

STA 4103/5107: Homework Assignment #1

(Wednesday, January 11)

Due: Wednesday, January 18

1. Write a matlab program to simulate a discrete-time, finite-state Markov process with the following transition matrix:

$$\Pi = \begin{bmatrix} 0.2 & 0.2 & 0.5 & 0.1 \\ 0.2 & 0.3 & 0.4 & 0.1 \\ 0.4 & 0.2 & 0.3 & 0.1 \\ 0.1 & 0.0 & 0.0 & 0.9 \end{bmatrix}$$

One way to simulate this Markov chain is the following:

- (a) Set  $i = 1$ , and choose  $X_i$  uniformly among the four states.
- (b) Given  $X_i$ , select  $X_{i+1}$  using the  $i$ -th row of the transition matrix.
- (c) Set  $i = i + 1$  and go to Step b.

Generate 3 sample paths with 200 time steps and display them on 3 plots.

2. Write a matlab program to simulate a discrete-time, finite-state Markov process with the following transition matrix:

$$\Pi = \begin{bmatrix} 0.2 & 0.2 & 0.1 & 0.5 \\ 0.1 & 0.3 & 0.4 & 0.2 \\ 0.3 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0.3 & 0.1 & 0.5 \end{bmatrix}$$

- (a) Generate four sample paths of this Markov chain with four different initial conditions. For each path, plot the relative frequencies (versus  $i$ ) with which the path visits the four states for  $i$  up to 1000.
- (b) Compare the estimated relative frequencies with the dominant eigenvector of the transition matrix. Rescale the dominant eigenvector by the sum of its entries for this comparison.

3. Repeat Problem 2 for a Markov chain having transition matrices:

$$\Pi = \begin{bmatrix} 0.5 & 0.5 & 0.0 & 0.0 \\ 0.1 & 0.9 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.3 & 0.7 \\ 0.0 & 0.0 & 0.2 & 0.8 \end{bmatrix}, \text{ and } \Pi = \begin{bmatrix} 0.0 & 0.5 & 0.0 & 0.5 \\ 0.5 & 0.0 & 0.5 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.5 \\ 0.5 & 0.0 & 0.5 & 0.0 \end{bmatrix}$$

Try different initial conditions for each case.