Name: Jian Wang Date: 2010-01-20

MAP 6437-4 - Monte Carlo Methods in Financial Mathematics - Spring 2010

Homework 1: Pseudorandom Generators

Due Date: Jan 20, 2010

1. Given an LCG with parameters a; c; m; prove that

Which shows that the (n+k) th term can be computed directly from the nth term.

Answer:

From the common formula of LCG, we can got that

Which is equal to

(1)

Besides, we can also know that

Substitute in (1), we can obtain

Similar, we can get

……..

Thus we can obtain

Then we can know that

The (n+k)th term can be computed directly by the nth term. The Proof is completed.

2. (a) If U and V are independently distributed random variables from the uniform

distribution U(0; 1) show that U + V (mod 1) is also U(0; 1):

Answer: The U and V are independently distributed random variables from the uniformly distribution U(0,1), Which shows that

Then

Since the U and V is uniformly distributed, then . Besides, the U+V is uniformly distributed in (0,2). And when U+V is belongs to (1,2), then U+V mod 1 equals to U+V-1 which is uniformly distributed in (0,1).

Hence we can obtain,

P(u+v <x) =

P(u+v<x) =

Hence

Hence the U+V(mod 1) is uniformly distributed.

(b) A random number generator is designed by

+

Where , what is the period of the generator {Rn}

Hence, we can get the Rn which equals to

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| R0 | R1 | R2 | R3 | R4 | R5 | R6 |
| 0.1428571 | 0.8035714 | 0.0357143 | 0.9821429 | 0.0714286 | 0.5892857 | 0.3928571 |
| R7 | R8 | R9 | R10 | R11 | R12 | R13 |
| 0.0535714 | 0.2857143 | 0.2321429 | 0.3214286 | 0.8392857 | 0.6428571 | 0.3035714 |
| R14 | R15 | R16 | R17 | R18 | R19 | R20 |
| 0.5357143 | 0.4821429 | 0.5714286 | 0.0892857 | 0.8928571 | 0.5535714 | 0.7857143 |
| R21 | R22 | R23 | R24 |  |  |  |
| 0.7321429 | 0.8214286 | 0.3392857 | 0.1428571 |  |  |  |

We can see, the period of Xn is 8 and the Period of Yn is 6, hence the period of Rn is

LCM(8,6) which is 24.

From the results I generated, we can see the R24=R0, which means that the period of Rn is 24.

3. Write a code that would implement RANDU. For debugging purposes, print x1000 when the seed is x0 = 1:

(a) Using RANDU generate u1,…, u20002, where un = xn/M: For all triplets in your

sequence, (ui; ui+1; ui+2);in which 0:5<=ui+1<= 0:51; plot ui versus ui+2: Comment

on the pattern of your scatterplot.

Answer:

