1. Study Group

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2. (a) $\min \sum_i c_i x_i \iff \max \sum_i -c_i x_i$

(b) $x_1 \geq b_1 \iff -x_1 \leq -b_1$

(c) $b_1 \leq x_1 \leq b_2 \iff \begin{cases} -x_1 \leq -b_1 \\ x_1 \leq b_2 \end{cases}$

(d) $x_2 = b_2 \iff b_2 \leq x_2 \leq b_2 \iff \begin{cases} -x_2 \leq -b_2 \\ x_2 \leq b_2 \end{cases}$

(e) same as (b), $\begin{cases} -(x_1+x_2+x_3) \leq -b_2 \\ x_1+x_2+x_3 \leq b_3 \end{cases}$

(f) $|x_1+x_2| \leq b_2 \iff -b_2 \leq x_1+x_2 \leq b_2$

$\iff \begin{cases} -(x_1+x_2) \leq b_2 \\ x_1+x_2 \leq b_2 \end{cases}$

(g) Impossible

(h) ~~Assign~~ $y = \max(x_1, x_2)$

then $\min \max(x_1, x_2) \iff \begin{cases} \max -y \\ x_1 - y \leq 0 \\ x_2 - y \leq 0 \end{cases}$ (Natually $y \geq 0$ since $x_1, x_2 \geq 0$)

(i) $x_4 = x_5 - x_6$ and $x_5, x_6 \geq 0$

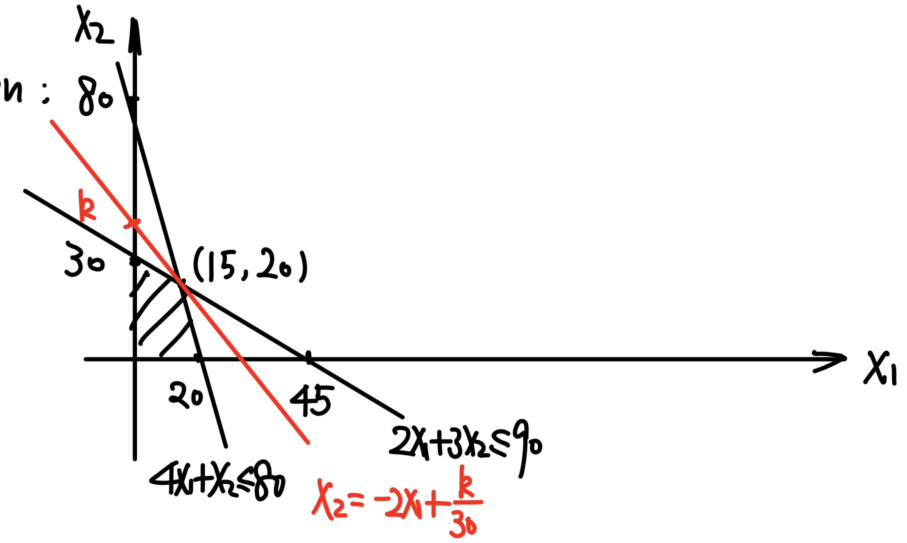
3. (a) $x_1$ batches of brownies

$x_2$ batches of cookies

LP: $\max \; 60x_1 + 30x_2$

$$\text{s.t.} \begin{cases} 4x_1 + x_2 \le 80 \\ 2x_1 + 3x_2 \le 90 \\ x_1, x_2 \ge 0 \end{cases}$$

feasible region:



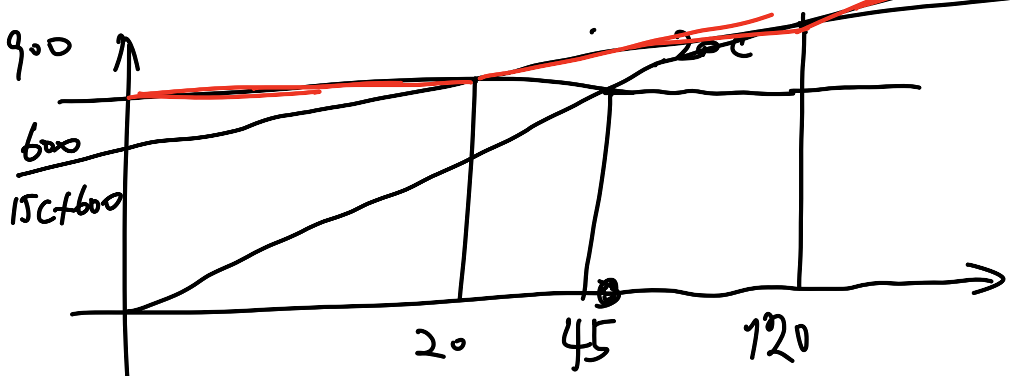

Simplex Alg:

| vertex | (0, 30) | (15, 20) | (20, 0) | (0, 0) |
|---|---|---|---|---|
| k | 900 | 1500 | 1200 | 0 |

So solution is $\begin{cases} x_1 = 15 \\ x_2 = 20 \end{cases}$, max object is 1500

(b) $\max \; cx_1 + 30x_2$

| vertex | (0, 0) | (0, 30) | (20, 0) | (15, 20) |
|---|---|---|---|---|
| k | 0 | 900 | 20c | 15c + 600 |




| Range | optimal vertex |
|---|---|
| $0 \leq c \leq 20$ | $(0,30)$ |
| $20 < c \leq 120$ | $(15,20)$ |
| $c > 120$ | $(20,0)$ |

4.(a) Buy $x_1$ pounds of Salmon

$x_2$ pounds of Bread

$x_3$ pounds of Squid.

