

Limit-order book resiliency after effective market orders: Empirical facts and applications to high-frequency trading

Hai-Chuan Xu^{a,b,c}, Wei Chen^d, Xiong Xiong^e, Wei Zhang^e, Wei-Xing Zhou^{a,c,f}

^aDepartment of Finance, East China University of Science and Technology, Shanghai 200237, China

^bPostdoctoral Research Station, East China University of Science and Technology, Shanghai 200237, China

^cResearch Center for Econophysics, East China University of Science and Technology, Shanghai 200237, China

^dShenzhen Stock Exchange, 5045 Shennan East Road, Shenzhen 518010, China

^eCollege of Management and Economics, Tianjin University, Tianjin 300072, China

^fDepartment of Mathematics, East China University of Science and Technology, Shanghai 200237, China

Abstract

In order-driven markets, limit-order book (LOB) resiliency is an important microscopic indicator of market quality when the order book is hit by a liquidity shock and plays an essential role in the design of optimal submission strategies of large orders. However, the evolutionary behavior of LOB resilience around liquidity shocks is not well understood empirically. Using order flow data sets of Chinese stocks, we quantify and compare the LOB dynamics characterized by the bid-ask spread, the LOB depth and the order intensity surrounding effective market orders with different aggressiveness. We find that traders are more likely to submit effective market orders when the spreads are relatively low, the same-side depth is high, and the opposite-side depth is low. Such phenomenon is especially significant when the initial spread is 1 tick. Although the resiliency patterns show obvious diversity after different types of market orders, the spread and depth can return to the sample average within 20 best limit updates. The price resiliency behavior is dominant after aggressive market orders, while the price continuation behavior is dominant after less-aggressive market orders. Moreover, the effective market orders produce asymmetrical stimulus to limit orders when the initial spreads equal to 1 tick. Under this case, effective buy market orders attract more buy limit orders and effective sell market orders attract more sell limit orders. The resiliency behavior of spread and depth is linked to limit order intensity. Finally, we present applications for high-frequency arbitrage based on LOB resiliency analysis and find that both long and short arbitrage strategies we design can achieve significantly positive returns.

Keywords: Limit order book; Liquidity; Resiliency; Bid-ask spread; LOB depth; Order intensity; High-frequency trading

JEL classification: G10, G12.

1. Introduction

Resiliency is an important measure of market liquidity. Kyle (1985) defines a market where prices recover quickly after a liquidity shock as a resilient market. Now with the popularity of electronic order-driven market, the definition of resiliency is extended. A limit order book is called resilient when it reverts to its normal shape promptly after trades (Large, 2007).

Studies on LOB resiliency have been carried out on different time horizons. Many researchers focus on minutely and daily time scales. Coppejans et al. (2004) study the resiliency of the Swedish stock index futures market and find that shocks to depth are restored in less than 60 minutes. Zawadowski et al. (2006) examine the dynamics of volatility, volume and bid-ask spread after experiencing a large liquidity shock, using data from the NYSE and the NASDAQ. Ponzi et al. (2009) study the resiliency dynamics of the bid-ask spread after a sudden bid-ask spread change

Email addresses: xxpeter@tju.edu.cn (Xiong Xiong), wxzhou@ecust.edu.cn (Wei-Xing Zhou)

in a double auction financial market and find that the spread decays as a power law on a seconds scale. Anand et al. (2013) study the impact of institutional trading on stock resiliency during the financial crisis of 2007-2009 from a long horizon perspective. They suggest that liquidity-supplying institutions are critical to recovery patterns after a shock. Jiang et al. (2013) investigate the time-reversal asymmetry of large-volatility dynamics in financial markets, based on the minutely and daily data of Chinese Stock Market Indices and German DAX. Xu et al. (2014) study the dynamics of absolute return, trading volume and bid-ask spread after the trading halts using minutely data from the Shanghai Stock Exchange and find a power-law recovery behavior.

Other researchers analyze market resiliency at order-event scales. Generally, There are two methods to conduct resiliency analysis on this shortest scale, Hawkes processes and global average measures. The first method views price changes, order submissions and cancelations as a single-variable or multi-variable Hawkes point process. Large (2007) estimates the intensities with a ten-variable point process model and finds that the order book does not replenish reliably after a large trade in over 60 percent of cases. Tock and Pomponio (2012) model trades-through using a bivariate Hawkes process and suggest that the cross-exciting effect of buy and sell trades-through is weak. Bacry et al. (2012) estimate Hawkes processes for high frequency financial price data using non-parametric estimation. They find slowly decaying (power-law) kernel shapes, suggesting a slowly resiliency. Filimonov and Sornette (2012) and Filimonov et al. (2014, 2015) calibrate the self-excited conditional Poisson Hawkes models to quantify how much of price reflexivity is due to endogenous feedback processes.

Although Hawkes process methods are more quantified, they can only characterize one dimension of liquidity, named intensity of order events. In contrast, global average measures can include different dimensions of liquidity. Degryse et al. (2005) investigate the evolution of depth and spreads as well as the prices and durations at the best prices around aggressive orders. They show that the depth and spread return to their initial levels within 20 best limit updates after the shock. Mu et al. (2010) study the dynamics of order flows around large intraday price changes using ultra-high-frequency data from the Shenzhen Stock Exchange. They uncover a significant reversal of price and find that the volatility, the volume of orders, the bid-ask spread, and the volume imbalance decay slowly as a power law. Differently, using intraday event study methodology, Gomber et al. (2015) find that resiliency to “normal levels” is fast after large transactions. In addition, Wuyts (2012) simulates a very aggressive market order shock based on a VAR model that captures various dimensions of liquidity and finds that all dimensions of liquidity revert to their steady-state values within 15 orders after a shock.

We note that these empirical studies focus mainly on aggressive orders, usually defined as the set of market orders that move the best price, like “large transactions”, “trades-through” and “extreme order events”, etc. This paper contributes to this literature by performing multi-dimension analysis (bid-ask spread, depth and intensity) of limit order book resiliency around effective market orders. Our research differs in three ways from previous papers. First, we include all effective market orders, not only aggressive orders, considering that less aggressive orders (Type 3 and Type 9 orders below) account for the largest proportion of effective market orders. More interestingly, according to our applications to arbitrage strategies, it is these less aggressive market orders that are most suitable as high-frequency trading signals. Second, seeing that Foucault et al build an equilibrium strategies model of an order-driven market and show that the spread is negatively related to the proportion of patient traders and their order arrival rate (Foucault et al., 2005), which indicates predictable relation between trading intensity and spread, we try to examine empirically the relationship between spread/depth and order intensity. Third, due to various intra-day seasonality, we remove the intra-day seasonality using the Fourier Flexible Form (FFF) regression before the global average measure.

2. Materials and methods

2.1. Dimensions of limit-order book resiliency

We describe the limit order book first. The order book right before the t -th event can be described as follows

$$\begin{array}{ccccccc} \cdots, & b_2, & b_1, & a_1, & a_2, & \cdots \\ \cdots, & B_2, & B_1, & A_1, & A_2, & \cdots \end{array} \quad (1)$$

where b_i and a_i are respectively the i -th bid and ask prices and B_i and A_i are the associated volumes at the corresponding quotes.

Kyle (1985) defines market liquidity along three dimensions: (i) tightness, “the cost of turning around a position over a short period of time”, measured by bid-ask spread $\tilde{s}(t) = a_1(t) - b_1(t)$, (ii) depth, “the size of an order flow innovation required to change prices a given amount”, measured by pending volume at the best quotes (B_1 and A_1) if the given amount is 1 tick, and (iii) resiliency, “the speed with which prices recover from a random, uninformative shock”. For order-driven markets, the definition of resiliency is extended as the speed with which the LOB reverts to its normal shape. Hence, LOB resiliency after shocks can be characterized by the evolution of bid-ask spread, depth and intensity, which is defined by the expected number of events in a unit time interval.

2.2. Data sets

Our data sets were extracted from the order flow data of 31 A-share stocks and 12 B-share stocks traded on the Shenzhen Stock Exchange in 2003. The key distinction is that A-shares are denominated in Renminbi and B-shares in Hong Kong Dollar. The A-shares market was open only to domestic investors in 2003. The market consists of three time periods on each trading day, namely, the opening call auction, the cooling period, and the continuous double auction period. Here we only consider the order flow occurring in the continuous double auction period (9:30 AM to 11:30 AM and 1:00 PM to 3:00 PM, 240 mins for each day). Because the results for different stocks are very similar, we only present the results of one A-share stock (000858) and one B-share stock (200002).

2.3. Order types

We present the classification of orders. Assume that, right before the arrival of an effective market order, the sequences of prices and volumes on the bid side of the LOB are $\{b_i : i = 1, 2, \dots\}$ and $\{B_i : i = 1, 2, \dots\}$ and those on the ask side are $\{a_i : i = 1, 2, \dots\}$ and $\{A_i : i = 1, 2, \dots\}$, respectively. Without loss of generality, assume that $b_m < \dots < b_2 < b_1 < a_1 < a_2 < \dots < a_n$, where b_m and a_n are respectively the minimal bid price and maximal ask price. These four sequences determine the current status of the LOB right before the arrival of an effective market order.

Consider an effective market order of price π and size ω . This order can be decomposed into two parts, the executed part and the remaining part that is not executed, such that

$$\omega = \omega_e + \omega_r, \quad (2)$$

where ω_e is the size of the executed part and ω_r is the size of the remaining part. When $\omega_r = 0$, all shares of the order are filled. This type of orders is termed filled effective market orders, or *filled orders* for short. When $\omega_r \neq 0$, only part of the order is filled and we can call this type of orders as partially filled effective market orders, or *partially filled orders* for short. Zhou (2012b) has shown that partially filled orders have remarkably larger market impact than filled orders. Hence, partially filled orders are more aggressive than filled orders.

