Classical bits: 0 or 1

Quantum bits: $|0\rangle$ or $|1\rangle$

(qubit) "ket 0" "ket 1"

or "superposition" of $|0\rangle$ and $|1\rangle$

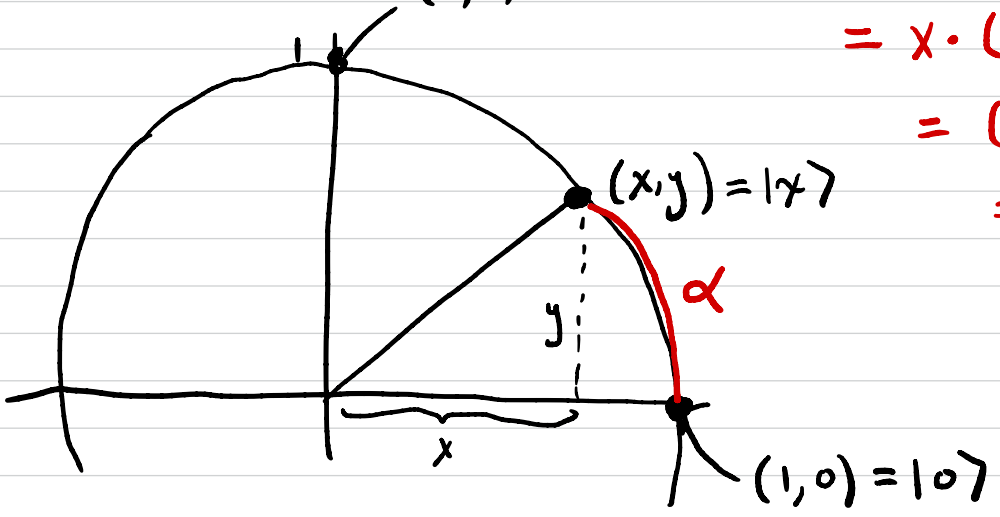
$$|\psi\rangle = x \cdot |0\rangle + y \cdot |1\rangle \qquad x, y \in \mathbb{R}$$

$$x^2 + y^2 = 1$$

(actually can be complex...)

e.g. $$|+\rangle = \frac{1}{\sqrt{2}} \cdot |0\rangle + \frac{1}{\sqrt{2}} \cdot |1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}} \cdot |0\rangle - \frac{1}{\sqrt{2}} \cdot |1\rangle$$



$|\psi\rangle = x\cdot|0\rangle + y\cdot|1\rangle$

$= x\cdot(1,0) + y\cdot(0,1)$

$= (x,0) + (0,y)$

$= (x,y)$

$|\psi\rangle = \cos(\alpha)\cdot|0\rangle + \sin(\alpha)\cdot|1\rangle$

e.g. $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \cos\frac{\pi}{4}\cdot|0\rangle + \sin\frac{\pi}{4}\cdot|1\rangle$

Quantum Measurements

Given $|\psi\rangle = x \cdot |0\rangle + y \cdot |1\rangle$ $\quad (x^2 + y^2 = 1)$
we can "measure" it

w/ prob x^2: "observe" 0
$|\psi\rangle$ becomes $|0\rangle$ "collapse of the wave function"

w/ prob y^2: "observe" 1
$|\psi\rangle$ becomes $|1\rangle$

e.g. measure $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$, observe 0 w/prob 1/2
observe 1 w/prob 1/2

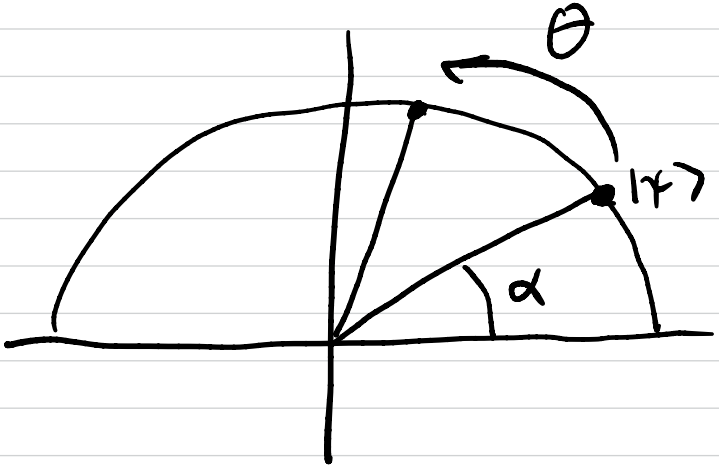
measure $|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$, observe 0 w/prob 1/2
observe 1 w/prob 1/2

measure $|0\rangle$ always observe 0

Quantum operations

(I'll tell about general operations later)

