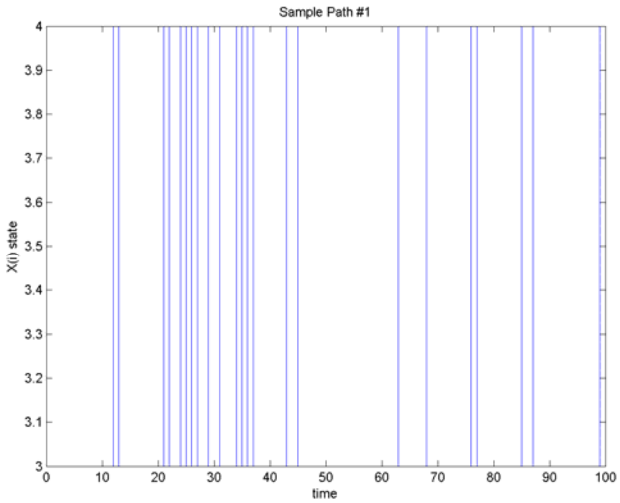
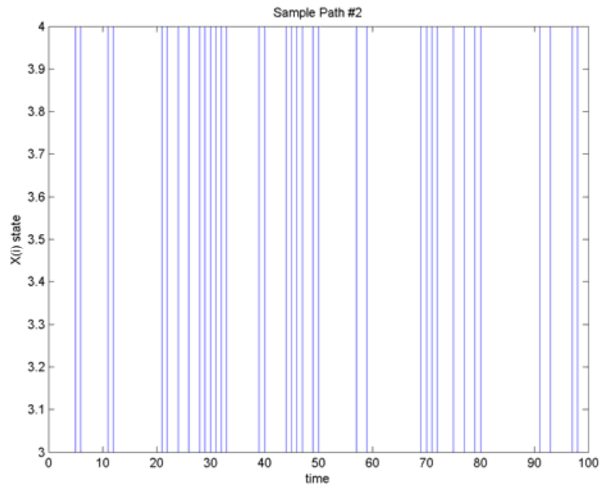
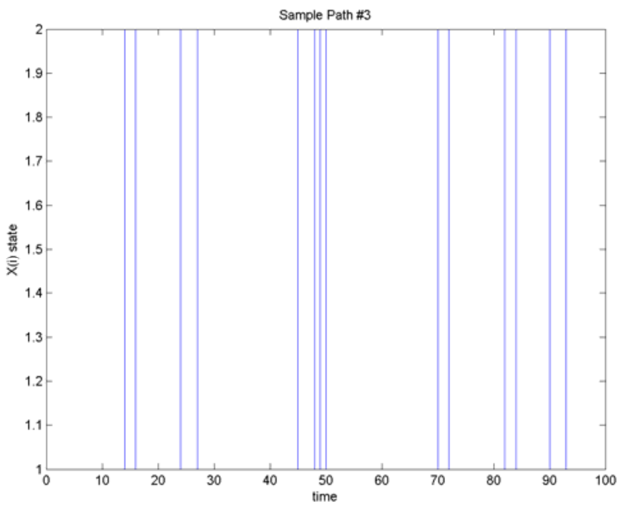
STA5107: Dr. Srivastava
Homework 2
Jaime Frade

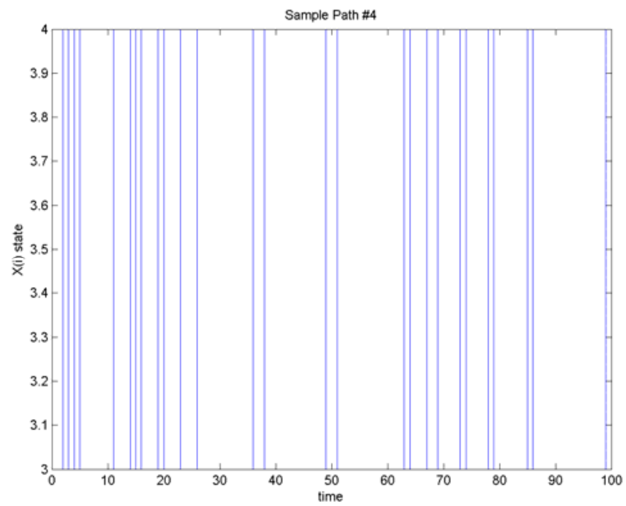
1.

