南京大学
NANJING UNIVERSITY

计算机视觉表征与识别
Chapter 9: Alignment and Transformation

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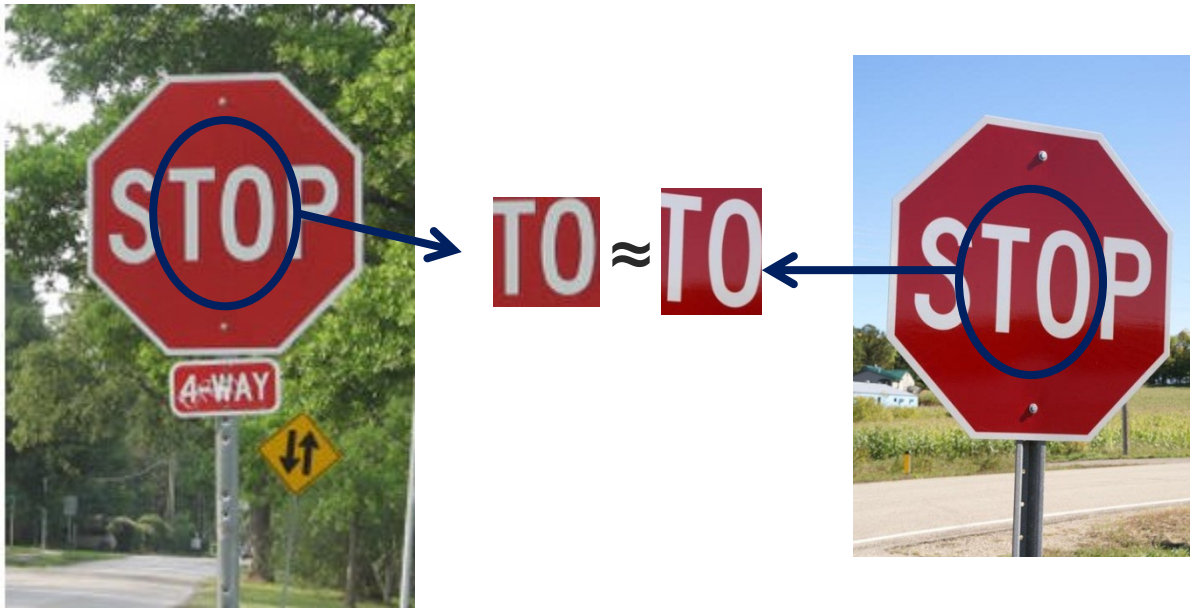
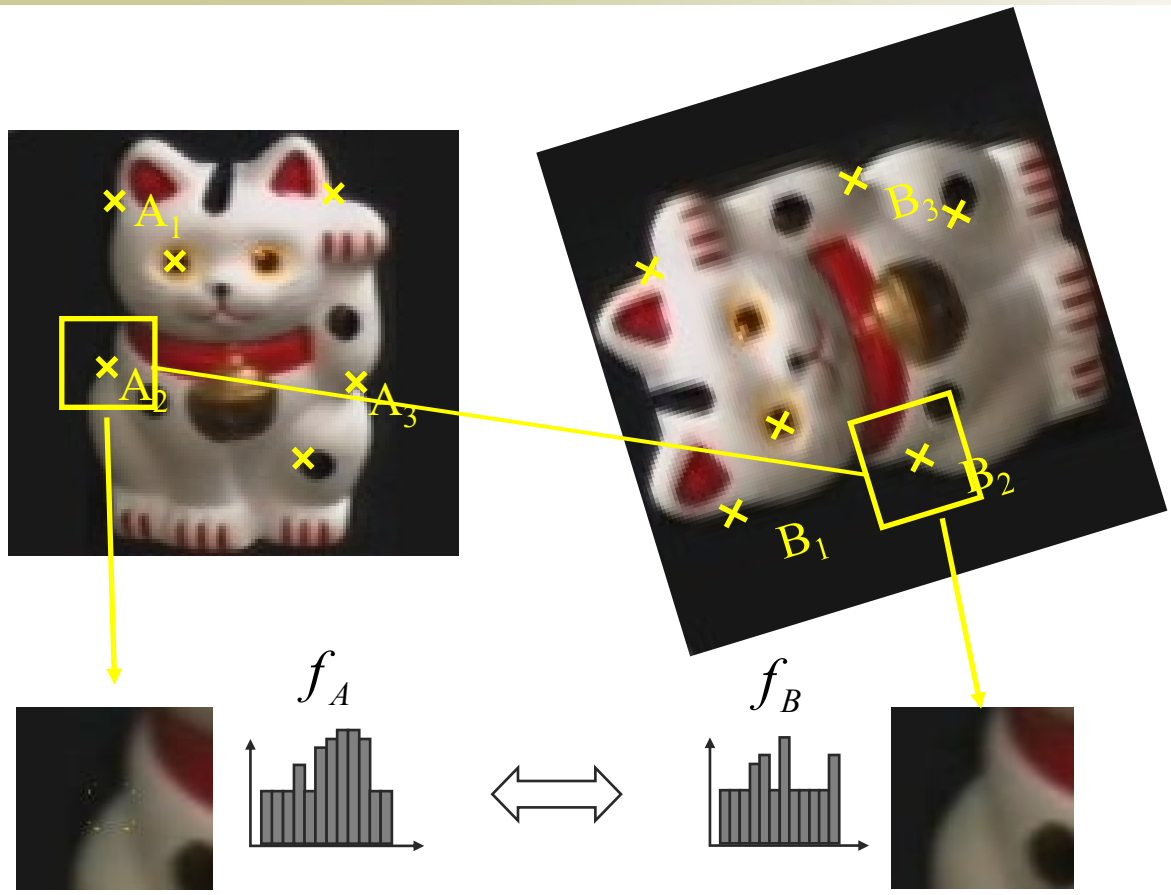
Correspondence and alignment

