

Performance of Passive Hedge Fund Replication Strategies

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Abstract

In this paper we extend Hasanhodzic and Lo (2007) by assessing the out-of-sample performance of various non-linear and conditional hedge fund replication models. We find that going beyond the linear case does not necessarily enhance the replication power. On the other hand, we find that selecting factors on the basis of an economic analysis can lead to a substantial improvement in out-of-sample replication quality, whatever the underlying form of the factor model. Overall, we confirm the findings in Hasanhodzic and Lo (2007)—the performance of the replicating strategies is systematically inferior to that of the actual hedge funds.

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1. Introduction

Following recent initiatives by major investment banks, the financial industry has expressed renewed interest in so-called passive hedge fund replication. These initiatives are meant to enable investors, all while paying significantly lower fees, to achieve returns similar to those of hedge funds by investing in a set of rules-based strategies based on liquid underlying assets aiming to replicate hedge fund performance, or at least the systematic factor exposure in hedge fund returns, i.e., their (traditional and alternative) beta components,¹ as opposed to their alpha components. The "Merrill Lynch Factor Index" and the "Goldman Sachs Absolute Return Tracker Index" were the first such initiatives; other attempts to introduce heuristic trading rules whose aim is to replicate the decisions made by hedge fund manager decisions have since followed.

The emergence of these products has taken place even though there is as yet no consensus on whether the technology that could allow satisfactory hedge fund replication is in place. In fact, academic research on hedge fund replication has been relatively limited. Although many papers have shown that the presence of systematic risk factors accounts for a significant share of hedge fund returns (Fung and Hsieh 1997, 2002, 2004, 2007 or Agarwal and Naik 2004), they have focused mostly on an *ex-post* in-sample evaluation of hedge fund manager risk-adjusted performance, as opposed to an out-of-sample assessment of the replication properties of hedge fund factor models. To the best of our knowledge, the only academic attempt to provide an empirical out-of-sample testing of hedge fund replication is a recent paper by Hasanhodzic and Lo (2007). Using monthly returns data for 1,610 hedge funds in the TASS database from 1986 to 2005, the authors estimate linear factor models for individual hedge funds using six common factors and find that the performance of linear clones is often inferior to that of their hedge-fund counterparts, and very significantly so for some hedge fund strategies (for example, the annualised mean returns of replication strategies for Emerging Market funds is 5.17%, to be compared to the 21.12% average annualised mean return for the funds themselves). Overall, the authors argue that the transparency and capacity advantages of the linear clones might earn them consideration as lower-cost alternatives to hedge funds in spite of their reported inferior performance.

In this paper, we extend the analysis done by Hasanhodzic and Lo (2007) in several important directions. First, we examine the hedge fund replication problem beyond the linear case by testing the performance of a non-linear factor model for hedge fund replication recently introduced in the context of *ex-post* performance evaluation by Diez de los Rios and Garcia (2007). Academic research that hedge fund returns exhibit non-linear dependencies with respect to underlying risk factors, and the introduction of non-linear payoffs has been found to improve the in-sample performance of hedge fund factor models (see Fung and Hsieh 2002 for fixed-income arbitrage strategies, Fung and Hsieh 2004 for trend-following strategies, Gatev, Goetzmann, and Rouwenhorst 2006 for pair-trading strategies, Mitchell and Pulvino 2001 for event-arbitrage strategies, as well as Schneeweis and Spurgin 2000 or Agarwal and Naik 2004 for a systematic test on all strategies). Because these non-linear dependencies often arise from the presence of dynamic trading strategies inherent to hedge fund manager performance and risk management processes, we also consider various conditional factor models as alternatives to the non-linear model in our attempt to improve upon the performance of the linear unconditional model in Hasanhodzic and Lo (2007). We test the performance of a Markov regime-switching approach as well as a Kalman filtering approach, both of which allow us to capture state-dependencies in the relationship between hedge fund returns and underlying risk factors (see Kazemi and Li 2007 for an analysis of conditional properties of hedge fund return distributions). In the end, given the profound correspondence between dynamic trading strategies and non-linear payoffs (Merton 1973) replicating portfolio interpretation of the Black and Scholes (1973) formula), deciding whether non-linear models or conditional models allow the more robust replication of hedge fund performance is pure econometrics. Finally, we also extend the approach in Hasanhodzic and Lo (2007) by considering the possible improvement created by the introduction of a specific set of factors for each strategy, as opposed to the use of a single set of systematic factors for all funds.

1 - This stands in contrast to the active replication approach, in which the performance of some non-investable hedge fund index is replicated through a portfolio of active hedge fund managers (see Goltz et al. 2007 for empirical evidence that a fair replication of hedge fund index performance can be achieved with a relatively limited number of funds in the replicating portfolio).

Given the concern over data mining that would arise from a statistical search for the best factors, we have constrained ourselves to a purely economic selection of factors.

By applying these factor models to the replication of CSFB/Tremont indices over the ten-year period from early 1997 to the end of 2006, we find that selecting factors on the basis of an economic analysis is an important step that can lead to substantial improvement in out-of-sample replication quality. On the other hand, introducing non-linear and/or conditional factor models does not necessarily increase the quality of replication, with the notable exception of the Kalman filtering approach, which allows a significant increase in replication power for several strategies. Overall, we confirm the findings in Hasanhodzic and Lo (2007)—the performance of the replicating strategies is systematically inferior to that of the actual hedge funds, whatever the model we use.

The rest of the paper is organised as follows. In section 2 we describe the various implementations of the factor model approach. Section 3 is devoted to the presentation of the empirical results.

2. Non-Linear and Dynamic Factor Models for Hedge Fund Return Replication

The factor replication approach involves looking for a portfolio composed of long and/or short positions in a set of suitably selected risk factors that minimises the tracking error with respect to the individual hedge fund, or hedge fund index, to be replicated. The best mimicking portfolio estimated from the in-sample analysis is then passively held in the out-of-sample period and out-of-sample performance is recorded and compared to the performance of the hedge fund target.

