Inverse Transform Method

Example

- Let
$$X \sim p(x) = \begin{cases} 2, & 0 < x < 1/4 \\ 2/3, & 1/4 < x < 1 \end{cases}$$
Evaluate $\mathbb{E}[X]$ by OMC.

Pseudocode

procedure INTEGRAL(N) ▷ N is total number of samples
    $s \leftarrow 0$ ▷ s is the sum of samples
    for $i = 1 \ldots N$ **do**
        generate $Y \sim p$
        $s \leftarrow s + Y$
    return $\frac{s}{N}$ ▷ return the average

- (Q.) How to generate $Y \sim p$?

Inverse Transform Method - 1

- (Prop) Suppose X has its CDF F and its inverse F^{-1} exists, then $F^{-1}(U) \sim X$, where $U \sim U(0,1)$.
- (proof)

$$\mathbb{P}(F^{-1}(U) \leq x) = \mathbb{P}(U \leq F(x)) = F(x).$$

Inverse Transform Method - 2

- Inverse transform method provides exact sampling as long as the inverse of CDF is explicitly available.
- ITM sample generation for $X \sim F$ given F^{-1}

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procedure ISINTEGRAL(F^{-1})
    generate $Y \sim U(0, 1)$ ▷ use *numpy.random.uniform*
    $X = F^{-1}(Y)$
    return X

HW

Let

$$X \sim p(x) = \begin{cases} 2, & 0 < x < 1/4 \\ 2/3, & 1/4 < x < 1 \end{cases}$$

- Evaluate its expectation
- Implement OMC to estimate its expectation.