Many empirical studies (Biais et al., 1995; Griffiths et al., 2000; Ranaldo, 2004; Tseng and Chen, 2016) measure order aggressiveness by the position of the order price relative to that of the latest best quotes. However, more precisely, the aggressiveness of an effective market order can also be partly captured by its penetrability. The penetrability p of an effective market order can be defined as the number of levels on the opposite LOB side it consumes:

$$p = \min_j \{j : \omega_e \leq A_1 + \dots + A_j\} \quad (3)$$

for an effective buy market order and

$$p = \min_j \{j : \omega_e \leq B_1 + \dots + B_j\} \quad (4)$$

for an effective sell market order. Therefore, effective market orders are orders with $p \geq 1$ and effective limit orders are characterized by $p = 0$. We find that, for an effective market order of penetrability p , $A_1 + \dots + A_{p-1} < \omega \leq A_1 + \dots + A_p$ and $\pi \geq a_p$ for filled buy orders, $\omega > A_1 + \dots + A_p$ and $\pi = a_p$ for partially filled buy orders, $B_1 + \dots + B_{p-1} < \omega \leq B_1 + \dots + B_p$ and $\pi \leq b_p$ for filled sell orders, and $\omega > B_1 + \dots + B_p$ and $\pi = b_p$ for partially filled sell orders.

We utilize the classification scheme consistent with that of Degryse et al. (2005). Degryse et al. (2005) classify incoming orders into 12 types based on order price and penetrability. Specifically, the effective buy (and sell) market orders are classified into three types (types 1, 2 and 3 for buys and types 7, 8 and 9 for sells). Orders of Type-1 in

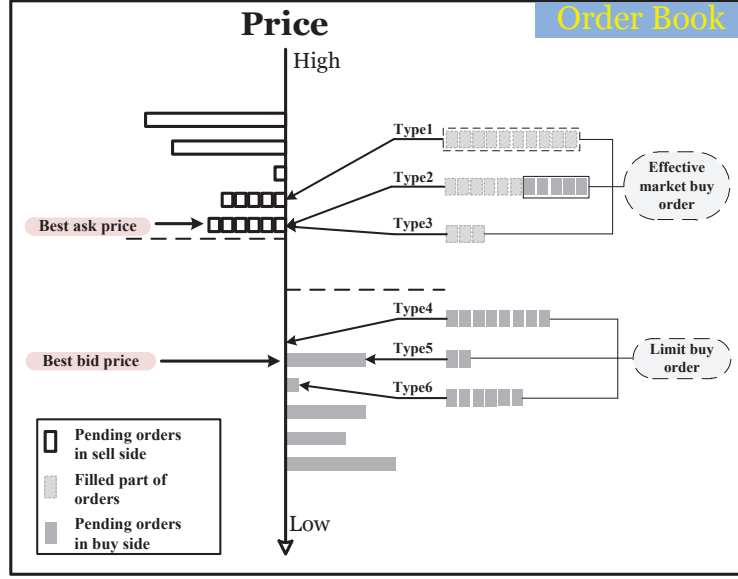


Figure 1: Classification of buy orders. The order prices are pointed by arrows. The number of small rectangles indicates order size.

Degryse et al. (2005) corresponds to bid orders with the order size greater than the best ask size and the order price higher than the best ask price. In our terminology, a Type-1 order is an effective buy market order whose penetrability p is greater than 1. This means that these orders walk up the limit order book and result in multiple trades. An order of Type-2 in Degryse et al. (2005) is an order with the size greater than the best ask size, but it does not walk up the limit order book above the best ask. In other words, a Type-2 order is a partially filled order with $p = 1$. Orders of Type-3 in Degryse et al. (2005) are bid market orders whose order size is lower than the best ask size. Speaking differently, a Type-3 order is a filled order with $p = 1$. Similarly, Type-7 orders are effective sell orders with $p > 1$, Type-8 orders are partially filled sell orders with $p = 1$, and Type-9 orders are filled sell with $p = 1$. Obviously, orders of types 1, 2 and 3 form the whole set of effective buy market orders and those of types 7, 8 and 9 constitute the whole set of effective sell market orders. Degryse et al. (2005) focus on the orders of types 1, 2, 7 and 8.

The remaining order types are not executed immediately. The prices of Type-4 orders are lower than the best ask, but higher than the best bid price, that is, Type-4 orders are buy limit orders placed in the spread. The prices of Type-5 orders are exactly at the best bid. The prices of Type-6 orders are lower than the best bid. Symmetrically, Type-10 orders are sell limit orders placed in the spread; Type-11 orders are sell limit orders placed exactly at the best ask; The remaining orders are Type-12 orders. Fig. 1 illustrates all the 6 types of orders on the buy side.

Table 1 shows the numbers of orders of the 12 types for stocks 000858 and 200002. We can find that Type-3 orders and Type-9 orders are most popular in the effective market orders. In addition, for all types of orders, the vast majority are placed when the spread is 1 tick. These features provide population guarantee for the high-frequency trading discussed below.

2.4. Methods

To quantify the limit order book resiliency, we average the spread, depth and intensity around effective market orders of the same type and compare the results among different types. Before doing the averaging measure, we should remove the intra-day seasonality of these 3 resiliency proxies. We use the Fourier Flexible Form (FFF) regression framework introduced by Gallant (Gallant, 1981) to characterize periodic patterns, which has been applied for modelling intra-day periodic returns, volatilities and quantities of financial securities (Andersen and Bollerslev, 1997; Andersen et al., 2001; Härdle et al., 2012). We employ the FFF framework to regress the intra-day periodic component $x(\tau) \in \{s(\tau), d(\tau), \lambda(\tau)\}$ for spread \tilde{s} , depth at best bid/ask \tilde{d} and intensity $\tilde{\lambda}$. Note that this paper only pay attention to the intensity of Type-4, Type-5, Type-10 and Type-11 orders, considering that the limit order book

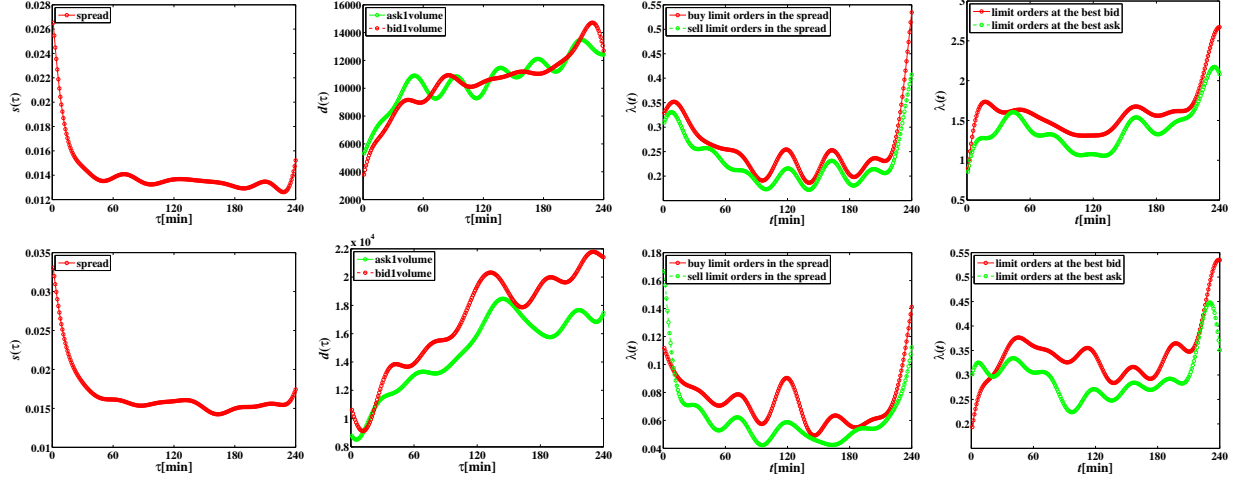


Figure 2: (Color online) Estimated 1-min intra-day seasonality factors for 000858 (top panel) and 200002 (bottom panel) traded from January to December, 2003. The columns from left to right correspond to the intra-day seasonality patterns of the bid-ask spread $s(\tau)$, the depth $d(\tau)$, the intensity $\lambda(\tau)$ of Type-4/10 orders, and the intensity $\lambda(\tau)$ of Type-5/11 orders.

resiliency after an effective market order is mainly achieved by these 4 types orders. The intra-day periodic component can be expressed as follows,

$$x(\tau) = \sum_{q=0}^Q \alpha_q \left(\frac{\tau}{T}\right)^q + \sum_{p=1}^P \left[\beta_{c,p} \cos\left(2\pi p \frac{\tau}{T}\right) + \beta_{s,p} \sin\left(2\pi p \frac{\tau}{T}\right) \right], \quad (5)$$

where τ presents the τ -th intra-day interval and $T = 240$ is the maximum number of the 1-min intervals. The parameters Q and P are the orders of the Fourier expansion. Andersen et al. suggest that the Fourier terms in Eq. (5) beyond $Q = 2$ and $P = 6$ produce insignificant estimates for any additional α_q , $\beta_{c,p}$, and $\beta_{s,p}$ coefficients (Andersen et al., 2000). We thus choose $Q = 2$ and $P = 6$. The regression will produce the unique set of the periodic intra-day estimates $x(1), x(2), \dots, x(240)$.

Fig. 2 displays the estimated 1-min intra-day seasonality for bid-ask spread, depth, intensity of Type-4/10 orders and intensity of Type-5/11 orders. We can observe that the intra-day seasonality of bid-ask spread appears a reversed

Table 1: The numbers of 12 different types of orders for stock 000858 and 200002 with different values of bid ask spreads.

