Asset price under $\mathbb{Q}$ follows

$$S_t = S_0 \exp\{\mu t + \sigma W_t\}.$$

Consider Digital put with its payoff

$$h(S_T) = I(S_T < S_0 e^{-b}).$$

We want to find the forward price:

$$v = \mathbb{E}^{\mathbb{Q}}[h(S_T)].$$

Parameters are given as

$$r = .03, \sigma = .2, \mu = r - \frac{1}{2}\sigma^2 = .01, T = 1, b = .39.$$

- Prove that the exact price is 0.02275.
- Use OMC find the price
- Use IS(α) find the price.
- Can you show your approach is optimal?
- Prove or demonstrate IS is more efficient to OMC.