An investigation of Brazil nut effect by Monte Carlo method

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Abstract

—The following paper presents a computational approach to investigate the cause of Brazil nut effect (BNE) using Monte Carlo method. One of the theories, size segregation, which can explain this phenomenon under certain criteria was examined in this experiment.

Having performed simulations of the random motion of particles in a container while vibrating them vertically with different conditions, size segregation has been proven to be a valid theory to allow BNE to occur. A further investigation in the factors which affect the rate of segregation which is a measure of how quickly BNE occurs having particles vibrated in a box was conducted. Lift distance and size of particles were found to be the two major factors.

A potential convection model was posited to explain several unexpected results obtained due to size segregation.

Index Terms— Brazil nut effect (BNE), size segregation, Monte Carlo method, Lift distance, size of particles

1. Background

Well - motivated

Brazil nut effect, which has the largest sized particles rise up to the surface of a packet of granular material after multiple shaking motions.[1] This daily phenomenon cannot be explained completely by any existing theories. Despite of the complexity, the applications of this effect were widely used in different areas in manufacturing, astronomy and geology.

(a) (b) (c)

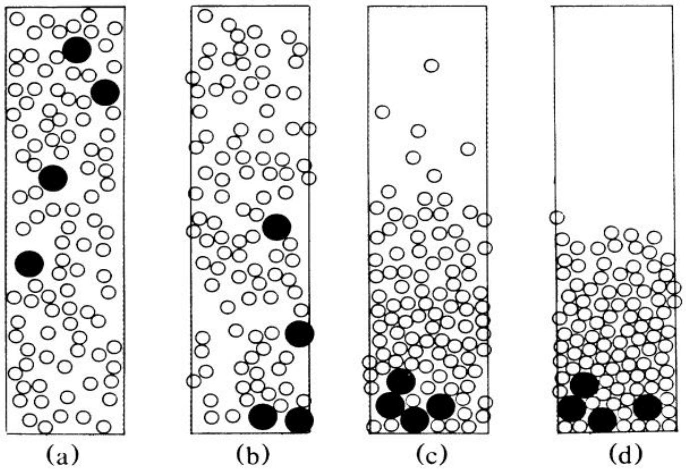
Fig.1 bigger sized nut flows up to the surface gradually after shaking.

The discovery of this effect was rather surprising, an intuitive guess would have particles randomly mixed after shaking. If there must be a clear motion pattern, the bigger and denser nuts should have moved to the bottom of the container to minimize the global potential energy of the system based on the fact that denser material are more likely to sink. For instance, a rock will sink in water. However, the global potential energy of the system has been increased after shaking as a result to have Brazil nut being pushed to the surface.

This peculiar motion was described by a theory size segregation, which was introduced and simulated

computationally for the first time in 1987.
Monte Carlo method has a great efficiency to simulate the random motion of particles in a container and study the dynamics of energy in a system. Therefore, it was used in the paper in 1987 to study size segregation.
Two different Monte Carlo methods were used, they are described in more details below.

A. A standard Monte Carlo cooling simulation



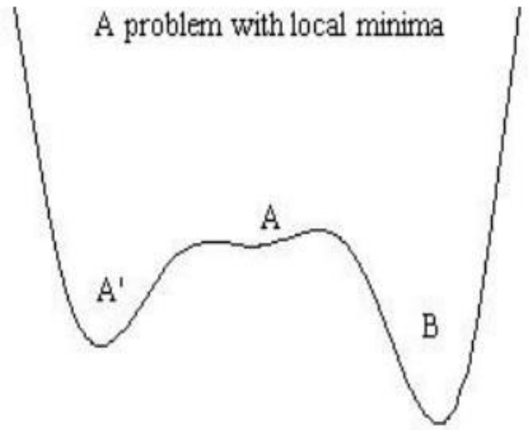


Fig.2 a standard Monte Carlo cooling simulation tends to the global equilibrium configuration with the large particles on the bottom. (a) The initial random placement of the particles. (b)— (d) The configurations at T* =k a T/m, gd, =33.3, 2.2, and 0.6, respectively. [1]

Fig.3 A and A' are local minimums, B is a global minimum. [2]

The first method is a standard Monte Carlo method which can cool down particles to reach global potential energy minimum in a system which has the bigger particles at the bottom of the container. As shown in fig.2 (d), the configuration of disks allows the minimum global potential energy of the system to occur. The result was achieved by using a standard Monte Carlo method which includes a small probability of accepting a move that does not improve the energy, but offers the chance to escape from a local minimum such as A and A', until the global minimum at B is found as shown in fig.3.[1]

B. Shaking simulation with modified Monte Carlo method

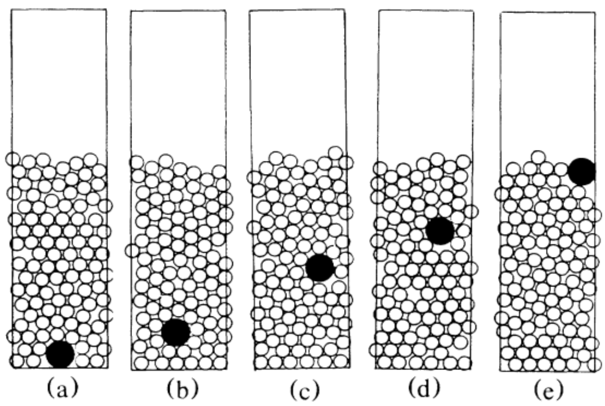


Fig.3 Computational simulation of Brazil nut effect in 1987 paper. In the shaking simulation the large particles rise to the top. (a) The initial configuration. (b)- (e) The configurations obtained after 10, 30, 40, and 60 "shakes," respectively. Approximately 3000 Monte Carlo steps per particle were performed during each shake. [1]

A modified Monte Carlo method was designed to simulate BNE, it is an algorithm whereby only certain configurations are chosen after random reconfigurations. Namely, configurations that decrease the energy of the system. In fig.3 (e), a Brazil nut flows to the surface after 60 “shakes” with a non-equilibrium global potential energy.

In this paper, the Monte Carlo method was simulated computationally, incorporating the segregation model, to determine the factors affecting the rate of size segregation. There are three potential factors, particle sizes, mass ratio and lift distance.

2. Method

General method

Initial condition

The simulations of BNE in this project took place in a two dimensional system, where Brazil nuts and cornflakes in muesli are circular discs which have different radius. A simple model of particles with only two possible radii was considered. In addition, size segregation depends on the ratio of the particle sizes, so the radius ratio between a Brazil nut and muesli was set to be 2:1.
The container is a rectangular shaped, infinitely tall box where ensures that no particles will reach the top of the box. Moreover, the height of the box was defined to be ten times more than its width, in result, the box can be regarded as an infinite system at the first approximation.
In this two dimensional system, the position of each nut which is randomly generated within the boundary of the box is a two dimensional array. In the simulation, no-overlap criterion for the particles was introduced.

