

STA 4103/5107: Homework Assignment #6

(Thursday, February 16)
Due: Thursday, February 23

1. Write a matlab program to simulate two processes, $\{x_t\}$ and $\{y_t\}$ defined as follows:

$$\begin{aligned}x_t &= Ax_{t-1} + w_t \\ y_t &= Hx_t + q_t\end{aligned}$$

where

$$A = \begin{bmatrix} 0.9 & 0.1 & -0.2 \\ -0.3 & 0.8 & 0.1 \\ 0.2 & 0.1 & 0.9 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0.5 & 0.2 \\ 0.5 & 1 & 0.1 \end{bmatrix}.$$

Also, w_t and q_t are vectors of Gaussian random variables with mean zeros and variances $W = I_3$ and $Q = 0.5I_2$, respectively.

- Use $x_1 = \text{randn}(3, 1)$ as the initial condition. Show the plots of $\{x_t\}$ and $\{y_t\}$ for $t = 1, \dots, 500$.
- Assuming A, H, W, Q are unknown, identify them using the simulated $\{x_t\}$ and $\{y_t\}$ in (a). Denote the estimates as $\hat{A}, \hat{H}, \hat{W}, \hat{Q}$.
- Simulate another set of $\{\tilde{x}_t\}$ and $\{\tilde{y}_t\}$ using the true parameters A, H, W, Q . Then use the Kalman filter algorithm to estimate $\{\tilde{x}_t\}$ using $\{\tilde{y}_t\}$ and $\hat{A}, \hat{H}, \hat{W}, \hat{Q}$. Plot the true and estimated states in the same plot for each component. Do you have similar estimation accuracy in all components? If not, why?
- Compute the R^2 -Error of each component in the filtering estimation (i.e. measurement update). Compare the results with the prediction estimation (i.e. time update).

2. Repeat Problem 1 except that $Q = 50I_2$. In part (d), do you observe significant difference between filtering estimation and prediction estimation? If not, what is the reason for that?

3. Repeat part (a) in Problem 1 except that

$$A = \begin{bmatrix} 0.9 & -0.2 & -0.1 \\ -0.2 & 0.9 & 0.3 \\ 0.0 & 0.3 & 0.6 \end{bmatrix}$$

Do you see any problem in the simulated processes? What is the reason for the problem?