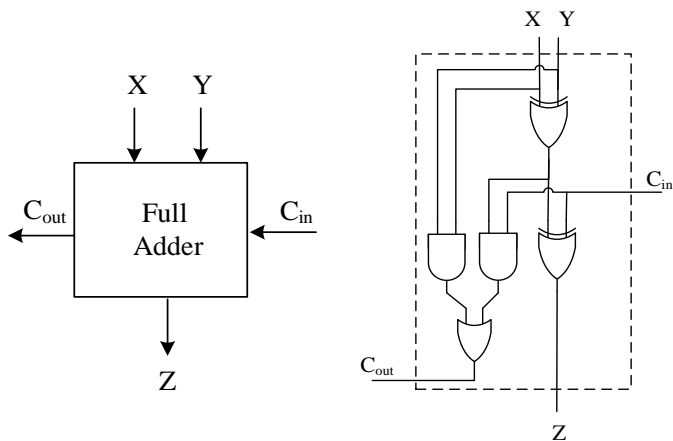
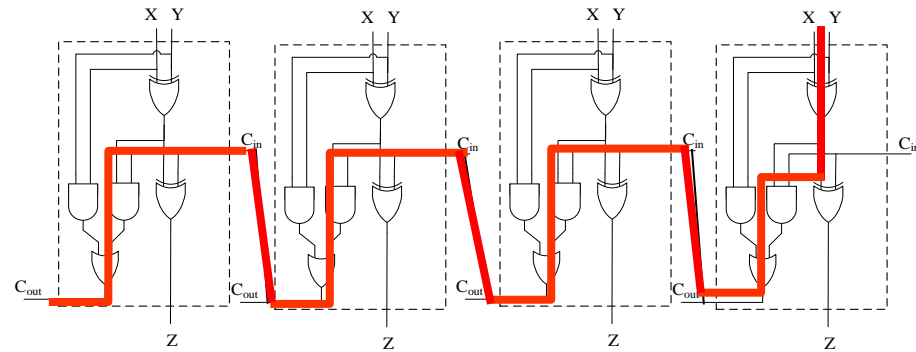


and an adder-subtractor

采用多路复用器的加减器，控制信号S选择做加法还是做减法

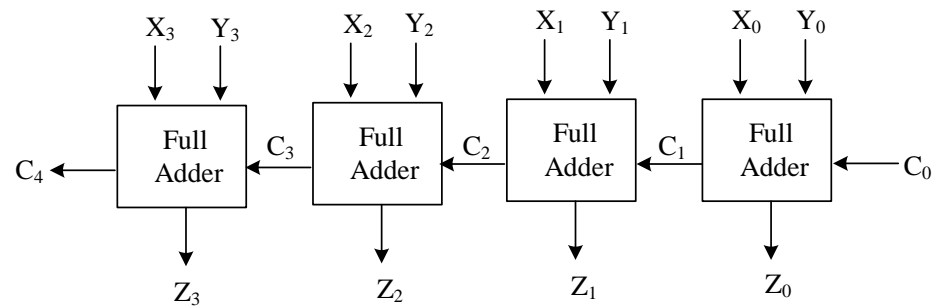
通过实例理解模块化的一般原理

- 模块是采用信息隐藏原理的抽象
 - 全加器模块是一个组合电路抽象，隐藏了虚框内的逻辑电路细节
- 系统中的两个模块可以连接，但不重叠
 - 波纹进位加法器中的两个全加器可连接，但两个全加器的内部电路不重叠
- 模块化是艺术，需要人的创造力
 - 波纹进位加法器比较直接简单
 - 并行进位加法器则需要艺术：
体现在中间表示P，G两种模块（留作练习）



Full adder symbol
全加器是模块

Its implementation by gates



A 4-bit ripple-carry adder
4位波纹进位加法器是系统

4.3 Sequential circuits

- **Sequential circuit = combinational circuit + state circuit**

时序电路 = 组合电路 + 状态电路，以实现多步计算过程

- With states, a system can execute multi-step computational processes
- Each step computes two types of values
 - The current output values 当前输出
 - The next state values 下一状态
- Both are computed from
 - The current input values 当前输入
 - The current state values 当前状态

- Correspondences to logic thinking abstractions 实现逻辑思维

- A combinational circuit is equivalent to a Boolean expression
- A sequential circuit is equivalent to an automaton
- 组合电路实现布尔表达式；时序电路实现自动机

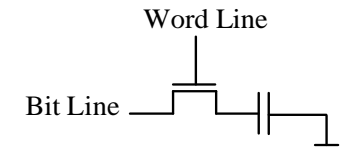
- States in hardware circuits are implemented by two types of basic circuits