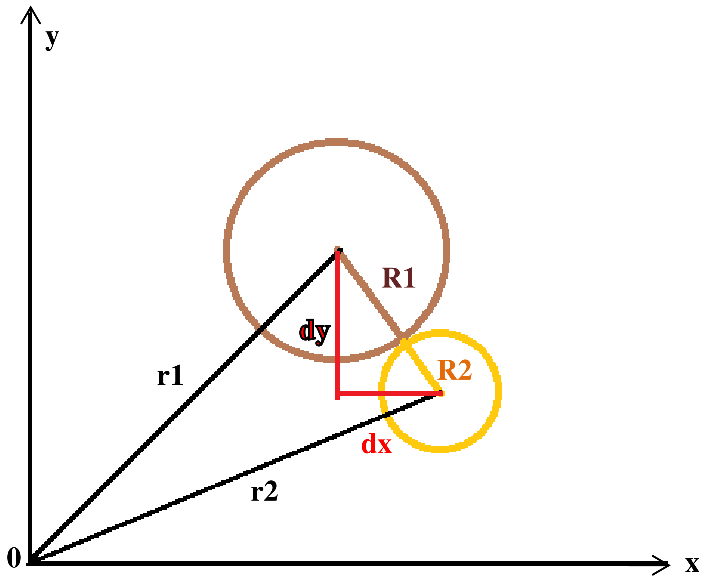


Fig.4 Minimum distance between two particles without overlapping.”r1”,”r2” are the position of each particle from center to (0,0) in a Cartesian system. “dx” and “dy ” are the difference in x and y coordinates for the two particles respectively. “R1” and “R2” represents the radius of each particle.

Overlap will not occur if the minimum distance between the centers of two particles is greater or equal to the sum of their radius. The following equation (1) is an equation which illustrates this condition.

$$\sqrt{dx^2+dy^2} \geq R1+R2 \quad (1)$$

The potential energy of each particle is given by:

$$PE = \mathrm{mgh} \quad (2)$$

The global potential energy of the system can be determined by the following equation:

$$PE_{\mathrm{global}} = \sum_{i=1}^{n} \mathrm{m_i g h_i} \quad (3)$$

Equilibrium simulation after ‘pouring’

Within the simulation, the Monte Carlo method was implemented to demonstrate the process of ‘pouring’, a process of pouring particles into a container until they are settled by gravity.
In the two dimensional system that was defined, disks should fall to the bottom of the box and reach local minimum energy where the potential energy can no longer decrease further for a random configuration. The procedures are described as follows:
(I) Instead of calculating the motion of particles, such as their velocities and displacements, they are randomly reconfigured under the condition ~~that the energy of the system is improved. Such procedure is defined to be a~~ cycle called “jump”[2]
(II) The new position should met all the criteria in initial condition that was defined above.
(III) Only gravitational potential energy is considered. Particle collisions are disregarded. Equation (1), (2) calculate the potential energy of individual particle and system respectively.
(IV) The step length of a ‘jump’ should be controlled by the largest nearest-neighbor distance (LNND). The nearest-neighbor distance is the distance between a given particle and the particle that is in closest proximity. The largest neighbor-distance is therefore the maximum value of the nearest neighbor distances.
(V) The process of “jump” should stop when all the particles will have reached the ground. A big value such as 1000 “jumps” was initialized for the Monte Carlo method to begin with, the iteration is stopped once the change in potential energies is insignificant.

Shaking simulation

Brazil nut effect occurs only when shaking process takes place, except sifting in fig.6.
In reality, particles can be lifted inside of a box if the acceleration of the box is greater than gravitational constant while being shaken. By the influence of gravity, they fall back to the bottom of the box. This single process is defined as a ‘shake’. Since particles moves randomly in the process, Monte Carlo method is an appropriate approach to be used.
Within the simulated model it was assumed that all the particles are lifted after shaking. To vibrate a particle in a vertical direction, the y coordinate of the disk has to be multiplied by a lifting constant which is greater than 1. Lifting constant is directly related to lift distance, it is a constant which is multiplied to the height of a particle to simulate a shaking process. For simplicity, I chose to increase the height of each particle by a factor of 2, so the lifting constant is equal to 2 in this case.

My initial code for shaking processes has one Brazil nut place on the bottom of the box and 10 muesli positioned above the Brazil nut with a steady increasing height [2]. See fig.7 (a). The radius ratio between the nut and muesli is 2:1. Having finishing the code for shaking simulation, Brazil nut effect has truly occurred as shown in fig.7 and fig.8.

Optimizing code

My earliest Monte Carlo method was to write loops to check if the new position for each particle will overlap with the existing particles. The model could save the new position if all the conditions are satisfied, otherwise it was discarded. The code was inefficient, particularly at scale, because the position of each new particle was compared to all other particles in the system iteratively. Therefore, the increase in particle numbers had increased the time for code to run exponentially.
An updated version of Monte Carlo modelled the box as a grid, and the presence of a particle excluded its position from the grid to represent the fact that this position was occupied.
Compared to the old version, the newest one has made sure no particles would overlap with each other without the use of loops. How it works is to delete all of the integer coordinates inside of each particles from the system to eliminate the options for the new particles to select.
Furthermore, the step length in the last version was set to be 10-15 times of the largest nearest distance to improve the efficiency of each "jumps".
In comparison, the final code could save more than 90% of the time to complete the same task as using the oldest version.

Investigation of the rate of size segregation

Size segregation is a result of the geometrical feature of particles inside of one system where the smaller particles are more likely to fill up the voids created from shaking. Therefore, mass is not an important factor in size segregation. Particle size and lift distance are the two parameters that were emphasized in this paper.

Particle size was the first factor that was investigated in the paper. To study the behavior of Brazil nuts and their motion explicitly, a Brazil nut was initially added to the bottom of the container accompany with a group of same sized particles. By controlling the amount of particles in the system and varying their sizes, numbers of "shakes" (n) and total displacement (d_T) for the Brazil nut were able to be measured upon the simulated diagrams. Therefore, the mean displacement of the Brazil nut for each "shake" d_s can be calculated as follows:

$$d_s = \frac{d_T}{n} \tag{4}$$

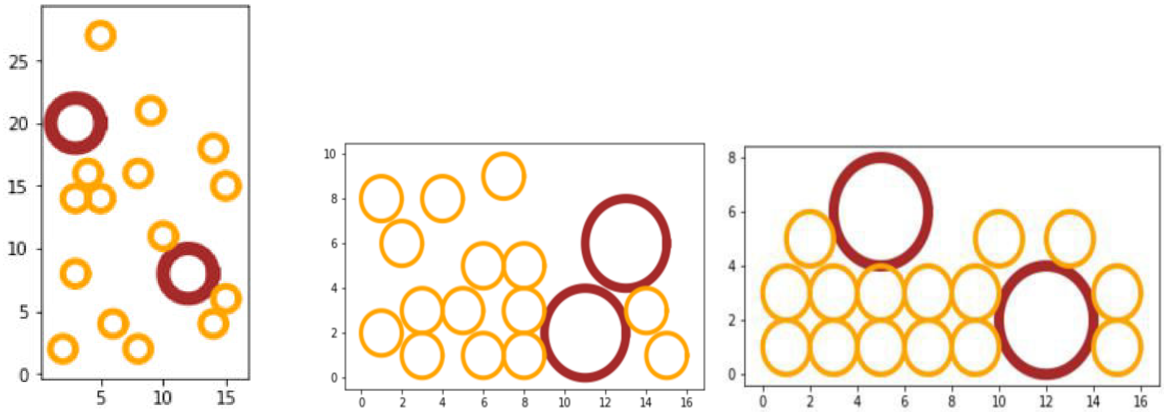
The mean displacement of the Brazil nut for each "shake" d_s is a constant which shows the rate of size segregation. A high value of d_s means a higher rate of size segregation.

The second factor, lift distance, was studied as the following procedures:
(1) Lift distance is directly dependent on lifting constant (2) 3 experiments were taken place with the exact initial configuration which had one Brazil nut placed at the bottom of the system with a group of same sized particles. (3) Each test has a different lifting constant, they are 1.5, 2.0, and 2.5. (4)The amount of "shakes" for each sample was measured as well as the total displacement of the Brazil nut. (5) The mean displacement of the Brazil nut for each "shake" d_s was calculated by using equation (4) to determine the rate of size segregation.
"Random seed" is a computational approach which can generate exactly the same configuration with respect to a given random number. This method was applied in the code to eliminate uncertainties or other potential factors which may affect the results. For instance, the height of the sample should be the same to ensure fixed amount of total displacement for the Brazil nut.

