# Time Series Analysis of FOF

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# Overview

# Overview

# Introduction

- Fund has been one of the most popular investment vehicle for people, including mutual funds and hedge funds.
- Fund of funds (FOF) is a popular investment product during the past twenty years.
  - Invest in a variety of fund categories.
  - Higher fees but fewer risks.
- American employees switched from DB plan to DC plan, which benefits the FOF market.

# Introduction

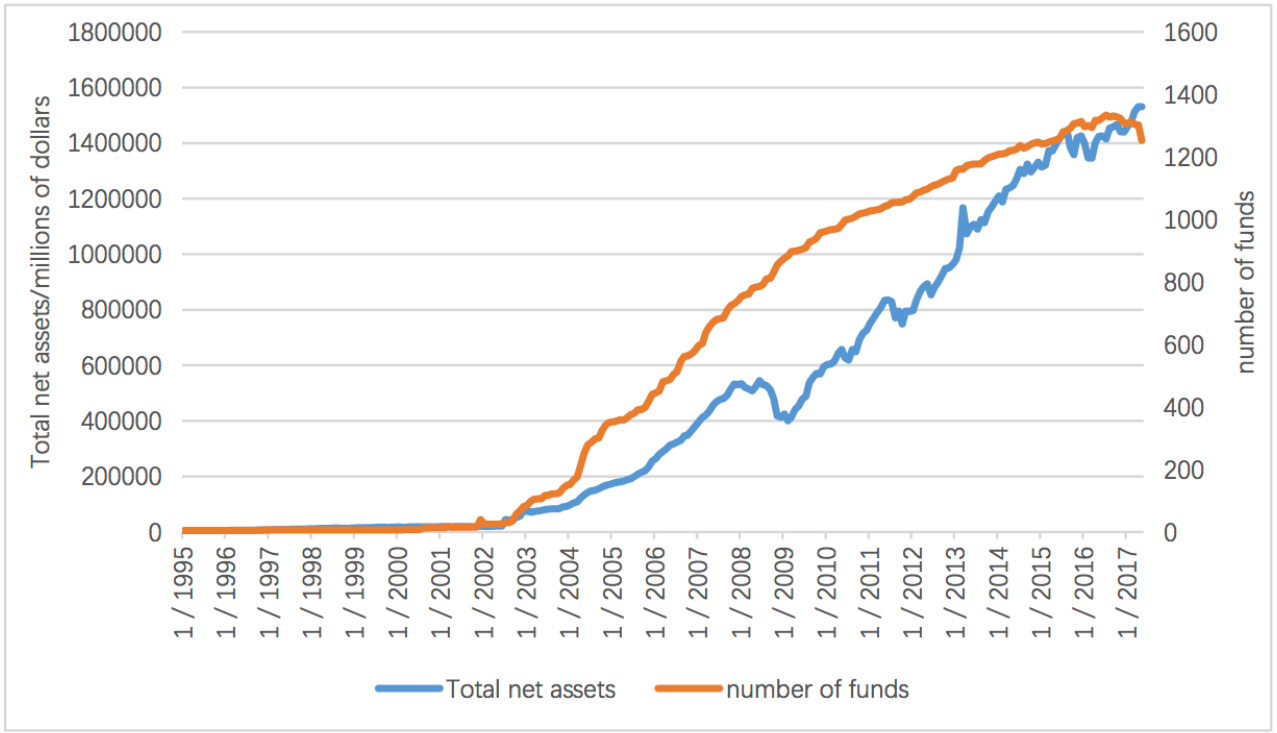



Figure: Funds of funds: Total net assets, number of funds (monthly)

# Introduction

- In this report, we studied:
  - The time trend of the number and the total net assets of FOFs in America in past 20 years.
  - The cointegration relationship between the fund of funds market and the retirement market.
- Data:
  - The monthly asset data of every American fund of funds, from 1/1/1995 to 5/1/2017, using Bloomberg database.
  - The quarterly data of pension in defined contribution plans, from Q1-2007 to Q4-2006, using Bloomberg database.

