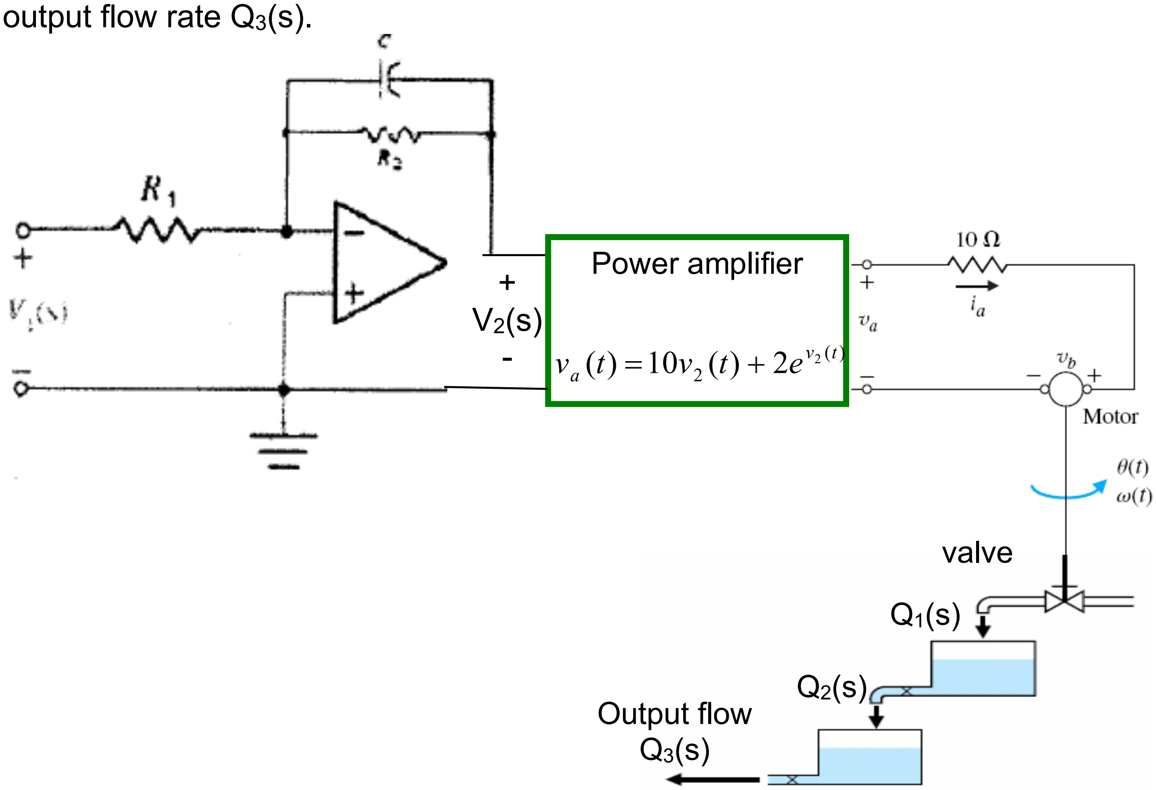
University of British Columbia
Department of Electrical & Computer Engineering
ELEC341 – Signals & Systems
Jan. 2020 – Apr. 2020

Take-home Assignment - 1
Contents covered: chapter 2 (focus: linearization, differential equation, transfer function, Mason's gain formula)
Due day: Jan. 27, 2020

Mark distribution:
Textbook problem part: 16+6+10+8+10=**50** points in total.
Matlab assignment part: 8+8=**16** points in total.

Textbook problems:

Problem 1: In the following figure, a two-tank system is shown. an ideal operational amplifier is used to provide the input voltage, $v_2(t)$, to a nonlinear power amplifier. The power amplifier may be approximately represented by $v_a(t)=10v_2(t)+2\exp(v_2(t))$. The DC motor is driven by the output current of the power amplifier, $i_a(t)$, and the resulting motor torque turns a shaft. The armature inductance of the DC motor is negligible, so that $L_a=0$, and the rotational friction of the motor shaft is also negligible. The motor torque constant and back-emf constant are equal and given by $K_m=K_b=10$. The inertia of the motor shaft is $J=0.01$ kg.m^2. The motor adjusts the input valve and ultimately varying the output flow rate $Q_3(s)$.

