1. Study Group

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2. Assume we have $G = (V, E)$ as an instance of Vertex Cover. Then we construct a Min Set Cover instance as below:

$U := E$

For each $u \in V$, $S_u := \{e \in E \mid e \text{ contains } u\}$.

$S = \{S_u\}, u \in V$.

Running Min Set Cover Alg on this instance, we get a cover $C_S = \{S_{u_1}, \cdots, S_{u_m}\}$.

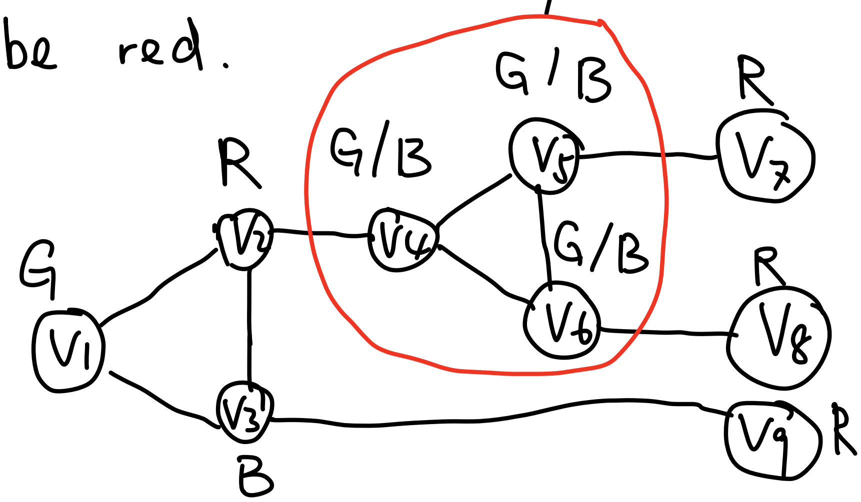
Obviously $\{u_1, \cdots, u_m\}$ is a vertex cover of G. Because for each $e \in E = U$, there must be a $S_{u_i} \ni e$, meaning that u_i successfully covers e.

It's a bijection from each set cover to each vertex cover. So Min Set Cover $\Longleftrightarrow$ Min Vertex Cover.

3. (a) Adding edges $(X_i, \neg X_i)$, (X_i, V_{BASE}), $(\neg X_i, V_{BASE})$

Since X_i and $\neg X_i$ are adjacent to V_{BASE}, they should be same color as V_{TRUE} or V_{FALSE}. Also, they cannot have same color. So X_i and $\neg X_i$ will have one as V_{TRUE} and one as V_{FALSE}.

(b) Assume $\{V_7, V_8, V_9\}$ are all not green. Then they must be red.



Then V_3 must be blue, then V_2 must be red, then $\{V_4, V_5, V_6\}$ should all not be assigned as red.

But V_4, V_5, V_6 are connected to each other, so any same color of two of them will lead to contradiction!

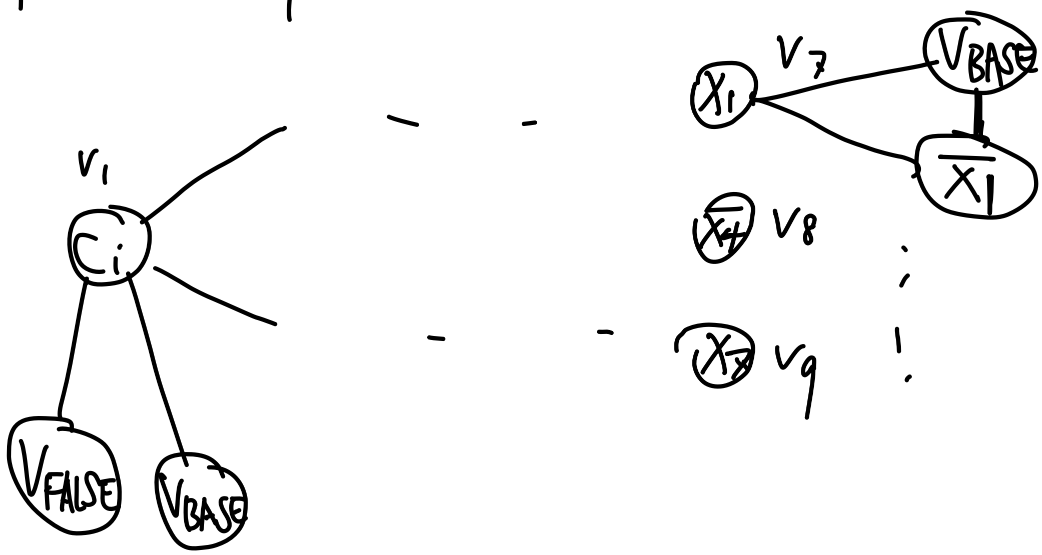
So at least one of $\{V_7, V_8, V_9\}$ should be assigned as green.

