Algorithms PART I: Embarrassingly Parallel

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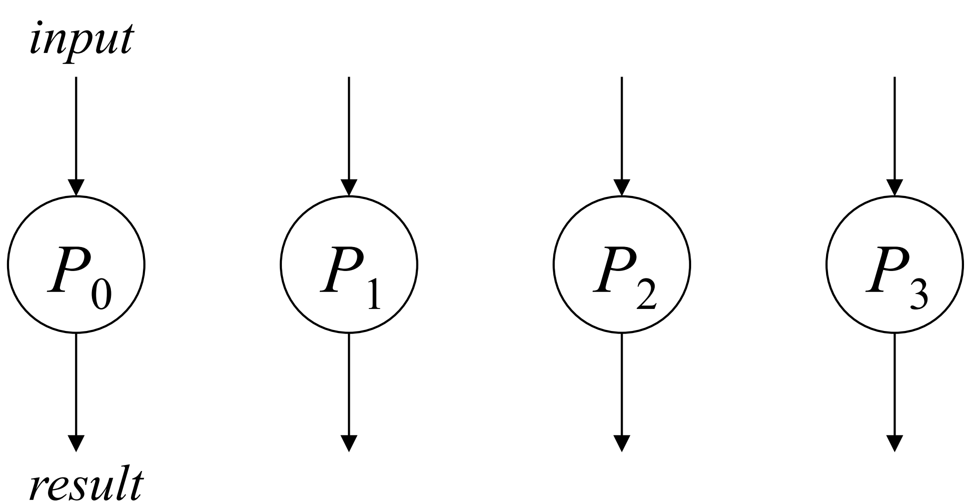
Overview

- Ideal parallelism
- Master-worker paradigm
- Processor farms
- Examples
  - Geometrical transformations of images
  - Mandelbrot set
  - Monte Carlo methods
- Load balancing of independent tasks
- Further reading

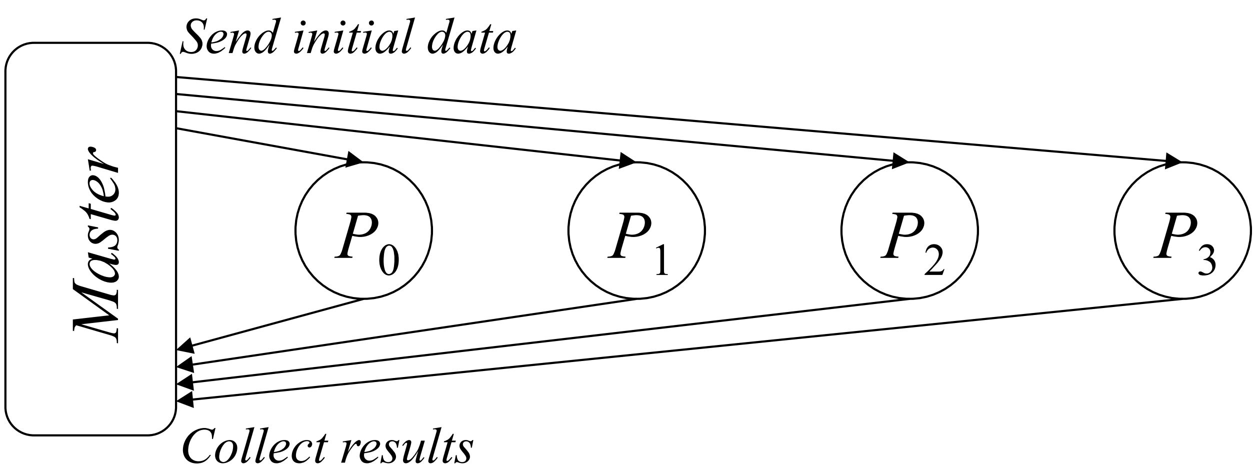
Ideal Parallelism

- An *ideal parallel computation* can be immediately divided into completely independent parts
  - "Embarrassingly parallel"
  - "Naturally parallel"
- No special techniques or algorithms required