# Overview

# Total Net Assets of FOF

## Data Characteristics

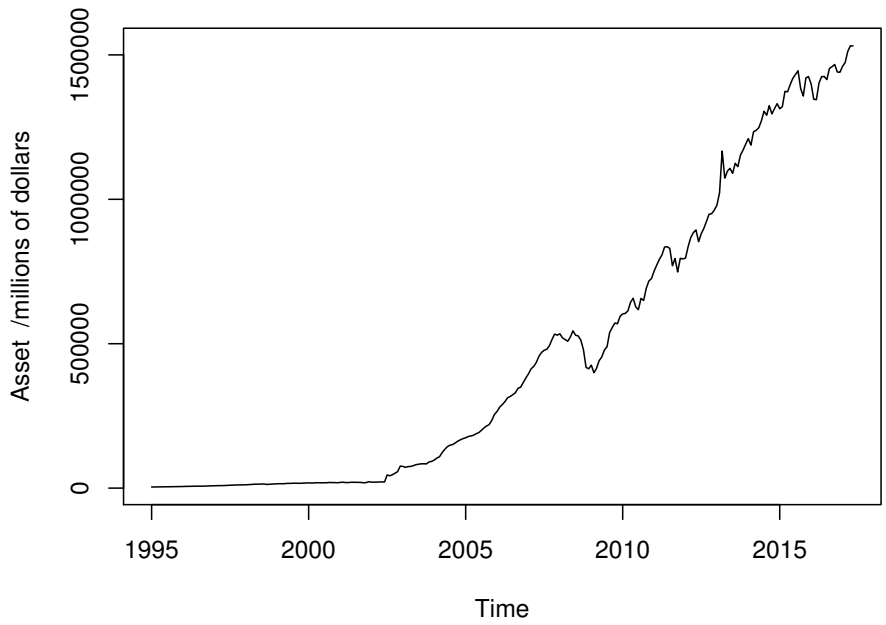



Figure: Funds of Funds: Total Net Assets of FOF

ADF Test of ast: p-value=0.8158 alternative hypothesis: stationary

# Total Net Assets of FOF

## Data Characteristics

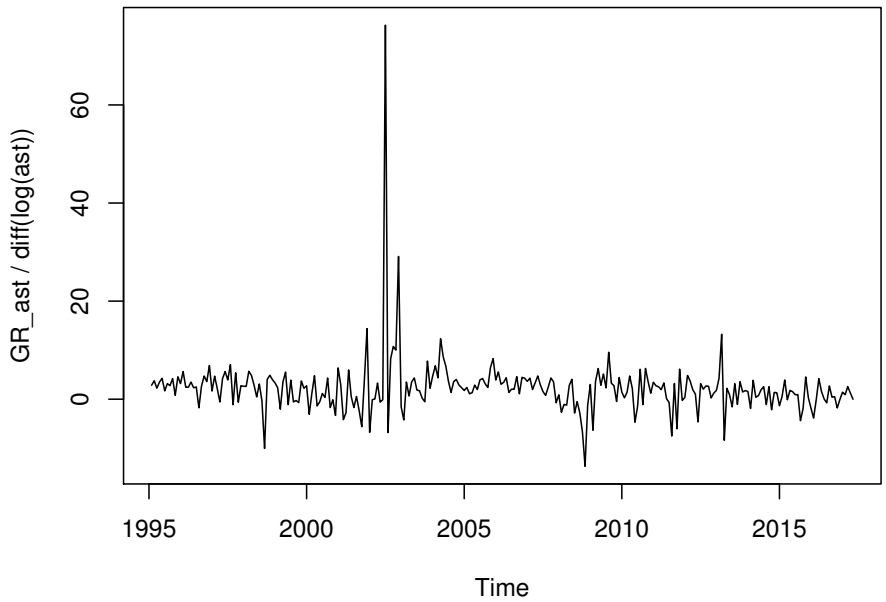


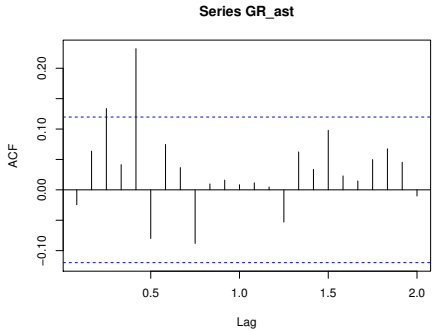

Figure: Growth Rate of FOF's Total Net Assets

ADF Test of GR_ast: p-value$<$0.01 alternative hypothesis: stationary

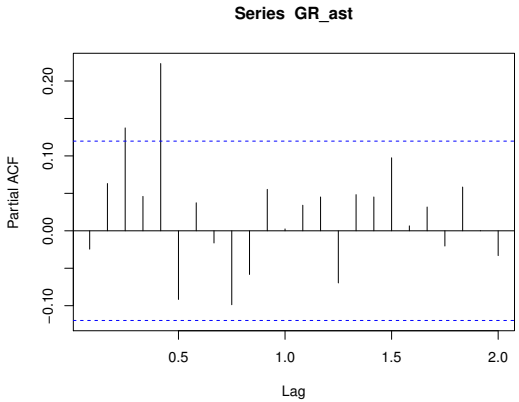
So,the GR_ast is a stationary series

# Total Net Assets of FOF

## MA Model for GR_ast




(a) ACF of GR_ast




(b) PACF of GR_ast

```
AR/MA
  0 1 2 3 4 5 6 7 8 9 10 11 12 13
0 o o x o x o o o o o o  o  o  o
1 x o o o x o o o o o o  o  o  o
2 x x o o x x o o o o o  o  o  o
3 x x o o x o o o o o o  o  o  o
4 x x o x x o x o o o o  o  o  o
5 x o o x x o x o o o o  o  o  o
6 x x o x x o x o o o o  o  o  o
7 x o o o x o o o o o o  o  o  o
```

(c) EACF of GR_ast

```
Call:
arima(x = GR_ast, order = c(0, 0, 5), fixed = c(0, 0, NA, 0, NA, NA))

Coefficients:
      ma1  ma2     ma3  ma4     ma5  intercept
        0    0  0.1352    0  0.2156     2.2333
s.e.    0    0  0.0616    0  0.0556     0.4705
```

(d) MA Model

Figure: Identify MA Model

# Total Net Assets of FOF

## Model Diagnostic Check

We found that the MA Model is so adequate that the squared residuals series have no correlation

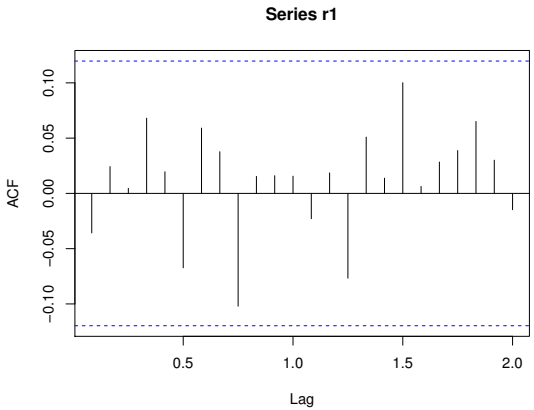



Figure: ACF of residuals

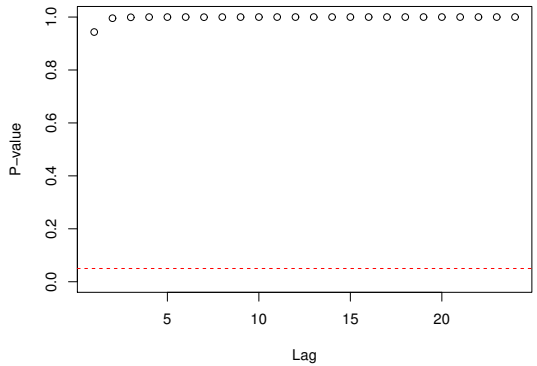



Figure: McLeod.Li.test of residuals

Box.test of residuals:

p-value=0.8388644

computing the Box.test with the squared data

# Total Net Assets of FOF

## Model Diagnostic Check

There is an outlier much bigger than others and it makes others' volatility so small. So we should find the outlier and adjust it.

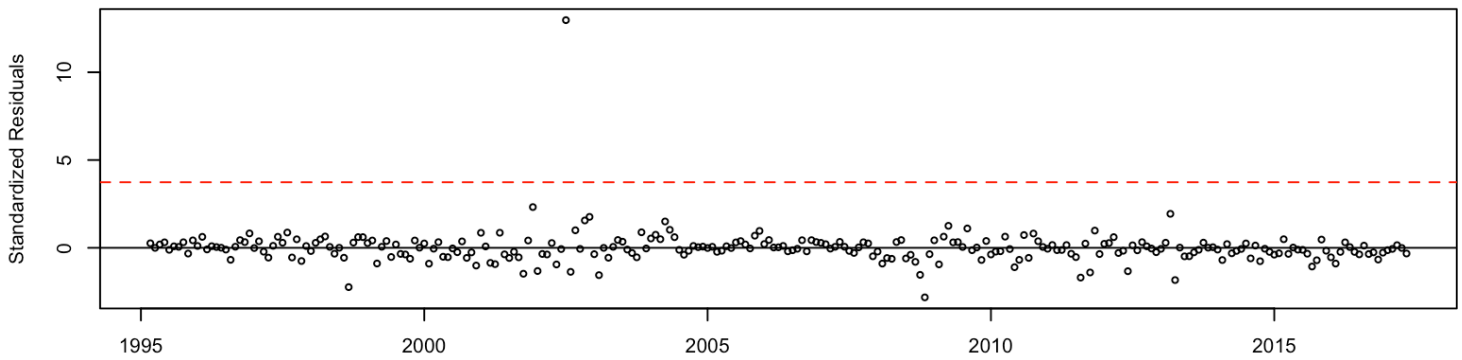



Figure: Standardized Residuals of MA Model for GR_ast

# Total Net Assets of FOF

## Model Diagnostic Check

We used function detectAO() and detectIO() (by Bonferroni Law ) to check the model and find the oulier, as figure.