Assume there's $n$ penguins,

LP:

$$\max \quad -6x_1 - x_2 - 8x_3$$

$$\text{s.t.} \begin{cases} -(400x_1 + 50x_2 + 300x_3) \leq -600n \\ -(300x_2 + 100x_3) \leq -800n \\ -(150x_1 + 25x_2 + 200x_3) \leq -500n \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

(b) dual LP:

$$\max \quad (600y_1 + 800y_2 + 500y_3)n$$

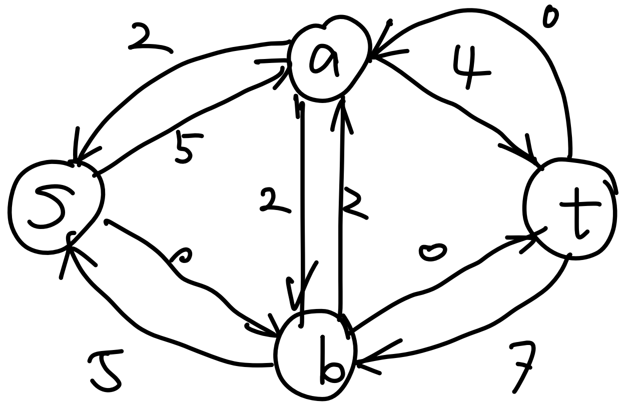
$$\text{s.t.} \begin{cases} 400y_1 + 150y_3 \leq 6 \\ 50y_1 + 300y_2 + 25y_3 \leq 1 \\ 300y_1 + 100y_2 + 200y_3 \leq 8 \\ y_1, y_2, y_3 \geq 0 \end{cases}$$

(c) $y_i$ is price of pill $i$.

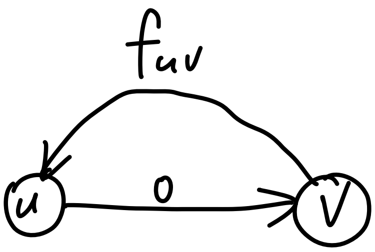
$(600y_1 + 800y_2 + 500y_3)n$ is profit the pharmacist can make. It's expected to be max.

Constraints means the pharmacist should guarantee that for same nutrition his combo pills should be cheaper than food.

5. (a)

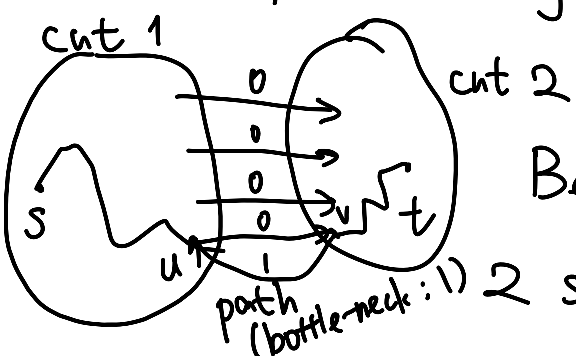



(b)




<u>Alg Description</u>: Modify the dual edge (v, u) to has 1 allowed instead of $f_{uv}$. Run BFS in which we always choose (v, u) to find a angment path from t to s, make reduction of 1 on related edges so we get a fixed max flow!

<u>Correctness Proof</u>: To prove the fix flow is still max flow after $c_{uv} \to c_{uv} - 1$, we are to prove that it's unable to find an augment path from s to t.




Before change of edge, we can get 2 seperated cuts of graph. Cut 1 has no positive edge to cut 2. (max-flow = min-cut)

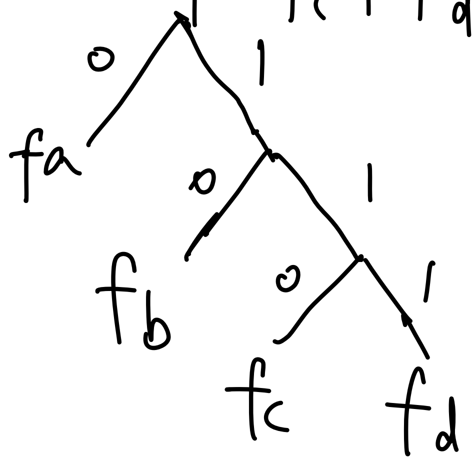
So the argument path we find won't change the cut and max flow is cost($f_{origin}$) − 1. (it's bounded actually)

Runtime Analysis: The cost is a BFS.

Total time: $O(|E| + |V|)$.

6. LP: $\max\ f_a + 2f_b + 3f_c + 3f_d$

s.t.
$$\begin{cases} f_a + f_b + f_c + f_d = 1 \\ f_a, f_b, f_c, f_d \geq 0 \\ f_c - f_b \leq 0 \\ f_d - f_b \leq 0 \\ f_b - f_a \leq 0 \\ f_c + f_d \leq f_a \end{cases}$$



7. Alg Description:

$i = 0$

while cost$(f) > 0$:

① find an augmenting path from s to t using only edges $(u,v)$ s.t. $f(u,v) > 0$

② find bottle-neek flow value $f'$

③ Reduce the remaining flow allowed of edges on the path by $f'$ to get a new $f$. $f_i(u,v) = f'$ for $(u,v)$ on path and 0 otherwise.

④ $i += 1$

Explanation: Each time we make at least 1 edge "full" to capacity. So it will take at most $|E|$ iterations to decomposite $f$ into flows $\{f_i\}$. $(i \leq |E|)$.

It can be reached since there's no positive cycle in graph.