Ideal Parallelism and the Master-Worker Paradigm

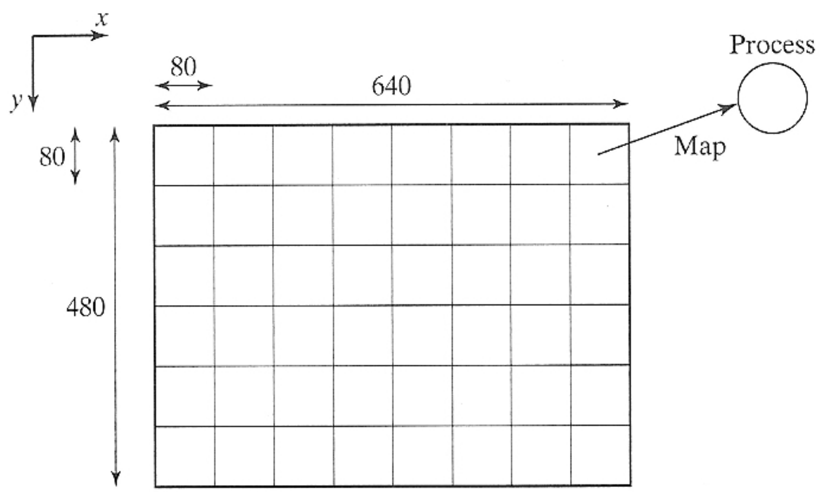
- Ideally there is no communication
  - Maximum speedup
- Practical embarrassingly parallel applications have initial communication and (sometimes) a final communication
  - Master-worker paradigm where master submits jobs to workers
  - No communications between workers



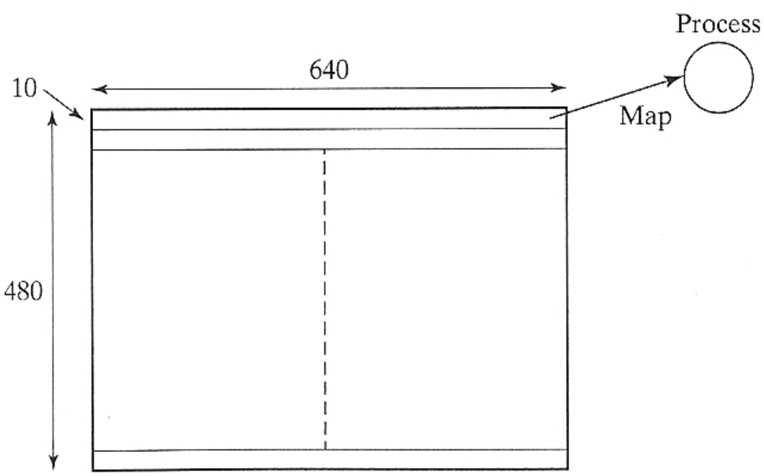
Parallel Tradeoffs

- Embarrassingly parallel with perfect load balancing:
  $$t_{comp} = t_s / P$$
  assuming P workers and sequential execution time t_s
- Master-worker paradigm gives speedup only if workers have to perform a reasonable amount of work
  - Sequential time > total communication time + one workers' time
    $t_s > t_p = t_{comm} + t_{comp}$
  - Speedup $S_P = t_s / t_P = P\, t_{comp} / (t_{comm} + t_{comp}) = P / (r^{-1} + 1)$
    where $r = t_{comp} / t_{comm}$
  - Thus $S_P \rightarrow P$ when $r \rightarrow \infty$
- However, communication t_{comm} can be expensive
  - Typically $t_s < t_{comm}$ for small tasks, that is, the time to send/recv data to the workers is more expensive than doing all the work
  - Try to overlap computation with communication to hide t_{comm} latency

Example 1: Geometrical Transformations of Images



(a) Square region for each process



(b) Row region for each process

- Partition *pixmap* into regions
  - By block (row & col block)
  - By row
- Pixmap operations
  - Shift

    $x' = x + \Delta x$

    $y' = y + \Delta y$
  - Scale

    $x' = S_x\, x$

    $y' = S_y\, y$
  - Rotation

    $x' = x \cos \theta + y \sin \theta$

    $y' = -x \sin \theta + y \cos \theta$
  - Clip

    $x_l \leq x' = x \leq x_h$

    $y_l \leq y' = y \leq y_h$

Example 1: Master and Worker Naïve Implementation

Master

```
row = 0;
for (p = 0; p < P; p++)
{ send(row, p);
  row += 480/P;
}
for (i = 0; i < 480; i++)
  for (j = 0; j < 640; j++)
    temp_map[i][j] = 0;
for (i = 0; i < 480; i++)
{ recv(&oldrow, &oldcol, &newrow, &newcol, anyP);
  if (!(newrow < 0 || newrow >= 480 || newcol < 0 || newcol >= 640))
    temp_map[newrow][newcol] = map[oldrow][oldcol];
}
for (i = 0; i < 480; i++)
  for (j = 0; j < 640; j++)
    map[i][j] = temp_map[i][j];
```

Worker

```
recv(&row, master);
for (oldrow = row; oldrow < row + 480/P; oldrow++)
{ for (oldcol = 0; oldcol < 640; oldcol++)
  { newrow = oldrow + delta_x;
    newcol = oldcol + delta_y;
    send(oldrow, oldcol, newrow, newcol, master);
  }
}
```

