exercise 1:
Let $S$ be the square $[0,1]\times[0,1]$ and $f$ be the function defined almost everywhere on $S$ by the formula $f(x,y)=\dfrac{x^2-y^2}{(x^2+y^2)^2}$. Show by a direct calculation that $\int_S |f| = +\infty$.

exercise 2:
Using Fubini's theorem find

$$\int_0^\infty \frac{e^{-at}-e^{-bt}}{t}dt,$$

for $a>0, b>0$.

exercise 3:
Let $A=[0,1]$ and $B=[b_1,b_2]$ be the intervals in $\mathbb{R}$ such that $b_1<b_2$.
(i). Setting $F(z)=\int_{-\infty}^{z} 1_B$, sketch the graph of $F$.
(ii). Show that $1_A * 1_B$ is piecewise linear, continuous, and has compact support (you may want to use the function $F$). Sketch the graph of $1_A * 1_B$.

exercise 4:
Let $f$ and $g$ be two measurable functions on $\mathbb{R}^d$ valued in $[0,\infty]$. Let $A$ and $B$ be two measurable subsets of $\mathbb{R}^d$ such that if $x\notin A$, $f(x)=0$, and if $x\notin B$, $g(x)=0$. Show that if $x\notin (A+B)$, $f*g(x)=0$.

exercise 5:
(i). Let $X$ be a metric space and $f$ and $g$ to continuous functions on $X$ valued in $\mathbb{R}$. Show that $\min(f,g)$ and $\max(f,g)$ are continuous functions on $X$ . **Hint:** simplify $\max(f,g)+\min(f,g)$ and $\max(f,g)-\min(f,g)$.
(ii). Let $f$ be in $L^1(\mathbb{R}^d)\cap L^\infty(\mathbb{R}^d)$. Show that there is a sequence $f_n$ in $C_c(\mathbb{R}^d)$ such that $f_n(x)$ converges to $f(x)$ for almost all $x$ in $\mathbb{R}^d$, $\int |f_n-f|$ converges to zero and $|f_n(x)|\le \|f\|_\infty$, for all $x$ in $\mathbb{R}^d$.
(iii). With $f_n$ and $f$ as in the previous question and $g$ in $L^1(\mathbb{R}^d)$ show that $\int f_n(x-y)g(y)dy$ converges to $\int f(x-y)g(y)dy$ for almost all $x$ in $\mathbb{R}^d$.

exercise 6:
Let $(X,\mathcal{A},\mu)$ be a finite measure space. Let $f_n$ be a sequence of measurable functions on $X$ and $f$ a measurable function on $X$. We say that $f_n$ converges to $f$ in measure if for every positive $\epsilon$,

$$\mu(\{x\in X : |f_n(x)-f(x)|>\epsilon\})$$

converges to zero.
(i). Show that if $f_n$ converges to $f$ almost everywhere, then $f_n$ converges to $f$ in measure.

Is the converse true?
(ii). Show that if $f_n$ converges to $f$ in $L^1$ norm, then $f_n$ converges to $f$ in measure. Is the converse true?