Type	000858				200002			
	$s = 0.01$	$s = 0.02$	$s = 0.03$	$s \geq 0.04$	$s = 0.01$	$s = 0.02$	$s = 0.03$	$s \geq 0.04$
Type 1	9346	1981	635	421	1688	467	162	203
Type 2	12568	1801	442	232	3673	598	152	116
Type 3	97038	14916	3735	2473	16026	2795	782	682
Type 4	0	8302	3370	2957	0	1934	993	1265
Type 5	69485	14331	4536	3189	12844	3858	1384	1458
Type 6	135210	32123	10145	7281	23297	7282	2740	3441
Type 7	10593	2311	672	447	1764	507	192	189
Type 8	12063	1935	451	252	3751	716	211	140
Type 9	90510	13412	3346	1815	15378	2718	771	657
Type 10	0	7520	3065	2487	0	1600	807	1066
Type 11	61318	12451	3474	2343	11722	3117	982	1215
Type 12	192156	44715	13863	9711	30682	8913	3352	4088

J -shaped pattern, which is consistent with many other limit-order markets. On the contrary, due to the accumulation of limit orders, the volumes at the best quotes show an increasing trend. For the intensities of limit orders in the spread, their seasonality generally show a U -shaped pattern, indicating that higher order-in-spread intensity emerge near the opening time and the closing time. Note that the time at 120th min is also a opening/closing time. We also observe that the intensity of buy orders and sell orders fluctuates synchronously.

In order to account for the intra-day seasonality effects, we adjust the spread and depth correspondingly as follows,

$$S(t) = \frac{100\langle \tilde{s}(t)/s(\tau_t) \rangle}{\langle \tilde{s}(0)/s(\tau_0) \rangle}, t = -20, -19, \dots, 19, 20 \quad (6)$$

$$D(t) = \frac{100\langle \tilde{d}(t)/d(\tau_t) \rangle}{\langle \tilde{d}(0)/d(\tau_0) \rangle}, t = -20, -19, \dots, 19, 20 \quad (7)$$

where t means the best limit updates around the effective market order. The best limit updates are defined as the updates of either the best quotes or the depth at these quotes (or a combination of both). Time $t = 0$ corresponds to the spread/depth just before the effective market order. τ_t means the 1-min interval in which the t -th best limit update occurs. This adjustment also includes a normalized process compared with the value at time $t = 0$, and we set the average value as 100 at $t = 0$.

The adjustment for intensity is slightly different:

$$\Lambda([t]) = \langle 2\tilde{\lambda}([t]) / (\lambda(\tau_{[t]}^-) + \lambda(\tau_{[t]}^+)) \rangle, t = -30, -29, \dots, -1, 1, \dots, 29, 30, \quad (8)$$

where $[t]$ presents the t -th 1-min interval away from the event time $t = 0$. Because $[t]$ may intersect with two neighboring τ -intervals, we average two λ 's of the covered intervals ($\tau_{[t]}^-$ and $\tau_{[t]}^+$) as the intra-day seasonality. Note that, for intensity, neither $\tilde{\lambda}$ nor Λ has definition on $t = 0$, because intensity must be defined over an interval.

3. LOB resiliency analysis

3.1. Resiliency of bid-ask spread

Fig. 3 illustrates the average resiliency behavior of the bid-ask spread before and after the six types of effective market orders with different aggressiveness. The first feature is that the resilience behaviors for the A-share stock and the B-share stock are very similar. By definition, the relative spread is 100 right before the effective market orders. We notice that the relative spread $S(0^-)$ is approximately minimal in almost all cases, which indicates that submitting market orders are more likely when the liquidity is high. What is intriguing is the obvious difference between the resiliency behavior for different types of orders.

In the left panel of Fig. 3, we show the results for buy market orders with the penetrability $p > 1$ (Type-1) and sell market orders with the penetrability $p > 1$ (Type-7). Before the microscopic liquidity shock, the bid-ask spread increases slightly and then decreases. An effective market order of Type-1 or Type-7 consumes at least all the orders on the opposite best and the bid-ask spread soars abruptly to a peak. On average, subsequent orders are less aggressive, be they limit orders or market orders. The spread narrows gradually and relaxes to its normal level after about 20 incoming orders. The evolutionary trajectories of bid-ask spread almost overlap around buy market orders and sell market orders.

In the middle panel of Fig. 3, we show the results for partially filled buy market orders with the penetrability $p = 1$ (Type-2) and partially filled sell market orders with the penetrability $p = 1$ (Type-8). About 20 orders before the microscopic liquidity shock, the bid-ask spread starts to decrease with an acceleration when approaching to $t = 0$. Right after the partially filled order at $t = 0^+$, the spread narrows further. When the shock is from a buy order, the spread starts to resile and comes back to the normal level in a few updates. On the contrary, when the shock is from a sell order, the first subsequent order further narrows the spread and the resiliency begins since the second subsequent order. This phenomenon of buy-sell asymmetry is also observed for other A-share and B-share stocks.

In the right panel of Fig. 3, we show the results for filled buy market orders with the penetrability $p = 1$ (Type-3) and filled sell market orders with the penetrability $p = 1$ (Type-9). The spread decreases before the microscopic liquidity shock and increases immediately after the shock. The spread at $t = 0$ is only slightly narrower than that at $t = -1$, which is due to the fact that the size of the order at $t = 0^+$ is no larger than the standing volume on the

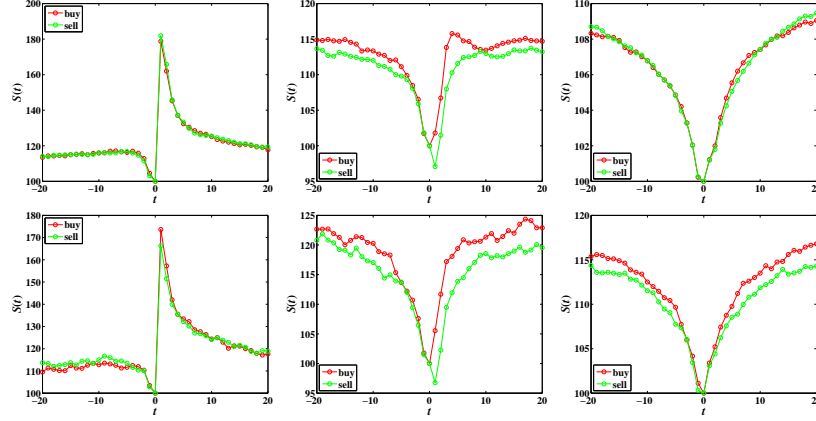


Figure 3: (Color online) LOB resiliency behavior of bid-ask spread around different types of effective market orders for stock 000858 (top) and 200002 (bottom). The market orders are of Type-1 (buy orders with the penetrability $p > 1$) and Type-7 (sell orders with the penetrability $p > 1$) in the left panel, of Type-2 (partially filled buy orders with $p = 1$) and Type-8 (partially filled sell orders with $p = 1$) in the middle panel, and of Type-3 (filled buy orders with $p = 1$) and Type-9 (filled sell orders with $p = 1$) in the right panel. Time $t = 0$ corresponds to the status of the LOB right before the arrival of an effective market order.

opposite best. The resilience speed after orders of Type-3 and Type-9 is slower than those in the middle panel. This observation is not surprising because the first price gap between the best price and the second best price after orders of Type-2 and Type-8 is much larger than the price gap after the orders of Type-3 and Type-9 (Zhou, 2012a,b).

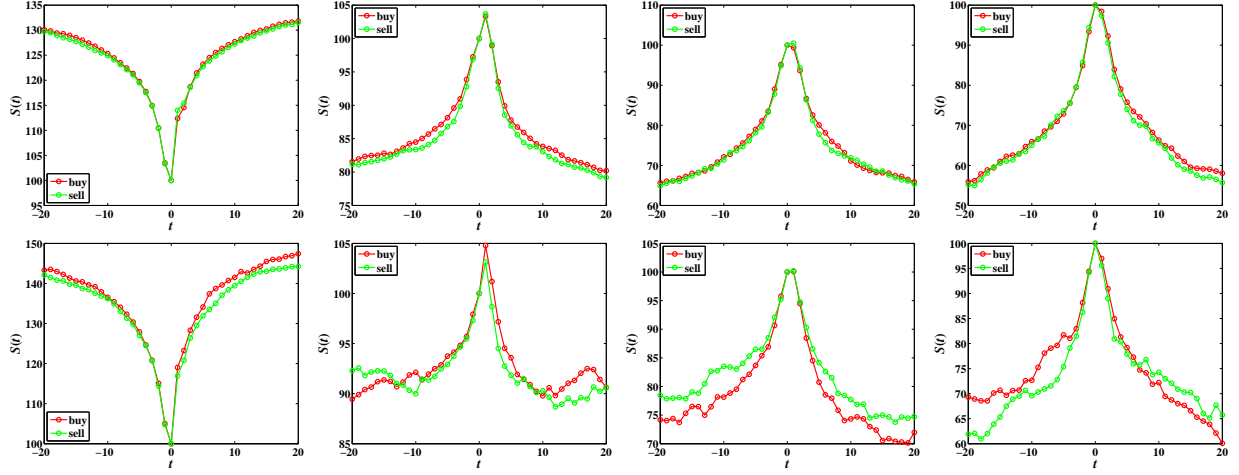


Figure 4: (Color online) LOB resiliency behavior of bid-ask spread around effective market orders with different initial spreads at $t = 0^-$ for stock 000858 (top) and 200002 (bottom). The plots in the four columns from left to right correspond to $s(0^-) = 0.01$, $s(0^-) = 0.02$, $s(0^-) = 0.03$ and $s(0^-) \geq 0.04$. Time $t = 0$ corresponds to the status of the LOB right before the arrival of an effective market order.

