STA 4103/5107: Homework Assignment #7

(Wednesday, March 14)
Due: Wednesday, March 21

1. Write a matlab program to simulate a random walk, for a given value of T and $s = \alpha\sqrt{T}$. Plot the sample paths of the X_t versus the real time t, for $\alpha = 1.0$ and T = 1, 0.1, 0.01, and 0.001. In each case, choose the total number of steps n to be $10/T$, so that you go up to time $t = 10$ irrespective of T. Use the "stairs" command in matlab to plot the sample paths.

2. Write a matlab program to simulate a homogeneous Poisson process over the interval [0, 100]. Generate 50 sample paths for intensity $\lambda = 0.1$ and display them on the same plot. Count the number of events occurring in the interval [10, 60]. Plot a histogram of 50 realizations of this random number. Does this sample follow a Poisson distribution (hint: use a Kolmogorov-Smirnov test)?

3. Write a matlab program to simulate a inhomogeneous Poisson process over the interval [0, 10] where the rate function

$$\lambda(t) = 2 + \sin(t) + \sin(2t)/2.$$

Plot the rate function versus time t. On the same figure, generate 30 sample paths for this process and display them on the same plot (use **subplot(4,1,1)** for the rate function, and **subplot(4,1,2:4)** for the sample paths).
(hint: a Newton-Raphson procedure can be used to estimate the values of F^{-1}).

4. Prove that the following simulation generates a homogeneous Poisson process with rate λ on [0, T]:

Step 1: Sample k from Poisson distribution with mean λT.
Step 2: Sample $s_1, \ldots, s_k$ i.i.d. from uniform [0, T].

That is, demonstrate that for any time interval [t, $t+\Delta t$] in [0, T],

$$P\{k \text{ events in } [t, t+\Delta t]\} = \frac{\exp(-\lambda\Delta t)(\lambda\Delta t)^k}{k!}$$