Project 6: High-Dynamic Range Imaging

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Abstract

We Implemented Debevec-Malik method of recovering high dynamic range radiance maps from photographs taken with different amounts of exposure. Once radiance maps was computed, we could compress multiple photographs into one low dynamic range gamut by tone mapping. We first demonstrated the algorithm with known exposure time and aligned image, then we dropped the the assumption that exposure time was known. Finally, we extended the method to images taken without tripods using homography-based registration of the sequence.

Introduction

Modern cameras cannot capture as many details as the human eye can see in any given scene, especially under challenging light conditions. Photographic sensor or film can only capture a range of radiance value that is less than the nature world contains. To recover a full dynamic range image, people can take a set of bracketed exposures photographs. Because the mapping between rediance to pixal value is non-linear and unknonw, the challenge is that how we can combine these images together.

We used Debevec-Malik method to estimate radiometric response function and irradiance value together. Since it is expensive and unnessarly to estimate all pixels, we chose random sampling, uniform sampling and variance-driven sampling and compared their result. Then, we recovered the radiance map by blending pixels from diffrent exposures with the weight function proposed by Mitsunaga and Nayar(1999), which maximized the signal-to-noise ratio. Finally, we use both global mapping and local mapping to display the radiance map on a lower dynamic range.

Even if Debevec-Malik method assume that exposure time is known, we could also estimate exposure time along with radiometric response function by adding constraint terms, as long as we knew the relative relationship of exposure among images. To make our approach more general, we could also pre-process multi-exposure images taken without tripod by homography transform. Furthermore, it is possible to extend it to HDR panorama.

The project is structured as following:

1.0 The Algorithm

- 1.1 Estimate the radiometric response function from the aligned images
- 1.2 Estimate a radiance map by selecting of blending pixels from different exposures
- 1.3 Tone map the resulting high dynamic range (HDR) image back into a displayable gamut

2.0 Extension:

- 2.1 Drop the assumption that exposure is known
- 2.2 Images taken without tripod

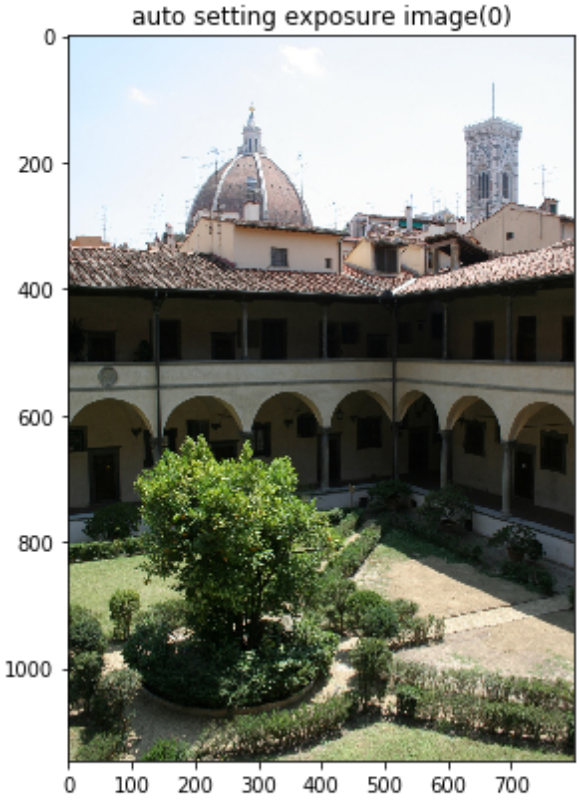
Contributions

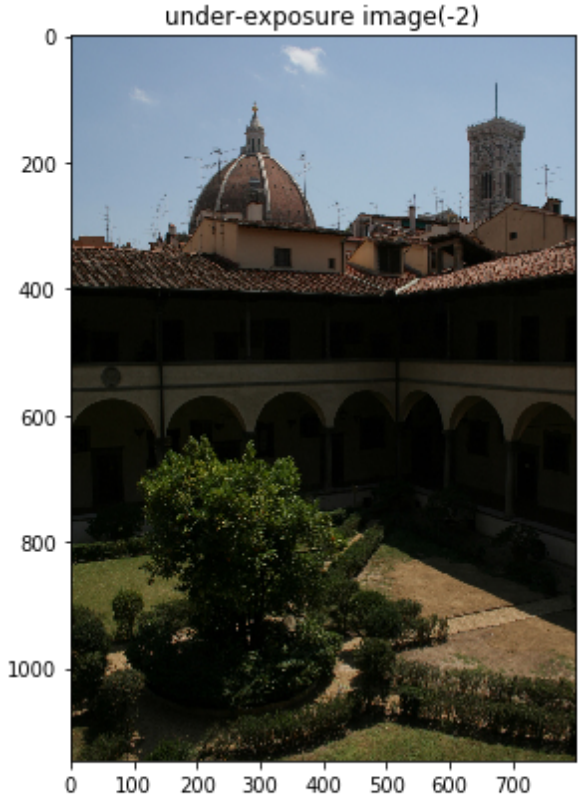
Author	Contribution
Jiamin Shi 20649720	1.1(estimate response function), 1.3, 2.1, 2.2, abstract&introduction, pipeline integration
Benjamin M. Winger 20571047	1.2(improvement of weight function), version control configuration, part of the conclusion and various misc fixes
Jingye Xu 20705168	1.2, 1.1(implementation/analysis sample methods)

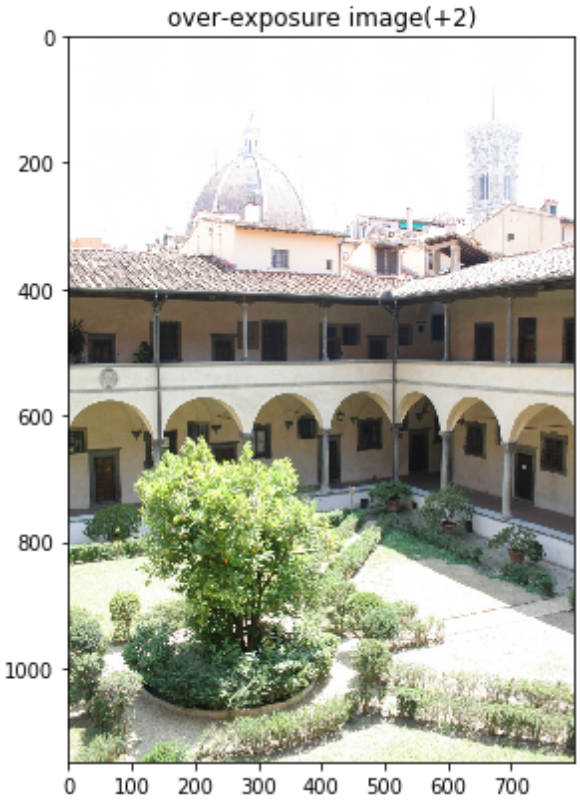
1. The Algorithm

In [1]:
```
import numpy as np

import skimage
from skimage import color
from skimage.util import img_as_ubyte
from skimage.transform import warp, ProjectiveTransform, SimilarityTrans
form
from skimage.color import rgb2gray

import matplotlib.image as image
import matplotlib.pyplot as plt

import warnings
warnings.filterwarnings('ignore')
```

In [2]:
```
# Load images
# source: https://www.easyhdr.com/examples
im_auto_expose = image.imread("images/laurenziana_0.jpg")
im_under_expose = image.imread("images/laurenziana_-2.jpg")
im_over_expose = image.imread("images/laurenziana_+2.jpg")
im_shape = im_auto_expose.shape[0:2]

imagebracket = np.array([im_under_expose,im_auto_expose,im_over_expose])

log_t = [-2,0,2]

# plot
fig = plt.figure(figsize=(10,16))
plt.subplot(221)
plt.imshow(im_auto_expose)
plt.title("auto setting exposure image(0)")
plt.subplot(222)
plt.imshow(im_under_expose)
plt.title("under-exposure image(-2)")
plt.subplot(223)
plt.imshow(im_over_expose)
plt.title("over-exposure image(+2)")
plt.show()
```


auto setting exposure image(0)
0
200
400
600
800
1000
0
100
200
300
400
500
600
700


under-exposure image(-2)
0
200
400
600
800
1000
0
100
200
300
400
500
600
700


over-exposure image(+2)
0
200
400
600
800
1000
0
100
200
300
400
500
600
700

1.1 Estimate the radiometric response function from the aligned images

Estimate irradiance values E_i and the radiometric response function f at the same time.

$$z_{ij} = f(E_i, t_j)$$

where t_j is the exposure time for the jth image.

The inverse response curve f^{-1} is given by

$$f^{-1}(z_{ij}) = E_i t_j$$

Taking logarithms of both sides

$$g(z_{ij}) = \log f^{-1}(z_{ij}) = \log E_i + \log t_j$$

(g maps pixel values z_{ij} into log irradiance)

Also, we need make the reponse curve smooth by adding a second-order smoothness constraint

$$\lambda \sum_k g''(k)^2 = \lambda \sum [g(k-1) - 2g(k) + g(k+1)]^2$$

Since pixel values are more reliable in the middile of their range, they also add a weight function

$$w(z) = \begin{cases} z - z_{min} & z \le (z_{min} + z_{max})/2 \\ z_{max} - z & z > (z_{min} + z_{max})/2 \end{cases}$$

Put all together we get a least squares problem to estimate the radiometric response function g and irradiance values E_i

$$E = \sum_i \sum_j w(z_{i,j})[g(z_{i,j}) - \log E_i - \log t_j]^2 + \lambda \sum_k g''(k)^2$$

In other word, we are solve the two equations

$$w(z_{i,j})g(z_{i,j}) - w(z_{i,j}) \log E_i = w(z_{i,j}) \log t_j$$
$$\lambda[g(z_{i,j} - 1) - 2g(z_{i,j}) + g(z_{i,j} + 1)] = 0$$

together

Assume that t_j is known

The response value $g_k = g(k)$, where g can be discretized according to the 256 pixel values commonly observed in eight-bit images. (The response curves are calibrated separately for each color channel.)

In [3]:
```
'''
Modified from gsolve.m

Solve for imaging system response function

Given a set of pixel values observed for several pixels in several
images with different exposure times, this function returns the
imaging system's response function


Z(i,j): the pixel values of pixel location number i in image j

B(j): the log delta t, or log shutter speed, for image j

l: lamdba, the constant that determines the amount of smoothness

w(z): the weighting function value for pixel value z

'''
zmin = 0.
zmax = 255.

def weight_hat(z):
    return min(z-zmin, zmax-z)

def gsolve(Z, B, lmd, w=weight_hat):

    locations = Z.shape[0]
    sequences = Z.shape[1]
    n = 256  # [0, 255]
    A = np.zeros((locations * sequences + n - 1, locations + n), dtype=f
loat)
    b = np.zeros(A.shape[0], dtype=float)

    #  Include the data-fitting equations
    k = 0
    for i in range(locations):
        for j in range(sequences):
            wij = w(Z[i, j])
            A[k, int(Z[i, j])] = wij
            A[k, n + i] = -wij
            b[k] = wij * B[j]
            k += 1

    # Fix the curve by setting its middle value to 0, i.e. g(128) = 0
    A[k, 128] = 1
    k += 1

    # Include the smoothness equations
    for i in range(n-2): #(0, 253) 254 equations
        wi = w(i + 1)
        A[k, i] = lmd * wi
        A[k, i+1] = -2 * lmd * wi
        A[k, i+2] = lmd * wi
        k += 1
```

```
    # Solve the system
    x = np.linalg.lstsq(A, b)[0]
    g = x[:n]
    lnE = x[n:]

    return (g, lnE)
```

In [4]:
```
# imagepack is to flat each input RGB image into a vector for each chann
le and pack them into three 2d array
# input: np-array
# return three Z(i,j)(see above) for each channel
def imagepack(imagearray):
    num_image = imagearray.shape[0]
    # imagesize refer to 1-d image size
    imagesize = imagearray[0].shape[0]*imagearray[0].shape[1]
    imagepack = np.zeros((3, imagesize,num_image))
    # naive loop ; can be optimized later
    for i in range(num_image):
        for j in range(3):
            imagepack[j,:,i] = np.ndarray.flatten(imagearray[i][:,:,j])
    return imagepack
```

In [5]:
```
RP,GP,BP = imagepack(imagebracket)
```

Sample

Question: how do we sample the pixel value z_{ij}? What is the sample ratio?

