

Consider 2-d PDE

$$\frac{1}{2}\Delta v(x) - v(x) + x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2} = 0, \quad x \in O = (0, 1)^2$$

with its boundary data

$$v(x) = (x_1 - \frac{1}{2})^2 + (x_2 - \frac{1}{2})^2, \quad x \notin O.$$

The exact solution is

$$v(x) = (x_1 - \frac{1}{2})^2 + (x_2 - \frac{1}{2})^2.$$

- Identify MRP with CFD in the form of

$$v(x) = \gamma \left\{ \ell^h(x) + \sum_{i=1}^d p^h(x + he_i|x)v(x + he_i) + p^h(x - he_i|x)v(x - he_i) \right\}.$$

- For  $h = 1/8$ , compute CFD solution by value iteration.
- For  $h = 1/8$ , compute CFD solution by Monte-Carlo method.
- For  $h = 1/8$ , compute CFD solution by TD method.
- Compare above three methods and conclude your observations.