Correspondence: matching points, patches, edges, or regions across images



Recap: Keypoint Matching



1. Find a set of distinctive key-points

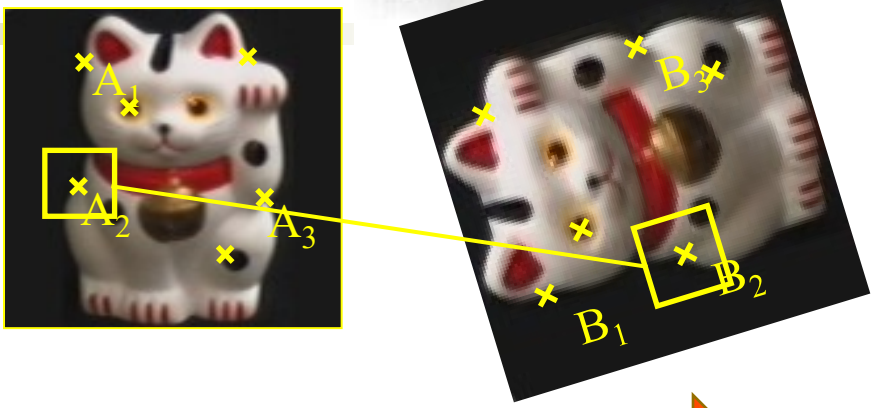
2. Define a region around each keypoint

3. Extract and normalize the region content

4. Compute a local descriptor from the normalized region

5. Match local descriptors

Recap: Key trade-offs



Detection

More Repeatable

Robust detection
Precise localization

More Points

Robust to occlusion
Works with less texture

Description

More Distinctive

Minimize wrong matches

More Flexible

Robust to expected variations
Maximize correct matches

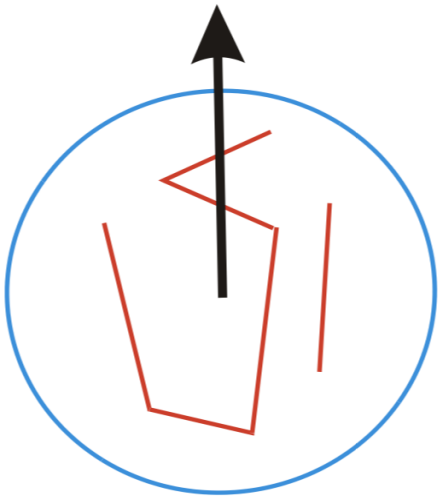
Summary: Scale Invariant Detection

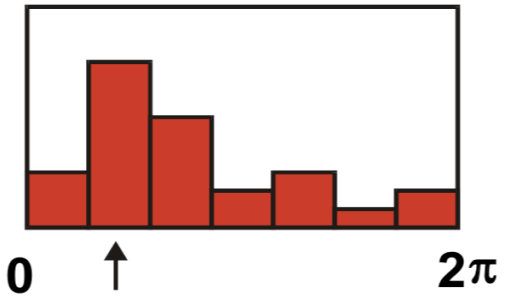
- **Given:** Two images of the same scene with a large *scale difference* between them.
- **Goal:** Find *the same* interest points *independently* in each image.
- **Solution:** Search for *maxima* of suitable functions in *scale* and in *space* (over the image).

- Two strategies
  - Laplacian-of-Gaussian (LoG)
  - Difference-of-Gaussian (DoG) as a fast approximation
  - *These can be used either on their own, or in combinations with single-scale keypoint detectors (Harris, Hessian).*

Orientation Normalization

- **Compute orientation histogram** [Lowe, SIFT, 1999]
- **Select dominant orientation**
- **Normalize: rotate to fixed orientation**





Slide adapted from David Lowe

Iterative Adaption

1. Detect keypoints, e.g. multi-scale Harris
2. Automatically select the scales
3. Adapt affine shape based on second order moment matrix
4. Refine point location

K. Mikolajczyk and C. Schmid, Scale and affine invariant interest point detectors, IJCV 60(1):63-86, 2004.

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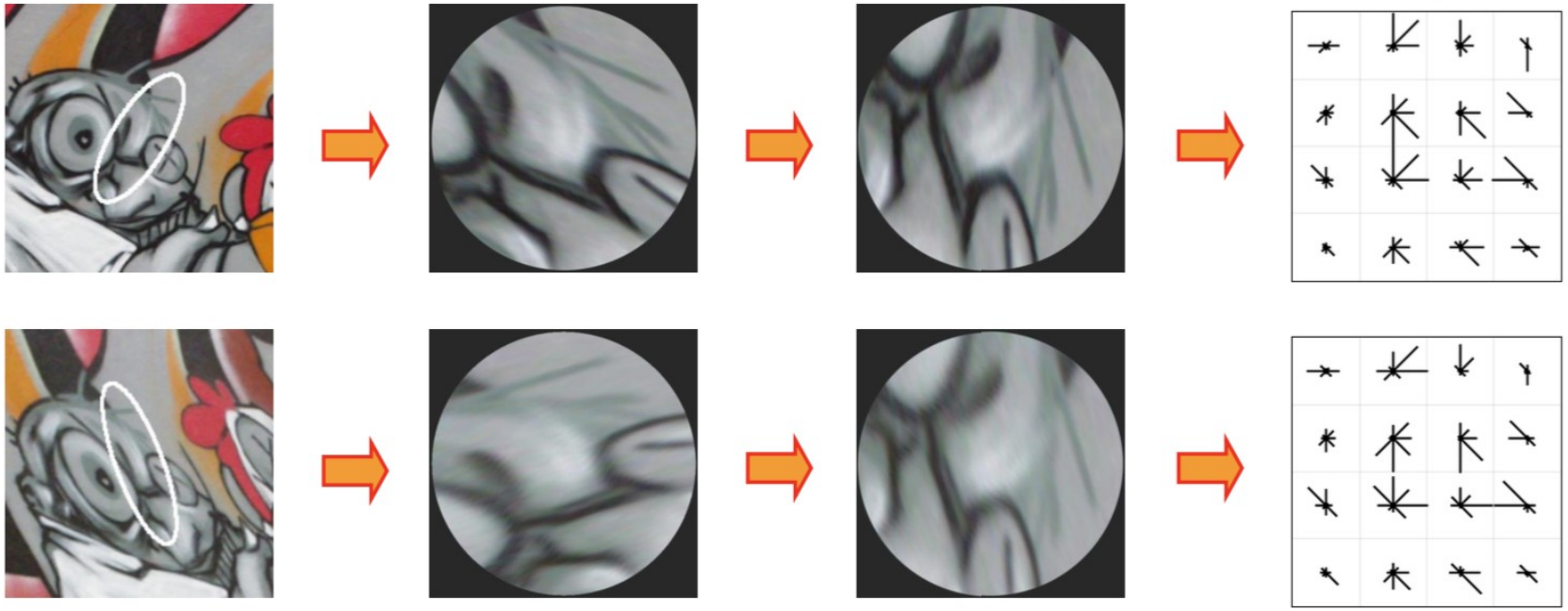
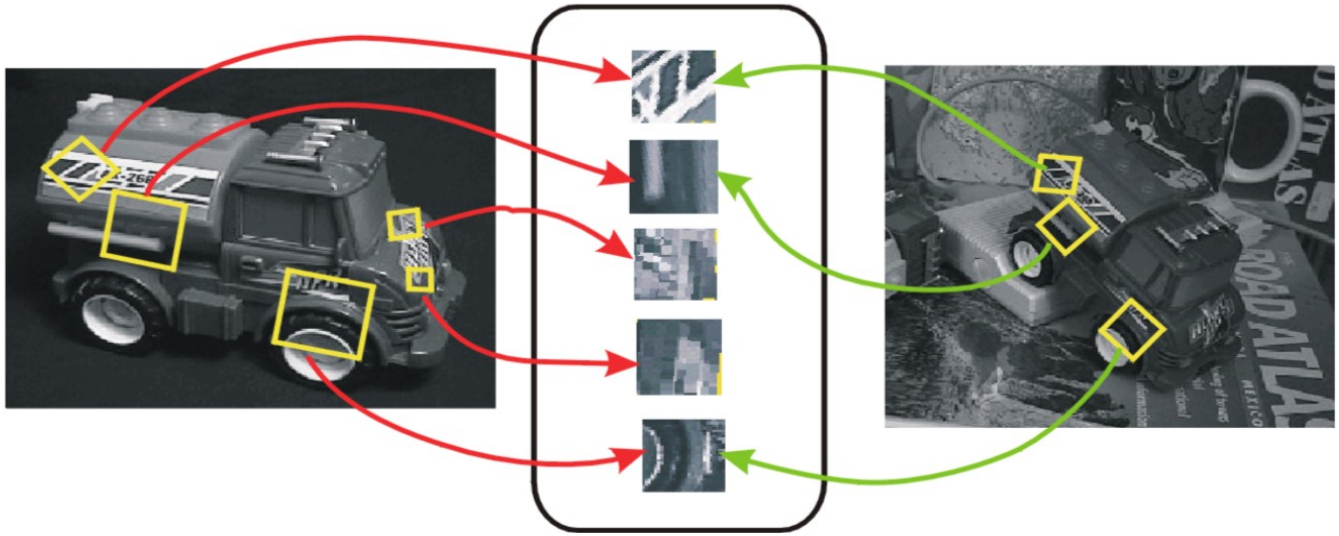
Slide credit: Tinne Tuytelaars

