

STA 4103/5107: Homework Assignment #6

(Wednesday, February 29)

Due: Wednesday, March 14

1. Consider the nonlinear dynamic system described by the equations:

$$\begin{aligned}x_t &= \sqrt{|x_{t-1}|} + u_t \\y_t &= x_t^2 + v_t\end{aligned}$$

where $x_1 \sim U[0, 1]$ (uniform on $[0, 1]$), $u_t \sim N(0, 0.1)$ (normal with mean 0 standard deviation 0.1), and $v_t \sim N(0, 0.1)$.

- Write a matlab program to simulate the state x_t and the observation process y_t for $t = 1, \dots, 200$.
- Write a matlab program to implement sequential Monte Carlo algorithm for $n = 100$. Plot the true and estimated states (using posterior mean). Compute the R^2 -error for the state from time step 11 to 200 (not considering the error around the initial value).
- Plot the histograms of the samples from the posterior at times $t = 50, 100, 150, 200$. Are these samples normally distributed (hint: use a Kolmogorov-Smirnov test)?

2. Consider the nonlinear dynamic system described by the equations:

$$\begin{aligned}x_t &= \frac{1}{2}x_{t-1} + \frac{25x_{t-1}}{1 + x_{t-1}^2} + 8\cos(1.2t) + u_t \\y_t &= \frac{x_t^2}{20} + 2x_t + v_t\end{aligned}$$

where $x_1 \sim U[0, 10]$, $u_t \sim N(0, 10)$, and $v_t \sim N(0, 1)$.

- Write a matlab program to simulate the state x_t and the observation process y_t for $t = 1, \dots, 100$.
- Write a matlab program to implement sequential Monte Carlo algorithm for $n = 500$. Plot the true and estimated states (using posterior mean). Compute the R^2 -error for the state from time step 11 to 100 (not considering the error around the initial value).
- Plot the histograms of the samples from the posterior at times $t = 25, 50, 75, 100$. Are these samples normally distributed?