状态电路有两种实现方法：存储单元，触发器

- Memory cells
- flip-flops: logic circuits with feedback wires
 - We discuss only the D flip-flop

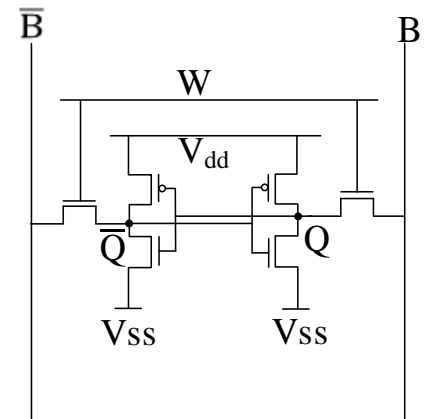
4.3.1 Types of memory technology 存储器类别

- Non-volatile memory (**NVM**): content is retained when power is off
 - 计算机下电后, 非易失存储器中的内容不会丢失
 - **ROM** (read-only memory) 只读存储器 例子: 开机后读取数据与指令
 - Read-write NVM 可读可写的非易失存储器 例子: U盘
- Volatile memory: content is lost when power is off
 - 计算机下电后, 易失存储器中的内容会丢失
 - **DRAM** (dynamic random access memory) 动态随机访问存储器
 - Simple and inexpensive 成本低
 - Needs to constantly refresh its content (once every 7.8-128 μ s)
 - Because the capacitor leaks electricity after charging
 - **SRAM** (static random access memory) 静态随机访问存储器
 - More complex but avoid refreshing overhead
 - Faster but more expensive than DRAM



DRAM cell

1 transistor and
1 capacitor
只需一个晶体管



SRAM cell

6 transistors
需要6个晶体管

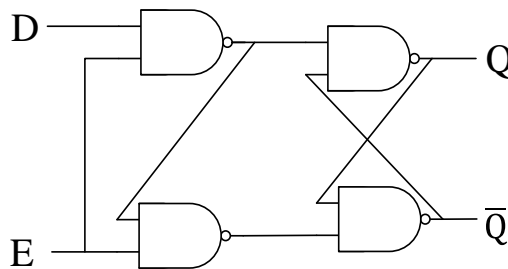
Type	Latency	Price \$/GB
Register	100s ps	N/A
SRAM	100s ps ~ 10 ns	\$100's ~1000s /GB
DRAM	10s ~ 100 ns	\$2~4 /GB
NVM: Main Memory	100 ns ~ 10 μ s	\$6 / GB
NVM: SSD	10 μ s ~ 1 ms	\$0.1~0.2 / GB
Hard Disk: HDD	> A few ms	\$0.02 / GB

4.3.2 D flip-flop D触发器

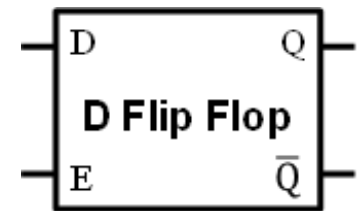
- We can also use gates to form state circuits
 - With feedback wires 触发器是 包含反馈线路的逻辑门电路
- A common type is the **delay flip-flop**, or D flip-flop
 - 2-input-2-output, where the 2 states (outputs) are negation of each other
 - Input D is data input; Input E (enable signal) is often the clock signal CLK
状态Q值是数据输入D一个时钟周期前的值，即D延迟一周期后变成Q
 - Functionality: when enabled, Q is D delayed one clock cycle
 - When E=0, Q remains the same; when E=1, the next Q will be the current D