Summary：Affine Invariance



Invariance vs. Covariance

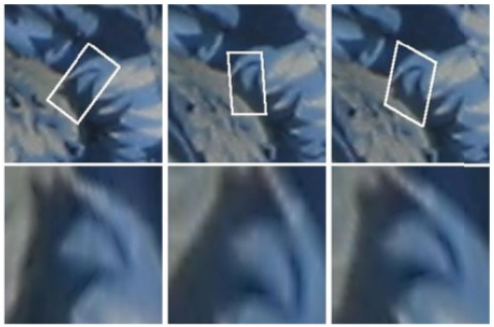
- **Invariance:**
  - features(transform(image)) = features(image)
- **Covariance:**
  - features(transform(image)) = transform(features(image))



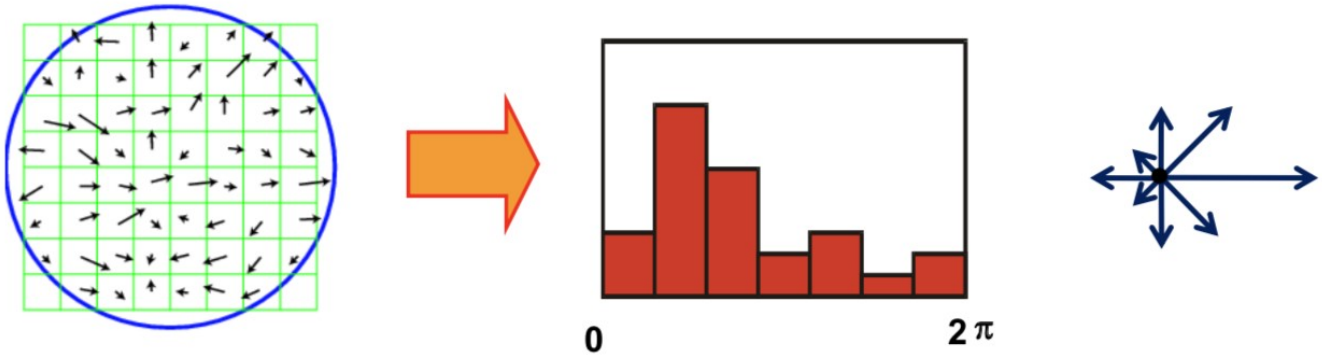
Covariant detection ⇒ invariant description

Feature Descriptor

- **Disadvantage of patches as descriptors:**
  - Small shifts can affect matching score a lot

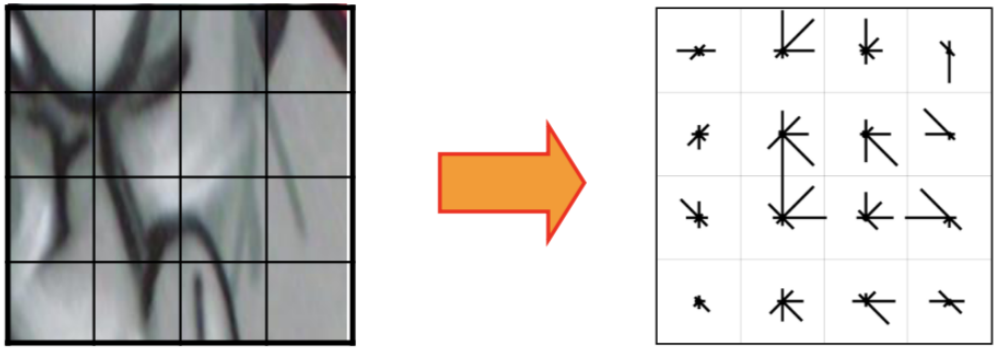


- **Solution: histograms**



Feature Descriptor：SIFT

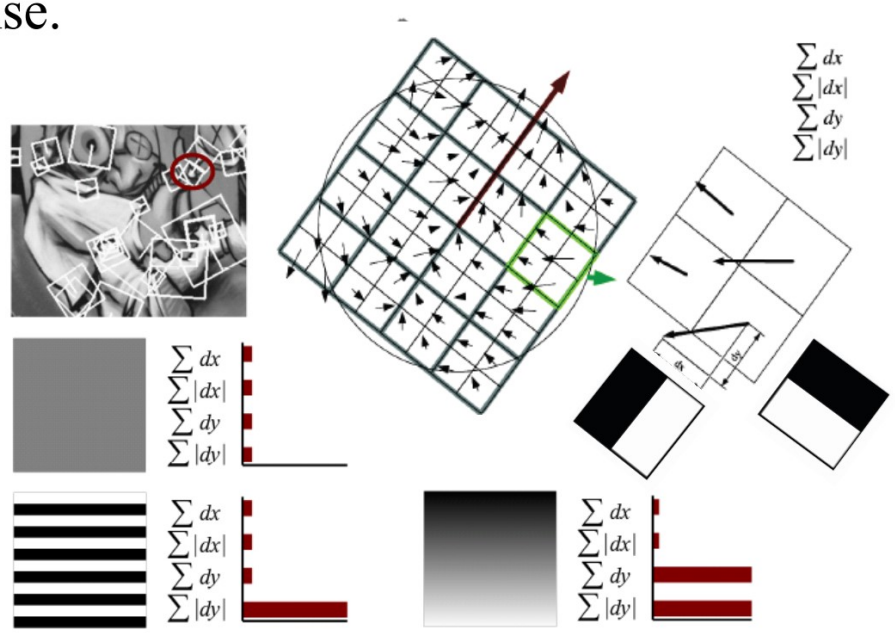
- Scale Invariant Feature Transform
- Descriptor computation:
  - Divide patch into 4x4 sub-patches: 16 cells
  - Compute histogram of gradient orientations (8 reference angles) for all pixels inside each sub-patch
  - Resulting descriptor: 4x4x8 = 128 dimensions



