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Problem 1

Answer:

The formula of the tilted mass function is as follows:

Where .

In this case: x will be 0 at the probability p and be 0 at the probability (1-p)

So the M(t) = (1-p)+p

Then we can find the   
= here x=0 or 1

We can see from the above formula, when x= 1

And when x= 0

The sum of the , so we can conclude that the tilted mass function follows a bernouli distribution with p=

Problem 2:

Answer(a) : From the definition of the expectation, we can know that

=

so

Answer(b): Var(x) =

Answer (c ) : From the inequality, we know that

Since both greater than 0

Then

Then

Form the question (b); we can derived that Var(x) =

The proof is completed.

Problem 3:

Answer:

Firstly, since we know that time can’t inverse, so when i<j then therefore, when i<j then min(=

Secondly, in the time status matrix

For each , it can be appeared in the row or column, so the max numbers can appear is 2\*n-1, the reason for minus 1 is that there will be an overlap in the row and j column. Besides for each i when the status of factor in both the row and the column is less than i, then its value of T will less than . Since it will occurred in both the row and the column, then we should minus 2\*(i-1) from the 2\*n-1.

So the total number for i is 2\*n-1-2\*(i-1)=2n-(2i-1)

Therefore, we can get that

I also give an example when i,j=5

We can see the number of is 9 = 2\*5-(2\*1-1)

the number of is 7 = 2\*5-(2\*2-1)

the number of is 5 = 2\*5-(2\*3-1)

the number of is 3 = 2\*5-(2\*4-1)

the number of is 1 = 2\*5-(2\*5-1)

which matches the identity.

Problem 4:

Answer:

We can see that Var(

The first order is 2\*

We take the first order equals to zero and get the =

The second order is 2=2Var(Y-C) is always greater than 0.

So the

If C=E(Y|Z)

Then Var(C)= )- we know that E(c)=E(Y)

And E(

So E(

Then we can get Var(c)=E(= E(YC)-=Cov(Y,C)

From the formula =

We can see that when C=E(Y|Z) then which means that no further improvement is possible by combining Y and E(Y|Z)

Problem 5:

Answer (a):

The sample mean estimator for is

Where E(X)=

We have

E()=E(X)=

Var(

In this case:

Let Y= where x follows the U(0,1) distribution

Then E(Y)=which equals to

So the Var (Y) is equal to 0.5(=0.242036

The sample mean estimator is

Var(=0.242036/n

1. The antithetic variates estimator is as follows:

Z=1/2\*(X+Y) where X and Y is an unbiased estimator for

In this case, we let Y equals to and Z equals to here, x follows the U(0,1)

The antithetic variates estimator is

W=1/2\*(Y+Z)

Var(W)=1/4\*Var(Y)+1/4\*Var(Z)+1/2\*cov(Y,Z)

Var (Y) =Var(Z)

E(YZ)=

So cov(Y,Z)= E(YZ)-E(Y)E(Z)=e-=3\*e--1<0

Then the Var(W) = e-0.25+1.5e-0.5=2.5e-0.75=0.003912

So the Var(Z)=1/n\*var(W)=0.003912/n

**From the above calculation, we can know that 0.003912/0.242036=0.016165, which means 98.4% of variance has been reduced**

Answer(b):

The method of the control variates estimator is as follows:

)

Here the is the mean value of C

In this case， we use the f(U)=U as the control variate, so the mean is 0.5

And the var(c) is 1/12

And the var() is Var(Y)-2\*

E(YC)=1 so the cov(Y,C)=1-0.5\*(e-1) =1.5-0.5\*e

We also used the =(1.5-0.5e)\*12

So var()= -2\*12\*(1.5-0.5e)^2+12\*(1.5-0.5e)^2=0.00394

SO the Var(Z)= var()/n=0.00394/n

**Compared with the variance in (a), we can know that the control variates estimator in this case is better than the sample mean estimator but worse than antithetic variates method.**

**We can see that it is less than the value of the other two method.**

Answer(C):

I used the Randu method(Which is known not a good random generator) to generate the random numbers and calculate the expectation under the following three method:

For the sample mean estimator:

Us e the

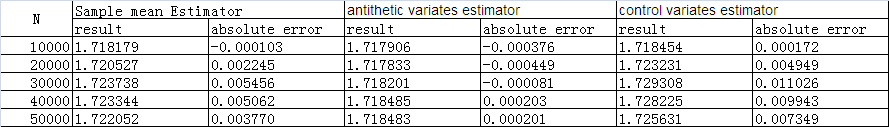
For the control variate estimator:

Use:

For the antithetic variates estimator

Use

Use N equals to 10000,20000,30000,40000,50000 and compare the results and the true value is e-1



From the above result, we can see that the result of the antithetic method is the best.