Each worker computes:

$\forall\ row \le x < row + 480/P;\ 0 \le y < 640:$

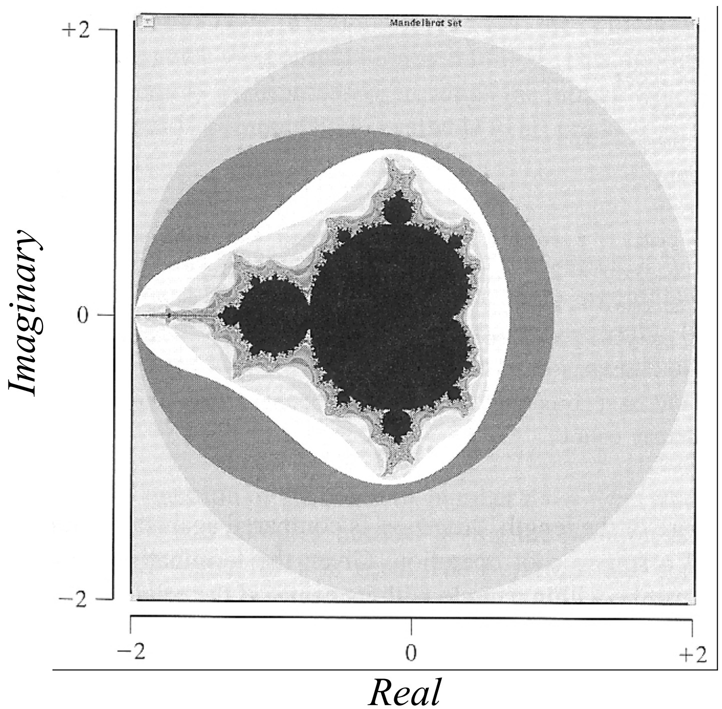
$$x' = x + \Delta x$$

$$y' = y + \Delta y$$

Example 1: Geometrical Transformation Speedups?

- Assume in the general case the pixmap has n^2 points
- Sequential time of pixmap shift $t_s = 2n^2$
- Communication
  $t_{comm} = P(t_{startup} + t_{data}) + n^2(t_{startup} + 4t_{data}) = O(P + n^2)$
- Computation
  $t_{comp} = 2n^2 / P = O(n^2/P)$
- Computation/communication ratio
  $r = O((n^2 / P) / (P + n^2)) = O(n^2 / (P^2 + n^2P))$
- **This is not good!**
  - The asymptotic computation time should be an order higher than the asymptotic communication time, e.g. $O(n^2)$ versus $O(n)$
  - … or there must be a very large constant in the computation time
- Performance on shared memory machine can be good
  - No communication time

Example 2: Mandelbrot Set



The number of iterations it takes for z to end up at a point outside the complex circle with radius 2 determines the pixmap color

- A pixmap is generated by iterating the complex-valued recurrence

$$z_{k+1} = z_k^2 + c$$

  with $z_0=0$ and $c=x+yi$ until $|z|\geq 2$

- The Mandelbrot set is shifted and scaled for display:

$$x = x_{min} + x_{scale}\ row$$
$$y = y_{min} + y_{scale}\ col$$

  for each of the pixmap's pixels at *row* and *col* location

Example 2: Mandelbrot Set Color Computation

```
int pix_color(float x0, float y0)
{
  float x = x0, y = y0;
  int i = 0;

  while (x*x + y*y < (2*2) && i < maxiter)
  {
    float xtemp = x*x - y*y + x0;
    float ytemp = 2*x*y + y0;

    x = xtemp;
    y = ytemp;

    i++;
  }

  return i;
}
```

Example 2: Mandelbrot Set Simple Master and Worker

Master

```
row = 0;
for (p = 0; p < P; p++)
{ send(row, p);
  row += 480/P;
}
for (i = 0; i < 480 * 640; i++)
{ recv(&x, &y, &color, anyP);
  display(x, y, color);
}
```

Send/recv (x,y) pixel colors

Worker

```
recv(&row, master);
for (y = row; y < row + 480/P; y++)
{ for (x = 0; x < 640; x++)
  { x0 = xmin + x * xscale;
    y0 = ymin + y * yscale;
    color = pix_color(x0, y0);
    send(x, y, color, master);
  }
}
```

Example 2: Mandelbrot Set Better Master and Worker

Master

```
row = 0;
for (p = 0; p < P; p++)
{ send(row, p);
  row += 480/P;
}
for (i = 0; i < 480; i++)
{ recv(&y, &color, anyP);
  for (x = 0; x < 640; x++)
    display(x, y, color[x]);
}
```