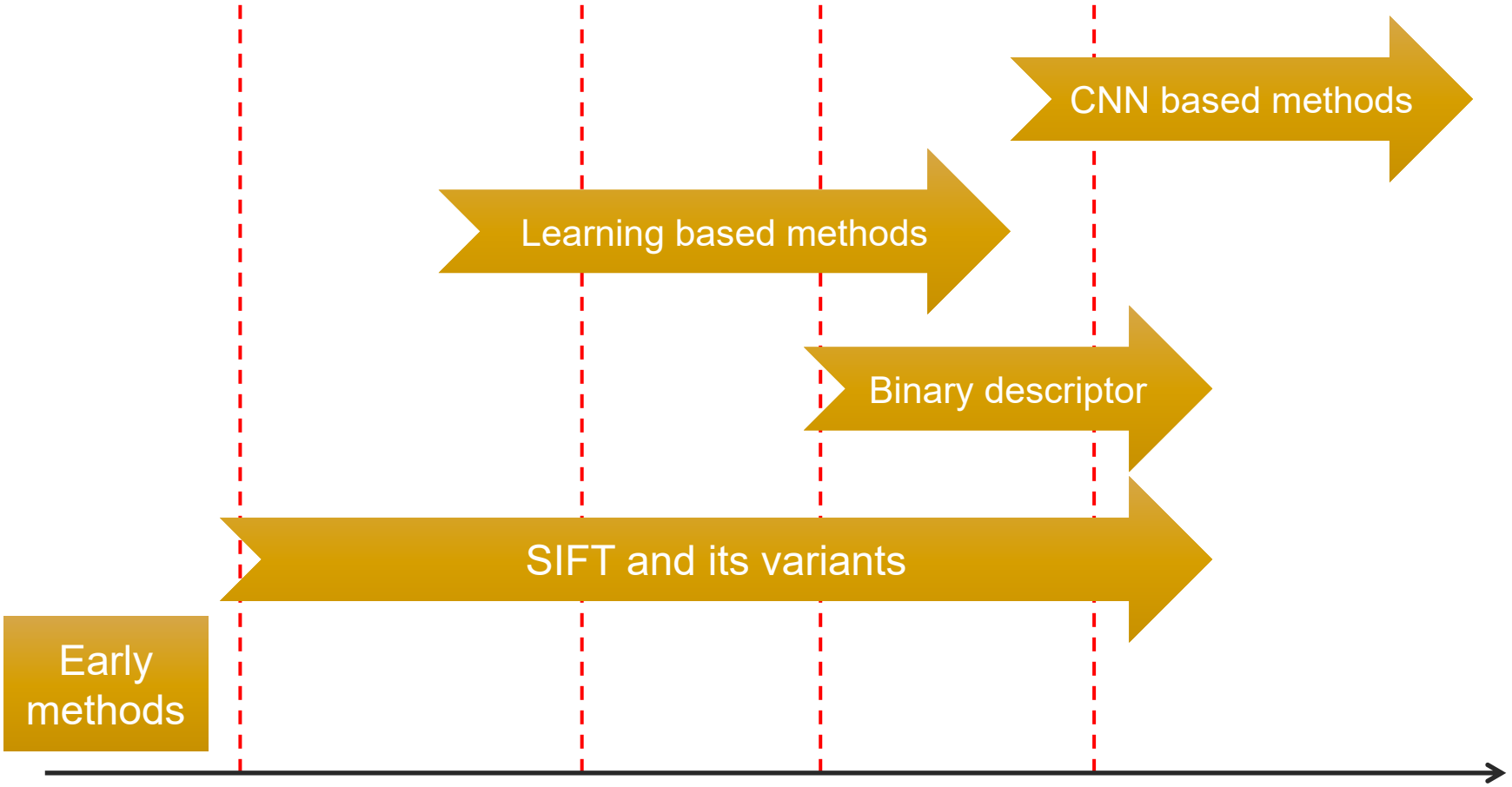
David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

SURF: Descriptor Extraction

1. Split the interest region (20s x 20s) into 4 x 4 square sub-regions.
2. Calculate Haar wavelet responses dx and dy, and weight the responses with a Gaussian kernel.
3. Sum the response over each sub-region for dx and dy, then sum the absolute value of response.
4. Concatenate summation results in all sub-regions, forming a 64D SURF descriptor.

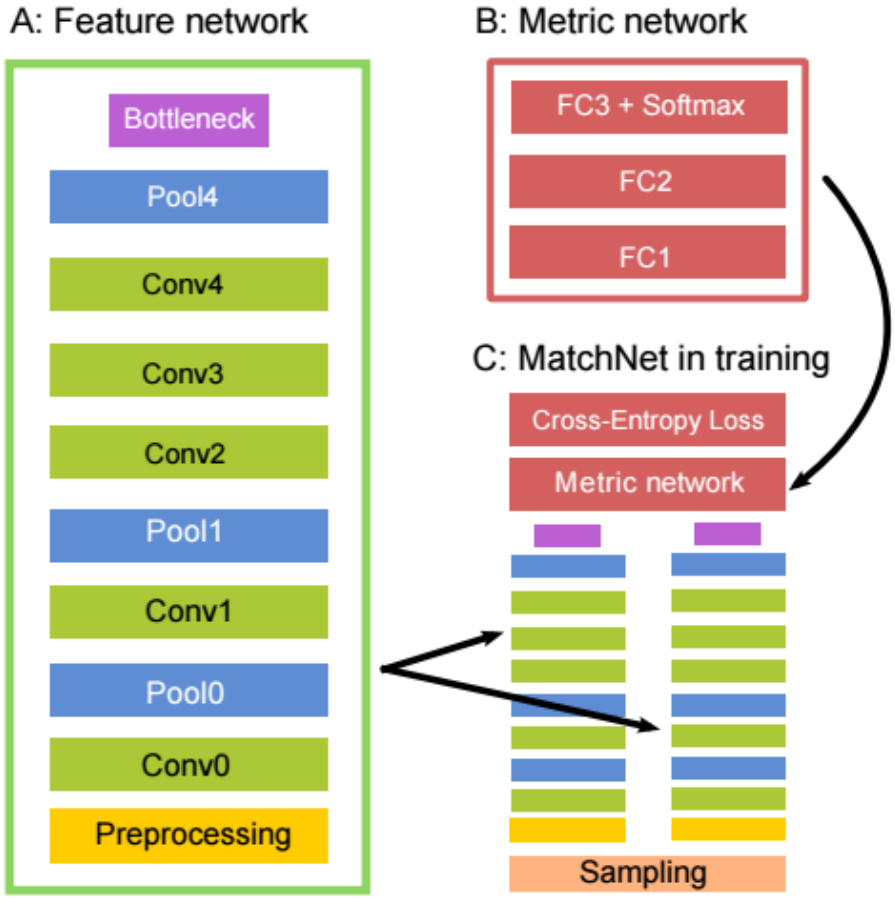


Local Descriptors: Trend



X. Han et al. Matchnet: Unifying feature and metric learning for patch-based matching. In CVPR 2015.

MatchNet



- Simultaneously learn the descriptor and the metric
- Siamese Feature descriptor network
- Metric network on top
- Cross-entropy loss, transfer matching problem to classification problem
- Train time：1 day – 1 week