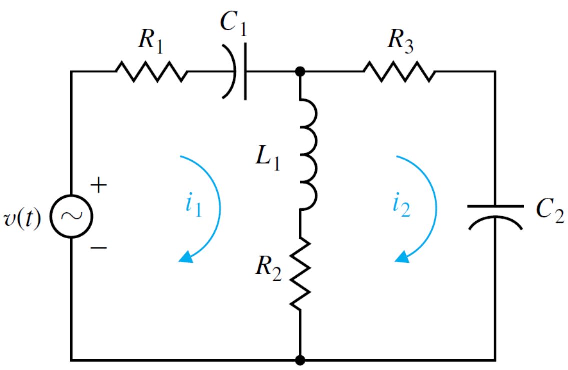


a. Find the transfer function $V_2(s)/V_1(s)$ of the op amp circuit, assuming. an ideal op-amp. Assume $R_1=R_2=100\ k\Omega$, and $c=1\ \mu F$.
b. Assuming that the operating point is $v_{20}=3$ volts, determine the linearized gain $V_a(s)/V_2(s)$ near the operating point.
c. Determine the differential equation relating $\theta(t)$ and $v_a(t)$. Assume the output $y(t)$ is the motor speed, $y(t)= d\theta(t)/dt$. Obtain the overall transfer function $Y(s)/V_1(s)$ near the operating point.

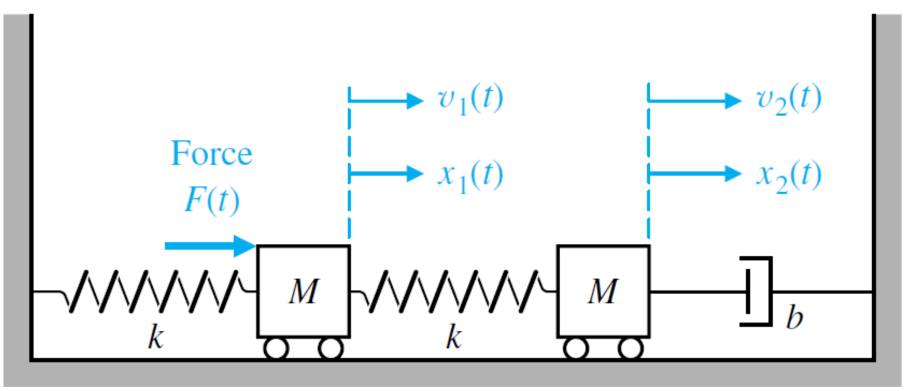
d. The input flow rate for the tank is controlled by the valve as $\frac{dq_1(t)}{dt} = \theta(t) - 4q_1(t)$; the flow $Q_2(s)$ and $Q_1(s)$ has the relationship $\frac{dq_2(t)}{dt} = q_1(t) - q_2(t)$; and the output flow $Q_3(s)$ is $\frac{dq_3(t)}{dt} = 2q_2(t) - 3q_3(t)$.

Determine the transfer function between the output flow and the motor speed $Q_3(s)/V_1(s)$.

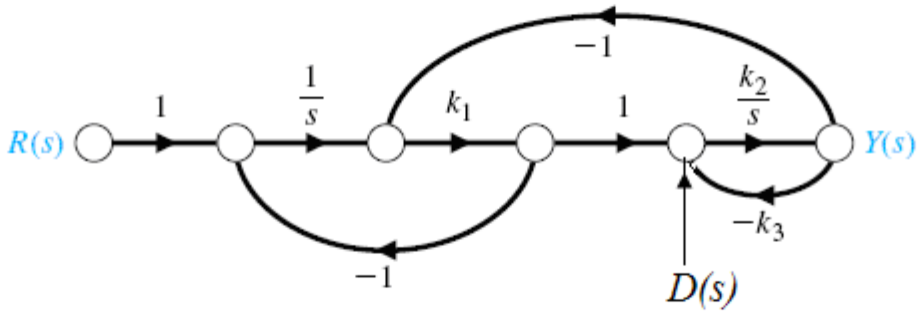
Problem 2 (P2.1 in the 12th edition): Write down the differential equations in terms of $i_1(t)$ and $i_2(t)$ for the circuit.