One operation: Rotate(θ) ← parameter

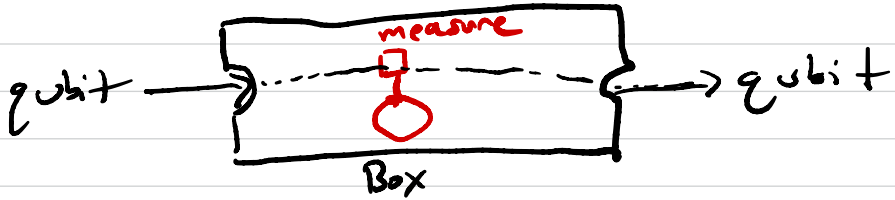


If $|\psi\rangle = \cos(\alpha)|0\rangle + \sin(\alpha)|1\rangle$

moves it to

$\cos(\alpha+\theta)|0\rangle + \sin(\alpha+\theta)|1\rangle$

Elitzur – Vaidman Bomb Algorithm



Box is either (1) empty or (2) contains a bomb

If (1) empty, and we send $|\psi\rangle = x|0\rangle + y|1\rangle$,
nothing happens & $|\psi\rangle$ comes out on other end

If (2) bomb, it measures $|\psi\rangle$

If observes 0, nothing happens
($|\psi\rangle = |0\rangle$ comes out other end)

If observes 1, bomb explodes

Classical strategy

If we send in 107,

no info

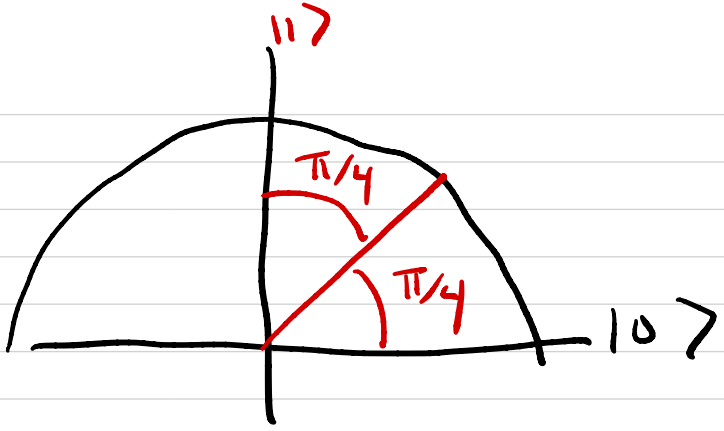
If we send in 117

and box has bomb,

explosion!

Quantum strategy

$|\chi\rangle = |0\rangle$
Rotate($\pi/4$, $|\chi\rangle$) (45°)
Send $|\chi\rangle$ through box
Rotate($\pi/4$, $|\chi\rangle$)
Measure $|\chi\rangle$
If observe 0, output "Bomb!"
If observe 1, output "Not sure"



If no bomb: $|0\rangle \xrightarrow[\pi/4]{\text{rotate}} \cos(\frac{\pi}{4})|0\rangle + \sin(\frac{\pi}{4})|1\rangle$

$\rightarrow$ [empty] $\rightarrow |\chi\rangle = \cos(\frac{\pi}{4})|0\rangle + \sin(\frac{\pi}{4})|1\rangle$

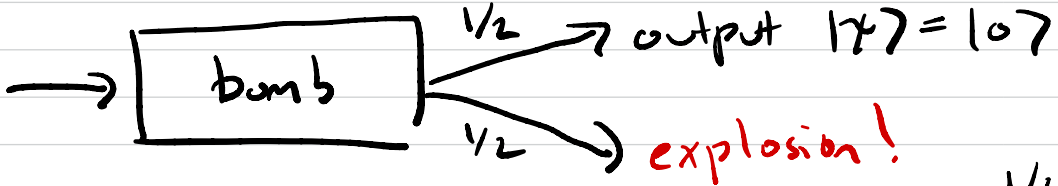
$\xrightarrow[\pi/4]{\text{rotate}} \cos(\pi/2)|0\rangle + \sin(\pi/2)|1\rangle$

$= 0\cdot|0\rangle + 1\cdot|1\rangle$

$\therefore$ Always output "Not sure!"

If bomb:

$|0\rangle \xrightarrow[\pi/4]{\text{rotate}} \cos(\pi/4)|0\rangle + \sin(\pi/4)|1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$



∴ Bomb!"

In first case: $|0\rangle \xrightarrow[\pi/4]{\text{rotate}} \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

- 1/2 → observe 0
- 1/2 → observe 1 "not sure"

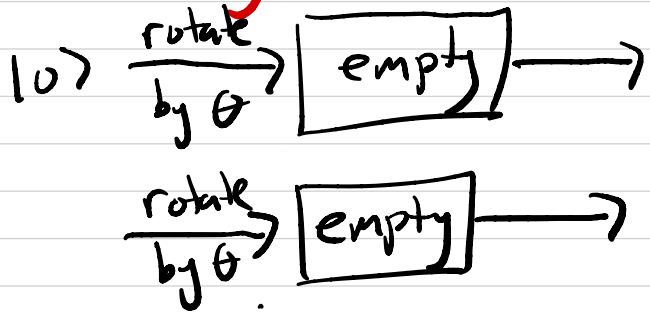
w/prob	1/2	1/4	1/4
	explosion	"Bomb!"	"not sure"