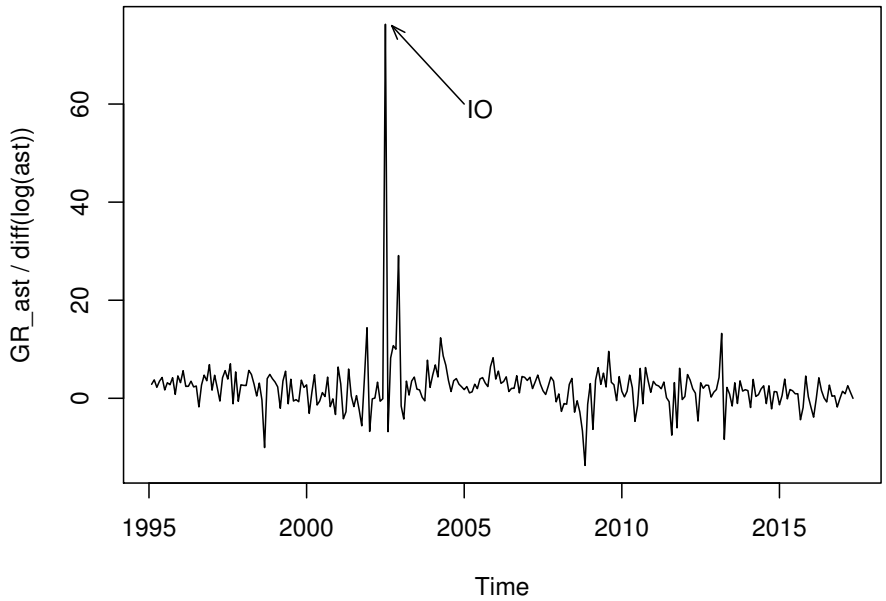


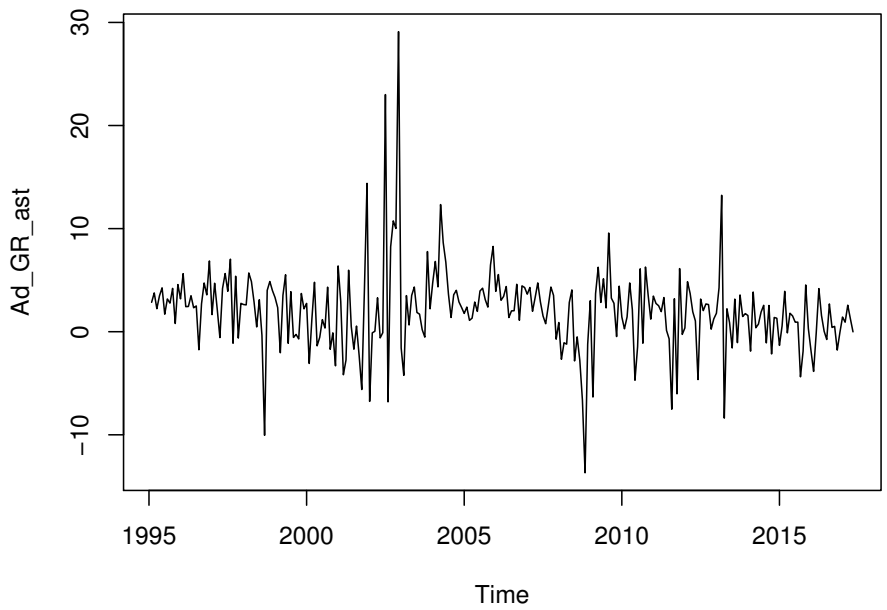

Figure: Detected outlier

# Total Net Assets of FOF

## Model Diagnostic Check

We adjusted the outlier :

$$Ad\_GR\_ast[90] = \frac{1}{3}(GR\_ast[89] + GR\_ast[90] + GR\_ast[91])$$



# Total Net Assets of FOF

## Model Diagnostic Check

Modeling a new MA model for the adjusted series and we found the residuals have conditional heteroskedasticity.

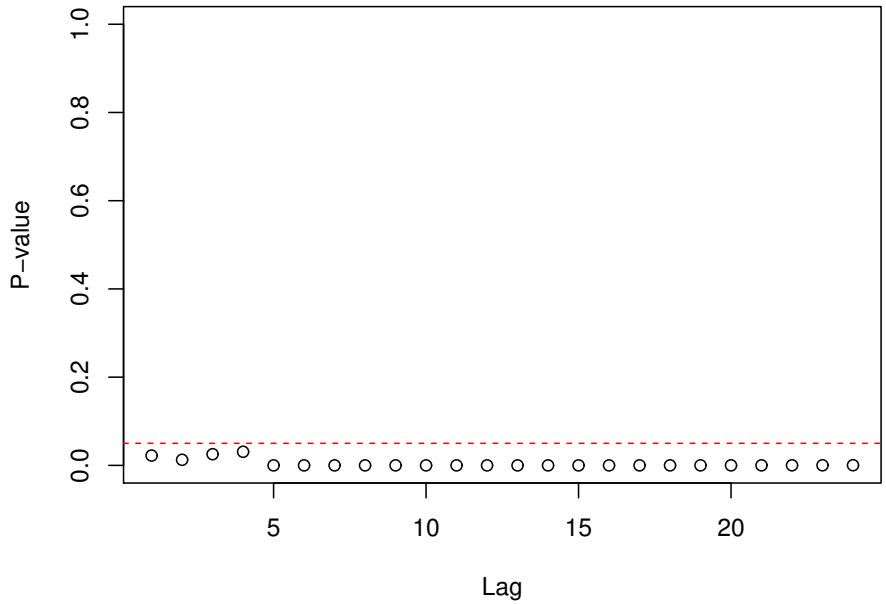



Figure: McLeod.Li.test for the residuals of adjusted series

# Total Net Assets of FOF

## ARMA(0,5)-GARCH(1,1) Model

After trying some times, we fit an ARMA(0,5)-GARCH(1,1) model (ma1=ma2=ma4=0, not significant) for the adjusted series.

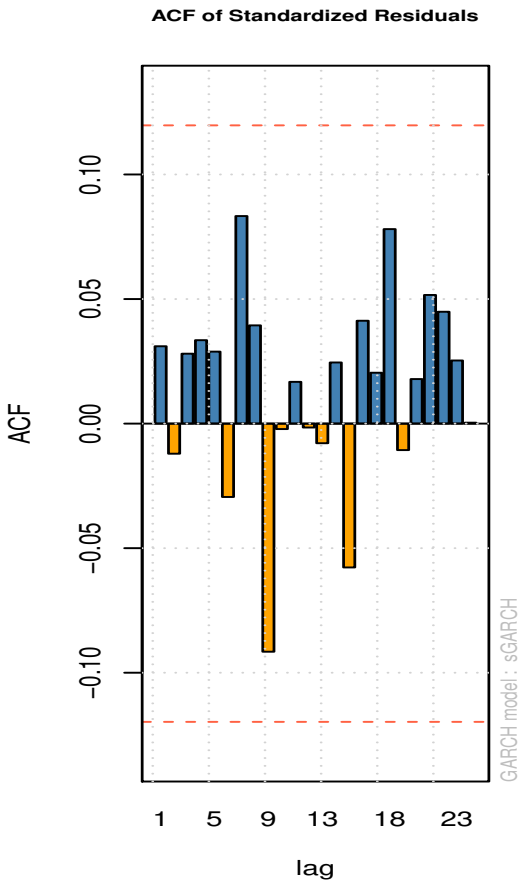
| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 2.072883 | 0.202273 | 10.2480 | 0.000000 |
| ma1 | 0.000000 | NA | NA | NA |
| ma2 | 0.000000 | NA | NA | NA |
| ma3 | 0.069979 | 0.051529 | 1.3581 | 0.174447 |
| ma4 | 0.000000 | NA | NA | NA |
| ma5 | 0.157267 | 0.050609 | 3.1075 | 0.001887 |
| omega | 7.802612 | 6.165227 | 1.2656 | 0.205662 |
| alpha1 | 0.558752 | 0.420160 | 1.3299 | 0.183566 |
| beta1 | 0.440247 | 0.130388 | 3.3764 | 0.000734 |
| shape | 2.488051 | 0.516139 | 4.8205 | 0.000001 |

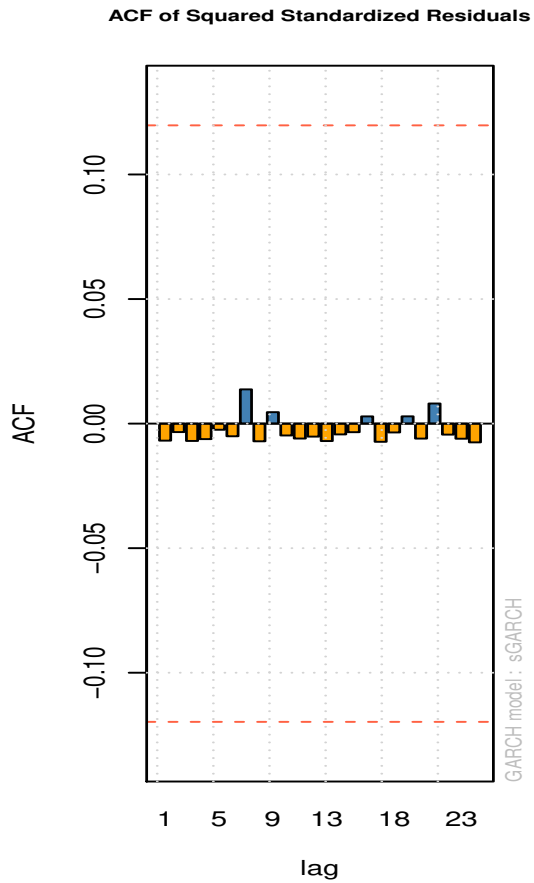
Figure: Parameters and t-value, Distribution : std