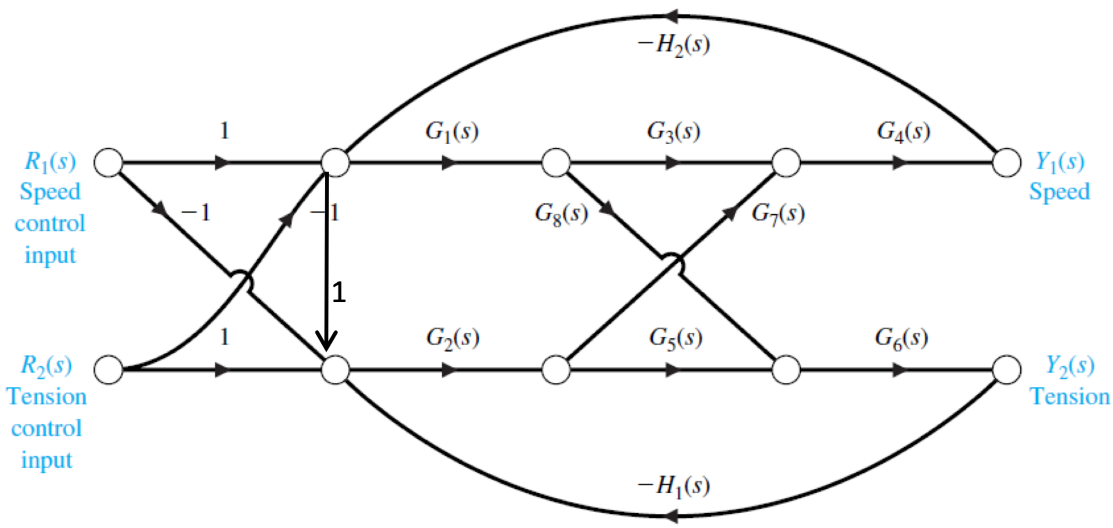
Problem 3 (modified from P2.3 in the 12th edition): A coupled spring-mass system is shown in the following figure. The masses and springs are assumed to be equal. Obtain the differential equations describing the system. And find the transfer functions of $X_1(s)/F(s)$ and $X_2(s)/F(s)$.



Problem 4: Determine the transfer functions $T_R(s) = Y(s)/R(s)$ and $T_D(s)=Y(s)/D(s)$ for the system in the figure.



Problem 5 (modified from P2.32 in the 12th edition): A model of the coupled motor drives is shown in the following figure. Find $Y_1(s)/R_1(s)$ and $Y_2(s)/R_2(s)$.



Matlab Assignments:

CP2.1 and **CP2.2** (same as MP2.2. in the 10th edition) in the 12th edition.

ELEC 341 Assignment 1 Jian Gao [redacted]

Problem 1

a) $$\frac{V_1(s)}{R_1(s)} = \frac{-V_2(s)}{\frac{R_2 \cdot \frac{1}{sC}}{R_2+\frac{1}{sC}}} \Rightarrow \frac{V_2}{V_1} = \frac{10}{s+10}$$

b) $$V_a = 10 \cdot 3 + 2e^3 = 70.17\ V$$
$$\frac{V_a}{V_2} = 23.39\ V/V$$

c) $$\frac{Y(s)}{V_1(s)} = \frac{V_2}{V_1} \cdot \frac{V_a}{V_2} \cdot \frac{V_b}{V_a} \cdot \frac{Y}{V_b} = \frac{10}{s+10} \cdot 23.39 \cdot \frac{1}{10+0.01s}$$

d) $$\frac{Q_3(s)}{V_1(s)} = \frac{Q_1}{V_1} \cdot \frac{Q_2}{Q_1} \cdot \frac{Q_3}{Q_2} = \boxed{\frac{2339 \times 2}{s(s+4)(s+1)(s+3)(s+10)(s+1000)}}$$

$$\begin{cases} Q_3(s)(s+3) = 2Q_2 \\ Q_2(s)(s+1) = Q_1 \\ s\,Q_1(s) = V_1(s)\dfrac{2339}{s(s+1000)(s+10)(s+4)} \end{cases}$$

Problem 2

Loop 1: $v(t) = v_{R_1} + v_{C_1} + v_{L_1} + v_{R_2}$

$$\Rightarrow v(t) = R_1 \cdot i_1 + \frac{1}{C_1}\int_0^t i_1(\tau)d\tau + L_1\frac{di_1(t)}{dt} + R_2 i_1$$

Loop 2: $0 = v_{R_3} + v_{C_2} + v_{R_2} + v_{L_1}$

$$\Rightarrow 0 = R_3 i_2 + \frac{1}{C_2}\int_0^t i_2(\tau)d\tau + R_2 \cdot i_2 + L_1\frac{di_2(t)}{dt}$$

Problem 3

$$\begin{cases} M_1\ddot{x}_1(t) = F(t) - k\,x_1(t) + k(x_2(t) - x_1(t)) & ① \\ M_2\ddot{x}_2(t) = -k(x_2(t) - x_1(t)) - b\,\dot{x}_2(t) & ② \end{cases}$$