Send/recv array of colors[x] for each row y

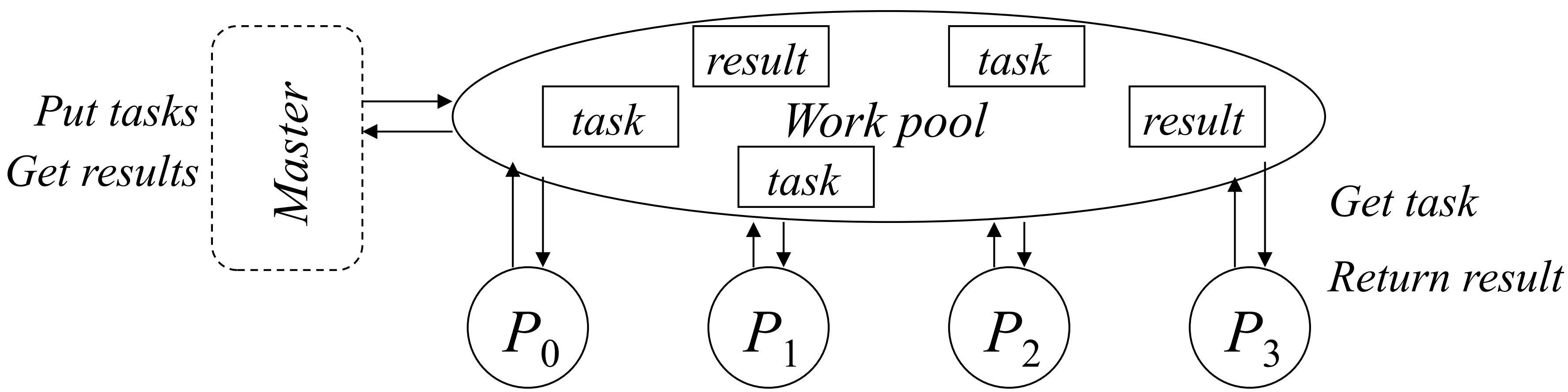
Worker

```
recv(&row, master);
for (y = row; y < row + 480/P; y++)
{ for (x = 0; x < 640; x++)
  { x0 = xmin + x * xscale;
    y0 = ymin + y * yscale;
    color[x] = pix_color(x0, y0);
  }
  send(y, color, master);
}
```

Assume n×n pixmap, n iterations on average per pixel, and P workers:
Communication time?
Computation time?
Computation/communication ratio?
Speedup?

Processor Farms

- *Processor farms* (also called the *work-pool approach*)
- A collection of workers, where each worker repeats:
  - Take new task from pool
  - Compute task
  - Return results into pool
- Achieves *load balancing*
  - Tasks differ in amount of work
  - Workers can differ in execution speed (viz. heterogeneous cluster)



Example 2: Mandelbrot Set with Processor Farm

Master

Assuming synchronous send/recv

```
count = 0;
row = 0;
for (p = 0; p < P; p++)
{ send(row, p);
  count++;
  row++;
}
do
{ recv(&y, &color, anyP);
  count--;
  if (row < 480)
  { send(row, anyP);
    row++;
    count++;
  }
  else
    send(-1, anyP);
  for (x = 0; x < 640; x++)
    display(x, y, color[x]);
} while (count > 0);
```

Keeps track of how many workers are active

Send row to workers

Recv colors for row y

Send next row

Send sentinel

Worker

Recv row

Compute color for (x,y)

Send colors

Recv next row

```
recv(&y, master);
while (y != -1)
{ for (x = 0; x < 640; x++)
  { x0 = xmin + x * xscale;
    y0 = ymin + y * yscale;
    color[x] = pix_color(x0, y0);
  }
  send(y, color, master);
  recv(&y, master);
}
```

Example 2: Mandelbrot Set with Processor Farm

Master

```
count = 0;
row = 0;
for (p = 0; p < P; p++)
{ send(row, p);
  count++;
  row++;
}
do
{ recv(&rank, &y, &color, anyP);
  count--;
  if (row < 480)
  { send(row, rank);
    row++;
    count++;
  }
  else
    send(-1, rank);
  for (x = 0; x < 640; x++)
    display(x, y, color[x]);
} while (count > 0);
```

Assuming asynchronous send/recv

Recv a row y and worker rank and send to worker (using rank)

