
Shared-memory Parallel Programming with Cilk Plus

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Outline for Today

- Threaded programming models
- Introduction to Cilk Plus
 - tasks
 - algorithmic complexity measures
 - scheduling
 - performance and granularity
 - task parallelism examples
 - vector addition using divide and conquer
 - nqueens: exploratory search

What is a Thread?

- **Thread: an independent flow of control**
 - software entity that executes a sequence of instructions
- **Thread requires**
 - program counter
 - a set of registers
 - an area in memory, including a call stack
 - a thread id
- **A process consists of one or more threads that share**
 - address space
 - attributes including user id, open files, working directory, ...

An Abstract Example of Threading

A sequential program for matrix multiply

```
for (row = 0; row < n; row++)  
  for (col = 0; col < n; col++)  
    c[row][col] =  
      dot_product(get_row(a, row), get_col(b, col))
```

can be transformed to use multiple threads

```
for (row = 0; row < n; row++)  
  for (col = 0; col < n; col++)  
    c[row][col] =  
      spawn dot_product(get_row(a, row), get_col(b, col))
```

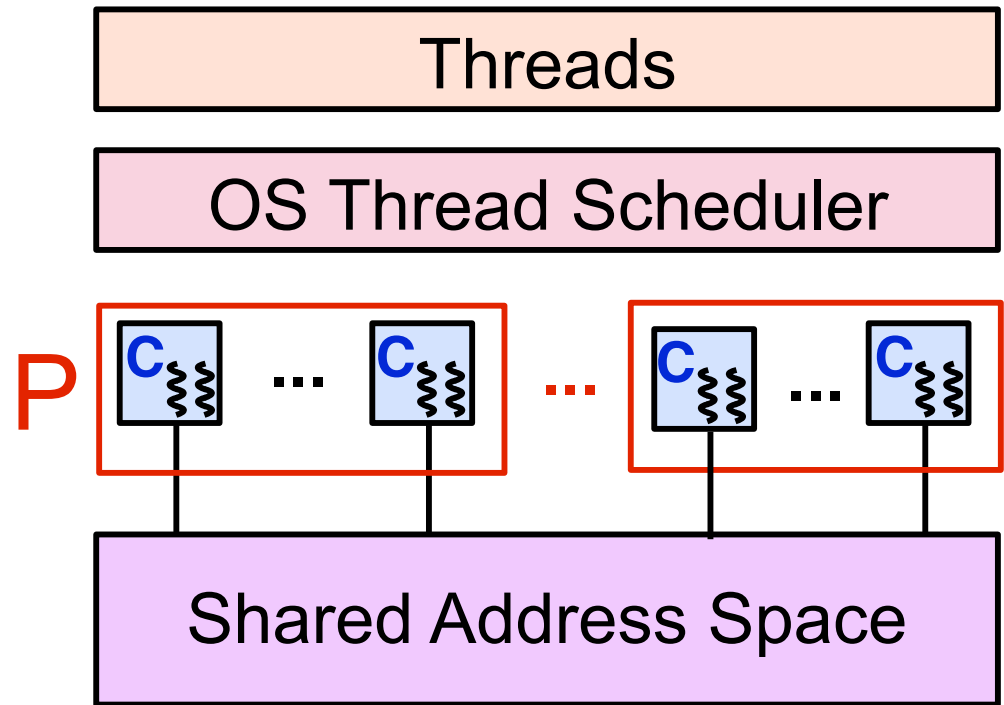
Why Threads?

Well matched to multicore hardware

- **Employ parallelism to compute on shared data**
 - boost performance on a fixed memory footprint (strong scaling)
- **Useful for hiding latency**
 - e.g. latency due to memory, communication, I/O
- **Useful for scheduling and load balancing**
 - especially for dynamic concurrency
- **Relatively easy to program**
 - easier than message-passing? you be the judge!

Threads and Memory

- All memory is globally accessible to every thread
- Each thread's stack is treated as local to the thread
- Additional local storage can be allocated on a per-thread basis
- Idealization: treat all memory as equidistant



Schema for SMP Node

Targets for Threaded Programs

Shared-memory parallel systems

- **Multicore processor**
- **Workstations or cluster nodes with multiple processors**
- **Xeon Phi accelerator**
 - about 250 threads
- **SGI UV: scalable shared memory system**
 - up to 4096 threads

Threaded Programming Models

- **Library-based models**
 - all data is shared, unless otherwise specified
 - examples: Pthreads, C++11 threads, Intel Threading Building Blocks, Java Concurrency Library, Boost
- **Directive-based models, e.g., OpenMP**
 - shared and private data
 - pragma syntax simplifies thread creation and synchronization
- **Programming languages**
 - Cilk Plus (Intel, GCC)
 - CUDA (NVIDIA)
 - Habanero-Java (Rice)

Toward Standard Threading for C/C++

At last month's meeting of the C standard committee, WG14 decided to form a study group to **produce a proposal for language extensions for C to simplify parallel programming**. This proposal is expected to **combine the best ideas from Cilk and OpenMP, two of the most widely-used and well-established parallel language extensions for the C language family**.

As the chair of this new study group, named CPLEX (C Parallel Language Extensions), I am announcing its organizational meeting:

June 17, 2013 10:00 AM PDT, 2 hours

Interested parties should join the group's mailing list, to which further information will be sent:

<http://www.open-std.org/mailman/listinfo/cplex>

Questions can be sent to that list, and/or to me directly.

--

Clark Nelson
Intel Corporation

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Vice chair, PL22.16 (ANSI C++ standard committee)
Chair, SG10 (WG21 study group for C++ feature-testing)
Chair, CPLEX
(WG14 study group for C parallel language extensions)

Cilk Plus Programming Model

- A simple and powerful model for writing multithreaded programs
- Extends C/C++ with three new keywords
 - cilk_spawn**: invoke a function (potentially) in parallel
 - cilk_sync**: wait for a procedure's spawned functions to finish
 - cilk_for**: execute a loop in parallel
- Cilk Plus programs specify logical parallelism
 - what computations can be performed in parallel, i.e., tasks
 - not mapping of work to threads or cores
- Faithful language extension
 - if Cilk Plus keywords are elided → C/C++ program semantics
- Availability
 - Intel compilers
 - GCC (some in 4.8, full in 5)

Cilk Plus Tasking Example: Fibonacci

Fibonacci sequence



- Computing Fibonacci recursively

```
int fib(int n) {  
    if (n < 2) return n;  
    else {  
        int n1, n2;  
        n1 = fib(n-1);  
        n2 = fib(n-2);  
        return (n1 + n2);  
    }  
}
```

Cilk Plus Tasking Example: Fibonacci

Fibonacci sequence



- Computing Fibonacci recursively **in parallel with Cilk Plus**

```
int fib(int n) {  
    if (n < 2) return n;  
    else {  
        int n1, n2;  
        n1 = cilk_spawn fib(n-1);  
        n2 = fib(n-2);  
        cilk_sync;  
        return (n1 + n2);  
    }  
}
```

Cilk Plus Terminology

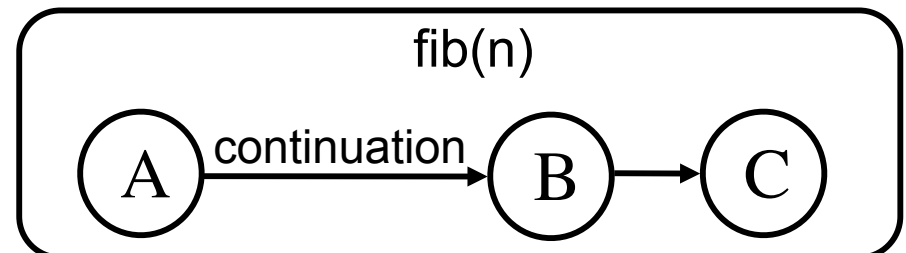
- Parallel control
 - cilk_spawn**, **cilk_sync**
 - return** from spawned function
- Strand
 - maximal sequence of instructions not containing parallel control

```
int fib(n) {  
  if (n < 2) return n;  
  else {  
    int n1, n2;  
    n1 = cilk_spawn fib(n - 1);  
    n2 = cilk_spawn fib(n - 2);  
    cilk_sync;  
    return (n1 + n2);  
  }  
}
```

