STA 4103/5107: Homework Assignment #3

(Thursday, January 26)
Due: Thursday, February 2

1. Write a matlab program to simulate a discrete-time, finite-state Markov process with the following transition matrix:

$$\Pi = \begin{bmatrix} 0.2 & 0.2 & 0.5 & 0.1 \\ 0.2 & 0.3 & 0.4 & 0.1 \\ 0.4 & 0.2 & 0.3 & 0.1 \\ 0.1 & 0.0 & 0.0 & 0.9 \end{bmatrix}$$

One way to simulate this Markov chain is the following:

(a) Set $i = 1$, and choose X_i uniformly among the four states.
(b) Given X_i, select X_{i+1} using the i-th row of the transition matrix.
(c) Set $i = i + 1$ and go to Step b.

Generate 3 sample paths with 200 time steps and display them on 3 plots.

2. Write a matlab program to simulate a discrete-time, finite-state Markov process with the following transition matrix:

$$\Pi = \begin{bmatrix} 0.2 & 0.2 & 0.1 & 0.5 \\ 0.1 & 0.3 & 0.4 & 0.2 \\ 0.3 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0.3 & 0.1 & 0.5 \end{bmatrix}$$

(a) Generate four sample paths of this Markov chain with four different initial conditions. For each path, plot the relative frequencies (versus i) with which the path visits the four states for i up to 1000.
(b) Compare the estimated relative frequencies with the dominant eigenvector of the transition matrix. Rescale the dominant eigenvector by the sum of its entries for this comparison.

3. Repeat Problem 2 for a Markov chain having transition matrices:

$$\Pi = \begin{bmatrix} 0.5 & 0.5 & 0.0 & 0.0 \\ 0.1 & 0.9 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.3 & 0.7 \\ 0.0 & 0.0 & 0.2 & 0.8 \end{bmatrix}, \text{ and } \Pi = \begin{bmatrix} 0.0 & 0.5 & 0.0 & 0.5 \\ 0.5 & 0.0 & 0.5 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.5 \\ 0.5 & 0.0 & 0.5 & 0.0 \end{bmatrix}$$

Try different initial conditions for each case.