PDE methods for option pricing

Jian Wang

FSU ID: jw09r

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Problem

[Summary]

Data: this text
Result: how to write algorithm with LaTeX2e
initialization;
while *not at end of this document* **do**
    read current;
    if *understand* **then**
        go to next section;
        current section becomes this one;
    else
        go back to the beginning of current section;
    end
end

Algorithm 1: How to write algorithms

[Statement of the problem]

1.Compute the values of the European vanilla put for $E^* = \$10$, $r^* = 0.05/yr$, $\sigma^* = 0.20/yr$ and with a six month expiry with and without Rannacher smoothing. Report the error as a function of ΔS and $\Delta\tau$. Compute the greeks, Δ and Γ and their errors. Propose and implement a technique to compute the $v = \partial V/\partial\sigma$ and report on its performance. Test the effects of the outer boundary on the solution in the range $[0, K]$.
2. Redo the previous task for the European binary put. In particular, examine the solution for large $\Delta\tau$ and no smoothing

[Mathematics Tools]

The BSM and boundary condition for European put option is:

$$\begin{cases} V_t + \frac{\sigma^2 S^2}{2} V_{SS} + rSV_s - rV = 0 \\ V(0,t) = Ke^{-r(T-t)} \\ V(\infty, t) = 0 \\ V(S,T) = max(K-S, 0) \end{cases}$$

(1)We use backward Euler for a few n¿=2 time steps
(2)Use Crank-Nicolson after that: given second order accuracy

combine : $\delta_t^+ V_j^n + L_h[\alpha V_j^{n+1} + (1-\alpha)V_j^n] = 0$
$\alpha = 1$: Back Euler
$\alpha = \frac{1}{2}$: Crank-Nicolson method

Crank-Nicolson method:

$$\frac{f_{ij} - f_{i,j-1}}{\Delta t} + \frac{ri\Delta S}{2} + (\frac{f_{i+1,j-1} - f_{i-1,j-1}}{2\Delta S} + \frac{ri\Delta S}{2}(\frac{f_{i+1,j} - f_{i-1,j}}{2\Delta S}) +$$
$$\frac{\sigma^2 i^2 (\Delta S)^2}{4}(\frac{f_{i+1,j-1} - 2f_{i,j-1} + f_{i-1,j-1}}{(\Delta S)^2}) +$$
$$\frac{\sigma^2 i^2 (\Delta S)^2}{4}(\frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{(\Delta S)^2}) = \frac{r}{2} f_{i,j-1} + \frac{r}{2} f_{ij}$$

We can rewrite the above equation as:

$$-\alpha_i f_{i-1,j-1} + (1-\beta_i) f_{i,j-1} - \gamma_i f_{i+1,j-1} = \alpha_i f_{i-1,j} + (1+\beta_i) f_{i,j} + \gamma_i f_{i+1,j}$$

Where:

$$\alpha_i = \frac{\Delta t}{4}(\sigma^2 i^2 - ri)$$
$$\beta_i = -\frac{\Delta t}{2}(\sigma^2 i^2 + r)$$
$$\gamma_i = \frac{\Delta t}{4}(\sigma^2 i^2 + ri)$$