ELEC 341 HW2

Jian Gao [redacted]

Problem 1 (a)

a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ $\quad \det(sI - A) = 0 \quad (s-1)(s-2)=0$ ✗

b) No matrix / eigenvalues ✗

$\boxed{\text{c) } (s+3)(s+2)=0 \quad ✓}$

d) $(s+1)(s^2+4s+5)=0$ ✗

(b) Standard form: $\dfrac{4s+3}{s^2+8s+12}$

Control canonical form: $A = \begin{bmatrix} 0 & 1 \\ -8 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = \begin{bmatrix} 3 & 4 \end{bmatrix} \quad D = 0$

(c)

$A = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = 0$

$$\frac{Y(s)}{U(s)} = C[sI-A]^{-1}B + D$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s & 1 \\ -1 & s-2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \frac{1}{s^2-2s+1} \begin{bmatrix} s-2 & -1 \\ 1 & s \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{s^2-2s+1} \begin{bmatrix} 1 & s \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{s^2-2s+1}$$

Unit step input: $U(s) = \frac{1}{s} \quad (s>0)$

$$Y(s) = \frac{1}{s^3-2s^2+s} = \frac{1}{s(s-1)^2}$$

$$y(t) = (t-1)e^t + 1$$

Problem 2

$$\begin{cases} v_1 = L\frac{di_L}{dt} + v_R \\ v_R = v_c + v_2 = (i_L - i_c)R \\ i_c = C\frac{dv_c}{dt} \end{cases}$$

a) $$\begin{cases} \dot{i}_L = \frac{v_1 - v_c - v_2}{L} \\ \dot{v}_c = \frac{R i_L - v_c - v_2}{RC} \end{cases}$$

$$\dot{x} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & -\frac{1}{L} \\ 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{L} & -\frac{1}{L} \\ 0 & -\frac{1}{RC} \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = 0}$$

b) $$\begin{cases} \dot{i}_L = \frac{v_1 - v_R}{L} \\ \dot{v}_R = \dot{v}_c = \frac{i_L - \frac{v_R}{R}}{C} \end{cases}$$

$$\dot{x} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_R \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_R \end{bmatrix} + \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

c) $$\begin{cases} \dot{i}_L(t) = \frac{v_L}{L} \\ \dot{v}_L(t) = -\dot{v}_R(t) = -\dot{v}_c(t) \\ \quad = \frac{-i_L + \frac{v_1 - v_L}{R}}{C} \end{cases}$$

$$\dot{x} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_L \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{RC} & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} i_L \\ v_L \end{bmatrix} + \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

d) $$\begin{cases} \dot{i}_L(t) = \frac{v_1 - i_R \cdot R}{L} \\ \dot{i}_R(t) = (i_L - i_R)\frac{R}{C} \end{cases}$$

$$\dot{x} = \begin{bmatrix} 0 & -\frac{R}{L} \\ \frac{R}{C} & -\frac{R}{C} \end{bmatrix} \begin{bmatrix} i_L \\ i_R \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & R \end{bmatrix} \begin{bmatrix} i_c \\ i_R \end{bmatrix} + \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

e) $$\begin{cases} \dot{i}_c(t) = \frac{v_1 - v_2 - v_c}{L} - \frac{i_c}{CR} \\ \dot{v}_c(t) = \frac{i_c}{C} \end{cases}$$

$$\dot{x} = \begin{bmatrix} -\frac{1}{RC} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_c \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & -\frac{1}{L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_c \\ v_c \end{bmatrix}$$

f) Pluggin in $\begin{cases} R = 0.4\Omega \\ L = 0.5\,H \\ C = 0.5\,F \end{cases}$ to a):

$$A = \begin{bmatrix} 0 & -2 \\ 2 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -2 \\ 0 & -5 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = 0$$

$$\phi(s) = (sI - A)^{-1} = \begin{bmatrix} s & -2 \\ 2 & s-5 \end{bmatrix}^{-1}$$

$$= \frac{1}{s^2 - 5s + 4} \begin{bmatrix} s-5 & 2 \\ -2 & s \end{bmatrix}$$

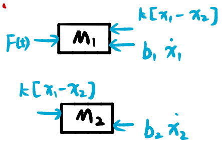
$$\phi(t) = \begin{bmatrix} \frac{4}{3}e^t - \frac{1}{3}e^{4t} & 2(-\frac{1}{3}e^t + \frac{1}{3}e^{4t}) \\ \frac{2}{3}e^t - \frac{2}{3}e^{4t} & -\frac{1}{3}e^t + \frac{4}{3}e^{4t} \end{bmatrix}$$

Free-response solution:

$$x(t) = \phi(t)\,x(0) = \begin{bmatrix} \frac{4}{3}e^t - \frac{1}{3}e^{4t} \\ \frac{2}{3}e^t - \frac{2}{3}e^{4t} \end{bmatrix}$$

Problem 3

1.



$$M_1\ddot{x}_1 + b_1\dot{x}_1 + k(x_1-x_2) = F(t) \quad (1)$$

$$M_2\ddot{x}_2 + b_2\dot{x}_2 + k(x_1-x_2) = 0 \quad (2)$$

2. $$G(s) = \frac{sX_2(s)}{F(s)} = \frac{ks}{(M_1s^2+b_1s+k)(M_2s^2+b_2s+k)-k^2} \quad (1)\&(2)$$

3. $$\begin{cases} x_1(t) = x_1(t) \\ x_2(t) = x_2(t) \\ x_3(t) = \dot{x}_1(t) \\ x_4(t) = \dot{x}_2(t) \end{cases}$$

4. (1)&(2)

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{M_1} & \frac{k}{M_1} & \frac{-b_1}{M_1} & 0 \\ \frac{k}{M_2} & -\frac{k}{M_2} & 0 & \frac{-b_2}{M_2} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_1} \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \qquad D = 0$$

5. $f_{spring}(y) = -ky - a\sin y$ (a is a constant)

We take $-ky - ay$ for $f_{spring}(y)$ when $y \to 0$.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k+a}{M_1} & \frac{k+a}{M_1} & \frac{-b_1}{M_1} & 0 \\ \frac{k+a}{M_2} & \frac{-k-a}{M_2} & 0 & \frac{-b_2}{M_2} \end{bmatrix},$$ BCD remain the same.

Problem 4

a) $$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad D = 0$$

b) $$\phi(s) = [sI - A]^{-1} = \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}^{-1} = \frac{1}{s^2+3s+2}\begin{bmatrix} s+3 & -1 \\ 2 & s \end{bmatrix}$$

$$\phi(t) = \mathcal{L}^{-1}\{\phi(s)\} = \begin{bmatrix} 2e^{-t} - e^{-2t} & -e^{-t} + e^{-2t} \\ 2(e^{-t} - e^{-2t}) & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

c) Free-response solution: $$x(t) = \phi(t)x(0) = \begin{bmatrix} 3e^{-t} - 2e^{-2t} \\ 3e^{-t} - 4e^{-2t} \end{bmatrix}$$