Homework 2

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January 23, 2011

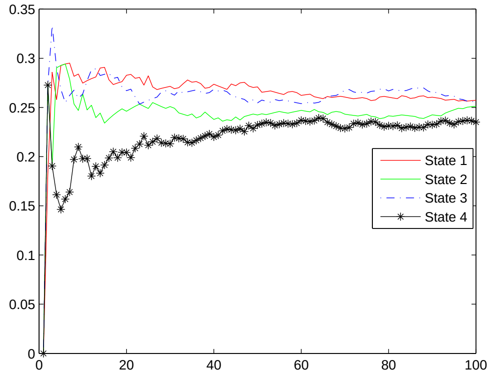

Figure 1: Averages along one path converge to the stationary probability

Problem 1.

a)

(i) X_t is irreducible, because every elements in the transition matrix Π is big than 0.

(ii) Since $d(x_1) = 1$, $d(x_2) = 1$, $d(x_3) = 1$, $d(x_4) = 1$, this irreducible Markov Chain is aperiodic.

(iii) The stationary probability vector X_t =[0.5 0.5 0.5 0.5], matlab code:

```
% for homework 2
% problem 1

clear; close all;

PI = [0.1 0.3 0.4 0.2; 0.2 0.1 0.3 0.4; 0.4 0.2 0.1 0.3; 0.3 0.4 0.2 0.1];
% transition matrix

[V, D] = eig(PI');
ind = find(abs(diag(D)-1)< 1e-6);
P = V(:,ind)/sum(V(:,ind));
```

b)

```
% for homework 2
```

```
% problem 1

clear; close all;

PI = [0.1 0.3 0.4 0.2; 0.2 0.1 0.3 0.4; 0.4 0.2 0.1 0.3; 0.3 0.4 0.2 0.1];
% transition matrix

M = 4;      % number of chains
N = 4;      % number of states
K = 1000;    % number of time steps in each chain

for m = 1:M
    %x(1,m) = m;  % random initial
     x(1,m) = ceil(4*rand);
    for k = 2:K
        % generate a chain
        P = PI(x(k-1,m),:); %pick i-th row
        U = rand;
        if U < P(1)
            x(k,m) = 1;

        elseif (P(1)<U&&U<(P(1)+P(2)))
            x(k,m) = 2;

        elseif((P(1)+P(2))<U&&U<(P(1)+P(2)+P(3)))

                        x(k,m) = 3;
        else
            x(k,m) = N;
        end
        for n=1:N
        p0(m,n,k) = sum(x(:,m)==n)/k;
        end;
    end
end

plot(squeeze(p0(1,1,1:10:1000)),'r');
hold on,plot(squeeze(p0(1,2,1:10:1000)),'-g');
hold on,plot(squeeze(p0(1,3,1:10:1000)),'-.b');
hold on,plot(squeeze(p0(1,4,1:10:1000)),'-*k');
```

c)

```
% for homework 2
```

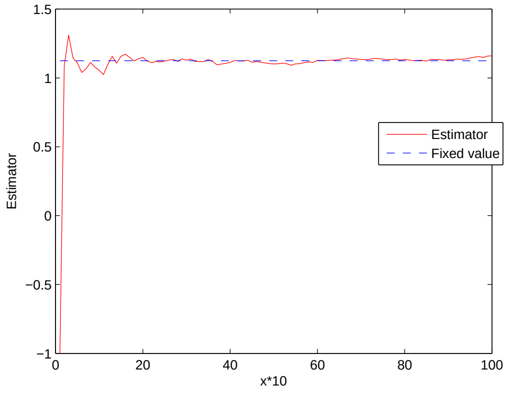

Figure 2: Simulation

```
% problem 1

clear; close all;

PI = [0.1 0.3 0.4 0.2; 0.2 0.1 0.3 0.4; 0.4 0.2 0.1 0.3; 0.3 0.4 0.2 0.1];
% transition matrix

f=[2.0 1.0 2.5 -1.0];

N=4; % state #
K=1000;

%generate the chain
 x(1,1) = ceil(4*rand);
    for k = 2:K
        % generate a chain
        P = PI(x(k-1,1),:); %pick i-th row
        U = rand;
        if U < P(1)
            x(k,1) = 1;

        elseif (P(1)<U&&U<(P(1)+P(2)))
            x(k,1) = 2;

        elseif((P(1)+P(2))<U&&U<(P(1)+P(2)+P(3)))

                    x(k,1) = 3;
```

```
        else
            x(k,1) = N;
        end
    end

[V, D] = eig(PI');
ind = find(abs(diag(D)-1)< 1e-6);
P = V(:,ind)/sum(V(:,ind));

% generate the estimator

for i=1:10:1000
Est1(i)=(1/i)*sum(f(x(1:i,1)));
Est2(i)=sum(f*P);
end;

plot(Est1(1:10:1000),'r');
hold on, plot(Est2(1:10:1000),'--b')
```

Problem 2

a)

```
>> PI = [0.1 0.3 0.4 0.2; 0.2 0.4 0 0.4; 0 0.3 0.5 0.2; 0.5 0.3 0.2 0];
>> PI*PI

ans =

    0.1700    0.3300    0.2800    0.2200
    0.3000    0.3400    0.1600    0.2000
    0.1600    0.3300    0.2900    0.2200
    0.1100    0.3300    0.3000    0.2600
```

So the elements in Π^2 is all big than 0, X_t is irreducible. Since $d(x_1) = 1$, $d(x_2) = 1$, $d(x_3) = 1$, $d(x_4) = 1$, X_t is aperiodic. $P =$[0.1975 0.3333 0.2469 0.2222]

b)

See figure 3.

c)

See figure 4

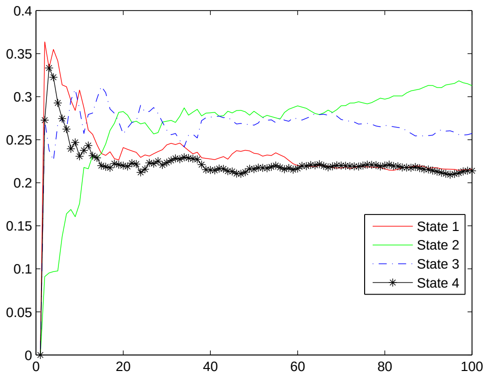

Figure 3: Averages along one path converge to the stationary probability

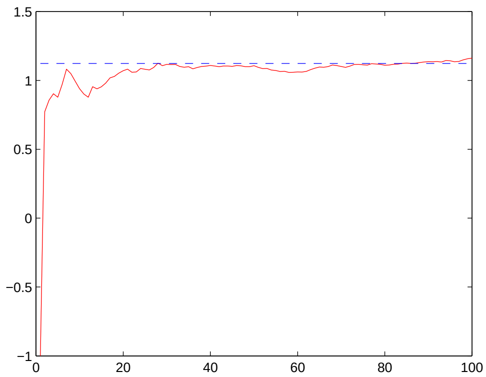

Figure 4: Simulation