July 27, 2021

Content: Solvability of McKean-Vlasov FBSDEs by Schauder's theorem

- The equivalence of two norms in Hölder space
- The full-fledged McKean-Vlasov FBSDE model (4.32)
- Assumptions: Nondegenerate MKV FBSDE
- Idea to show the existence
- Some preparation results:
  - Lemma 4.33: Existence result of FBSDE; the regularity of decoupling field
  - Lemma 4.34 and remark 4.35: the senstivity of the FBSDE; the bound of W_2 distance between two given measures

Questions:

- Prove the inequality: for $x, y > 0$ and $\gamma > 1$, we have $(x+y)^\gamma \geq x^\gamma + y^\gamma$.
- (Carmona, Delarue), Page 245. Assumption (Nondegenerate MKV FBSDE): why we need the growth condition with respect to the square root of the second moment of the measure? And why the growth condition of F is independent to the law of X?
- (Carmona, Delarue), Page 247. Lemma 4.33: why the decoupling field u is Lipschitz to x? In the assumption (A2), F is linear growth w.r.t. y, z.

August 03, 2021

Content: Solvability of McKean-Vlasov FBSDEs by Schauder's theorem. Fixed point argument in some space under the bounded case.

- The assumptions and spaces.
- Main result: Lemma 4.35 and Proposition 4.37 (Carmona, Delarue).
- The proof of Lemma 4.35.

Questions:

- For any bounded continuous function $\varphi : [0,T] \times \mathbb{R} \to \mathbb{R}$ and for any flow of probability measures $\boldsymbol{\mu} = (\mu_t)_{0\leq t\leq T} \in C([0,T]; \mathcal{P}_2(\mathbb{R}))$. Show that the map $[0,T] \ni t \mapsto \varphi(t,\cdot) \diamond \mu_t \in \mathcal{P}(\mathbb{R}\times\mathbb{R})$ is continuous.
- Why we need to use $\mathcal{M}^1_f(\mathbb{R})$ rather than $\mathcal{P}_1(\mathbb{R})$? In the Schauder's fixed point theorem, $(V, \|\cdot\|)$ is a normed linear vector space.
- The distance induced by the Kantorovich-Rubinstein norm $\|\cdot\|_{KR*}$ coincides with the 1-Wasserstein metric W_1.
- Let

  $$\mathcal{E} = \left\{ \boldsymbol{\mu} \in C([0,T]; \mathcal{P}_4(\mathbb{R})) : \sup_{t\in[0,T]} \int_{\mathbb{R}} |x|^4 \, d\mu_t(x) \leq \gamma \right\},$$

  where γ is a constant. Let $(\boldsymbol{\mu}^n)_{n\geq 1}$ be a sequence in $\mathcal{E}$. Show that the measures $(\mu^n_t)_{n\geq 1}$ are tightness for any $t \in [0,T]$.