MA 503 TEST 2 Name:__________________________

Professor Larsen
December 9, 2020

1. Show that if $f_n, f\colon E \to [0,\infty)$ are measurable with $m(E) < \infty$ and $f_n \to f$ uniformly, then

$$\int f_n \to \int f.$$

2. Suppose $f \in L^1([0,1])$ with $|f| \leq 1$ a.e. Find, with proof,

$$\lim_{n\to\infty} \int |f|^n.$$

3. Let $f, g \in L^2(\mathbb{R})$. Set $f_n(x) := \frac{1}{n} f(x+n)$, $n \in \mathbb{N}$. Show that

$$\int f_n g \to 0.$$