

# High frequency data trading

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# Table of contents

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# Contents

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## High frequency trading

High frequency trading is a specialized case of algorithmic trading involving the frequent turnover of many positions of a security.

### Positive impact

- Increased liquidity
- Narrowing spreads
- Improve market efficiency
- Increase fees for Exchanges

### Negative impact

- Impact on the institutional investors.
- Increase volatility
- Disadvantages to the small Investors

## HFT Strategies:

### Market Making

place bets on both sides of the trade by placing a limit order to sell slightly above the current market price, or to buy slightly below the current market price, thereby profiting from the difference between the two.

### Statistical Arbitrage

Firms and traders looking to make profits from market arbitrage essentially exploit the momentary inconsistencies in factors such as rates, prices, and other conditions between different exchanges or asset classes

### Liquidity Rebate Trading

look for large orders, fill a part of that order, and then offer these shares back to the market by placing a limit order, which makes them eligible to collect the rebate fee for providing liquidity, with or without them making a capital gain.

### Momentum Ignition

ignition strategies involve initiating and canceling a number of trades and orders with a certain security in a particular direction, which may ignite a rapid market price movement.

# Contents

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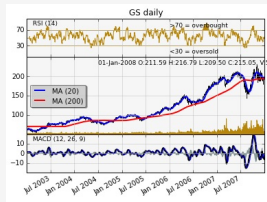
# Dataset

The dataset is named stockdata which from huge package in R. It contained data that were originally obtained from Yahoo! Finance. There are 1,258 observations representing 1,258 sequential trading days(form Jan 1 2003 to Jan 1 2008) and 452 variables, each of which was the day's closing price for a different stock within the Standard & Poor's 500. We also added two index into the data set, one is S&P 500 and another is Nasdaq(so totally 454 variables). Among all the stock data , we used Goldman Sachs stock return series as our response variable and other stocks as the predictors to analyze the stock return series movement.

**Company::**



**Price::**



# Contents

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# Methodology

## Logistic regression

$$\ln \frac{F(x)}{1-F(x)} = \beta_0 + \sum_i \beta_i x_i$$

## Ridge regression

$$\hat{\beta}^{ridge} = \operatorname{argmin}_{\beta} \left\{ \sum_{i=1}^p (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^p \beta_j^2 \right\}$$

## Lasso regression

$$\hat{\beta}^{lasso} = \operatorname{argmin}_{\beta} \left\{ \sum_{i=1}^p (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^p |\beta_j| \right\}$$

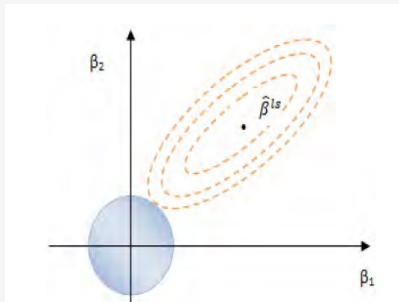
# Methodology

## Comparison of L1 and L2 Penalized Model

### Ridge regression

$$\hat{\beta}^{ridge} = \operatorname{argmin}_{\beta} \left\{ \sum_{i=1}^p (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^p \beta_j^2 \right\}$$

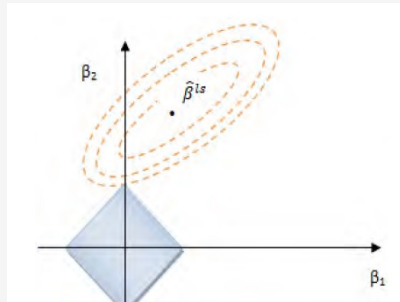
**Coefficients:**



### Lasso regression

$$\hat{\beta}^{lasso} = \operatorname{argmin}_{\beta} \left\{ \sum_{i=1}^p (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^p |\beta_j| \right\}$$