The formula of Randu is xn = 65539xn-1 mod ,

From the progam, I obtain that the x1000 is 649091873. I generate the total 20002 un, and select the pair (ui, ui+1,ui+2) in which 0.5<=ui+1<=0.51. I listed them as follows:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| i | ui | ui+1 | ui+2 | i | ui | ui+1 | ui+2 |
| 1 | 0.393468 | 0.501968 | 0.470595 | 109 | 0.936732 | 0.504564 | 0.59679 |
| 2 | 0.129442 | 0.507559 | 0.880372 | 110 | 0.1485 | 0.509006 | 0.717538 |
| 3 | 0.464327 | 0.501714 | 0.831344 | 111 | 0.703131 | 0.504687 | 0.699944 |
| 4 | 0.330422 | 0.505212 | 0.057474 | 112 | 0.061986 | 0.505813 | 0.477003 |
| 5 | 0.160996 | 0.5067 | 0.591236 | 113 | 0.694312 | 0.505818 | 0.786099 |
| 6 | 0.402959 | 0.509688 | 0.431499 | 114 | 0.110888 | 0.503349 | 0.022103 |
| 7 | 0.714498 | 0.503938 | 0.593143 | 115 | 0.652154 | 0.509471 | 0.187439 |
| 8 | 0.123949 | 0.508414 | 0.93494 | 116 | 0.815187 | 0.50809 | 0.711862 |
| 9 | 0.486542 | 0.505861 | 0.656287 | 117 | 0.428119 | 0.507594 | 0.19249 |
| 10 | 0.485078 | 0.501955 | 0.646033 | 118 | 0.077534 | 0.505826 | 0.33715 |
| 11 | 0.513061 | 0.505278 | 0.414119 | 119 | 0.997994 | 0.505791 | 0.052805 |
| 12 | 0.662911 | 0.504205 | 0.059031 | 120 | 0.677543 | 0.502448 | 0.916797 |
| 13 | 0.614955 | 0.506271 | 0.503036 | 121 | 0.594234 | 0.505694 | 0.686058 |
| 14 | 0.506271 | 0.503036 | 0.461775 | 122 | 0.776858 | 0.506142 | 0.045129 |
| 15 | 0.20674 | 0.500864 | 0.14453 | 123 | 0.854873 | 0.507337 | 0.350166 |
| 16 | 0.196013 | 0.504818 | 0.26479 | 124 | 0.584499 | 0.508656 | 0.79144 |
| 17 | 0.373953 | 0.508181 | 0.683509 | 125 | 0.176605 | 0.504271 | 0.436184 |
| 18 | 0.640176 | 0.508267 | 0.288014 | 126 | 0.569974 | 0.50292 | 0.887758 |
| 19 | 0.566434 | 0.500906 | 0.907535 | 127 | 0.931255 | 0.508883 | 0.672004 |
| 20 | 0.649331 | 0.505458 | 0.188767 | 128 | 0.292414 | 0.503309 | 0.388132 |
| 21 | 0.661431 | 0.50953 | 0.104305 | 129 | 0.428806 | 0.508128 | 0.189514 |
| 22 | 0.217756 | 0.504189 | 0.065333 | 130 | 0.13997 | 0.502766 | 0.756863 |
| 23 | 0.675499 | 0.502906 | 0.937947 | 131 | 0.884443 | 0.502297 | 0.053797 |
| 24 | 0.157349 | 0.507051 | 0.626164 | 132 | 0.311242 | 0.50584 | 0.233858 |
| 25 | 0.10686 | 0.508172 | 0.087291 | 133 | 0.291636 | 0.50244 | 0.389918 |
| 26 | 0.482926 | 0.504473 | 0.680504 | 134 | 0.33625 | 0.508842 | 0.026802 |
| 27 | 0.372122 | 0.50458 | 0.678383 | 135 | 0.704718 | 0.500231 | 0.658928 |
| 28 | 0.616251 | 0.503814 | 0.476622 | 136 | 0.079563 | 0.502942 | 0.30158 |
| 29 | 0.989159 | 0.501993 | 0.109525 | 137 | 0.934261 | 0.50258 | 0.607137 |
| 30 | 0.6726 | 0.505439 | 0.979239 | 138 | 0.45711 | 0.506857 | 0.927158 |
| 31 | 0.401921 | 0.503462 | 0.403483 | 139 | 0.325081 | 0.505548 | 0.107556 |
| 32 | 0.125704 | 0.506048 | 0.904955 | 140 | 0.590084 | 0.507648 | 0.735132 |
| 33 | 0.148652 | 0.500918 | 0.667641 | 141 | 0.882215 | 0.506601 | 0.099666 |
| 34 | 0.382101 | 0.502107 | 0.573734 | 142 | 0.761585 | 0.505793 | 0.180496 |
| 35 | 0.427295 | 0.502436 | 0.16896 | 143 | 0.430042 | 0.502863 | 0.146804 |
| 36 | 0.552946 | 0.504815 | 0.052378 | 144 | 0.392049 | 0.502655 | 0.487487 |