In brief, factor-based replication is based on the following two-step process:

Step 1: Calibration of a satisfactory factor model for hedge fund returns:

$$r_{t,i} = \sum_{k=1}^K \hat{\beta}_{ik} f_{kt} + \hat{\varepsilon}_{it}$$

where $r_{t,i}$ is the return at date t on hedge fund (or hedge fund style) i , $\hat{\beta}_{ik}$ the (potentially time-varying) estimated exposure of the return on hedge fund i to factor k , f_{kt} is the return at date t on factor k , and $\hat{\varepsilon}_{it}$ is the estimated specific risk in the return of hedge fund i at date t .

Step 2: Identification of the replicating portfolio strategy, or the clone, as:

$$\hat{r}_{t,i} = \sum_{k=1}^K \hat{\beta}_{ik} f_{kt}$$

The possible limits in the analysis are the following:

- If the explanatory power (e.g., measured by a traditional R^2 measure) of the in-sample regression analysis is weak, a significant part of hedge fund returns will not be captured in the returns of the clone. Poor explanatory power can be explained either by the fact that a significant fraction of hedge fund returns is not attributable to systematic risk exposures, or by the presence of specification risk, i.e., the fact that the chosen functional form for the factor model and/or the choice of factors may not satisfactorily capture the time variations in hedge fund returns. In our benchmark case, we use the model by Hasanhodzic and Lo (2007), which is a linear model involving the following six factors for all hedge fund strategies: the US Dollar Index, the Lehman Corporate AA Intermediate Bond Index, the spread between the Lehman Corporate BAA Bond Index and the Lehman Treasury Index, the S&P 500, the Goldman Sachs Commodity Index.²
- Even if the in-sample explanatory power is relatively high, the out-of-sample quality of replication could be low, because of a robustness problem, induced by the presence of noise in the calibration

sample (the model is fitted to reflect a sample-specific pattern and will not perform well out-of-sample) and/or non-stationary series. In particular, as recalled in the introduction, there is strong evidence that hedge fund managers dynamically manage their exposure to various risk factors in the context of either active factor timing strategies or of risk management strategies.

Of course, whether or not a particular strategy is a decent hedge fund replication strategy raises the issue of how to measure the quality of fit between the hedge fund and its clone. A first measure, naturally, is the out-of-sample correlation of the time-series for both the hedge fund and the clone. Although this measure is a very natural first estimate of the replication power of a given model, it has a number of drawbacks. First, it relies solely on second order co-movements and completely abstracts away from differences in the first order moments that are obviously very important to the investor. Hence, a given clone can be perfectly correlated with the hedge fund it seeks to replicate while yielding consistently lower returns. Secondly, and perhaps more importantly, the correlation coefficient is a directional measure that does not depend on the strength of the signal. Multiplying the clone by a given number and thus increasing the standard deviation by the same number has no impact on the correlation coefficient.

So as to complement the information provided by the correlation coefficient, we therefore choose to report the annualised root mean squared error (RMSE), which can also be interpreted as a measure of the tracking error of the clone, as well as the annualised geometric average excess return (AER):

$$RMSE = \sqrt{\frac{12}{T} \sum_{t=1}^T (\hat{r}_t - r_t)^2}$$

$$AER = \left[\prod_{t=1}^T (1 + \hat{r}_t - r_t) \right]^{\frac{12}{T}} - 1$$

A good replication results in a low RMSE. If the fit is not perfect, the AER tells us whether the deviation between the clone and the index leads to under- or over-performance of the clone relative to the index.

We now turn to the description of the hedge fund replication models we test. These models may be divided into two categories: conditional factor models and non-linear factor models. We first discuss the implementation of the linear unconditional model of Hasanhodzic and Lo (2007), which we regard as our benchmark model.

2.1 Linear Factor Model

This model specification has been proposed by Hasanhodzic and Lo (2007). The authors test two types of replicating portfolios, or clones, fixed-weight portfolios and linear clones based on rolling-window regressions. For the first type, factor exposures are estimated over the entire period and they remain constant over the whole period. This, however, introduces a look-ahead bias that drives the model out of the set of possible replication models. In what follows, then, we rely instead on the 24-month rolling-window approach, which generates truly out-of-sample results.

Formally, these clones are defined in Hasanhodzic and Lo (2007) as:

$$\hat{r}_t = \gamma \left(\sum_{k=1}^K \hat{\beta}_k f_{kt} \right)$$

with

$$\sum_{k=1}^K \hat{\beta}_k = 1 \quad \text{and} \quad \gamma = \sqrt{\frac{\sum_{l=1}^{24} (r_{t-l})^2}{\sum_{l=1}^{24} (\hat{r}_{t-l})^2}}$$

Obviously, this specification implies that all risk factor exposures are leveraged with the same unique factor (γ) and that the sum of all factor exposure is equal to one (budget constraints). In what follows, we implement a slightly more general version of the model, where we allow for different leverage levels for various factors, and define factor returns as excess return over cash (risk-free asset denoted as r_{ft}).³

We first calibrate the multi-factor model by performing unconstrained regressions on the risk factors over the sample window (where the asterisk denotes a self-financed return):

$$r_t - r_{ft} = r_t^* = \sum_{k=1}^K \beta_k f_{kt}^* + \varepsilon_t$$

The replication step then consists of allocating the estimated exposures to the corresponding risk factors and investing the remainder in cash. Accordingly, the linear clone strategy delivers:⁴

$$\hat{r}_t^L = r_{ft} + \sum_{k=1}^K \hat{\beta}_k f_{kt}^*$$

where $\hat{\beta}$ denotes the least-squares estimate of the above regression. As noted above, the striking difference between this specification and the Hasanhodzic and Lo (2007) model is that the leverage factor is allowed to vary across the risk factors. One unresolved question at this stage is whether this lower degree of structure has a negative impact on the stability of the estimated model parameters.⁵

The next sections introduce possible extensions to this unconditional linear factor model, where similar assumptions have been maintained regarding the budget constraint and the way the model is calibrated.