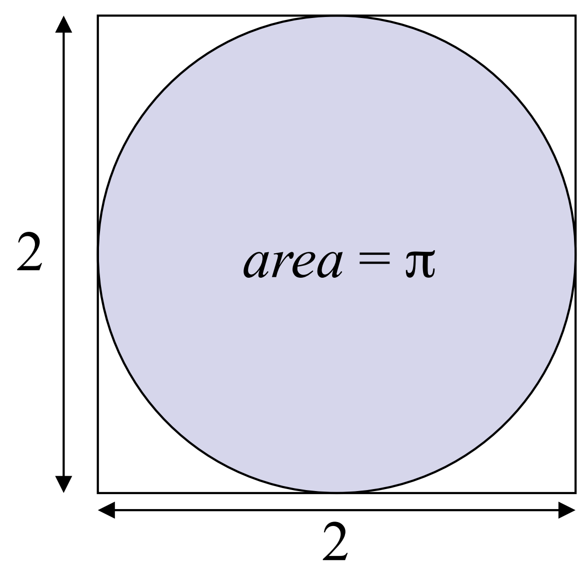
Worker

```
recv(&y, master);
while (y != -1)
{ for (x = 0; x < 640; x++)
  { x0 = xmin + x * xscale;
    y0 = ymin + y * yscale;
    color[x] = pix_color(x0, y0);
  }
  send(myrank, y, color, master);
  recv(&y, master);
}
```

Send colors and rank

Example 3: Monte Carlo Methods

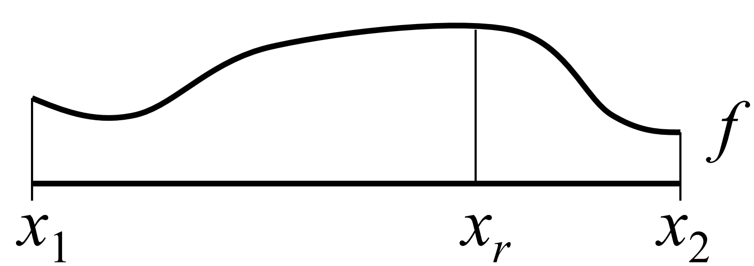
- Perform random selections to sample the solution
- Each sample is independent
- Example
  - Compute π by sampling the [-1..1,-1..1] square that contains a circle with radius 1
  - The probability of hitting the circle is $\pi/4$



Example 3: Monte Carlo Methods

- General Monte Carlo methods sample inside and outside the solution space
- Many Monte Carlo methods do not sample outside solution space
- Function integration by sampling the function values over the integration domain

$$\int_{x_1}^{x_2} f(x)dx = \lim_{N\to\infty} \frac{x_2 - x_1}{N} \sum_{r=1}^{N} f(x_r)$$



Example 3: Monte Carlo Methods and Parallel RNGs

- Approach 1: master sends random number sequences to the workers
  - Uses one random number generator (RNG)
  - Lots of communication
- Approach 2: workers produce independent random number sequences
  - Communication of sample parameters only
  - Cannot use standard pseudo RNG (sequences are the same)
  - Needs parallel RNG
- Parallel RNGs (e.g. SPRNG library)
  - Parallel pseudo RNG
  - Parallel quasi-random RNG

Example 3: Monte Carlo Methods and Parallel RNGs

- *Linear congruential generator* (pseudo RNG):

  $$x_{i+1} = (a\, x_i + c) \bmod m$$

  with a choice of a, c, and m
  - Good choice of a, c, and m is crucial!
  - Cannot easily segment the sequence (for processors)
- A parallel pseudo RNG with a "jump" constant k

  $$x_{i+k} = (A\, x_i + C) \bmod m$$

  where $A = a^k \bmod m$, $C = c(a^{k-1} + a^{k-2} + \ldots + a^1 + a^0) \bmod m$

Parallel computation of sequence

$$x_0 \quad x_1 \ \ldots \ x_{k-1} \quad x_k \quad x_{k+1} \ \ldots \ x_{2k-1} \quad x_{2k} \ \ldots$$

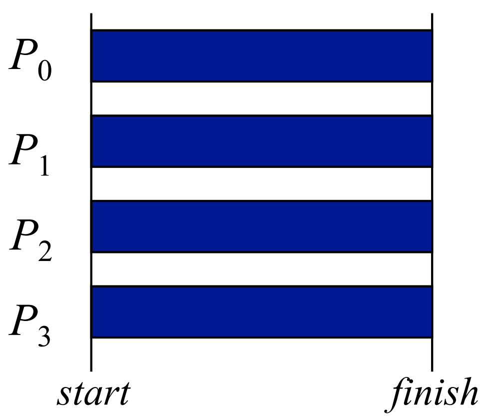
The sequences per processor

$$\begin{array}{ccccc} x_{ik} & x_{ik+1} & x_{ik+2} & & x_{ik+k-1} \\ P_0 & P_1 & P_2 & \cdots\cdots & P_{k-1} \end{array}$$