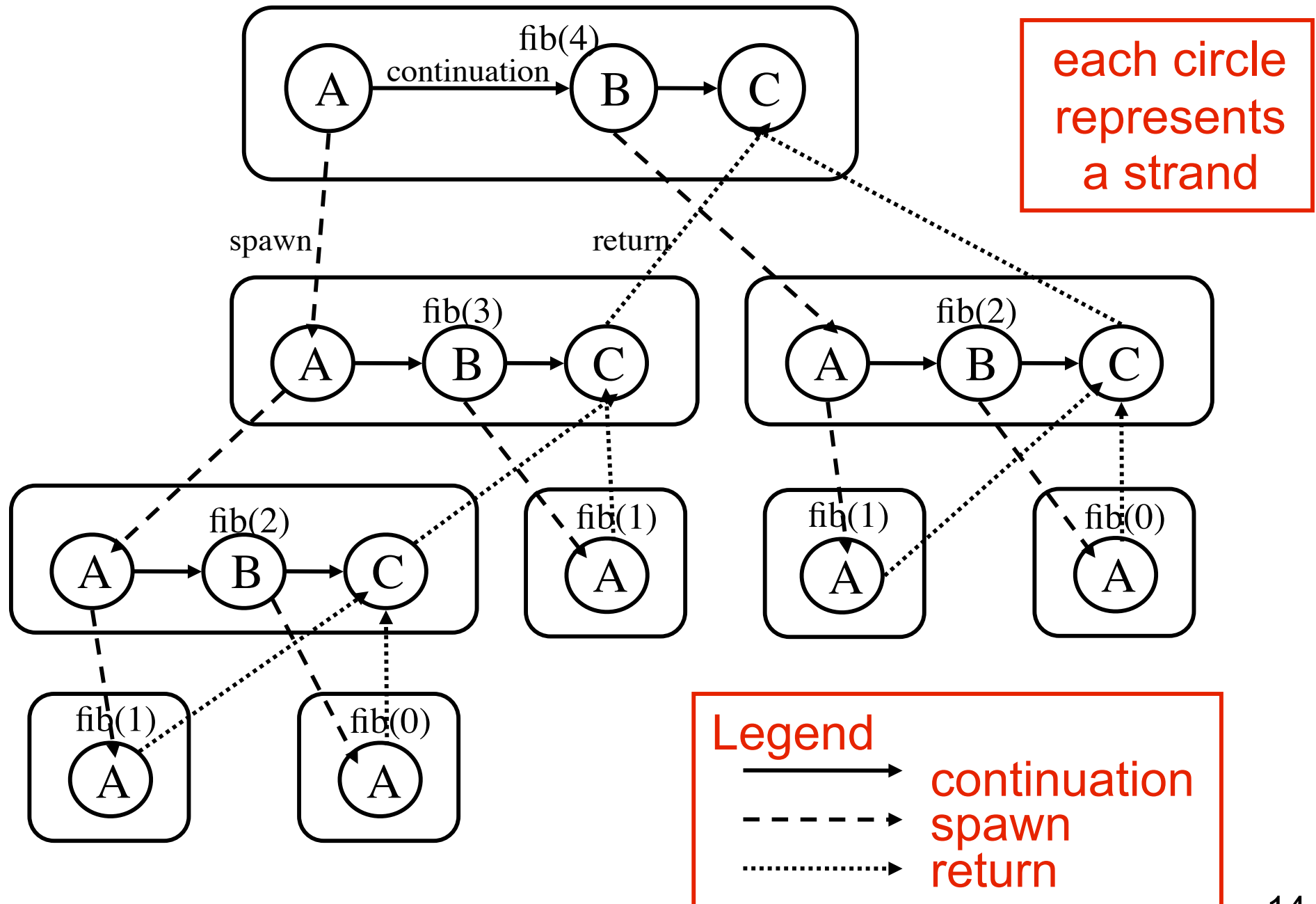
Strand A: **code before first spawn**

Strand B: **compute n-2 before 2nd spawn**

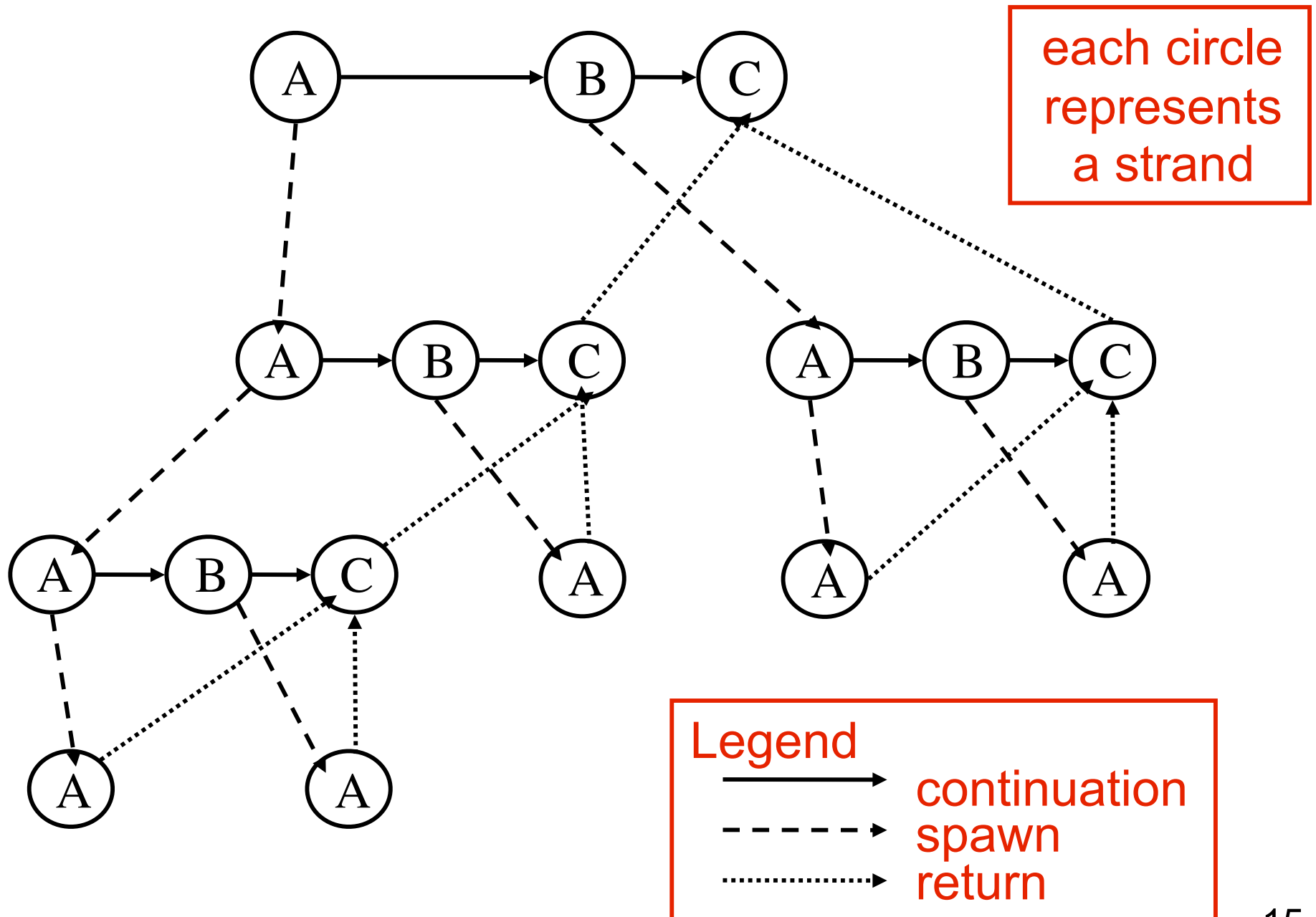
Strand C: **n1+ n2 before the return**



Cilk Program Execution as a DAG

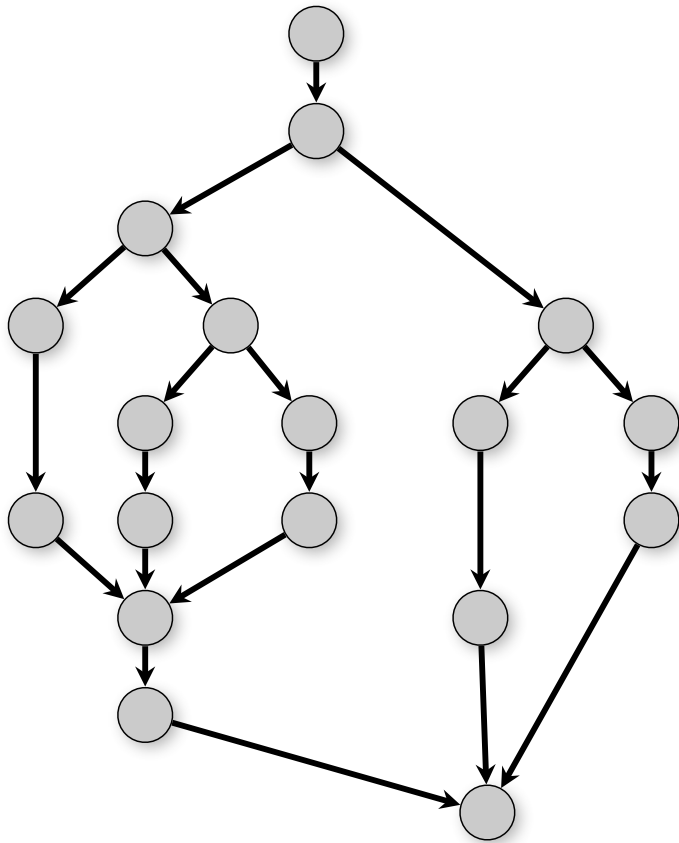


Cilk Program Execution as a DAG



Algorithmic Complexity Measures

T_P = execution time on P processors



PROC₀ ... PROC_{P-1}

Computation graph abstraction:

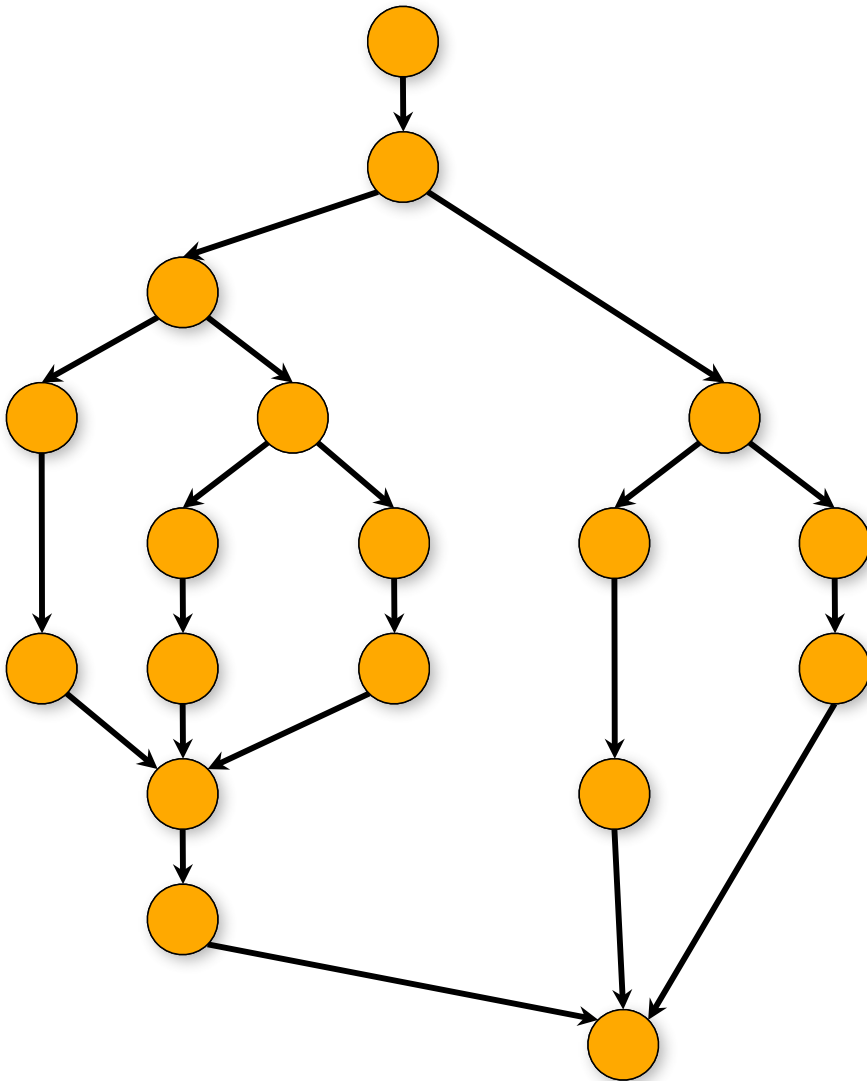
- node = arbitrary sequential computation
- edge = dependence (successor node can only execute after predecessor node has completed)
- Directed Acyclic Graph (DAG)