2.2 Option-Based Factor Model

As recalled in the introduction, previous research has motivated the introduction of new regressors with non-linear exposure to standard asset classes to capture the non-linear dependency of hedge fund returns with respect to systematic underlying risk factors. In this context, there is a key distinction between the two following approaches: i) heuristic attempts to introduce *ad-hoc* option portfolios to improve the performance of a hedge fund factor model; and ii) statistical models whose aim is to extract implied option payoffs from hedge fund return observations.

Although it is insightful and can improve the in-sample performance of factor models of hedge fund returns (see the introduction for a literature review, as well as Fung and Hsieh 2004 for a detailed summary of this particular literature), the first approach suffers from one major shortcoming: concern over the efficiency of heuristic option portfolios in hedge fund return modelling. Hence, even if the introduction of arbitrary option portfolios can improve the in-sample explanatory power, nothing guarantees that the chosen underlying assets and levels of moneyness accurately represent the true state-dependent factor exposure of hedge fund managers. As an alternative, the second approach introduced in a recent paper by Diez de los Rios and Garcia (2007) suggests that suitably designed statistical techniques can be used to estimate implicit option positions in hedge fund returns. The authors argue that suitably designed statistical techniques can be used to (a) determine the portfolio of options that best approximates the returns of a given hedge fund, (b) use options on any benchmark portfolio deemed to best characterise the strategies of the fund (and not simply traded options on an equity index), (c) estimate the corresponding moneyness of the options that best characterise the returns of a particular fund, and (d) assess whether the presence of the estimated non-linearities is statistically significant.

3 - This is valid for all factors except for the credit spread, which is already defined as a self-financed long-short position.

4 - ^L indicates that linear clone.

5 - See Jagannathan and Ma (2003), who show that imposing mistaken constraints might lead to better out-of-sample results due to a shrinkage effect imposed by the structure in the estimators.

We review here the methodology used in Diez de los Rios and Garcia (2007). The functional form of the hedge fund factor model is inspired by Glosten and Jagannathan (1994), who suggest a piecewise linear approximation of the non-linear payoff structure. In addition to the multiple risk factors consistent with the model in Hasanhodzic and Lo (2007), Diez de los Rios and Garcia (2007) introduce an option of the first factor, an equity index. Consequently, the payoff of the hedge fund may be written as:

$$r_t = \beta_0 + \sum_{k=1}^K \beta_k f_{kt} + \delta \max(f_{1t} - s, 0) + \varepsilon_t$$

where factor f_1 denotes the return on the equity index, and a suitably-designed econometric procedure (see below for more details) is used to assess the significance of the option exposure (through a formal test whether $\delta = 0$).

While Diez de los Rios and Garcia (2007) find that none of the reported exposures with respect to the non-linear factor turns out to be statistically significant, their analysis suggests that non-linear patterns in the payoff function are consistent with both intuition and previous results (see Agarwal and Naik 2004) and are a useful addition in the context of an in-sample performance analysis, while leaving open the question of whether the model is suitable in a hedge fund return replication framework.

The choice of the strike price s is obtained in a data-dependent procedure, not exogenously assumed. In the spirit of Diez de los Rios and Garcia (2007), we consider the following Wald-test statistic that corresponds to the null hypothesis that all parameters are jointly equal to zero:

$$W(s) = \hat{\beta}(s)' \hat{\Sigma}^{-1}(s) \hat{\beta}(s)$$

Here, $\hat{\beta}$ denotes the least-squares factor exposure estimates and $\hat{\Sigma}$ the estimated beta-covariance matrix given as $\hat{\Sigma}(s) = (X'X)^{-1} X' \hat{\varepsilon}(s) \hat{\varepsilon}(s)' X (X'X)^{-1}$, with X the T times $K+1$ matrix of factor returns, including the non-linear factor in the last column.

We follow Hansen (1996), and more specifically Diez de los Rios and Garcia (2007), and compute this test statistic for all possible strike prices s .⁶ The optimal strike price s^* is then obtained as the strike price that maximises the Wald statistic over all possible strike prices:⁷

$$s^* = \underset{s \in [s_{\min}, s_{\max}]}{\operatorname{argmax}} W(s)$$

In the context of the option-based factor model, the implementation of the budget constraint is given as:⁸

$$\hat{r}_t^0 = rf + \sum_{k=1}^5 \hat{\beta}_k f_{kt}^* + \hat{\delta} [\max(f_{1t} - s_t^*, 0) - rf * C_{t-1}]$$

Here, C_{t-1} is the Black-Scholes price at date $t-1$ for the option generating the payoff $\max(f_{1t} - s_t^*, 0)$, written on the return on the equity index as opposed to its value. This Black-Scholes price can be given by (where σ is an estimate of the index equity volatility):

$$C_{t-1} = \exp(-rf) + \Phi(d_1) - (1 + s_t) \exp(-rf) \Phi(d_2)$$

$$d_1 = \frac{-\log(1 + s_t) + rf + \sigma^2 / 2}{\sigma} \quad d_2 = d_1 - \sigma$$

The next sections introduce models that explicitly account for time-varying factor exposures through state-space models.

6 - Like Diez de los Rios and Garcia (2007), we limit our search to strike prices lying between the 15th and the 85th percentiles of observed values.

7 - The methodology described above is a slight modification of the original methodology in Diez de los Rios and Garcia (2007), who had in fact considered the Wald statistics related to the null hypothesis that the option factor loading be equal to zero, as opposed to all factor loadings being equal to zero simultaneously. Our motivation is that the former approach seems more consistent with the spirit of a multi-factor model.