Fig. 4 shows the evolution of bid-ask spread around market orders for different spreads at $t = 0^-$. Because the average spread is between 0.01 CNY (1 tick) and 0.02 CNY (2 ticks), $S(t)$ decreases before the market orders at $t = 0$ and increases afterwards, as shown in the left column. For larger $s(0^-)$, the bid-ask spread increases first and then decreases. For $s(0^-) = 0.02$, the market orders further widen the spread and the maximum spread is at $t = 1^-$. For $s(0^-) = 0.03$, the market orders at $t = 0$ have minor impacts on the bid-ask spread and $s(0^-) \approx s(1^-)$. For $s(0^-) \geq 0.04$, the maximum spread is at $t = s(0^-)$ and the market orders usually narrow the spread, indicating that under this case, the greater falls of spread caused by Type-2 and Type-8 orders dominate the spread expansions caused by Type-1 and Type-7 orders on average, even though the absolute number of Type-2/8 orders is smaller than that of Type-1/7 orders.

3.2. Resiliency of LOB depth

Fig. 5 shows the average resiliency behavior of depth at the best quotes around the six types of effective market orders with different aggressiveness. Both the A-share stock and the B-share stock display similar resilience behaviors. We can find that the same side depth increases and the opposite side depth decreases before time $t = 0$ in almost all cases, which means that effective market orders are more likely to take place when the same side depth is high and the opposite side depth is low. This is due to the consistence of market orders. Like the analysis of bid-ask spread above, what is interesting is the obvious difference between the resiliency behavior for different types of orders.

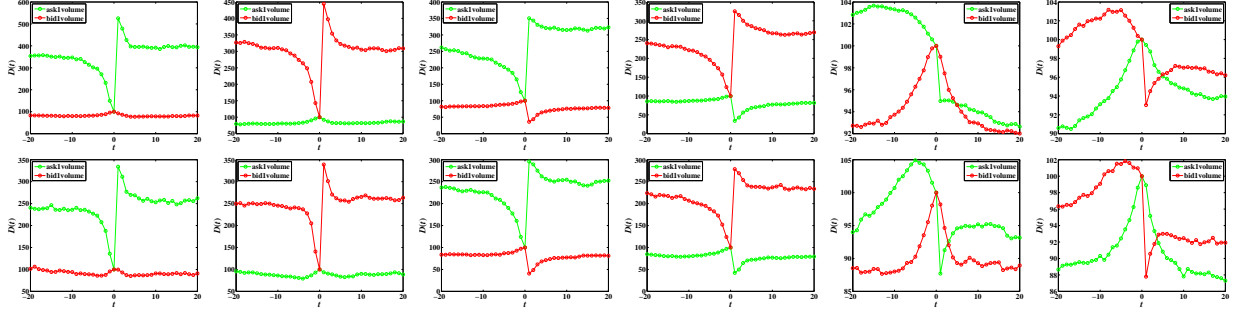


Figure 5: (Color online) LOB resiliency behavior of depth at the best bid (red line) and the best ask (green line) around different types of effective market orders for stock 000858 (top) and 200002 (bottom). The plots in the six columns from left to right correspond to the orders of Type-1 (buy orders with the penetrability $p > 1$), Type-7 (sell orders with the penetrability $p > 1$), Type-2 (partially filled buy orders with $p = 1$), Type-8 (partially filled sell orders with $p = 1$), Type-3 (filled buy orders with $p = 1$) and Type-9 (filled sell orders with $p = 1$). Time $t = 0$ corresponds to the status of the LOB right before the arrival of an effective market order.

In the first and second columns of Fig. 5, we show the results for buy market orders with the penetrability $p > 1$ (Type-1) and sell market orders with the penetrability $p > 1$ (Type-7), indicating the symmetry between these two types of orders. An effective market order of Type-1 or Type-7 penetrates at least one price level on the opposite order book and the neighboring level bears the new depth at the best. Empirical analysis has shown that the shape function of limit order book increases first and then decreases with respect to the distance of the price level to the best price for the Chinese stocks (Gu et al., 2008), which has been also observed in other markets (Challet and Stinchcombe, 2001; Maslov and Mills, 2001; Weber and Rosenow, 2005; Bouchaud et al., 2002; Potters and Bouchaud, 2003; Eisler et al., 2007; Gould et al., 2013). Hence, the new depth on the opposite side right after the market order of Type-1 or Type-7 is markedly higher than the normal value, while the same-side depth does not change. After the shock, the depths on the same side and the opposite side will reverse to their normal values within about 20 best limit updates.

In the third and fourth columns of Fig. 5, we show the results for partially filled buy market orders with the penetrability $p = 1$ (Type-2) and partially filled sell market orders with the penetrability $p = 1$ (Type-8), indicating the symmetry between these two types of orders. These effective market orders also lead to over-resiliency of the opposite depth. This is because the depth on the second best level before the shock becomes the depth on the best price level after the shock. More interestingly, Type-2 and Type-8 orders shocks can cause over-resiliency at the same side depth. This is because the unexecuted part of the order will reside on the order book forming the new same-side best. The depths at both sides relax to their normal levels after about 20 best limit updates.

In the fifth and sixth columns of Fig. 5, we show the results for filled buy market orders with the penetrability $p = 1$ (Type-3) and filled sell market orders with the penetrability $p = 1$ (Type-9). The patterns around these two types of orders are generally symmetric, except for the after-shock dynamics of opposite depth for stock 000858. Immediately after $t = 0$, the opposite side depth shows a sharp decline, which reflects the liquidity consumption by an effective market order of Type-3 or Type-9. For stock 000858, after the shock of Type-3 orders, the opposite side depth continues to decline; while after the shock of Type-9 orders, the opposite side depth starts its resiliency slowly. However, this asymmetry does not appear for stock 200002, that is, the opposite side depths show a reverse pattern consistently after either a Type-3 order's shock or a Type-9 order's shock. This indicates that the asymmetry for 000858 maybe just a special case. As for the same side depths, they all gradually reverse to their normal levels like other cases.

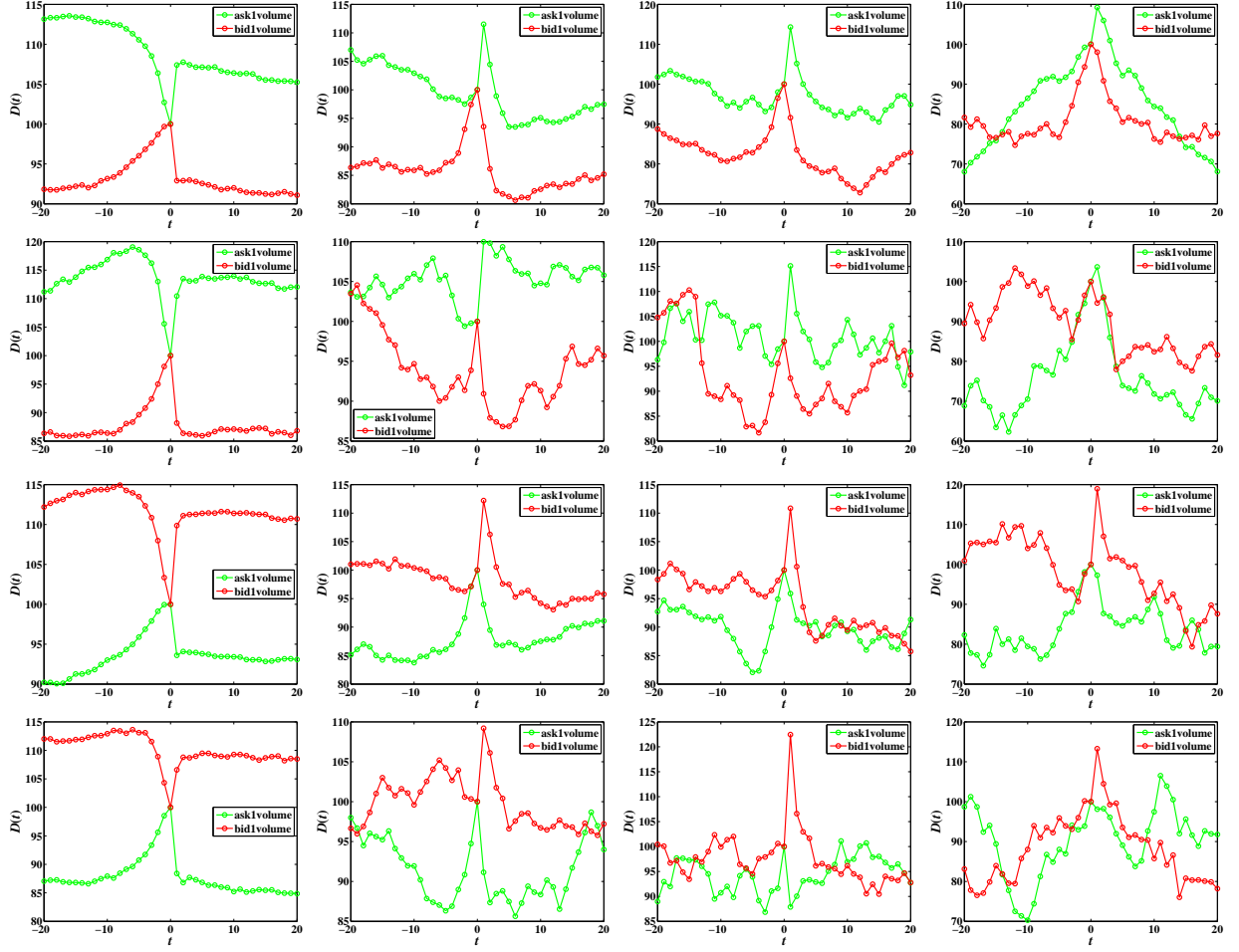


Figure 6: (Color online) LOB resiliency behavior of depth at the best bid (red line) and the best ask (green line) around effective market orders with different initial spread at $t = 0^-$ for stock 000858 and 200002. The plots in the four columns from left to right correspond to $s(0^-) = 0.01$, $s(0^-) = 0.02$, $s(0^-) = 0.03$ and $s(0^-) \geq 0.04$. The plots in the top two rows correspond to the dynamics around effective buy market orders for stock 000858 and 200002. The plots in the bottom two rows correspond to the dynamics around effective sell market orders for stock 000858 and 200002. Time $t = 0$ corresponds to the status of the LOB right before the arrival of an effective market order.