The linear system should be overdetermined. For N sample pixels in each image and P images, we need $N \times P > (Z_{max} - Z_{min}) + N$ (i.e. number of given parameters is greater than number of unknowns)

Suppose we have 3 images, $2N > 255$, $N > 128$ should be sufficient.

Weakness/Strength Of Difference Sample Methods

For randomSample method - pick pixel randomly

- Weakness: not stable - some random result will lead to very bad estimate, for example, only sampling pixels with intensity 10
- Strength: very quick and if our sample size is big enough, the result will be representative

For windowSample method - pick pixel randomly:

- Weakness: not so stable and having no idea how intensity distributes among the whole images. In some extreme case, it may fail to sample some pixel with certain intensity. Also, as Debevec and Malik 1997 indicated we better avoid highly-variance areas, this schema may visit those bad areas
- Strength: very quick and it also goes through the whole images and in some way, it will be representative.

For pixelCoverSample method - random select k pixels for each pixel value ([0,255])

- Weakness: slow and have to go through the whole image to know the capacity of the k. Also it require shuffle/couting schema to do randomly sample. Fataly sometimes this schema does not work if any intensity is missing
- Strength: all different intensity pixels get reported. Much more representative

In [6]:
```
# sample function
# general template:
# inputimg: np.array - like Z(i,j)(see above) ; parameters for different
sampling schemas
# output: sampled input


# random chose "outputsize" pixels
def randomSample(inputimg,outputsize):
    outputsize = int(outputsize)
    indexrange = inputimg.shape[0]
    num_image = inputimg.shape[1]
    output = np.zeros((outputsize,num_image))
    sampleindex = np.random.choice(indexrange,size=outputsize,replace=Fa
lse)
    for i in range(num_image):
        output[:,i] = inputimg[sampleindex,i]
    return output


# choose pixels per windowsize where windowsize = floor(num of pixel / o
utputsize)
def windowSample(inputimg,outputsize):
    outputsize = int(outputsize)
    indexrange = inputimg.shape[0]
    num_image = inputimg.shape[1]
    windowsize = int(indexrange/outputsize)
    output = np.zeros((outputsize,num_image))
    sampleindex = np.arange(outputsize) * windowsize
    for i in range(num_image):
        output[:,i] = inputimg[sampleindex,i]
    return output

######## warning:
######## pixelCoverSample is not so good since sometimes, some certain i
ntensity value will be absent
######## for exmaple, int the toy image, we don't have 0
######## might try some more tolerant sampling schema

# random select k pixels for each pixel value ([0,255])
# base: using which image as a standard to pick index
# if k is too larger, the output will base on the maxmum possible k valu
e
def pixelCoverSample(inputimg,k,base = 1):
    indexrange = inputimg.shape[0]
    num_image = inputimg.shape[1]
    baseimg = inputimg[:,base]
    print(baseimg)
    pixelrange = 256
    # i copy the inputimg since i will call shuffle later, shuffle is a
 in-place function
    imgcopy = np.copy(inputimg)

    # check whether input k is feasible
    freqcount = np.zeros(pixelrange)
    for i in range(indexrange):
```

```
        freqcount[int(baseimg[i])] = freqcount[int(baseimg[i])] + 1
    maxk = np.amin(freqcount)
    kval = k if maxk >= k else maxk
    if kval != k:
        print("warning: the given k is more than the capacity!")
    if kval == 0:
        print("warning: the capacity is zero")
        return
    # sample part:
    # i used an inelegant method ; can improve later
    # better idea - store some info while check whther k is feasible
    # Or we can use some buind-in liabary - I did not find yet

    output = np.zeros((kval * pixelrange,num_image))
    # shuffle the imgs so that we have a random behavior
    np.apply_along_axis(np.random.shuffle,1,imgcopy)
    coutmap = np.full((pixelrange*kval),kval)
    # i is the index of imgcopy
    # j is the number of elements we already have in output
    i = 0
    j = 0
    while True:
        if j == pixelrange*kval:
            break
        else:
            tempval = imgcopy[i]
            if coutmap[int(tempval[base])] != 0:
                output[j,:] = imgcopy[i,:]
                coutmap[int(tempval[base])] = coutmap[int(tempval[base
])] - 1
                j = j + 1
            i = i + 1
    return output
```

Variance-driven sampling

Debevec and Malik(1997) indicated "Furthermore, the pixels are best sampled from regions of the image with low intensity variance so that radiance can be assumed to be constant across the area of the pixel, and the effect of optical blur of the imaging system is minimized".

In this, we can introduce a biased sample schema that the probablity P of each point X_{ij} get sampled is based on its local gradient

$$W_{i,j} = 1 - sigmoid(||\nabla X_{ij}||)$$

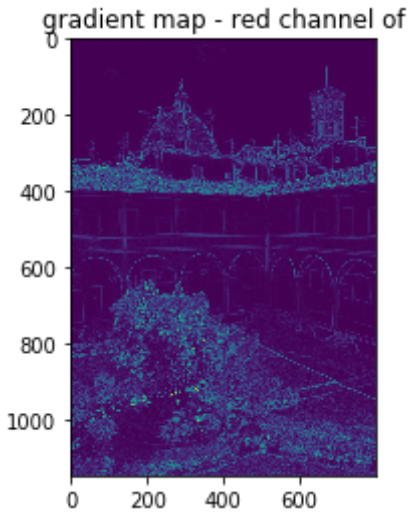
$$p(X_{ij}) = \frac{W_{ij}}{\sum_{p,q} W_{p,q}}$$

In [7]:
```
# we use sigmoid-like function to map our gradient into probality
# subjective to change
# maybe slow due to floating number
def decreaseSigmoid(a):
    return 1 - 1/(1 + np.exp(-1*a))

vfunc = np.vectorize(decreaseSigmoid)

x = np.matrix( [[1,2],[2,3]])
y = vfunc(x)
y = np.sum(y)
```

In [8]:
```
# input:
# imgpack: data from imagePace
# base: choose which image as basis to caculate local variance
# shape: orignal shape
# output:
# 1-d array - the gradient of the image
def GradientMap(imgpack,shape,base=1):
    baseimg = imgpack[:,base]
    baseimg = baseimg.reshape(shape)
    yGadient = np.gradient(baseimg,axis =0)
    xGadient = np.gradient(baseimg,axis =1)
    result = np.sqrt(yGadient*yGadient + xGadient*xGadient)
    return result

# input:
# Gradient:data from GradientMap
# functionvector: function vector applying to each pixel in the image; s
ee numpy.vectorize
# output:
# the weight map for each pixel
def ProbMap(Gradient, functionvector):
    result = functionvector(Gradient)
    result = result / np.sum(result)
    return result
```

In [9]:
```
# test block - visualize result
GradientResult = GradientMap(RP,im_shape)

plt.figure(figsize = (6, 4))
plt.title("gradient map - red channel of")
plt.imshow(GradientResult)
```

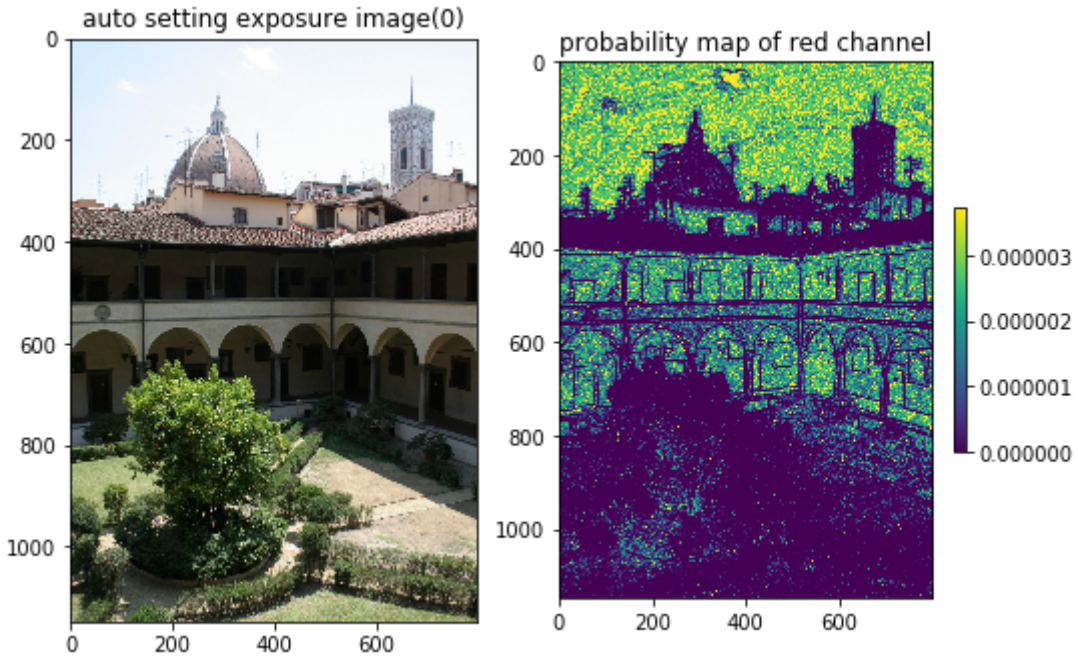
Out[9]:
```
<matplotlib.image.AxesImage at 0x10e5e0790>
```



The following block visualizes the result of probability map. Bright points have more chance to get sampled.

In [10]:
```
# visualize result
promap = ProbMap(GradientResult,vfunc)
plt.figure(figsize = (8, 6))
plt.subplot(121)
plt.title("auto setting exposure image(0)")
plt.imshow(im_auto_expose)
plt.subplot(122)
plt.title("probability map of red channel")
im = plt.imshow(promap)
plt.colorbar(im,fraction=0.030)
```

Out[10]: <matplotlib.colorbar.Colorbar at 0x1c2ca77a50>



In [11]:
```
#### sampling block
# input:
# imgPack: data from imagePack
# probmap: data from ProbMap
# ouputsize: size of the sample
# output:
# sampled imagePack
def gradientDrivenSample(imgPack,probmap,outputsize):
    pvector = np.ndarray.flatten(probmap)
    outputsize = int(outputsize)
    indexrange = imgPack.shape[0]
    num_image = imgPack.shape[1]
    output = np.zeros((outputsize,num_image))
    sampleindex = np.random.choice(indexrange,size=outputsize,replace=Fa
lse,p=pvector)
    for i in range(num_image):
        output[:,i] = imgPack[sampleindex,i]
    return output
```

Print the log response function and result image

If the images are noise free, we can use any non-saturated pixel value to estimate the corresponding radiance by mapping it through the inverse response curve $E = g(z)$.

Debevec and Malik (1997) use a hat function (10.7) which accentuates mid-tone pixels while avoiding saturated values.