8 - ⁰ indicates the option-based clone.

2.3 Markov Regime-Switching Model

In a first attempt to account for dynamics in factor exposures, we introduce a state-space model with a discrete number of states.⁹ Following Hamilton (1989), we implement a Gaussian Markov regime-switching model (MRS henceforth) for the time series of hedge fund index returns and the dependant variables.

The model may be written as:

$$\hat{r}_t = \sum_{k=1}^K \beta_k(S_t) f_{kt}^* + \varepsilon_t$$

where the residuals are Gaussians with $\varepsilon_t \sim N(0, \sigma^2(S_t))$ and S_t is a discrete number that denotes the state of nature at time t . Both factor exposures and residual volatilities are assumed to depend on the state of nature.

In our analysis, due to the required number of parameter estimates and the related estimation risk,

we restrict the number of states to two. The probability matrix P is written as: $P = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix}$,

where entry p_{ij} denotes the probability that the state of nature is j at time $t+1$ when it is i at time t and $p_{i1} = 1 - p_{i2}$. The MRS model is closely related to the piece-wise linear approximation of the non-linear factor model studied in Diez de los Rios and Garcia (2007) (see previous section). Indeed, the former model also consists of two states of nature, state one characterised by the option's being out of the money ($f_{1,t} - s_t \leq 0$) and state two by the option's being in the money ($f_{1,t} - s_t > 0$). The striking difference between the two models is that within the Markov switching model, changes in the regime are not determined solely by the level of the dependent variables. Instead, they are a function of unobservable state variables modelled through a Markov chain characterised by the above transition probability matrix.

The return of the MRS clone is then given as:¹⁰

$$R_t^{M*} = rf + \sum_{k=1}^K \tilde{\beta}_{kt} F_{kt}^* \quad \text{with} \quad \tilde{\beta}_{kt} = \tilde{p}_{1,t} \hat{\beta}_k^1 + \tilde{p}_{2,t} \hat{\beta}_k^2$$

where $\tilde{p}_{1,t}$ and $\tilde{p}_{2,t}$ are the smoothed state probabilities with $\tilde{p}_{1,t} + \tilde{p}_{2,t} = 1$. Further, $\hat{\beta}_k^1$ and $\hat{\beta}_k^2$ denote the estimated risk factor exposures in states $S=1$ and $S=2$ respectively. Details of the estimation procedure related to the involved parameters are shown in appendix A.

2.4 Kalman Filter Model

A competing approach to modelling dynamic risk factor exposures through state-space variables is the Kalman filter (Kalman 1960). This approach linearly links the observable variables with the state variables and has been shown to be the best of the one-sided linear filters (Hamilton 1994).

The vector representation of a linear state space factor model is given by:

$$\begin{aligned} \beta_t &= A\beta_{t-1} + \eta_t \quad (\text{Transition equation}) \\ \hat{r}_t &= \beta_t' F_t + \varepsilon_t \quad (\text{Measurement equation}) \end{aligned}$$

Here β_t is the vector of factor exposures at time t to the various risk factors, F_t the vector of factor returns at t and A the transition matrix. We further suppose that η and ε are normally distributed with zero mean and covariance matrices H and G respectively. Accordingly, we obtain the Kalman filter clones as:¹¹

$$\hat{r}_t^K = rf + \beta_{t|t-1}' f_{kt}^*$$

Details on the Kalman filtering estimation can be found in appendix B.

9 - An example of Markov regime-switching models for hedge returns is given in Billioy, Getmansky, and Pelizzonx (2006).

10 - ^M indicates the Markov regime-switching clone.

11 - ^K indicates the Kalman-filter clone.

3. Empirical Results

As in Hasanhodzic and Lo (2007), we assess the performance of competing replicating strategies using the TASS database. More precisely, we collect monthly return data of CSFB/Tremont hedge fund indices for the following twelve strategies: Convertible Arbitrage, Managed Futures, Distressed Securities, Emerging Markets, Equity Market Neutral, Event Driven, Fixed-Income Arbitrage, Global Macro, L/S Equity, Risk Arbitrage, Dedicated Short Bias, and Fund of Funds.¹² The initial calibration period starts in January 1997 and ends in December 1998. For the unconditional clones (linear model and option-based model), a 24-month estimation window is rolled over every month. As far as the conditional models (MRS and Kalman filter) are concerned, we add new arriving information to the calibration window instead of rolling the calibration window over.¹³ According to the different replication strategies described in section 2, we obtain a total of 96 monthly out-of-sample returns for the different clones from January 1999 through December 2006.

In order to address the factor selection process reviewed in section 2, we distinguish between two sets of risk factors. The first set of results is based on the factors proposed in Hasanhodzic and Lo (2007) and will henceforth be labelled *Full Factor Model* (section 3.1). The second set of empirical results is based on an economic choice of specific factors for each strategy and will be labelled *Economic Factor Model* (section 3.2).

3.1 Full Factor Model

Like Hasanhodzic and Lo (2007), we choose the following risk factors: the US Dollar Index, the Lehman Corporate AA Intermediate Bond Index, the spread between the Lehman Corporate BAA Bond Index and the Lehman Treasury Index, the S&P 500 and the Goldman Sachs Commodity Index. Based on these risk factors and according to the described methodology, we obtain in-sample and out-of-sample results for each of the four factor models.

Exhibit 1 reports the average adjusted R^2 over all in-sample periods. The results suggest that the introduction of a statistical option-based factor significantly enhances the in-sample fit of the model. Quite unsurprisingly, we first note that whatever the model we use the explanatory power is higher for directional strategies such as Long/Short Equity or Dedicated Short Bias than for less directional strategies such as Risk Arbitrage or Equity Market Neutral, where a lower degree of systematic risk exposure is expected. The numbers in exhibit 1 further indicate that the Kalman filter model clearly outperforms all other models in terms of explanatory power, including the MRS model, which probably was also to be expected given its greater flexibility.