Fig. 6 shows the evolution of depth around effective market orders for different spreads at $t = 0^-$. For the opposite side depths in each plot, they show the combined effects of over-resiliency (Type-1, Type-7, Type-2 or Type-8) and sharp declining (Type-3 or Type-9). However, these combined effects are different for different spreads at $t = 0^-$. For $s(0^-) = 0.01$, they show full-resiliency or partial-resiliency immediately after the effective market orders and then present flat trends. For $s(0^-) > 0.01$, they show over-resiliency immediately after the effective market orders and then present relaxation trends. For the same side depths in each plot, when $s(0^-) = 0.01$, the market orders cause sharp decline; when $s(0^-) > 0.01$, the market orders cause gradually decline. This indicates that orders of Type-2 and Type-8 are more likely submitted when $s(0^-) = 0.01$, which is consistent with Table 1. We should also notice that, the phenomenon that effective market orders take place when the same side depth is high and the opposite side depth is low, can get the best expression when the initial spread is 0.01. In other words, the first column of Fig. 6 is significantly different from the others, which is also confirmed by Fig. 4 for spreads and Fig. 8 for intensity.

3.3. Resiliency of order intensity

The resiliency behavior of bid-ask spread/depth analyzed above can be attributed to the order flow resiliency essentially. For example, is the bid-ask spread resiliency mainly due to the placement of buy limit orders or sell

limit orders in the spread? Is the depth resiliency consistent with the intensity of limit orders placed at the best price level? Here we investigate the evolution of limit order intensity around different types of effective market orders to answer these questions. The empirical results for a representative A-share stock 000858 are illustrated in Fig. 7 for the intensity of limit orders placed in the spread (Type-4 and Type-10) and limit orders at the best price (Type-5 and Type-11). The results are very similar for other A-share and B-share stocks we investigated.

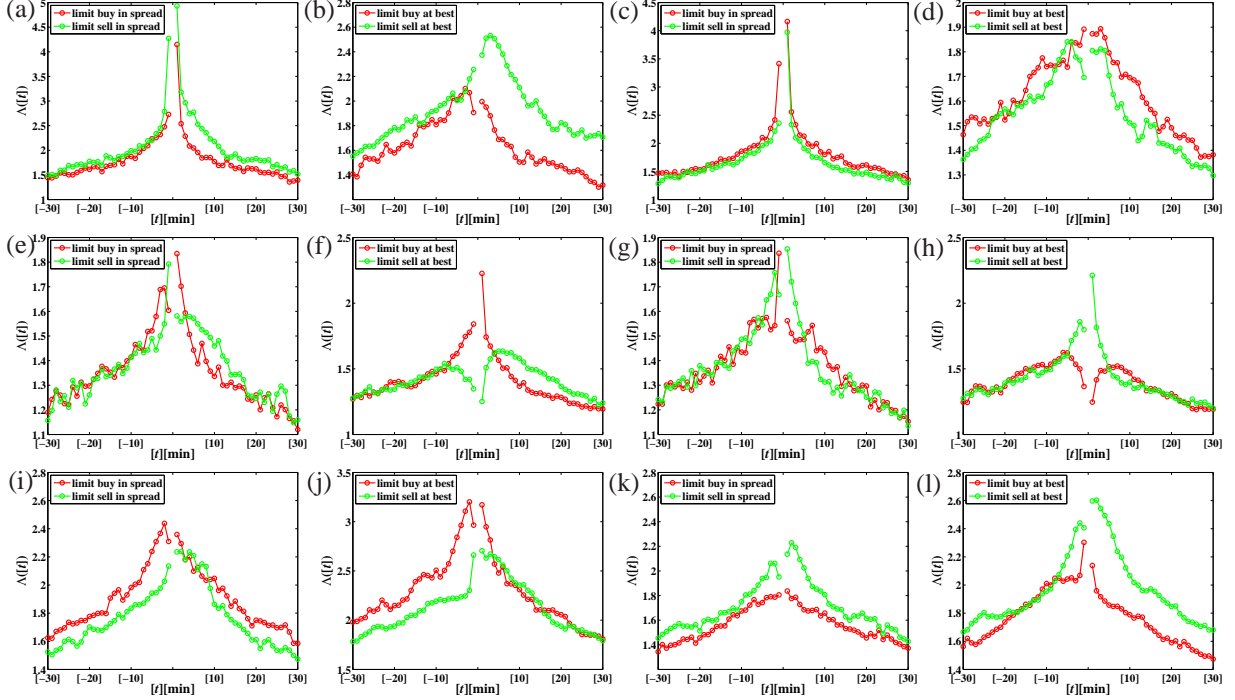


Figure 7: (Color online) Intensity of limit orders for stock 000858 around effective market orders of different types: Type 1 in plots (a) and (b), Type 2 in plots (c) and (d), Type 2 in plots (e) and (f), Type 8 in plots (g) and (h), Type 3 in plots (i) and (j), and Type 9 in plots (k) and (l).

Fig. 7(a) presents the evolution of intensity of limit orders placed in the spread (Type-4 for buys and Type-10 for sells) around Type-1 buy market orders with the size greater than the outstanding volume on the best ask and the price higher than the best ask, while Fig. 7(b) presents the evolution of intensity of limit orders placed at the best price (Type-5 for buys and Type-11 for sells) around Type-1 buy market orders. A Type-1 buy market order appears usually when the sell limit order intensity increases rapidly and is remarkably higher than the buy limit order intensity. This observation indicates that the most aggressive liquidity takers enter the market when there are more liquidity providers on the opposite side. After the arrival of a Type-1 buy market order, the bid-ask spread widens (Zhou, 2012a). More limit orders are placed in one or two minutes and the limit order intensity decreases gradually afterwards to its average level within 30 minutes. Moreover, the intensities of Type-10 (and Type-11) sell limit orders are higher than those of Type-4 (and Type-5) buy limit orders in the after-period of Type-1 orders shock. This means that, Type-10 (and Type-11) sell limit orders contribute more to the resiliency of spread and LOB depth than Type-4 (and Type-5) buy limit orders. Hence the price reversal behavior is dominant after Type-1 buy market orders. In addition, the intensity of Type-4 buy limit orders increases more than Type-10 sell limit orders right after the Type-1 buy market orders, which can be probably viewed as an indicator of herding behaviors among traders. According to Fig. 7(c) and Fig. 7(d), the behaviors around Type-7 sell market orders are similar.

In Fig. 7(e) and Fig. 7(f), we present respectively the evolution of intensity of limit orders placed in the spread (Type-4 for buys and Type-10 for sells) and at the best price (Type-5 for buys and Type-11 for sells) around Type-2 buy market orders with the size greater than the outstanding volume on the best ask and the price higher than the best ask. Both intensities of Type-4 and Type-5 buy limit orders increase before Type-2 buy market orders. They continue increasing in the first minute after Type-2 buy market orders and then decay to the average level within about 30 minutes, which implies a herding behavior of buyers, similar as Type-4 and Type-5 buy limit orders after Type-1

buy market orders shown in Fig. 7(a). The patterns of sell limit orders around Type-2 buy market orders are very different. Before Type-2 buy market orders, the intensity of Type-10 sell limit orders increases, while the intensity of Type-11 sell limit orders increases first and starts to decrease about 10 minutes before Type-2 buy market orders. After Type-2 buy market orders, the intensity of Type-10 sell limit orders that are placed in the spread decreases continuously to the average level within 30 minutes, while the intensity of Type-11 sell limit orders submitted at the best ask decreases immediately in the first minute and then increases in the subsequent a few minutes before decaying to the average level. This phenomenon indicates that buy limit orders contribute more to the resiliency of spread than sell limit orders after Type-2 buy limit orders and the price continues to rise. It is also worth noting that, comparing the intensity of limit orders at at best quotes shown in Fig. 7(f) and Fig. 7(h) with the depth at the best quotes (column 3 and column 4 in Fig. 5) after Type-2 and Type-8 market orders, the limit order intensity play an obvious effect on the bid-ask depth balance since more limit orders are placed to the side with lower depth. The patterns of limit order intensity around Type-8 sell market orders in Fig. 7(g) and Fig. 7(h) are similar.

In Fig. 7(i) and Fig. 7(j), we illustrate respectively the evolution of intensity of limit orders placed in the spread (Type-4 for buys and Type-10 for sells) and at the best price (Type-5 for buys and Type-11 for sells) around Type-3 buy market orders with the size less than the outstanding volume on the best ask and the price not less than the best ask. Before Type-3 buy market orders, the intensities of buy and sell limit orders increase continuously and buy limit orders have higher intensity than sell limit orders. After the entering of Type-3 buy market orders, the intensities of different types of limit orders increase slightly and then decay to the average level within 30 minutes. In the first a few minute after Type-3 buy market orders, buy limit orders have higher intensity than sell limit orders, indicating again that buy limit orders contribute more to the resiliency of spread than sell limit orders and the price continues to rise. The observations for limit order intensity around Type-9 sell market orders in Fig. 7(k) and Fig. 7(l) are qualitatively similar. However, the intensity difference between sell limit orders and buy limit orders after Type-9 sell market orders in Fig. 7(k) and Fig. 7(l) is larger than the intensity difference between buy limit orders and sell limit orders after Type-3 buy market orders in Fig. 7(i) and Fig. 7(j), suggesting that traders are more sensitive to bad news and have stronger herding behaviors.