Processor abstraction:

- P identical processors
- each processor executes one node at a time

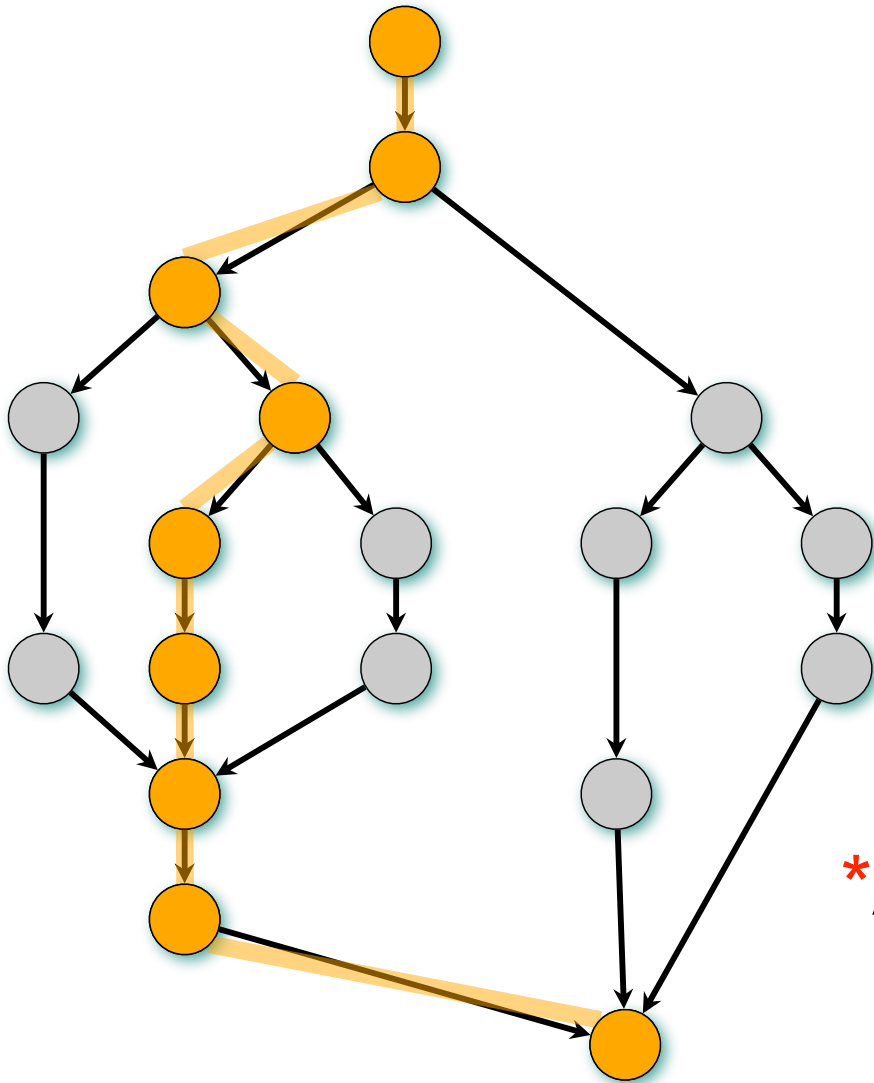
T_P = execution time on P processors

$T_1 = work$



Algorithmic Complexity Measures

T_P = execution time on P processors



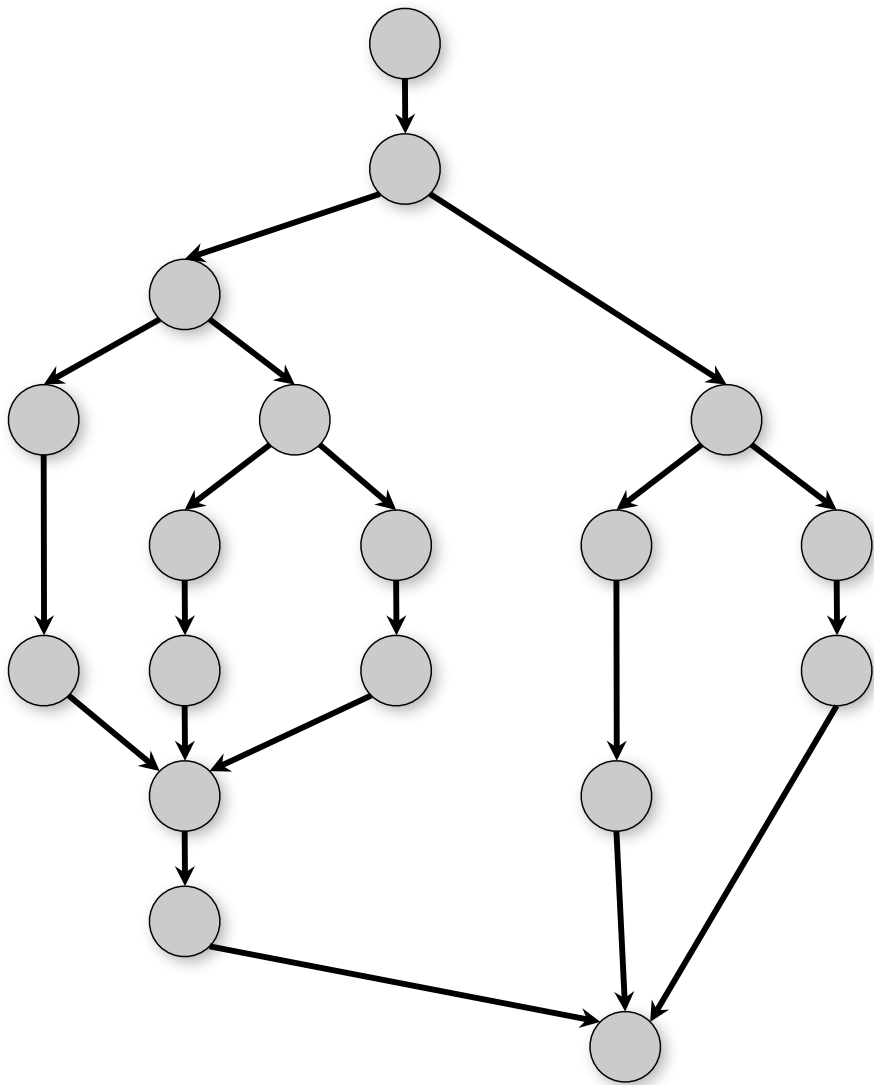
$T_1 = \text{work}$

$$T_\infty = \text{span}^*$$

*Also called *critical-path length*

Algorithmic Complexity Measures

T_P = execution time on P processors



$$T_1 = \text{work}$$

$$T_\infty = \text{span}$$

LOWER BOUNDS

- $T_P \geq T_1/P$
- $T_P \geq T_\infty$

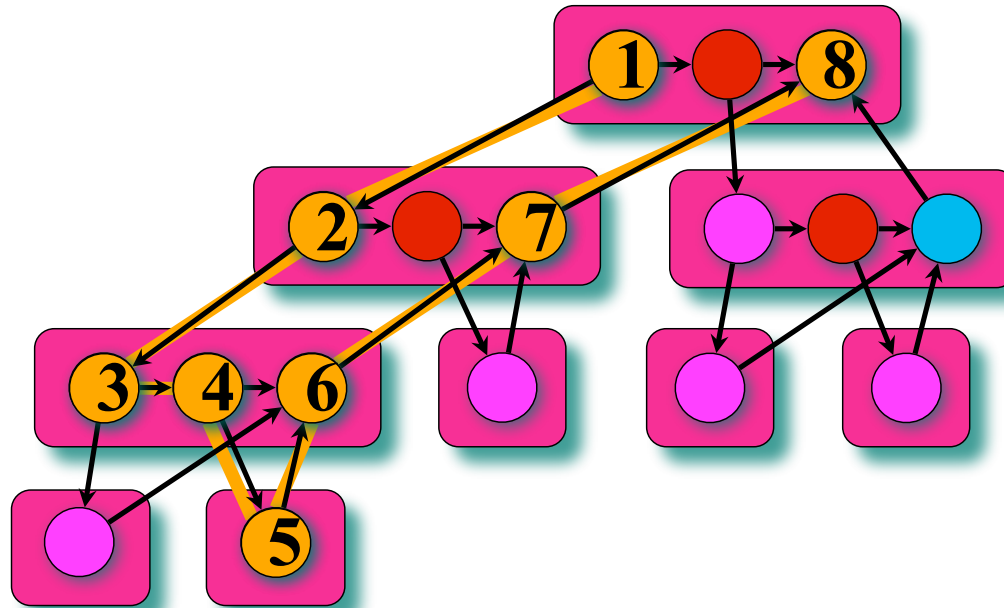
Speedup

Definition: T_1/T_P = *speedup* on P processors

If $T_1/T_P = \Theta(P)$, we have *linear speedup*;
= P , we have *perfect linear speedup*;
> P , we have *superlinear speedup*,

Superlinear speedup is not possible in this model because of the lower bound $T_P \geq T_1/P$, but it can occur in practice (e.g., due to cache effects)

Example: `fib(4)`

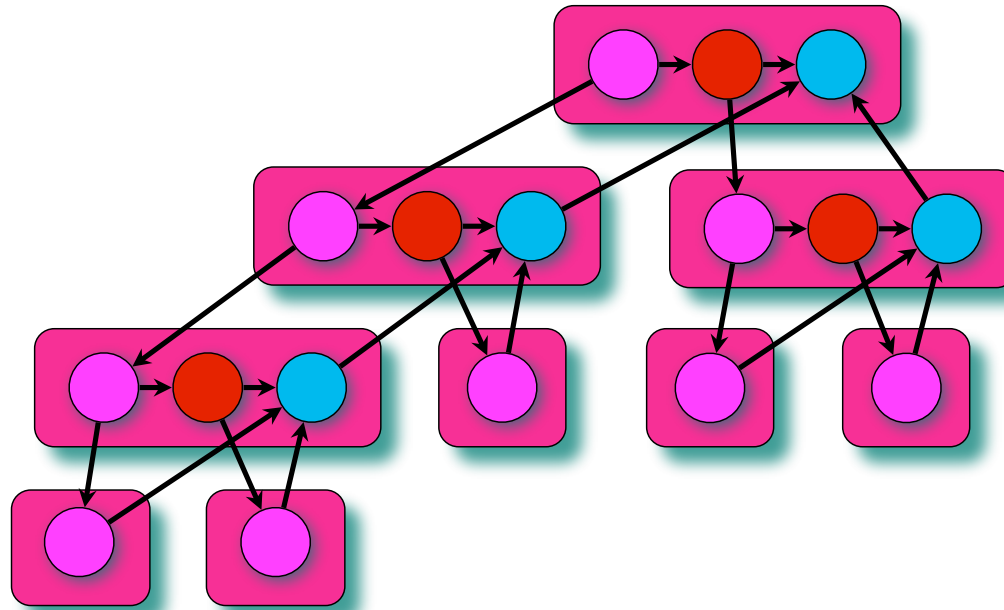


Assume for simplicity that each strand in `fib()` takes unit time to execute.

