# 240 Mini Project

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## Contents

# 1 Introduction

Baosteel is one of the largest and most prominent steel producers in China and the world. It is headquartered in Shanghai, China. Baosteel is a state-owned enterprise and a subsidiary of China Baowu Steel Group Corporation Limited, which is the largest steel company in China and the second-largest globally.

Steel sheet production is a process where flat steel sheets are manufactured from raw materials such as iron ore, coal, and limestone. Designing a specific and detailed steel sheet production plan ensures efficient resource allocation, cost control, optimizing manufacturing processes, and enhancing customer satisfaction. It provides a strategic framework for adapting to market changes, mitigating risks, and aligning with sustainability goals, fostering the long-term business resilience.

# 2 Problem Statement

Baosteel is making a steel sheets production plan for the next year. For each of the four periods in one year, there is a demand (tons), regular production cost ($/ton), overtime production cost ($/ton), production shortage cost ($/ton), and maximum inventory capacity (tons). Additionally, the regular production capacity ($C$) for each period is 4200 tons, the inventory holding cost ($H$) is $150 per ton, and the set-up cost ($T$) is $50 per ton. The explanation of each element is shown below:

1. Demand ($D_i$): the demand for steel sheets for each period $i$
2. Regular production cost ($P_i$): the cost of steel sheets production per ton below the production capacity for each period $i$
3. Overtime production cost ($O_i$): the cost of steel sheets production per ton above the production capacity for each period $i$
4. Production shortage cost ($S_i$): the cost of steel sheets per ton for not meeting the demand (backlogging) for each period $i$
5. Maximum inventory capacity ($M_i$): the maximum allowable inventory level at the end of each period $i$
6. Production capacity ($C$): the production capacity for each period
7. Inventory holding cost ($H$): the cost per ton of holding the steel sheets in store for each period
8. Set-up cost ($T$): the cost per ton of changing production levels between each period

Moreover, the last period production in the previous year is 4300 tons, the starting inventory is 1200 tons, and the ending inventory should be at least 1800 tons. The data for each period is shown in the Table 1 below, and we are going to minimize the total cost while trying to meet the demand.

| Detailed Data for Steel Sheets Production | | | | | |
|---|---|---|---|---|---|
| Element | $D_i$ (tons) | $P_i$ ($/ton) | $O_i$ ($/ton) | $S_i$ ($/ton) | $M_i$ (tons) |
| Period 1 | 4000 | 7400 | 7500 | 150 | 3000 |
| Period 2 | 4500 | 7500 | 7600 | 160 | 3500 |
| Period 3 | 4200 | 7600 | 7800 | 170 | 4000 |
| Period 4 | 4400 | 8000 | 8200 | 180 | 4500 |

Table 1: Detailed Data for Steel Sheets Production

# 3 Assumption

- Assume backlogging is allowed in the production process.
- The data are generated randomly according to the common sense and Baosteel production report.

## 4 Mathematical Formulation

Let $x_i$ denote the production amount in the period $i$, and $y_i$ denote the inventory amount at the end of period $i$. The cost includes regular production cost, overtime production cost, production shortage cost, inventory holding cost, and setup cost. Our goal is to minimize the total cost while trying to meet the demand, which can be expressed as:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{4} \left(P_i x_i + O_i \max(0, x_i - C) + S_i \max(0, D_i - x_i - y_{i-1}) + H y_i + T|x_i - x_{i-1}|\right) \\
\text{s.t.} \quad & y_i = y_{i-1} + x_i - D_i \quad \text{(inventory of each period)} \\
& x_0 = 4300 \quad \text{(last production in the previous year)} \\
& y_0 = 1200 \quad \text{(starting inventory)} \\
& y_4 \geq 1800 \quad \text{(ending inventory)} \\
& x_i \geq 0 \\
& 0 \leq y_i \leq M_i \quad \text{(maximum inventory capacity)}, \quad i \in \{1,2,3,4\}
\end{aligned}
$$

Turn it into a linear program, we set variables $w_i = \max(0, x_i - C)$, $v_i = \max(0, D_i - x_i - y_{i-1})$, $z_i = |x_i - x_{i-1}|$, and we have:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{4} (P_i x_i + O_i w_i + S_i v_i + H y_i + T z_i) \\
\text{s.t.} \quad & y_i = y_{i-1} + x_i - D_i \\
& y_0 = 1200 \\
& y_4 \geq 1800 \\
& x_i \geq 0 \\
& 0 \leq y_i \leq M_i \\
& w_i \geq 0 \\
& w_i \geq x_i - C \\
& v_i \geq 0 \\
& v_i \geq D_i - x_i - y_{i-1} \\
& x_0 = 4300 \\
& z_i \geq x_i - x_{i-1} \\
& z_i \geq x_{i-1} - x_i, \quad i \in \{1,2,3,4\}
\end{aligned}
$$