Plots (a-h) of Fig. 8 show the evolution of intensity of limit orders in the spread (Type-4 and Type-10 orders) around effective market orders with different initial spreads. Similar with that of spread and depth, the evolutionary patterns of limit orders intensity when $s(0^-) = 0.01$ are significantly different from other cases. The intensity curves of buy limit orders and sell limit orders basically overlap for $s(0^-) \geq 0.02$ in Fig. 8(b-d) and Fig. 8(f-h). This indicates that no matter the effective market order shock is buy or sell, the intensity of the following buy limit orders has no obvious difference from that of following sell limit orders for the case of $s(0^-) \geq 0.02$. Both of them show higher intensities within one minute after the shock than their respective parts within one minute before the shock, and then these intensities of limit orders slowly decay to their normal levels within 30 minutes. This means that the effective market orders produce *symmetrical* stimulus to Type-4 orders and Type-10 orders under the case of $s(0^-) \geq 0.02$. However, for the case of $s(0^-) = 0.01$, the intensity curve of buy limit orders does not overlap with that of sell limit orders. After effective buy market orders, the intensity of Type-4 buy limit orders is higher than or comparable to the intensity of Type-10 sell limit orders in Fig. 8(a). Accordingly, effective sell market orders are followed by more Type-10 sell limit orders than Type-4 buy limit orders, as shown in Fig. 8(e). We find that the corresponding intensity curves in Fig. 8(a,e) and in Fig. 7(i,k) are quite similar, indicating that the price continuation behavior after Type-3 and Type-9 effective market orders is especially significant when the initial spread is 0.01 CNY.

Similar analysis can be performed on the limit orders placed at the best quotes (Type-5 and Type-11 orders). Plots (i-p) of Fig. 8 show the intensity of limit orders at the best quotes around effective market orders with different initial spreads. Similar with Fig. 8(a-h), the evolutionary patterns of limit orders intensity when $s(0^-) = 0.01$ is significantly different from the other cases with $s(0^-) \geq 0.02$. As shown in Fig. 8(j-l) and Fig. 8(n-p), although the intensity curves of buy limit orders and sell limit orders do not fully overlap when $s(0^-) \geq 0.02$, their differences are not marked. Both curves in each plot slowly decay to their normal levels within about 30 minutes. This means that the effective market orders produce *symmetrical* stimulus to Type-5 buy limit orders and Type-11 sell limit orders under the case of $s(0^-) \geq 0.02$. However, for the case of $s(0^-) = 0.01$, the intensity curves of buy limit orders and sell limit orders show significant discrepancy. Fig. 8(i) shows that, within 3 minutes after effective buy market orders, the intensity of Type-5 orders is higher than that of Type-11 orders. Fig. 8(m) illustrates that there are more Type-11 sell limit orders than Type-5 buy limit orders after effective sell market orders. This suggests that effective market orders produce *asymmetrical* stimulus to Type-5 orders and Type-11 orders under the case of $s(0^-) = 0.01$. Specifically, effective

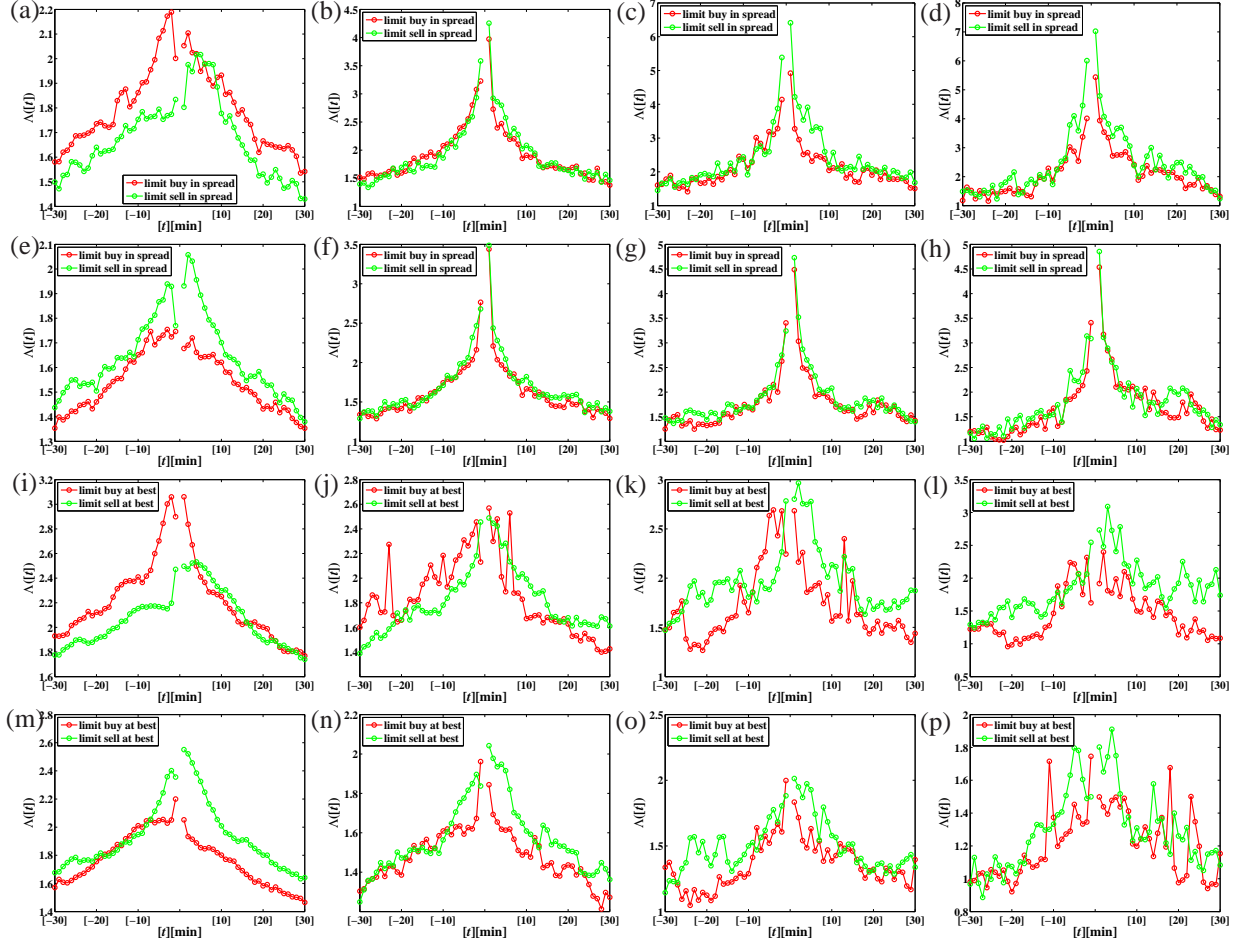


Figure 8: (Color online) Impact of initial bid-ask spread on the intensity of limit orders around effective market orders for stock 000858. The initial spreads for the four columns are $s(0^-) = 0.01$, $s(0^-) = 0.02$, $s(0^-) = 0.03$ and $s(0^-) \geq 0.04$, respectively. (a-d) Intensity of limit orders in the spread around effective buy market orders. (e-h) Intensity of limit orders in the spread around effective sell market orders. (i-l) Intensity of limit orders at the best quotes around effective buy market orders. (m-p) Intensity of limit orders at the best quotes around effective sell market orders. Time $t = 0$ corresponds to the transaction caused by the effective market order.

buy market orders attract more buy limit orders at best bid (Type-5 orders), while effective sell market orders attract more sell limit orders at best ask (Type-11 orders). These observations can be used to explain the price continuation behavior after less-aggressive effective market orders (Type-3 and Type-9 orders). Indeed, the intensity curves in Fig. 8(i,m) and in Fig. 7(j,l) share similar patterns.

4. Applications to high-frequency trading

The resiliency described above may be used as building blocks to construct high-frequency trading strategies. Such strategies can be seen as statistical arbitrages in the sense that it is uncertain whether one will win or loss money in a single run, but it expects positive returns and bounded losses statistically. Similar ideas can be found in Cont et al. (2010). Here we provide some strategic implications based on results of limit-order book resiliency. Taking short arbitrages as an example, when the current spread is 0.01 CNY (in which case the price continuation behavior is remarkable), if one wants to achieve profits, the best ask price in the near future is expected to be lower than the current best bid price. This favorable situation first requires effective sell market orders to push the best bid price down such that the bid-ask spread is widened, and then requires sell limit orders falling into the bid-ask spread such that the best ask price goes down. According to Fig. 5, sell market orders take place more often when the sell

side depth is high and the buy side depth is low. Hence, we identify instantaneous time satisfying some filtration limitations after Type-9 orders as short arbitrage opportunity. The Type-9 orders happen when the buy side depth is low and lead depth further lower after their execution, which create a favorable condition for another sell market order. Furthermore, according to the price continuation behavior after Type-9 orders, both Type-10 orders (sell limit orders in the spread) and Type-11 orders (sell limit orders at best ask) have stronger intensity than Type-4 orders (buy limit orders in the spread) and Type-5 orders (buy limit orders at best bid), which is another suitable condition for short arbitrage. Similarly, we can identify instantaneous time satisfying some filtration limitations after Type-3 orders as long arbitrage opportunity. The detailed strategies for long arbitrage and short arbitrage are described as follows.

For long arbitrage, we enter and exit the position as follows. If a Type-3 order just arrived, the spread is 0.01 CNY, the depth at the best ask $A_1 \leq \lceil A_1 \rceil$ and the depth at the best bid $B_1 \geq \lfloor B_1 \rfloor$, then we submit a buy market order. Here we call $\lceil A_1 \rceil$ and $\lfloor B_1 \rfloor$ the filter parameters. We submit a sell market order at the first time t when one of the following conditions is fulfilled:

- (1) $b_1(t) > a_1(0) + u$, in which case we are selling at a price that is not only strictly greater than our buying price, but also strictly archive our earning target. Here we call u the stop-profit parameter.
- (2) $b_1(t) < a_1(0) - v$, which results in a strictly loss. We call v the stop-loss parameter.
- (3) $b_1(t) \geq a_1(0) - v$ and $A_1(t) > cB_1(t)$, in which case although it does not trigger the stop-loss condition, the situation is extremely bad. We call c the control parameter.