Work: $T_1 = 17$ (T_P refers to execution time on P processors)

Span: $T_\infty = 8$ (Span = “critical path length”)

Example: `fib(4)`



Assume for simplicity that each strand in `fib()` takes unit time to execute.

Work: $T_1 = 17$

Span: $T_\infty = 8$

Ideal Speedup: $T_1 / T_\infty = 2.125$

Using more than
2 processors
makes little sense

Task Scheduling

- Popular scheduling strategies
 - work-sharing**: task scheduled to run in parallel at every spawn
 - benefit: maximizes parallelism
 - drawback: cost of setting up new tasks is high → should be avoided
 - work-stealing**: processor looks for work when it becomes idle
 - lazy parallelism: put off work for parallel execution until necessary
 - benefits: executes with precisely as much parallelism as needed
 - minimizes the number of tasks that must be set up
 - runs with same efficiency as serial program on uniprocessor
- Cilk uses **work-stealing** rather than **work-sharing**

Cilk Execution using Work Stealing

- Cilk runtime maps logical tasks to compute cores
- Approach:
 - lazy task creation plus work-stealing scheduler
 - **cilk_spawn**: a potentially parallel task is available
 - an idle thread steals a task from a random working thread

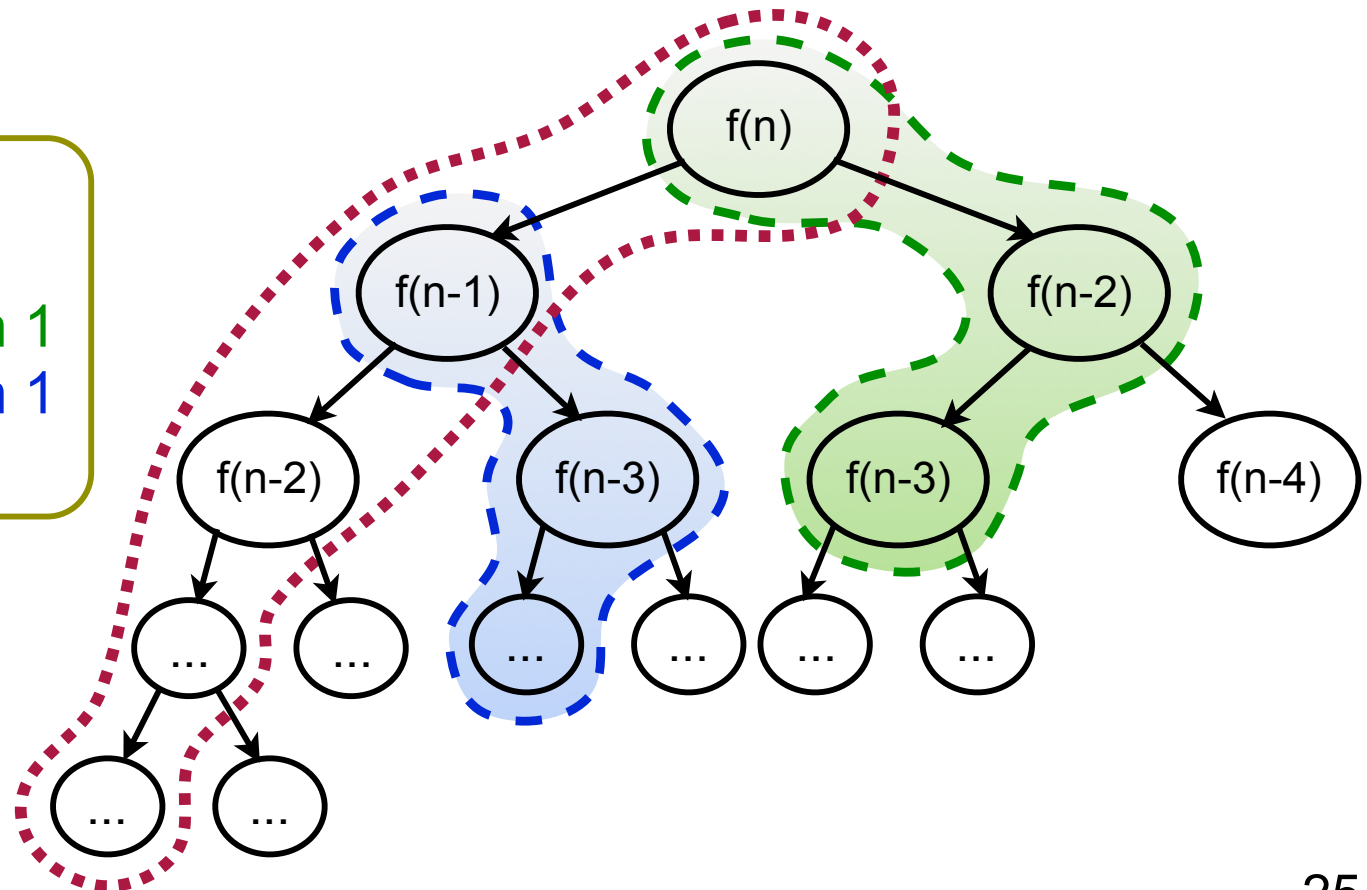
Possible Execution:

thread 1 begins

thread 2 steals from 1

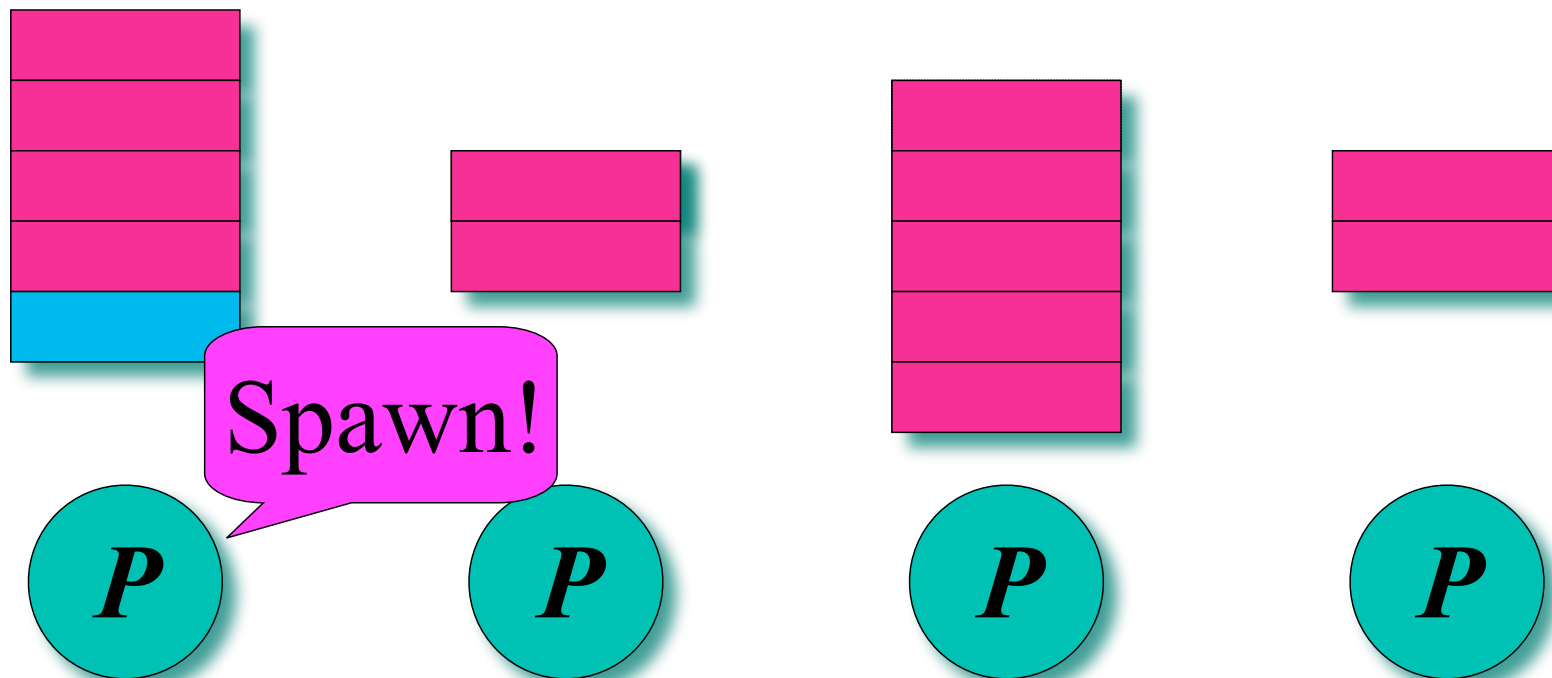
thread 3 steals from 1

etc...



