Experiment 1: Communications and Filter Technology

Pre-Laboratory Assignment

M. Ostermann, M.Sc. Technische Kybernetik, 3539468
E. Rommel, M.Sc. Technische Kybernetik, 3552157
J. Yu, M.Sc. Autonome Systeme, 3532601
Z. Zhou, M.Sc. Autonome Systeme, 3524224

28.06.2022

V1.1

Assume that the error has a uniform distribution in each step of quantizer and the variance can be computed in the interval between $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$.

$$\begin{aligned}
{\sigma_e}^2 &= \int_{-\Delta/2}^{\Delta/2} e^2 p(e) \mathrm{d}e \\
&= \int_{-\Delta/2}^{\Delta/2} e^2 (\frac{1}{\Delta}) \mathrm{d}e \\
&= \frac{e^3}{3\Delta}\Big|_{-\Delta/2}^{\Delta/2} \\
&= \frac{\Delta^2}{12}
\end{aligned}$$

The number of quantization Q is 2^{R_b},so the variance is:

$$\begin{aligned}
\Delta = \frac{\Delta u_1}{Q} &= \frac{x_{max} - x_{min}}{2^{R_b}} \\
{\sigma_e}^2 &= \frac{(\frac{x_{max}-x_{min}}{2^{R_b}})^2}{12}
\end{aligned}$$

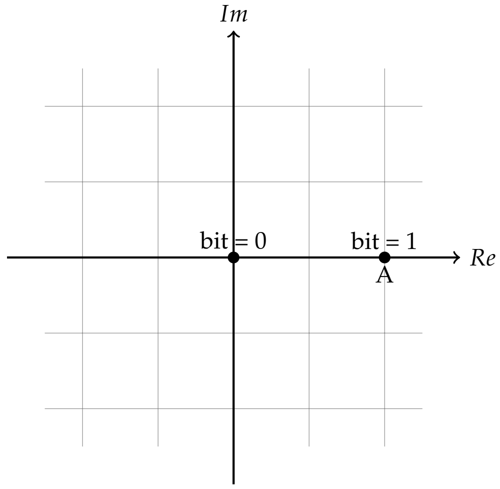
The variance of signal is computed with the equation in script and then the final SQNR can be calculated:

$$\begin{aligned}
SQNR &= \frac{{\sigma_s}^2}{{\sigma_e}^2} \\
&= \frac{\frac{(x_{max}-x_{min})^2}{12}}{\frac{(x_{max}-x_{min})^2}{12 \cdot 2^{2R_b}}} \\
&= 2^{2R_b}
\end{aligned}$$

V1.2

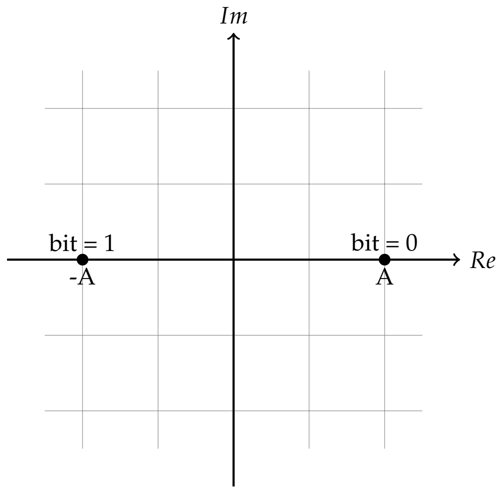
In constellations diagram the x-axis is the real number part of $A(t)e^{j\Phi t}$ and y-axis is the imaginary number part. so the two point locate in $(A, 0)$ and $(0, 0)$.

$$BASK : S(t) = \begin{cases} 0, & \text{bit} = 0 \\ A\cos(\omega t), & \text{bit} = 1 \end{cases}$$



The phase of two stages is either 0 or π, so two points locate in $(A, 0)$ and $(-A, 0)$.

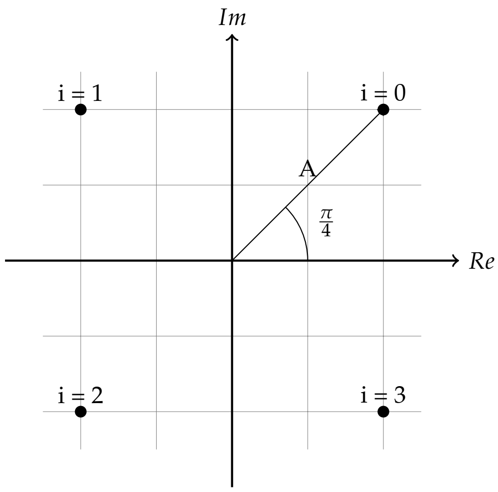
$$BPSK : S(t) = \begin{cases} A\cos(\omega t + 0), & \text{bit} = 0 \\ A\cos(\omega t + \pi), & \text{bit} = 1 \end{cases}$$



V1.3

4-PSK can switch between 4 stages, whose i equals 0,1,2,3 and meanwhile, the phase change between $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

$$S_i(t) = A\cos\omega t + \frac{2\pi i}{4} + \frac{\pi}{4} \qquad i = 0, 1, 2, 3$$



V1.4

1. With $d_{\min} = 3$ we get the following values for k, r and n:

$$\begin{aligned} r &= 3 \\ k &= 2^r - r - 1 = 2^3 - 3 - 1 = 4 \\ n &= 2^r - 1 = k + r = 2^3 - 1 = 7 \end{aligned}$$