3. Results and Analysis

Results of equilibrium simulation after ‘pouring’

A. Comparable size



(a) (b) (c)

Fig.5 two Brazil nut and 15 muesli with radius ratio 2:1 settled to the bottom of the box and system equilibrium has been reached. (a) The initial random distribution of particles after pouring. (b) The configurations of particles after 100 jumps. (c) After 200 jumps where the potential energy of the system has reached equilibrium.

Within the code, the model that was implemented kept decreasing the gravitational potential energy of the system until a local minima has been found in (c). In other words, to reach equilibrium, the total height of the center of mass of the system must be minimized. In part (c), the biggest nuts in the system were not both lying at the lowest level. Therefore, the system has not reached the global potential energy minimum, but the lowest potential energy for such specific configuration. Clearly, this result does not match with the standard Monte Carlo cooling simulation in fig.2 which has the largest particles at the bottom of the system to reach the equilibrium of global potential energy.

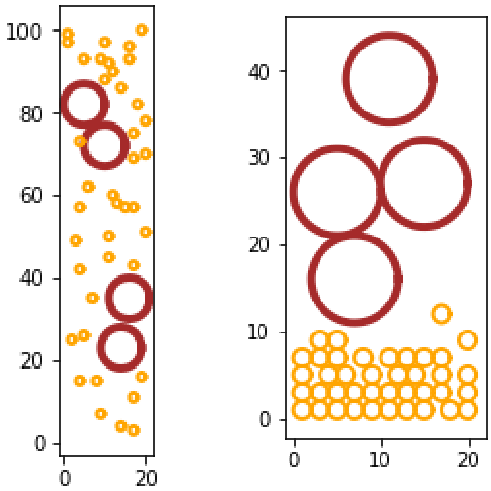
The underlying cause of this disparity in results is the Monte Carlo method that I have coded only allowed the potential energy of the particle to be less or equal to its original energy after moving. Once the bottom has been occupied with the smaller particles, the big ones can no longer reach the bottom of the container.

A slight change in the condition of shifting particles can lead my result to match with fig.2. While the last Monte Carlo method in this paper has only allowed one particle to be moved in a cycle of “jump”, an improved version should randomly pick two particles at the same time in a cycle of “jump”. Then move them to new positions as long as the total potential energy of the system is decreased. In that sense, the algorithm has a probability to accept a new particle position which increases the potential energy of the system, since the more massive particles have greater contributions towards the potential energy of the system. Denser particles are more likely to reach the bottom as shown in fig.2.

With this refined method, the computational model accounts for the effects of particle density. The largest particle will always sit at the bottom of a box if its density is relatively massive, because they are intended to minimize the potential energy of the system. Therefore the computational model in this paper holds only for situations in which the ratio of particle densities between Brazil nuts and muesli is approximately unitary. Where the ratio diverges from 1, the model becomes increasingly inaccurate.

The model that was developed in this paper can match with the phenomenon ‘pouring’ very well. The simulation modelled a situation where particles were poured into a 2 dimensional box. By influence of gravity, the distribution of the particles tends to be random and has a minimum energy for each specific configuration while system equilibrium occurs. Having a layer of denser particles embedded in the less dense one is not a possible configuration after pouring mixture to the bottle.

B. Sifting



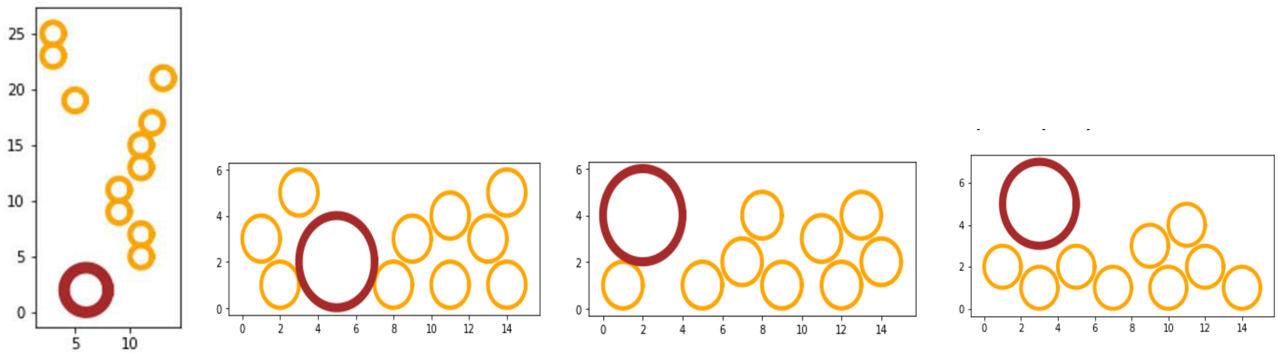
(a) (b)

Fig.6 (a) four Brazil nut and 40 muesli with ratio radius 5:1. (b) Brazil nut effect occurs without shaking

A sifting mechanism [1] was applied within the computational model, it occurs while the size difference between the two kind of particles are considerably large. The smaller particles will filter through the gaps between big ones to settle down and result in Brazil nut effect without even shaking.

Results of shaking simulation

A. shaking simulation



(a) (b) (c) (d)

Fig.7 A system of one Brazil nut and ten muesli under the conditions: the Brazil nut is on the bottom and 10 muesli are above it with increasing height and random x coordinates. (a) The initial distribution following the conditions were given above (b) The particles settled to the bottom in random order to reach global potential minima. (c) First shake applied (d) Second "shake" applied, resulting in the Brazil nut rising to the top. Approximately 500 “jumps” for each shakes were conducted to settle all the particles. The lifting constant was set to be 2. The Brazil nut moved upwards by approximately 1.75 units in the system each time.

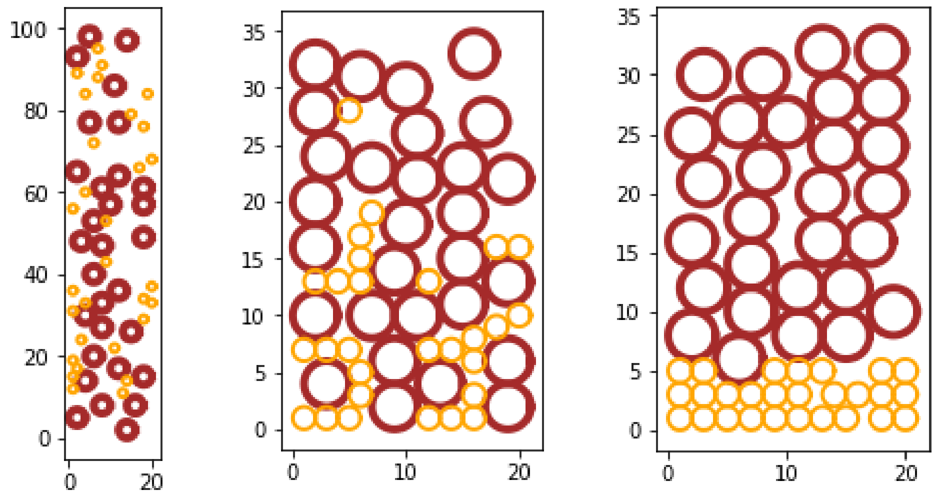
A Brazil nut and ten other equally sized muesli which are half the size of the nut were poured into a container. (b) is a configuration which has global potential energy minimum, because the height of the largest particle was minimized. (b) to (d) demonstrates the Brazil nut effect which has larger disc move gradually to the surface from the bottom of a cell after two “shakes”. After each “shake”, there was a falling process to ensure the configuration met the local minimum potential energy.