E	D	Q	Q _{next}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

D flip-flop: Truth table



Internal logic diagram



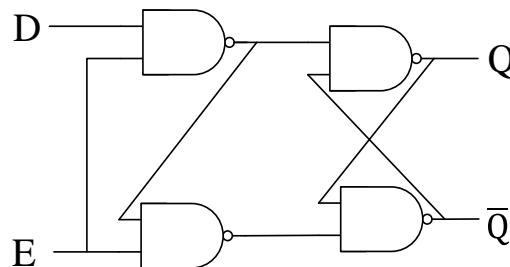
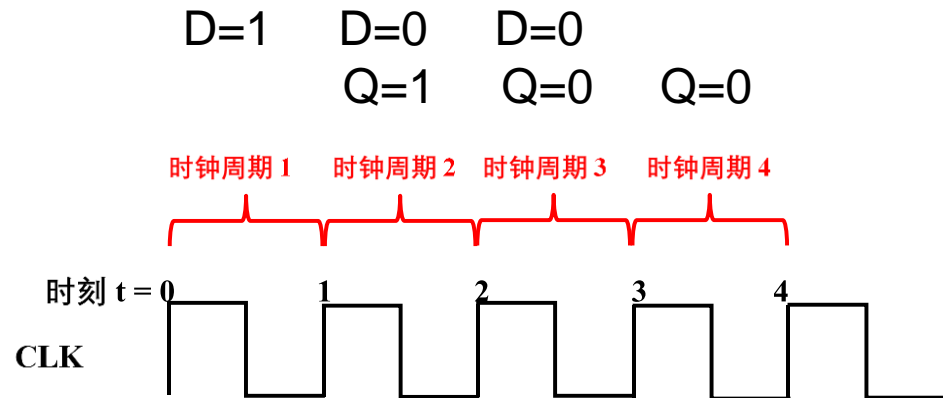
Symbol

4.3.2 D flip-flop D触发器

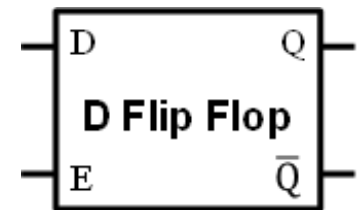
- Input D is data input; **Input E (enable signal) is often the clock signal CLK**
- D是数据输入，E常常是时钟信号CLK
- 状态Q值是数据输入D一个时钟周期前的值，即D延迟一周期后变成Q

