

Consider 2-d PDE

$$\frac{1}{2}\Delta v(x) - v(x) + x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2} = 0, \quad x \in O = (0, 1)^2$$

with its boundary data

$$v(x) = (x_1 - \frac{1}{2})^2 + (x_2 - \frac{1}{2})^2, \quad x \notin O.$$

- Show that exact solution is

$$v(x) = (x_1 - \frac{1}{2})^2 + (x_2 - \frac{1}{2})^2.$$

- Identify  $\gamma, \ell^h, p^h$  in the CFD solution given by

$$v(x) = \gamma \left\{ \ell^h(x) + \sum_{i=1}^d p^h(x + he_i|x)v(x + he_i) + p^h(x - he_i|x)v(x - he_i) \right\}.$$

- For  $\{h = 2^{-i}, i = 2, 3, 4, 5\}$ , compute CFD solution and find maxnorm of error.