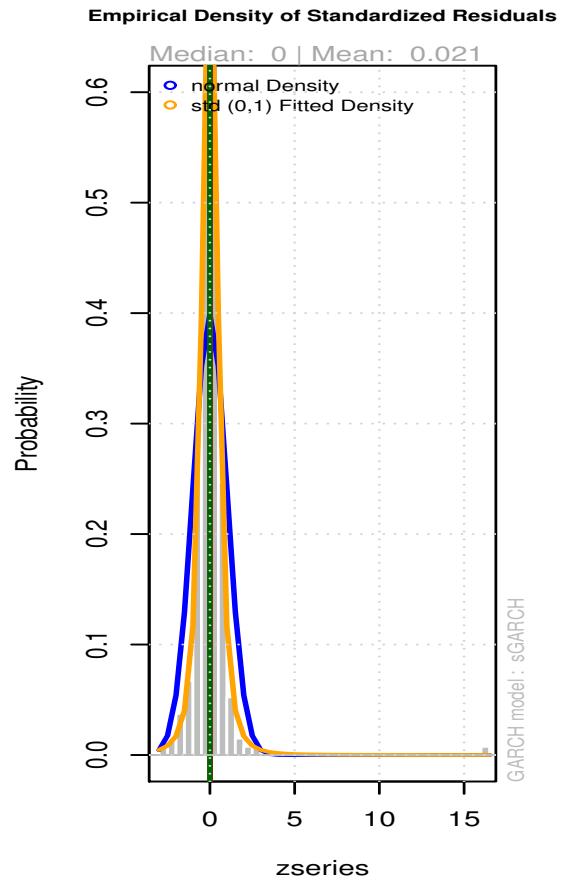
# Total Net Assets of FOF

## ARMA(0,5)-GARCH(1,1) Model

The ARMA(0,5)-GARCH(1,1) with std is adequate.

