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EXERCISE 1.

Given $\overleftrightarrow{\Sigma}$ $\vec{R}$ R_f

Show that (1) is consistent with $Z = \overleftrightarrow{\Sigma}^{-1}(\vec{R} - R_f\vec{1})$

At the point of tangency G.

$$E_G = \vec{R}^T \frac{\overleftrightarrow{\Sigma}^{-1}(\vec{R} - R_f\vec{1})}{\vec{1}^T \overleftrightarrow{\Sigma}^{-1}(\vec{R} - R_f\vec{1})} = \frac{\vec{R}^T \vec{Z}}{\vec{1}^T \vec{Z}}$$

Plug in E_G to equation (1).

$$R.H.S. = \frac{E_G - R_f}{(\vec{R} - R_f\vec{1})^T \overleftrightarrow{\Sigma}^{-1} (\vec{R} - R_f\vec{1})} \overleftrightarrow{\Sigma}^{-1} (\vec{R} - R_f\vec{1})$$

$$R.H.S. = \frac{(E_G - R_f)\cdot \vec{Z}}{(\vec{R} - R_f\vec{1})^T \cdot \vec{Z}} = \frac{\left(\frac{\vec{R}^T \vec{Z}}{\vec{1}^T \cdot \vec{Z}} - R_f\right)\vec{Z}}{(\vec{R} - R_f \cdot 1)^T \cdot \vec{Z}}$$

$$L.H.S = \vec{X} = \frac{\vec{Z}}{\vec{1}^T \cdot \vec{Z}}$$

To prove R.H.S. = L.H.S. we need to prove

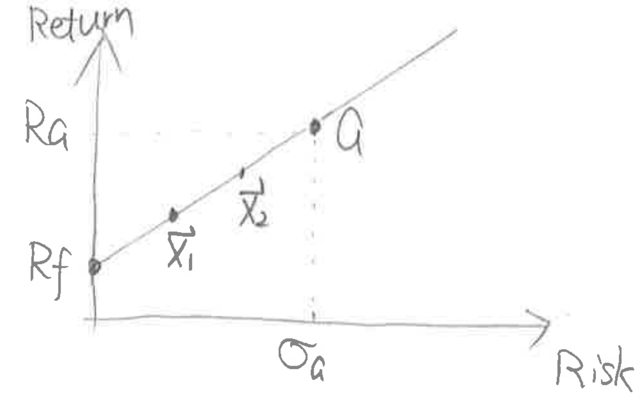
$$\vec{1}^T \cdot \vec{Z} = \frac{(\vec{R} - R_f \cdot \vec{1})^T \cdot \vec{Z}}{\frac{\vec{R}^T \cdot \vec{Z}}{\vec{1}^T \cdot \vec{Z}} - R_f}$$

$$\vec{R}^T \cdot \vec{Z} - R_f \vec{1}^T \cdot \vec{Z} = \vec{R}^T \cdot \vec{Z} - R_f \cdot \vec{1}^T \cdot \vec{Z} \quad \checkmark$$

which is true

EXERCISE 2.

Capital allocation line



Any portfolio on the CAL can be expressed as a combination $\vec{X} = \begin{pmatrix} x_f \\ x_G \end{pmatrix}$

here $x_f + x_G = 1$ $\quad$ x_f means the portion on R_f

x_G means the portion on R_G

Then $\overleftrightarrow{\Sigma} = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_G^2 \end{pmatrix}$

Now suppose we have $\vec{x_1}$, $\vec{x_2}$ two portfolio on the CAL.

$$\frac{\vec{x_1}^T \overleftrightarrow{\Sigma} \vec{x_2}}{\sqrt{\vec{x_1}^T \overleftrightarrow{\Sigma} \vec{x_1}} \sqrt{\vec{x_2}^T \overleftrightarrow{\Sigma} \vec{x_2}}} = \frac{x_{1G} \sigma_G^2 x_{2G}}{\sqrt{x_{1G}^2 \sigma_G^2} \sqrt{x_{2G}^2 \sigma_G^2}} = 1$$

perfectly correlated

EXERCISE 3.

$$Z_{it} = \underbrace{\alpha_i + \beta_i R_{mt}}_{\text{constant}} + \underset{\substack{\uparrow \\ \text{Gaussian } N(0,\sigma^2)}}{\epsilon_{it}}$$

$$E(\hat{y}_{it}) = E\left(e^{\alpha_i + \beta_i R_{mt} + \epsilon_{it}}\right) = e^{\alpha_i + \beta_i R_{mt}} E\left(e^{\epsilon_{it}}\right)$$

$$= e^{\hat{Z}_{it}} \int_{-\infty}^{+\infty} dx \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} e^{x}$$

$$E(y_{it}) = \hat{y}_{it} e^{\frac{\sigma^2}{2}}$$

EXERCISE 4.

The fact: for simple linear regression R^2 equals the square of correlation ρ

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \qquad SST = SSE + SSR$$

$$\Sigma(R_{it} - \bar{R}_i)^2 = \Sigma(R_{it} - \hat{R}_{it})^2 + \Sigma(\hat{R}_{it} - \bar{R}_i)^2$$

$$\rho^2 = \frac{\left\{E\left[(R_{it} - \bar{R}_i)(\hat{R}_{it} - \bar{\hat{R}}_{it})\right]\right\}^2}{E(R_{it} - \bar{R}_i)^2 \cdot E(\hat{R}_{it} - \bar{\hat{R}}_{it})^2} = \frac{\left[\sum_t (R_{it} - \bar{R}_t)(\hat{R}_{it} - \bar{\hat{R}}_{it})\right]^2}{\sum_t (R_{it} - \bar{R}_{it})^2 \; \Sigma(\hat{R}_{it} - \bar{\hat{R}}_{it})^2}$$

$$\underset{R_{it} = \hat{R}_{it} + \varepsilon_i}{=\!=} \frac{\sum_t (\hat{R}_{it} - \bar{\hat{R}}_{it})^2}{\sum_t (R_{it} - \bar{R}_{it})^2} = R^2$$