**Coefficients:**



# Methodology

## Comparison of L1 and L2 Penalized Model

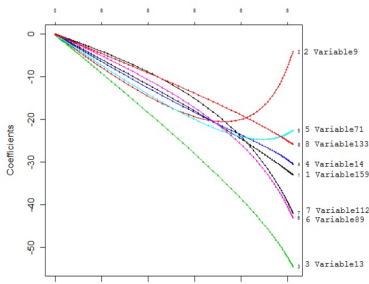
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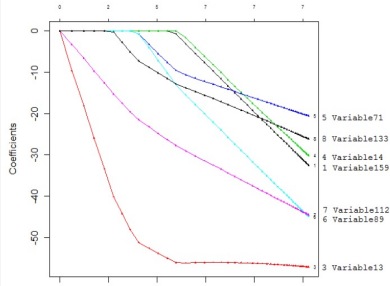
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Path::



Path::



# Methodology

## Support vector machine

- Introduced in COLT-92 by Boser, Guyon & Vapnik. Became rather popular since.
- Theoretically well motivated algorithm: developed from Statistical Learning Theory (Vapnik & Chervonenkis) since the 60s.
- Empirically good performance: successful applications in many fields (bioinformatics, text, image recognition, . . . )

Try to maximize the margin:

$$r = 1/\|w\|, y_j = 1, -1$$

Primal form:

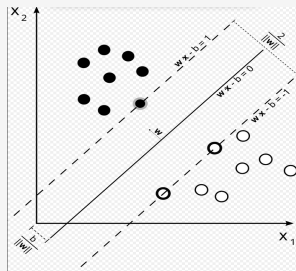
$$\max_{W, b} r = 1/\|W\|$$

$$s.t. (W^T x_j + b) y_j \geq 1$$

Dual form:

$$\max_{\alpha_1, \dots, \alpha_M} \sum \alpha_l - \frac{1}{2} \sum_{j=1}^M \sum_{k=1}^M \alpha_j \alpha_k y_j y_k \langle X_j, X_k \rangle$$

$$s.t. \alpha_l \geq 0, \sum_{l=1}^M \alpha_l y_l = 0$$



# Methodology

## Kernel functions

We can use the kernel function to calculate the inner product in high dimensional cases in its original feature spaces.

## Example: two dimension polynomial

$$\begin{aligned} k(x, z) &= (x^T z)^2 \\ &= (x_1^2, \sqrt{2}x_1x_2, x_2^2)^T (z_1^2, \sqrt{2}z_1z_2, z_2^2) \\ &= \Phi(x)^T \Phi(z) \end{aligned}$$

## Kernel functions that we used

- Linear kernel:  $k(x, y) = x^T y + c$
- Polynomial Kernel:  $k(x, y) = (\alpha x^T y + c)^d$
- Radial basis function kernel (RBF):  $k(x, y) = \exp(-\gamma \|x - y\|^2)$

# Methodology

## Glasso:

Suppose we have  $N$  multivariate normal observations of dimension  $p$ , with mean  $\mu$  and covariance  $\Sigma$ . Let  $\Theta = \Sigma^{-1}$  and  $S$  be the empirical covariance matrix, the problem is to maximize the log-likelihood

$$\ln P(X|u, \Sigma) = -\frac{N}{2} \ln |\Sigma| - \frac{1}{2} \sum (x_n - u)^T \Sigma^{-1} (x_n - u)$$

combined with the  $L_1$  penalty

$$\ln |\Theta| - \text{tr}(S\Theta) - \lambda \|\Theta\|_1$$

## Algorithm

Many algorithms for this problem, The following might be the oldest and simple one by Meinshausen and Bühlmann(2006)

- Estimate a sparse graphical model by fitting a lasso model to each variable, using others as predictors
- Set  $\Sigma_{ii}^{-1}$  to be non zero, if either the estimated coefficient of variable  $i$

# Methodology

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# Bayesian-Glasso model

For the high dimensional problem, it is not very easy to built the Bayesian network due to its exponentially increasing complexity.

Our idea is to first use the Glasso model to conduct the model selection and then use Bayesian network structure learning process to define the network structure.