## 5 Result

Solved by Gurobi, we have the following result:

| | | | | |
|---|---|---|---|---|
| $x_1 = 4500$ | $y_1 = 1700$ | $w_1 = 300$ | $v_1 = 0$ | $z_1 = 200$ |
| $x_2 = 4500$ | $y_2 = 1700$ | $w_2 = 300$ | $v_2 = 0$ | $z_2 = 0$ |
| $x_3 = 4500$ | $y_3 = 2000$ | $w_3 = 300$ | $v_3 = 0$ | $z_3 = 0$ |
| $x_4 = 4200$ | $y_4 = 1800$ | $w_4 = 0$ | $v_4 = 0$ | $z_4 = 300$ |

We need to produce 4500 tons per period for periods 1, 2 and 3, and 4200 tons for period 4. We need to work extra hours for the first three periods, but there is no backlogging in the production process. The minimized total cost is 1.4278e+08 dollars. The ending inventory is 1800 tons.

Variable Sensitivity Analysis:

| Name | Solution | Reduced Cost | Objective Coef | Min Obj Coef | Max Obj Coef |
|---|---|---|---|---|---|
| x1 | 4500.0 | 0.0 | 7400.0 | 7350.0 | 7425.0 |
| x2 | 4500.0 | 0.0 | 7500.0 | 7450.0 | 7600.0 |
| x3 | 4500.0 | 0.0 | 7600.0 | 7550.0 | 7625.0 |
| x4 | 4200.0 | 0.0 | 8000.0 | 7266.67 | 15466.67 |
| y1 | 1700.0 | 0.0 | 150.0 | 100.0 | 175.0 |
| y2 | 1700.0 | 0.0 | 300.0 | 275.0 | 350.0 |
| y3 | 2000.0 | 0.0 | 0.0 | -7466.67 | 733.33 |
| y4 | 1800.0 | 0.0 | 150.0 | -15416.67 | inf |
| w1 | 300.0 | 0.0 | 7500.0 | 7450.0 | 7525.0 |
| w2 | 300.0 | 0.0 | 7600.0 | 7550.0 | 7700.0 |
| w3 | 300.0 | 0.0 | 7800.0 | 7750.0 | 7825.0 |
| w4 | 0.0 | 733.33 | 8200.0 | 7466.67 | inf |
| v1 | 0.0 | 150.0 | 150.0 | 0.0 | inf |
| v2 | 0.0 | 160.0 | 160.0 | 0.0 | inf |
| v3 | 0.0 | 170.0 | 170.0 | 0.0 | inf |
| v4 | 0.0 | 180.0 | 180.0 | 0.0 | inf |
| z1 | 200.0 | 0.0 | 50.0 | 0.0 | 75.0 |
| z2 | 0.0 | 33.33 | 50.0 | 16.67 | inf |
| z3 | 0.0 | 16.67 | 50.0 | 33.33 | inf |
| z4 | 300.0 | 0.0 | 50.0 | 0.0 | 75.0 |

Sensitivity Analysis:

| Name | Shadow Price | Slack | Constraint RHS | Min RHS | Max RHS |
|---|---|---|---|---|---|
| c1 | -14966.67 | 0.0 | -2800.0 | -5350.0 | -2200.0 |
| c2 | -15116.67 | 0.0 | -4500.0 | -8400.0 | -3900.0 |
| c3 | -15416.67 | 0.0 | -4200.0 | -6900.0 | -3600.0 |
| c4 | -15416.67 | 0.0 | -4400.0 | -6400.0 | -3800.0 |
| c5 | 15566.67 | 0.0 | 1800.0 | 1200.0 | 3800.0 |
| c6 | 7500.0 | 0.0 | -4200.0 | -4500.0 | inf |
| c7 | 7600.0 | 0.0 | -4200.0 | -4500.0 | inf |
| c8 | 7800.0 | 0.0 | -4200.0 | -4500.0 | inf |
| c9 | 7466.67 | 0.0 | -4200.0 | -4425.0 | -2200.0 |
| c10 | 0.0 | -1700.0 | 2800.0 | -inf | 4500.0 |
| c11 | 0.0 | -1700.0 | 4500.0 | -inf | 6200.0 |
| c12 | 0.0 | -2000.0 | 4200.0 | -inf | 6200.0 |
| c13 | 0.0 | -1800.0 | 4400.0 | -inf | 6200.0 |
| c14 | 50.0 | 0.0 | -4300.0 | -4500.0 | inf |
| c15 | 0.0 | -400.0 | 4300.0 | -inf | 4700.0 |
| c16 | 0.0 | -0.0 | 0.0 | -inf | 0.0 |
| c17 | 16.67 | 0.0 | 0.0 | -900.0 | 0.0 |
| c18 | 0.0 | -0.0 | 0.0 | -inf | 0.0 |
| c19 | 33.33 | 0.0 | 0.0 | -450.0 | 0.0 |
| c20 | 0.0 | -600.0 | 0.0 | -inf | 600.0 |
| c21 | 50.0 | 0.0 | 0.0 | -300.0 | inf |

