ELEC 341 Jian Gao

Assignment 4 [redacted]

Problem 1

1) $T(s) = \frac{Ks}{(s+1)(0.1s+1)(0.01s+1)}$

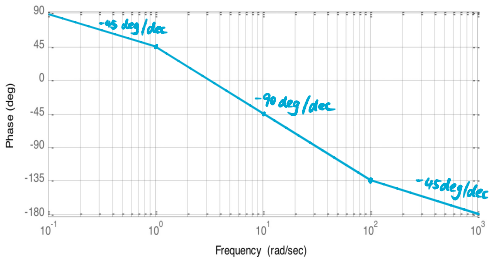
Plug in $w = 0.1 \Rightarrow K \cdot 0.1 \approx 1$

$\Rightarrow K = 10$

$T(s) = \frac{10s}{(s+1)(0.1s+1)(0.01s+1)}$

poles: 1, 10, 100

2)



Problem 2

a) $T_s = \frac{4}{\zeta w_n} = 2$ sec

From the figure: $M_{pw} = 1.3$

O.P. = 30% $\Rightarrow \zeta = 0.36$

$w_n = \frac{2}{0.36} = 5.56$

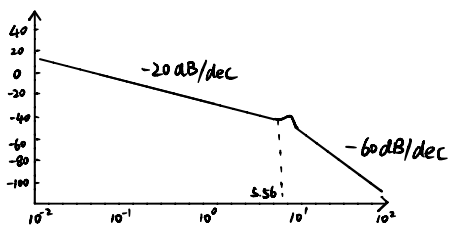
$Q(s) = s^2 + 2\zeta w_n s + w_n^2$

$= s^2 + 4s + 30.91$

$G(s) = \frac{1}{s^2+4s+30.91}$

b) $G_c(s)G(s) = \frac{K}{s \cdot (s^2+4s+30.91)}$

(K=1) Bode Plots: $s = 0.01 \Rightarrow G_c(s)G(s) > 1$ $s = 0.1 \Rightarrow G_c(s)G(s) < 1$ $w_n = 5.56$



d) $G_c(s)G(s) = \frac{K}{s \cdot (s^2+4s+30.91)}$

$Q(s) = s^3 + 4s^2 + 30.91s + K$

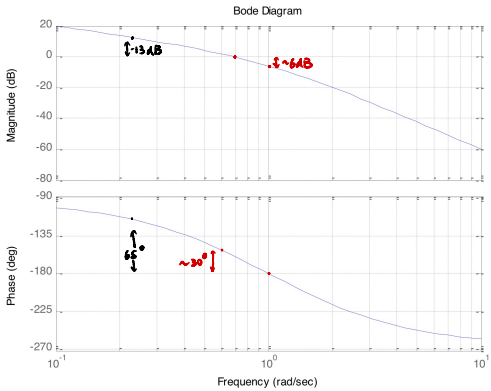
s^3	1	30.91
s^2	4	K
s^1	$30.91 - \frac{K}{4}$	0
s^0	K	0

$4s^2+K \overline{)\ s^3+30.91s}$; $\frac{1}{4}s$; $s^3 + \frac{K}{4}s$; $(30.91 - \frac{K}{4})s$

Stable $\Rightarrow \begin{cases} K > 0 \\ 30.91 - \frac{K}{4} > 0 \end{cases} \Rightarrow 0 < K < 123.64$

Problem 3

a)



Gain Margin : 6dB

Phase Margin : 30°

b) Phase Margin: 65°

=> Gain Margin : -13dB

crossover freq: $\omega_m = 0.23$ rad/s

Problem 4

a) $m\ddot{y}(t) = f(t) - b\dot{y}(t) - Ky(t)$

$\vec{x}(t) = \begin{bmatrix} ky(t) \\ b\dot{y}(t) \end{bmatrix}$ $\quad \dot{\vec{x}}(t) = \begin{bmatrix} k\dot{y}(t) \\ b\ddot{y}(t) \end{bmatrix}$

$k\dot{y}(t) = \frac{k}{b} \cdot b\dot{y}(t)$

$b\ddot{y}(t) = \frac{b}{m}(f(t) - b\dot{y} - ky)$

SVM: $\dot{\vec{x}} = \begin{bmatrix} 0 & \frac{k}{b} \\ -\frac{b}{m} & -\frac{k}{m} \end{bmatrix}_A \vec{x} + \begin{bmatrix} 0 \\ \frac{k}{m} \end{bmatrix}_B f(t)$

$y = \begin{bmatrix} \frac{1}{k} & 0 \end{bmatrix}_C \vec{x} + \underbrace{0}_D \cdot f(t)$

b) $A = \begin{bmatrix} 0 & 25 \\ -0.2 & -0.2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$

$C = \begin{bmatrix} \frac{1}{500} & 0 \end{bmatrix}$ $\quad D = 0$

$T(s) = C[sI - A]^{-1}B + D$

$[sI - A]^{-1} = \begin{bmatrix} s & -25 \\ 0.2 & s+0.2 \end{bmatrix}^{-1} = \frac{1}{s^2+0.2s+5}\begin{bmatrix} s+0.2 & +25 \\ -0.2 & s \end{bmatrix}$

$= \frac{0.01}{s^2+0.2s+5}$

$Q(s) = s^2 + 0.2s + 5 = 0 \Rightarrow \zeta = 0.045 \Rightarrow O.P. = 85\%$

$T_s = \frac{4}{\zeta\omega_n} = 40$ s

c) $A_{new} = A - BK = \begin{bmatrix} 0 & 25 \\ -0.2 & -0.2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}\begin{bmatrix} K_1 & K_2 \end{bmatrix}$

$Q(s) = \det(sI - A_{new}) = s^2 + (0.2 + 0.2K_2)s + 25(0.2 + 0.2K_1) = 0$

$\begin{cases} P.O. \% \approx 16.3\% \\ T_s = \frac{4}{\zeta\omega_n} = 1 \end{cases} \Rightarrow \begin{cases} \zeta = 0.5 \\ \omega_n = 8 \end{cases}$

$\Rightarrow K = \begin{bmatrix} 11.8 & 39 \end{bmatrix}$

$u(t) = \begin{bmatrix} -11.8 & -39 \end{bmatrix} x(t)$