For short arbitrage, we enter and exit the position as follows. If a Type-9 order just arrived, the spread is 0.01 CNY, the depth at the best bid $B_1 \leq \lceil B_1 \rceil$ and the depth at the best ask $A_1 \geq \lfloor A_1 \rfloor$, then we submit a sell market order. Here we call $\lceil B_1 \rceil$ and $\lfloor A_1 \rfloor$ the filter parameters. We submit a buy market order at the first time t when one of the following conditions is fulfilled:

- (1) $a_1(t) < b_1(0) - u$, in which case we are buying at a price that is not only strictly less than our selling price, but also strictly archive our earning target. Here u is the stop-profit parameter.
- (2) $a_1(t) > b_1(0) + v$, which results in a strictly loss. We call v the stop-loss parameter.
- (3) $a_1(t) \leq b_1(0) + v$ and $B_1(t) > cA_1(t)$, in which case although it does not trigger the stop-loss condition, the situation is extremely bad. We call c the control parameter.

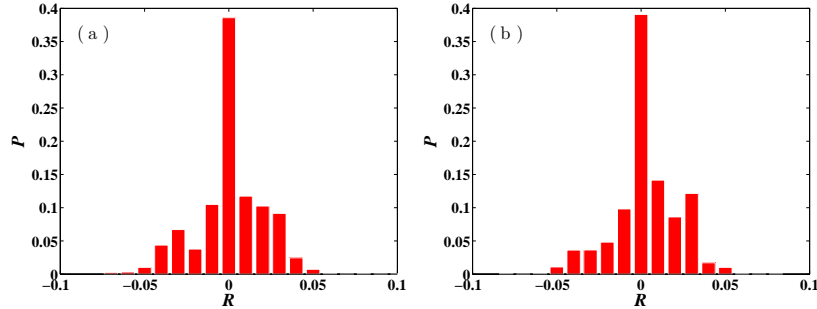


Figure 9: Probability distribution of profit and loss per round-trip trade for two example strategies. (a) Long arbitrages after filtered Type-3 orders, where the strategy parameters are settled as $\lceil A_1 \rceil = 400$, $\lfloor B_1 \rfloor = 10000$, $u = 0.02$, $v = 0.03$, and $c = 30$. As result, there are 4711 signals for entering the position. We simulate all 4711 round-trip trades and find that $P_+ = 34.49\%$ trades can archive positive returns, 7.57% higher than the negative returns, which is $P_- = 26.92\%$. We also calculate the average return $\langle R \rangle = 0.001$, which is significantly positive with the p -value $[p] = 5.68 \times 10^{-4}$. (b) Short arbitrages after filtered Type-9 orders, where the strategy parameters are settled as $\lceil B_1 \rceil = 400$, $\lfloor A_1 \rfloor = 10000$, $u = 0.02$, $v = 0.03$, and $c = 30$. As a result, there are 4503 signals for entering the position. We simulate all 4503 round-trip trades and find that $P_+ = 37.71\%$ trades can archive positive returns, 14.5% higher than the negative returns, which is $P_- = 23.21\%$. We also calculate the average return $\langle R \rangle = 0.0028$, which is significantly positive with the p -value $[p] = 2.61 \times 10^{-23}$.

Now we test these two strategies based on the order flow sequence of stock 000858. First, we run two example strategies to obtain the Profit & Loss distribution, as shown in Fig. 9. For the long arbitrage example, the strategy parameters are settled as $\lceil A_1 \rceil = 400$, $\lfloor B_1 \rfloor = 10000$, $u = 0.02$, $v = 0.03$, and $c = 30$. There are thus 4711 signals for entering the position. We simulate all 4711 round-trip trades and find that $P_+ = 34.49\%$ trades can achieve

positive returns, 7.57% higher than the negative returns, which is $P_- = 26.92\%$. We also calculate the average return $\langle R \rangle = 0.001$, which is significantly positive with the p -value $[p] = 5.68 \times 10^{-4}$. For the short arbitrage example, the strategy parameters are settled as $[B_1] = 400$, $[A_1] = 10000$, $u = 0.02$, $v = 0.03$, and $c = 30$. As result, there are 4503 signals for entering the position. We simulate all 4503 round-trip trades and find that $P_+ = 37.71\%$ trades can achieve positive returns, 14.5% higher than the negative returns, which is $P_- = 23.21\%$. We also calculate the average return $\langle R \rangle = 0.0028$, which is significantly positive with the p -value $[p] = 2.61 \times 10^{-23}$. The transaction costs are not took into account in the analysis of arbitrage strategy, which can however be easily included in this analysis.

Further, we examine the profitability of our arbitrage strategies with different parameters. The first three rows of Fig. 10 display the average returns $\langle R \rangle$, positive return percentage P_+ and negative return percentage P_- of long arbitrages after filtering Type-3 orders. The filter parameter $[A_1]$ ranges from 200 to 2000 and $[B_1]$ ranges from 6000 to 15000. Intuitively, we find that lower $[A_1]$ and higher $[B_1]$ lead to higher average returns and lower negative return percentage, as verified by Fig. 10(a-c). However, according to Fig. 10(d-f), positive return percentage is not significantly influenced by the filter parameters. In addition, with the increase of the stop-profit/stop-loss parameter pairs $[u, v]$, the average returns gradually increase (see Fig. 10(a-c)), while the positive return percentage and the negative return percentage show no significant changes except for $[u = 0, v = 0.01]$. Fig. 10(j-l) also confirm the results with different stop-profit/stop-loss parameters. Besides, Fig. 10(j-l) display the profitability of long arbitrages with different control parameters c . We find that the average returns are not monotonous and reach relatively high values when the control parameters are between 30 and 60. This is because both positive return percentage and negative return percentage synchronously increase with the increase of the control parameters.

The analysis of short arbitrages gives similar results. The first three rows of Fig. 11 illustrate the average returns $\langle R \rangle$, the positive return percentage P_+ and the negative return percentage P_- of the short arbitrages after filtering Type-9 orders. The filter parameter $[B_1]$ ranges from 200 to 2000 and $[A_1]$ ranges from 6000 to 15000. Intuitively, we find that lower $[B_1]$ and higher $[A_1]$ lead to higher average return and lower negative return percentage. However, Fig. 11(d-f) show that positive return percentage seems not to be significantly influenced by filter parameters. In addition, according to Fig. 11(a-c), the average returns gradually increase with the increase of the stop-profit/stop-loss parameter pairs $[u, v]$. In contrast, positive return percentage and negative return percentage do not exhibit significant changes except for $[u = 0, v = 0.01]$. Fig. 11(j-l) also confirm the results with different stop-profit/stop-loss parameters. Moreover, Fig. 11(j-l) display the profitability of short arbitrages with different control parameters c . We find that the average returns basically have increasing trends, which may be due to the faster increases of positive return percentages than negative return percentages.

5. Conclusions

In this paper, we quantify and compare the resiliency of limit-order book after the submission of different types of effective market orders, using ultrahigh frequency data sets from the Chinese stock market. The orders are classified by their aggressiveness and the resiliency is analyzed based on three dimensions, namely the bid-ask spread, the LOB depth, and the intensity of incoming orders. We adopt a traditional approach to filter the intra-day seasonality of these indicators and then take average of the same type orders. Our results suggest that the evolutionary consistency between bid-ask spread/depth and order intensity. In addition, we develop high-frequency arbitrage strategies based on our resiliency analysis to test the profitability of limit-order book resiliency.

First, the relative spread before the arrival of effective market orders is approximately minimal in almost all cases, which indicates that submitting market orders is more likely when the liquidity is high. The resiliency patterns of bid-ask spread show obvious diversity among different types of market orders. However, they all can return to the sample average within 20 best limit updates.

Second, effective market orders are prone to take place when the same side depth is high and the opposite side depth is low. This phenomenon is especially significant when the initial spread is 0.01 CNY (1 tick). After a market order shock, the LOB depth will also recover within about 20 best limit updates. Furthermore, the LOB resiliency behavior of depth is quite symmetric for Type-1 order shock vs. Type-7 order shock, Type-2 order shock vs. Type-8 order shock, and Type-3 order shock vs. Type-9 order shock.

Third, aggressive market orders do attract more placements of limit orders. After an aggressive buy (sell) market order shock, sell (buy) limit orders contribute more than buy (sell) limit orders to the resiliency of spread. In other