| 37 | 0.669609 | 0.50251 | 0.988578 | 145 | 0.01879 | 0.503484 | 0.851792 |
| 38 | 0.447543 | 0.500862 | 0.977286 | 146 | 0.29014 | 0.506255 | 0.426265 |
| 39 | 0.519958 | 0.50015 | 0.321281 | 147 | 0.050482 | 0.507493 | 0.590625 |
| 40 | 0.042791 | 0.502245 | 0.628347 | 148 | 0.595485 | 0.500414 | 0.643119 |
| 41 | 0.722509 | 0.507645 | 0.54329 | 149 | 0.761966 | 0.507914 | 0.189789 |
| 42 | 0.364127 | 0.500675 | 0.726908 | 150 | 0.09708 | 0.505442 | 0.158934 |
| 43 | 0.992058 | 0.508005 | 0.119504 | 151 | 0.650979 | 0.501551 | 0.150496 |
| 44 | 0.626291 | 0.505674 | 0.397425 | 152 | 0.830551 | 0.507554 | 0.57036 |
| 45 | 0.173248 | 0.502258 | 0.454313 | 153 | 0.525451 | 0.508084 | 0.319449 |
| 46 | 0.250805 | 0.508061 | 0.791119 | 154 | 0.372854 | 0.504824 | 0.673256 |
| 47 | 0.75333 | 0.5088 | 0.272831 | 155 | 0.23956 | 0.509787 | 0.902681 |
| 48 | 0.865233 | 0.5079 | 0.260303 | 156 | 0.371206 | 0.500125 | 0.659889 |
| 49 | 0.743611 | 0.508207 | 0.356743 | 157 | 0.604213 | 0.505784 | 0.59679 |
| 50 | 0.034003 | 0.503955 | 0.717707 | 158 | 0.717077 | 0.505265 | 0.577899 |
| 51 | 0.053548 | 0.50937 | 0.574282 | 159 | 0.386388 | 0.503678 | 0.544572 |
| 52 | 0.363303 | 0.502842 | 0.747325 | 160 | 0.734334 | 0.504851 | 0.420101 |
| 53 | 0.605998 | 0.501923 | 0.557559 | 161 | 0.360343 | 0.508305 | 0.806745 |
| 54 | 0.602626 | 0.501817 | 0.587269 | 162 | 0.985284 | 0.509593 | 0.190002 |
| 55 | 0.59489 | 0.509127 | 0.700753 | 163 | 0.009468 | 0.508903 | 0.968206 |
| 56 | 0.672722 | 0.501105 | 0.952138 | 164 | 0.519073 | 0.505735 | 0.362755 |
| 57 | 0.693763 | 0.505207 | 0.787381 | 165 | 0.737492 | 0.508711 | 0.414836 |
| 58 | 0.149308 | 0.504351 | 0.682335 | 166 | 0.171921 | 0.503341 | 0.472761 |
| 59 | 0.705343 | 0.500643 | 0.655769 | 167 | 0.009559 | 0.502646 | 0.929844 |
| 60 | 0.644815 | 0.505336 | 0.228686 | 168 | 0.06043 | 0.503341 | 0.476179 |
| 61 | 0.960993 | 0.507582 | 0.396555 | 169 | 0.391134 | 0.500091 | 0.480346 |
| 62 | 0.891141 | 0.506279 | 0.017403 | 170 | 0.453188 | 0.50785 | 0.968404 |
| 63 | 0.88101 | 0.503167 | 0.089916 | 171 | 0.082478 | 0.504605 | 0.285329 |
| 64 | 0.600078 | 0.500276 | 0.600957 | 172 | 0.002174 | 0.505577 | 0.013893 |
| 65 | 0.082386 | 0.504574 | 0.28597 | 173 | 0.818436 | 0.507586 | 0.679588 |
| 66 | 0.3312 | 0.509988 | 0.079127 | 174 | 0.679588 | 0.509253 | 0.939228 |
| 67 | 0.833939 | 0.505814 | 0.529435 | 175 | 0.232587 | 0.501563 | 0.916095 |
| 68 | 0.484635 | 0.506671 | 0.678306 | 176 | 0.734334 | 0.505522 | 0.424129 |
| 69 | 0.678306 | 0.509803 | 0.95406 | 177 | 0.042547 | 0.50786 | 0.664237 |
| 70 | 0.8228 | 0.500963 | 0.600575 | 178 | 0.414677 | 0.50993 | 0.32749 |
| 71 | 0.375235 | 0.505373 | 0.655127 | 179 | 0.112841 | 0.500175 | 0.985481 |
| 72 | 0.622721 | 0.506438 | 0.434139 | 180 | 0.047369 | 0.508104 | 0.622304 |
| 73 | 0.097248 | 0.507593 | 0.170333 | 181 | 0.805711 | 0.507557 | 0.793942 |
| 74 | 0.165543 | 0.508439 | 0.560747 | 182 | 0.787325 | 0.507087 | 0.956593 |
| 75 | 0.218717 | 0.50805 | 0.079844 | 183 | 0.382101 | 0.504243 | 0.586551 |
| 76 | 0.110995 | 0.507332 | 0.045037 | 184 | 0.214201 | 0.50744 | 0.116833 |
| 77 | 0.793322 | 0.505789 | 0.894838 | 185 | 0.343193 | 0.505683 | 0.945363 |