In [13]:
```
%%time
%matplotlib notebook

lmd = 100
#### we use window sample here
targetsize = 1000 #(im_auto_expose.shape[1] * im_auto_expose.shape[0]) /
100
sampledRP = windowSample(RP,targetsize)
sampledGP = windowSample(GP,targetsize)
sampledBP = windowSample(BP,targetsize)
Rg, RinE = gsolve(sampledRP, log_t, lmd)
Gg, GinE = gsolve(sampledGP, log_t, lmd)
Bg, BinE = gsolve(sampledBP, log_t, lmd)
```

```
CPU times: user 3.23 s, sys: 59.1 ms, total: 3.29 s
Wall time: 1.88 s
```

In [14]:
```
%%time
%matplotlib notebook
#### use gradient drive sample schema here

RPpromap = ProbMap(GradientMap(RP, im_shape), vfunc)
GPpromap = ProbMap(GradientMap(GP, im_shape), vfunc)
BPpromap = ProbMap(GradientMap(BP, im_shape), vfunc)

vDsampledRP = gradientDrivenSample(RP,RPpromap,targetsize)
vDsampledGP = gradientDrivenSample(GP,GPpromap,targetsize)
vDsampledBP = gradientDrivenSample(BP,BPpromap,targetsize)
```

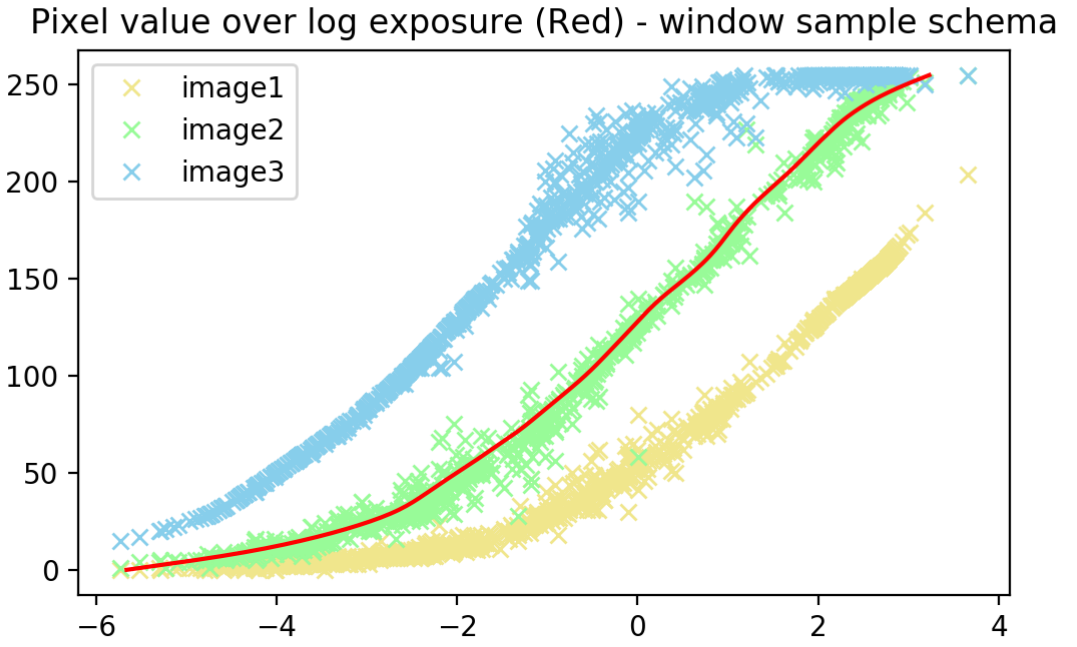
```
CPU times: user 5.67 s, sys: 189 ms, total: 5.86 s
Wall time: 5.53 s
```

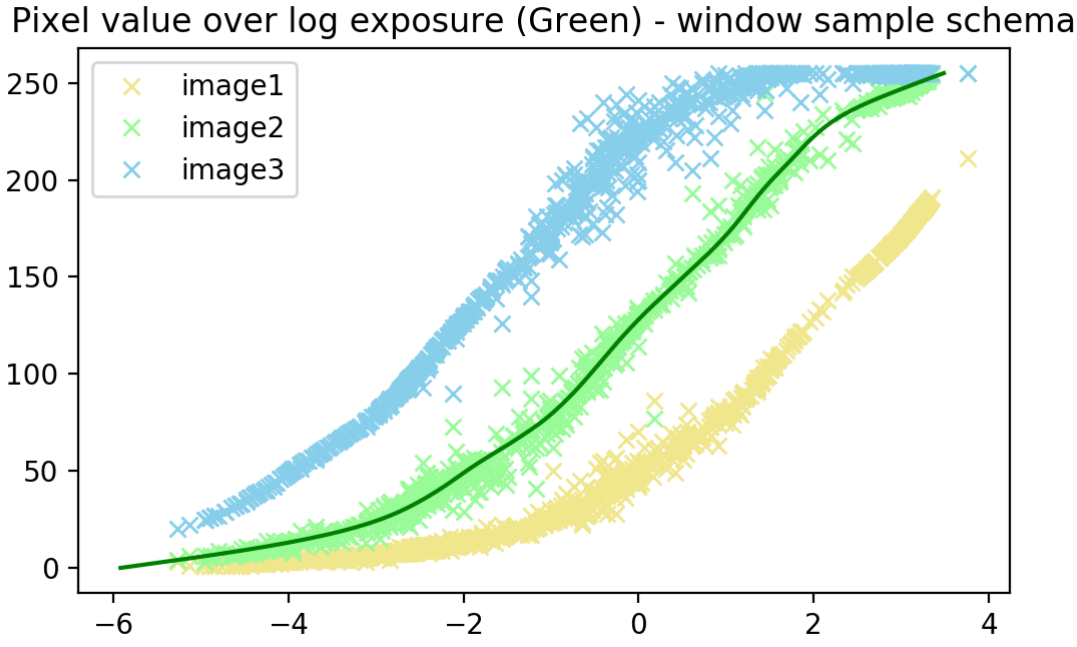
Reconstructed curves by window sample schema

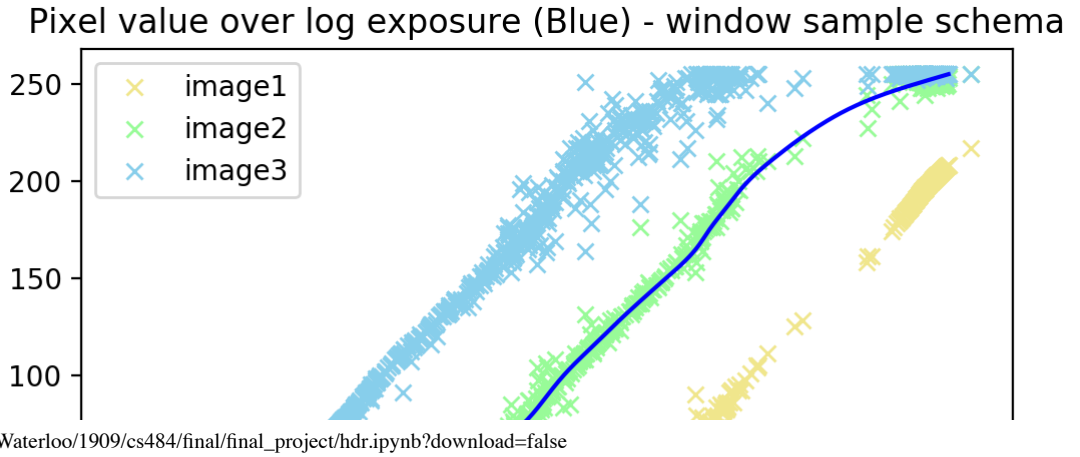
In [15]:
```
%%time
%matplotlib notebook
#### we use radient drive sample here
#### notice this can be very slow
vDRg, vDRinE = gsolve(vDsampledRP,[-2,0,2], lmd)
vDGg, vDGinE = gsolve(vDsampledGP,[-2,0,2], lmd)
vDBg, vDBinE = gsolve(vDsampledBP,[-2,0,2], lmd)
```

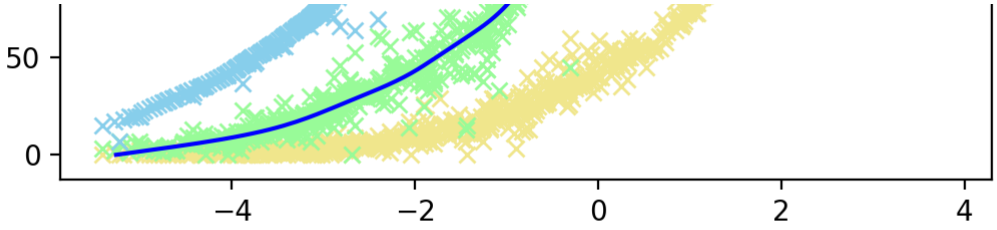
```
CPU times: user 3.64 s, sys: 67.2 ms, total: 3.71 s
Wall time: 2.2 s
```

In [16]:
```
pixelrange = np.arange(256)

fig = plt.figure(figsize=(6,12))
plt.subplot(311)
plt.plot(RinE, sampledRP[:,0], 'x', color = 'khaki', label='image1')
plt.plot(RinE, sampledRP[:,1], 'x', color = 'palegreen', label='image2')
plt.plot(RinE, sampledRP[:,2], 'x', color = 'skyblue', label='image3')
plt.plot(Rg, pixelrange, color='red')
plt.legend()
plt.title("Pixel value over log exposure (Red) - window sample schema")

plt.subplot(312)
plt.plot(GinE, sampledGP[:,0], 'x', color = 'khaki', label='image1')
plt.plot(GinE, sampledGP[:,1], 'x', color = 'palegreen', label='image2')
plt.plot(GinE, sampledGP[:,2], 'x', color = 'skyblue', label='image3')
plt.plot(Gg, pixelrange, color='green')
plt.legend()
plt.title("Pixel value over log exposure (Green) - window sample schema"
)

plt.subplot(313)
plt.plot(BinE, sampledBP[:,0], 'x', color = 'khaki', label='image1')
plt.plot(BinE, sampledBP[:,1], 'x', color = 'palegreen', label='image2')
plt.plot(BinE, sampledBP[:,2], 'x', color = 'skyblue', label='image3')
plt.plot(Bg, pixelrange, color='blue')
plt.legend()
plt.title("Pixel value over log exposure (Blue) - window sample schema")
```


Pixel value over log exposure (Red) - window sample schema
image1
image2
image3
250
200
150
100
50
0
−6
−4
−2
0
2
4


Pixel value over log exposure (Green) - window sample schema
image1
image2
image3
250
200
150
100
50
0
−6
−4
−2
0
2
4

