STA 4103/5107: Homework Assignment #4

(Wednesday, February 1)
Due: Wednesday, February 8

1. Write a matlab program implementing a Gibbs sampler to sample from the Markov Random Field model for binary images introduced in the class. Choose the image size to be 10 × 10. Use a random image of 1s and (-1)s to initialize the program. Study the following cases:

a) H = - 1, 0, 1, and J = 0.
b) H = 0, and J = - 1, 1.
c) H = 4, and J = - 1, - 2.

Show a sequence (up to 9) of images in each combination of H and J.

Help on image plotting: Assume XX is a 3-dimensional array with size NxNxK, which denotes a sequence of K NxN binary images. Then this sequence of images can be plotted as:

```
for k = 1:K
    figure(k);
    if (sum(sum(XX(:,:,k)))==N^2)
        image(255*ones(N,N));
    elseif (sum(sum(XX(:,:,k)))==-N^2)
        image(zeros(N,N));
    else
        imagesc(XX(:,:,k));
    end
    colormap(gray);
    title(sprintf('%d-th run', k));
end
```

2. Let $X \in R^{m \times n}$ be a matrix of random variables such that they form a Markov Random Field. The conditional density of an element is dependent only on the values of its vertical and horizontal neighbors (except for the boundaries where the neighbors are limited). Let the conditional density of a pixel be Gaussian with mean μ and variance 0.1, where μ is the mean of its neighbors. That is:

$$f(X_{i,j} \mid \text{all other pixels}) = f(X_{i,j} \mid X_{i,j-1}, X_{i,j+1}, X_{i-1,j}, X_{i+1,j}) = N(\mu, 0.1)$$

where $\mu = (X_{i,j-1} + X_{i,j+1} + X_{i-1,j} + X_{i+1,j})/4$.

Write a matlab program to generate samples from this distribution using a Gibbs sampler. To set as initial condition download the four 100 × 100 matrices from the class website. For each initial condition, run the Markov chain for 5 sweeps. Show the matrix after every sweep (you can use "imagesc" in matlab) and comment on the effect of sampling on the images.