MA 503 TEST 1 Name:_____

Professor Larsen
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1. Show that if $f: \mathbb{R} \to \mathbb{R}$ satisfies

$$\forall \alpha \in \mathbb{Q}, \{x : f(x) > \alpha\} \text{ is measurable,}$$

then f is measurable.

2. Show that if $S \subset C \subset \mathbb{R}$ and C is closed, then $\bar{S} \subset C$.

3. Show that if $f\colon \mathbb{R} \to [0, \infty)$ is continuous and $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$, then f has a maximum.