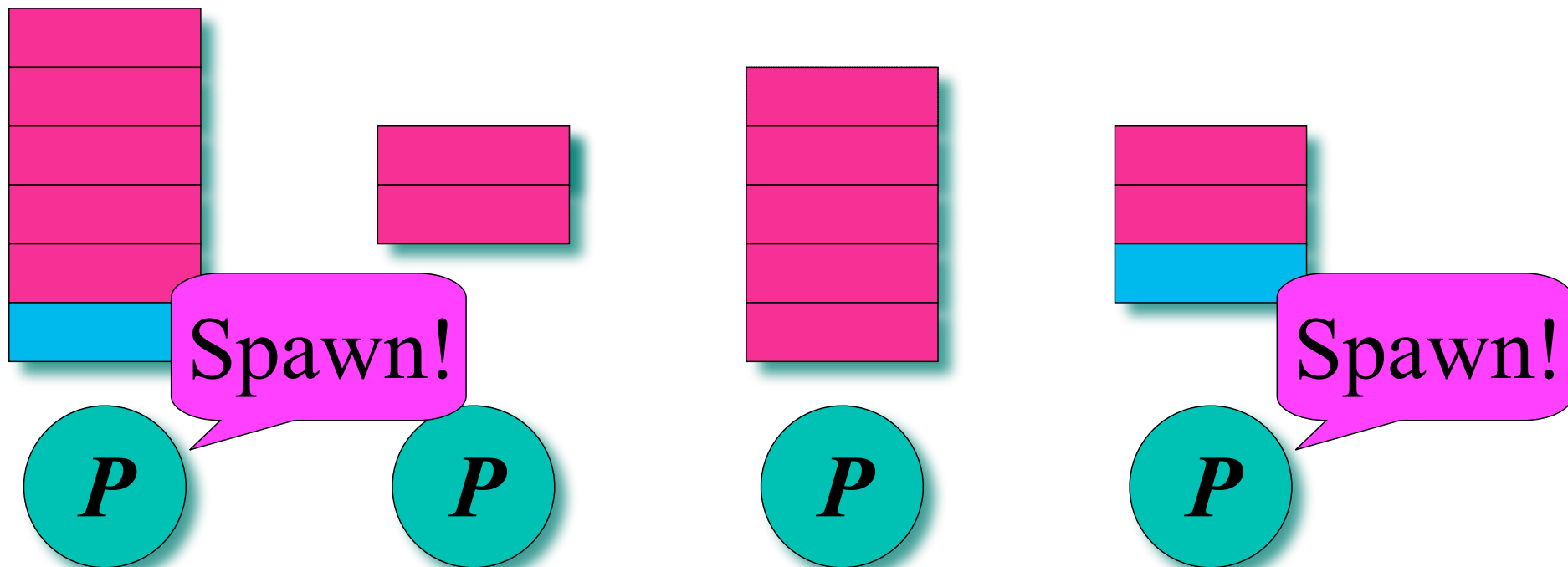
Cilk's Work-Stealing Scheduler

Each processor maintains a **work deque** of ready strands, and it manipulates the bottom of the deque like a stack.



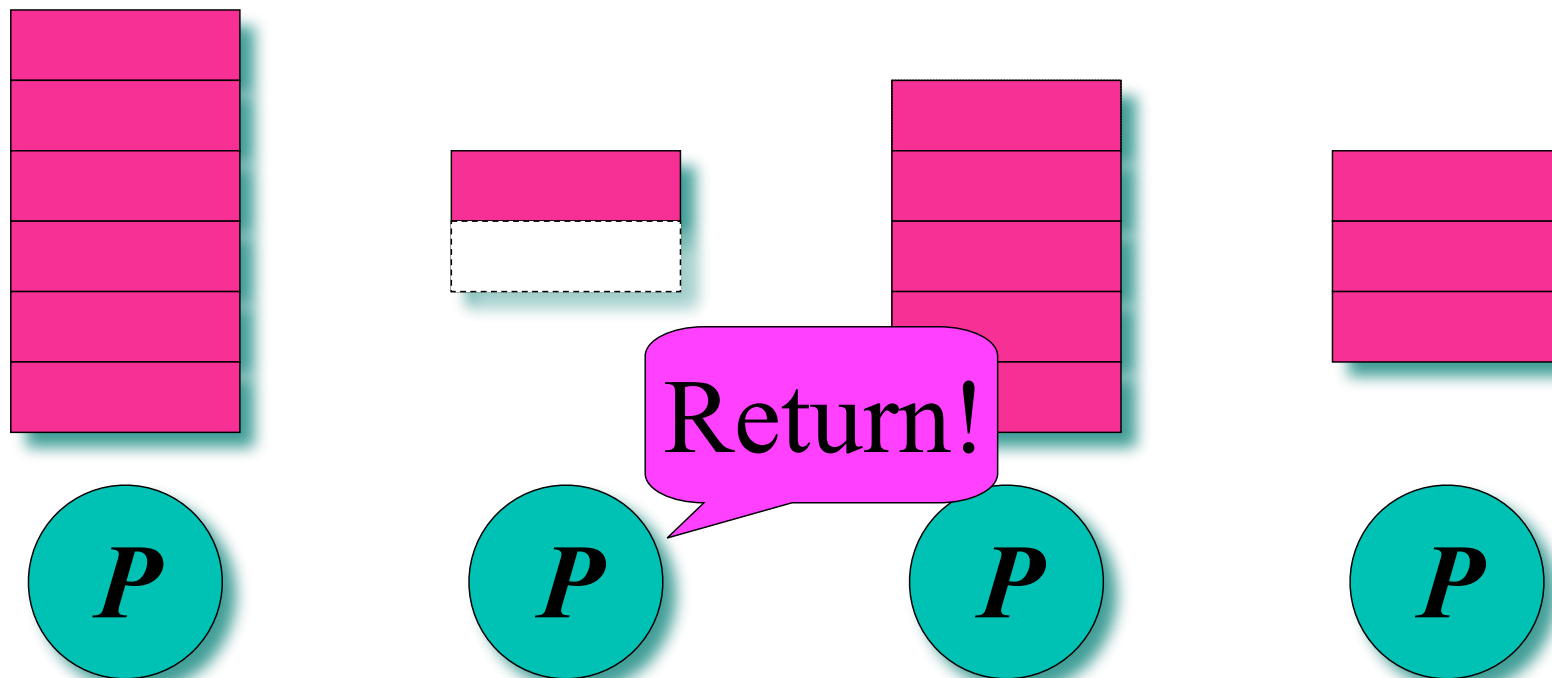
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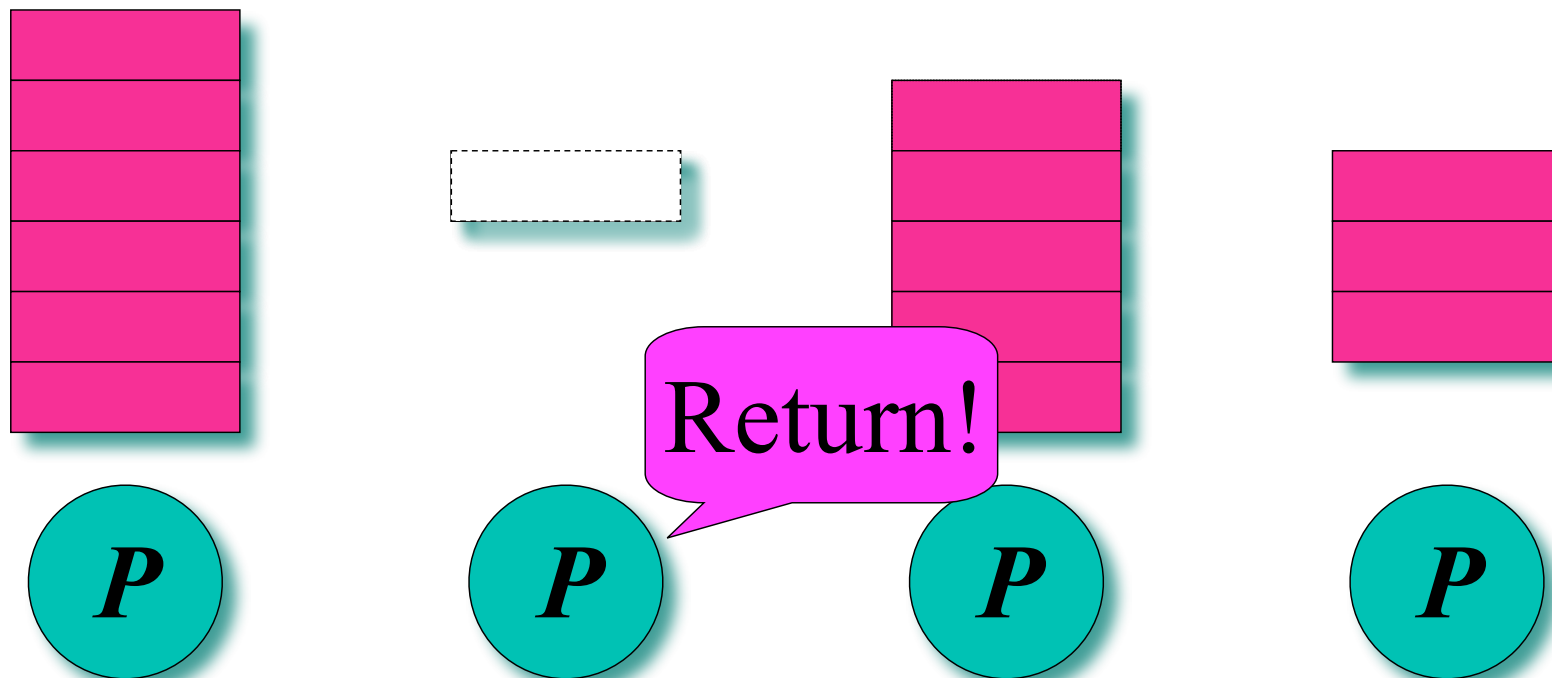
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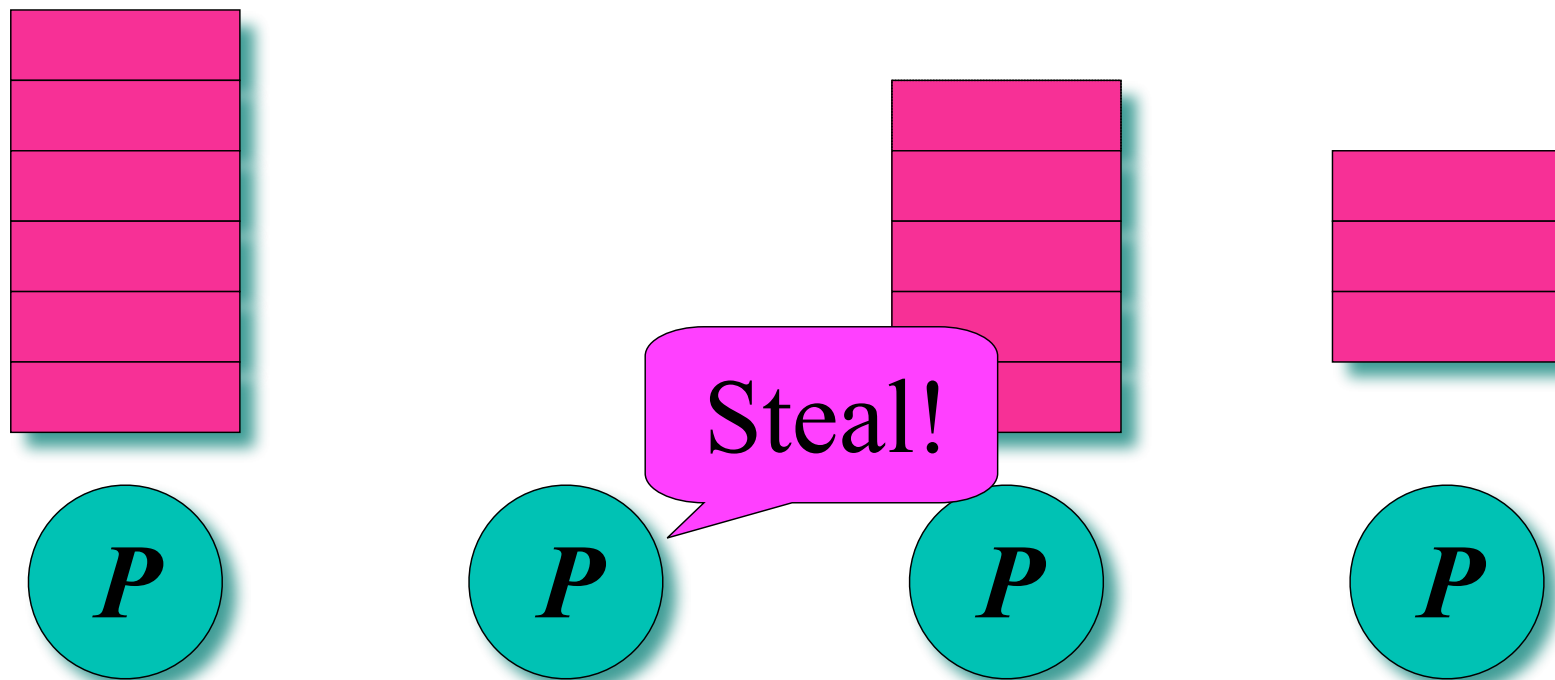
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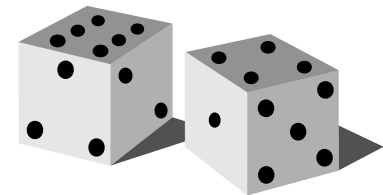