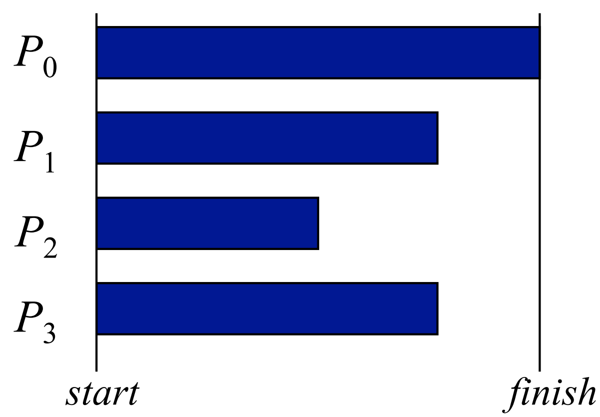
Load Balancing

- Load balancing attempts to spread tasks evenly across processors
- Load imbalance is caused by
  - Tasks of different execution cost, e.g. Mandelbrot example
  - Processors operate with different execution speeds or are busy
- When tasks and processors are not load balanced:
  - Some processes finish early and sit idle waiting
  - Global computation is finished when the slowest processor(s) completes its task

Load balanced

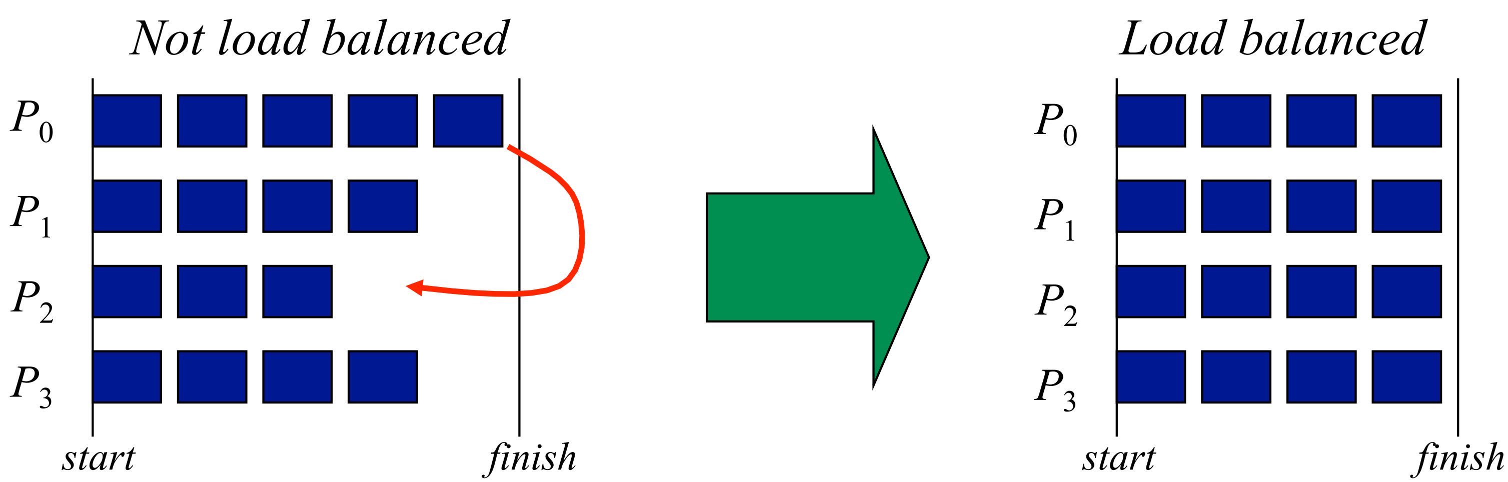


Not load balanced



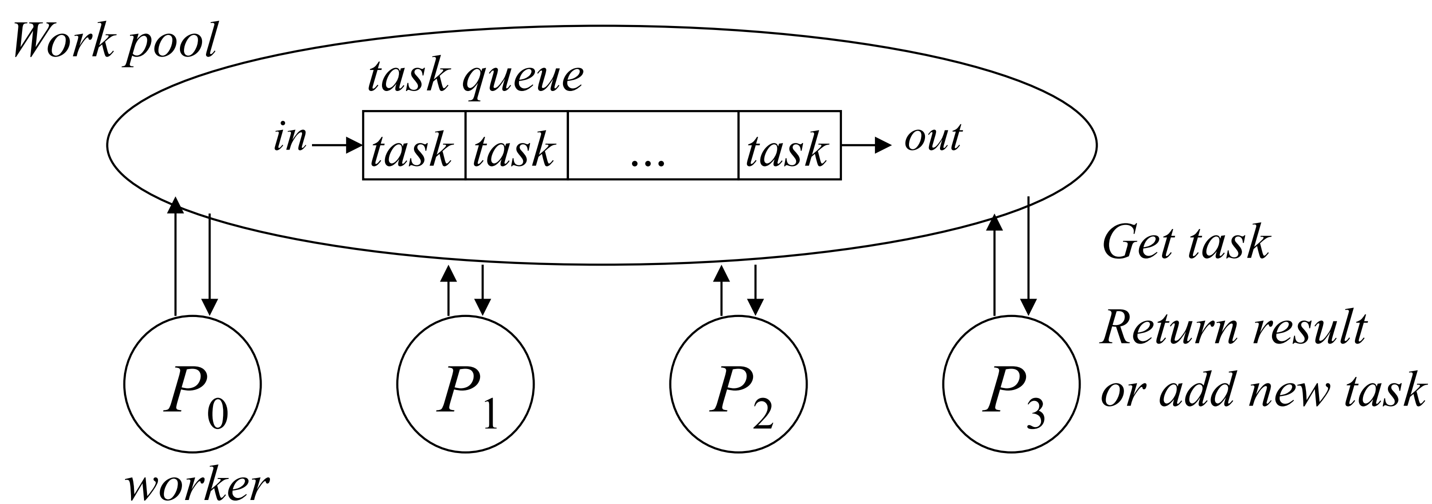
Static Load Balancing

- Load balancing can be viewed as a form of "bin packing"