X. Han et al. Matchnet: Unifying feature and metric learning for patch-based matching. In CVPR 2015.

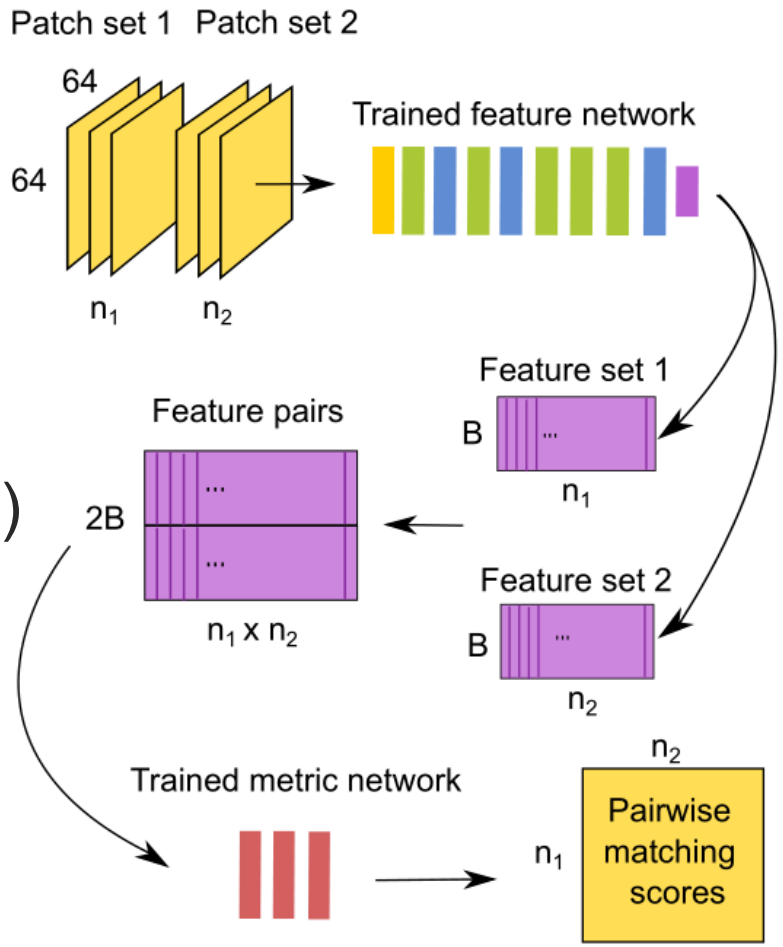
Training MatchNet

- Cross-entropy error

$$E = -\frac{1}{n}\sum_{i=1}^{n}[y_i \log(\hat{y}_i) + (1-y_i)\log(1-\hat{y}_i)]$$

- Stochastic gradient descent (SGD)

- A special reservoir sampler for negative sampling



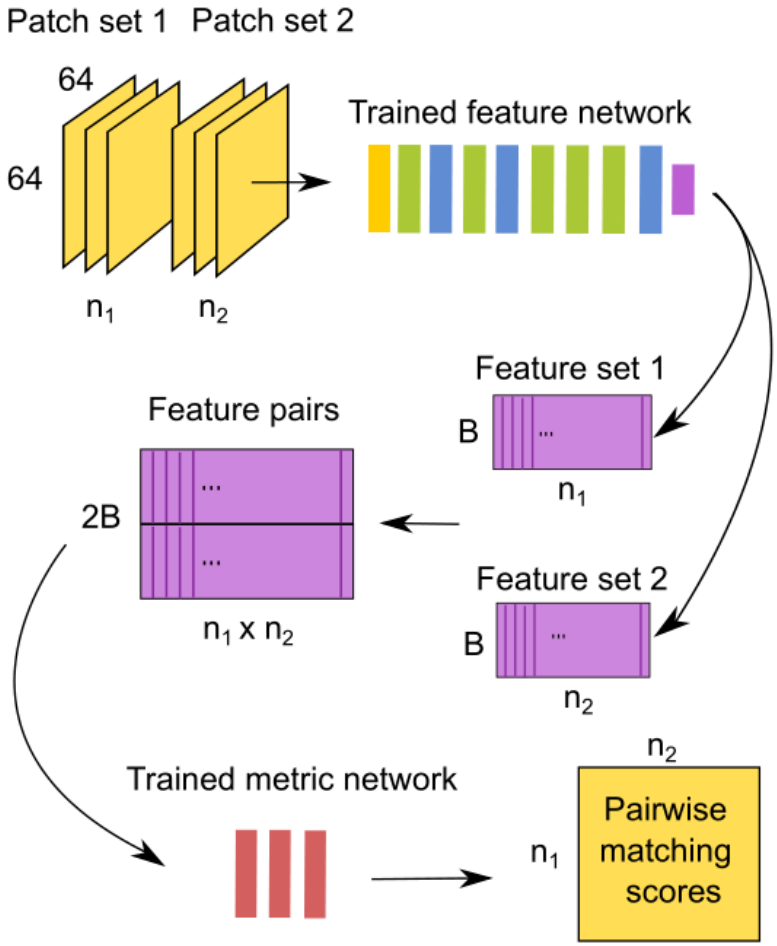
X. Han et al. Matchnet: Unifying feature and metric learning for patch-based matching. In CVPR 2015.

Testing MatchNet

A two-stage prediction pipeline:

1. Generate feature descriptors for all patches.

2. Pair the features and push them through the metric network to get the scores.



Today's class

- Introduction to alignment
- Alignment methods
  - Global methods
  - Hypothesize and test
- Image Transformation
  - Common image transformations
  - Examples of solving image alignment
- Homework: Mosaics

Alignment

Alignment: solving the transformation that makes two things match better

T

Parametric (global) warping

p = (x,y)

p' = (x',y')

Transformation T is a coordinate-changing machine:

p' = *T*(p)

What does it mean that *T* is global?

- Is the same for any point p
- can be described by just a few numbers (parameters)

For linear transformations, we can represent T as a matrix

p' = **T**p

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

Fitting and Alignment

Fitting: find the parameters of a model that best fit the data.

Alignment: find the parameters of the transformation that best align matched points.

Fitting and Alignment

- Design challenges
  - Design a suitable **goodness of fit** measure
    - Similarity should reflect application goals
    - Encode robustness to outliers and noise
  - Design an **optimization** method
    - Avoid local optima
    - Find best parameters quickly

Fitting and Alignment: Methods

- Global optimization / Search for parameters
  - Least squares fit
  - Robust least squares
  - Iterative closest point (ICP)
- Hypothesize and test
  - Generalized Hough transform
  - RANSAC

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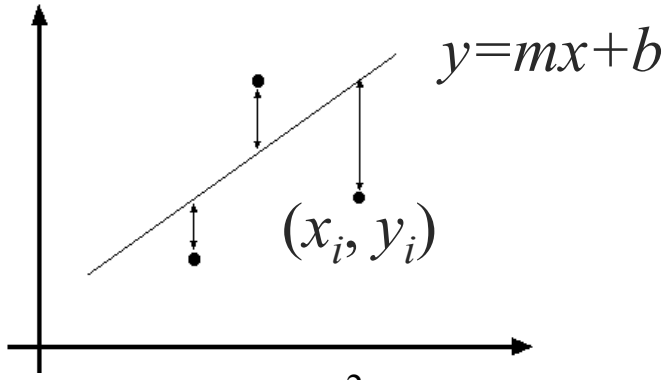
Least squares line fitting

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m x_i + b$
- Find (m, b) to minimize



$$E = \sum_{i=1}^{n} (y_i - m x_i - b)^2$$

$$E = \sum_{i=1}^{n} \left(\begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - y_i \right)^2 = \left\| \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \right\|^2 = \|\mathbf{Ap} - \mathbf{y}\|^2$$

$$= \mathbf{y}^T \mathbf{y} - 2(\mathbf{Ap})^T \mathbf{y} + (\mathbf{Ap})^T (\mathbf{Ap})$$

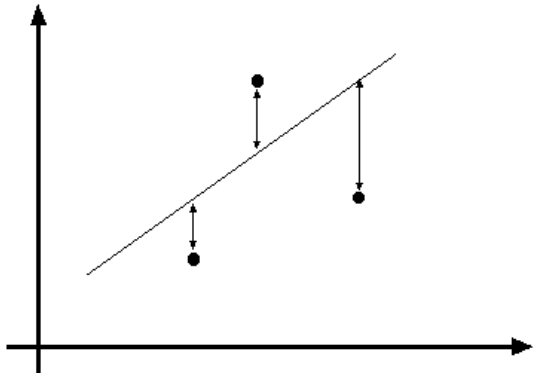
$$\frac{dE}{dp} = 2\mathbf{A}^T \mathbf{Ap} - 2\mathbf{A}^T \mathbf{y} = 0$$

$$\mathbf{A}^T \mathbf{Ap} = \mathbf{A}^T \mathbf{y} \Rightarrow \mathbf{p} = \left(\mathbf{A}^T \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{y}$$