Indeed, while the number of states is restricted to two for the MRS, the Kalman filter model implicitly allows for an infinite number of states driven by instantaneous shocks.

Exhibit 1. Average adjusted in-sample R^2 are reported based on rolling (unconditional models) and growing (conditional models) window regressions based on Hasanhodzic and Lo (2007) risk factors. Sample horizon as of January 1997 through December 2006.

	unconditional clone		conditional clone	
	linear	option	MRS	Kalman
Convertible Arbitrage	-0.12	0.14	0.23	0.62
Managed Futures	0.37	0.41	0.25	0.66
Distressed Securities	0.20	0.30	0.24	0.77
Emerging Markets	0.39	0.45	0.47	0.73
Equity Market Neutral	-0.38	0.03	0.16	0.51
Event Driven	0.31	0.40	0.47	0.80
Fixed Income Arbitrage	0.10	0.18	0.50	0.71
Global Macro	-0.28	0.07	0.31	0.63
Long/Short Equity	0.49	0.54	0.54	0.71
Risk Arbitrage	0.23	0.31	0.29	0.63
Dedicated Short Bias	0.63	0.63	0.61	0.82
Fund of Funds	0.40	0.42	0.33	0.69

12 - The CSFB/Tremont hedge fund indices are based on funds listed in the TASS database. For construction details, see <http://www.hedgeindex.com/>. We have also performed similar tests on EDHEC composite hedge fund indices, and obtained very similar results.

13 - Indeed, the main motivation for the rolling-window approach is to account for non-stationary exposures and distributions. Since conditional approaches allow explicit accounting for regime shifts and time-varying exposures, "growing calibration windows" seem more appropriate as the increase in sample size should then lead to a corresponding decrease in estimation risk.

However, as has been stated in the previous section, the real challenge of hedge fund replication is the robustness of the models and their ability to deliver improved out-of-sample fits. In fact, due to the increasing number of parameters, conditional and option-based models are less parsimonious and hence *a priori* more prone to estimation risk than the unconditional linear model.

To test for this, we now turn to the out-of-sample analysis. For each hedge fund strategy, we compare the four clones (corresponding to the four kinds of factor models) and the index over the out-of-sample period. As underlined in section 2, a natural measure of replication quality is the root-mean squared error (RMSE) since it subsequently penalises instantaneous deviations between the clone and the index. Additionally, the correlation coefficient may be of interest. Exhibit 2 shows both the annualised RMSE and the correlation coefficients over the out-of-sample period from January 1999 through December 2006 for the various indices and replication strategies.

Several comments are in order. First, we obtain that the replication quality does not vary significantly across the different factor model implementations. Second, the option augmented model performs poorly out-of-sample when compared to the linear model, as evidenced by consistently higher RMSE for the option-based model compared to the corresponding linear model. The notable exception is Long/Short Equity, for which the replication performance is roughly equivalent. Obviously, the additional factor and the related estimation risk have a negative effect on the out-of-sample fit. In particular, the statistical nature of the kink, i.e., the strike level determined over past returns, seems to be critical. Although the methodology is statistically appealing, a serious robustness problem arises with respect to the estimation of the kink, which leads to a reduction in out-of-sample replication power. On the other hand, the superior in-sample fit of the Kalman filter model is not confirmed in the out-of-sample analysis since using that model does not generate a significantly better replication quality, measured in terms either of RMSE or of correlation coefficient. In other words, while the in-sample analysis confirms the presence of dynamic factor exposure, capturing such time-varying exposures in a robust manner is a formidable challenge.

Exhibit 2. Annualised RMSE and, in parentheses, correlation coefficients are given for each strategy and each replication model based on Hasanhodzic and Lo (2007) risk factors. Monthly out-of-sample returns from January 1999 through December 2006 have been considered.

	unconditional clone				conditional clone			
	linear		option		MRS		Kalman	
Convertible Arbitrage	0.05	(0.10)	0.06	(-0.05)	0.05	(0.13)	0.05	(0.21)
Managed Futures	0.12	(0.42)	0.13	(0.35)	0.12	(0.36)	0.12	(0.40)
Distressed Securities	0.05	(0.41)	0.06	(0.34)	0.05	(0.40)	0.05	(0.39)
Emerging Markets	0.11	(0.51)	0.12	(0.46)	0.12	(0.53)	0.11	(0.48)
Equity Market Neutral	0.03	(0.37)	0.03	(0.05)	0.03	(0.27)	0.03	(0.39)
Event Driven	0.04	(0.47)	0.05	(0.39)	0.05	(0.46)	0.05	(0.41)
Fixed Income Arbitrage	0.03	(0.22)	0.05	(0.14)	0.04	(0.13)	0.03	(0.22)
Global Macro	0.08	(-0.00)	0.08	(0.02)	0.09	(0.07)	0.09	(-0.09)
Long/Short Equity	0.09	(0.47)	0.09	(0.50)	0.19	(0.33)	0.10	(0.38)
Risk Arbitrage	0.04	(0.15)	0.05	(0.02)	0.05	(0.05)	0.05	(0.08)
Dedicated Short Bias	0.12	(0.68)	0.12	(0.68)	0.12	(0.69)	0.12	(0.67)
Fund of Funds	0.06	(0.40)	0.06	(0.40)	0.06	(0.45)	0.06	(0.32)

As a general comment, the misspecifications measured by the RMSE seem to be driven by specific exogenous factors for each strategy such as differences in the idiosyncratic variance of the strategy. In this context, in spite of the limitations reported in section 2, it may be worth looking at the correlation coefficient as a complement, since it is a standardised measure and invariant to linear transformations. To some extent, a high RMSE together with a high correlation coefficient may indicate the presence of abnormal returns. For instance, comparing the replication quality of unconditional linear clones for the Event Driven and the Long/Short Equity index relying on the correlation coefficient suggests a similar fit (0.47 respectively). However, the RMSE is much higher for the Long/Short Equity clone (0.09) than for the Event Driven clone (0.04).