A reasonable explanation for BNE in this result is that the voids created beneath the large particles were occupied by smaller particles until an equilibrium of local potential energy has been reached. However, there is a very small chance for a large nut to move back down if a void beneath it can fit itself in addition to the fact that no smaller particles are "willing" to fill up the gap. This configuration has been eliminated in the experiment considering the insignificant probability for it to happen. Therefore, the Brazil nut can move upwards by approximately 1.75 units each time until flows to the surface.

Overall, the potential energy of the system changed from global minimum to local minimum. Such local minimum is a crucial condition for BNE to occur as shown in (d).

The lifting constant which determines the displacement of each particles after shaking was set to be 2. It is arguable that particles with different mass should have a varied displacement. However, mass is not a factor to size segregation in the model that was created.

B. Binary size segregation



(a) (b) (c)

Fig.8 segregation of a 30/30 Brazil nut and muesli mixture. Radius ratio between muesli and Brazil nuts is 1:2 (a) Random arrangement (b) particles settles to reach equilibrium after 1000 'jumps'. (c) Brazil nuts flows to the top after shaking for 4 times.

Size segregation occurs in fig.7 which has 1:10 Brazil nut and muesli ratio. To investigate if particles ratio is a factor for Brazil nut effect, a binary system where a 30/30 Brazil nut and muesli mixture has was undergoing the same procedure used for fig.7. Size segregation occurs between (b) and (c), and all of the Brazil nuts flowed to the top of the layer of muesli after the 4^{th} shake. Therefore, size segregation is independent to the ratio of particles with different sizes.

In fig.8 (b), the muesli didn't full fill the gaps between Brazil nuts and the Brazil nuts had a greater tendency to rise to the surface, compared to the muesli. This may affect the validity of the result in (c). A future improvement cam use a smaller ratio of particle radii between the Brazil nuts and the muesli, so that the mixture will be more randomly distributed after "pouring".

Results of studying the rate of size segregation

A. Behaviour of a brazil nut in 5 groups of particles

Types	Particle type 1	Particle type 2	Particle type 3	Particle type 4	Particle type 5	Brazil nut
Radius	2	3	4	5	6	5
"Shakes" (n)	1	2	5	114	∞	
Total displacement (d_T)	20	32	65	∞	∞	
Displacement per "shake" (d_S)	20	16	13	≈10		

Table.1 there are 5 groups of particles with varied sizes and a brazil nut which has radius equal to 5. Each group contains 20 same sized particles. a distribution of radius ratio of each group with respect to the radius of a brazil nut are respectively 2:5, 3:5,4:5,5:5,6:5. The Brazil nut was initial added to the bottom of the container. Amount of shakes and total displacement for each group were measured to find their displacements per "shake". Lift distance is 20 units which is the diameter of 2 Brazil nuts.

A brazil nut was added to the five groups of different sized particles separately to investigate the relationship between particle sizes and the rate of size segregation. Overall, five experiments were conducted to study the behavior of the Brazil nut and each groups of particles.
The same initial conditions were applied to each experiment, such as the same lift distance and the initial position of the Brazil nut. After that, number of "shakes" (n) and total displacement for the Brazil nut were measured upon the simulations.
The rate was determined by measuring the mean displacement of the Brazil nut for each "shake" d_s with equation (4). Suggested by the table.1, the final result was that the Brazil nut would be lifted to the surface if the radius ratio is smaller and equal to 1. The rate of size segregation increases when the radius ratio becomes bigger in a range of 4:5 and 2:5.
When the radius ratio was equal to 1, the amounts of "shakes" required to take the Brazil nut to the top of all the particles were random, because each particle has an equal probability to be at the surface.
For the case when radius ratio overcame 1, the Brazil nut couldn't rise to the surface no matter how many shakes were taken.

A further study can focus on the behavior of the Brazil nut when the radius ratio is in-between 4:5 and 1.

B. Add one brazil nut to a distribution of particles with varied sizes

Types	radius	number	Lifting constant
Particles	2,3,4	20	2
Brazil nut	3	1	2

Table.2 Information of particles in a 2 dimensional system. Radius, mass of particles and lifting constant have no unit, they are written as ratio.

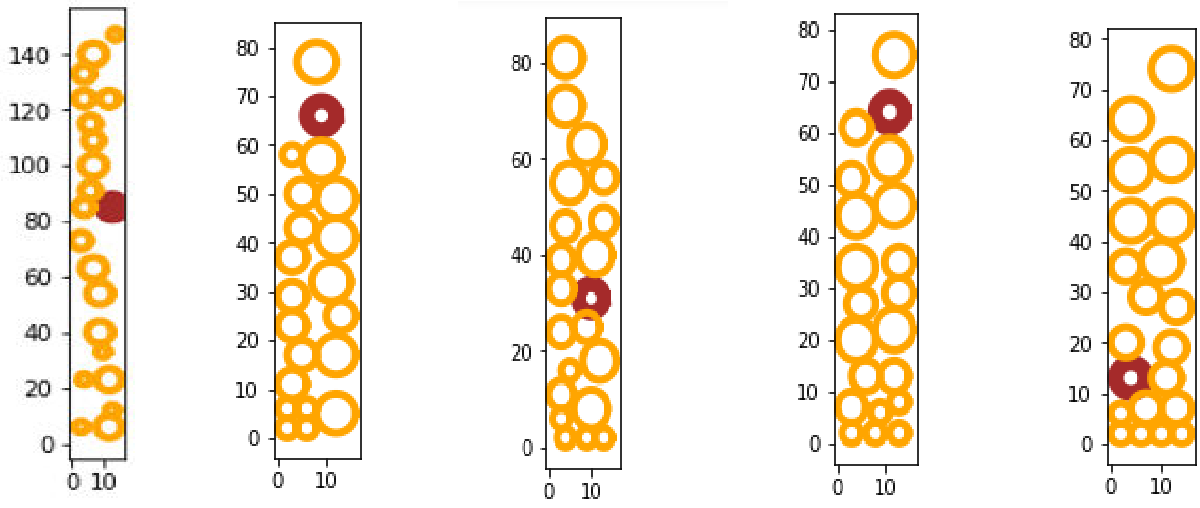


Fig.9 One Brazil nut with radius 3 and twenty muesli with random size in the range of 2 to 4. (a) Initial random distribution. (b) Local minimum after pouring. (c) Results after shaking by 3 times. (d) results after shaking by 4 times.(e) results after shaking by 15 times. Each" shake" has 500 "jumps". Lifting constant is 2.

In figure.9, the system contained the biggest particle with a radius of 4 units, while the Brazil nut had radius 3 units. The nut was intended to go down to the bottom of the container from the top after shaking by 3 times. It had been pushed upwards by the end of the fourth shake. Between (d) and (e), the Brazil nut gradually moved to the bottom of the container and lost the tendency of flowing upwards.