Matlab: `p = A \ y;`

Problem with “vertical” least squares

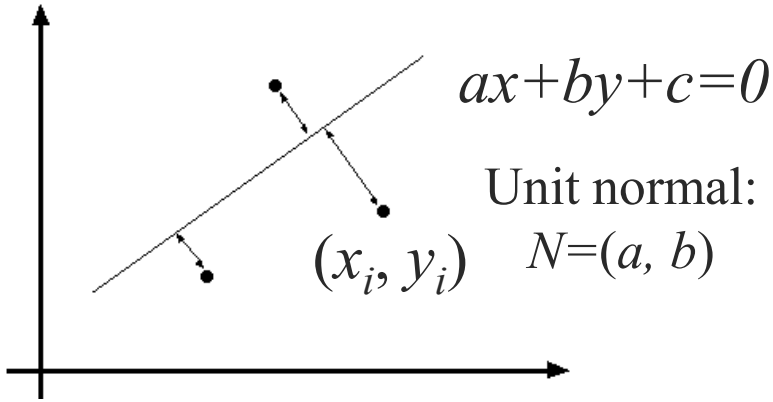
- Not rotation-invariant
- Fails completely for vertical lines



Total least squares

If $(a^2+b^2=1)$ then

Distance between point (x_i, y_i) and line $ax+by+c=0$ is $|ax_i + by_i + c|$

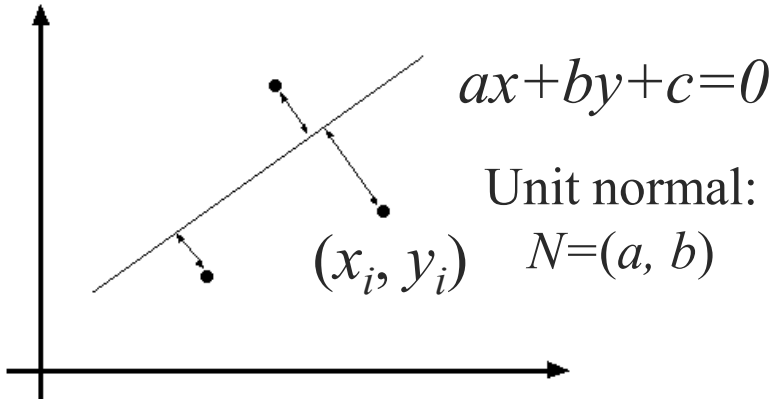


proof:
http://mathworld.wolfram.com/Point-LineDistance2-Dimensional.html

Total least squares

If ($a^2+b^2=1$) then

Distance between point (x_i, y_i) and line $ax+by+c=0$ is $|ax_i + by_i + c|$



Find (a, b, c) to minimize the sum of squared perpendicular distances

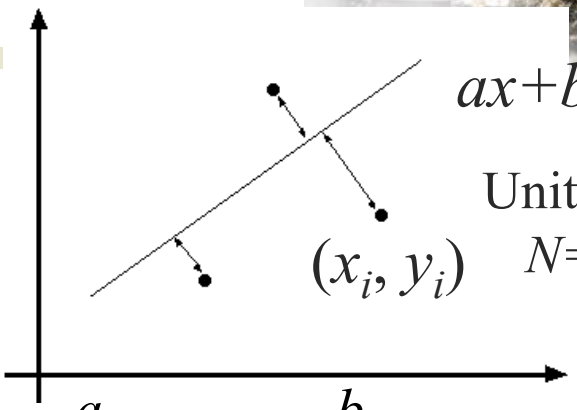
$$E = \sum_{i=1}^{n} (a x_i + b y_i + c)^2$$

Slide modified from S. Lazebnik

Total least squares

Find (a, b, c) to minimize the sum of squared perpendicular distances

$$E = \sum_{i=1}^{n} (a x_i + b y_i + c)^2$$



$$\frac{\partial E}{\partial c} = \sum_{i=1}^{n} 2(a x_i + b y_i + c) = 0$$

$$c = -\frac{a}{n}\sum_{i=1}^{n} x_i - \frac{b}{n}\sum_{i=1}^{n} y_i = -a\bar{x} - b\bar{y}$$

$$E = \sum_{i=1}^{n} (a(x_i - \bar{x}) + b(y_i - \bar{y}))^2 = \left\| \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2 = \mathbf{p}^T \mathbf{A}^T \mathbf{A} \mathbf{p}$$

$$\text{minimize } \mathbf{p}^T \mathbf{A}^T \mathbf{A} \mathbf{p} \quad \text{s.t. } \mathbf{p}^T \mathbf{p} = 1 \quad \Rightarrow \quad \text{minimize } \frac{\mathbf{p}^T \mathbf{A}^T \mathbf{A} \mathbf{p}}{\mathbf{p}^T \mathbf{p}}$$

Solution is eigenvector corresponding to smallest eigenvalue of A^TA

See details on Raleigh Quotient: http://en.wikipedia.org/wiki/Rayleigh_quotient

Recap: Two Common Optimization Problems

Problem statement

$$\text{minimize} \quad \|\mathbf{Ax}-\mathbf{b}\|^2$$

least squares solution to $\mathbf{Ax}=\mathbf{b}$

Solution

$$\mathbf{x}=\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{b}$$

$\mathbf{x}=\mathbf{A}\backslash\mathbf{b}$ (matlab)

Problem statement

$$\text{minimize} \quad \mathbf{x}^T\mathbf{A}^T\mathbf{Ax} \quad \text{s.t.} \quad \mathbf{x}^T\mathbf{x}=1$$

$$\text{minimize} \ \frac{\mathbf{x}^T\mathbf{A}^T\mathbf{Ax}}{\mathbf{x}^T\mathbf{x}}$$

non - trivial lsq solution to $\mathbf{Ax}=0$

Solution

$$[\mathbf{v},\lambda]=\text{eig}(\mathbf{A}^T\mathbf{A})$$

$$\lambda_1<\lambda_{2..n}:\mathbf{x}=\mathbf{v}_1$$

Least squares (global) optimization

Good

- Clearly specified objective
- Optimization is easy

Bad

- May not be what you want to optimize
- Sensitive to outliers
  - Bad matches, extra points
- Doesn’t allow you to get multiple good fits
  - Detecting multiple objects, lines, etc.