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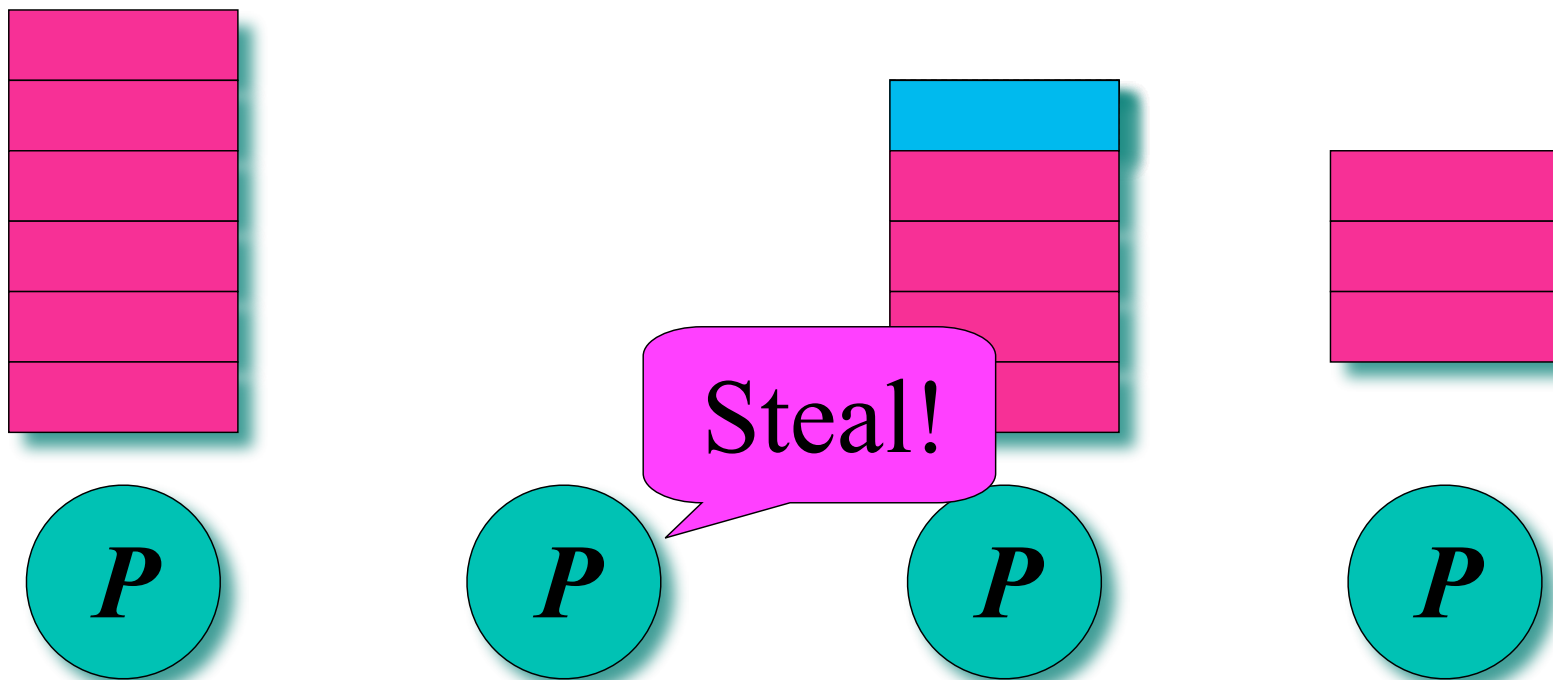


When a processor runs out of work, it **steals** a strand from the top of a **random** victim's deque

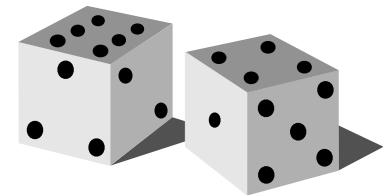


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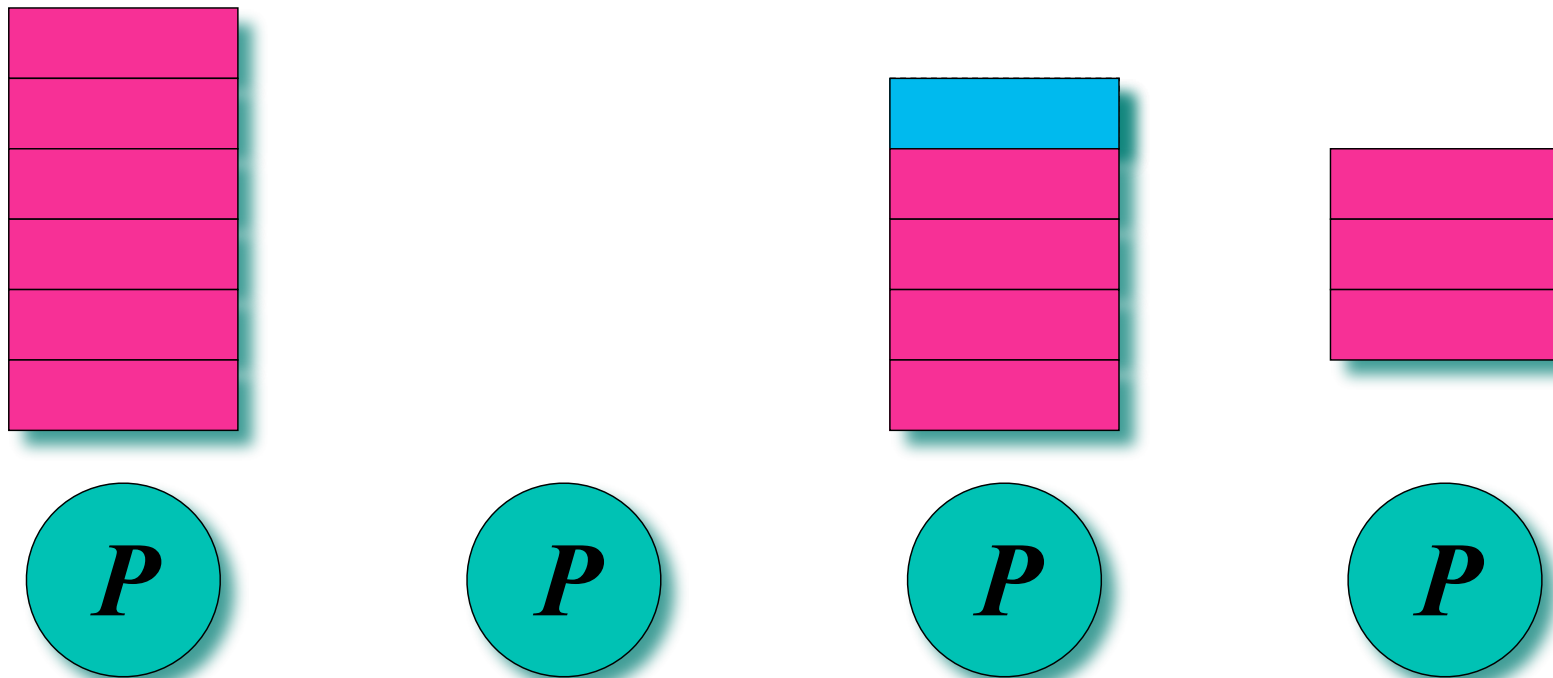


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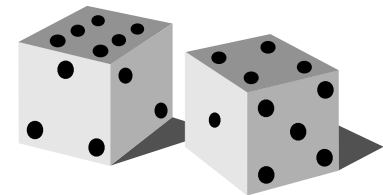