The fluctuated positions of the Brazil nut inside of the box seemed to be rather peculiar, but Brazil nut effect was still valid here, the bigger nut has more tendency to flow upwards. The Brazil nut from fig.9 was poured into the container with a distribution of 20 20 particles with various radii. Some of the particles have bigger radius compared to Brazil nut, such as the particles right below the nut in (b) have radius 4 units. As a result, the Brazil nut dropped by 35 units after 3 times of shaking. In (c), the particles surrounding the Brazil nut had relatively smaller radius, they push the nut upwards by approximately 35 units. After the fourth shake, the Brazil nut kept sinking, until the particles were distributed such that the particles with greater radius occupied the highest position in the box.
The substantial displacement from (c) to (d) with a difference of one "shake" does not appear to accurately model a physical system. The problem was that I have set the lifting constant to be 2, so that the lift distance for particles will be excessively high if they are further away from the bottom of the system. Therefore, lifting constant worked better for the system which has a low center of mass. To improve the accuracy of the model, adding a certain lift distance for each disks can be implemented to simulate the process of shaking box.

The following result is an improved version for the experiment in fig.10.

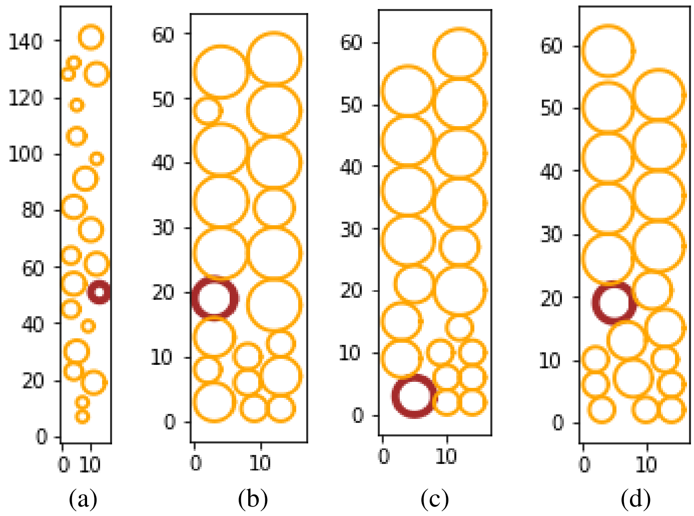


Fig.10 the initial conditions were set to be the same as the conditions in fig.10. (b) Initial configuration (c) results after shaking by 3 times. (d) results after shaking by 6 times.(e) results after shaking by 15 times. Each" shake" has 500 "jumps". Lifting distance is the diameter of the Brazil nut.

Rather than using the lifting constant, each particle was brought up by a lift distance equal to 6 units inside of the system. By simple calculation, the displacement of Brazil nut after each "shake" is approximately equal to 6 units which is equal to the lift distance. The maximum displacement was 20 unit which is 3.5 lift distance. This result more accurately models a physical system

From (b) to (d), the Brazil nut cannot rise up to the surface, but reaches a point where all the particles above it have bigger size. The furthest height that the Brazil nut could go was 20 units above the bottom of the system. From the diagrams, the particles with sizes bigger than the Brazil nut has barely moved and stayed at the top surface all the time. These results have well demonstrated the behavior of the smaller particles underneath of the bigger ones which have already moved to the surface.
Intuitively, the expected result would have particles being separated into obvious layers after considerable amount of "shakes", whereas the behavior of the Brazil nut was particularly unexpected. While shaking the box, the nut was fluctuating regularly with a sinusoidal pattern between the bottom of the box and 20 units further above. See appendix A.

This result is rather surprising, the first reason is that the Brazil nut went to the bottom of the system from a higher position within a group of particles which are smaller or equal to its size. The motion of the Brazil nut in this case is opposite to any of the results that were obtained before. Since the bigger sized particles are above the Brazil nut, so they have no effect on the reversed direction of Brazil nut effect. Since the mass ratio is not a factor in this paper, a reversible Brazil nut effect has occurred in size segregation.

The second unexplained observation was the cause of the sinusoidal pattern. The amplitude of the wave pattern is 20 units, and half wavelength is approximately equal to 5 "shakes". This regular pattern may be a result of granular convection.

C. Potential Convection theory

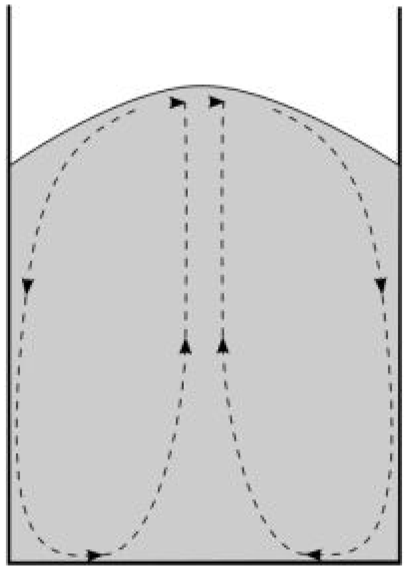
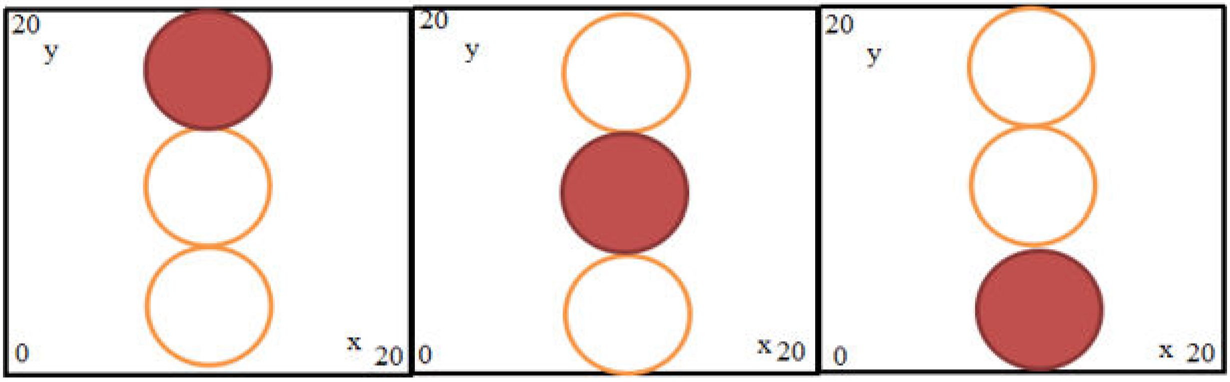


Fig.11 [3] the diagram shows the granular convection within a rectangular shaped box. This is a model which simulates the vibration-induced convection flow which has the particle at the middle of the box flowing upwards to the surface and goes down to the bottom next to the side walls.

Granular convection suggests that the particles move in vibration-induced convection flow when shaken; individual particles move up through the middle, across the surface, and down the sides. If a large particle is involved, it will be moved up to the top by convection flow. Once at the top, the large particle will remain stationary because the convection currents are too narrow to sweep it down along the wall. [4]

The major difference between size segregation and granular convection is the former doesn't consider the frictions of the wall or the particles. The situation in fig.11 has the friction from the bigger sized particles which has blocked the way for Brazil nut to flow upward, therefore a convection pattern was created.
The convection pattern in fig.11 is different to a standard convection model which moves the Brazil nut to the top and keeps it there, whereas the convection pattern found in my results allows the nut to move downwards within a sinusoidal shape.

The bigger sized particles above 20 units' height in the system has no interaction with the particles below 20 units. Therefore, those bigger sized particles have no contribution towards the creation of convection. Having the particles above 20 units' height eliminated, a new local system within the square which has a length of 20 units can be studied. In Appendix A, there are only two kind of particles in the new local system, namely; 2-3 particles with the same radius of the Brazil nut and 7 smaller particles.
By observation, the motion of the Brazil nut is highly dependent on the motion of the other particles, mainly the ones that have the same size. Three horizontal lines across each picture in Appendix A at height 20 units, 15 units and 10 units were drawn to investigate the distribution of each kind of particles. Having counted and studied the amount of different kind of particles, the following diagrams have shown the three extreme cases which can model how the behavior of Brazil nut is affected by the other same sized particles in a simplified manner.

