Monte Carlo MAP 5615 HW1

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a) The LCG can only produce at most m distinct values. If (a, m) = 1, the maximum number of distinct values should be m.

b) If (a, m) = d, $\left(\frac{a}{d}, \frac{m}{d}\right) = 1$. Thus the maximum number of distinct values is $\frac{m}{d}$.

To make the period of LCG as large as possible, it is better to choose (a, m) = 1

3

$x_{n+1} = ax_n + c \ (mod\ m)$

$\Rightarrow x_{n+2} = a(ax_n + c) + c \ (mod\ m) = a^2 x_n + (a+1)c \ (mod\ m)$

$\Rightarrow x_{n+3} = a^3 x_n + (a^2 + a + 1)c \ (mod\ m)$

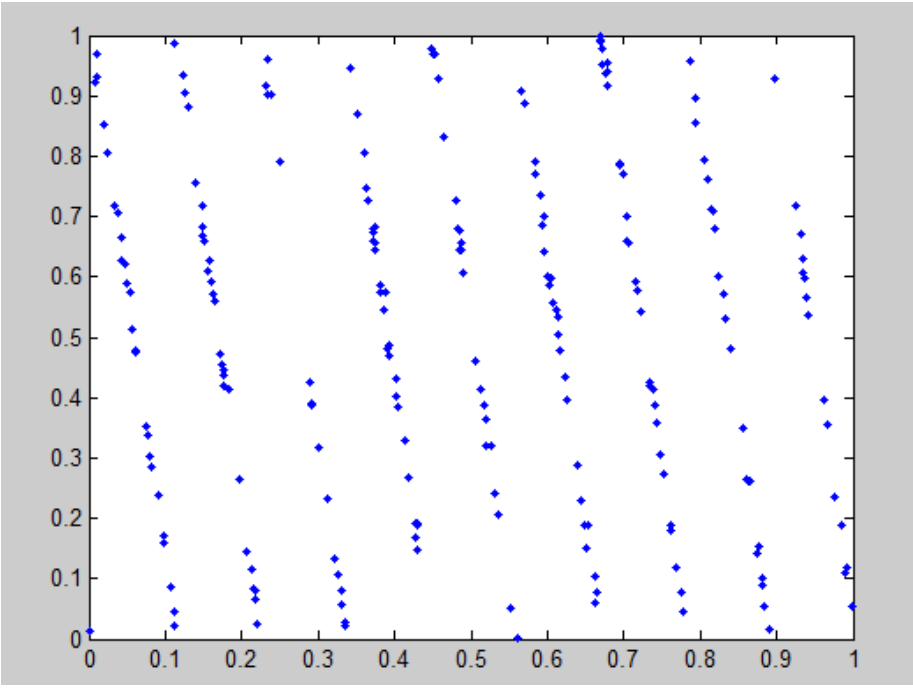
$\Rightarrow \cdots$

$\Rightarrow x_{n+k} = a^k x_n + (a^k + \cdots a + 1)c \ (mod\ m)$

$\Rightarrow x_{n+k} \equiv a^k x_n + \frac{a^k - 1}{a - 1} c \ (mod\ m); (a \geq 2, k \geq 0)$