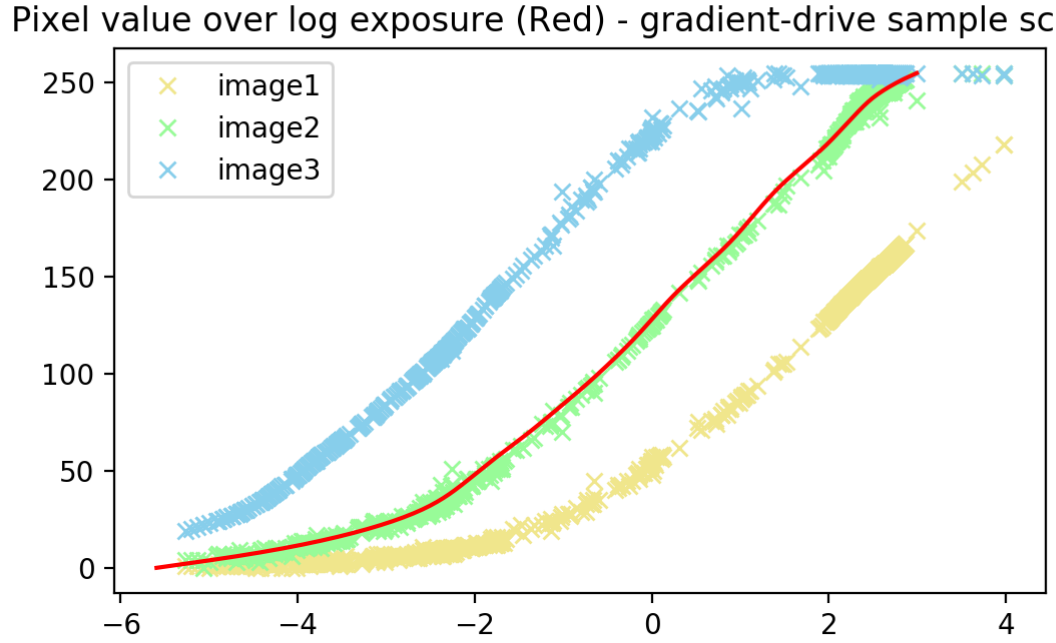

Pixel value over log exposure (Blue) - window sample schema
image1
image2
image3
250
200
150
100

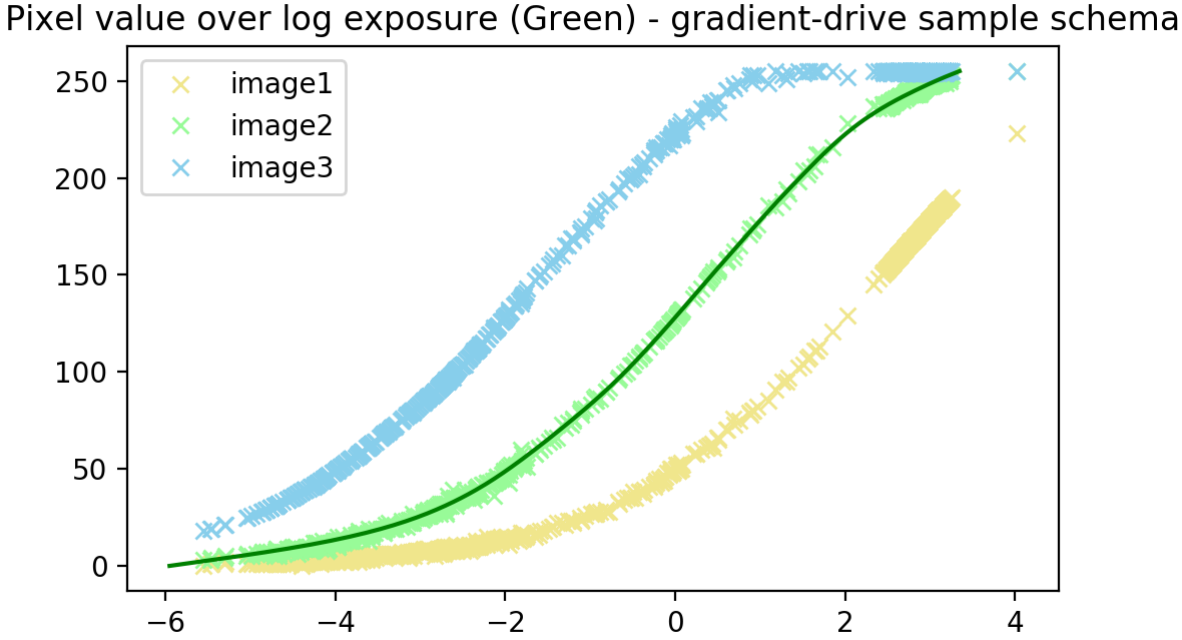


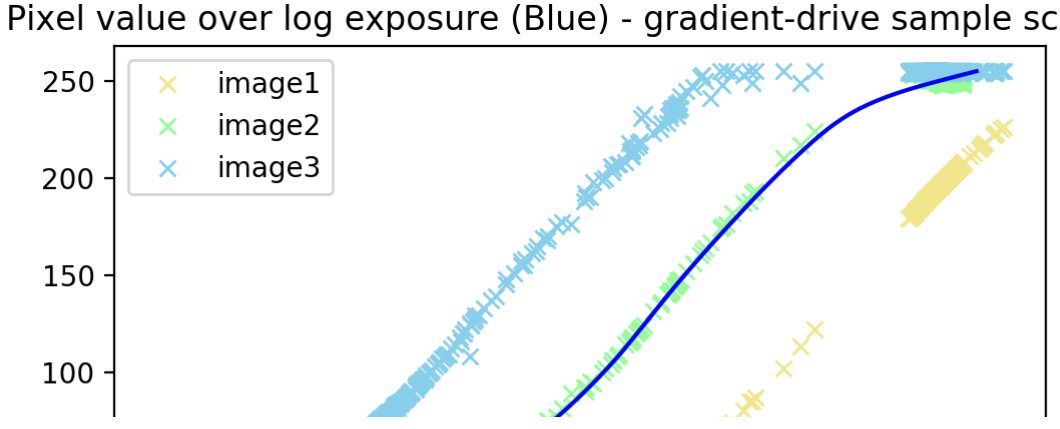
Out[16]: Text(0.5, 1.0, 'Pixel value over log exposure (Blue) - window sample schema')

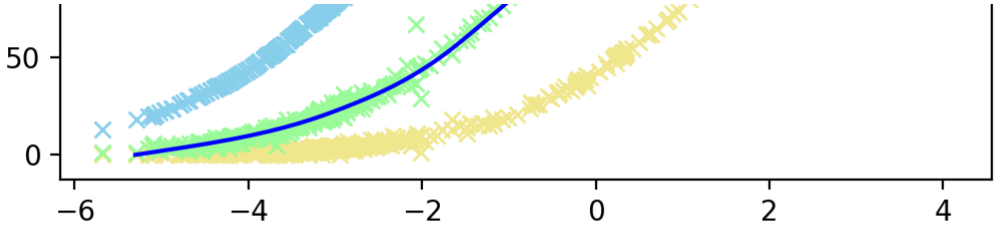
Reconstructed curves by drive sample schema

In [17]:
```
fig = plt.figure(figsize=(6,12))
plt.subplot(311)
plt.plot(vDRinE, vDsampledRP[:,0], 'x', color = 'khaki', label='image1')
plt.plot(vDRinE, vDsampledRP[:,1], 'x', color = 'palegreen', label='imag
e2')
plt.plot(vDRinE, vDsampledRP[:,2], 'x', color = 'skyblue', label='image
3')
plt.plot(vDRg, pixelrange, color='red')
plt.legend()
plt.title("Pixel value over log exposure (Red) - gradient-drive sample s
chema")

plt.subplot(312)
plt.plot(vDGinE, vDsampledGP[:,0], 'x', color = 'khaki', label='image1')
plt.plot(vDGinE, vDsampledGP[:,1], 'x', color = 'palegreen', label='imag
e2')
plt.plot(vDGinE, vDsampledGP[:,2], 'x', color = 'skyblue', label='image
3')
plt.plot(vDGg, pixelrange, color='green')
plt.legend()
plt.title("Pixel value over log exposure (Green) - gradient-drive sample
schema")

plt.subplot(313)
plt.plot(vDBinE, vDsampledBP[:,0], 'x', color = 'khaki', label='image1')
plt.plot(vDBinE, vDsampledBP[:,1], 'x', color = 'palegreen', label='imag
e2')
plt.plot(vDBinE, vDsampledBP[:,2], 'x', color = 'skyblue', label='image
3')
plt.plot(vDBg, pixelrange, color='blue')
plt.legend()
plt.title("Pixel value over log exposure (Blue) - gradient-drive sample
 schema")
```


Pixel value over log exposure (Red) - gradient-drive sample schema
image1
image2
image3
250
200
150
100
50
0
−6
−4
−2
0
2
4


Pixel value over log exposure (Green) - gradient-drive sample schema
image1
image2
image3
250
200
150
100
50
0
−6
−4
−2
0
2
4


Pixel value over log exposure (Blue) - gradient-drive sample schema
image1
image2
image3
250
200
150
100



Out[17]: Text(0.5, 1.0, 'Pixel value over log exposure (Blue) - gradient-drive sample schema')

1.2 Estimate a radiance map by selecting of blending pixels from different exposures

The naive way to measure exposures is

$$\log E_i = g(z_{ij}) - \log t_j$$

Unfortunately, pixels are noisy, especially under low-light conditions when fewer photons arrive at the sensor.

Mitsunaga and Nayar (1999) show that in order to maximize the signal-to-noise ratio (SNR), the weighting function must emphasize both higher pixel values and larger gradients in the transfer function

$$w(z) = f^{-1}(z)/f'^{-1}(z)$$

(Note that there is some confusion of the notation in the book, the measurement of weight function should based the inverse of reponse function **without logarithm**. Also, weight function should always be **positive**)

the weights w are used to form the final irradiance estimate

$$\log E_i = \frac{\sum_j w(z_{ij})[g(z_{ij}) - \log t_j]}{\sum_j w(z_{ij})}$$

We also multiply it by a hat distribution to further deter the use of clipped highlights and shadows from source images.

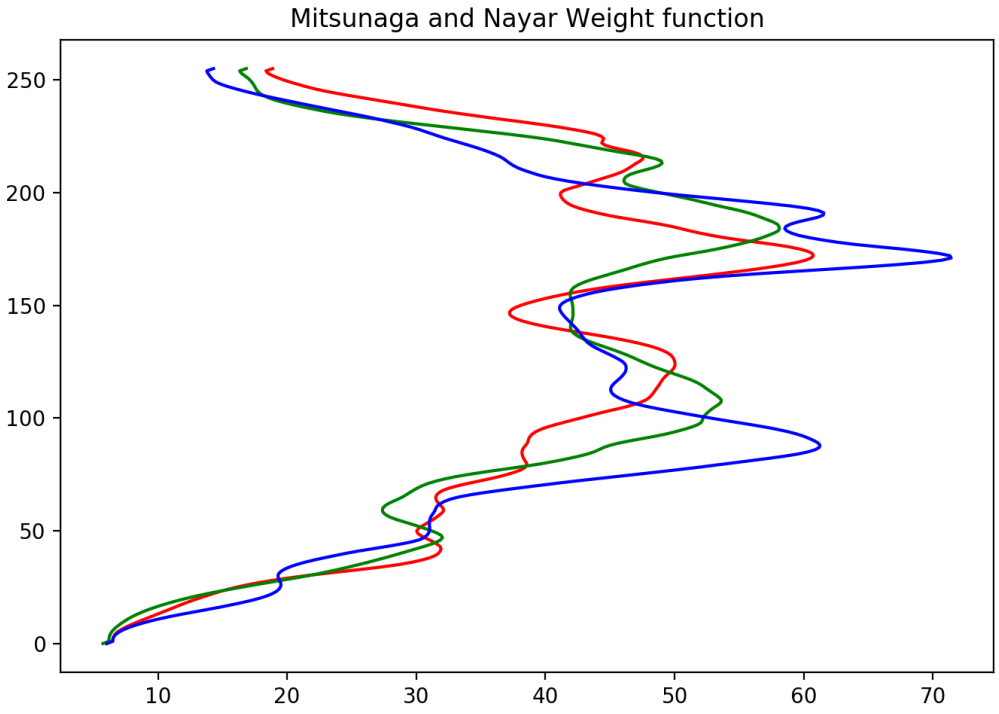
In [18]:
```
# input:
# g : the return data from g-solver
# output:
# result: the weight map
def weightgenerate(g, clamp_extremes=True):
    trans = np.exp(g)
    gprime = np.gradient(trans)
    result = np.array(trans/gprime)
    weights = np.zeros(g.shape[0])
    gmax = len(g)
    midpoint = weights.size * 1/2
    for i in range(weights.size):
        if i < midpoint:
            weights[i] = i
        else:
            weights[i] = weights.size - i
    # Add a small value to ensure all weights are positive
    # Otherwise, we may encounter division by zero in logEEstimate
    if clamp_extremes:
        return result * (weights + 1e-8)
    else:
        return result
```