One striking result in Hasanhodzic and Lo (2007) is that the performance of hedge fund clones is significantly poorer than that of the hedge fund index that they try to replicate. To analyse this question, exhibit 3 shows annualised average excess returns (AER). With the exception of the Dedicated Short Bias strategy, all excess returns are found to be negative, ranging up to -14% on an annual basis. Again, the option augmented payoff is dominated by the linear model, as evidenced by mostly lower excess returns for the option clones. As far as the conditional models are concerned, the results seem to suggest that the Kalman filter clearly outperforms the competing conditional model (MRS) but also the unconditional model. This is line with previous statements on RMSE and correlation coefficients and explained by the persistently negative bias between hedge fund clones and the indices they try to replicate. In this case, improved replication quality, measured by lower RMSE, leads directly to higher performance.

As a general comment, these results confirm the findings in Hasanhodzic and Lo (2007) and illustrate that risk premia associated with common risk factors do not account for hedge fund returns.

From the investor's standpoint, another relevant question is whether the inferior performance of the clones is accompanied by a corresponding decrease in volatility, in such a way that the clones might prove to be attractive investments from a mean-variance point of view even if they turn out to be poor replicators of the corresponding hedge fund indices. Exhibit 4 shows annualised Sharpe ratios for the hedge fund indices and the corresponding clones.

Exhibit 3. Annualised average excess returns (AER) are reported for each strategy and each replication model based on Hasanhodzic and Lo (2007) risk factors. Monthly out-of-sample returns from January 1999 through December 2006 have been considered.

	unconditional clone		conditional clone	
	linear	option	MRS	Kalman
Convertible Arbitrage	-0.06	-0.05	-0.06	-0.05
Managed Futures	-0.03	-0.09	-0.09	-0.04
Distressed Securities	-0.07	-0.08	-0.08	-0.06
Emerging Markets	-0.11	-0.11	-0.14	-0.09
Equity Market Neutral	-0.05	-0.05	-0.06	-0.05
Event Driven	-0.05	-0.07	-0.07	-0.04
Fixed Income Arbitrage	-0.03	-0.05	-0.04	-0.02
Global Macro	-0.09	-0.07	-0.10	-0.08
Long/Short Equity	-0.05	-0.06	-0.07	-0.03
Risk Arbitrage	-0.01	-0.03	-0.02	-0.01
Dedicated Short Bias	0.04	0.03	-0.01	0.02
Fund of Funds	-0.05	-0.07	-0.06	-0.04

Exhibit 4. Sharpe ratios for hedge fund indices and their clones are given. US treasury bills (three months) are used to model the risk-free asset. Clones are based on Hasanhodzic and Lo (2007) risk factors. Monthly out-of-sample returns from January 1999 through December 2006 have been considered.

	Hedge Fund Index	unconditional clone		conditional clone	
		linear	option	MRS	Kalman
Convertible Arbitrage	1.76	0.37	0.42	0.37	0.53
Managed Futures	0.20	0.02	-0.61	-0.68	-0.10
Distressed Securities	2.19	0.69	0.34	0.41	0.91
Emerging Markets	1.03	0.00	0.01	-0.29	0.30
Equity Market Neutral	3.16	0.59	0.69	0.02	0.76
Event Driven	2.19	0.85	0.29	0.21	1.04
Fixed Income Arbitrage	1.44	0.28	-0.36	-0.10	0.49
Global Macro	1.34	-0.04	0.36	-0.20	0.13
Long/Short Equity	0.78	0.41	0.19	0.00	0.63
Risk Arbitrage	0.94	1.03	0.18	0.43	0.83
Dedicated Short Bias	-0.40	-0.11	-0.18	-0.40	-0.23
Fund of Funds	1.12	0.33	-0.17	0.12	0.61

These numbers strengthen the former results, as the underperformance of the clones systematically leads to severe deteriorations in the Sharpe ratios, whatever the underlying factor model used. In fact, the inferior performance of the clones is not compensated for by a parallel decrease in volatility.¹⁴

Overall, the results suggest that hedge fund clones under-perform the corresponding hedge fund indices. They also suggest that differences in replication quality are more pronounced across hedge fund strategies than across model specifications. Finally, it turns out that conditional and non-linear models, which are less parsimonious than their linear counterparts, do not necessarily lead to improved out-of-sample replication. In an attempt to improve this replication, the next section focuses on the factor selection phase. We argue that the customisation of factor sets for each strategy may reduce the estimation risk while enhancing the specification fit for each strategy.

3.2 Economic Selection of Factors

In this section, we test whether selecting specific sets of factors for each strategy leads to an improvement in the replication performance. Based on an economic analysis and in accordance with Fung and Hsieh (2007), who provide a comprehensive summary of factor based risk analyses over the past decade, we select potentially significant risk factors for each strategy.