6

a) $x_{1000} = 649091873$

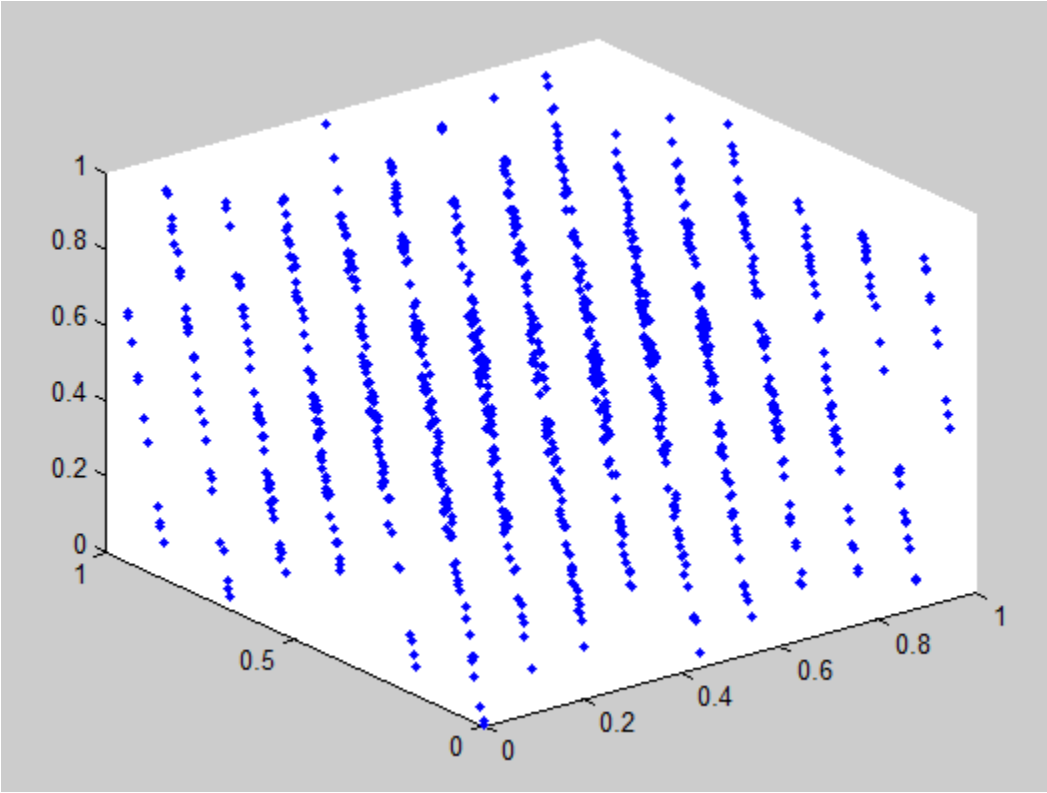


We already know $u_{i+2} - 6u_{i+1} + 9u_i = k$.

To plot u_{i+2} *versus* u_i, we have $u_{i+2} + 9u_i = k$, where $0 < u_i < 1$.

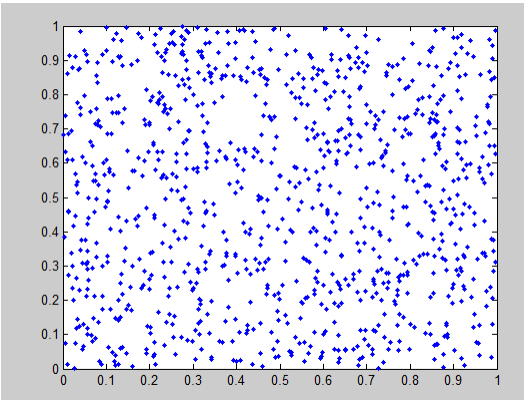
Then 0<k<10, and thus the pairs must lie on no more than 9 lines on R^2.

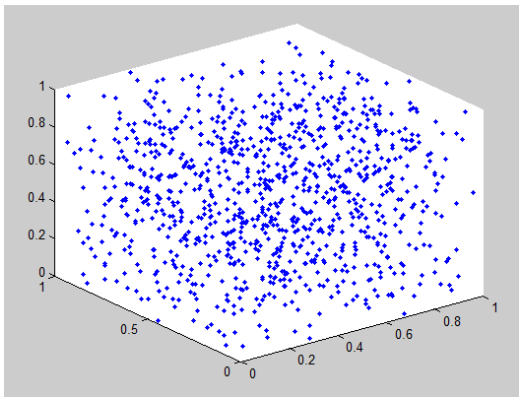
b)



From the graph we can see there are 15 planes.

7





In the graph of pairs and triples there is no regular order. Mersenne twister provides for generation of high quality pseudorandom numbers.

Appendix

Code for problem 6

function I=RANDU(x0,N)

```
x=[];
x(1)=x0;
for i = 2:N;
       x(i)= mod(65539*x(i-1), 2^31);
end
I=x;
u=x/2^31;
%for i = 2:20003;
     % if u(i+1)<=0.51 && u(i+1)>=0.5;
              %plot(u(i), u(i+2), '.');
              %hold on
       %end
%end

u1=u(1:1000);
u2=u(2:1001);
u3=u(3:1002);

plot3(u1,u2,u3, '.');

end
```