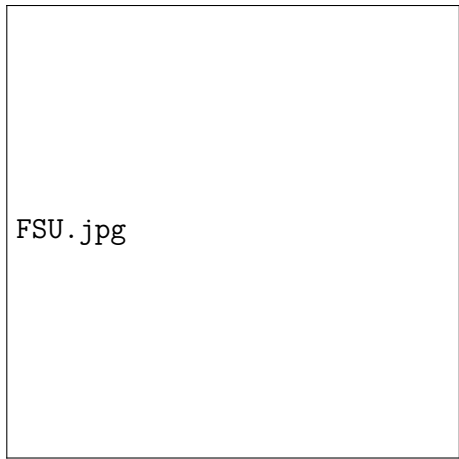




# Hidden Markov Model for Financial Economics

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## Introduction

A Hidden Markov model (HMM) is a stochastic signal model which has three assumptions:

- 1 The observation at time  $t$ ,  $O_t$ , was generated by some process whose state,  $S_t$ , is **hidden**.
- 2 The hidden process satisfies the first-order Markov property: given  $S_{t-1}$ ,  $S_t$  is independent of  $S_i$  for any  $i < t - 1$ .
- 3 The hidden state variable is discrete.

## Elements of HMM

- 1 Observation data,  $O = (O_t), t = 1, \dots, T$
- 2 Hidden states,  $S = (S_i), i = 1, 2, \dots, N$
- 3 Hidden state sequence:  $Q = (q_t), t = 1, \dots, T$
- 4 Transition matrix  $A$

$$a_{ij} = P(q_t = S_j | q_{t-1} = S_i), \quad i, j = 1, 2, \dots, N$$

- 5 Observation symbols per state,  $V = (v_k), k = 1, 2, \dots, M$
- 6 The observation probability

$$B : b_i(k) = P(O_t = v_k | q_t = S_i), \quad i = 1, 2, \dots, N; \quad k = 1, 2, \dots, M$$

- 7 Initial probabilities, vector  $p$ , of being in state  $S_i$  at  $t = 1$

$$p_i = P(q_1 = S_i), \quad i = 1, 2, \dots, N$$

## Three problems and corresponding solutions for HMMs

- 1 Given  $(O, \lambda)$ , compute the probability of observations,  $P(O|\lambda)$   
**Forward, backward algorithm**
- 2 Given  $(O, \lambda)$ , simulate the most likely hidden states,  $Q$   
**Viterbi algorithm**
- 3 Given  $O$ , calibrate HMM parameters,  $\lambda$   
**Baum-Welch algorithm**