| 78 | 0.037848 | 0.507678 | 0.705437 | 186 | 0.974862 | 0.501766 | 0.236835 |
| 79 | 0.428928 | 0.508738 | 0.192078 | 187 | 0.877256 | 0.508326 | 0.154646 |
| 80 | 0.698447 | 0.509495 | 0.770946 | 188 | 0.531981 | 0.50497 | 0.241992 |
| 81 | 0.651543 | 0.502512 | 0.151183 | 189 | 0.375448 | 0.504122 | 0.645697 |
| 82 | 0.156403 | 0.502687 | 0.608495 | 190 | 0.233655 | 0.5008 | 0.901904 |
| 83 | 0.769183 | 0.506799 | 0.118146 | 191 | 0.535033 | 0.503383 | 0.205003 |
| 84 | 0.935237 | 0.507951 | 0.630574 | 192 | 0.56088 | 0.508415 | 0.00257 |
| 85 | 0.896985 | 0.500068 | 0.92754 | 193 | 0.816209 | 0.50896 | 0.707879 |
| 86 | 0.486207 | 0.505526 | 0.657294 | 194 | 0.161438 | 0.503877 | 0.570315 |
| 87 | 0.090702 | 0.509014 | 0.237764 | 195 | 0.450472 | 0.504096 | 0.970327 |
| 88 | 0.214689 | 0.502557 | 0.083141 | 196 | 0.481492 | 0.509936 | 0.72619 |
| 89 | 0.403752 | 0.503035 | 0.384439 | 197 | 0.793932 | 0.500295 | 0.856385 |
| 90 | 0.009147 | 0.500678 | 0.921742 | 198 | 0.668968 | 0.50309 | 0.997825 |
| 91 | 0.74149 | 0.509901 | 0.385995 | 199 | 0.182617 | 0.509306 | 0.412287 |
| 92 | 0.92625 | 0.50925 | 0.719247 | 200 | 0.490067 | 0.502702 | 0.605611 |
| 93 | 0.74857 | 0.507214 | 0.306158 | 201 | 0.417072 | 0.503689 | 0.268483 |
| 94 | 0.055562 | 0.502411 | 0.514405 | 202 | 0.149796 | 0.501421 | 0.660362 |
| 95 | 0.583569 | 0.503971 | 0.77171 | 203 | 0.611933 | 0.508744 | 0.54506 |
| 96 | 0.613871 | 0.509918 | 0.534668 | 204 | 0.938655 | 0.502274 | 0.565753 |
| 97 | 0.862792 | 0.504665 | 0.262867 | 205 | 0.860961 | 0.502407 | 0.265797 |
| 98 | 0.387258 | 0.509949 | 0.574373 | 206 | 0.075489 | 0.505308 | 0.35244 |
| 99 | 0.809877 | 0.508579 | 0.762584 | 207 | 0.487595 | 0.505358 | 0.643789 |
| 100 | 0.774371 | 0.507714 | 0.076945 | 208 | 0.66404 | 0.509057 | 0.077982 |
| 101 | 0.352256 | 0.506506 | 0.868729 | 209 | 0.516555 | 0.506239 | 0.388437 |
| 102 | 0.175872 | 0.500304 | 0.418972 | 210 | 0.025199 | 0.505559 | 0.806562 |
| 103 | 0.322198 | 0.505258 | 0.131773 | 211 | 0.334953 | 0.506111 | 0.022087 |
| 104 | 0.965723 | 0.507856 | 0.355629 | 212 | 0.175293 | 0.503753 | 0.444882 |
| 105 | 0.839828 | 0.506637 | 0.481367 | 213 | 0.220701 | 0.501732 | 0.024087 |
| 106 | 0.874876 | 0.502588 | 0.141646 | 214 | 0.232892 | 0.509498 | 0.960957 |
| 107 | 0.670189 | 0.504005 | 0.992332 | 215 | 0.300455 | 0.503446 | 0.31658 |
| 108 | 0.941905 | 0.502076 | 0.53531 |  |  |  |  |

The picture of ui versus ui+2 is

From the above chart, we can see that the distribution of ui versus ui+2 mainly lies in the 10 patterns.

(b) Generate a sequence of length 1002. Use a program that plots points in 3 dimensions and rotates the axes to rotate the points until you can see the 15 planes.

(S-Plus function spin may be helpful.)

Answer:

4. Download a code for Mersenne twister written by Mutsuo Saito and Makoto Matsumoto

(www.math.sci.hiroshima-u.ac.jp/~m-mat/MT/emt.html)1. Generate 1002 numbers,

and plot pairs and triples of successive numbers for a visual inspection of randomness.

Discuss your conclusions.

The program I downloaded can not work on my computer. I tried many methods but still did not succeed.