STA 4103/5107: Homework Assignment #4

(Thursday, February 2)
Due: Thursday, February 9

1. Let X_t be a Markov chain generated using some initial probability P[1] and the transition matrix:

$$\Pi = \begin{bmatrix} 0.1 & 0.3 & 0.4 & 0.2 \\ 0.2 & 0.4 & 0.0 & 0.4 \\ 0.0 & 0.3 & 0.5 & 0.2 \\ 0.5 & 0.3 & 0.2 & 0.0 \end{bmatrix}.$$

(a) Verify if X_t is (i) irreducible, and (ii) aperiodic, and then find the stationary probability vector for X_t.
(b) Run a simulation to verify that the averages along a sample path converge to the stationary probability.

2. Let Π be an $m \times m$ transition matrix of an irreducible, homogeneous Markov chain on a finite state space. Suppose Π is idempotent, i.e. $\Pi^2 = \Pi$. Prove that (1) the Markov chain is aperiodic, and (2) all rows of Π are identical.

3. Write a matlab program implementing the Metropolis-Hastings algorithm to sample a random variable X with the density

$$f(x) = \frac{x^2 |\sin(\pi x)| e^{-x^3}}{\int_0^\infty x^2 |\sin(\pi x)| e^{-x^3} \, dx}, \quad x > 0.$$

You have to decide what q (proposal density) you want to use. Choose positive numbers to start the Markov chain.
(a) Plot the density function $f(x)$.
(b) Histogram the values attained by Markov chain and compare it to the plot of $f(x)$.
(c) Estimate the value of $E[X]$ and $var(X)$ using values of the Markov chain.