The parity bits for $k = 4$ are calculated with the equations

$$\begin{aligned} P_1 &= X_1 \oplus X_2 \oplus X_3, \\ P_2 &= X_2 \oplus X_3 \oplus X_4, \\ P_3 &= X_1 \oplus X_3 \oplus X_4, \end{aligned}$$

where $\oplus$ represents the modulo-2 addition and X_1 to X_4 are the transmitted information bits of block length k. Therefore, the generator matrix G, the parity check matrix H and the decoding matrix D are

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix},$$
$$H = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix},$$
$$D = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}^T.$$

2. For the code word $x = 1100$ the code word c is calculated as

$$\begin{aligned} c &= G_{(1,:)} \oplus G_{(2,:)} \\ &= 1000101 \oplus 0100110 \\ &= 1100011. \end{aligned}$$

3. If the wrong word $c_1 = 1101011$ is being received, the error can be detected and also corrected. By calculating the parity vector

$$\begin{aligned} p_1 &= c_1 H^T \\ &= (1\,1\,0\,1\,0\,1\,1) \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= (0\,1\,1) \neq 0 \end{aligned}$$

we know that an error exists as the vector is nonzero. The vector p_1 is the same as the fourth row of the matrix H^T, therefore the error occurred in the fourth bit of c. By flipping this bit, the error can be corrected to $c'_1 = (1\ 1\ 0\ 0\ 0\ 1\ 1)$. With the decoding matrix D we receive the correct transmitted information without the parity bits. The decoded data is:

$$\begin{aligned} x'_1 &= c'_1 D \\ &= (1\,1\,0\,0\,0\,1\,1) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ &= (1\,1\,0\,0) \end{aligned}$$

4. If the word $c_2 = 1101010$ with two wrong bits is received, the parity vector p_2 is

$$\begin{aligned} p_2 &= c_2 H^T \\ &= (1\,1\,0\,1\,0\,1\,0) \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= (0\,1\,0) \neq 0, \end{aligned}$$

which also shows that an error exists. If we compare p_2 to the matrix H^T, the 6th bit would have to be corrected. We get the wrongly corrected code word $c'_2 = (1\,1\,0\,1\,0\,0\,0)$. The decoded data would be $x'_2 = (1\ 1\ 0\ 1)$. This shows that two errors can still be detected, but none of them can be corrected.

V1.5

In the following, the diagrams for Exercise 1.-4. are depicted. Applying the input "110"to the described system generates the output code word $c = \{11, 01, 11\}$ (marked blue in the trellis diagram).

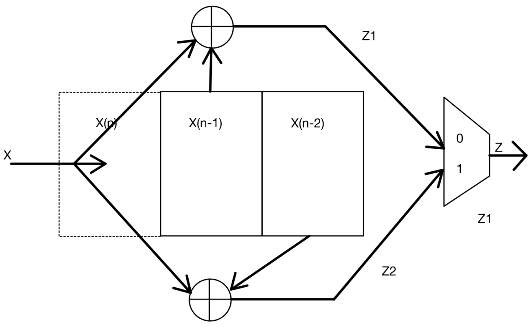


Abb. 1: Encoder structure for generator vectors $g_1 = [110]$ and $g_2 = [101]$

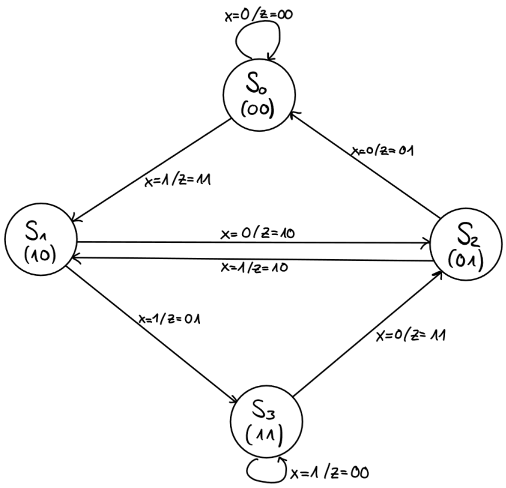


Abb. 2: State diagram for generator vectors $g_1 = [110]$ and $g_2 = [101]$

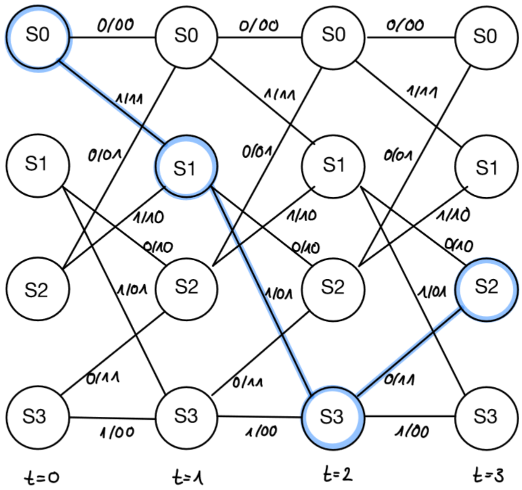


Abb. 3: Trellis diagram for generator vectors $g_1 = [110]$ and $g_2 = [101]$. The blue path corresponds to the input "110".