Proof of Search in Derivate Space

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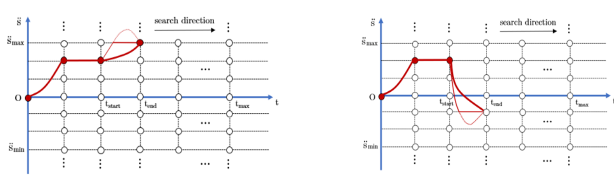
\begin{aligned}
\min_{\ddot{s}[1],\ldots,\ddot{s}[n_s]} \ & w_2 \sum_{k=1}^{n_s} (\ddot{s}[k])^2 + w_3 \sum_{k=0}^{n_s} (\ddot{s}[k+1] - \ddot{s}[k])^2 \\
\text{s.t.} \quad & \ddot{s}[0] = \ddot{s}_{\text{start}}, \ \ddot{s}[n_s+1] = \ddot{s}_{\text{end}},
\end{aligned} \tag{1}
$$

where w_2 and w_3 are tunable parameters. The solution of the problem in Eq. (1) is also guaranteed to satisfy the constraints of $\ddot{s} \in [\ddot{s}_{\text{min}}, \ddot{s}_{\text{max}}]$, as shown in the following proposition.

Proposition 1. *Suppose that $\ddot{s}_{start}, \ddot{s}_{end} \in [\ddot{s}_{min}, \ddot{s}_{max}]$. Then, the solution of problem (1) satisfies the constraints of $\ddot{s}$, i.e., $\ddot{s}[1], \ldots, \ddot{s}[n_s] \in [\ddot{s}_{min}, \ddot{s}_{max}]$.*

Proof. We complete the proof by contradiction. Suppose there exsits $\ddot{s}[k] > \ddot{s}_{\text{max}}$ for the optimal solution of Eq. (1) (corresponding to the dotted line in Fig.1(a)). A smaller value of the objective function will be achieved by replacing $\ddot{s}[k]$ with $\ddot{s}_{\text{max}}$, which renders a contradiction. Hence, we have $\ddot{s}[1], \ldots, \ddot{s}[n_s] \in [\ddot{s}_{\text{min}}, \ddot{s}_{\text{max}}]$. $\square$



(a) (b)

Fig. 1: Illustrations of the range of fitting points.