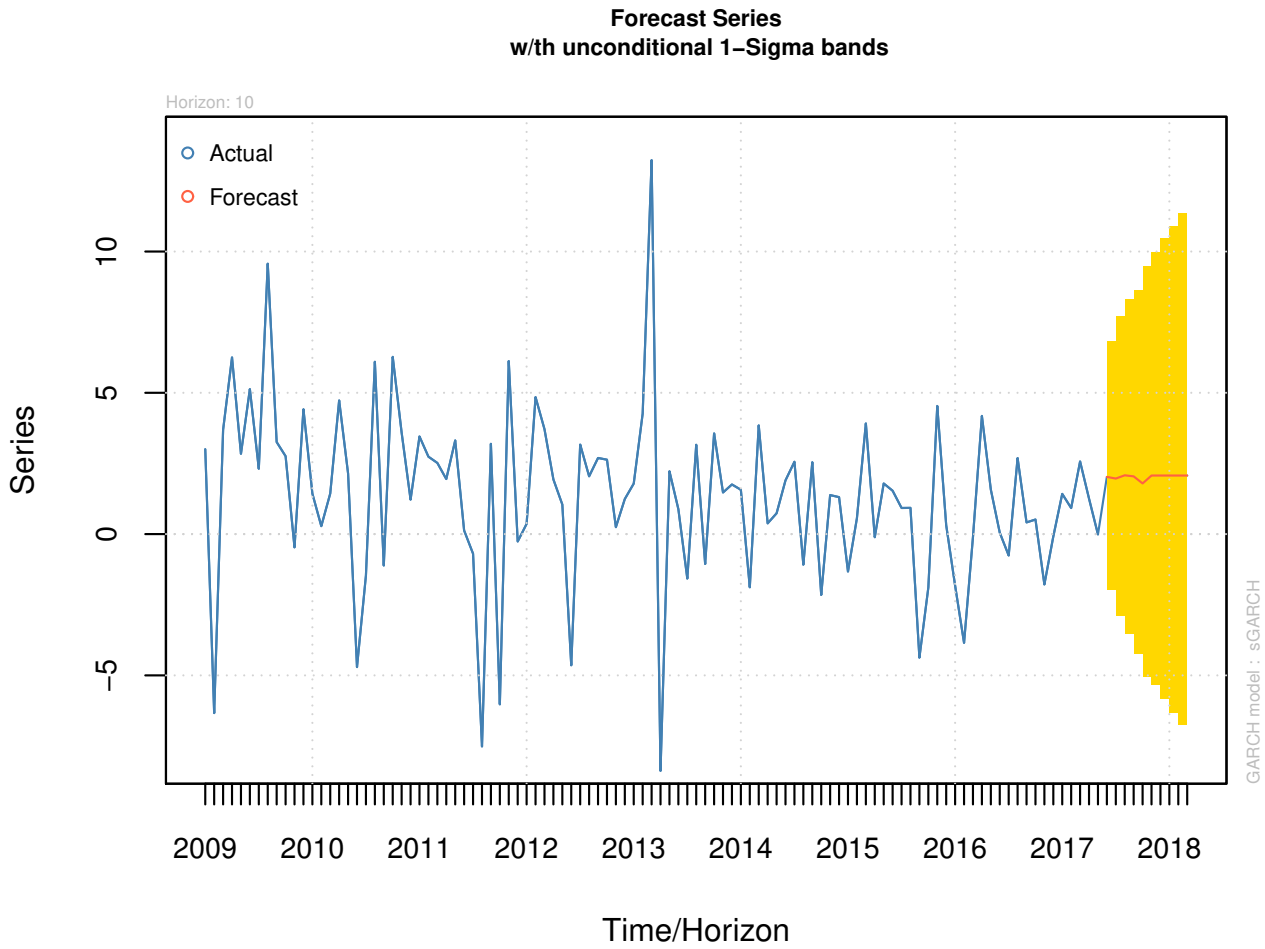


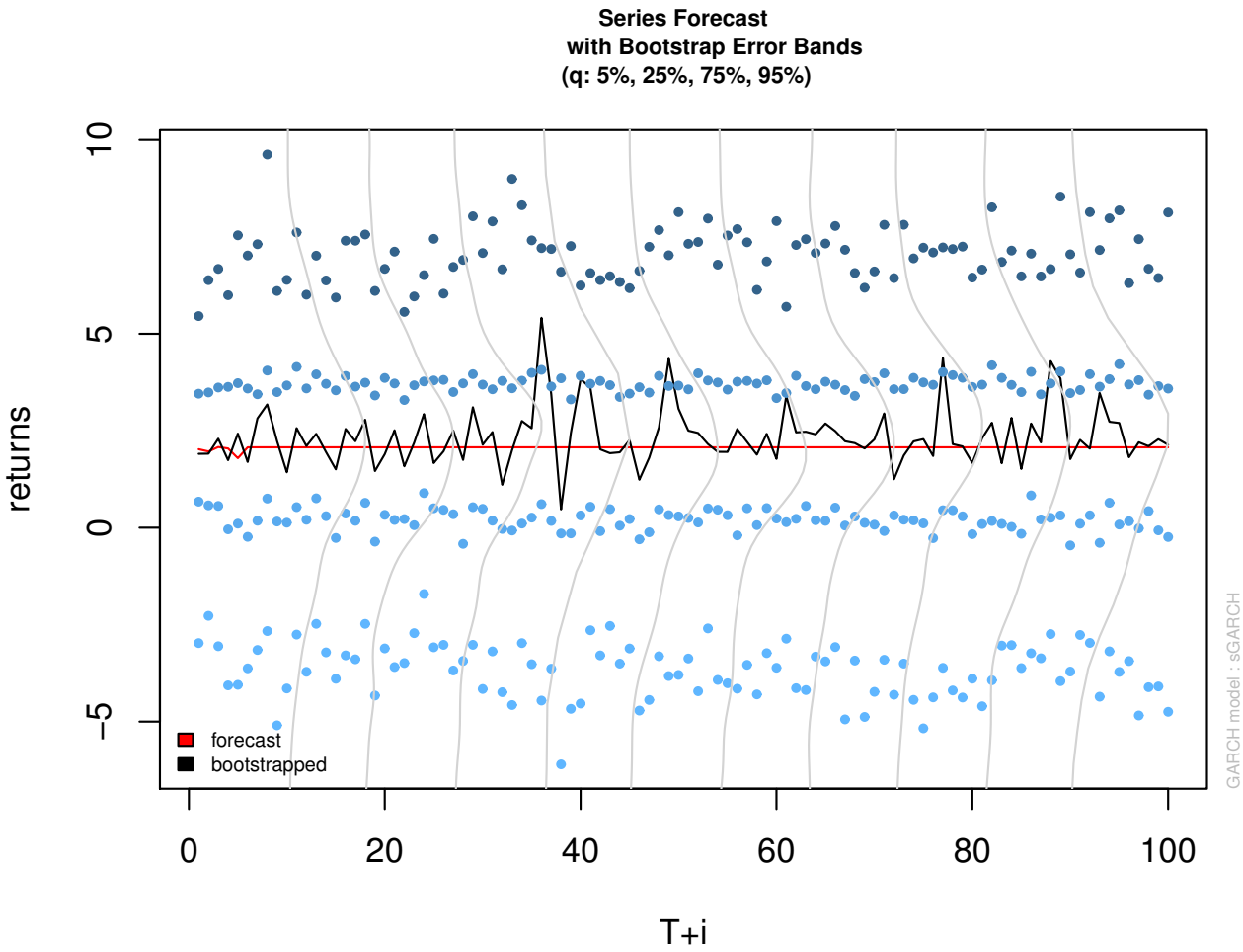
# Total Net Assets of FOF

## ARMA(0,5)-GARCH(1,1) Model



# Total Net Assets of FOF

## ARMA(0,5)-GARCH(1,1) Model



# Overview

# Number of FOFs

## Data Characteristics

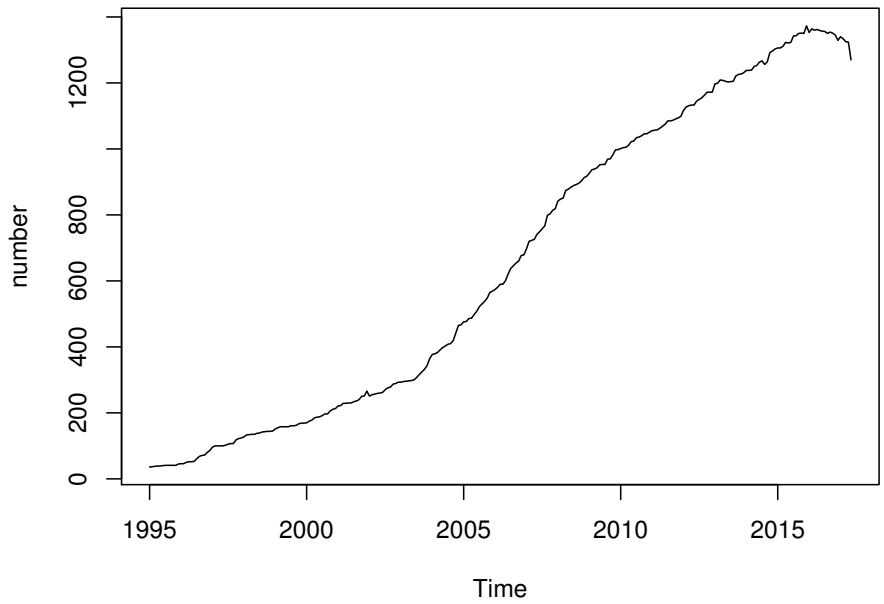



Figure: Funds of Funds: Number of FOFs

ADF Test of ast: p-value=0.985 alternative hypothesis: stationary

# Number of FOFs

## Data Characteristics

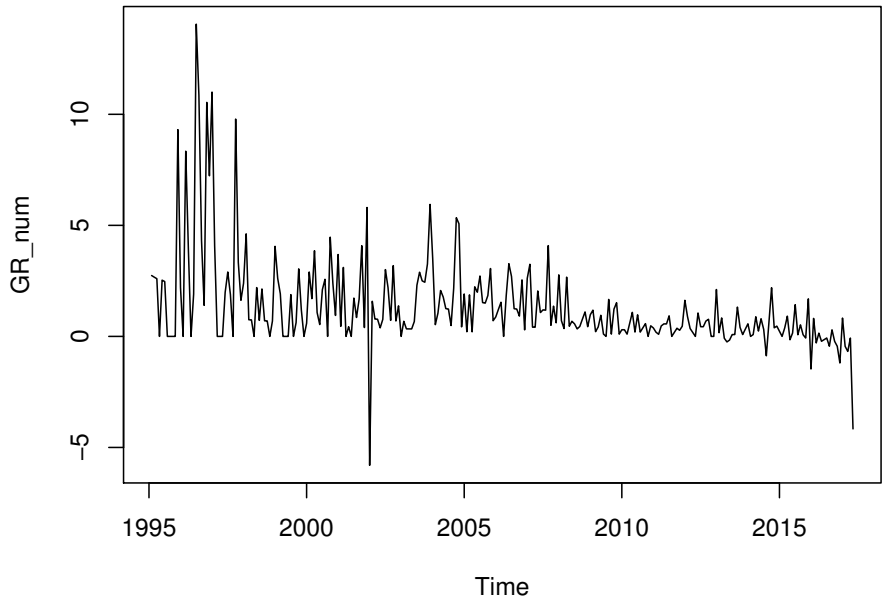



Figure: Growth Rate of FOF's Number

ADF Test of GR_ast: p-value$<$0.01 alternative hypothesis: stationary

So,the GR_num is a stationary series

# Number of FOFs

## ARMA(3,5)-eGARCH(1,1) Model

We firstly fit a ARMA(3,3) model for GR_num series and the Box.test(residuals) : p-value=0.5521436. And the McLeod.Li.test indicates that there is a ARCH effect.

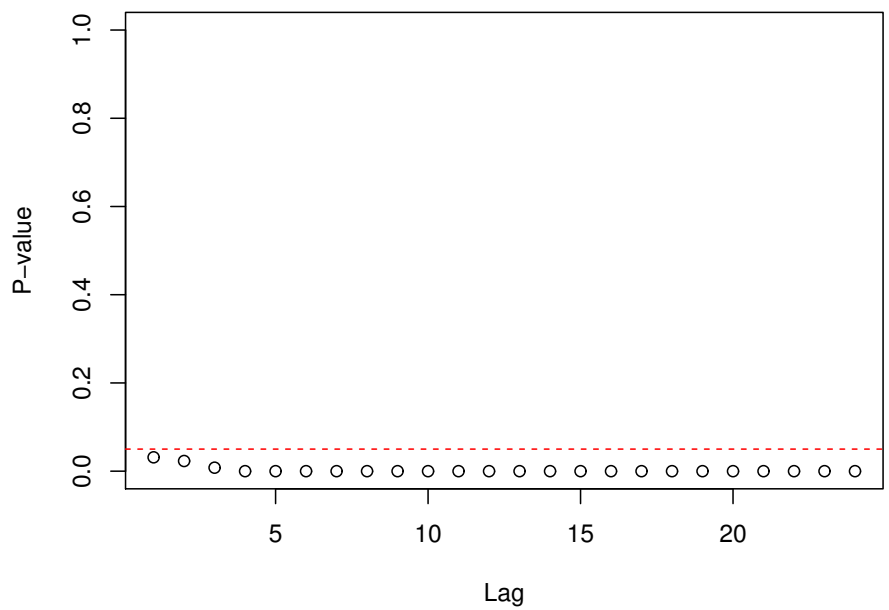



Figure: McLeod.Li.test for the residuals

# Number of FOFs

## ARMA(3,5)-eGARCH(1,1) Model

After trying, we fit an ARMA(3,5)-eGARCH(1,1) model for the adjusted series.