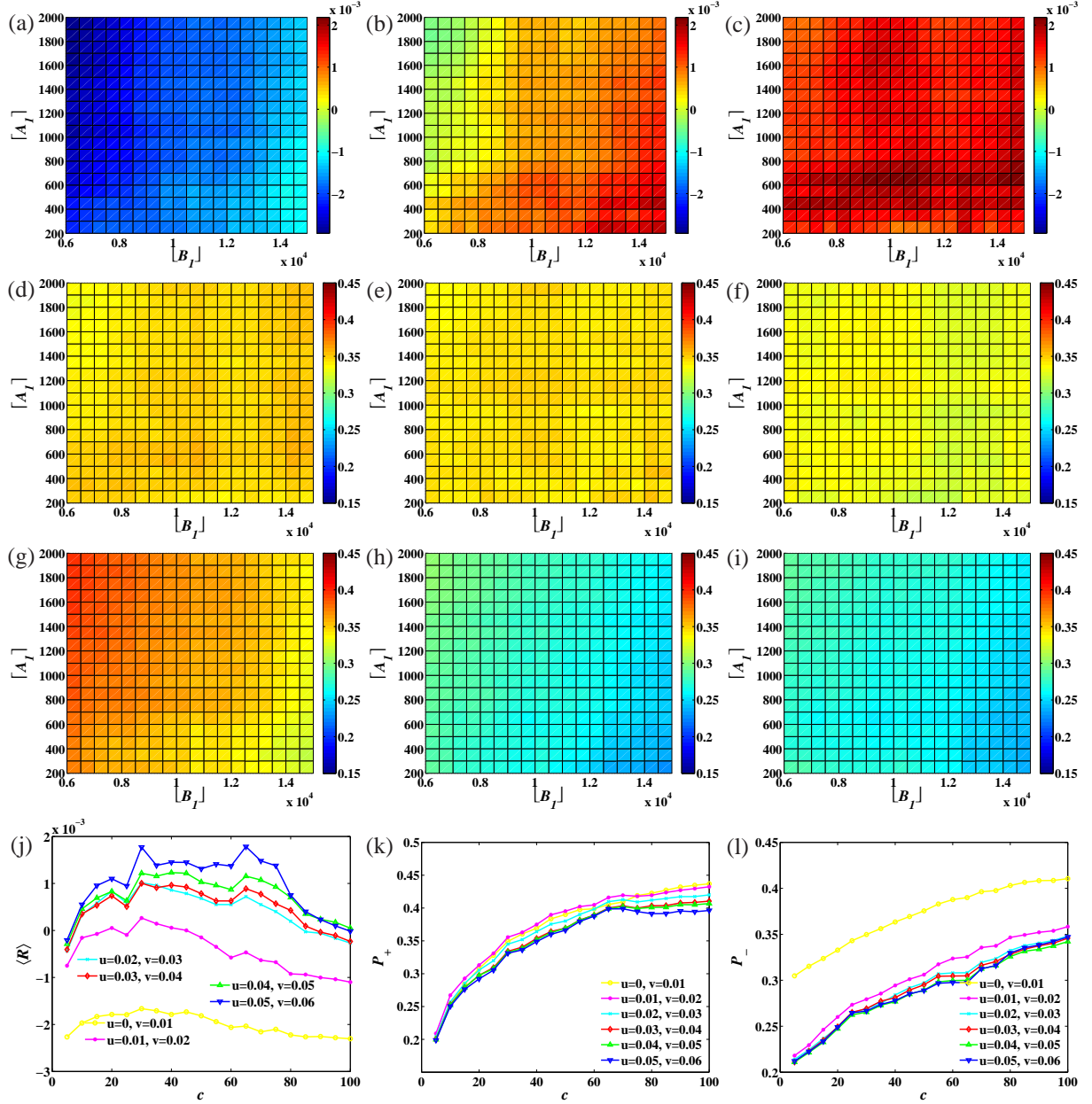


Figure 10: (Color online) Profitability of long arbitrages after filtering Type-3 orders. (a-c) Average returns. (d-f) Positive return percentage. (g-i) Negative return percentage. In these 9 plots, we fix the control parameter $c = 30$, the filter parameter $[A_1]$ ranges from 6000 to 15000 with a step of 1000, and the filter parameter $[B_1]$ ranges from 6000 to 15000 with a step of 1000. In each of the first three rows, the pair values of the stop-profit parameter u and the stop-loss parameter v are (0, 0.01), (0.02, 0.03) and (0.05, 0.06) from left to right. (j-l) Simulation results of long arbitrages after filtering Type-3 orders for different control parameter: average returns (j), positive return percentage (k), and negative return percentage (l). In these plots, we fix the filter parameters $[A_1] = 400$ and $[B_1] = 10000$. The lines in each present different stop-profit/stop-loss parameter pairs.

words, the price resiliency behavior is dominant after Type-1 and Type-7 orders. However, after a relatively less-aggressive market buy (sell) order shock, limit buy (sell) orders contribute more to the resiliency of spread than limit sell (buy) orders. This means that, after Type-2, Type-8, Type-3 and Type-9 orders, the price continuation behavior is dominant. Moreover, effective market orders produce *symmetrical* stimulus to limit orders when the initial spreads are larger. On the contrary, effective market orders produce *asymmetrical* stimulus to limit orders when the initial

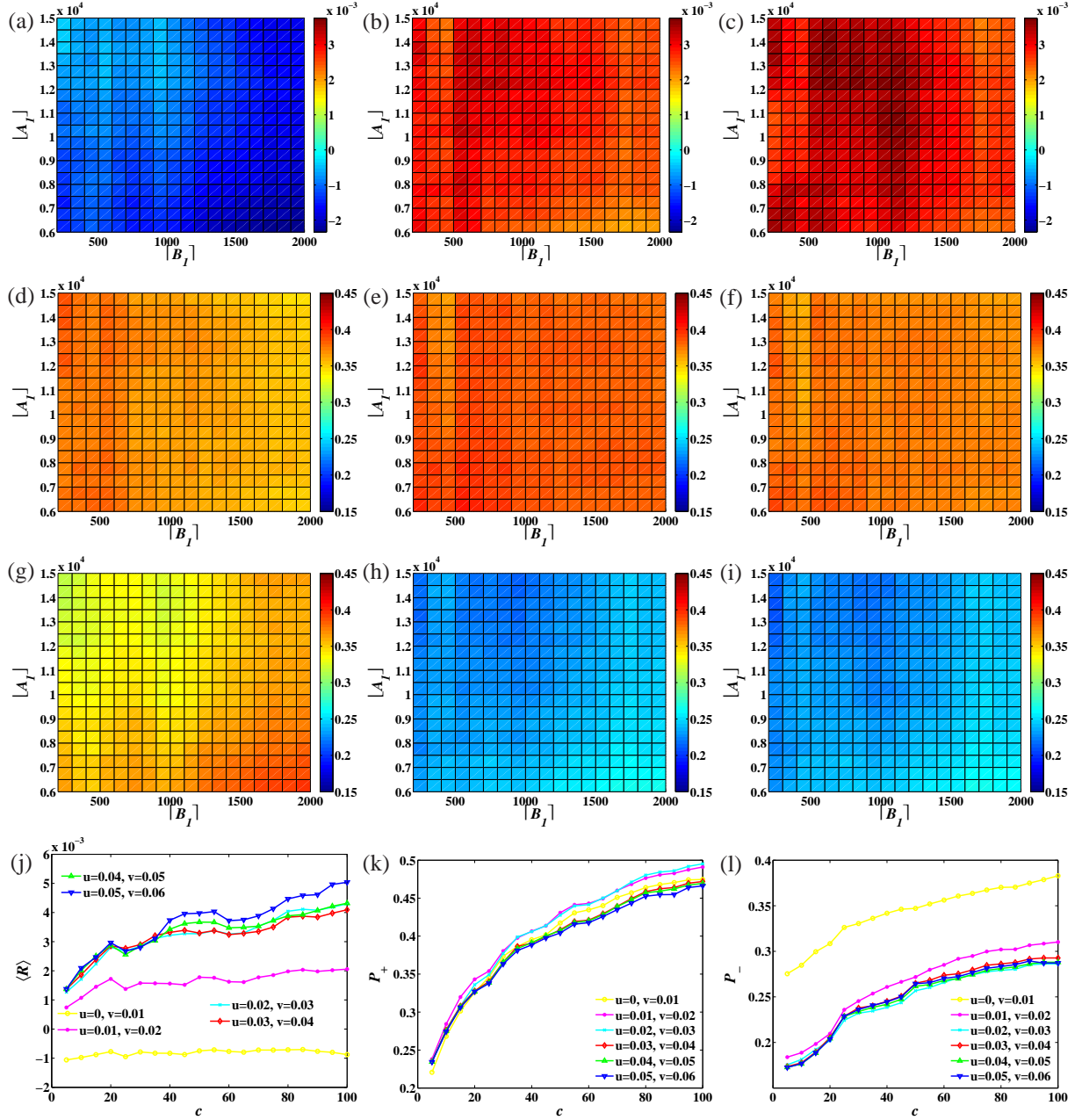


Figure 11: (Color online) Profitability of short arbitrages after filtering Type-9 orders. (a-c) Average returns. (d-f) Positive return percentage. (g-i) Negative return percentage. In these 9 plots, we fix the control parameter $c = 30$, the filter parameter $[B_1]$ ranges from 200 to 2000 with a step of 100, and the filter parameter $[A_1]$ ranges from 6000 to 15000 with a step of 1000. In each of the first three rows, the pair values of the stop-profit parameter u and the stop-loss parameter v are (0, 0.01), (0.02, 0.03) and (0.05, 0.06) from left to right. (j-l) Simulation results of long arbitrages after filtering Type-9 orders for different control parameter: average returns (j), positive return percentage (k), and negative return percentage (l). In these plots, we fix the filter parameters $[B_1] = 400$ and $[A_1] = 10000$. The lines in each present different stop-profit/stop-loss parameter pairs.

spreads equal to 0.01 CNY. In this case, effective buy market orders excite more buy limit orders and effective sell market orders attract more sell limit orders. This observation is important because it can reasonably explain the price continuation behavior after less-aggressive effective market orders (Type-3 and Type-9 orders).

Finally, the price continuation behavior after Type-3 and Type-9 orders can be explored to develop high-frequency statistical arbitrage strategies. Both long arbitrage strategy and short arbitrage strategy we designed can achieve

significantly positive returns. We also analyze the profitability of our arbitrage strategies with different parameters. We find that depth filter parameters, stop-profit/stop-loss parameters, and control parameters can affect the performance of the arbitrage strategies. The good news is that the effects of these trading parameters have regular patterns, so that we are able to pursue profits as high as possible under suitable parameters.

Besides the application to high-frequency trading, the analysis of LOB resiliency can also be applied in many aspects. For example, we notice that very recently Besson and Lasnier (2015) study the resiliency properties across different European markets and improve the standard market impact model using their resiliency indicator. In addition, we believe that LOB resiliency can also be applied to improve the estimation of the transient or permanent price impact (Bouchaud et al., 2004, 2006; Farmer et al., 2006), to solve the optimal trade execution problem more precisely (Alfonsi et al., 2010; Obizhaeva and Wang, 2013), and to design and calibrate computational models for order-driven markets (Mike and Farmer, 2008; Gu and Zhou, 2009; Li et al., 2014; Sornette, 2014).

Acknowledgments

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