(c) If we have a 3-SAT instance $I = C_1 \wedge C_2 \wedge \cdots C_k$ (C_i is a 3-SAT clause), then ~~m~~ construct as (a) for each variable x_i. For each C_i, we construct as below:

Construct vertex C_i

Add edge (C_i, V_{FALSE}), (C_i, V_{BASE})

Add edges to construct a gadget making V_1 as C_i, $\{V_7, V_8, V_9\}$ as the 3 literals in clause C_i.



Proof of correctness: If 3-SAT instance I is satisfiable,

green $\mapsto$ TRUE
red $\mapsto$ FALSE
blue $\mapsto$ BASE

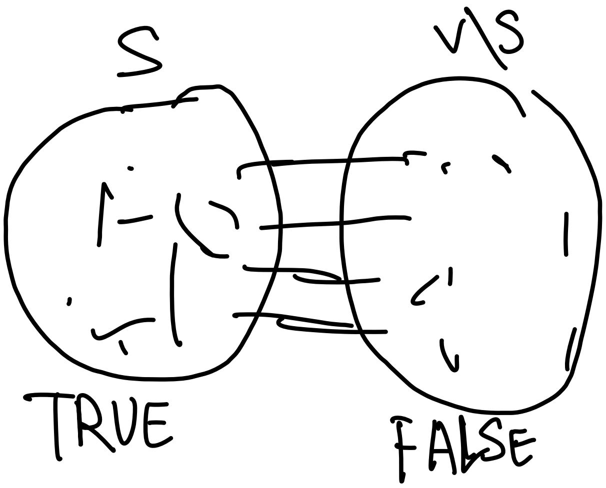
then for each clause, assign each literal as green if it's true in 3-SAT, otherwise red.

Also, assign C_i as green. By the observation, we can get a valid 3-color solution for our graph constructed.

If we have a 3-color solution for our graph constructed, then C_i must be green and at least one literal should be green by (b). This gives a 3-SAT solution since each clause can be satisfied by assigning $X_i/\overline{X_i}$ by 1 if green, 0 otherwise.

$\in \{v_7, v_8, v_9\}$

4. (a) Reduction: For each $(u,v) \in E$, construct clause $(X_u \oplus X_v)$. We assign $c=r$ then run 2-XOR on the clauses, the TRUE set and FALSE set give the 2 parts of the cut of G.



Proof of correctness: After running 2-XOR, we can decide if there's a cut with c cross-edges.

The clause $(X_u \oplus X_v)$ is TRUE $\Leftrightarrow$ X_u and X_v are in 2 dif parts of the cut and $(u,v) \in E$. If there's at least c cross-edges, then we can find the cut by 2-XOR. Also, if there's at least $r=c$ clauses satisfiable, then making $S = \{u \mid X_u \text{ is TRUE}\}$ and $V \setminus S$ gives a cut having at least c clauses.

(b) Adding a new but same variable p to each clause of 3-XOR then we get 4-XOR!

<u>Proof of correctness:</u> If there's an instance of 3-XOR, then solution for 3-XOR with r unioning {p = FALSE} gives the solution for 4-XOR with r.

Also, given a solution for 4-XOR.

① if p = FALSE, then solution for 4-XOR with r substracting {p = FALSE} gives the solution for 3-XOR with r

② if p = TRUE, then we have a psedo solution for 3-XOR with r by substracting {p = TRUE} from solution for 4-XOR with r. Then we negates all the variables in the psedo solution and we successfully get a correct solution for 3-XOR with r.