Figure 1: Sensitivity Analysis

# 6 Discussion and Conclusion

## 6.1 Variable Sensitivity Analysis

From the sensitivity analysis outcome in Figure 1, we have variables of two categories. One has non-zero value and zero reduced cost, the other has zero value and non-zero reduced cost. We will choose one example of each category in our analysis discussion.

We first take variable $x_1$ as an example. $x_1$ represents that the optimal production amount in period 1 is 4500 tons. With the reduced cost of zero, we can say the decision variable $x_1$ is basic. We will always produce 4500 tons in period 1 as long as its regular production cost is within the range of 7350 (Min Obj Coef) and 7425 (Max Obj Coef).

Then, consider the variable $w_4$. $w_4$ measures that if the production amount in period 4 exceeds the production capacity in period 4. With a value of zero and a reduced cost of 733.33, we can say $w_4$ is a non-basic decision variable. If the production amount exceeds the production capacity in period 4 by 1 ton, the total cost is expected to increase by 733.33.

We have noticed that all $v_i$ values for $i = 1, 2, 3, 4$ are zero (non-basic decision variable). $v_i$ is the number of tons of the demands that are not accomplished. This means that all demands are met at the optimal value, and the reduced costs are equal to the value of production shortage costs of each period. The company should not miss any demand unless the production shortage cost at each period drops to zero.

To minimize the overall costs, we would prefer to reduce the set-up costs for changing the production levels. The variables $z_i$ for $i = 1, 2, 3, 4$ represent the change of production level between each period. $z_1$ and $z_4$, which are basic variables, are 200 and 400 respectively, and $z_2$ and $z_3$ are zero. The optimal value of $z_i$ shows that the company should avoid changing the production levels with exception of when the production costs are very high. The reduced costs for $z_2$ and $z_3$ are 33.33 and 16.67, which means that the company may consider change the production level between period 1 and 2, and period 2 and 3 if the fixed set-up cost can be decreased by the amount of the reduced costs.

## 6.2 Constraint Sensitivity Analysis

From the sensitivity analysis outcome, we have constraints of two categories. One has non-zero shadow price and zero reduced cost, the other has zero shadow price and non-zero reduced cost. We will choose one example of each category in our analysis discussion.

We first take constraint $c_5$ as an example. This constraint represents that the ending inventory of the last period should not be less than 1800 tons. As shown in the sensitivity analysis table, this constraint has a slack of 0, which simply means this constraint is binding. And if we require more production of ending inventory, we will be forced to increase the total cost by 15566.67, which is also the shadow price of this constraint. Also, if we do not change the RHS of $c_5$ beyond the interval of 1200.0 (Min RHS) to 3800.0 (Max RHS), we can always expect the total cost to increase by this amount when we increase the RHS value by 1. This result is explainable because if we want to keep a higher inventory level in our production process, we will encounter the risk of having a higher cost.

Then consider constraint $c_{10}$. This constraint represents that the shortage of production from period 1 should be the larger value between 0 and the actual positive shortage. As shown in the sensitivity analysis table, this constraint has a slack of -1700.0 and a shadow price of 0, which simply means this constraint is not binding. And if we change the amount of production in period 1 within the Min RHS and Max RHS, we should not expect a lower total cost. This means under our assumptions, we will never want to set our production level not meeting the demand. And this could be explained by the existence of production shortage cost and the set-up cost which requires a cost of changing production levels between the periods. Since we have our last period production in the previous year of 4300 tons, we are not inclined to change the production level too much due to the set-up cost. And a similar production amount of around 4300 can already easily help us meet the demand. Therefore, there is no production shortage occurring in our modeling.

## 6.3 Further Improvement and Conclusion

In conclusion, for Baosteel, a well-structured production plan contributes to operational excellence, enabling the company to meet demand effectively while minimizing cost and maximizing profitability.

Moreover, we are imagining some possible situations where our current optimal plan will have a fundamental change. For instance, if the maximum inventory is very small and overtime production is discouraged, production shortages will occur. In this way, by tuning the production shortage cost according to the reduced cost in the sensitivity analysis, the shortage amount would be a non-zero value for each month.