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 2.615540 | 0.625937 | 4.17860 | 0.000029 |
| ar1 | 1.372733 | 0.007493 | 183.20082 | 0.000000 |
| ar2 | -1.261653 | 0.006375 | -197.90744 | 0.000000 |
| ar3 | 0.889158 | 0.006031 | 147.42112 | 0.000000 |
| ma1 | -1.322301 | 0.018999 | -69.59889 | 0.000000 |
| ma2 | 1.268756 | 0.021915 | 57.89419 | 0.000000 |
| ma3 | -0.795558 | 0.015635 | -50.88312 | 0.000000 |
| ma4 | -0.061642 | 0.018559 | -3.32148 | 0.000895 |
| ma5 | 0.083884 | 0.023332 | 3.59523 | 0.000324 |
| omega | 0.047724 | 0.026200 | 1.82153 | 0.068526 |
| alpha1 | -0.011091 | 0.064722 | -0.17136 | 0.863942 |
| beta1 | 0.959178 | 0.018981 | 50.53461 | 0.000000 |
| gamma1 | 0.459072 | 0.123103 | 3.72918 | 0.000192 |

Figure: Parameters and t-value, Distribution : norm

# Number of FOFs

## ARMA(3,5)-eGARCH(1,1) Model

The mean model and variance model are both adequate.

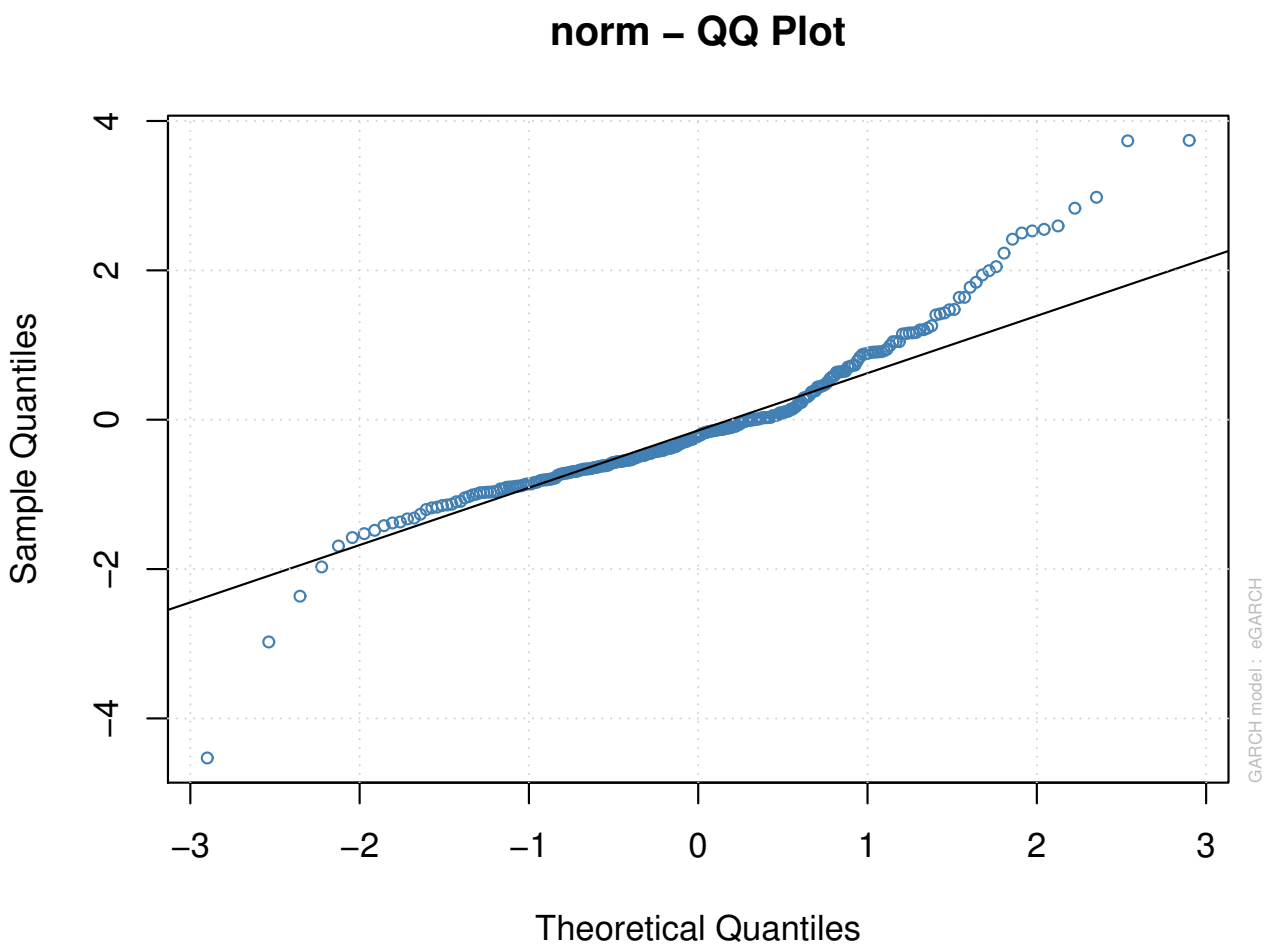
```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                     0.04726  0.8279
Lag[2*(p+q)+(p+q)-1][23]   7.97262  1.0000
Lag[4*(p+q)+(p+q)-1][39]  16.08229  0.8872
d.o.f=8
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                         statistic p-value
Lag[1]                     0.03929  0.8429
Lag[2*(p+q)+(p+q)-1][5]    0.27847  0.9859
Lag[4*(p+q)+(p+q)-1][9]    1.72204  0.9355
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.09952 0.500 2.000  0.7524
ARCH Lag[5]   0.30191 1.440 1.667  0.9398
ARCH Lag[7]   1.90223 2.315 1.543  0.7380
```

Figure: Model Diagnostic Check

# Number of FOFs

## ARMA(3,5)-eGARCH(1,1) Model

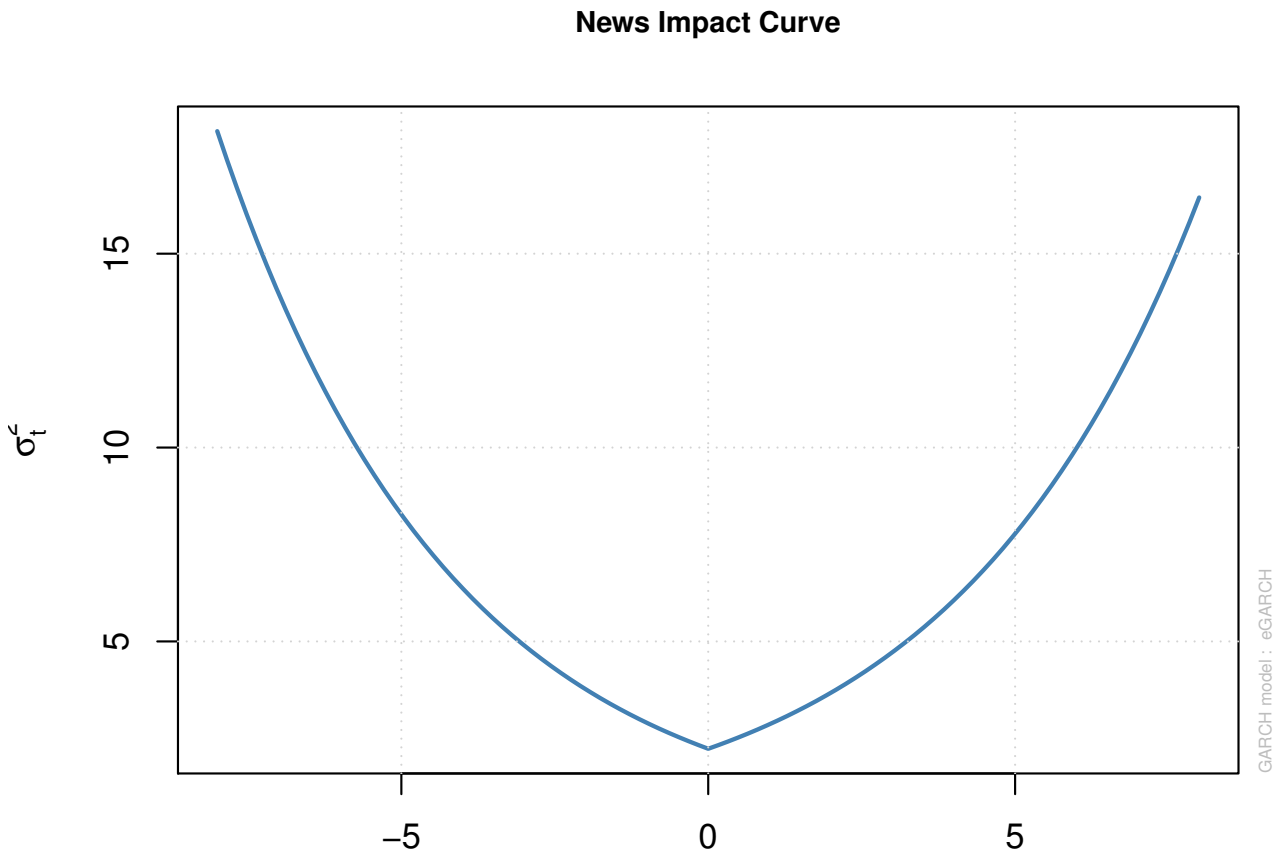
Standardized Residuals have a fatter tail distribution