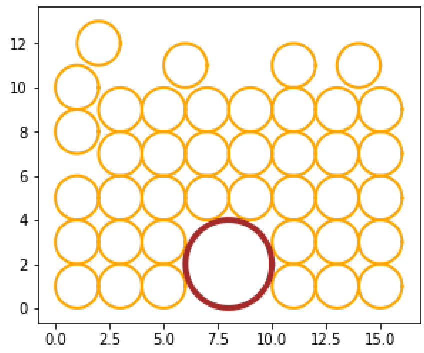
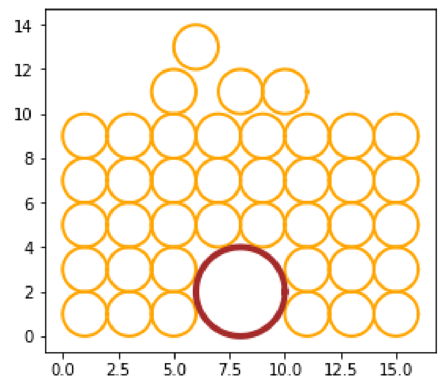
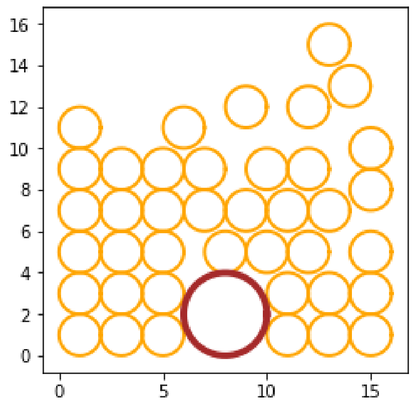
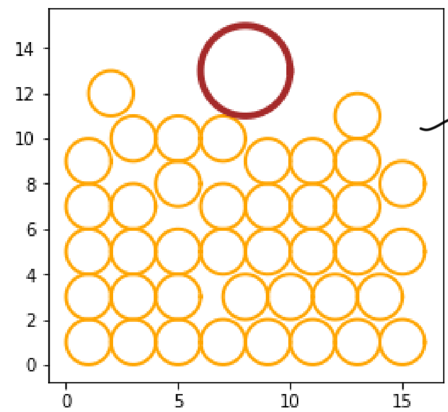


(a) (b) (c)

Fig.12 the Brown fully filed disks are Brazil nuts, empty disks are muesli. The square has length 20 units which is the new local system that was defined earlier (a) the Brazil nut rise to the top if the space below is occupied (b)if the upper and lower space are occupied, the Brazil nut can full fill the center. (c) The Brazil nut will go down to the bottom if the space above it is occupied.

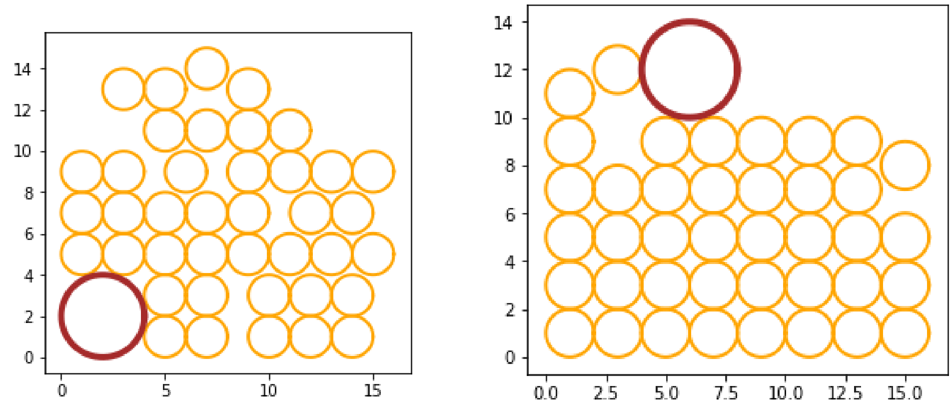
In fig.12, smaller particles were ignored, because they were at the bottom or attached to the side walls most of the time. A reasonable guess for the convection inside of this new local system would be the fact that bigger sized disks has the tendency to flow upwards due to size segregation. On account of the limited size of the local system, the particles at the top were pushed downward slowly by the other risen particles. Due to size segregation, all the particles strictly follow a specific pattern which can maximize their chances to flow upwards. The behavior of Brazil nut might be a result of the phase differences between these motion patterns.

D. Relationship of segregation rate and lift distance

Lifting constant	**Initial global potential energy equilibrium**	**Final state after multiple 'shakes'**
1.5	 (a)	 (b)
2.0		

(c) (d)

2.5



(e) (f)

Fig.13 40 muesli and one Brazil nut with radius ratio 1:2 sitting on the bottom of the combiner. (a)(b) The initial and final configuration after 10 “shakes” which has lift constant equals to 1.5. (c)(d) The initial and final configuration after 4 “shakes” which has lifting constant equals to 2.0. (e)(f) The initial and final configuration after 3 “shakes” which has lifting constant equals to 2.5

The Brazil nut was not able to rise to the surface when the lift constant is equal to 1.5, meaning the heights were multiplied by a factor of 1.5 to simulate the process of shaking. Owing to the low lifting constant, the displacement for each particle after shaking was pretty small. In fig.13 (b), the voids created underneath the Brazil nut can fit no more than one muesli. Since the Brazil nut was close to the bottom, therefore it is unlikely for a muesli to fill up the gap under the Brazil nut to lift it upwards.
Comparing the cases when the lifting constant was set to be 2.0 and 2.5, the one had lifting constant equaled to 2.5 took less “shakes” to reach the same amount of height. According to the equation (4), displacement per “shake” for the one with higher lifting constant will be larger, meaning a higher rate of size segregation.
For the same initial condition, the higher the lifting constant was set, the less shaking processes were needed to have size segregation occurring which results in a higher rate of size segregation. This is because the displacement of the Brazil nut increases with lifting constant, so more steps will be required for the nut to rise.

4. Discussion and conclusion

Conclusion

Size segregation, a model which explain the cause of BNE was computationally simulated by using Monte Carlo method in this paper.
Particles were set to have comparable sizes in this paper to investigate the size segregation. The intention was to eliminate the occurrence of sifting which gives rise to Brazil nut effect without the process of shaking. Size segregation could well model and explain most of the cases in this paper such as the binary system, behavior of the Brazil nut in a distribution of particles with smaller sizes.
In those cases, Brazil nuts were the biggest particles in the system and all of particles had experienced random motions. Size segregation occurs when smaller particles filled up the voids beneath the Brazil nut in a random manner while shaking, until Brazil nuts rose to the top. The whole size segregation process can be explained geometrically, so mass or density is not a factor for this model.
However, the behavior of the Brazil nut was no longer random when a range of bigger sized particles were poured into the box. The situation which the surface of the material was fully occupied by those bigger particles after shaking, can be treated as having a strong friction applied to the system. Studied the motion of the Brazil nut in a new local system underneath the bigger sized particles, its motion was modeled by a convection theory.
A further investigation in finding the rate of the size segregation was well completed. By conducting a considerable amount of tests, size of the particle, lift distance after each shaking process were proven to be the two major factor which have impact on the rate of size segregation. The two conclusions are as follows:
(1) When the radius ratio between a kind of particle and a brazil nut is less than or equal to one, the Brazil nut

will rise to the surface. Decreasing the radius ratio between two kinds of particles can cause a decline in the rate of segregation. When the ratio is one, the rate of size segregation is undermined since the Brazil nut has an equal probability of being lifted upwards to the others. However, the Brazil nut cannot rise to the surface when the radius ratio is greater than 1.