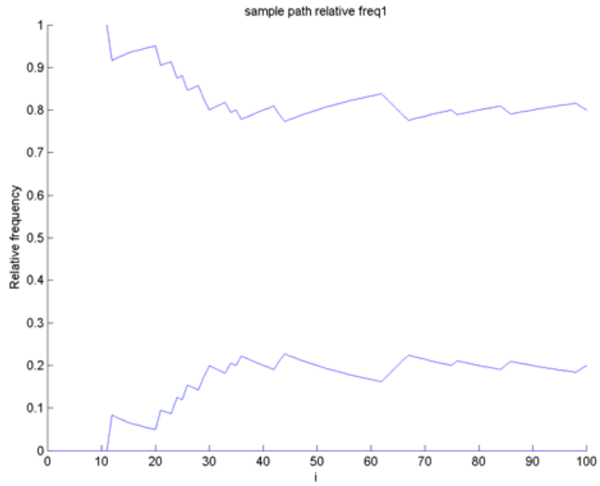
OUTPUT

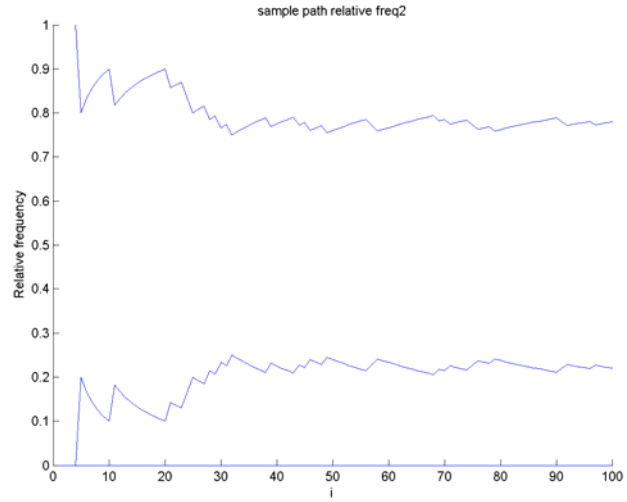


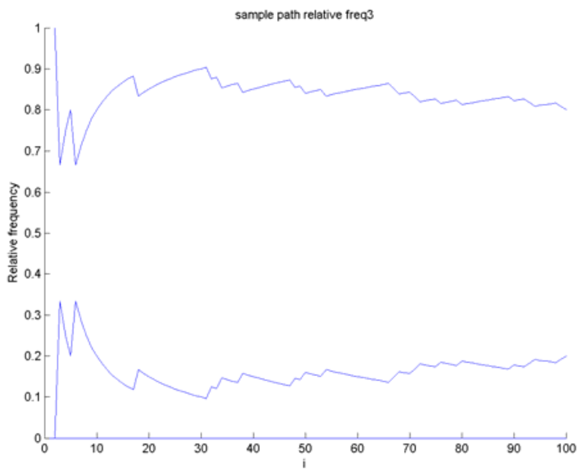


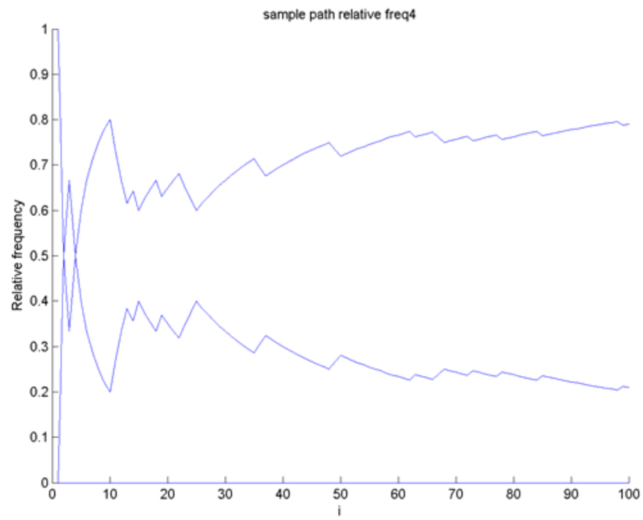





sample path relative freq1
Relative frequency
i


sample path relative freq2
Relative frequency
i


sample path relative freq3
Relative frequency
i


sample path relative freq4
Relative frequency
i

Sample path relative Frequency at i=100

<u>When state= 1</u>
v2_Rel_Freq =

0 0 0.2000 0.8000

<u>When state= 2</u>
v2_Rel_Freq =

0 0 0.2200 0.7800

<u>When state= 3</u>
v2_Rel_Freq =

0.1600 0.8400 0 0

<u>When state= 4</u>

v2_Rel_Freq =

0.1600 0.8300 0 0

```
>> vectoreig=v(1:4,2)/sum(v(1:4,2))
```

vectoreig1 =

0.1667
0.8333
0
0

```
>> vectoreig=v(1:4,4)/sum(v(1:4,4))
```

vectoreig2 =

0
0
0.2222
0.7778

Sample path relative Frequency at i=10000, to display convergence.

<u>When state= 1</u>

v2_Rel_Freq =

0	**0**	**0.2234**	**0.7766**

<u>When state= 2</u>

v2_Rel_Freq =

0	**0**	**0.2264**	**0.7736**

<u>When state= 3</u>

v2_Rel_Freq =

0.1627	**0.8373**	**0**	**0**

<u>When state= 4</u>

v2_Rel_Freq =

0.1647	**0.8353**	**0**	**0**

<u>Comment</u>
From the setup, one can notice that not all states communicate with each other. The transition matrices is irreducible when separated into two disjoint matrices that within the matrices do communicate with each other. Therefore if the state enters into one of these two matrices, it will continue and remain in the state, as seen in relative frequencies graphs.

