MA 503 TEST 1 Name:__________________________

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October 31, 2014

1. Show that if $m(A) = 0$, then $m^*(A \cup B) = m^*(B)$ for all $B \subset \mathbb{R}$.

2. For $E \subset \mathbb{R}$, $E \neq \emptyset$, and $x \in \mathbb{R}$, define $d(x, E) := \inf\{|x - y| : y \in E\}$. Show that for every $E \subset \mathbb{R}, E \neq \emptyset$, the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) := d(x, E)$ is measurable.

3. Show that if $f\colon E \to \mathbb{R}$ is measurable with $m(E) < \infty$, then $\forall \varepsilon > 0$, $\exists A \subset E$ with $m(A) < \varepsilon$ such that f is bounded on $E \setminus A$.

4. Define $\delta_0 \colon \mathcal{P}(\mathbb{R}) \to [0, \infty]$ by

$$\delta_0(A) := \begin{cases} 1 & \text{if } 0 \in A \\ 0 & \text{if } 0 \notin A. \end{cases}$$

Show that δ_0 is a (countably additive) measure on $\mathcal{P}(\mathbb{R})$ (the power set of $\mathbb{R}$).

5. Prove or give a counterexample: if f^2 is measurable, then f is measurable.