E	D	Q	Q _{next}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

D flip-flop: Truth table



Internal logic diagram

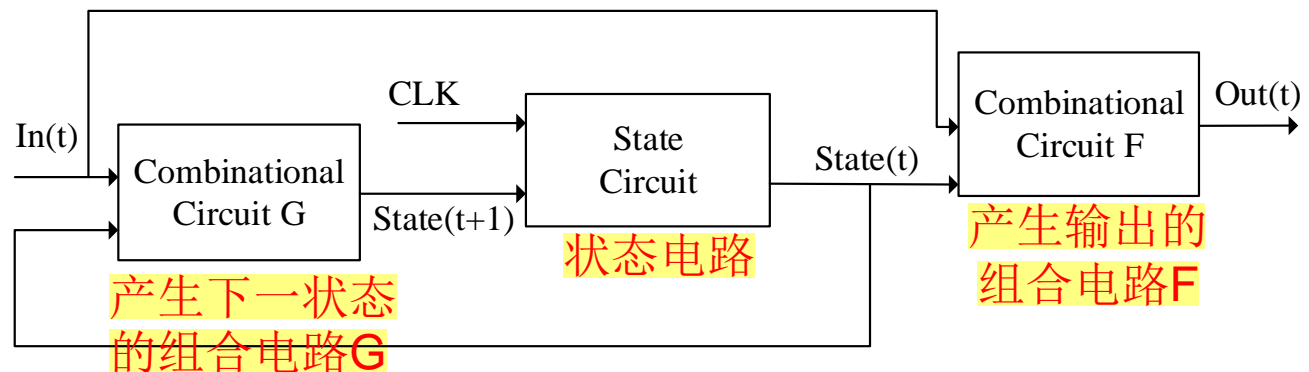


Symbol

4.3.3 General organization of sequential circuits

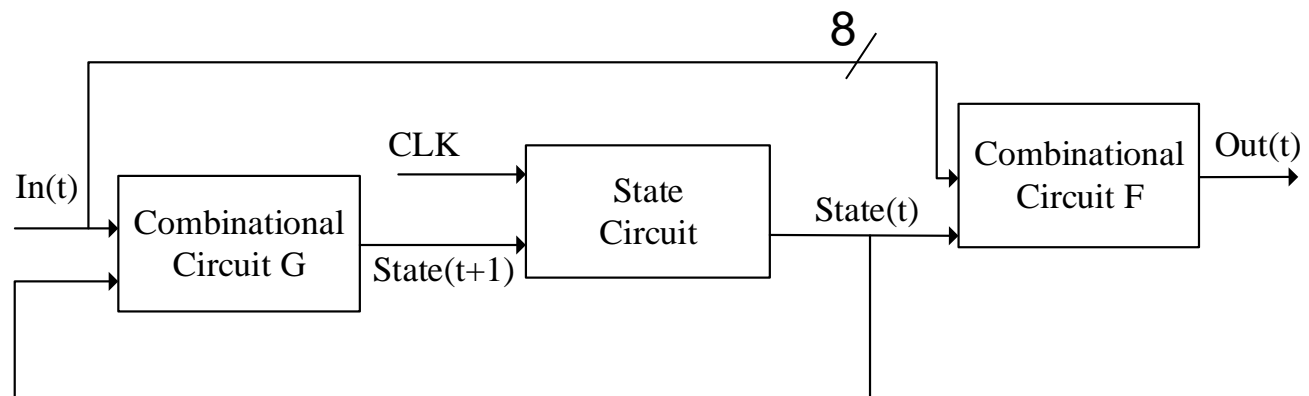
时序电路的一般组成：两个组合电路，一个状态电路；时钟信号驱动

- Any sequential circuit can be organized as shown below
 - Comprised of two combinational circuits and a state circuit
 - Driven by a clock signal CLK
 - The state circuit consists of one or more D flip-flops
 - The output circuit F: $\text{Out}(t) = F(\text{In}(t), \text{State}(t))$ 输出电路
 - The next-state circuit G: $\text{State}(t+1) = G(\text{In}(t), \text{State}(t))$ 下一状态电路
- Logic diagram of a sequential circuit 时序电路的逻辑线路图
 - Also called synchronous sequential circuit
 - Because the sequential circuit is synchronized by the clock signal



4.3.3 General organization of sequential circuits

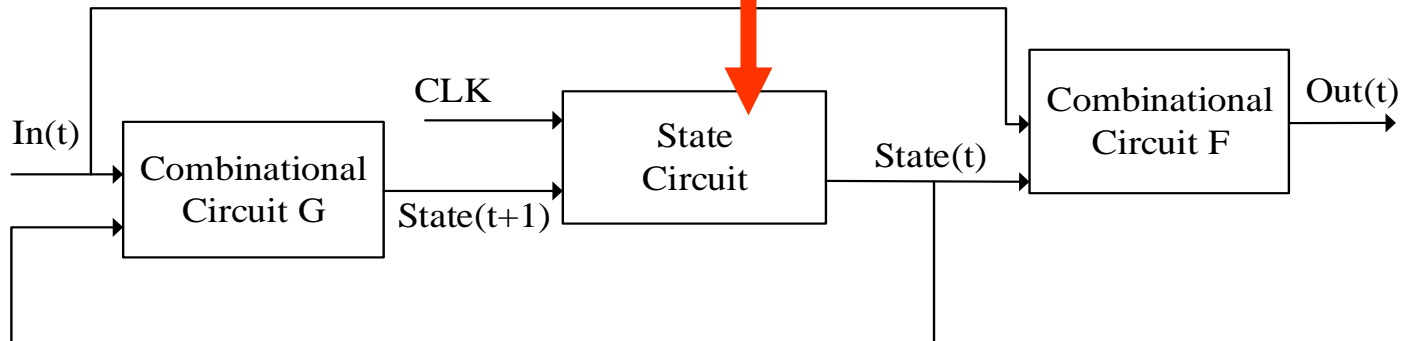
- Any sequential circuit can be organized as shown below
 - Comprised of two combinational circuits and a state circuit
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 - The state circuit consists of one or more D flip-flops
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 - The next-state circuit G: $\text{State}(t+1) = G(\text{In}(t), \text{State}(t))$
- An input or output may have multiple bits （此处输入In是8条线）



We will go through the process of **designing a 4-bit serial adder**

4.3.4 Design a 4-bit serial adder 设计4位串行加法器

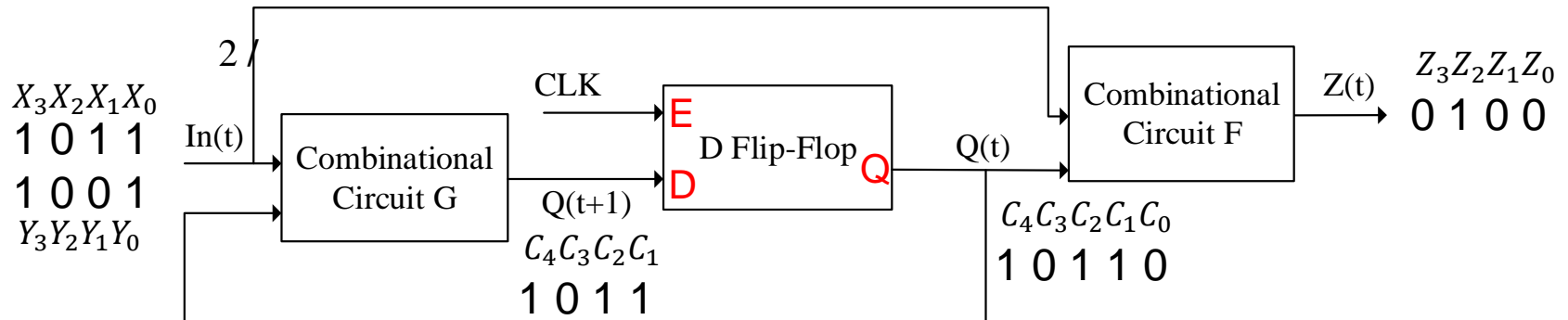
- Perform $Z_3Z_2Z_1Z_0 = X_3X_2X_1X_0 + Y_3Y_2Y_1Y_0$ in 4 steps
需要四步（四个时钟周期）
 - Each step performs a full addition 每一步执行一个全加操作
- Verify: $11_{10} + 9_{10} = 1011_2 + 1001_2 = 10100_2 = 20_{10} = 4_{10}$ and overflow
用一个实例验证：输入 $X=1011$ ， $Y=1001$ ；则输出 $Z=0100$ 且产生溢出
- Design process 设计过程
 - Q: how many bits (or D flip-flops) are needed for the state circuit?
 - 首先确定状态电路：需要多少个D触发器？