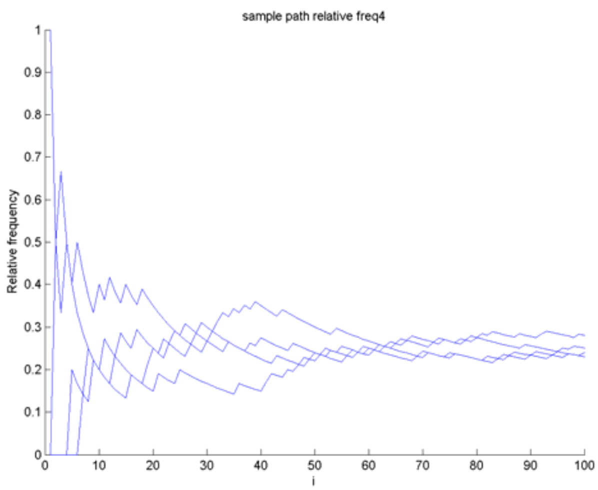
CODE

```
%prob1
clear all
clc
trans = [0.5 0.5 0.0 0.0;
        0.1 0.9 0.0 0.0;
        0.0 0.0 0.3 0.7;
        0.0 0.0 0.2 0.8];
 n=100;
 state = zeros(4,n);
 path=zeros(n);
 for i=1:4;
     rand1 = rand/2;
     rand2 = rand/2;
     state(:,1)=[rand1 0.5-rand1 rand2 0.5-rand2];
     for j=1:n;
        x = rand;
        y = find(x<cumsum(state(:,j)));
        path(j,i) = min(y);
        state(:,j+1)=trans(path(j,i),:)';
         for k=1:4;
             count(j,i,k)=length(find(path(1:j,i)==k))/j;
         end;
     end;
 end;

 for i=1:4;
     figure(i);
     stairs(path(:,i));
     xlabel('time')
     ylabel('X(i) state')
     title(['Sample Path #' int2str(i)]);
     saveas(figure(i),['Prob1 Sample path ' int2str(i) '.png'])
 end;
 x1 = [1:1:100];
 for i=1:4;
     figure(i+4);
     hold on
     plot(x1,count(:,i,1));
     plot(x1,count(:,i,2));
     plot(x1,count(:,i,3));
     plot(x1,count(:,i,4));
     xlabel('i');ylabel('Relative frequency');
     title(['sample path relative freq' int2str(i)]);
     saveas(figure(i+4),['Prob1 sample path rel freq ' int2str(i) '.png'])
     hold off
 end;
 for i=1:4;
 [v,d] = eig(trans');
 v1_eigen = v(1:4,1)/sum(v(1:4,1))
 v2_Rel_Freq = [count(n,i,1), count(n,i,2), count(n,i,3), count(n,i,4)]
 end;

 vectoreig1=v(1:4,2)/sum(v(1:4,2))
 vectoreig2=v(1:4,4)/sum(v(1:4,4))
```