Other ways to search for parameters

- **Line search**
  1. For each parameter, step through values and choose value that gives best fit
  2. Repeat (1) until no parameter changes
- **Grid search**
  1. Propose several sets of parameters, evenly sampled in the joint set
  2. Choose best (or top few) and sample joint parameters around the current best; repeat
- **Gradient descent**
  1. Provide initial position (e.g., random)
  2. Locally search for better parameters by following gradient

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  - **Hypothesize and test**
- Image Transformation
  - Common image transformations
  - Examples of solving image alignment
- Homework: Mosaics

Hypothesize and test

1. Propose parameters
   - Try all possible
   - Each point votes for all consistent parameters
   - Repeatedly sample enough points to solve for parameters
2. Score the given parameters
   - Number of consistent points, possibly weighted by distance
3. Choose from among the set of parameters
   - Global or local maximum of scores
4. Possibly refine parameters using inliers

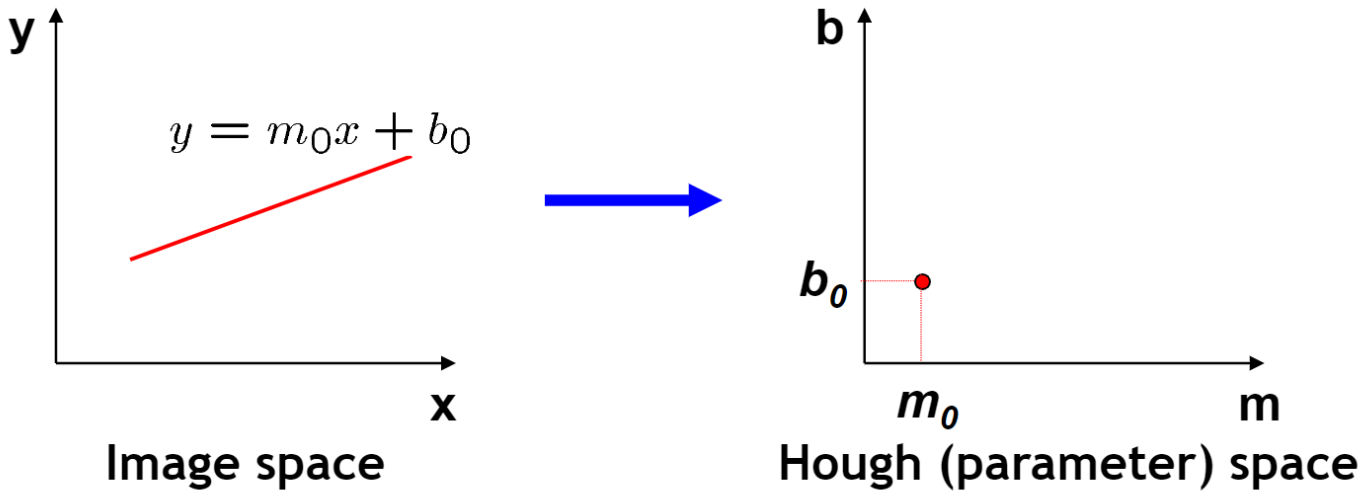
Hough Transform: Outline

1. Create a grid of parameter values

2. Each point votes for a set of parameters, incrementing those values in grid

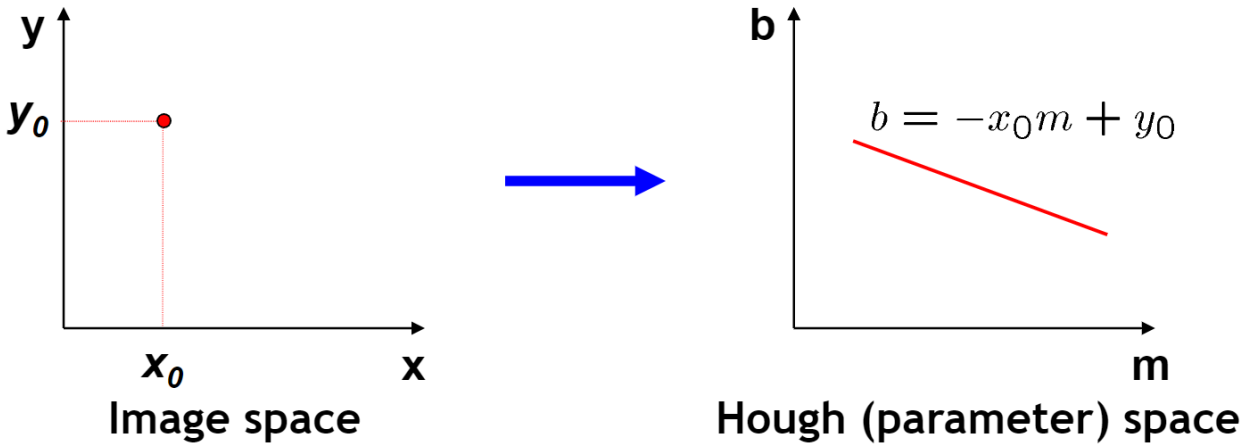
3. Find maximum or local maxima in grid

Hough Space



- **Connection between image (x,y) and Hough (m,b) spaces**
  - **A line in the image corresponds to a point in Hough space.**
  - **To go from image space to Hough space:**
    - **Given a set of points (x,y), find all (m,b) such that $y = mx + b$**

Hough Space

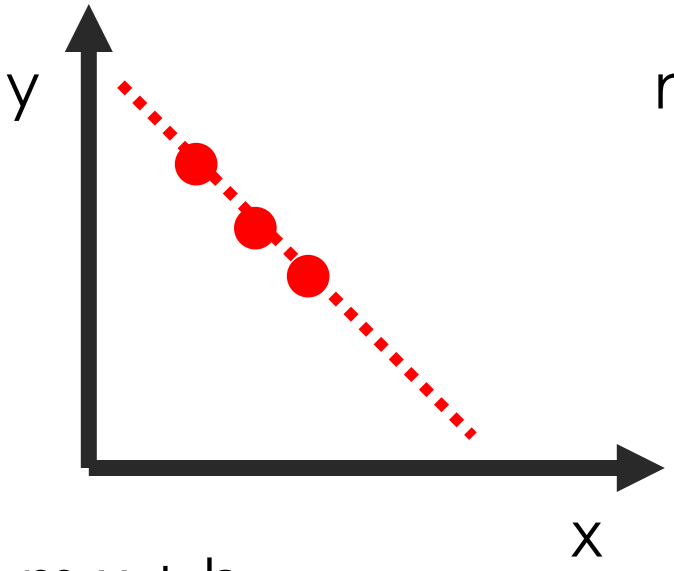


- Connection between image (x,y) and Hough (m,b) spaces
  - A line in the image corresponds to a point in Hough space.
  - To go from image space to Hough space:
    - Given a set of points (x,y), find all (m,b) such that $y = mx + b$
  - What does a point (x_0, y_0) in the image space map to?
    - Answer: the solutions of $b = -x_0 m + y_0$
    - This is a line in Hough space.

