

STA 4103/5107: Homework Assignment #3

(Wednesday, January 25)

Due: Wednesday, February 1

1. Write a matlab program implementing the Metropolis-Hastings algorithm to sample a random variable X with the density

$$f(x) = \frac{x^2 |\sin(\pi x)| e^{-x^3}}{\int_0^\infty x^2 |\sin(\pi x)| e^{-x^3} dx}, \quad x > 0.$$

You have to decide what q (proposal density) you want to use. Choose positive numbers to start the Markov chain.

- (a) Plot the density function $f(x)$.
- (b) Histogram the values attained by Markov chain and compare it to the plot of $f(x)$.
- (c) Estimate the value of $E[X]$ and $var(X)$ using values of the Markov chain.

2. Consider the problem of sampling from a posterior density when it is easy to sample from the prior density. For two continuous random variables X and Y , the Bayes' rule can be stated as:

$$f_{X|Y}(x|y) = \frac{f_{Y|X}(y|x)f_X(x)}{f_Y(y)}.$$

$f_{X|Y}(x|y)$ is the posterior density, $f_X(x)$ is the prior density, $f_{Y|X}(y|x)$ is called the likelihood function (as a function of x), and $f_Y(y)$ is a normalizer. Let X be a normal random variable with mean 5 and variance 4, i.e. $X \sim N(5, 4)$ and the conditional density of Y given $X = x$ is given by:

$$f_{Y|X}(y|x) = \frac{1}{Z} e^{-|x-y|^{0.5}}.$$

Choose the prior $f_X(x)$ as the proposal density and the posterior $f_{X|Y}(x|y)$ as the target density.

- (a) Write a matlab program implementing Metropolis-Hastings algorithm to sample from the posterior for $y = 6$. Show evolution of this chain for several starting points.
- (b) Use the Markov chain to estimate the posterior mean and variance.