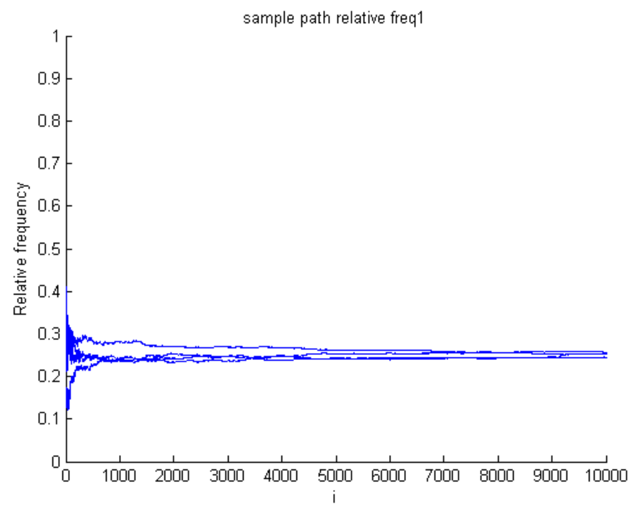
2.

- The process is irreducible because there exists positive probability which is associated with each transition from any state, say i, to any other state, j
- The process is aperiodic. This can be shown by nothing that there exists positive probability that is associated with traveling from state 1 and returning to state 1 in one step. Therefore, the greatest common factors is 1 and thus by definition the chain is aperiodic.

OUTPUT



When i=100



When i=10000

The stationary probability is
v1_eigen =

0.2500
0.2500
0.2500
0.2500

Sample path relative Frequency at i=10000, to display convergence.

<u>When state= 1</u>

v2_Rel_Freq =

0.2540 0.2420 0.2530 0.2510

<u>When state= 2</u>

v2_Rel_Freq =

0.2440 0.2500 0.2570 0.2490

<u>When state= 3</u>

v2_Rel_Freq =

0.2490 0.2640 0.2430 0.2440

<u>When state= 4</u>

v2_Rel_Freq =

0.2430 0.2470 0.2640 0.2460

total2 =

1.1250

avg1 =

1.1244

<u>Comment</u>
For each state, the averages along that path converge to the stationary probability. The above two values are approximately close to one another. Obtained 1.125 by the formula and through simulation, obtained 1.124

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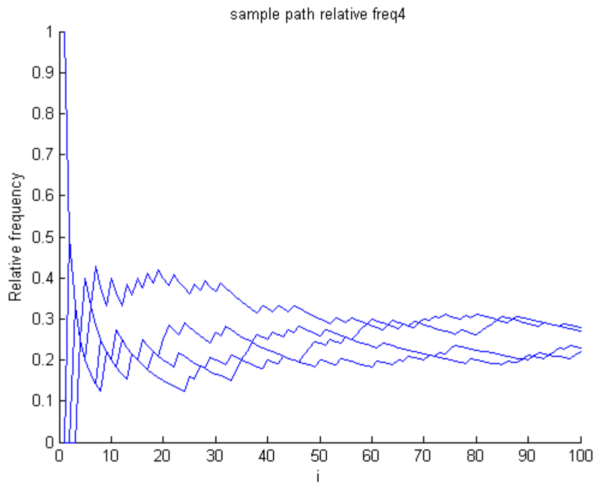
CODE