Design process

- Design process

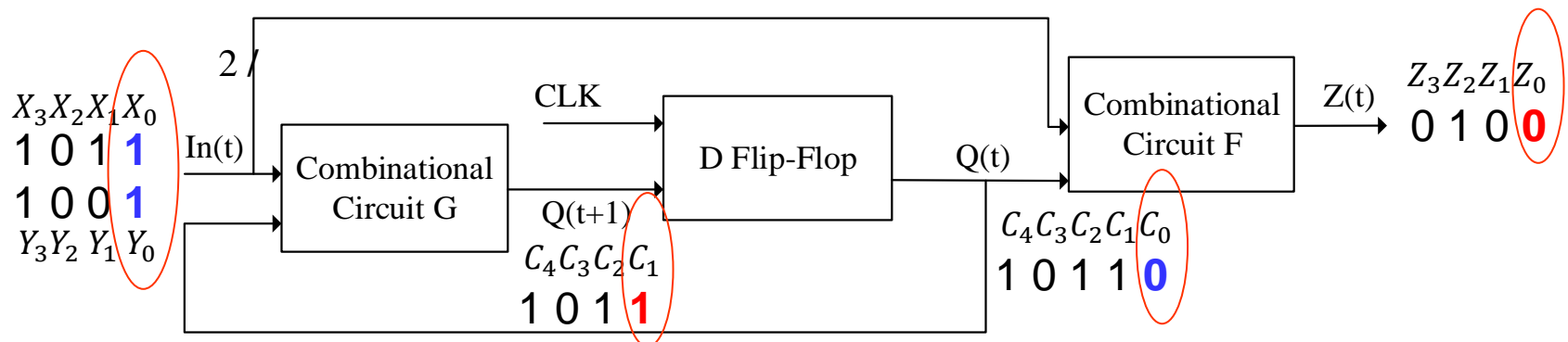
- Q: how many bits (or D flip-flops) are needed for the state circuit?
 - A: 1 bit to hold the carry bit. **Only one D flip-flop is needed.** Denote the state as Q.
Clock signal as the Enable signal
只需要一个 D 触发器，其状态 Q 表示当前进位，其 Enable 端连接时钟信号 CLK
- Draw the logic diagram for the sequential circuit
确定状态电路后，只需要画出组合电路 G、F（确定它们的布尔表达式）
 - How?
使用加法原理理解串行加法器的时序电路
 - 某些同学更喜欢使用实例来理解



Design process

- Draw the logic diagram for the sequential circuit
 - 确定组合电路G、F的布尔表达式
 - How?
 - 使用实例和加法原理理解串行加法器的时序电路
 - Step 1: inputs in blue, outputs in red. Note that $C_0 = 0$

