Monte Carlo MAP 5615 HW2

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$$D^+ = max\left(\frac{k}{N} - F(x_k)\right) = max\left(\frac{2}{15}, \frac{1}{15}, 0.3\right) = 0.3$$

$$D^- = max\left(F(x_k) - \frac{k-1}{N}\right) = max\left(0.2, \frac{4}{15}, \frac{1}{30}\right) = \frac{4}{15}$$

$D_N = max(D^+, D^-) = 0.3$

D_N measures the distance between functions F_N and F.

D^+measures the maximum distance between functions F_N and F at the left end point of the interval $[x_k, x_{k+1})$.

D^-measures the maximum distance between functions F_N and F at the right end point of the interval $[x_k, x_{k+1})$.

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$Left\ hand\ side$

$$= max\left[F(x_1) - \frac{1-1}{N}, \max_{k=1,\dots,N-1}\left(F(x_{k+1}) - \frac{k}{N}, \frac{k}{N} - F(x_k)\right), \frac{N}{N} - F(x_N)\right]$$

$$= max\left[\max_{k=1,\dots,N}\left(\frac{k}{N} - F(x_k)\right), \max_{k=1,\dots,N}\left(F(x_k) - \frac{k-1}{N}\right)\right]$$

$= Right\ hand\ sid$

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I tried RANDU generator and the LCG $x_n \equiv x_{n-1}\ mod\ 10000$.

RANDU is bad random number generator since the triplets of random numbers RANDU generated lie on no more than 15 planes in R^3. This doesn't look random at all.

The other LCG generator is bad since it generates integers from 1 to 10000. This is not a sequence of random numbers.

Following is the result for RANDU. See code at appendix [1]

RANDU

>> chiKS

c =

14.2000 8.2400 9.5000 12.1400 14.0800 6.0400 8.5200 9.7400 14.8000 15.0000

DN =

0.3898

Since 0.3898<0.41, we accept that c follows chi square distribution with degree 9 at the 5% significance level.

Note: 0.41 comes from the table of KS test with N=10 and 5% significant level. See appendix [2]

LCG

```
>> chiKS
c =
   Columns 1 through 8

        1000        1000        1000        1000        1000
1000        1000        1000

   Columns 9 through 10

        1000        1000
DN =
    1
```

Since 1>0.41, we reject that c follows chi square distribution with degree 9 at the 5% significance level.

By this test RANDU are good random number generators while the LCG is a bad one. This also means the test is weak.

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In the chi square test, we have 2 assumptions. The random experiment has mutually exclusive and exhaustive outcomes. The $nP_i > 5$ for all i.

Assuming we have the probability of run up of length i P_i, we still cannot make sure $nP_i > 5$ if i is very large. So I don't think we can use chi square test directly. We may set the probability of run up of length t or more as a outcome. Then we can use chi square test.

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Design a test:

Given a sequence of random numbers. Consider the interval L=(0, 0.5) and R= (0.5, 1) , the equ-head-tail of length p is by giving an example. Consider the numbers 0.3,0.6,0.7,0.4,0.8,.1,0.9,0.8,0.3,0.2,0.7. Say a random number lie in L is tail and one lie in R is head. Put a vertical line when the total number of heads and tails are equal.

0.3,0.6 | 0.7,0.4 | 0.8,0.1 | 0.9,0.8,0.3,0.2 | 0.7

Count the numbers in each block. We have a equ-head-tail length 2,2,2,4.

Apply chi square test as following:

Pick a value t. Let u(i) be the number of equ-head-tail of length 2i for i = 1, 2, 3. . . t-1; let u(t) be the number of length 2t or more. The outcomes of the test are: "equ-head-tail length of 2", "equ-head-tail length of 4"…… "equ-head-tail length of 2t-2" and "equ-head-tail length of 2t or more". We know the probability of each outcome occurring, write as P_i. Pick a value n such that $nP_i > 5$. Given a sequence of random number, go through the sequence to identify blocks as "equ-head-tail length of i", for some i. Stop when there are a total of n length found. Count the number of equ-head-tail length of i, and t or more. These counts are the Y_i value in chi square test.

For example, apply the test on RANDU generator.
Pick t = 4, n = 100. See code in appendix [3]

Equ-head-tail length	2	4	6	8 or more
P_i	0.5	0.125	0.0625	0.3125
Y_i	50	15	5	30
nP_i	50	12.5	6.25	31.25

Q_3=0.8, P(x<0.8)=0.1505, so accept RANDU generator at the significant level 5%. We can see that this test is a weak test since RANDU is a bad generator.

Appendix

[1] code of problem 5

```
function chiKS
x=RANDU(1,10000);

%x=[1:10000];
M=10;
k=10;
p=1/k;
c = zeros(1,M);
for m = 1:M
    for i = 1:k
        s=x(1000*(m-1)+1:1000*m);
        Y(i)=0;
        for j = 1:1000
            if ((i-1)/k) <= s(j) && s(j) < (i/k)
                Y(i) = Y(i) +1;
            end
```

```
            end
            c(m)=c(m)+(Y(i)-1000*p)^2/(1000*p);
        end
end
for i = 1:M
        DU(i)=i/M-chi2cdf(c(i),9);
        DD(i)=chi2cdf(c(i),9)-(i-1)/M;
end
DU=sort(DU);
DD=sort(DD);
DN=max(DU(M),DD(M));
c
DN
```

[2] table of KS test
http://www.eridlc.com/onlinetextbook/appendix/table7.htm

[3] code of problem 9

```
function updown
s=RANDU(1,10000);
c=0;
U=0;
D=0;
for j = 1:10000
        if s(j) < 0.5
            U = U +1;
        else
            D = D +1;
        end
        j = j+1;
        if U == D
            c=[c, j-1];
            n = size(c);
            if n(2)>100
                break
            end
        end
end

count=zeros(1,4);
C=[];
```

```
for i = 1:100
    C(i)= c(i+1)-c(i);
end
for i = 1:100
    if C(i)==2
        count(1)=count(1)+1;
    elseif C(i) == 4
        count(2)=count(2)+1;
    elseif C(i)== 6
            count(3) = count(3)+1;
    else
            count(4)=count(4)+1;
    end
end
count
end
```