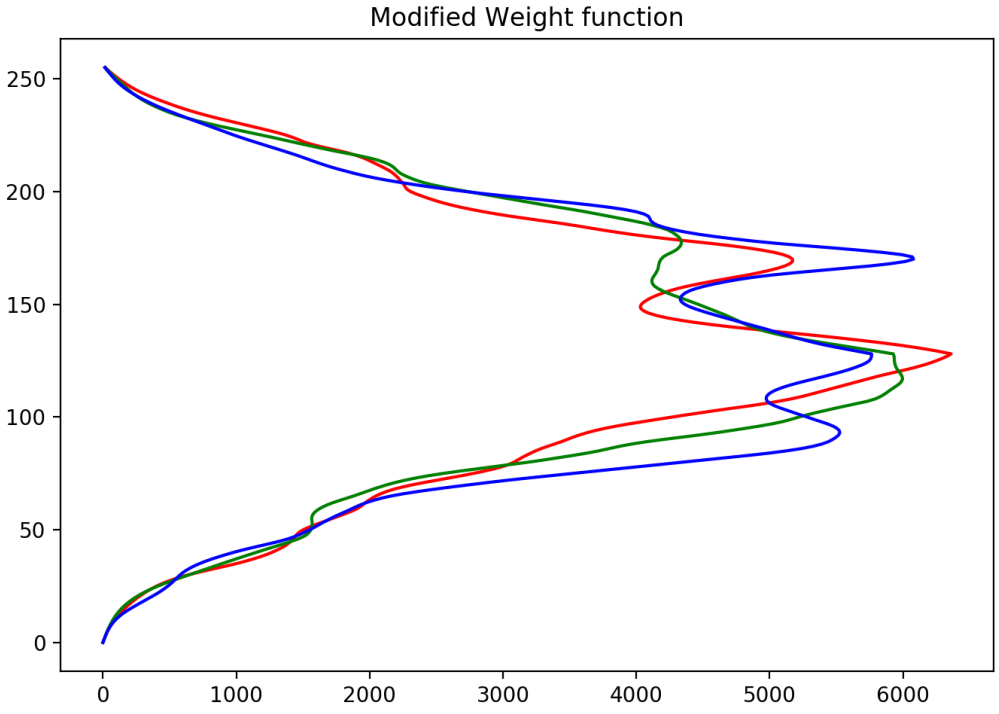
In [19]:
```
# input:
# g : the return data from g-solver
# weight: the return data from weightgenerate correspond to g
# t: log shutter speed, for image j
# imgPack: the return data from imagepack
# output:
# logE_i for each pixel - np array
def logEEstimate(g,weight,t,imgPack):
    size = imgPack.shape[0]
    num_img = imgPack.shape[1]
    result = np.zeros(size)
    for i in range(size):
        temp = 0
        weightsum = 0
        for j in range(num_img):
            temp = temp + weight[int(imgPack[i,j])] * (g[int(imgPack[i,j
])] - t[j])
            weightsum = weightsum + weight[int(imgPack[i,j])]
        result[i] = temp/weightsum
    return result
```

In [20]:
```
%%time
%matplotlib notebook
## estimate g prime g' for each channel
Rweight = weightgenerate(Rg)
Gweight = weightgenerate(Gg)
Bweight = weightgenerate(Bg)
origRweight = weightgenerate(Rg, False)
origGweight = weightgenerate(Gg, False)
origBweight = weightgenerate(Bg, False)
RglogE = logEEstimate(Rg,Rweight,[-2,0,2],RP)
GglogE = logEEstimate(Gg,Gweight,[-2,0,2],GP)
BglogE = logEEstimate(Bg,Bweight,[-2,0,2],BP)
```

```
CPU times: user 18.8 s, sys: 96 ms, total: 18.9 s
Wall time: 19.2 s
```

In [21]:
```
pixelrange = np.arange(256)
plt.figure(figsize = (8, 12))
plt.subplot(211)
plt.plot(origRweight, pixelrange, color='red')
plt.plot(origGweight, pixelrange, color='green')
plt.plot(origBweight, pixelrange, color='blue')
plt.title("Mitsunaga and Nayar Weight function")
plt.subplot(212)
plt.plot(Rweight, pixelrange, color='red')
plt.plot(Gweight, pixelrange, color='green')
plt.plot(Bweight, pixelrange, color='blue')
plt.title("Modified Weight function")
```





Out[21]: Text(0.5, 1.0, 'Modified Weight function')

1.3 Tone map the resulting high dynamic range (HDR) image back into a displayable gamut

It is usually necessary to display the HDR image on a lower gamut screen.

1. Global Transfer Curve (i.e Gamma Curve) (Larson, Rushmeier, and Pattanaik 2005)
   - If Gamma curve is applied seperate to each channel, then the color is less saturated
   - If Gamma curve is applied to the luminance channel, then result is better. (the image is splited up into luminance and chrominance components $L^*a^*b^*$
2. If the image has wide range of exposures, we can divide each pixel by the average brightness in a region around it, like dodging and burning.

Reconstruct image

In [22]:
```
result_img = np.zeros(im_auto_expose.shape)
logEs = [RglogE, GglogE, BglogE]
for i in range(3):
    result_img[:,:,i] = (logEs[i].reshape(im_shape))
```

In [23]:
```
def normal2image(img, max_value = 255.):
    ret = img.copy()
    ret -= img.min()
    ret *= max_value / ret.max()
    return ret

def image2normal(img):
    min_value = img.min()
    max_value = img.max()
    normal = (img - img.min()) / (max_value - min_value)
    return normal
```

Linearly maped

In [24]:
```
img_linearMap = normal2image(result_img).astype(np.uint8)
```

In [25]:
```
def compute_gamma(Vin, gamma = 0.5):
    return np.power(Vin, 1 / gamma)
```

Gamma correction is, in the simplest cases, defined by the following power-law expression:

$$V_{\text{out}} = AV_{\text{in}}^{\gamma}$$

where the non-negative real input value V_{in} is raised to the power γ and multiplied by the constant A, to get the output value V_{out}. In the common case of A = 1, inputs and outputs are typically in the range 0–1.

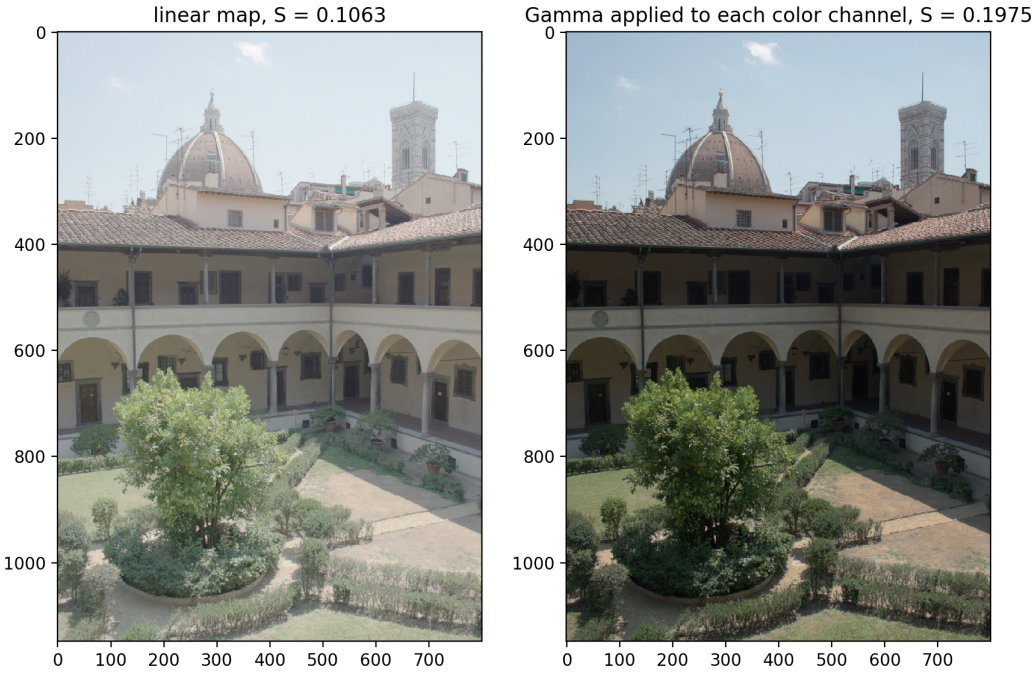
In [26]:
```
img_normal = image2normal(result_img)
img_gamColor = compute_gamma(img_normal)
img_gamColor = normal2image(img_gamColor).astype(np.uint8)
```

Gamma applied to intensity

In [27]:
```
def tonemap_gamma_intensity(img):
    lab = color.rgb2lab(img)
    l_channel = lab[:,:,0]
    l_max = l_channel.max()
    l_gamma = normal2image(compute_gamma(image2normal(l_channel)), l_max
)
    lab[:,:,0] = l_gamma
    return img_as_ubyte(color.lab2rgb(lab))

img_gamIntensity = tonemap_gamma_intensity(img_normal)
```

In [28]:
```
## Dodging and burning (Linear Filters)
def tonemap_dab(img):
    lab2 = color.rgb2lab(img)
    l_channel = lab2[:,:,0]
    hh = np.log(l_channel)
    h_low = skimage.filters.gaussian(hh,  sigma=100 , truncate=2.0)
    h_high = hh - h_low
    h = h_low + h_high * 0.1
    l = np.exp(hh)
    lab2[:,:,0] = l
    return img_as_ubyte(color.lab2rgb(lab2))

img_DaB = tonemap_dab(img_normal)
```

In [29]:
```
def computeSaturation(img):
    return color.rgb2hsv(img)[:,:,1].mean()

fig = plt.figure(figsize=(10,16))
plt.subplot(221)
plt.imshow(img_linearMap)
plt.title('linear map, S = {:.4f}'.format(computeSaturation(img_linearMa
p)))

plt.subplot(222)
plt.imshow(img_gamColor)
plt.title('Gamma applied to each color channel, S = {:.4f}'.format(compu
teSaturation(img_gamColor)))

plt.subplot(223)
plt.imshow(img_gamIntensity)
plt.title('Gamma applied to intensity, S = {:.4f}'.format(computeSaturat
ion(img_gamIntensity)))

plt.subplot(224)
plt.imshow(img_DaB)
plt.title('Dodge and Burn, S = {:.4f}'.format(computeSaturation(img_DaB
)))
```


linear map, S = 0.1063
Gamma applied to each color channel, S = 0.1975
Gamma applied to intensity, S = 0.2242
Dodge and Burn, S = 0.1059
0
200
400
600
800
1000
0
100
200
300
400
500
600
700

