MA 503 TEST 2 Name:__________________________

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1. Suppose f_n, g_n are measurable, $f_n \to f$ a.e., $g_n \to g$ a.e., $|f_n| \leq M$ for all $n \in \mathbb{N}$ and some $M > 0$, and $\int |g_n| \to \int |g| < \infty$. Show that

$$\int f_n g_n \to \int fg.$$

2. Show that if $f \in L^1(\mathbb{R}) \cap L^\infty(\mathbb{R})$, then $f \in L^p(\mathbb{R})$ $\forall p \in (1, \infty)$.
(Hint: write $\mathbb{R} = A \cup A^c$, where $A := \{x \in \mathbb{R} : |f(x)| \leq 1\}$).

3. Show that if $f_n \to f$ in $L^p(\mathbb{R})$ for some $p \in [1, \infty)$, then $\forall \varepsilon > 0\ \exists N \in \mathbb{N}$ such that

$$\int_{[-N,N]^c} |f_n|^p < \varepsilon$$

for all $n \geq N$.