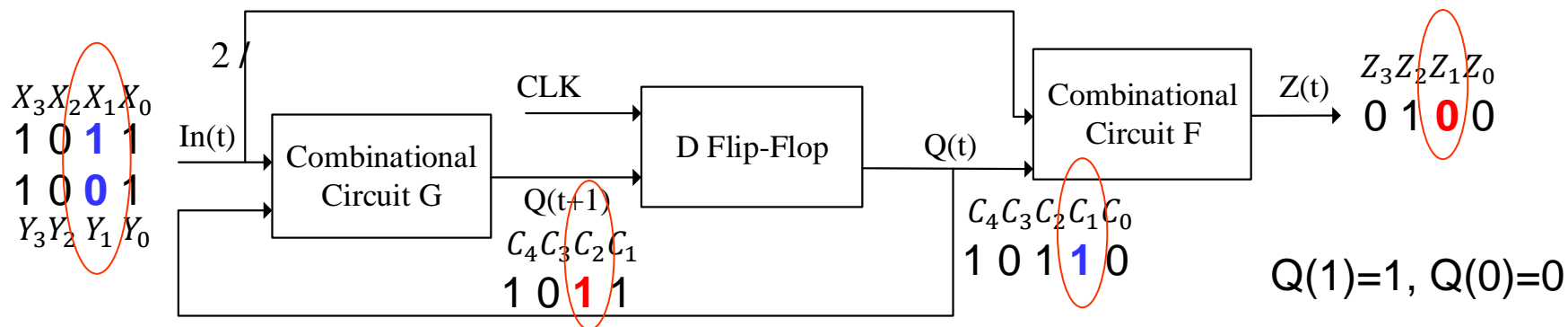
$$\begin{array}{r}
 101\text{ } \textcolor{blue}{1} = X \\
 + \quad 100\text{ } \textcolor{blue}{1} = Y \\
 \hline
 101\text{ } \textcolor{red}{1}\textcolor{blue}{0} = C \\
 010\text{ } \textcolor{red}{0} = Z
 \end{array}$$



Design process

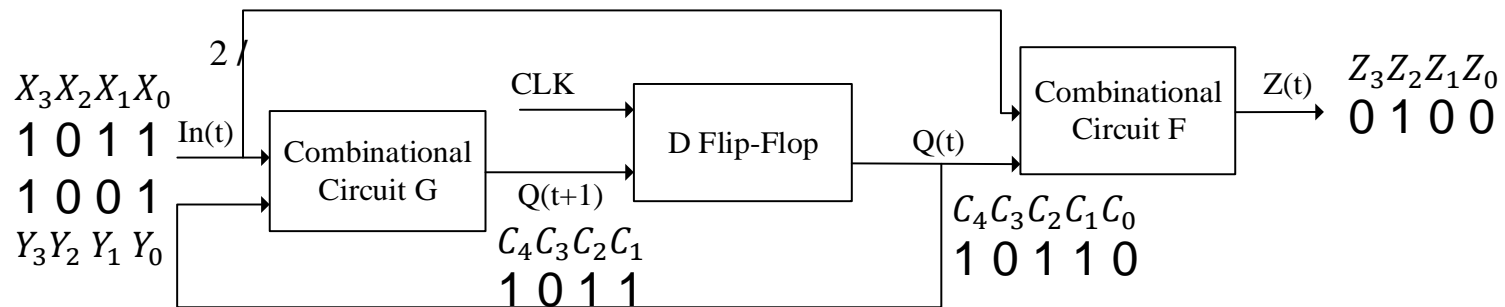
- Draw the logic diagram for the sequential circuit
 - 确定组合电路G、F的布尔表达式
 - How?
使用实例和加法原理理解串行加法器的时序电路
 - Step 2: inputs in blue, outputs in red.

$$\begin{array}{r} = X \\ + = Y \\ \hline 1 0 = C \\ 0 0 = Z \end{array}$$

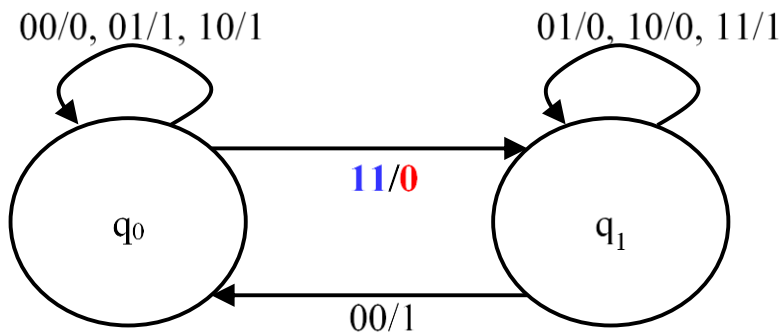


Design process

- Draw the logic diagram for the sequential circuit
 - Derive the state-transition diagram 画出时序电路的状态转换图



