MAP 6437-4 - Monte Carlo Methods in Financial Mathematics - Spring 2010
Homework 3: Generation of Random Variables
Due Date: Feb 22, 2010

1. Write a computer program that implements the Inverse Transformation Method for a discrete density. The program should take the density function $\{p_1, p_2, ..., p_n\}$ as input, and output a random integer from $\{1, 2, ..., n\}$ with this density. To test your program apply it to the density $\{0.2, 0.2, 0.4, 0.1, 0.1\}$. Generate 1000 random integers in $\{1, 2, 3, 4, 5\}$ from this density, and plot a histogram for the data you obtain. Visually inspect whether the histogram is a good approximation of the theoretical density.

2. Let T be the number of arithmetical operations $(\pm, \times, \div)$ and comparisons (checking if a number is less than another) that need to be computed to generate a value from $X \sim Poisson(\lambda)$ using Algorithm (Poisson). Find $E[T]$ and $Var(T)$. (Ignore any operations in initialization and assignments $a := b$.)

3. Write a computer program that implements the Box-Muller method. Your program will output independent standard normals X_1, X_2 when you input independent uniform variables U_1, U_2. Then generate 1000 pairs (X_{2i-1}, X_{2i}) $i = 1, ..., 1000$, and plot these pairs in $\mathbf{R}^2$, using the following two generators to compute the corresponding pairs (U_{2i-1}, U_{2i}) :

   (a) Mersenne twister,

   (b) a clearly poor LCG: $x_{n+1} \equiv 1229x_n + 1 (\mathrm{mod}\, 2048)$

   What conclusions do you make based on these graphs?

4. Write a computer program that implements the Beasley-Springer-Moro algorithm (see pg 67-68 of Glasserman's book or Blackboard). This algorithm gives an approximation to the inverse normal cdf. Then generate 2000 standard normals, $\mathbb{N}(0, 1)$, using this method. Test the normality of this data, and the Box-Muller data you generated in Problem 3 (a), using the Anderson-Darling test. (You will need a table for the percentiles for the Anderson-Darling statistic. See the paper "EDF Statistics for goodness of Fit and Some Comparisons" on Blackboard) Which data set is closer to the normal distribution?

5. Give a method for generating the Weibull distribution function

$$F(x) = 1 - e^{-\alpha x^\beta}, 0 < x < \infty$$

6. Give a method for generating a random variable with density function

$$f(x) = \begin{cases} e^{2x}, -\infty < x < 0 \\ e^{-2x}, 0 < x < \infty \end{cases}$$

7. Verify the equation

$$W(3T/4) = \frac{1}{2}\left(W(T/2) + W(T)\right) + \frac{\sqrt{2T}}{4} Z_4$$

using Lemma 5 in your lecture notes.