Homework 3

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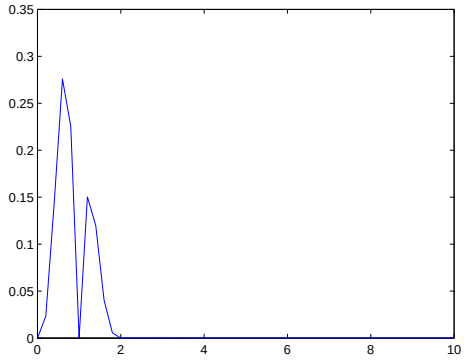
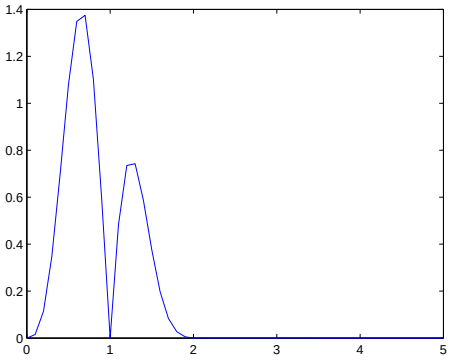


(a)



(b)

Figure 1: (a)Function of $x^2|sin(\pi x)|e^{-x^3}$. This implies that we could integrate the function from 0 to 3 (b) Density function of $f(x)$

0.1 Problem 1

(a) See figure 1
(b) See figure 2
(c) $E(x)$= 0.8616 $var(x)$= 0.1417
Matlab Code:

```
%homework 3
% we are going to use exponential distribution as proposal denstiy

f0=inline('x.^2.*abs(sin(pi*x)).*exp(-x.^3)','x');

c=quad(f0,0,4);

f=inline('x.^2.*abs(sin(pi*x)).*exp(-x.^3)/c','x','c');

%q function

t=0:0.1:5;
%plot the density function of f(x);
plot(t,f(t,c));

% the M-H algorithm
K=5000;
```

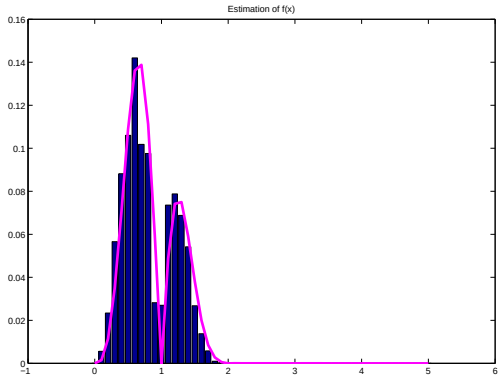


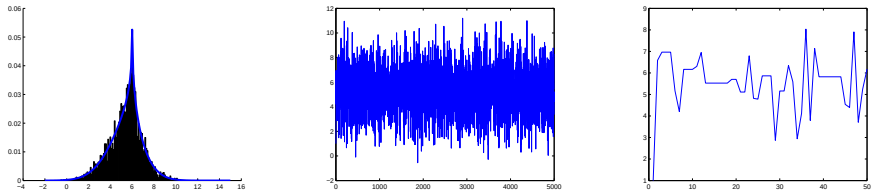
Figure 2: Histogram of Markov chain and $f(x)$

```
x=zeros(1,K);
x(1)=0.5;

for k=2:K
    y=exprnd(1);
    rho=min((f(y,c)*exp(-x(k-1)))/(f(x(k-1),c)*exp(-y)),1);
    U=rand;
    x(k)=y*(U<rho) + x(k-1)*(U>rho);
end;

%compute histogram

h=histc(x,t)/K;

figure(2);
bar(t,h);hold on;
plot(t,f(t,c)/sum(f(t,c)),'m','linewidth',3);
ex=mean(x);
varx=var(x);
```

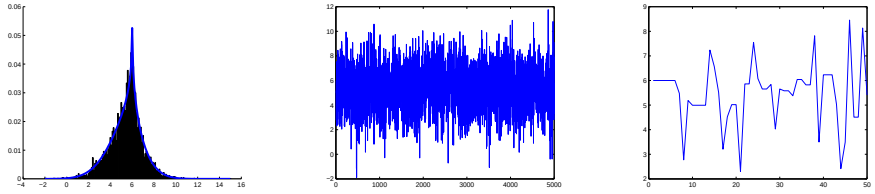
0.2 Problem 2

Solutions: $\rho(x,y) = \frac{f(y)*q(x)}{f(x)*q(y)} = \frac{f_{Y|X}(6|y)*f_x(y)*q(x)/f(6)}{f_{Y|X}(6|x)*f_x(x)*q(y)/f(6)} = \frac{f_{Y|X(6|y)}}{f_{Y|X}(6|x)}$;

The target density function $f_{X|Y}(x|y)$ when $y = 6$ is $f_{X|Y}(x|6) = \frac{e^{-|x-6|^{0.5}-(x-5)^2/8}}{\int e^{-|x-6|^{0.5}-(x-5)^2/8}}$



(a) (b) (c)



(a) (b) (c)

Figure 3: Top: (a)Sample from Posterior and original density function(Initial x=1).(b)5000 steps Evolution of the Chain (c) First 50 Steps Evolution; Bottom: Initial x=6

(1) Sample from the posterior for $y = 6$;

(2) Use the Markov chain to estimate the posterior mean and variance Initial $x = 1$:

$E(x) =$

5.4595

$Variance(x) =$

2.2726

Initial $x = 6$: $E(x) =$

5.4579

$Variance(x) =$

2.3152

Matlab Code:

```
%homework 3
% Problem 2

f=inline('exp(-abs(x-6).^0.5)','x');

f0=inline('exp(-abs(x-6).^0.5-(x-5).^2./8)','x');
```

```
c=quad(f0,-2,14);

f1=inline('exp(-abs(x-6).^0.5-(x-5).^2./8)/c','x','c');
% the M-H algorithm
K=5000;
x=zeros(1,K);
x(1)=6;
t=-2:0.1:15;

for k=2:K
    y=5+2*randn(1); %nomal distribution with mu=5, var=4;
    rho=min(f(y)/f(x(k-1)),1);
    U=rand;
    x(k)=y*(U<rho) + x(k-1)*(U>rho);
end;

%compute histogram

h=histc(x,t)/K;

figure(2);
hold on; bar(t,h);hold on;
plot(t,f1(t,c)/sum(f1(t,c)),'b','linewidth',3);

ex=mean(x);
varx=var(x);
```