XY/Z



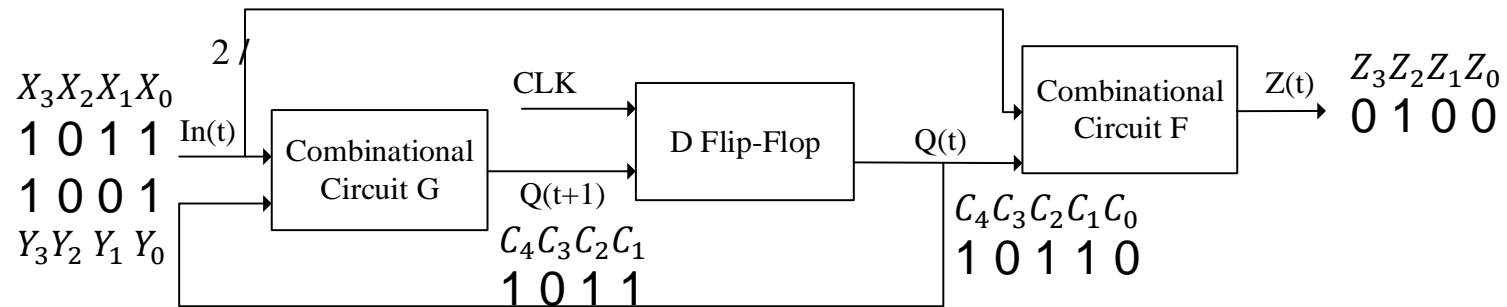
画出时序电路的状态转换图

q_0 denotes $C=0$
 q_1 denotes $C=1$

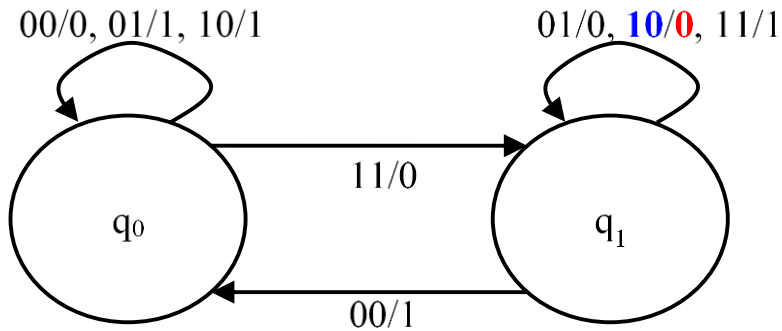
$$\begin{array}{r}
 101\textcolor{blue}{1} = X \\
 100\textcolor{blue}{1} = Y \\
 \hline
 101\textcolor{red}{1}\textcolor{blue}{0} = C \\
 010\textcolor{red}{0} = Z
 \end{array}$$

Design process

- Draw the logic diagram for the sequential circuit
 - Derive the state-transition diagram



XY/Z



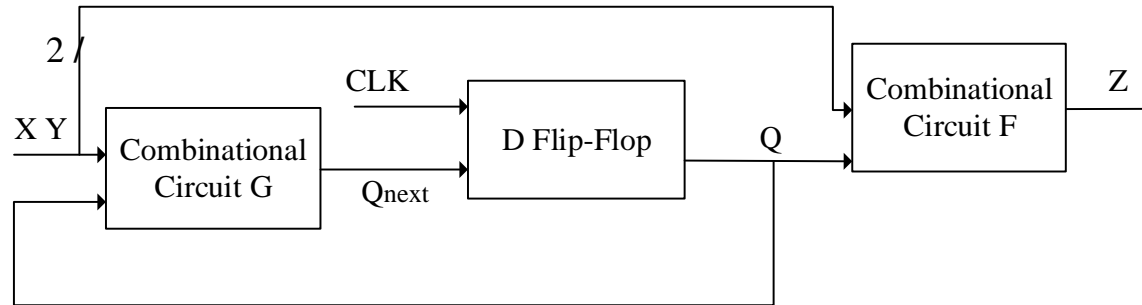
画出时序电路的状态转换图

q_0 denotes $C=0$
 q_1 denotes $C=1$

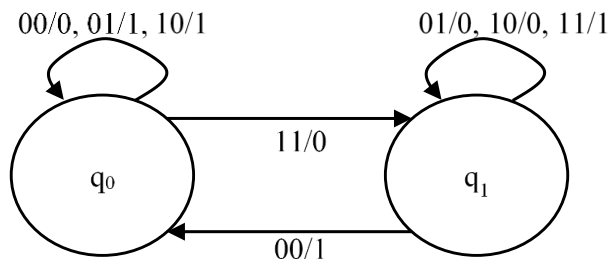
$$\begin{array}{r}
 1011 = X \\
 1001 = Y \\
 \hline
 10110 = C \\
 0100 = Z
 \end{array}$$

Design process

- Derive the state-transition diagram (remember that Q denotes carry C)
- Derive the truth table and Boolean expressions for F, G