- *Static scheduling* of tasks amounts to optimal bin packing
  - Round robin algorithm
  - Randomized algorithms
  - Recursive bisection
  - Optimized scheduling with simulated annealing and genetic algorithms
- Problem: difficult to estimate amount of work per task, deal with changes in processor utilization and communication latencies



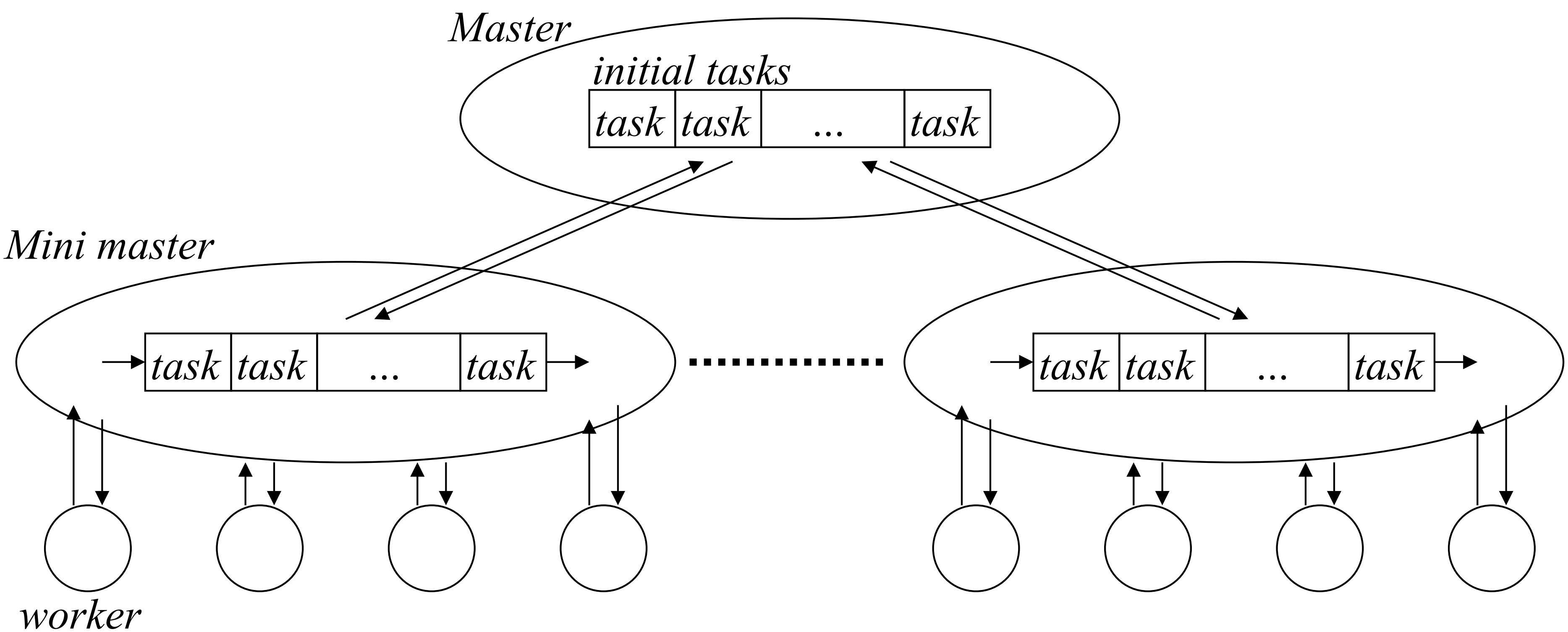
Centralized Dynamic Load Balancing

- Centralized: work pool with replicated workers
- Master process or central queue holds incomplete tasks
  - First-in-first-out or priority queue (e.g. priority based on task size)
- Terminates when queue is empty or workers receive termination signal