# Number of FOFs

## ARMA(3,5)-eGARCH(1,1) Model

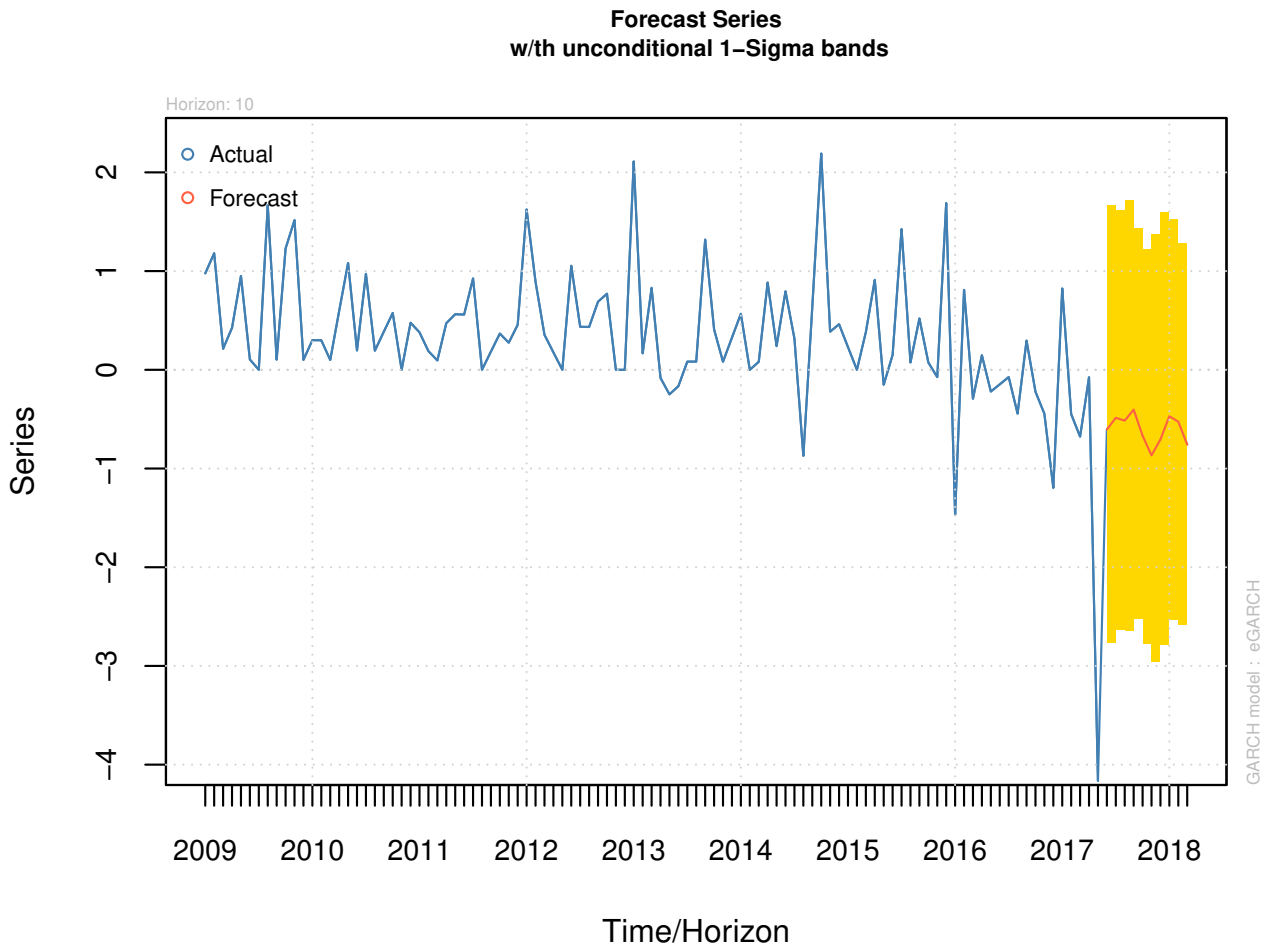
It means that the GR_num is more sensitive for negative impact than positive impact. It's EGARCH 's advantage.