Accordingly, we add the following potentially useful factors to the set of the five factors in Hasanhodzic and Lo (2007):

- Small/Large spread, proxied by the return differential between the S&P 600 Small Cap index and the S&P 500 Composite index
- FC Emerging Markets index
- Merrill Lynch 300 Global Convertible Bond index
- Default spread, proxied by the return differential between Lehman US Aggregate Intermediate Credit BAA and the Lehman US Aggregate Intermediate AAA indices
- Mortgage spread modelled by the excess return of the GNMA index over the Lehman US Treasury Bill index

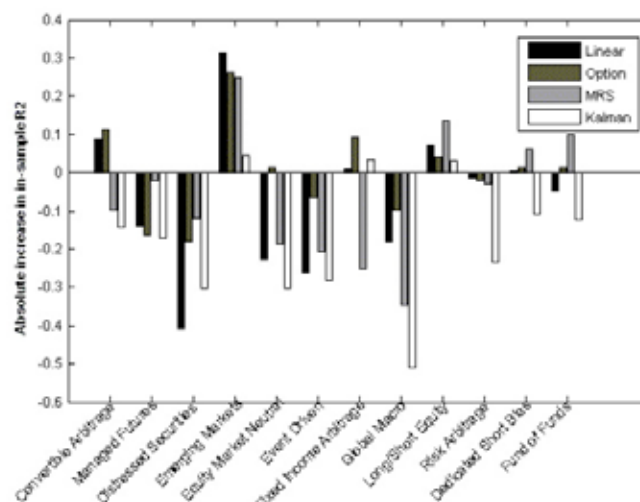
Exhibit 5 shows the factors for each strategy. The table further indicates the factor on which the option is written for the option-based replication model.

For some strategies, including Managed Futures, Distressed Securities, Event Driven, Global Macro, and Risk Arbitrage, the economic factor model merely consists of using a reduced set of factors as compared to the full factor model. Exhibit 6 shows the absolute increase in adjusted in-sample R² when switching from the full factor model to the economic factor model. As expected, we observe a decrease in in-sample fit for the large majority of these strategies. On the other hand, the remaining strategies (Convertible Arbitrage, Emerging Markets, Equity Market Neutral, Long/Short Equity, Dedicated Short Bias, Fixed-Income Arbitrage, Fund of Funds) benefit from additional factors as compared to the set of factors introduced in the full factor model, which leads to improved in-sample replication quality.

Exhibit 5. Factor selection according to economic criteria consistent with the academic literature (See Fung and Hsieh 2007 for a comprehensive summary). The asterisk indicates the non-linear factor for the option-augmented model (linear and non-linear instruments are retained for the corresponding factor).

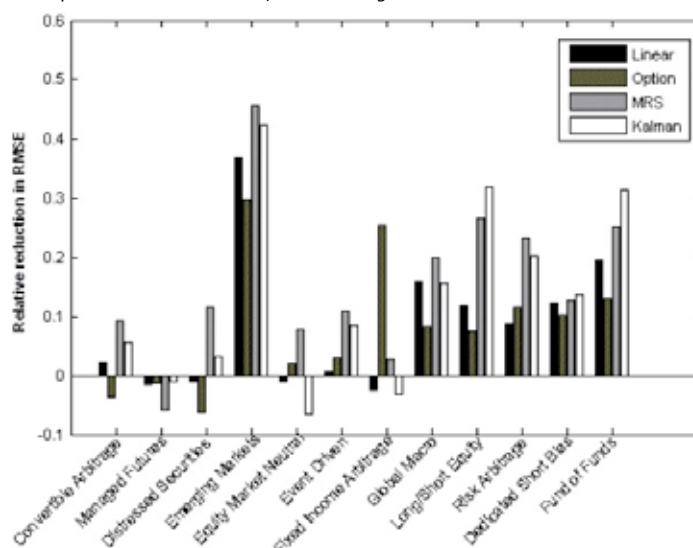
	S&P 500	Bond	Credit spread	Commodity	Dollar	S-L	Emerging Markets	Convertible Bonds	Default Spread	Mortgage Spread
Convertible Arbitrage	X*		X					X		
Managed Futures		X*		X	X					
Distressed Securities			X*							
Emerging Markets	X*						X			
Equity Market Neutral	X*					X				
Event Driven	X*									
Fixed Income Arbitrage			X*					X	X	X
Global Macro		X*								
Long/Short Equity	X*					X				
Risk Arbitrage	X*									
Dedicated Short Bias	X*					X				
Fund of Funds	X*					X				

Exhibit 6. Absolute increase in average adjusted in-sample R^2 when the economic set of factors (see exhibit 1) rather than the factors from Hasanhodzic and Lo (2007) is used.. Monthly out-of-sample returns from January 1999 through December 2006 have been considered.



As mentioned above, the main concern for an investor is the robustness of the estimated models. Because of the customisation of factor sets, one would expect a better out-of-sample fit for the economic factor model than for the full factor model. To check for this, we report in exhibit 7 the relative reductions in annualised RMSE when the sets of economic factors rather than the full factor model are used.

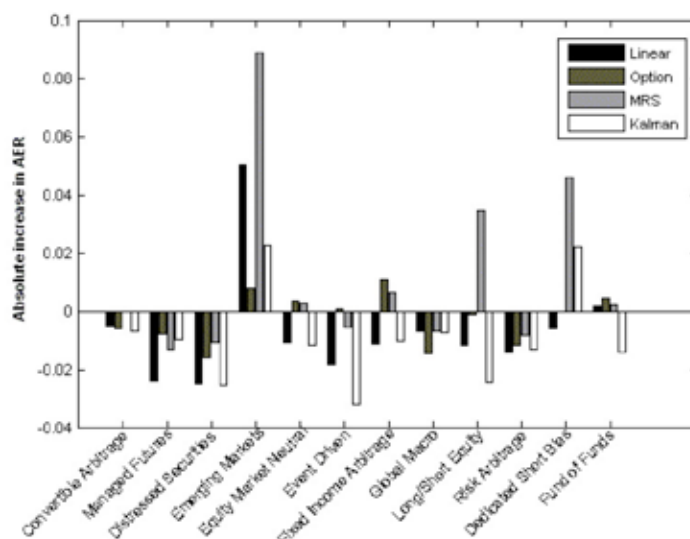
Exhibit 7. Reductions in annualised RMSE when the economic set of factors (see exhibit 2) rather than the factors from Hasanhodzic and Lo (2007) are used. Monthly out-of-sample returns from January 1999 through December 2006 have been considered.



