exercise 1:
(i). Let $f : [0,1] \to \mathbb{R}$, $f(x) = \sqrt{x}$. Show that f is uniformly continuous but not Lipschitz continuous.
(ii). Let $g : (0,1) \to \mathbb{R}$, $g(x) = \frac{1}{x}$. Show that g is not uniformly continuous.

exercise 2:
Find a function $f : \mathbb{R} \to \mathbb{R}$ which is continuous, bounded, but not uniformly continuous.

exercise 3:
Let X be a non-empty set and $B(X)$ be the space of bounded functions from X to $\mathbb{R}$ with the metric

$$\rho(f,g) = \sup_{x \in X} |f(x) - g(x)|.$$

Show that $B(X)$ is complete.

exercise 4:
Let (X, ρ) be a metric space and x_n be a sequence in X. Assume that there is a sequence a_n in $\mathbb{R}$ such that $\rho(x_{n+1}, x_n) \leq a_n$ and $\sum a_n$ converges. Show that x_n is a Cauchy sequence.

exercise 5:
Let (X, d) be a metric space and A a non empty subset of X. Define

$$d(x, A) = \inf\{d(x,a) : a \in A\}.$$

Show that A is closed if and only if $A = \{x \in X : d(x, A) = 0\}$.

exercise 6:
Let (X, d) be a metric space and A a non empty subset of X. Define

$$d(x, A) = \inf\{d(x,a) : a \in A\}.$$

Show that $f : X \to \mathbb{R}^+, f(x) = d(x, A)$ is Lipschitz continuous.

exercise 7:
Exercise 9.3.33 (from your textbook).

exercise 8:
Let L be a linear function between the two normed spaces (V_1, N_1) and (V_2, N_2). Show that the following conditions are equivalent:

(i). L is continuous at 0.
(ii). L is Lipschitz continuous.
(iii). $\exists C > 0$, $\forall x \in V_1$, $N_2(Lx) \leq CN_1(x)$.

<u>exercise 9</u>:
Let A and B be two subsets of $\mathbb{R}^d$.
(i). If A is closed and B is compact, show that $A + B$ is closed.
(ii). If A is closed and B is closed, is $A + B$ is closed?

<u>exercise 10</u>:
Let X be the space $[0, 1)$ equipped with its usual metric. Find a cover of X by open sets which does NOT have a finite subcover.

<u>exercise 11</u>:
Let $S = \{x \in l^2 : \|x\| = 1\}$.
(i). Show that S is closed and bounded.
(ii). Find with proof $\epsilon > 0$ such that S cannot be covered by finitely many balls with radius ϵ.