Final Project:Summary of Methos of Concentration Inequalities

Proposal

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Abstract

In the lecture of concentration inequality, we use martingale method(see the surveys of McDiarmid [2,3]) to show the proof of centration inequalities bound tail probabilities of general functions of independent random variables. There are several other methods have been known to prove such inequalities, including information-theoretic methods (see Alhswede, Gacs and Korner, Marton, Dembo, Massart and Rio), Talagrands induction method , and various problem-specific methods; see Janson, uczak and Rucin ski for a survey. A novel way of deriving powerful inequalities, the entropy method,(see the surveys of Boucheron[1]) based on logarithmic Sobolev inequalities, was developed by Ledoux, Bobkov and Ledoux, Massart, Rio and Bousquet for proving sharp concentration bounds for maxima of empirical processes. Recently Boucheron, Lugosi and Massart pointed out that the methodology may be used effectively outside of the context of empirical process theory as well.

In this project, I will review the martingale method first, then discuss the entrophy-based method.

Entrophy-based Method These inequalities may be considered as exponential versions of the well-known EfronStein inequality.

EfronStein inequality:

$$Z = f(X_1, ..., X_n) \tag{1}$$

$$Z^{(i)} = f(X_1, ..., X_{i-1}, X_{i'}, X_{i+1}, ..., X_n) \tag{2}$$

$$Var(Z) = \frac{1}{2}E\left\{\sum_{i=1}^{n}(Z - Z^{(i)})^2\right\} \tag{3}$$

Logarithmic Sobolev inequalities: For any function f:χ_n →R, noting $Z = f(X_1, ..., X_n)$ and for all λ ⊂R,

$$\lambda E[Z\exp^{\lambda Z}] - E[\exp^{\lambda Z}]\log E[\exp^{\lambda Z}] \leq \frac{1}{2}\sum_{i=1}^{n} E[\exp^{\lambda Z}\,\psi(-\lambda(Z - Z^{(i)}))1_{Z>Z^{(i)}}] \tag{4}$$

$$\lambda E[Z\exp^{\lambda Z}] - E[\exp^{\lambda Z}]\log E[\exp^{\lambda Z}] \leq \frac{1}{2}\sum_{i=1}^{n} E[\exp^{\lambda Z}\,\psi(\lambda(Z^{(i)} - Z))1_{Z<Z^{(i)}}] \tag{5}$$

Also, by reviewing the relationship of the new results with some of the existing work, we may find some other inequalities may be recovered easily from some of the new inequalities, for example, Talagrands convex distance inequality.

We also will show some application of new inequalities.

Bibliography

[1] STÉPHANE BOUCHERON, GÁBOR LUGOSI, and PASCAL MASSART. "CONCENTRATION INEQUALITIES USING THE ENTROPY METHOD". In: *The Annals of Probability* 31.3 (2003), pp. 1583–1614.

[2] MCDIARMID. "On the method of bounded differences." In: *In Surveys in Combinatorics (J. Siemons, ed.) Cambridge Univ. Press.* (1989), pp. 148–188.

[3] MCDIARMID and C. "Concentration. 195–248." In: *In Probabilistic Methods for Algorithmic Discrete Mathematics (M. Habib, C. McDiarmid, J. Ramirez-Alfonsin and B. Reed, eds.)* (Springer 1998), pp. 195–248.