STA 4103/5107: Homework Assignment #1

(Thursday, January 12)
Due: Thursday, January 19

1. Write a Matlab program to simulate a random walk, for a given value of T and $s = \alpha\sqrt{T}$. Plot the sample paths of the X_t versus the real time t, for $\alpha = 1.0$ and T = 1, 0.1, 0.01, and 0.001. In each case, choose the total number of steps n to be 10/T, so that you go up to time $t = 10$ irrespective of T. Use the "stairs" command in Matlab to plot the sample paths.

2. Prove that the standard Wiener process is a Gaussian process that has mean $m(t) = 0$ and a non-stationary covariance function $k(t, s) = \min(t, s)$.

3. Let $W(t)$, $t \geq 0$, be a standard Wiener process. For a positive constant α define a new stochastic process $X(t)$ as

$$X(t) = \alpha W\left(\frac{t}{\alpha^2}\right)$$

Prove that $X(t)$ also is a standard Wiener process.

4. Let $W(t)$, $t \geq 0$, be a standard Wiener process. Define a new stochastic process $Z(t)$ as

$$Z(t) = e^{W(t) - \frac{1}{2}t}, \qquad t \geq 0$$

Show that $E[Z(t)] = 1$ and use this result to compute the covariance function of $Z(t)$.