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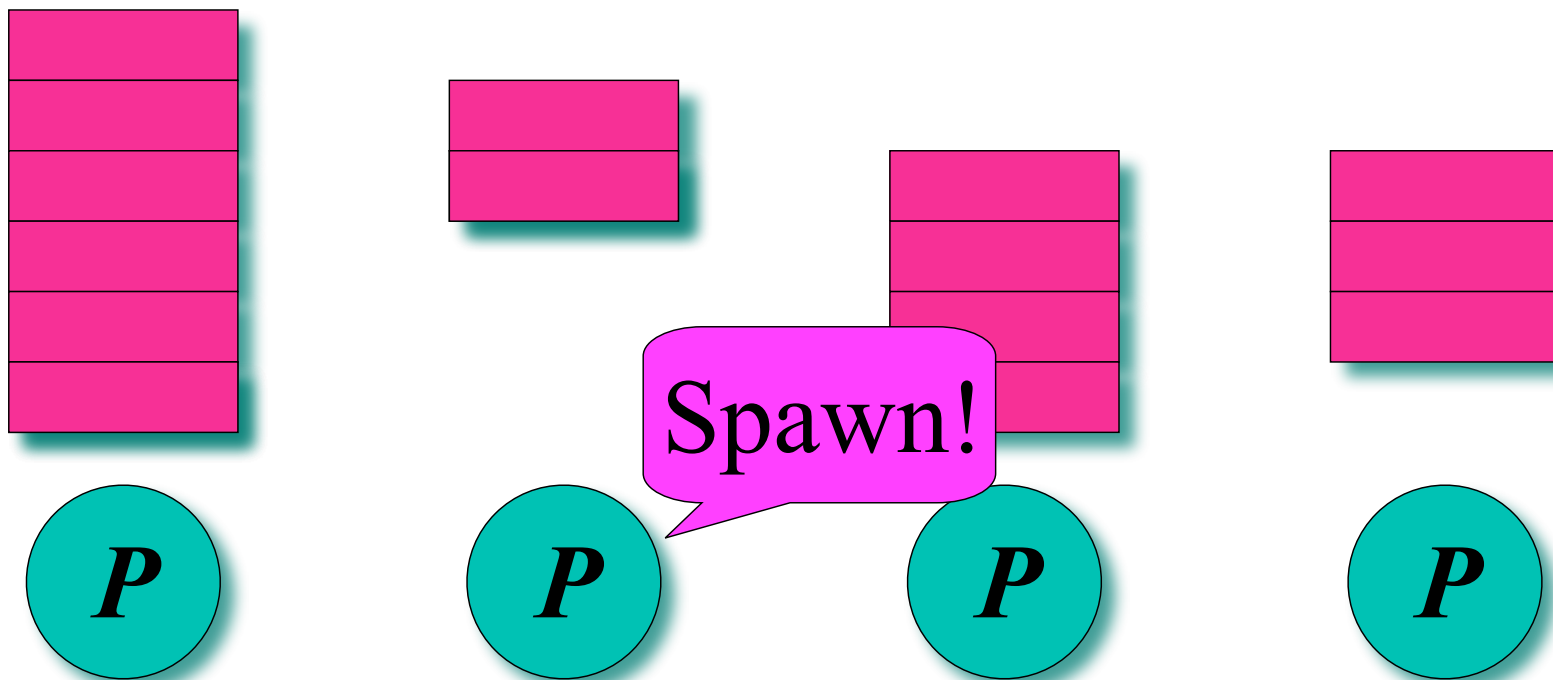


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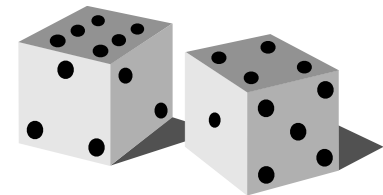


Cilk's Work-Stealing Scheduler

Each processor maintains a **work deque** of ready strands, and it manipulates the bottom of the deque like a stack.



When a processor runs out of work, it **steals** a strand from the top of a **random** victim's deque



Performance of Work-Stealing

Theorem: Cilk's work-stealing scheduler achieves an expected running time of $T_P \leq T_1/P + O(T_\infty)$ on P processors

Greedy Scheduling Theorem

- Types of schedule steps
 - complete step
 - at least P operations ready to run
 - select any P and run them
 - incomplete step
 - strictly $< P$ operation ready to run
 - greedy scheduler runs them all

Theorem: On P processors, a greedy scheduler executes any computation G with work T_1 and critical path of length T_∞ in time $T_p \leq T_1/P + T_\infty$

Proof sketch

- only two types of scheduler steps: complete, incomplete
- cannot be more than T_1/P complete steps, else work $> T_1$
- every incomplete step reduces remaining critical path length by 1
 - no more than T_∞ incomplete steps

Parallel Slackness Revisited

critical path overhead = smallest constant c_∞ such that

$$T_p \leq \frac{T_1}{P} + c_\infty T_\infty$$

$$T_p \leq \left(\frac{T_1}{T_\infty P} + c_\infty \right) T_\infty = \left(\frac{\bar{P}}{P} + c_\infty \right) T_\infty$$

Let $\bar{P} = T_1/T_\infty =$
parallelism =
max speedup on
 ∞ processors

Parallel slackness assumption

$$\bar{P} / P \gg c_\infty \quad \text{thus} \quad \frac{T_1}{P} \gg c_\infty T_\infty$$

$$T_p \approx \frac{T_1}{P}$$

linear speedup

“critical path overhead has
little effect on performance
when sufficient parallel
slackness exists”

Work Overhead

$$c_1 = \frac{T_1}{T_s} \quad \text{work overhead}$$

$$T_p \leq c_1 \frac{T_s}{P} + c_\infty T_\infty$$

“Minimize work overhead (c_1)
at the expense of a larger
critical path overhead (c_∞),
because work overhead
has a more direct impact
on performance”

$$T_p \approx c_1 \frac{T_s}{P} \quad \text{assuming parallel slackness}$$

You can reduce C_1 by increasing
the granularity of parallel work

Parallelizing Vector Addition

C

```
void vadd (real *A, real *B, int n){  
    int i; for (i=0; i<n; i++) A[i]+=B[i];  
}
```

Divide and Conquer

- **An effective parallelization strategy**
 - creates a good mix of large and small sub-problems
- **Work-stealing scheduler can allocate chunks of work efficiently to the cores, as long as**
 - not only a few large chunks
 - if work is divided into just a few large chunks, there may not be enough parallelism to keep all the cores busy
 - not too many very small chunks
 - if the chunks are too small, then scheduling overhead may overwhelm the benefit of parallelism

Parallelizing Vector Addition

C

```
void vadd (real *A, real *B, int n) {  
    int i; for (i=0; i<n; i++) A[i]+=B[i];  
}
```

C

```
void vadd (real *A, real *B, int n) {  
    if (n<=BASE) {  
        int i; for (i=0; i<n; i++) A[i]+=B[i];  
    } else {  
        vadd (A, B, n/2);  
        vadd (A+n/2, B+n/2, n-n/2);  
    }  
}
```

Parallelization strategy:

1. Convert loops to recursion.

Parallelizing Vector Addition

C

```
void vadd (real *A, real *B, int n){
    int i; for (i=0; i<n; i++) A[i]+=B[i];
}
```

**Cilk
Plus**

```
void vadd (real *A, real *B, int n){
    if (n<=BASE) {
        int i; for (i=0; i<n; i++) A[i]+=B[i];
    } else {
        vadd cilk_spawn vadd (A, B, n/2);
        vadd (A+n/2, B+n/2, n-n/2);
    } cilk_sync;
}
```

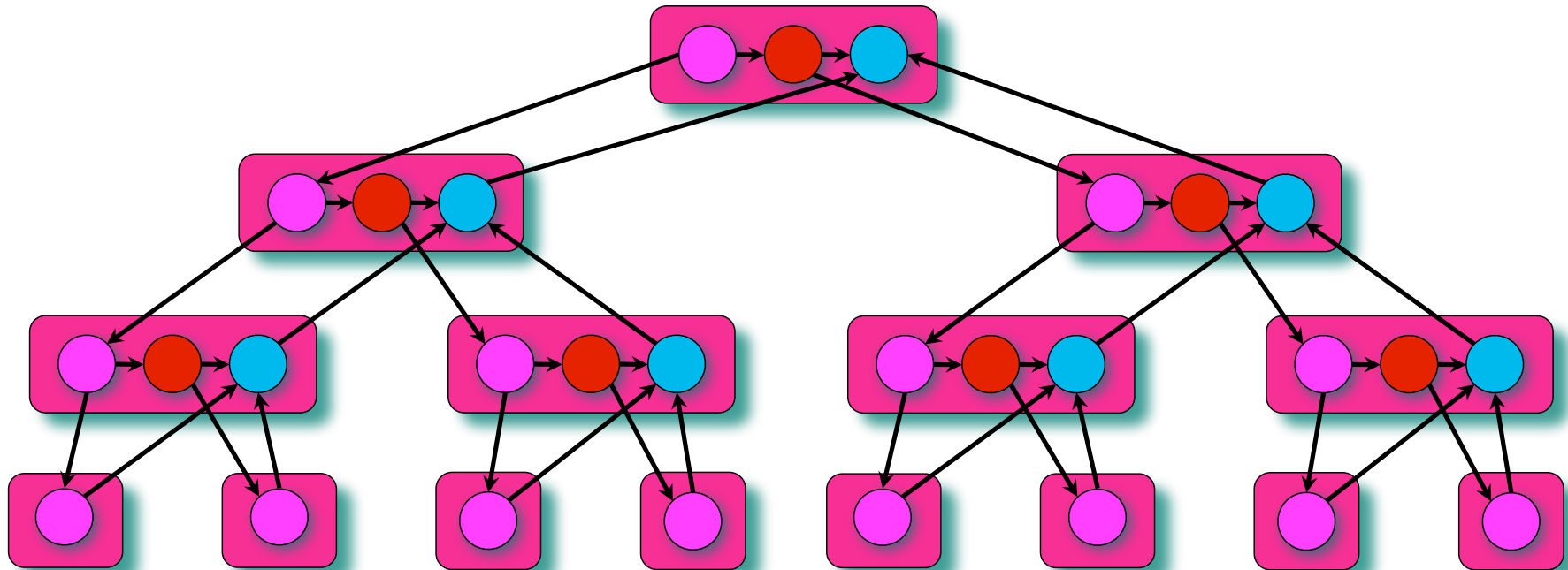
Parallelization strategy:

1. Convert loops to recursion.
2. Insert Cilk Plus keywords.

Side benefit:
D&C is generally
good for caches!