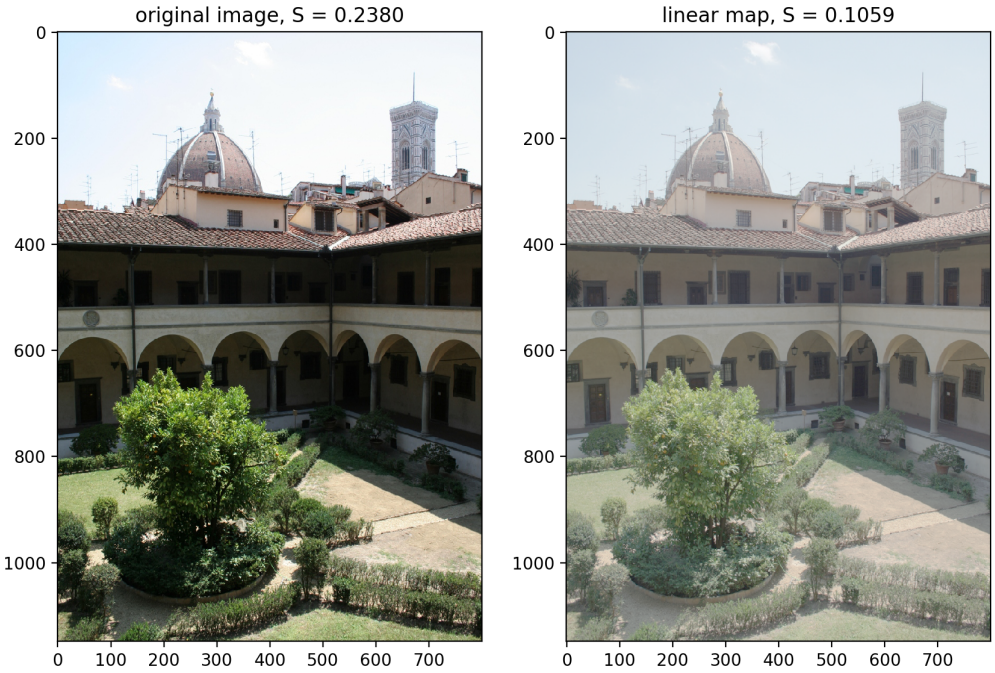
Out[29]: Text(0.5, 1.0, 'Dodge and Burn, S = 0.1059')

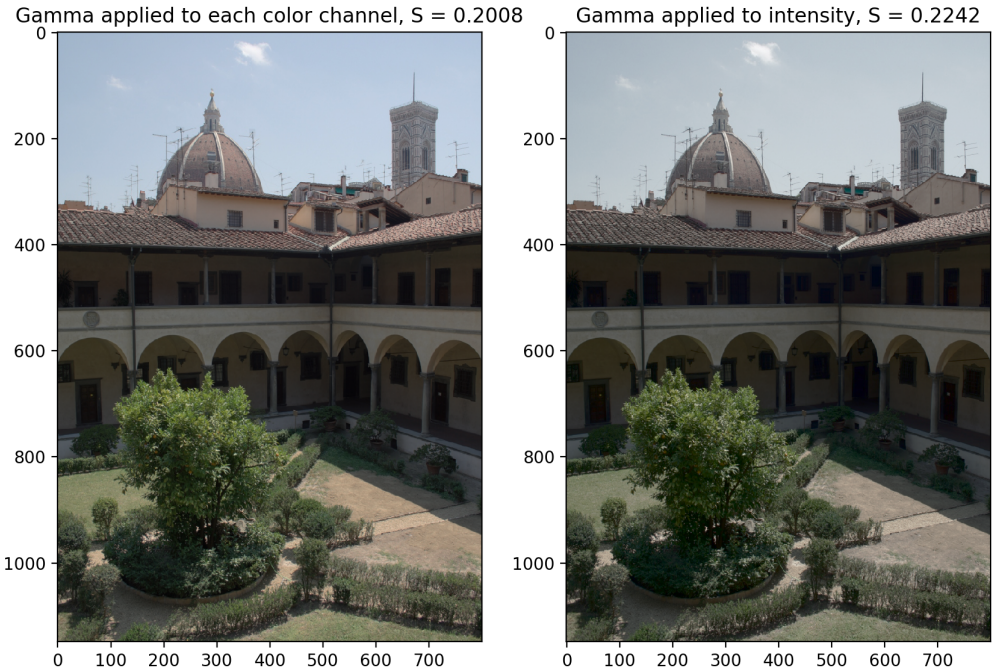
Variance-Driven Sample

In [30]:
```
radiances = [vDRg, vDGg, vDBg]
samples = [RP, GP, BP]
result_img2 = np.zeros(im_auto_expose.shape)

for i in range(len(radiances)):
    weight = weightgenerate(radiances[i])
    result_img2[:,:,i] = (logEEstimate(radiances[i],weight,[-2,0,2],samp
les[i]).reshape(im_shape))
```

In [31]:
```
img_linearMap = normal2image(result_img2).astype(np.uint8)

def tonemap_gamma_colour(img, gamma=None):
    img_normal = image2normal(img)
    if gamma:
        img_gamColor = compute_gamma(img_normal, gamma)
    else:
        img_gamColor = compute_gamma(img_normal)
    return normal2image(img_gamColor).astype(np.uint8)

img_gamColor = tonemap_gamma_colour(result_img2)

## lab
def tonemap_gamma_luminosity(img, gamma=None):
    lab = color.rgb2lab(img)
    l_channel = lab[:,:,0]
    l_max = l_channel.max()
    if gamma:
        img_gamColor = compute_gamma(image2normal(l_channel), gamma)
    else:
        img_gamColor = compute_gamma(image2normal(l_channel))
    l_gamma = normal2image(img_gamColor, l_max)
    lab[:,:,0] = l_gamma
    return img_as_ubyte(color.lab2rgb(lab))

img_gamIntensity = tonemap_gamma_luminosity(img_normal)
```

In [32]:
```
fig = plt.figure(figsize=(10,16))


plt.subplot(221)
plt.imshow(im_auto_expose)
plt.title('original image, S = {:.4f}'.format(computeSaturation(im_auto_
expose)))

plt.subplot(222)
plt.imshow(img_DaB)
plt.title('linear map, S = {:.4f}'.format(computeSaturation(img_DaB)))

plt.subplot(223)
plt.imshow(img_gamColor)
plt.title('Gamma applied to each color channel, S = {:.4f}'.format(compu
teSaturation(img_gamColor)))

plt.subplot(224)
plt.imshow(img_gamIntensity)
plt.title('Gamma applied to intensity, S = {:.4f}'.format(computeSaturat
ion(img_gamIntensity)))
```


original image, S = 0.2380
linear map, S = 0.1059
0
200
400
600
800
1000
0 100 200 300 400 500 600 700


Gamma applied to each color channel, S = 0.2008
Gamma applied to intensity, S = 0.2242
0
200
400
600
800
1000
0 100 200 300 400 500 600 700

```
Out[32]: Text(0.5, 1.0, 'Gamma applied to intensity, S = 0.2242')
```

2. Extensions

2.1 Drop the assumption that exposure is known

Suppose $\log t_j$ are unknowns in our least squares problem.

$$E = \sum_i \sum_j w(z_{i,j})[g(z_{i,j}) - \log E_i - \log t_j]^2 + \lambda \sum_k g''(k)^2 + \eta \sum_j (t_j - \hat{t}_j)^2$$

In other word, we are solve the three equations together to estimate the radiometric response function g, irradiance values E_i and t_j

$$w(z_{i,j})g(z_{i,j}) - w(z_{i,j}) \log E_i - w(z_{i,j}) \log t_j = 0$$
$$\lambda[g(z_{i,j} - 1) - 2g(z_{i,j}) + g(z_{i,j} + 1)] = 0$$
$$\eta(t_j - \hat{t}_j) = 0$$

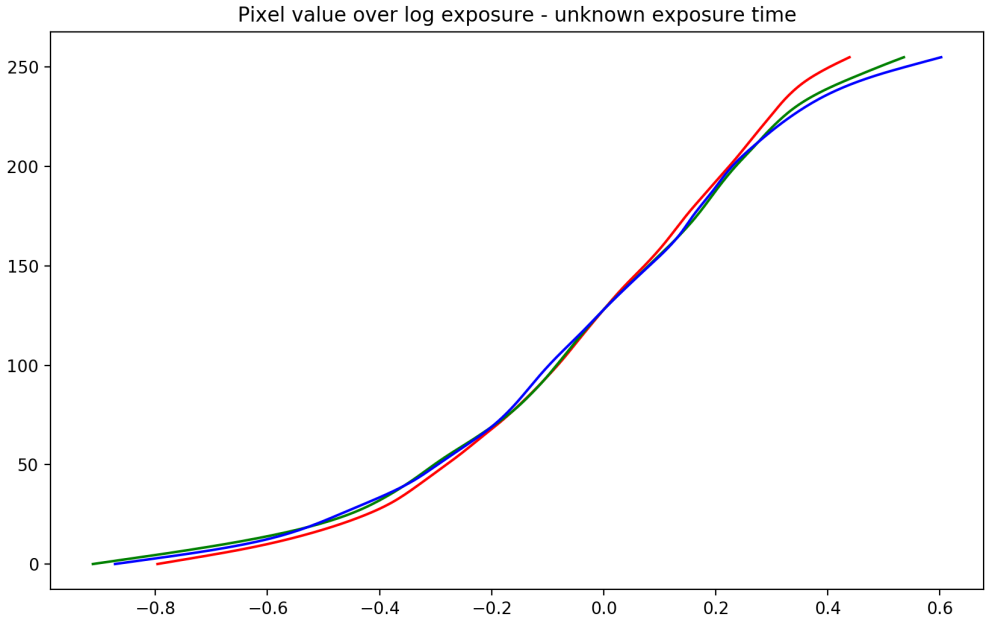
In [33]:
```
'''
Modified from gsolve.m

Solve for imaging system response function

Given a set of pixel values observed for several pixels in several
images with different exposure times, this function returns the
imaging system's response function


Z(i,j): the pixel values of pixel location number i in image j

l: lamdba, the constant that determines the amount of smoothness

w(z): the weighting function value for pixel value z

eta: the constraint for exposure time, should be positive for ascending
 exposure time sequence

t_hat: the nominal value of default image (middle image in image sequenc
e)
'''

def gsolve2(Z, lmd, eta = 10, t_hat = 0, w=weight_hat):

    locations = Z.shape[0]
    sequences = Z.shape[1]
    n = 256  # [0, 255]
    A = np.zeros((locations * sequences + n + sequences - 1, n + locatio
ns + sequences), dtype=float)
    b = np.zeros(A.shape[0], dtype=float)

    #  Include the data-fitting equations
    k = 0
    for i in range(locations):
        for j in range(sequences):
            wij = w(Z[i, j])
            A[k, int(Z[i, j])] = wij
            A[k, n + i] = -wij
            A[k, n + locations + j] = -wij #b[k] = wij * B[j]
            k += 1

    # Fix the curve by setting its middle value to 0, i.e. g(128) = 0
    A[k, 128] = 1
    k += 1

    # Include the smoothness equations
    for i in range(n-2):
        wi = w(i + 1)
        A[k, i] = lmd * wi
        A[k, i+1] = -2 * lmd * wi
        A[k, i+2] = lmd * wi
        k += 1

    # exposure time constraints
    for i in range(sequences):
```

```
        A[k, n + locations + i] = eta
        b[locations * sequences + n + i - 1] =  eta * (i - sequences//2
+ t_hat)
        k += 1

    # Solve the system
    x = np.linalg.lstsq(A, b)[0]
    g = x[:n]
    lnE = x[n:n + locations]
    lnt = x[n + locations:]

    return (g, lnE, lnt)
```

Silimilarly, for N sample pixels in each image and P images, we need $N \times P > (Z_{max} - Z_{min}) + N + P$
Suppose we have 3 images, $2N > 255 + 2$, $N > 129$ should be sufficient.

