12 Solutions

1. TB

2. Observe that $T = 6X + 1$, since the last step of the algorithm has only one operation. If, $i = i + 1$ is computed first in step 3, note that $T = 5X + 1$. Then $E[T] = 6\lambda + 1, Var(X) = 25\lambda$.

3. Observe that

$$F(x) = P\{X \leq x\} = \frac{\int_0^x \sqrt{1-u^2}du}{\int_0^1 \sqrt{1-u^2}du}$$

The denominator is the area of the quarter circle, $\pi/4$. For the numerator, use the substitution $u = \sin\theta$, to get:

$$F(x) = \frac{2}{\pi}\left(\arcsin x + x\sqrt{1-x^2}\right)$$

An alternative solution uses geometry. Let C be the circular region, and D be the triangle. Then, the area of the triangle is: $x\sqrt{1-x^2}/2$, and the area of the circular part is $\theta/2 = (\arcsin x)/2$. Dividing the sum of the areas by $\pi/4$ gives the desired answer. The probability of acceptance, i.e., "success" is $\pi/4$.Since the number of iterations a geometric random variable with $p = \pi/4$, the average number of times Step 1 is repeated is $4/\pi$.

4. Observe that $c = \max(p_i/q_i) = 1.4$. Then the algorithm is:

   (a) Generate u_1 from $U(0,1)$. Set $y = Floor(4u_1) + 1$.

   (b) Generate u_2 from $U(0,1)$

   (c) If $u_2 < \dfrac{4p_y}{1.4} = p_y/0.35$, set $X = y$ and stop. Otherwise return to step 1.

   The average complexity is: $E[T] = 1.4 \times 11 = 15.4$

5. TB

6. TB

7. Put $1 - e^{-\alpha x^\beta} = y \Rightarrow 1 - y = e^{-\alpha x^\beta} \Rightarrow \alpha x^\beta = -\log(1-y) \Rightarrow$

$$x = \left(-\frac{\log(1-y)}{\alpha}\right)^{1/\beta}$$

   Then, we have:

   (a) Generate u

   (b) Set $X = \left(-\frac{\log u}{\alpha}\right)^{1/\beta}$