

Let $D = \{X_i : i \in \mathbb{N}\}$ be a data set of iid sequence from a random generator of distribution $\mathcal{N}(b, \sigma^2)$ for some unknown parameters b and σ . Our goal is to estimate b using so called stochastic approximation (SA) with a given learning rate $\alpha \in (0, 1)$:

- initialize b_0
- iterate $b_{n+1} = b_n + \alpha(x_k - b_k)$.

We want to examine the convergence $b_n \rightarrow b$. For simplicity, let's fix $\alpha = 0.01$.

1. Write pseudocode for SA.
2. Generate a data set $D = \{x_i : 1 \leq i \leq 10000\}$ with $\mathcal{N}(1, 4)$.
3. Write a code to implement SA on D , and demonstrate $b_n \rightarrow b$ as $n \rightarrow \infty$.
4. Prove that $\lim_n \mathbb{E}b_n = b$.
5. Can you prove or disprove that $b_n \rightarrow b$ in L^2 ?