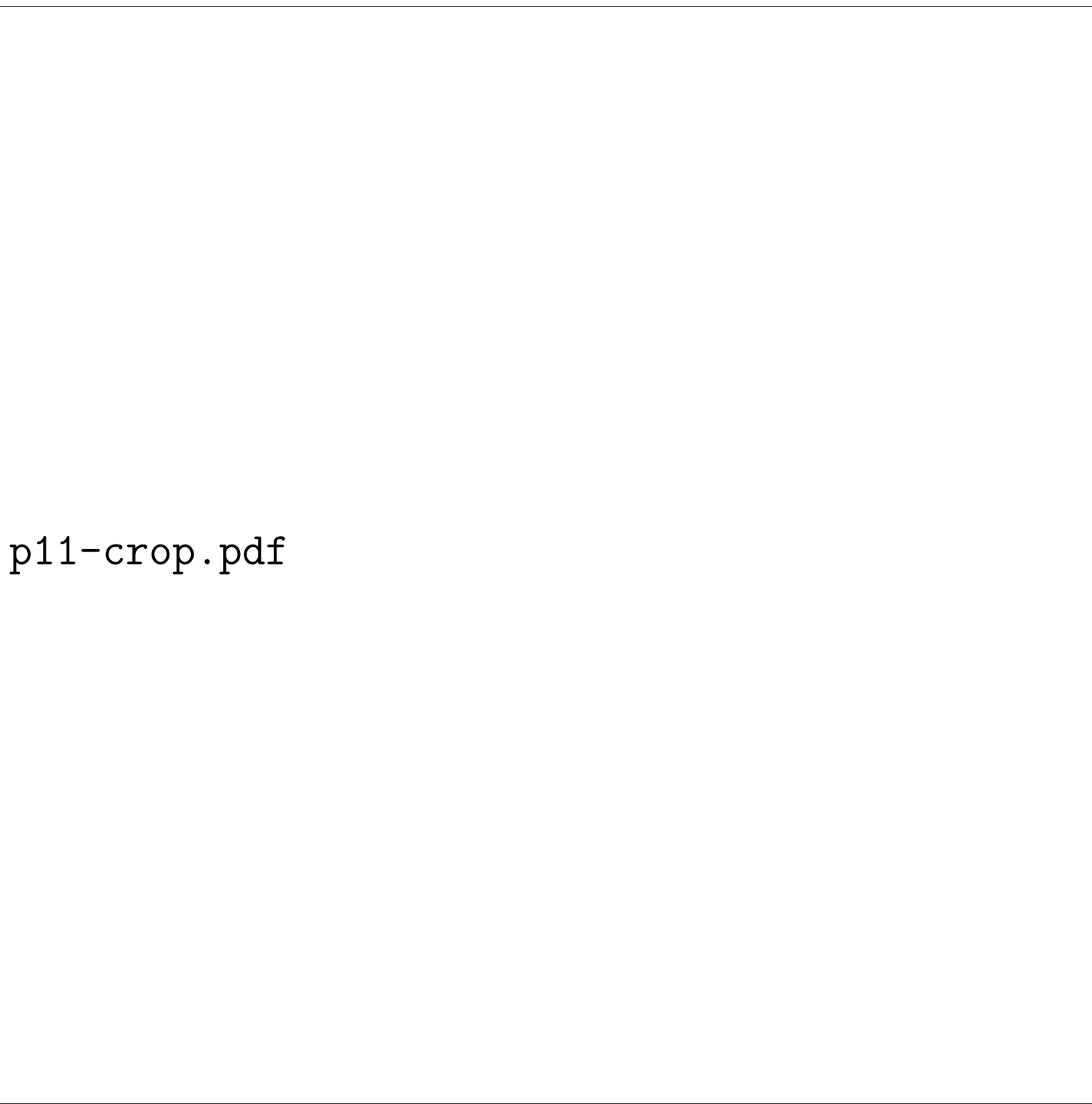
## Forward algorithm

- 1 Initialization,  $\alpha_1(i) = p_i b_i(O_1)$  for  $i = 1, \dots, N$
- 2 For  $t = 2, 3, \dots, T$ , for  $j = 1, \dots, N$

$$\alpha_t(j) = \left[ \sum_{i=1}^N \alpha_{t-1}(i) a_{ij} \right] b_j(O_t),$$

- 3  $P(O|\lambda) = \sum_{i=1}^N \alpha_T(i)$

## Hidden Markov Model



## Some Applications of HMMs

Figure 1: 1. Speech recognition 2. Bioinformatics 3. Finance

## Forecast economics regimes

Economics indicators

- 1 Credit Index
- 2 Yield Curve
- 3 Commodity
- 4 Dow Jones Industrial Average



Figure 2: HMMs for Bear Market Predictions

## Training and Predicting Process

- 1 Use HMM for single and multiple observation data with normal distributions.
- 2 Calibrate Markov-switching model parameters using Baum-Welch algorithm
- 3 Use the obtained parameters to predict stock prices for the next trading period.

