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Hidden Markov Model for Financial Economics

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Introduction

A Hidden Markov model (HMM) is a stochastic signal model which has three assumptions:

1. The observation at time t, O_t, was generated by some process whose state, S_t, is **hidden**.
2. The hidden process satisfies the first-order Markov property: given S_{t-1}, S_t is independent of S_i for any $i < t-1$.
3. The hidden state variable is discrete.

Elements of HMM

1. Observation data, $O = (O_t)$, $t = 1, .., T$
2. Hidden states, $S = (S_i), i = 1, 2, ..., N$
3. Hidden state sequence: $Q = (q_t)$, $t = 1, ..., T$
4. Transition matrix A

$$a_{ij} = P(q_t = S_j | q_{t-1} = S_i), \ i, j = 1, 2, ..., N$$

5. Observation symbols per state, $V = (v_k), k = 1, 2, ..., M$
6. The observation probability

$$B : b_i(k) = P(O_t = v_k | q_t = S_i), i = 1, 2, ..., N; k = 1, 2, ..., M$$

7. Initial probabilities, vector p, of being in state S_i at $t = 1$

$$p_i = P(q_1 = S_i), \ i = 1, 2, ..., N$$

Three problems and corresponding solutions for HMMs

1. Given (O, λ), compute the probability of observations, $P(O|\lambda)$
   Forward, backward algorithm
2. Given (O, λ), simulate the most likely hidden states, Q
   Viterbi algorithm
3. Given O, calibrate HMM parameters, λ
   Baum-Welch algorithm

Forward algorithm

1. Initialization, $\alpha_1(i) = p_i b_i(O_1)$ for $i = 1, ..., N$
2. For $t = 2, 3, ..., T$, for $j = 1, ..., N$

$$\alpha_t(j) = \left[\sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} \right] b_j(O_t),$$

3. $P(O|\lambda) = \sum_{i=1}^{N} \alpha_T(i)$

Hidden Markov Model

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Some Applications of HMMs

Figure 1: 1. Speech recognition 2. Bioinformatics 3. Finance

Forecast economics regimes

Economics indicators

1. Credit Index
2. Yield Curve
3. Commodity
4. Dow Jones Industrial Average

DJBear1-crop.pdf	NdataBear-crop.pdf

Figure 2: HMMs for Bear Market Predictions

Training and Predicting Process

1. Use HMM for single and multiple observation data with normal distributions.
2. Calibrate Markov-switching model parameters using Baum-Welch algorithm
3. Use the obtained parameters to predict stock prices for the next trading period.

HMMs for Stock Price Predictions

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Figure 3: Forecast $S\&P500$ close prices using single observation

HMMs for Stock Tradings

Symbol	Initial Investment ($)	Earning ($)	Earning %
SPY	9,000.00	2050.66	22.79
GOOG	30,000.00	29,036.4	96.79
FORD	250.00	10.10	4.04
AAPL	950.00	19.06	2.01
GE	1,700.00	490.00	28.82
TOTAL	41,900.00	31,606.22	75.43

Table 1: One year daily stock trading portfolio from December 2012 to December 2013

References

[1] Zoubin Ghahramani. An Introduction to Hidden Markov Models and Bayesian Networks. International Journal of Pattern Recognition and Artifical Interlligence, 15(1): 9-42, 2001.

[2] Lawrence Rabiner. A Tutorial on Hidden Markov Models and Seclected Applications in Speech Recognition, IEEE Vol 77-2, 1989.

[3] Przemyslaw Dymarski. Hidden Markov Models, Theory and Applications, InTech 2011.

[4] Xiaolin Li, Training Hidden Markov Models with Multiple Observations âĂŞ A Combinatorial Method. IEEE Transactions on PAMI, vol. 22(4): 371-377, 2000.

[5] Mark Kritzman, SÃlbastien Page, and David Turkington. Regime Shifts: Implications for Dynamic Strategies. Financial Analysts Journal, Vol.68(3), 2012 CFA Institute.