Decentralized Dynamic Load Balancing

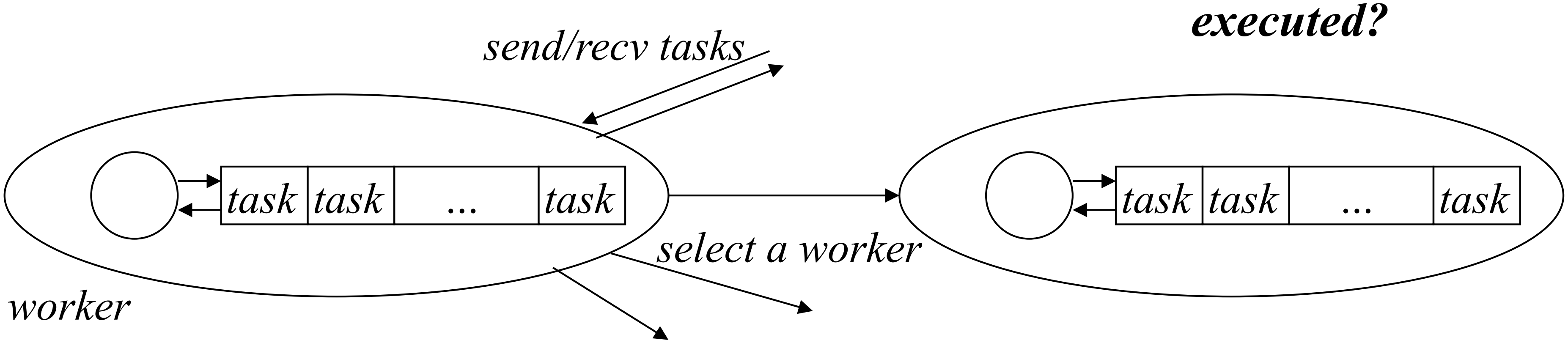
- Disadvantage of centralized approach is the central queue through which tasks move one by one
- Decentralized: distributed work pools



Fully Distributed Work Pool

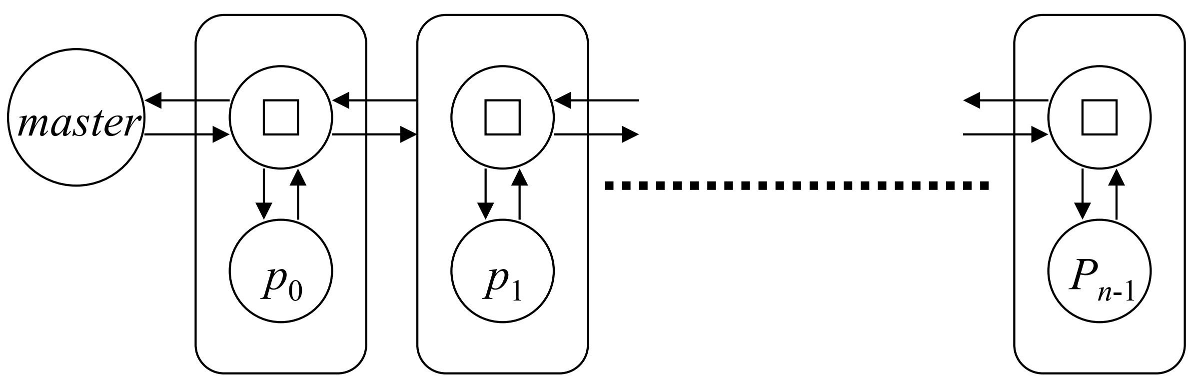
- *Receiver-initiated poll method*: (an idle) worker process requests a task from another worker process
- *Sender-initiated push method*: (an overloaded) worker process sends a task to another (idle) worker
- Workers maintain local task queues
- Process selection
  - Topology-based: select nearest neighbors
  - Round-robin: try each of the other workers in turn
  - Random polling/pushing: pick an arbitrary worker

Absence of starvation: assume ∞ tasks, how can we guarantee each one is eventually executed?



Worker Pipeline

- Workers are organized in an array (or ring) with the master on one end (or middle)
  - Master feeds the pipeline
  - When the buffer of a worker is idle, it sends a request to the left
  - When the buffer of a worker is full, incoming tasks are shifted to the worker on the right (passing task along until an empty slot)



Further Reading

- [PP2] pages 79-99, 201-210