Job interview quesions

practice

Dr. Jian Wang

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Part One

1. Wells fargo

1.1 Programming

1) Given two integers n and k, return all possible combinations of k numbers out of 1, ... ,n.

For example,
If n = 4 and k = 2, a solution is:

[
[2,4],
[3,4],
[2,3],
[1,2],
[1,3],
[1,4],
]

source: leet code 77
answer:
Can use recursive method, after join i, next time start from i+1 to n, till selected number equal to k, add to result.
source code can be found in leet code page 147.

2) Compute and return the square root of x.

source: note that only need to find the nearest integer,leet code 69,page 193

answer: hint: can use bisection and newton method to find the iterative solution, for the newton method, use $f(t) = t^2 - x$

3) find all the arbitrage based on the following currency rate list

For example:

[[dollar:yuan,6], [dollar:yen,30],[yuan:yen,5],[yen:dollar,1/50] [yen:won,1/2]] the arbitrage can be dollar to yuan , yuan to yen and yen to dollar:

source:

answer: hint:Use graph and if a product of a loop is not equal to 1, exits arbitrage

1.2 Stochastic differential equation

1) calculate : $P(w_{t+1} < 0 | w_t > 0)$

source:

answer:

See green book page 131. 2)what is ito formula:

source:

answer:

3) how to change the $\frac{X_t}{Y_t}$ to a martingale measure, assume that X_t and Y_t follows different ito process

source:

answer: hint: can use second order differential equation and let the dt term equal to 0

4)option pricing: calculate the second derivative according to K(strike price) in the equation $E(e^{-rt}(payoff))$

source:

answer:

5) A stochastic process, $\{W_t : 0 <= t <= \infty\}$, is a standard Brownian motion if:
1. $W_0 = 0$
2. It has continuous sample paths
3. It has independent,normally -distributed increments.

6) A stochastic process, $\{X_t : 0 <= t <= \infty\}$, is a martingale with respect to the filtration, f_t, and probability measure, P, if:
1) $E^P[|X_t|] < \infty$ for all $t \geq 0$
2) $E^p[X_{t+s}|F_t] = X_t$ for all $t, s >= 0$

7)Quadratic Variation Consider a partition of the time interval,[0,T] given by:

$$0 = t_0 < t_1 < t_2 < ... < t_n = T$$

Let X_t be a Brownian motion and consider the sum of squared changes:

$$Q_n(T) := \sum_{i=1}^{n} [\Delta X_{t_i}]^2$$

where $\Delta X_{t_i} = X_{t_i} - X_{t_{i-1}}$
The quadratic variation of a stochastic process X_t, is equal to the limit of $Q_n(T)$ as $\Delta_t := max(t_i - t_{i-1}) \to 0$

1.3 Numerical PDE

1)Use fourier transformation to find if a numerical differential equation scheme is converge

source:

answer:

2)How to find a numerical scheme to a stochastic differential equation:

source:

answer: for the stochastic term dW_t, can use the monte carlo method to to find $dW_t = (W_{t_{i+1}} - W_{t_i})/(t_{i+1} - t_i)$?

3)Given an integer, write a function to determine if it is a power of two.

source:leet code 231

answer:hint: can use the bit operation for n and (n-1), in python: n&(n-1)

4)Bisection convergence rate, newton method convergence rate? how to get it?

source:

answer:

newton method is quadratic and bisection is linear since $\varepsilon_n / \varepsilon_{n-1} = 1/2$

1.4 Partial differential equation

1)Separate variables method: Solve $u_{tt} = u_t - u_x$

source:

answer:hint:use separate variables.

2)For the composite function $f(g(x))$, $g(x)$ is one PDE equation solution, and also g(x) is the optimal solution for f(g(x)), find $g(x)$

source:

answer:not very clear question, need further discussion.

3)Some first order partial differential equation:

$u_t + u_x = 0$

source:

answer:hint:prepare all kinds of first order differential equation, homegeneous and non homegeneous

4)Wave equation:

$u_{tt} = au_{xx}, x \in [0,1], u(x,0) = 1$

what is solution for a>0 and a<0?
source:

answer:

1.5 Linear algebra

1) Calculate minimum polynomials of matrices

source:

answer:

2) Given a matrix A, find the A^{100}

source:

answer:hint:can use Jordan canonical form to find $A = UBU^{-1}$ where B is a diagonal matrix.

3) How the verify if a matrix is positive definite or not?

source:

answer:

a)A matrix is positive definite if it's symmetric and all its eigenvalues are positive
b)A matrix is positive definite if it's symmetric and all its pivots are positive.
c)A matrix is positive definite if $x^T Ax > O$ for all vectors x 0.

d)Hermitian matrix,leading principal submatrix is positive definite
e)Hermitian matrix,$A = A^*A$ and A is convertible.

4)Calculate eigen values and eigen vectors of a matrix, how to make a matrix become upper triangular or lower triangular.

source:

answer: Prepare basic materials of linear algebra.

1.6 Monte Carlo method

1) How to generate random numbers of a distribution, given that we can generate a uniform random numbers.

source:

answer: Prepare monte carlo course materials. Inverse function method and accept rejection method, see green book page 184.

2)What is the convergent rate of monte carlo method:

source:

answer:for monte carlo method, o($\frac{1}{\sqrt{n}}$), hint: central limit theorem

3)Variance reduction method:

source:

answer:hint:control variate, importance sampling,...,prepare materials

1.7 Statistics

1)Linear regression, how to define if a regression model is linear and why choose linear regression model.

source:

answer:

2) x is independent variable and y is dependent variable, $\hat{y}$ is the linear regression model of x, find the correlation of y and $\hat{y}$

source:

answer:

the correlation of y and $\hat{y}$ is same to the correlation of x and y, proof can be derived using definition.

3)X and Y are two iid exponential distribution, find the density of X+Y

source:

answer:

using the convolution method , $f(x+y) = \int_{-\infty}^{+infty} f_{X,Y}(x, z-x)dx$, the answer for the problem is $\frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \exp^{-\lambda_2 Z}$

4)n bulbs, each is a iid exponential distribution,light up all the bulbs, x_1 is the time when the first bulb become dark, x_2 is the time when the second bulb become dark,...,x_m is the time when the mth bulb become dark. find the maximum log likelihood of λ based on the above observation.

source:

answer:

Note, should use the pdf not the cdf to deal with this problem, since x_i is the exact time the bulb dark. The answer is the mean of x_i. 5)X, Y are two different normal distribution, find the variance of $X\dot{Y}$

source:

answer:hint: calculate from the definition.

6)1,2,3x,x^2,5x, the mean of the above numbers is 6, find the mode of them

source:

answer:hint: solve the one variable equation.

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