(Exercise /Fall 2003 Final)
Suppose f_n are measurable, $f_n \geq 0$, $f_n \to f$ a.e., and $f_n < f$ for all $n \in \mathbb{N}$. Prove or find a counterexample:

$$\int f_n \to \int f$$

(Exercise /Fall 2003 Final)
Find a sequence $\{f_n\} \subset L^1[0,1]$ such that $||f_n||_2 = 1$ and $f_n \to 0$ a.e.

(Exercise /Fall 2003 Final)
Suppose $\{f_n\}$ is a sequence of measurable functions that is equiintegrable, i.e., $\forall \epsilon > 0$, $\exists \delta > 0$ such that $m(A) < \delta \Longrightarrow \int_A |f_n| < \epsilon$ for all $n \in \mathbb{N}$. Show that if $m(E) < \infty$ and $f_n \to f$ a.e., then

$$\int_E f_n \to \int_E f$$

(Exercise /Fall 2003 Final)
Show that if $f_n \to f$ a.e. and $\int |f_n| \to \int |f| < \infty$, then $\{f_n\}$ is equiintegrable.