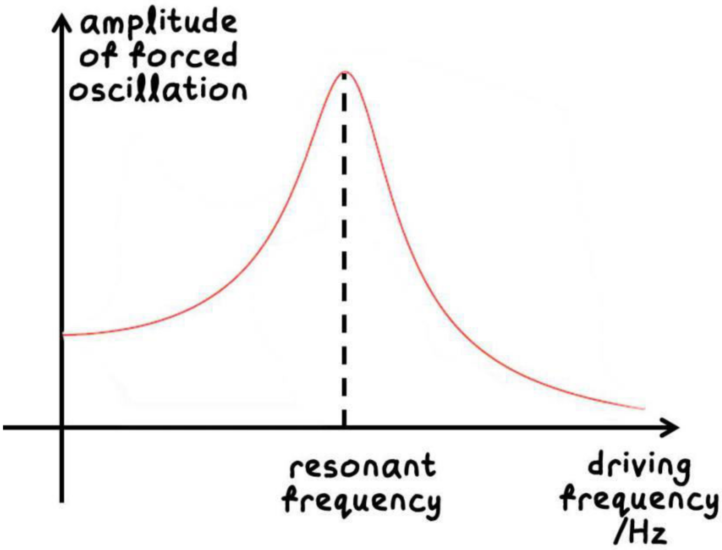
(2) Increasing lift distance can achieve a higher rate of size segregation. When he lift constant is equal to 1.5, BNE will not occur.

Further work

1. Shaking frequency

In this paper, I have multiply the height of each particle by 2 to simulate shaking process.

From observation, the frequency of shaking can affect the rate of size segregation in reality. For a given amount of particles, the change in frequency of shaking can result in the amplitude of particle vibration.



[5]

Fig.13 a graph of driving frequency against amplitude of oscillation. At resonant frequency, the amplitude of oscillation is maximized.

The highest lifting constant means the highest shaking amplitude which occurs at resonant shaking frequency. In the further work, how driving frequency affects the rate of size segregation could be conducted. The model in this paper can determine the rate of size segregation corresponding to lifting constant, thus the relationship between driving frequency and the size segregation rate can be easily plotted.

2. Density as a factor

The density of the particle was independent with size segregation. However, density of the particle is an important factor in BNE. For instance, the particle density was taken into account in Granular convection.

3. Granular convection

Granular convection, another important theory to model Brazil nut effect wasn't being investigated in this paper. Size segregation by Monte Carlo method does not take friction into account, thus random motion of particles can still lead BNE to occur even if some of the instant motions are unrealistic. Convection simulates the trajectory of particles and a convection flow inside of a box under the condition that friction is a major factor to cause BNE.

4. Reverse Brazil nut effect (RBNE)

Moreover, applying convection theory to a packet of particles in different shaped box could obtain Reverse Brazil nut effect (RBNE). [6]

Well written and stimulating. Impressive!

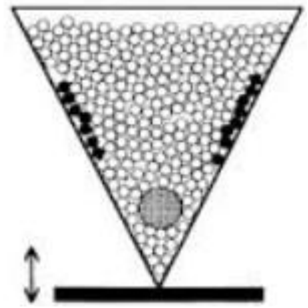


Fig.14 [7] RBNE in a triangular shaped container after shaking

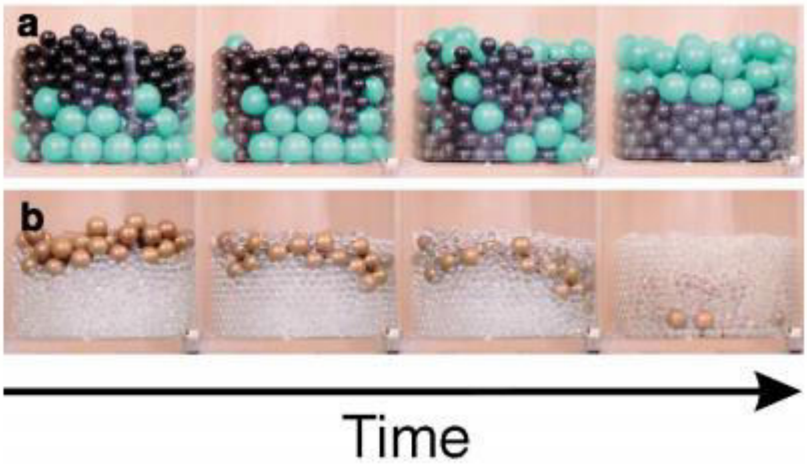


Fig.15 [6] (a) is BNE, (b) is RBNE where the bigger sized particles sink to the bottom

4. Oscillating box in horizontal or varied orientations

The box was only shacked in one direction in this paper. For future study, a relationship between the distribution of particles and oscillation orientation can be investigated.

5. Shape of the particles

The last one can change the shape of the particles in the system to investigate the type of shapes which has the highest rate of size segregation.

5. Reference

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(4) Rietz, Frank; Stannarius, Ralf (2008). "On the brink of jamming: Granular convection in densely filled containers". *Physical Review Letters*. **100** (7): 078002. arXiv:1706.04978. Bibcode:2008PhRvL.100g8002R. doi:10.1103/PhysRevLett.100.078002. PMID 18352597

(5) http://physicsnet.co.uk/a-level-physics-as-a2/further-mechanics/forced-vibrations-resonance/

(6) Reversing the Brazil-Nut Effect: Competition between Percolation and Condensation, A. P. J. Breu, H.-M. Ensner, C. A. Kruelle, and I. Rehberg, Phys. Rev. Lett. 10 JANUARY 2003