## Algorithm

- Use Glasso algorithm to find the edges among variables
- Use greedy search methods to change the direction only on those existed edges
- Choose the direction which has the optimal BIC score
- Finish when all the edges are reached or attain the maximum iteration numbers



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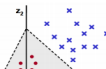
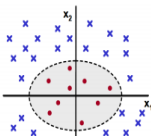
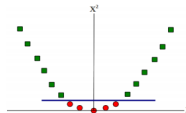
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# Methodology

## Higher dimensional situations

sometimes, in lower dimension we can not separate the data properly, so we need to project the data to the high dimensions

## Examples



# Methodology

## Reference

- Tibshirani, R. (1996). "Regression shrinkage and selection via the lasso". Journal of the Royal Statistical Society, Series B 58 (1): 267–288. JSTOR 2346178
- Hoerl, A.E. and Kennard, R. (1970). Ridge regression: Biased estimation for nonorthogonal problems. Technometrics, 12: 55-67
- Vapnik, V. (1995). "Support-vector networks". Machine Learning 20 (3): 273. doi:10.1007/BF00994018

## Packages

- R packages: glm, glmnet, e1071, bnlearn, huge
- Python packages: sklearn (svm, ridge, lasso, logistic)

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# Contents

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## Numerical results

- Same day stock return series analysis:  
we use the same day stock return series to build the machine learning models. For the Bayesian network, we only use the R package. For the logistic regression, ridge regression, lasso and svm, we used different languages(R and Python) and also compare the CPU time. To test the accuracy rate of model, we choose the first 1000 data as training data and the last remaining 257 data as testing. GS as response(discretized as 1 and -1) and the other 453 stocks as predictors.
- Predict the stock data:  
we used the last one day, two day,... to last five day stock returns as the predictors and today's GS return series as response to see if our model can be used to predict the stockdata.  
Still use the first 1000 data as training and the remaining 252 data as testing. GS as the response and the other 2270 variables as predictors.

# Numerical results

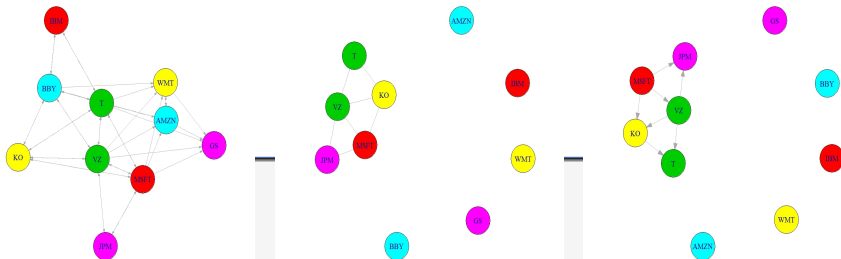
## Bayesian network

Table : 10 companies

Stock code	Industry	Company name
GS	Financials	Goldman Sachs Group
JPM	Financials	JPMorgan Chase & Co.
MSFT	Information Technology	Microsoft Corp.
IBM	Information Technology	International Bus. Machines
T	Telecommunications Services	AT&T Inc
VZ	Telecommunications Services	Verizon Communications
WMT	Consumer Staples	Wal-Mart Stores
KO	Consumer Staples	Coca Cola Co.
AMZN	Consumer Discretionary	Amazon.com Inc
BBY	Consumer Discretionary	Best Buy Co. Inc.