In the vast majority of the cases, out-of-sample replication quality is improved when the economic factor set is used; figures for RMSE are thus reduced. In some cases this reduction is very significant. For the Emerging Market index we obtain a reduction in RMSE of around 40% for all replication models, due to the introduction of an emerging market index in the set of explanatory factors. Interestingly, conditional factor models (MRS and Kalman filter) significantly outperform unconditional models for most strategies in terms of the relative reduction in RMSE. This confirms the intuition that the non-linearity in hedge fund returns may be captured by the use of statistical methods capturing dynamic factor exposures. On the contrary, in terms of relative replication improvements, the option-based clone is again dominated by the linear model.

Overall, these results clearly suggest some potential improvements associated with an *ex-ante* choice of customised factor sets for the various hedge fund strategies. So as to complement this analysis by the performance perspective, exhibit 8 shows the absolute increases in annualised average excess returns for the economic factor model compared to the full factor model.

Exhibit 8. Absolute increases in AER when economic set of factors (see Exhibit 3) is used instead the factors from Hasanhodzic and Lo (2007). Monthly out-of-sample returns from January 1999 through December 2006 have been considered.



These results clearly draw a less optimistic picture of the replication quality of economic factor models. Indeed, with the exception of Emerging Market clones, most other clones perform much less well than the full factor model clones.

4. Conclusion

In this paper, we analyse the out-of-sample properties of methodologies for hedge fund return replication and conclude that none of them generates fully satisfactory results.

In fact, the factor approach to hedge fund replication faces a series of formidable challenges. These challenges include identifying the right factors, as well as replicating in a robust manner the time- and state-dependent exposures of hedge fund managers with respect to these factors. This paper is an attempt to assess the performance of non-linear and conditional factor models in terms of replication ability. We find that going beyond the linear case does not necessarily enhance the replication performance. We also find that selecting factors on the basis of an economic analysis allows substantial improvement in out-of-sample replication quality, whatever the underlying form of the factor model. Overall, we confirm the findings in Hasanhodzic and Lo (2007)—the performance of the replicating strategies is systematically inferior to that of the actual hedge funds.

In conclusion, although the replication of hedge fund factor exposures may seem very attractive from a conceptual standpoint, one has to conclude that it still is very much a work in progress. In the end, the relevant question may not be: "Is it feasible to deliver hedge fund returns with lower risks?" to which the answer is a clear negative, but instead: "Can suitably designed mechanical trading strategies provide a cost-efficient way for investors to get access to alternative beta exposures?" With respect to the second question, there are reasons to believe that such low-cost alternatives to hedge funds might still be useful to investors and managers of funds of hedge funds, either for benchmarking or for risk management.

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Appendices

A. Estimation of the MRS-Model

The Gaussian hypothesis allows us to write the joint likelihood function as:

$$L(\theta) = \prod_{t=1}^T f(R_t | \theta) = \prod_{t=1}^T \left\{ \underbrace{\frac{\tilde{p}_{1,t}}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\varepsilon_t^2}{2\sigma^2}\right)}_{\eta_{1,t}} + \underbrace{\frac{\tilde{p}_{2,t}}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\varepsilon_t^2}{2\sigma^2}\right)}_{\eta_{2,t}} \right\}$$

where θ denotes the parameter vector, that is, factor exposures β_k and the standard deviations σ for both states of nature as well as the smoothed state probabilities \tilde{p} . Defining the filtered probabilities $\hat{p}_{i,t}$ by:

$$\hat{p}_{1,t} = \frac{\eta_{1,t}}{\eta_{1,t} + \eta_{2,t}} \quad \text{and} \quad \hat{p}_{2,t} = \frac{\eta_{2,t}}{\eta_{1,t} + \eta_{2,t}}$$

the smoothed state probabilities are obtained recursively from the filtered probabilities \hat{p} and the transition matrix:

$$\begin{pmatrix} \tilde{p}_{1,t} \\ \tilde{p}_{2,t} \end{pmatrix} = P \cdot \begin{pmatrix} \hat{p}_{1,t-1} \\ \hat{p}_{2,t-1} \end{pmatrix} = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix} \cdot \begin{pmatrix} \hat{p}_{1,t-1} \\ \hat{p}_{2,t-1} \end{pmatrix} \text{ for all } t > 1$$

In the absence of any prior information, we further set $\hat{p}_{1,1} = \hat{p}_{2,1} = 0.5$. The necessary condition $\tilde{p}_{1,t} + \tilde{p}_{2,t} = 1$ is fulfilled at each point in time. Finally, the set of parameters is obtained by maximising the logarithm of the above likelihood function.

B. The Kalman filtering technique

The Kalman filtering technique consists of two steps, the prediction and the updating step.

- Prediction:
$$\begin{cases} \beta_{t|t-1} = A\beta_{t-1|t-1} \\ P_{t|t-1} = AP_{t-1|t-1}A' + H \end{cases}$$

$$\begin{aligned} v_t &= R_t^{HF} - \beta_{t|t-1}' F_t \\ S_t &= F_t' P_{t|t-1} F_t + G \end{aligned}$$

- Updating
$$\begin{aligned} K_t &= P_{t|t-1} F_t' S_t^{-1} \\ \beta_{t|t} &= \beta_{t|t-1} + K_t v_t \\ P_{t|t} &= P_{t|t-1} - K_t S_t K \end{aligned}$$

In the absence of prior information on the dynamics of the vector of factor exposures β , we assume that A is the identity matrix. We further define the initial values $\beta_{1|0}$ and $P_{1|0}$ as the parameters stemming from an unconstrained least-squares estimation over the initial calibration period (01/1997 – 12/1998 in our case):

$$\beta_{1|0} = \hat{\beta}_{OLS} \quad \text{and} \quad P_{1|0} = \hat{\sigma}^2 (R_F' R_F)^{-1}$$