In [34]:
```
lmd = 100

targetsize = 1000
eta = 100
sampledRP = windowSample(RP,targetsize)
sampledGP = windowSample(GP,targetsize)
sampledBP = windowSample(BP,targetsize)
R_g, R_linE, R_lnt = gsolve2(sampledRP, lmd, eta = eta)
G_g, G_linE, G_lnt = gsolve2(sampledGP, lmd, eta = eta)
B_g, B_linE, B_lnt = gsolve2(sampledBP, lmd, eta = eta)
```

In [35]:
```
fig = plt.figure(figsize=(10,6))
plt.plot(R_g, np.arange(256), color = "r")
plt.plot(G_g, np.arange(256), color = "g")
plt.plot(B_g, np.arange(256), color = "b")
plt.title("Pixel value over log exposure - unknown exposure time")
```



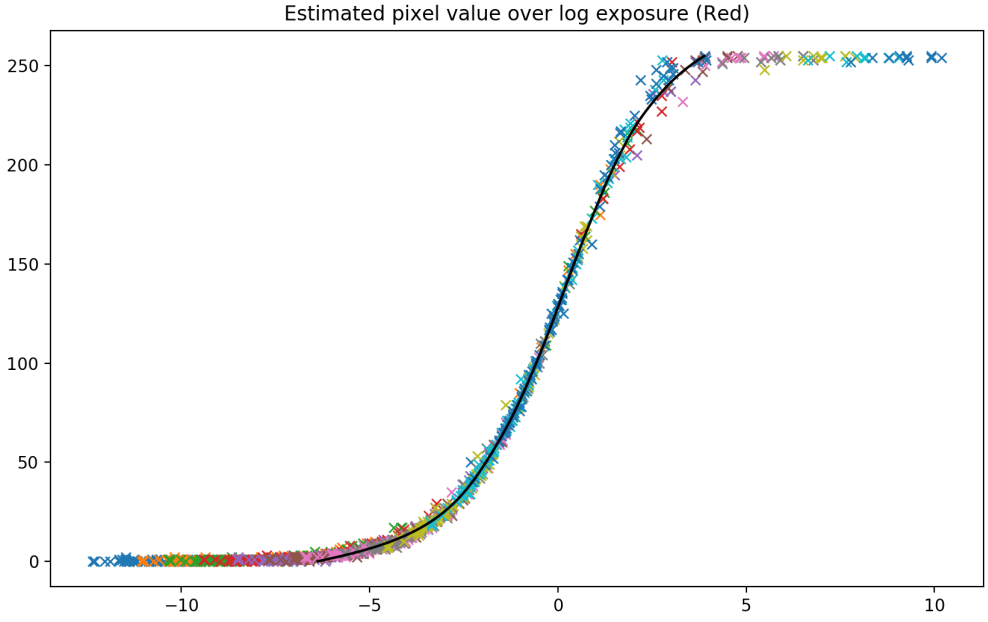
Out[35]: Text(0.5, 1.0, 'Pixel value over log exposure - unknown exposure time')

We will demo our apporoach also works with a longer sequence whose EV is not arithmetic sequence.

In [36]:
```
cave_prefix = 'images/cave/agia-sofia_'

cave_images = np.array([image.imread(cave_prefix + "-4.0.jpg"),
                        image.imread(cave_prefix + "-2.7.jpg"),
                        image.imread(cave_prefix + "-2.jpg"),
                        image.imread(cave_prefix + "-1.4.jpg"),
                        image.imread(cave_prefix + "-0.7.jpg"),
                        image.imread(cave_prefix + "0.jpg"),
                        image.imread(cave_prefix + "+1.jpg"),
                        image.imread(cave_prefix + "+2.jpg"),
                        image.imread(cave_prefix + "+3.jpg"),
                        image.imread(cave_prefix + "+4.jpg"),
                        image.imread(cave_prefix + "+5.jpg")])
```

In [37]:
```
cave_RP,cave_GP,cave_BP = imagepack(cave_images)

lmd = 100
eta = 100

cave_sampledRP = windowSample(cave_RP, 100)
cave_sampledGP = windowSample(cave_GP, 100)
cave_sampledBP = windowSample(cave_BP, 100)
cave_R_g, cave_R_linE, cave_R_lnt = gsolve2(cave_sampledRP, lmd, eta = e
ta)
cave_G_g, cave_G_linE, cave_G_lnt = gsolve2(cave_sampledGP, lmd, eta = e
ta)
cave_B_g, cave_B_linE, cave_B_lnt = gsolve2(cave_sampledBP, lmd, eta = e
ta)
```

In [38]:
```
fig = plt.figure(figsize=(10,6))

for i in range(cave_images.shape[0]):
    plt.plot(cave_R_linE + cave_R_lnt[i], cave_sampledRP[:,i], 'x')

plt.plot(cave_R_g, np.arange(256), color = 'black')
plt.title('Estimated pixel value over log exposure (Red)')
```



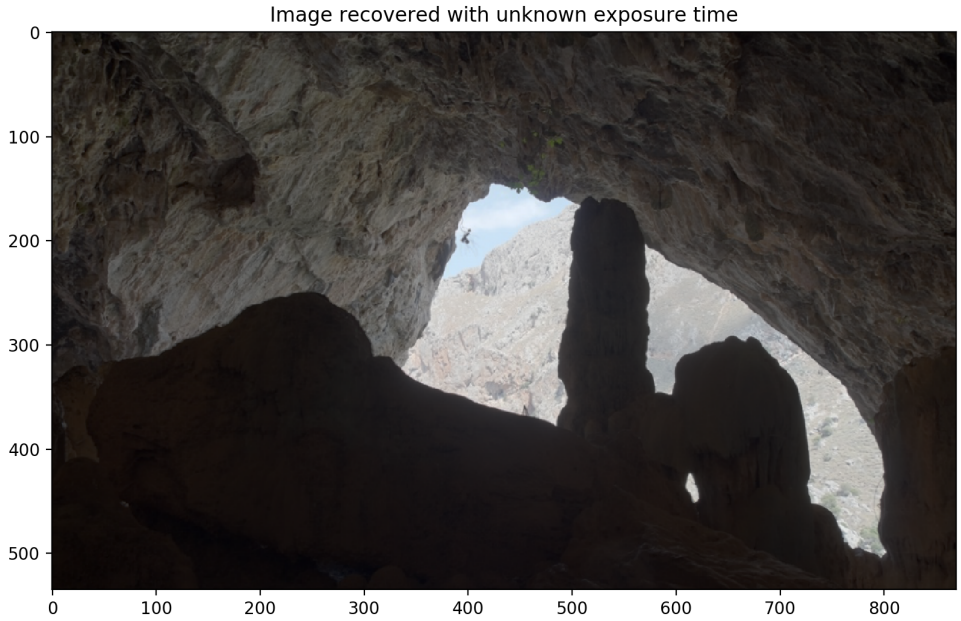
Out[38]: Text(0.5, 1.0, 'Estimated pixel value over log exposure (Red)')

In [39]:
```
cave_lnt = np.sum([cave_R_lnt, cave_G_lnt, cave_B_lnt], axis = 0) / 3.0
real_cave_lnt = np.array([-4.0, -2.7, -2.0, -1.4, -0.7, 0, 1, 2, 3,4 ,5
])
np.set_printoptions(precision=1)
print("The real lnt sequence are")
print(real_cave_lnt)
print("The estimated lnt are")
print(cave_lnt)
```

```
The real lnt sequence are
[-4.  -2.7 -2.  -1.4 -0.7  0.   1.   2.   3.   4.   5. ]
The estimated lnt are
[-4.9 -3.6 -2.9 -2.  -1.1 -0.2  0.8  1.9  3.   4.   5.1]
```

In [40]:
```
cave_R_weight = weightgenerate(cave_R_g)
cave_G_weight = weightgenerate(cave_G_g)
cave_B_weight = weightgenerate(cave_B_g)
cave_RglogE = logEEstimate(cave_R_g,cave_R_weight,cave_lnt,cave_RP)
cave_GglogE = logEEstimate(cave_G_g,cave_G_weight,cave_lnt,cave_GP)
cave_BglogE = logEEstimate(cave_B_g,cave_B_weight,cave_lnt,cave_BP)

cave_result_img = np.zeros(cave_images[0].shape)
cave_logEs = [cave_RglogE, cave_GglogE, cave_BglogE]
for i in range(3):
    cave_result_img[:,:,i] = (cave_logEs[i].reshape(cave_result_img.shap
e[0], cave_result_img.shape[1]))
```

In [41]:
```
cave_img_gamColor = tonemap_gamma_colour(cave_result_img, 0.7)

fig = plt.figure(figsize=(10,6))
plt.imshow(cave_img_gamColor)
plt.title("Image recovered with unknown exposure time")
image.imsave("image2.png", cave_img_gamColor)
```



2.2 Images taken without tripod

When multiple images are taken at varying orientations and exposures, we need to first align the input. The global alignment method should be tolerant to exposure differences.

We use Horris corner for feature matches, which is invariance to intensity shift. Then, we use RANSAC to make the matching images are geometrically consistent. The function is implemented as `produceMatches` .

In [42]:
```
from lib.feature import produceMatches
```

In [43]:
```
'''
Sample images are taken from https://www.ptgui.com/hdrtutorial.html
(C) Copyright 2006 by Joost Nieuwenhuijse
'''

pano_prefix = 'images/panoramas/IMG_0'
a = []
for i in range(475, 487):
    a.append(image.imread("images/panoramas/IMG_0{}.JPG".format(i)))

pano_images = np.array(a)
```

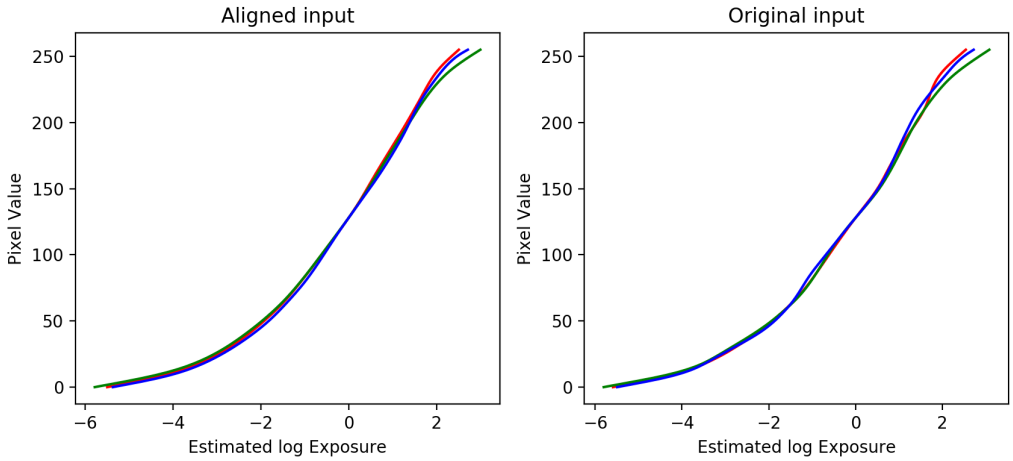
Then, we can perform a global alignment and crop invalid margin.