## HMMs for Stock Price Predictions

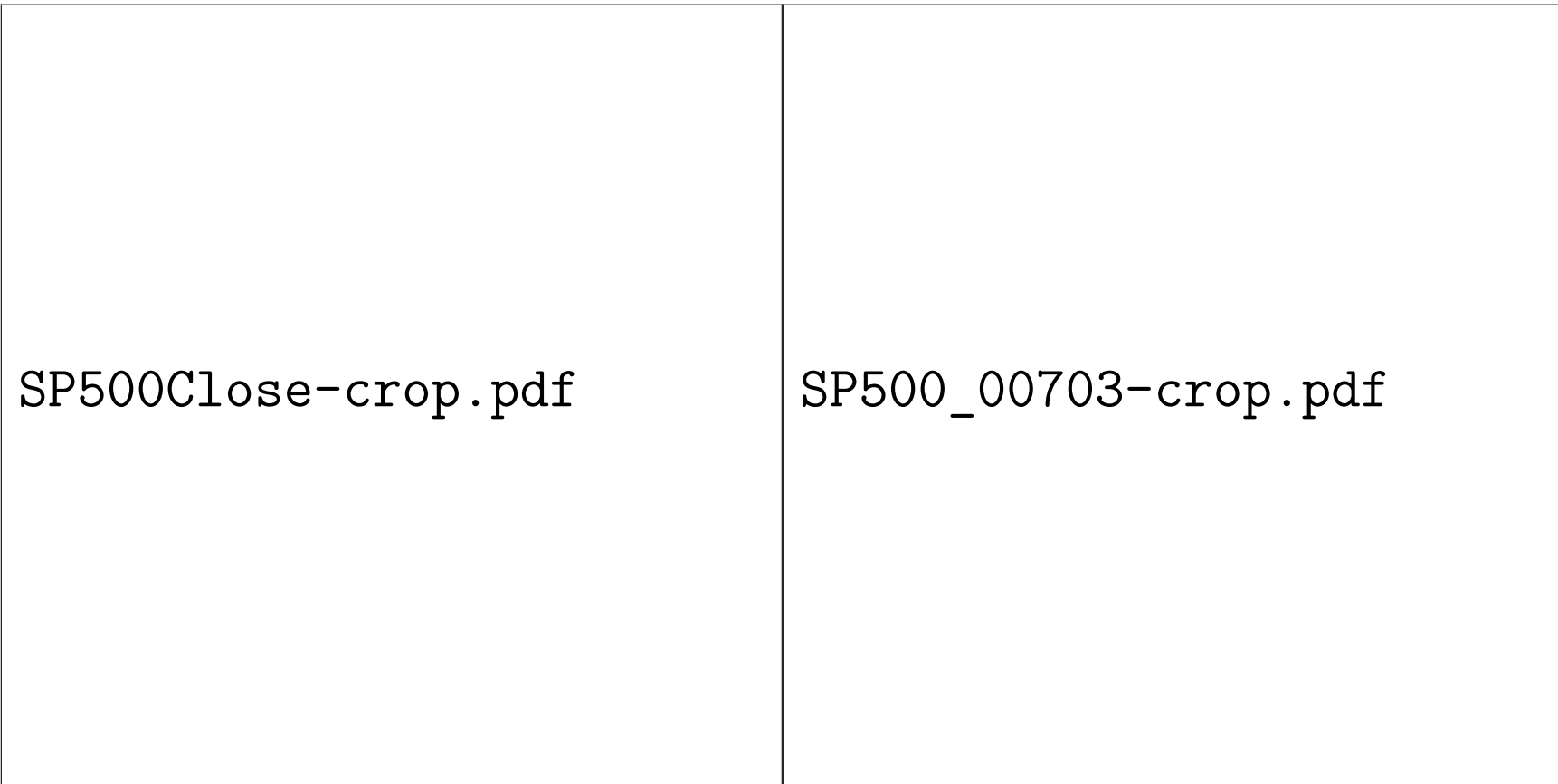


Figure 3: Forecast  $S\&P500$  close prices using single observation

## HMMs for Stock Tradings

Symbol	Initial Investment (\$)	Earning (\$)	Earning %
SPY	9,000.00	2050.66	22.79
GOOG	30,000.00	29,036.4	96.79
FORD	250.00	10.10	4.04
AAPL	950.00	19.06	2.01
GE	1,700.00	490.00	28.82
TOTAL	41,900.00	31,606.22	75.43

Table 1: One year daily stock trading portfolio from December 2012 to December 2013

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