Vector Addition

```
void vadd (real *A, real *B, int n){  
    if (n<=BASE) {  
        int i; for (i=0; i<n; i++) A[i]+=B[i];  
    } else {  
        cilk_spawn vadd (A, B, n/2);  
        vadd (A+n/2, B+n/2, n-n/2);  
        cilk_sync;  
    }  
}
```



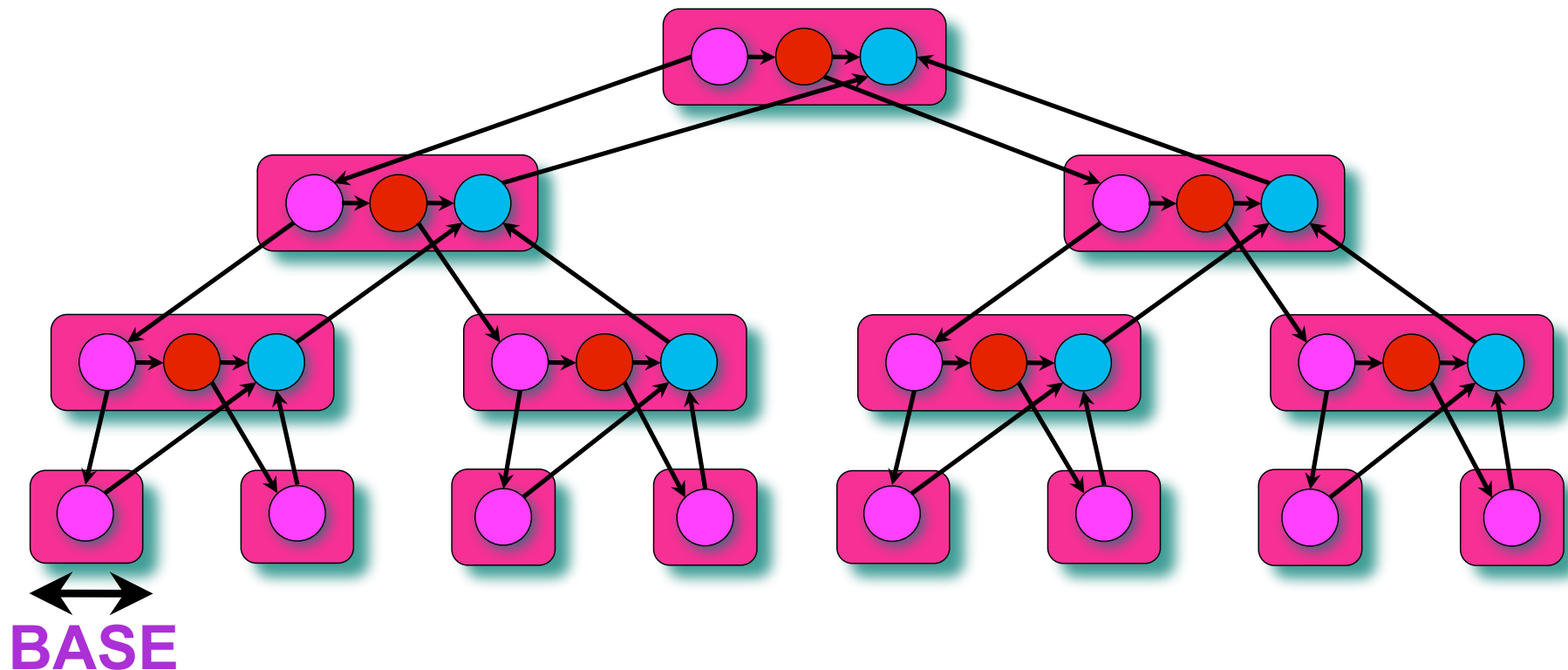
Vector Addition Analysis

To add two vectors of length n , where $\text{BASE} = \Theta(1)$:

Work: $T_1 = \Theta(n)$

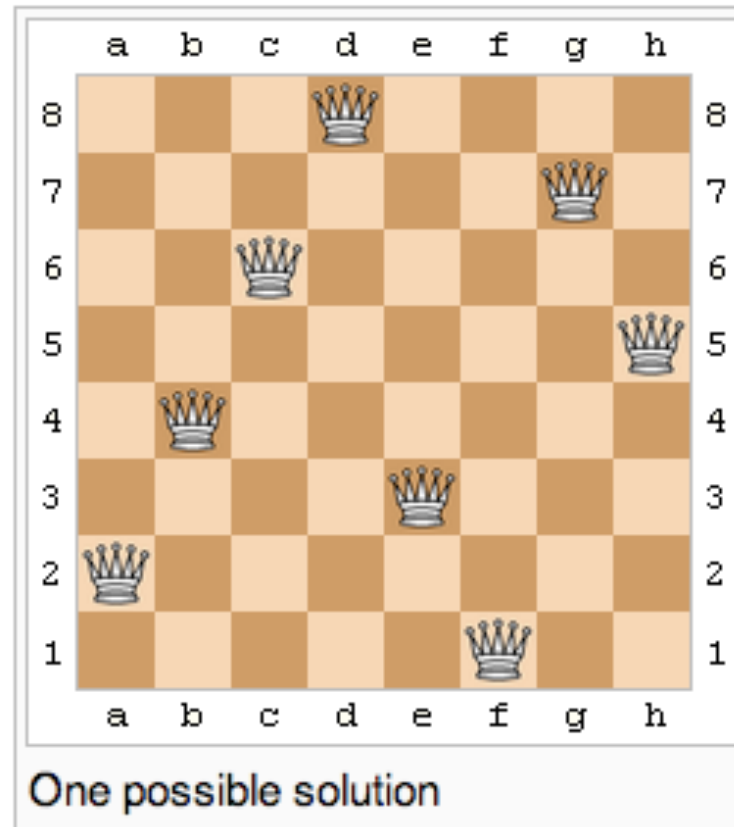
Span: $T_\infty = \Theta(\lg n)$

Parallelism: $T_1 / T_\infty = \Theta(n / \lg n)$



Example: N Queens

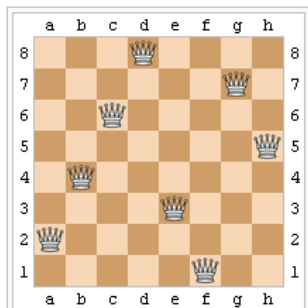
- **Problem**
 - place N queens on an $N \times N$ chess board
 - no 2 queens in same row, column, or diagonal
- **Example: a solution to 8 queens problem**



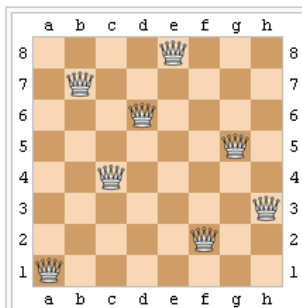
N Queens: Many Solutions Possible

Example: 8 queens

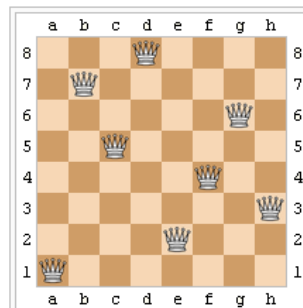
- 92 distinct solutions
- 12 unique solutions; others are rotations & reflections



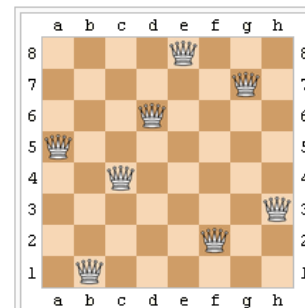
Unique solution 1



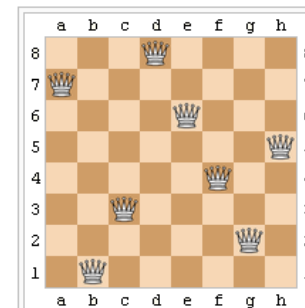
Unique solution 2



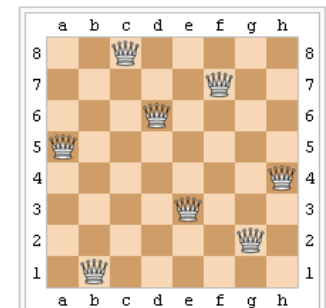
Unique solution 3



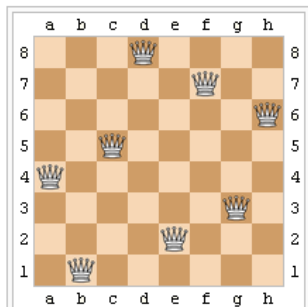
Unique solution 7



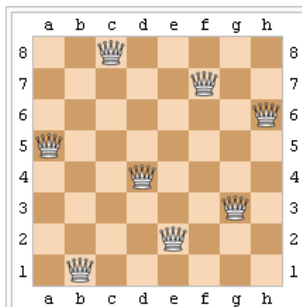
Unique solution 8



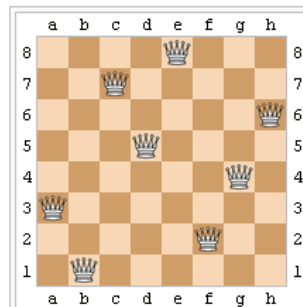
Unique solution 9



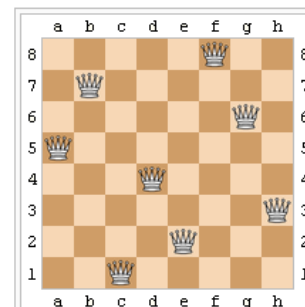
Unique solution 4



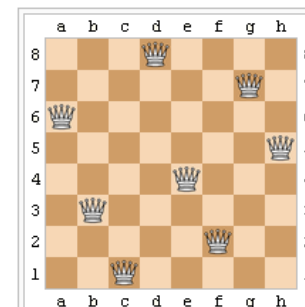
Unique solution 5



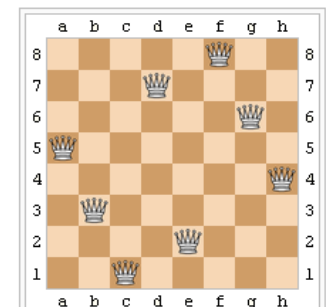
Unique solution 6



Unique solution 10



Unique solution 11



Unique solution 12

Image credit: http://en.wikipedia.org/wiki/Eight_queens_puzzle

N Queens Solution Sketch

Sequential Recursive Enumeration of All Solutions

```
int nqueens(n, j, placement) {  
    // precondition: placed j queens so far  
    if (j == n) { print placement; return; }  
    for (k = 0; k < n; k++)  
        if putting j+1 queen in kth position in row j+1 is legal  
            add queen j+1 to placement  
            nqueens(n, j+1, placement)  
            remove queen j+1 from placement  
}
```

- Where's the potential for parallelism?
- What issues must we consider?

Parallel N Queens Solution Sketch

```
void nqueens(n, j, placement) {  
    // precondition: placed j queens so far  
    if (j == n) { /* found a placement */ process placement; return; }  
    for (k = 1; k <= n; k++)  
        if putting j+1 queen in kth position in row j+1 is legal  
            copy placement into newplacement and add extra queen  
            cilk_spawn nqueens(n, j+1, newplacement)  
            cilk_sync  
            discard placement  
}
```

Issues regarding placements

- how can we report placements?
- what if a single placement suffices?
 - no need to compute all legal placements
 - so far, no way to terminate children exploring alternate placement

Approaches to Managing Placements

- **Choices for reporting multiple legal placements**
 - count them
 - print them on the fly
 - collect them on the fly; print them at the end
- **If only one placement desired, can skip remaining search**

References

- **“Introduction to Parallel Computing” by Ananth Grama, Anshul Gupta, George Karypis, and Vipin Kumar. Addison Wesley, 2003**
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- **Charles Leiserson, Bradley Kuzmaul, Michael Bender, and Hua-wen Jing. MIT 6.895 lecture notes - Theory of Parallel Systems. <http://bit.ly/mit-6895-fall03>**
- **Intel Cilk++ Programmer’s Guide. Document # 322581-001US.**