## Numerical results

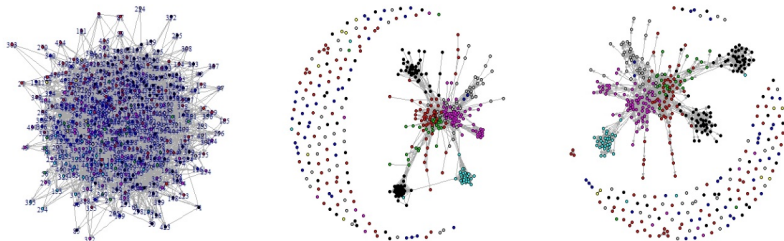
## Results for simple cases





# Numerical results

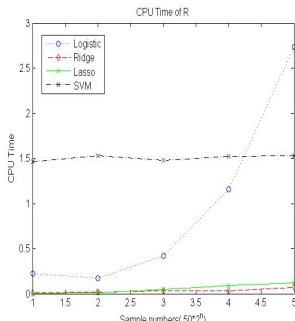
## Results for total stocks



# Numerical results

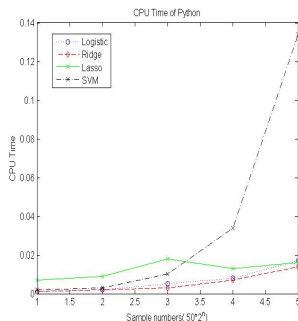
## CPU Time for R

We changed the number of samples from 50 to 800, doubled each time to test the running time for the different machine learning methods:



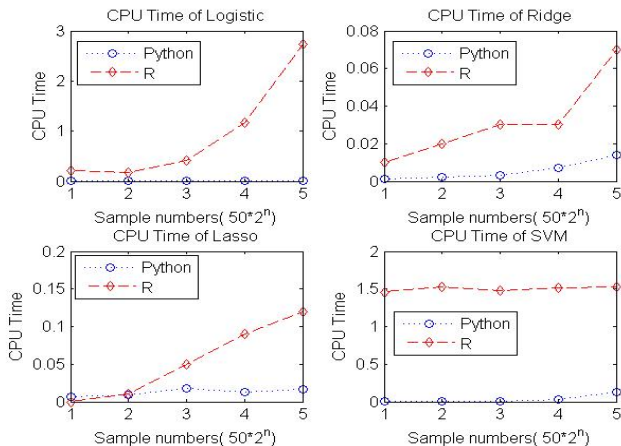
## CPU Time for Python

We changed the number of samples from 50 to 800, doubled each time to test the running time for the different machine learning methods:



# Numerical results

## Comparison of CPU time for python and R



## Numerical results:

### Accuracy rate:

Table : Accuracy rate

Methods	Python	R
Logistic	69.8%	68.4%
Ridge( $\lambda=1$ )	73.9%	77.4%
Lasso( $\lambda=0.01$ )	78.6%	79.0%
svm(linear)	72.4%	71.8%
svm(poly)	74.3%	71.2%
svm(rbf)	75.1%	72.8%

### Results:

- lasso ( $\lambda=0.01$ ) and svm (rbf) performed good for both two languages.
- Python performed better in most cases but the difference is not

# Numerical results:

## Predicted

$$R_t^{GS} = \sum_{i=1:5} \sum_{j=1:454} \beta_{i,j} R_{t-i}^j \quad (1)$$

## Predict:

Table : Accuracy rate and CPU time

Methods	Accuracy rate	CPU time
Logistic	51.2%	0.1210
Ridge( $\lambda=1$ )	54.0%	0.1230
Lasso( $\lambda=0.01$ )	49.2%	0.0940
svm(linear)	52.8%	1.1931
svm(poly)	45.6%	1.2800
svm(rbf)	47.2%	1.2921
Bayesian Glasso	51.6%	around two hours

# Contents

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## Future work

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- Compare with the time series model, such as Garch(machine learning methods can consider the whole economic environment while time series cannot)
- Deal with the high frequency data instead of daily data

# Contents

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Thanks a lot and Questions

# Contents

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# Contents

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- Introduction to  $\text{\LaTeX}$
- Course 2
- Termpapers and presentations with  $\text{\LaTeX}$
- Beamer class



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- Introduction to  $\text{\LaTeX}$

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- 1 Introduction to  $\text{\LaTeX}$
- 2 Course 2
- 3 Termpapers and presentations with  $\text{\LaTeX}$
- 4 Beamer class

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## 1 Introduction to $\text{\LaTeX}$

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1 Introduction to  $\text{\LaTeX}$

2 Course 2

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- 2 Course 2
- 3 Termpapers and presentations with  $\text{\LaTeX}$
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# Contents

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# Tables

Date	Instructor	Title
WS 04/05	Sascha Frank	First steps with $\text{\LaTeX}$
SS 05	Sascha Frank	$\text{\LaTeX}$ Course serial

## Tables with pause

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