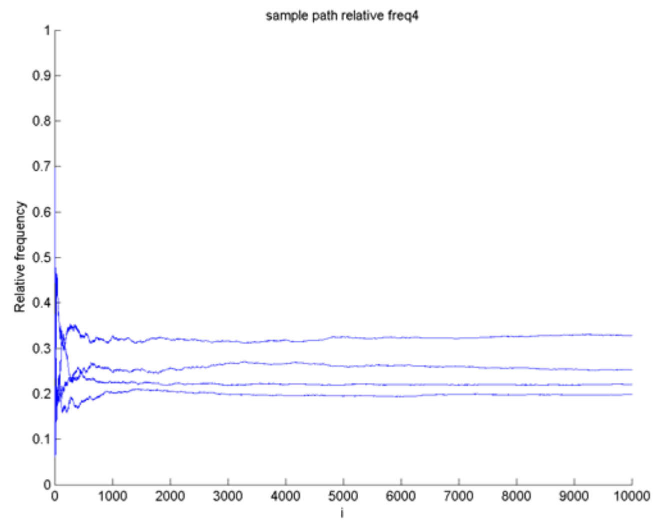
```
%prob1
clear all
clc

trans = [0.1 0.3 0.4 0.2;
        0.2 0.1 0.3 0.4;
        0.4 0.2 0.1 0.3;
        0.3 0.4 0.2 0.1];

 n=10000;
 state = zeros(4,n);
 path=zeros(n);

 for i=1:4;
     rand1 = rand/2;
     rand2 = rand/2;
     state(:,1)=[rand1 0.5-rand1 rand2 0.5-rand2];
     for j=1:n;
        x = rand;
        y = find(x<cumsum(state(:,j)));
        path(j,i) = min(y);
        state(:,j+1)=trans(path(j,i),:)';

         for k=1:4;
             count(j,i,k)=length(find(path(1:j,i)==k))/j;
         end;
     end;
 end;

%  for i=1:4;
%      figure(i);
%      stairs(path(:,i));
%      xlabel('time')
%      ylabel('X(i) state')
%      title(['Sample Path #' int2str(i)]);
%      saveas(figure(i),['Prob 2 -Sample path ' int2str(i) '.png'])
%  end;
%
 x1 = [1:1:100];
 for i=1:4;
     figure(i+4);
     hold on
     plot(x1,count(:,i,1));
     plot(x1,count(:,i,2));
     plot(x1,count(:,i,3));
     plot(x1,count(:,i,4));
     xlabel('i');
     ylabel('Relative frequency');
     title(['sample path relative freq' int2str(i)]);
     saveas(figure(i+4),['Prob 2 - sample path relative freq ' int2str(i) '.png'])
     hold off
 end;
```

```
 for i=1:4;
 [v,d] = eig(trans');
 v1_eigen = v/sum(v)
 v2_Rel_Freq = [count(n,i,1), count(n,i,2), count(n,i,3), count(n,i,4)]
 end;


f=[2 1 2.5 -1];
total2 = dot(f,v1_eigen)

n1=10000;

x(1) = ceil(4*rand);

for i=1:n1
[m,n]=sort(trans(x(i),:));
exitloop=0;
j=1;
u1=rand;
while exitloop==0;
    if u1<sum(m(1:j));
    x(i+1)=n(j);
    exitloop=1;
    else
    j=j+1;
    end
end
end


for i=1:n1;
    total1(i)=sum(f(x(i)));
end;

avg1=sum(total1)/n1
```

3.
OUTPUT



When i=100



When i=10000

The stationary probability is

v1_eigen =

0.1975
0.3333
0.2469
0.2222

Sample path relative Frequency at i=10000, to display convergence.

When state= 1

v2_Rel_Freq =

0.1991 0.3245 0.2541 0.2223

When state= 2

v2_Rel_Freq =

0.1986 0.3199 0.2591 0.2224

When state= 3
v2_Rel_Freq =

0.2002 0.3332 0.2453 0.2213

When state= 4
v2_Rel_Freq =

0.1980 0.3281 0.2536 0.2203

total2 =

1.1235

avg1 =

1.1200

Comment

For each state, the averages along that path converge to the stationary probability. The above two values are approximately close to one another. Obtained 1.1235 by the formula and through simulation, obtained 1.1200