# Number of FOFs

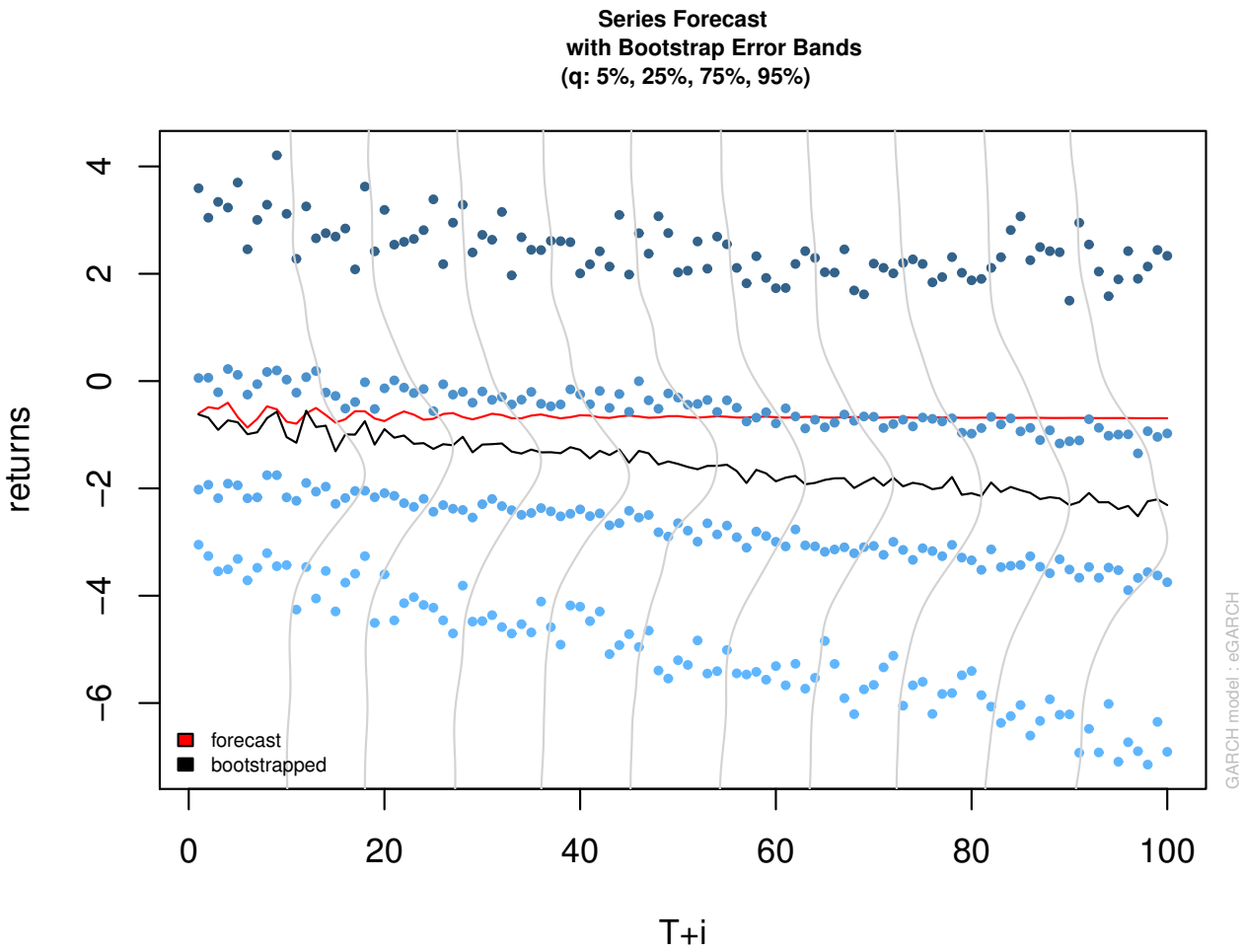
## ARMA(3,5)-eGARCH(1,1) Model

We also can make a forecast



# Number of FOFs

## ARMA(3,5)-eGARCH(1,1) Model

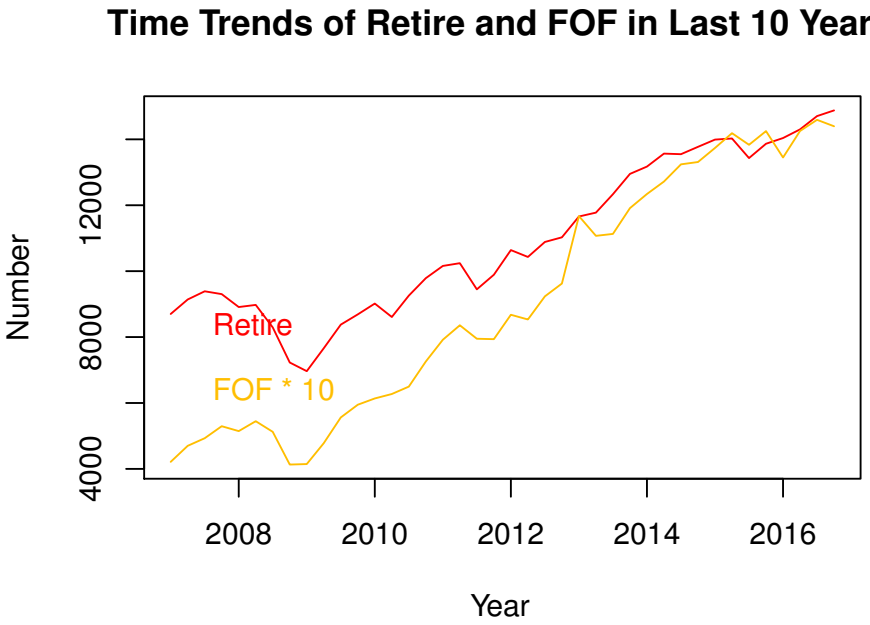


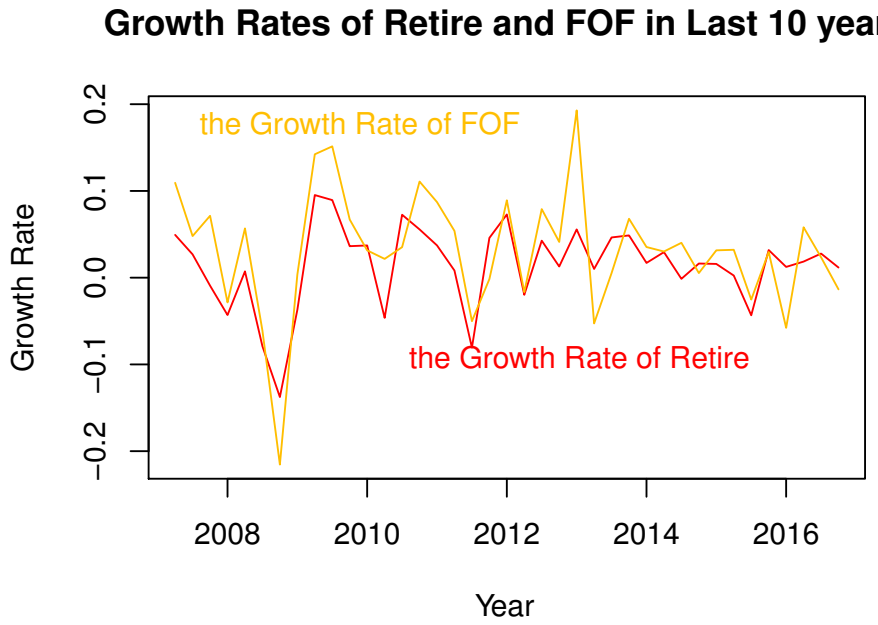
# Overview

# Cointegration with Retirement Market

- In this part, we would explore the relationship between the fund market and the retirement market.
- The Fund of Funds is favored by risk averter, especially for those who have retired.
- There might be cointegration relationships between the two markets.

# Cointegration with Retirement Market



# Cointegration with Retirement Market

## Unit Root Test

- We tried 3 tests to specify the stationarity of the data: augmented Dickey–Fuller test, Kwiatkowski–Phillips–Schmidt–Shin test and Phillips–Perron test.

Table

| TEST Method | ADF | KPSS | PP |
|---|---|---|---|
| FOF | 2.53 | 1.07 | -0.16 |
| diff(FOF) | -3.4 | 0.11 | 40 |
| Retire | 1.64 | 1.01 | 0.23 |
| diff(Retire) | -2.31 | 0.18 | 1.55 |
| 10pct | -1.61 | 0.35 | * |
| 5pct | -1.95 | 0.46 | 0.26 |
| 1pct | -2.62 | 0.74 | * |

# Cointegration with Retirement Market

## Cointegration Relationship

- First, estimate relationship between FOF and Retire.
- $FOF_t = \alpha + \beta * Retire_t + \mu_t$

| Coefficients: | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| (Intercept) | -7.552e+02 | 5.632e+01 | -13.41 | 5.51e-16 *** |
| Retire | 1.524e-01 | 5.042e-03 | 30.22 | <2e-16 *** |

- The results of Unit Root Test of $u_t$ show that $u_t$ is stationary, meaning that two I(1) processes generate one I(0) process.

| Tests | ADF-Test | KPSS-Test | PP-Test |
|---|---|---|---|
| Statistics | -3.1799 (<1pct | 0.2674(<10pct) | -10.0379 (<Z-tau) |

# Cointegration with Retirement Market

## Error Correction Model

- Let $y = diff(FOF)$ and $x = diff(Retire)$ . And set the ECM equation as:

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + \alpha_4 y_{t-4} + \beta_0 x_t + \beta_1 x_{t-1} + \gamma r_{t-1} + \epsilon_t$$

- The results indicate cointegration relationship between FOF and Retire, and the cointegration vector is (1, -0.15).

# Cointegration with Retirement Market

## Error Correction Model

$$y_t = \alpha_1 y_{t-1} \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + \alpha_4 y_{t-4} + \beta_0 x_t + \beta_1 x_{t-1} + + \gamma r_{t-1} + \epsilon_t$$

.

| Coefficients: | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| (Intercept) | 22.13335 | 11.14436 | 1.986 | 0.0573 |
| L(y, 1) | -0.46108 | 0.19994 | -2.306 | 0.029 * |
| L(y, 2) | -0.01601 | 0.12908 | -0.124 | 0.9022 |
| L(y, 3) | -0.03563 | 0.12999 | -0.274 | 0.7861 |
| L(y, 4) | -0.02875 | 0.13862 | -0.207 | 0.8373 |
| L(x, 1) | 0.05842 | 0.02549 | 2.292 | 0.03 * |
| L(x, 0) | 0.09517 | 0.01852 | 5.138 | 0.000021 *** |
| L(r, 1) | -0.38373 | 0.16855 | -2.277 | 0.0309 * |

# Overview

# Conclusion

- We adjusted the outlier and fit a ARMA(0,5)-GARCH(1,1) model for GR_ast
- We fit a ARMA(3,5)-eGARCH(1,1) model for GR_num
- We find cointegration relationship between the fund market and the pension market. And build an error correction medel.

# End

Thanks!
Q&A