STA 4103/5107: Homework Assignment #9

(Wednesday, March 28)
Due: Wednesday, April 4

1. Consider the function:

$$h(x, y) = \sin(2x + y) + 2\cos(x - 2y) + 0.1(x + y).$$

(a) Plot the function and the contour over the region $x \in [-3, 3]$, $y \in [-3, 3]$.

(b) **Find the local minima using Gradient Method**: let the step size $\alpha = 0.1$. Try several different initial points and show the minimum found at each run. Plot the search paths over the contour plot. Also try step size $\alpha = 0.01, 1$.

(c) **Finding the local maxima using Gradient Method:** let the step size $\alpha = 0.1$. Try several different initial points and show the maximum found at each run. Plot the search paths over the contour plot. Also try step size $\alpha = 0.01, 1$.

2. Consider the function:

$$h(x, y) = (x\sin(20y) + y\sin(20x))^2 \cosh(\sin(10x)x) + (x\cos(10y) - y\sin(10x))^2 \cosh(\cos(20y)y).$$

(a) Plot the function and the contour over the region $x \in [-1, 1]$, $y \in [-1, 1]$.

(b) **Optimization Using M-H Simulated Annealing:**
Write a program to solve for the maximizer of this function within the domain $[-1, 1] \times [-1, 1]$ using Metropolis-Hastings based simulated annealing algorithm.
The procedure is similar to the above problem except the candidate is generated from a general proposal density. Choose a Gaussian density, centered around the current point, as the proposal density. That is, set

$$q(x_2, y_2 \mid x_1, y_1) \propto \exp(-\tfrac{1}{2} \| (x_1, y_1) - (x_2, y_2) \|^2 / \sigma^2).$$

Use $T_{t+1} = \alpha T_t$ for some $\alpha = 0.99$ and $T_1 = 10$ for reducing the temperature. Plot the evolution of this process for several starting conditions and several values of σ.