CODE

```
%prob3

clear all
clc

trans = [0.1 0.3 0.4 0.2;
        0.2 0.4 0.0 0.4;
        0.0 0.3 0.5 0.2;
        0.5 0.3 0.2 0.0];

 n=10000;
 state = zeros(4,n);
 path=zeros(n);



 for i=1:4;
     rand1 = rand/2;
     rand2 = rand/2;
     state(:,1)=[rand1 0.5-rand1 rand2 0.5-rand2];
     for j=1:n;
        x = rand;
        y = find(x<cumsum(state(:,j)));
        path(j,i) = min(y);
        state(:,j+1)=trans(path(j,i),:)';

         for k=1:4;
             count(j,i,k)=length(find(path(1:j,i)==k))/j;
         end;
     end;
 end;

%  for i=1:4;
%      figure(i);
%      stairs(path(:,i));
%      xlabel('time')
%      ylabel('X(i) state')
%      title(['Sample Path #' int2str(i)]);
%      saveas(figure(i),['Prob3-Sample path prob3 ' int2str(i) '.png'])
%  end;

 x1 = [1:1:10000];
 for i=1:4;
     figure(i+4);
     hold on
     plot(x1,count(:,i,1));
     plot(x1,count(:,i,2));
     plot(x1,count(:,i,3));
     plot(x1,count(:,i,4));
     xlabel('i');
     ylabel('Relative frequency');
     title(['sample path relative freq' int2str(i)]);
     saveas(figure(i+4),['Prob3-sample path relative freq ' int2str(i) '.png'])
     hold off
```

```
 end;

 for i=1:4;
 [v,d] = eig(trans');
 v1_eigen = v/sum(v)
 v2_Rel_Freq = [count(n,i,1), count(n,i,2), count(n,i,3), count(n,i,4)]
 end;


f=[2 1 2.5 -1];
total2 = dot(f,v1_eigen)

n1=1000;

x(1) = ceil(4*rand);

for i=1:n1
[m,n]=sort(trans(x(i),:));
exitloop=0;
j=1;
u1=rand;
while exitloop==0;
    if u1<sum(m(1:j));
    x(i+1)=n(j);
    exitloop=1;
    else
    j=j+1;
    end
end
end


for i=1:n1;
    total1(i)=sum(f(x(i)));
end;

% for i=1:4
%     total2(i)=sum(f(i)*v1_eigen(i));
% end

avg1=sum(total1)/n1
```

- When $\prod$ is irreducible, this implies that $\exists$ an $n > 0$ s.t. for each $\prod^n$ has an element $\prod_{i,j} > 0$. For any i and any j.

- $\prod_{m\times m}$ is idempotent $\Rightarrow \prod^2 = \prod$

$$
\begin{aligned}
\prod \cdot \prod^2 &= \prod \cdot \prod \\
\prod^3 &= \prod^2 \\
&\vdots \\
\prod^n &= \prod^{n-1} \\
&= \prod
\end{aligned} \tag{1}
$$

Therefore $\prod_{i,j} > 0$ for $n = 1$, $\prod$ is <u>aperiodic</u>.

- Since $\prod$ is idempotent, consider the j^{th} column of two rows, m and n.

$$
\begin{aligned}
\prod_{m,j} &= \sum_{i=1}^{N} \prod_{m,i} \cdot \prod_{i,j} \\
1 &= \sum_{i=1}^{N} \frac{\prod_{m,i} \cdot \prod_{i,j}}{\prod_{m,j}} \\
1 &= \frac{\prod_{m,1} \cdot \prod_{1,j} + \prod_{m,2} \cdot \prod_{2,j} + \cdots + \prod_{m,j} \cdot \prod_{j,j} + \cdots + \prod_{m,N} \cdot \prod_{N,j}}{\prod_{m,j}}
\end{aligned}
$$

$$
\begin{aligned}
\prod_{n,j} &= \sum_{i=1}^{N} \prod_{n,i} \cdot \prod_{i,j} \\
1 &= \sum_{i=1}^{N} \frac{\prod_{n,i} \cdot \prod_{i,j}}{\prod_{n,j}} \\
1 &= \frac{\prod_{n,1} \cdot \prod_{1,j} + \prod_{n,2} \cdot \prod_{2,j} + \cdots + \prod_{n,j} \cdot \prod_{j,j} + \cdots + \prod_{n,N} \cdot \prod_{N,j}}{\prod_{n,j}}
\end{aligned}
$$

From above

$$\sum_{i=1}^{N} \frac{\prod_{m,i} \cdot \prod_{i,j}}{\prod_{m,j}} = 1 - \prod_{j,j}$$
$$\sum_{i=1}^{N} \frac{\prod_{n,i} \cdot \prod_{i,j}}{\prod_{n,j}} = 1 - \prod_{j,j}$$

Therefore $\prod_{m,j} = \prod_{n,j}$, thus all rows of the $\prod$ are identical