XY/Z



q_0 denotes $C=0$
 q_1 denotes $C=1$

Q	X	Y	Z	Q_{next}
q_0	0	0	0	q_0
q_0	0	1	1	q_0
q_0	1	0	1	q_0
q_0	1	1	0	q_1
q_1	0	0	1	q_0
q_1	0	1	0	q_1
q_1	1	0	0	q_1
q_1	1	1	1	q_1

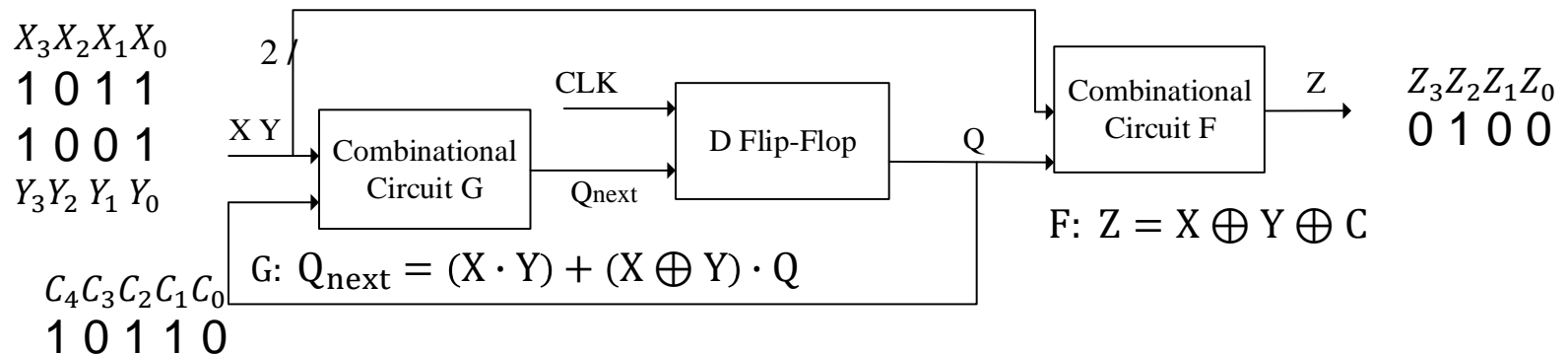
进而导出真值表
 进而求出组合电路F和G
 的布尔逻辑表达式

$$Z = F(X, Y, Q) = X \oplus Y \oplus C$$

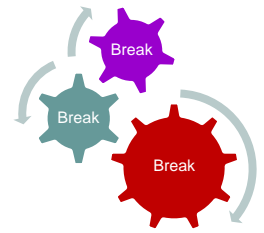
$$Q_{next} = G(X, Y, Q) = (X \cdot Y) + (X \oplus Y) \cdot Q$$

Design process

- Derive the state-transition diagram (remember that Q denotes carry C)
 - Derive the truth table and Boolean expressions for F, G
 - Verify the resulting sequential circuit,
given $X_3X_2X_1X_0 = 1011$, $Y_3Y_2Y_1Y_0 = 1001$, $C_0 = 0$
- Step 1: $X_0 = 1, Y_0 = 1, C_0 = 0$. $Z_0 = 0, C_1 = 1$
 - Step 2: $X_1 = 1, Y_1 = 0, C_1 = 1$. $Z_1 = 0, C_2 = 1$
 - Step 3: $X_2 = 0, Y_2 = 0, C_2 = 1$. $Z_2 = 1, C_3 = 0$
 - Step 4: $X_3 = 1, Y_3 = 1, C_3 = 0$. $Z_3 = 0, C_4 = 1$
- 最后，用实例验证
- The final result is $Z_3Z_2Z_1Z_0 = 0100$, with $C_4 = 1$ indicating overflow



Take-Home Messages



- Modularization
 - Divide a system into multiple subsystems (modules), and compose modules into a system (higher-level abstraction), using information hiding
 - Understand modularization via a design journey
- Combinational circuits are circuits of gates without feedback wires
 - Equivalent to Boolean expression
 - How many combinational circuits are there, of n -input-1-output?
 - Boolean expressions and logic diagrams are simpler than CMOS circuits
 - Examples: adders, adder-subtractor with multiplexer
- Sequential circuits = state circuits + combinational circuits
 - State can be represented by a memory cell or a flip-flop
 - There are different types of memory, with different latencies and costs
 - Non-volatile memory (NVM): ROM (read-only memory), Read-write NVM
 - Volatile memory: DRAM, SRAM
 - A flip-flop, such as D flip-flop, is a circuit of gates with feedback wires
 - **Design a serial subtractor** to see how a sequential circuit generates the output and the next state from the inputs and the current state