exercise 1:
Let X be a set and f and g two functions from X to $\overline{\mathbb{R}}$ such that $f(x) + g(x)$ is well defined for all x in X. Show that for any t in $\mathbb{R}$

$$\{x \in X : f(x) + g(x) < t\} = \bigcup_{q \in \mathbb{Q}} \{x \in X : f(x) < t - q\} \cap \{x \in X : g(x) < q\}$$

Infer that if $(X, \mathcal{A}, \mu)$ is a measure space and f and g are measurable functions from X to $\overline{\mathbb{R}}$ such that $f(x) + g(x)$ is well defined for all x in X, then $f + g$ is measurable.

exercise 2:
Let u_n be a sequence valued in $\overline{\mathbb{R}}$. Using the definition of $\liminf$ and of $\limsup$ given in class, show that $\liminf u_n \leq \limsup u_n$. Show that u_n converges in $\overline{\mathbb{R}}$ if and only if $\liminf u_n \geq \limsup u_n$.

exercise 3:
(i). Let f be a simple function from the measure space $(X, \mathcal{A}, \mu)$ to $[0, \infty]$. Define $\nu(A) = \int_A f$, for all A in $\mathcal{A}$. Show (by direct calculations) that ν is a measure on $(X, \mathcal{A})$.
(ii). Using the M.C.T. show that the result is still true if f is measurable and valued in $[0, \infty]$.

exercise 4:
(i). Compute $\int \liminf f_n$ and $\liminf \int f_n$ for the sequence of functions from $\mathbb{R}$ to $\mathbb{R}$ $f_n = 1_{[n,n+1]}$.
(ii). Find a sequence of measurable functions f_n from $[0, 1]$ to $[0, \infty]$ such that

$$\int \liminf f_n \neq \liminf \int f_n$$

(iii). Find a sequence of measurable functions f_n from $\mathbb{R}$ to $[0, \infty]$ such that f_n converges uniformly to zero and

$$\int \liminf f_n \neq \liminf \int f_n$$

exercise 5:
3.1.1.