MAP 6437-4 - Monte Carlo Methods in Financial Mathematics - Spring 2010
Homework 1: Pseudorandom Generators
Due Date: Jan 20, 2010

1. Given an LCG with parameters a, c, m, prove that

$$x_{n+k} \equiv a^k x_n + \frac{(a^k - 1)}{a - 1} c \,(\mathrm{mod}\, m);\ (a \geq 2, k \geq 0)$$

which shows that the $(n + k)$th term can be computed directly from the nth term.

2. (a) If U and V are independently distributed random variables from the uniform distribution $U(0, 1)$ show that $U + V (\mathrm{mod}\, 1)$ is also $U(0, 1)$.

   (b) A random number generator is designed by

$$R_n = \left(\frac{X_n}{8} + \frac{Y_n}{7} \right) \mathrm{mod}\, 1$$

where $X_0 = 0, Y_0 = 1, X_{n+1} = (9X_n + 3) \,\mathrm{mod}\, 8$, and $Y_{n+1} = 3Y_n \,\mathrm{mod}\, 7$ for $n = 0, 1, ...$ Calculate $R_0, R_1, ..., R_5$. What is the period of the generator $\{R_n\}$?

3. Write a code that would implement RANDU. For debugging purposes, print x_{1000} when the seed is $x_0 = 1$.

   (a) Using RANDU generate $u_1, ..., u_{20,002}$, where $u_n = x_n/M$. For all triplets in your sequence, (u_i, u_{i+1}, u_{i+2}),in which $0.5 \leq u_{i+1} \leq 0.51$, plot u_i versus u_{i+2}. Comment on the pattern of your scatterplot.

   (b) Generate a sequence of length 1002. Use a program that plots points in 3 dimensions and rotates the axes to rotate the points until you can see the 15 planes. (S-Plus function spin may be helpful.)

4. Download a code for Mersenne twister written by Mutsuo Saito and Makoto Matsumoto (www.math.sci.hiroshima-u.ac.jp/~m-mat/MT/emt.html)[1]. Generate 1002 numbers, and plot pairs and triples of successive numbers for a visual inspection of randomness. Discuss your conclusions.

5. Download (or, write) a code for one of the combined generators of L'Ecuyer. See pages 52 and 53 of **Monte Carlo Methods in Financial Engineering**, by Paul Glasserman, for C codes. (These pages will be uploaded on Blackboard. You do not need to turn in your code. However, you will use this code later.)

[1] Also see the links at Wikipedia for twister.