N = Input("Safety level?")
$|\psi\rangle = |0\rangle$
$\theta = (\pi/2)/N$

for $k = 1..N$
  Rotate(θ, $|\psi\rangle$)
  Send $|\psi\rangle$ through box

Measure $|\psi\rangle$
If observe 0, output "Bomb"
If observe 1, output "Empty"

If empty

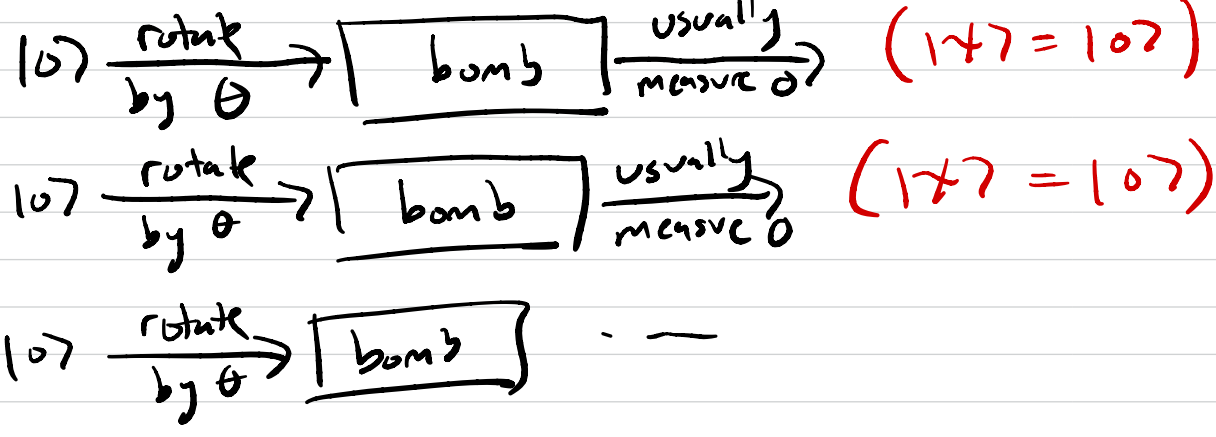


rotate by θ N times
= rotate by $N \cdot \theta = \pi/2$

At end $|\psi\rangle = \cos(\pi/2)|0\rangle + \sin(\pi/2)|1\rangle$
$= |1\rangle$

$\therefore$ Always see "empty"

If contains bomb



If no explosion, final state is $|0\rangle$ ∴ output "Bomb!"

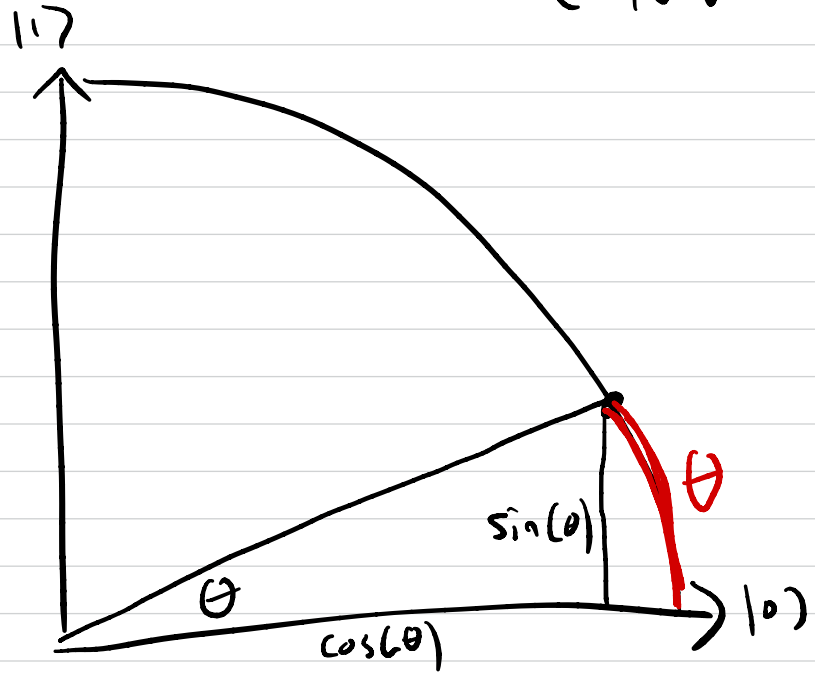
$$\Pr[\text{explosion}] = \Pr[\text{explosion at time 1}] + \Pr[\text{explosion at time 2}] + \dots$$

$$\Pr[\text{explosion at time } i] \le \Pr[\text{cos}(\theta)|0\rangle + \text{sin}(\theta)|1\rangle \text{ measures to } 1] = \sin(\theta)^2$$

$$\Pr[\text{explosion}] \le N \cdot \sin(\theta)^2$$

$$\Pr[\text{explosion}] \le N \cdot (\sin\theta)^2 \le N\theta^2$$

$$= N\left(\frac{\pi/2}{N}\right)^2 = \frac{(\pi/2)^2}{N} \le \frac{2.5}{N}$$



Summary

If empty, 100% chance to output "empty"

If bomb, explosion w/prob $\leq \frac{2.5}{N}$

otherwise, output "bomb!"