exercise 1:
Let x and y be in $\mathbb{R}^d$. Assume that $x \neq 0$. Set $e = \frac{x}{\|x\|}$, $P(y) =< y, e > e$, and $z = y - P(y)$.
(i). Show that $< z, P(y) >= 0$.
(ii). Show that $\|P(y)\| \leq \|y\|$.
(iii). Infer the Cauchy Schwartz inequality.

exercise 2:
Set for $x = (x_1, ..., x_d)$ in $\mathbb{R}^d$,

$$\|x\|_1 = \sum_{i=1}^{d} |x_i|, \quad \|x\|_\infty = \max\{|x_i| : i = 1, .., d\}$$

Show that ρ_1 and ρ_∞ define two distances on $\mathbb{R}^d$, where

$$\rho_1(x, y) = \|x - y\|_1, \quad \rho_\infty(x, y) = \|x - y\|_\infty$$

exercise 3:
Solve exercise 9.6 from your textbook. Infer the following result:
if (X, ρ) is a metric space and we set for all x and y in X,

$$d(x, y) = \frac{\rho(x, y)}{1 + \rho(x, y)}$$

then d is a distance on X.

exercise 4:
Let x_n be a convergent sequence in the metric space (X, ρ). Show that the limit of x_n is unique.

exercise 5:
Let (X, ρ) be a metric space. Suppose that X is a finite set. Show that any subset of X is both open and closed.

exercise 6:
Let (X, ρ) be a metric space. Let A be a subset of X. Show that V is an open subset of A if and only if there is an open subset W of X such that $V = A \cap W$.