E, X, Y, t —two→ ABt, BAt, AAt, BBt, AB, BA

t is time, others are all $L \times L$ matrices.

$$[ABt]_{ij} = \langle A_i(t) B_j(0) \rangle = \langle \left(C_i^\dagger(t) + C_i(t)\right)\left(C_j^\dagger(0) - C_j(0)\right) \rangle$$

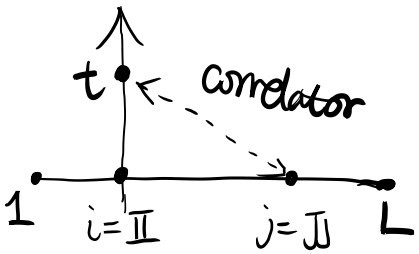
ground state average $\langle \ldots \rangle$

$\Pi, J\!J$
ABt, BAt, AAt, BBt, AB, BA —wick→ S

$(2\Pi + 2J\!J - 2) \times (2\Pi + 2J\!J - 2)$ matrix

$$\langle \sigma_i^x(t) \sigma_j^x(0) \rangle = \langle \underbrace{A_1 B A_2 B \cdots A B A_i}_{t} \underbrace{| A_1 B A B \cdots A B A_j}_{t=0} \rangle$$

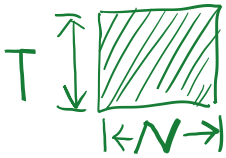
$2i + 2j - 2$ terms

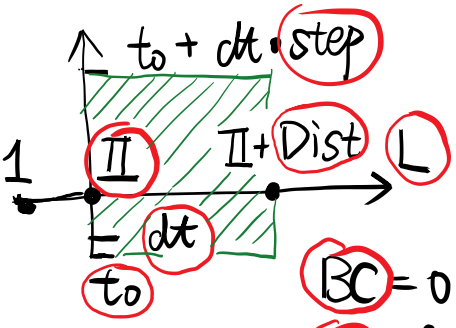


Using Wick's Theorem, express the $2i+2j-2$ correlators as a combination of 2 correlators,

This combination is called **Pfaffian**

N, T → convention1 →





BC = 0

J = (1, 1, ⋯, 1) (L)

L, BC, J, h → getH → H

$H_{2L\times 2L} =$

$$\begin{pmatrix} h_1 & -\frac{J_1}{2} & & & & & 0 & -\frac{J_1}{2} & & & & \\ \frac{J_1}{2} & h_2 & -\frac{J_2}{2} & & & & \frac{J_1}{2} & 0 & -\frac{J_2}{2} & & & \\ & -\frac{J_2}{2} & h_3 & \ddots & & & & \frac{J_2}{2} & 0 & \ddots & & \\ & & \ddots & \ddots & -\frac{J_{N-1}}{2} & & & & \ddots & \ddots & -\frac{J_{N-1}}{2} & \\ & & & -\frac{J_{N-1}}{2} & h_N & & & & & \frac{J_{N-1}}{2} & 0 & \\ 0 & \frac{J_1}{2} & & & & & -h_1 & \frac{J_1}{2} & & & & \\ -\frac{J_1}{2} & 0 & \frac{J_2}{2} & & & & \frac{J_1}{2} & -h_2 & \frac{J_2}{2} & & & \\ & -\frac{J_2}{2} & 0 & \ddots & & & & \frac{J_2}{2} & h_3 & \ddots & & \\ & & \ddots & \ddots & \frac{J_{N-1}}{2} & & & & \ddots & \ddots & \frac{J_{N-1}}{2} & \\ & & & -\frac{J_{N-1}}{2} & 0 & & & & & \frac{J_{N-1}}{2} & -h_N & \end{pmatrix}$$

$J = (J_1\ J_2 \cdots J_L)$

$h = (h_1\ h_2 \cdots h_L)$

row or column both okay

BC = 0 open Boundary Condition

H, typeflag → generateXYE → E, X, Y

To be Modified

$H\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} X & Y \\ Y & X \end{pmatrix}\begin{pmatrix} E \\ -E \end{pmatrix}$

X, Y 正交，归一　　E = (大 ⋯ 大 小 ⋯ 更小)