STA 4103/5107: Homework Assignment #2

(Wednesday, January 18)
Due: Wednesday, January 25

1. Let X_t be a Markov chain generated using some initial probability P[1] and the transition matrix:

$$\Pi = \begin{bmatrix} 0.1 & 0.3 & 0.4 & 0.2 \\ 0.2 & 0.1 & 0.3 & 0.4 \\ 0.4 & 0.2 & 0.1 & 0.3 \\ 0.3 & 0.4 & 0.2 & 0.1 \end{bmatrix}$$

a) Verify if X_t is (i) irreducible, and (ii) aperiodic, and then find the stationary probability vector for X_t.
b) Use your program from Homework 1 to verify that the averages along a sample path converge to the stationary probability.
c) If $f: \{x_1, x_2, x_3, x_4\} \to \mathbb{R}$ is as follows:
$$f(x_1) = 2.0,\ f(x_2) = 1.0,\ f(x_3) = 2.5,\ f(x_4) = -1.0\ ,$$
show through simulation that
$$\frac{1}{n}\sum_{i=1}^{n} f(X_i) \overset{n\to\infty}{\to} \sum_{j=1}^{4} f(x_j)P(x_j)$$
where P is the stationary probability.

2. Repeat Problem 1 with the transition matrix:

$$\Pi = \begin{bmatrix} 0.1 & 0.3 & 0.4 & 0.2 \\ 0.2 & 0.4 & 0.0 & 0.4 \\ 0.0 & 0.3 & 0.5 & 0.2 \\ 0.5 & 0.3 & 0.2 & 0.0 \end{bmatrix}.$$

3. Let Π be an $m \times m$ transition matrix of an irreducible, homogeneous Markov chain on a finite state space. Suppose Π is idempotent, i.e. $\Pi^2 = \Pi$. Prove that (1) the Markov chain is aperiodic, and (2) all rows of Π are identical.