STA 4103/5107: Homework Assignment #2

(Thursday, January 19)
Due: Thursday, January 26

1. Write a Matlab program to simulate a homogeneous Poisson process over the interval [0, 100]. Generate 50 sample paths for intensity $\lambda = 0.1$ and display them on the same plot. Count the number of events occurring in the interval [10, 60]. Plot a histogram of 50 realizations of this random number. Does this sample follow a Poisson distribution (hint: use a Kolmogorov-Smirnov test)?

2. Write a Matlab program to simulate an inhomogeneous Poisson process over the interval [0, 10] where the rate function

$$\lambda(t) = 2 + \sin(t) + \sin(2t)/2.$$

Plot the rate function versus time t. On the same figure, generate 30 sample paths for this process and display them on the same plot (use **subplot(4,1,1)** for the rate function, and **subplot(4,1,2:4)** for the sample paths).
(hint: a Newton-Raphson procedure can be used to estimate the values of F^{-1}).

3. Prove that the following simulation generates a homogeneous Poisson process with rate λ on $[0, T]$:

Step 1: Sample k from Poisson distribution with mean λT.
Step 2: Sample $s_1, \ldots, s_k$ i.i.d. from uniform $[0, T]$.

That is, demonstrate that for any time interval $[t, t+\Delta t]$ in $[0, T]$,

$$P\{k \text{ events in } [t, t+\Delta t]\} = \frac{\exp(-\lambda\Delta t)(\lambda\Delta t)^k}{k!}$$

4. Assume $s = (s_1, \ldots, s_n)$ is a simulation of an inhomogeneous Poisson process with rate function $\lambda(t)$, $0 \le t \le T$. Let γ be a mapping from $[0, T]$ to $[0, T]$ which satisfies:

$$i)\ \gamma(0) = 0, \quad ii)\ \gamma(T) = T, \quad iii)\ 0 < \dot{\gamma}(t) < \infty, \forall t \in [0, T]$$

where $\dot{\gamma}(t)$ denotes the derivative of $\gamma(t)$ with respect to t. Prove that $\gamma^{-1}(s) = (\gamma^{-1}(s_1), \cdots, \gamma^{-1}(s_n))$ is also an inhomogeneous Poisson process, with rate function

$$\lambda(\gamma(t))\dot{\gamma}(t).$$