(7) http://www.damtp.cam.ac.uk/user/nv253/teaching/Lecture7.pdf

Appendix A

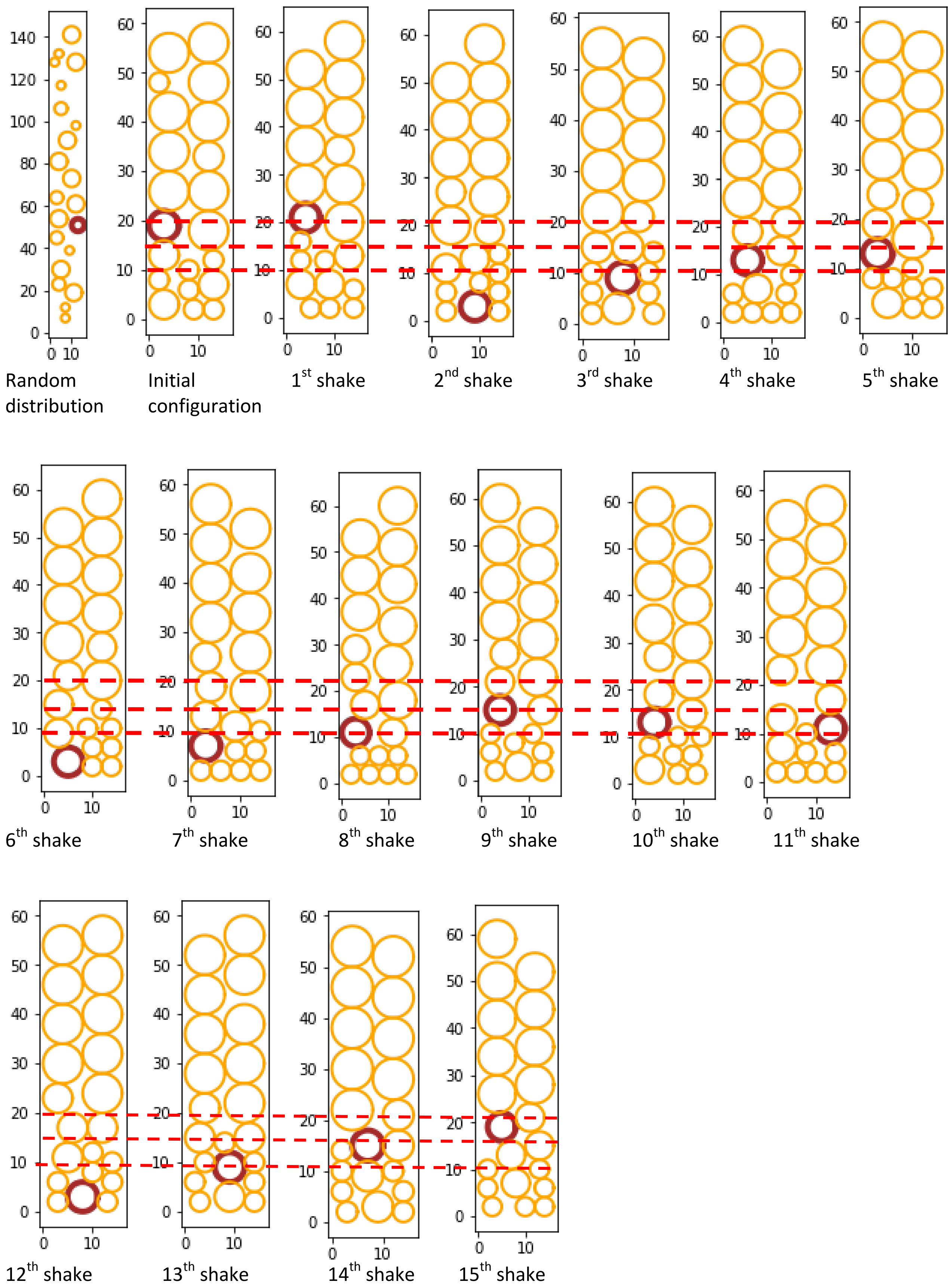
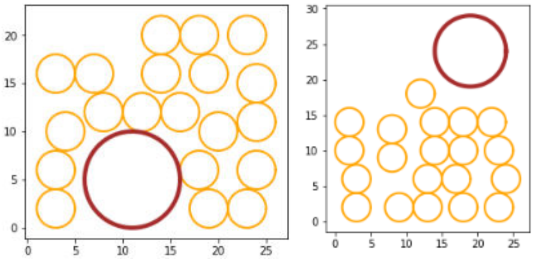


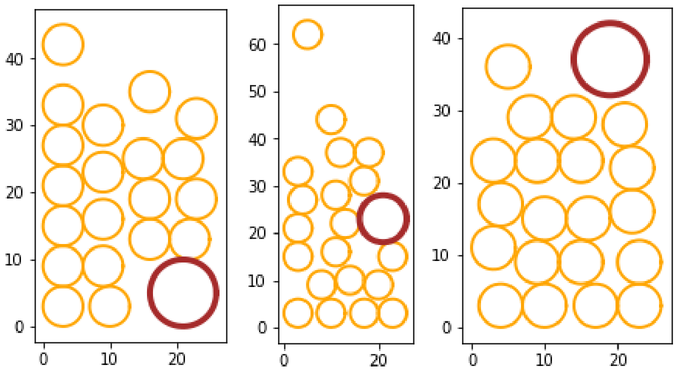
Fig.16 Full 15 “shakes” of fig.11 between (b) and (d), three red lines across the container horizontally with vertical coordinate, 10 units, 15 units, 20 units

Appendix B



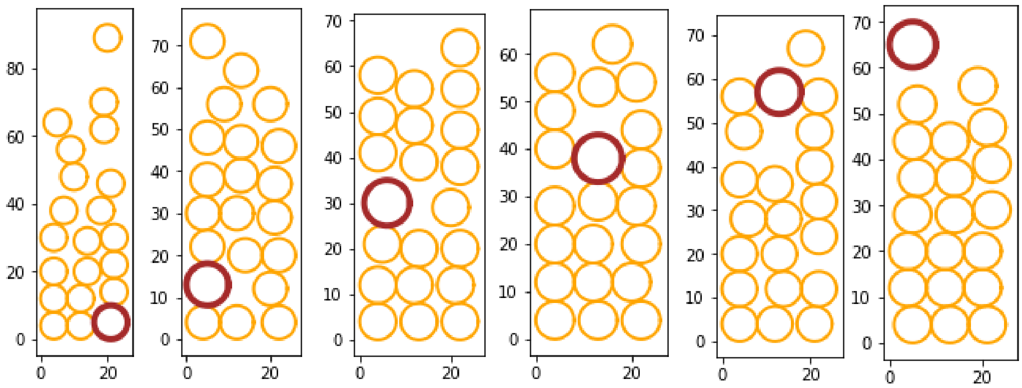
(a) (b)

Fig.17 the diagrams showed the behaviour of a brazil nut and twenty equally sized particles in a box with width 25 units. Radius ratio between the particle and Brazil nut is 2:5. (a) The initial configuration without shaking. (b) The Brazil nut reached the top after 1 “shake”



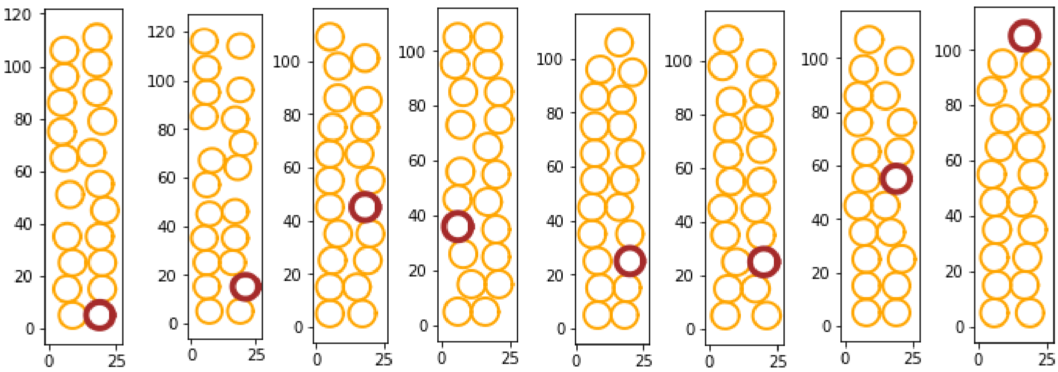
(a) (b) (c)

Fig.18 the diagrams showed the behaviour of a brazil nut and twenty equally sized particles in a box with width 25 units. Radius ratio between the particle and Brazil nut is 3:5. (a) The initial configuration without shaking. (b) 1 “shake”(c) The Brazil nut reached the top after 2“shake”



(a) (b) (c) (d) (e) (f)

Fig.19 the diagrams showed the behaviour of a brazil nut and twenty equally sized particles in a box with width 25 units. Radius ratio between the particle and Brazil nut is 4:5. (a) The initial configuration without shaking. (b)-(e) 1-4 “shake”(c) The Brazil nut reached the top after 5“shake”



(a) (b) (c) (d) (e) (f) (g) (h)

Fig.20 the diagrams showed the behaviour of a brazil nut and twenty equally sized particles in a box with width 25 units. Radius ratio between the particle and Brazil nut is 1:1. (a) The initial configuration without shaking. (b)-(g) 10,20,30,40,50,100 “shake”(c) The Brazil nut reached the top after 114 “shake”