# 7 Appendix (code)

```
from gurobipy import *

# Create a new model
m = Model("mp1")

# Create variables
x1 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="x1")
x2 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="x2")
x3 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="x3")
x4 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="x4")
y1 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, ub=3000, name="y1")
y2 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, ub=3500, name="y2")
y3 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, ub=4000, name="y3")
y4 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, ub=4500, name="y4")
w1 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="w1")
w2 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="w2")
w3 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="w3")
w4 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="w4")
v1 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="v1")
v2 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="v2")
v3 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="v3")
v4 = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name="v4")
z1 = m.addVar(vtype=GRB.CONTINUOUS, name="z1")
z2 = m.addVar(vtype=GRB.CONTINUOUS, name="z2")
z3 = m.addVar(vtype=GRB.CONTINUOUS, name="z3")
z4 = m.addVar(vtype=GRB.CONTINUOUS, name="z4")

# Set objective
m.setObjective(7400*x1 + 7500*x2 + 7600*x3 + 8000*x4 + 7500*w1 + 7600*w2 + 7800*w3 +
    8200*w4 + 150*v1 + 160*v2 + 170*v3 + 180*v4 + 150*y1 + 150*y2 + 150*y2 + 150*y4 +
    50*z1 + 50*z2 + 50*z3 +50*z4, GRB.MINIMIZE)

# Add constraint:

m.addConstr(y1 == 1200 + x1 - 4000, "c1");
m.addConstr(y2 == y1 + x2 - 4500, "c2");
m.addConstr(y3 == y2 + x3 - 4200, "c3");
m.addConstr(y4 == y3 + x4 - 4400, "c4");
m.addConstr(y4 >= 1800, "c5");

m.addConstr(w1 >= x1 - 4200, "c6");
m.addConstr(w2 >= x2 - 4200, "c7");
m.addConstr(w3 >= x3 - 4200, "c8");
m.addConstr(w4 >= x4 - 4200, "c9");

m.addConstr(v1 >= 4000 - x1 - 1200, "c10");
m.addConstr(v2 >= 4500 - x2 - y1, "c11");
m.addConstr(v3 >= 4200 - x3 - y2, "c12");
m.addConstr(v4 >= 4400 - x4 - y3, "c13");

m.addConstr(z1 >= x1 - 4300, "c14");
m.addConstr(z1 >= 4300 - x1, "c15");
m.addConstr(z2 >= x2 - x1, "c16");
m.addConstr(z2 >= x1 - x2, "c17");
m.addConstr(z3 >= x3 - x2, "c18");
m.addConstr(z3 >= x2 - x3, "c19");
m.addConstr(z4 >= x4 - x3, "c20");
m.addConstr(z4 >= x3 - x4, "c21");

# Optimize model
m.optimize()

# Output results
for v in m.getVars():
    print('%s %g' % (v.varName, v.x))

print('Objective: %g' % m.objVal)
```

```
print('\nVariable Sensitivity Analysis:')
print('Name\tSolution\tReduced Cost\tObjective Coef\tMin Obj Coef\tMax Obj Coef')
for v in m.getVars():
    print(f"{v.VarName}\t{round(v.x,2)}\t\t{round(v.RC, 2)}\t\t{round(v.Obj, 2)}\t\t{round(v.SAObjLow, 2)}\t\t{round(v.SAObjUp, 2)}")


print('\nSensitivity Analysis:')
print('Name\t\tShadow Price\tSlack\tConstraint RHS\t\tMin RHS\t\tMax RHS')
for c in m.getConstrs():
    print(f"{c.ConstrName}\t\t{round(c.Pi, 2)}\t\t{round(c.Slack, 2)}\t\t{round(c.RHS, 2)}\t\t{round(c.SARHSLow, 2)}\t\t{round(c.SARHSUp, 2)}")

from prettytable import PrettyTable

# Variable Sensitivity Analysis
variable_table = PrettyTable()
variable_table.field_names = ["Name", "Solution", "Reduced Cost", "Objective Coef", "Min Obj Coef", "Max Obj Coef"]

for v in m.getVars():
    variable_table.add_row([v.VarName, round(v.x, 2), round(v.RC, 2), round(v.Obj, 2), round(v.SAObjLow, 2), round(v.SAObjUp, 2)])

# Sensitivity Analysis
sensitivity_table = PrettyTable()
sensitivity_table.field_names = ["Name", "Shadow Price", "Slack", "Constraint RHS", "Min RHS", "Max RHS"]

for c in m.getConstrs():
    sensitivity_table.add_row([c.ConstrName, round(c.Pi, 2), round(c.Slack, 2), round(c.RHS, 2), round(c.SARHSLow, 2), round(c.SARHSUp, 2)])

# Print the tables
print('\nVariable Sensitivity Analysis:')
print(variable_table)

print('\nSensitivity Analysis:')
print(sensitivity_table)
```