①: $\mathcal{L}$: $M_1 s^2 X_1(s) = F(s) - kX_1(s) + kX_2(s) - kX_1(s)$

②: $\mathcal{L}$: $M_2 s^2 X_2(s) = -kX_2(s) + kX_1(s) - bsX_2(s)$

$\Rightarrow kX_1(s) = (M_2 s^2 + k + bs)X_2(s)$

$\Rightarrow X_2(s) = \dfrac{kX_1(s)}{M_2 s^2 + k + bs}$

$$\therefore \frac{X_1(s)}{F(s)} = \frac{1}{M_1 s^2 + 2k - \frac{k^2}{M_2 s^2 + k + bs}} \qquad \frac{X_2(s)}{F(s)} = \frac{k}{(M_2 s^2 + k + bs)\left(M_1 s^2 + 2k - \frac{k^2}{M_2 s^2 + k + bs}\right)}$$

Problem 4

① All loops:

$L_1 = \frac{1}{s} \cdot k_1 \cdot (-1)$

$L_2 = k_1 \cdot \frac{k_2}{s} \cdot (-1)$

$L_3 = \frac{k_2}{s} \cdot (-k_3)$

$$\Delta = 1 - (L_1 + L_2 + L_3) + L_1 L_3 = 1 + \frac{k_1 + k_1k_2 + k_2k_3}{s} + \frac{k_1k_2k_3}{s^2}$$

② Find P_k's & Δ_k's:

$R(s)$: $P_1 = \frac{1}{s} \cdot k_1 \cdot \frac{k_2}{s} = \frac{k_1k_2}{s^2}$

$\Delta_1 = 1$

$D(s)$: $P_1 = \frac{k_2}{s}$

$\Delta_1 = 1 + \frac{k_1}{s}$

③ Mason's Gain Formula:

$$T_R(s) = \frac{Y(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta} = \frac{k_1k_2}{s^2 + (k_1 + k_1k_2 + k_2k_3)s + k_1k_2k_3}$$

$$T_D(s) = \frac{Y(s)}{D(s)} = \frac{P_1\Delta_1}{\Delta} = \frac{\frac{k_2}{s}\left(1 + \frac{k_1}{s}\right)}{\Delta} = \frac{k_2(s + k_1)}{s^2 + (k_1 + k_1k_2 + k_2k_3)s + k_1k_2k_3}$$

Problem 5

① All Loops:

$$L_1 = G_1 G_3 G_4 (-H_2)$$

$$L_2 = G_2 G_5 G_6 (-H_1)$$

$$L_3 = G_2 G_7 G_4 (-H_2)$$

$$L_4 = G_6 (-H_1) G_2 G_7 G_4 (-H_2) G_1 G_8$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1 L_2$$

② P_k's & Δ_k's:

Y_1/R_1:

$$P_1 = G_1 G_3 G_4 \qquad \Delta_1 = 1 - L_2$$

$$P_2 = G_2 G_7 G_4 \qquad \Delta_2 = 1$$

$$P_3 = -G_2 G_7 G_4 \qquad \Delta_3 = 1$$

Y_2/R_2:

$$P_1 = G_2 G_5 G_6 \qquad \Delta_1 = 1 - L_1$$

$$P_2 = -G_1 G_8 G_6 \qquad \Delta_2 = 1 - L_3$$

③ Mason's Gain Formula:

$$T_1(s) = \frac{Y_1(s)}{R_1(s)} = \frac{P_1 \Delta_1 + P_2 + P_3}{\Delta} = \frac{G_1 G_3 G_4 (1 - L_2)}{\Delta} \longleftarrow \text{plug in ①}$$

$$T_2(s) = \frac{Y_2(s)}{R_2(s)} = \frac{G_2 G_5 G_6 (1 - L_1) - G_1 G_8 G_6 (1 - L_3)}{\Delta}$$

Matlab

CP2.1

```
>> p=[1 7 10];q=[1 2]

q =

     1     2

>> pq=conv(p,q)

pq =

     1     9    24    20

>> P=roots(p),Z=roots(q)

P =

    -5
    -2


Z =

    -2

>> value=polyval(p,-1)

value =

     4
```

CP2.2

```
>> numc=[1];denc=[1 1];sysc=tf(numc,denc)

sysc =

    1
  -----
  s + 1

Continuous-time transfer function.

>> numg=[1 2];deng=[1 3];sysg=tf(numg,deng)

sysg =

  s + 2
  -----
  s + 3

Continuous-time transfer function.

>> sys_s=series(sysc,sysg);
>> sys_cl=feedback(sys_s,[1])

sys_cl =

      s + 2
  -------------
  s^2 + 5 s + 5

Continuous-time transfer function.

>> step(sys_cl);grid on
```