In [44]:
```
def cropImage(imgs, margin = 25):
    min_r = 0
    min_c = 0
    max_r = imgs[0].shape[0]
    max_c = imgs[0].shape[1]

    for img in imgs:
        intensity = np.sum(img, axis = 2)
        image_co = np.argwhere(intensity > 0)
        min_cor = np.min(image_co, axis = 0)
        max_cor = np.max(image_co, axis = 0)
        min_r = max(min_r, min_cor[0])
        min_c = max(min_c, min_cor[1])
        max_r = min(max_r, max_cor[0])
        max_c = min(max_c, max_cor[1])

    crop_imgs = []
    for img in imgs:
        im = (np.uint8((img[min_r + margin : max_r - margin, min_c + mar
gin: max_c -margin, :])*255))
        crop_imgs.append(im)

    return np.array(crop_imgs)


def produceAlignedImages(imgs, output_shape = None, panoramas = False, o
verlap_size = None):
    ref_img = imgs[0]
    warp_imgs = []
    #warp_imgs.append(ref_img.astype(np.float64))
    for i in range(imgs.shape[0]):
        matchesLR, model_robust, inliers = produceMatches(ref_img, imgs[
i], panoramas, overlap_size)
        if (output_shape == None):
            warp_img = warp(imgs[i], model_robust)
        else:
            warp_img = warp(imgs[i], model_robust, output_shape = output
_shape)
        warp_imgs.append(warp_img)

    warp_imgs = np.array(warp_imgs)

    return cropImage(warp_imgs)
```

In [45]:
```
crop_images = produceAlignedImages(pano_images[6:9])
```

we can compare radiance reponse between aligned image input and original image input.

In [49]:
```
pano_lnt = [0, -2, 2]

lmd = 100
targetsize = 1000

pano_lnEs = []
pano_gs = []
pano_packs = imagepack(crop_images)
for pack in pano_packs:
    sample = windowSample(pack, targetsize)
    g, lnE = gsolve(sample, pano_lnt, lmd)
    pano_lnEs.append(lnE)
    pano_gs.append(g)

pano_packs_ori = imagepack(pano_images[6:9])
pano_lnEs_ori = []
pano_gs_ori = []
for pack in pano_packs_ori:
    sample = windowSample(pack, targetsize)
    g, lnE = gsolve(sample, pano_lnt, lmd)
    pano_lnEs_ori.append(lnE)
    pano_gs_ori.append(g)

fig = plt.figure(figsize=(10,4))
plt.subplot(121)
plt.plot(pano_gs[0], np.arange(256), color='r')
plt.plot(pano_gs[1], np.arange(256), color='g')
plt.plot(pano_gs[2], np.arange(256), color='b')
plt.xlabel('Estimated log Exposure')
plt.ylabel('Pixel Value')
plt.title('Aligned input')

plt.subplot(122)
plt.plot(pano_gs_ori[0], np.arange(256), color='r')
plt.plot(pano_gs_ori[1], np.arange(256), color='g')
plt.plot(pano_gs_ori[2], np.arange(256), color='b')
plt.xlabel('Estimated log Exposure')
plt.ylabel('Pixel Value')
plt.title('Original input')
```



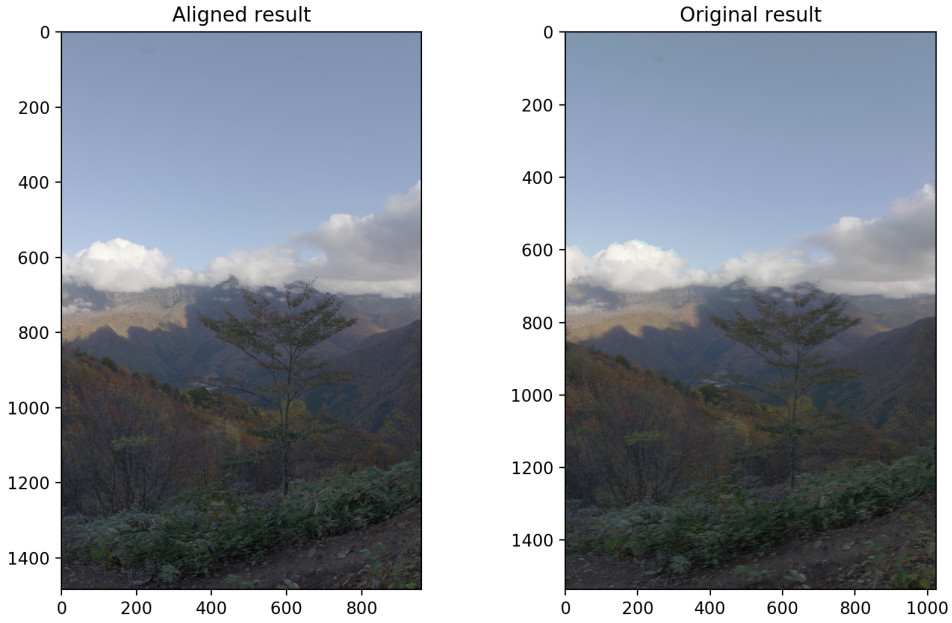
Out[49]: Text(0.5, 1.0, 'Original input')

In [47]:
```
def getResultImage(gs, packs, lnt, img_shape):
    glogEs = []
    for i in range(3):
        weight = weightgenerate(gs[i])
        glogE = logEEstimate(gs[i], weight, lnt, packs[i])
        glogEs.append(glogE)

    result_img = np.zeros(img_shape)
    for i in range(3):
        result_img[:,:,i] = (glogEs[i].reshape(img_shape[0], img_shape[1
]))

    return result_img
```

In [48]:
```
pano_result_img = getResultImage(pano_gs, pano_packs, [0,-2,2], crop_ima
ges[0].shape)
pano_unaligned_result_img = getResultImage(pano_gs_ori, pano_packs_ori,
[0,-2,2], pano_images[6].shape)
```

We can see there is blurriness in the unaligned images.

In [49]:
```
fig = plt.figure(figsize=(10,6))
plt.subplot(121)
plt.imshow(tonemap_gamma_colour(pano_result_img, 0.7))
plt.title('Aligned result')

plt.subplot(122)
plt.imshow(tonemap_gamma_colour(pano_unaligned_result_img, 0.7))
plt.title('Original result')
```



Out[49]: Text(0.5, 1.0, 'Original result')

Produce HDR Panoramas

To reproduce HDR Panoramas, we can compute each pixel by the radiance value response function from the overlapping images.

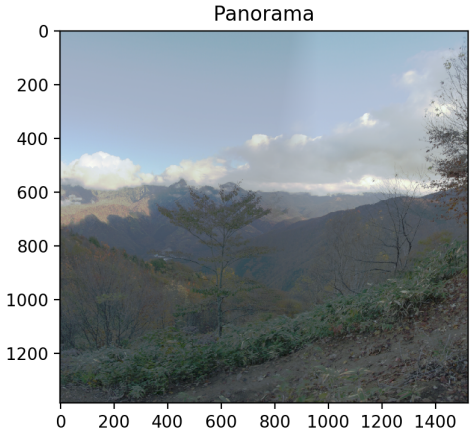
In [50]:
```
crop_images2 = produceAlignedImages(pano_images[9:12])
```

In [51]:
```
width = crop_images[0].shape[1] * 2
height = crop_images[0].shape[0]

_, model_pano, _ = produceMatches(crop_images[0], crop_images2[0], panor
amas = True, overlap_size = 400)

tform = SimilarityTransform(scale = 1.0)

crop_images12 = []
for img in crop_images:
    crop_images12.append(warp(img, tform, output_shape=(height, width)))

for img in crop_images2:
    crop_images12.append(warp(img, model_pano, output_shape = (height, w
idth)))

crop_images12 = np.array(crop_images12)

resultsss = cropImage(crop_images12)
```

In [52]:
```
pano_lnt2 = [0, -2, 2, 0, -2, 2]

lmd = 100
targetsize = 1000

pano_lnEs2 = []
pano_gs2 = []
pano_packs2 = imagepack(resultsss)
for pack in pano_packs2:
    sample = windowSample(pack, targetsize)
    g, lnE = gsolve(sample, pano_lnt2, lmd)
    pano_lnEs2.append(lnE)
    pano_gs2.append(g)
```

In [53]:
```
pano_result_img1 = getResultImage(pano_gs2, imagepack(crop_images), [0,-
2,2], crop_images[0].shape)
pano_result_img2 = getResultImage(pano_gs2, imagepack(crop_images2), [0,
-2,2], crop_images2[0].shape)
```

In [54]:
```
from lib.blend import boundaryDT, alpha

gam_result_img = tonemap_gamma_colour(pano_result_img1, 0.8)
gam_result_img2 = tonemap_gamma_colour(pano_result_img2, 0.8)

imWarpL = warp(gam_result_img, tform, output_shape=(height, width))
imWarpR = warp(gam_result_img2, model_pano, output_shape = (height, widt
h))

boundaryDTL = warp(boundaryDT(gam_result_img), tform, output_shape=(heig
ht, width))
boundaryDTR = warp(boundaryDT(gam_result_img2), model_pano, output_shape
=(height, width))
alphaValue = alpha(boundaryDTL, boundaryDTR, height, width)

imAlphaL = np.zeros(imWarpL.shape)
imAlphaR = np.zeros(imWarpR.shape)
for i in range (3):
    imAlphaL[:,:,i] = alphaValue[0]
    imAlphaR[:,:,i] = alphaValue[1]
fig = plt.figure(figsize=(10,4))
plt.imshow(np.multiply(imWarpL ,imAlphaL)[100:,:-400,:] + np.multiply(im
WarpR, imAlphaR)[100:,:-400,:])
plt.title("Panorama")
```



Out[54]: Text(0.5, 1.0, 'Panorama')

Conclusions

We have implemented the Debevec-Malik method for recovering high dynamic range radiance maps from ordinary photographs. Between the two implemented extensions, we are also able to remove two significant assumptions about our target data: that we know the relative exposure levels of the images, and that the images line up, in addition to making it possible to extend the field of view, as in a panorama, when suitable unaligned image data is provided.

When we use different sample methods to reconstruct the response function, we obtain slightly different results. For the window sampling schema, it more or less preserves some spatial information as we sample among each kernel of fixed size. Also, the probability of each pixel being sampled only depends on its location. There is no preference for pixels of a certain intensity. In this case, our reconstructed response function is a continuous curve, as pixels of almost all intensities get sampled and contribute to reconstruction. However, from the final result, the reconstructed curve is noisy. This is due to pixels from regions of high-variance having an optical blur effect, which disturbs our reconstruction. In this case, we use a biased sampling method, namely variance-driven sampling (detail see above). Since we have lower probability of sampling points of high-variance, our reconstructed curve is less noisy. However, in this sampling method, we are at risk of missing pixels of certain intensities. In this case, our reconstruction curve will not be entirely representative.

The resulting images are a little flat and undersaturated, however this can be compensated for with other post-processing tools (e.g. global boost to saturation and adjustment to the tone curve to give the image more of a pop). The colour data are there, just not optimally distributed for a perfect image. That being said, the distribution of image data in extremes of the image (e.g. the dark part of the cave in the Agia Sofia dataset) may be so close in value that much of the data is lost by conversion to an integer image representation. Improvements to tonemapping would be beneficial here to better distribute the colour data, however, as previously mentioned, this doesn't produce a perfect final image, and some post-processing would be required anyway, and this could be done on the floating-point representation rather than the integer representation.

Reference

Debevec, P. E., & Malik, J. (1997). Recovering high dynamic range radiance maps from photographs. Proceedings of the 24th Annual Conference on Computer Graphics and Interactive Techniques - SIGGRAPH 97. doi: 10.1145/258734.258884

Eden, A., Uyttendaele, M., & Szeliski, R. (2006). Seamless Image Stitching of Scenes with Large Motions and Exposure Differences. 2006 IEEE Computer Society Conference on Computer Vision and Pattern Recognition - Volume 2 (CVPR06). doi: 10.1109/cvpr.2006.268

Szeliski, R. (2011). Computer Vision Algorithms and Applications. London: Springer.