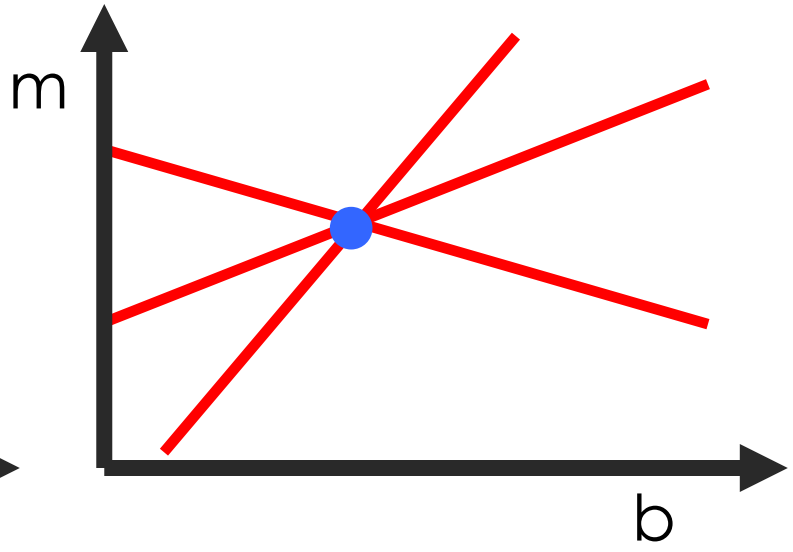
Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Given a set of points, find the curve or line that explains the data points best

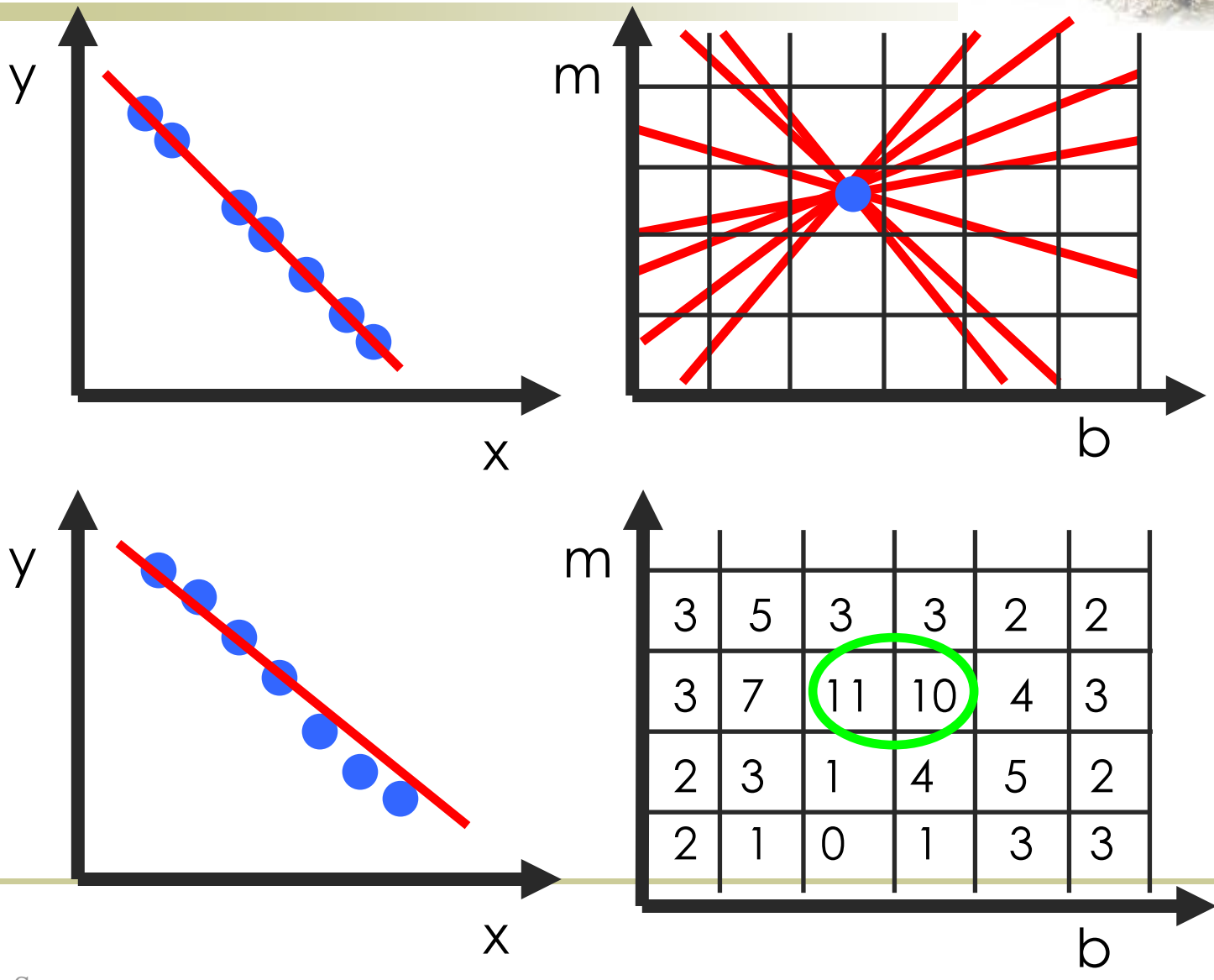


$y = m\,x + b$



Hough space

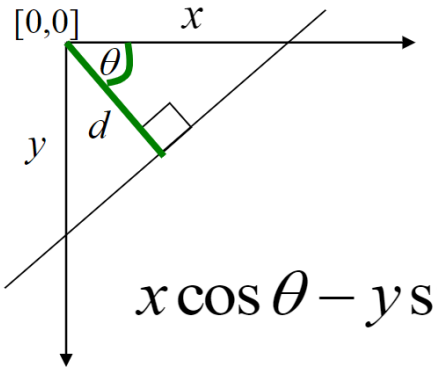
Hough transform



Slide from S. Savarese

Polar Representation for Lines

- Issues with usual (m,b) parameter space: can take on infinite values, undefined for vertical lines.

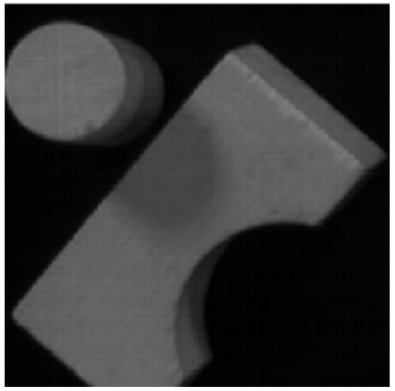


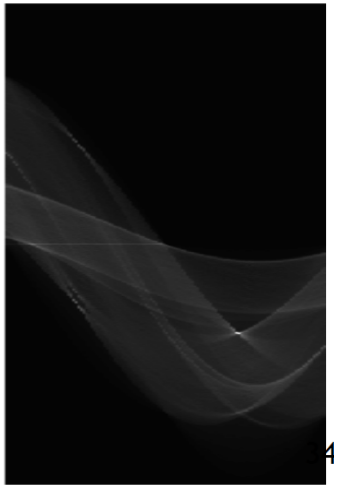
$$x\cos\theta - y\sin\theta = d$$

d : perpendicular distance from line to origin

θ : angle the perpendicular makes with the x-axis

- Point in image space
  ⇒ Sinusoid segment in Hough space





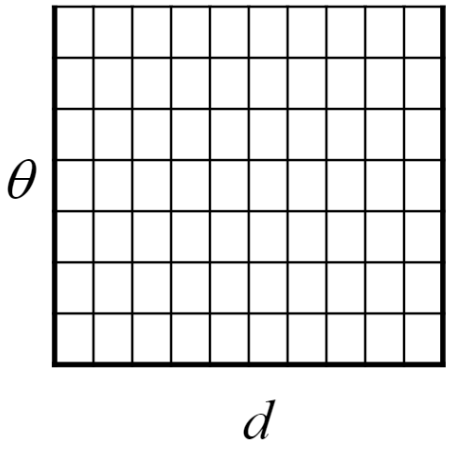
Slide adapted from Steve Seitz

Hough Transform Algorithm

Using the polar parameterization:

$$x\cos\theta + y\sin\theta = d$$

H: accumulator array (votes)



Basic Hough transform algorithm

1. Initialize $H[d,\theta] = 0.$
2. For each edge point (x,y) in the image

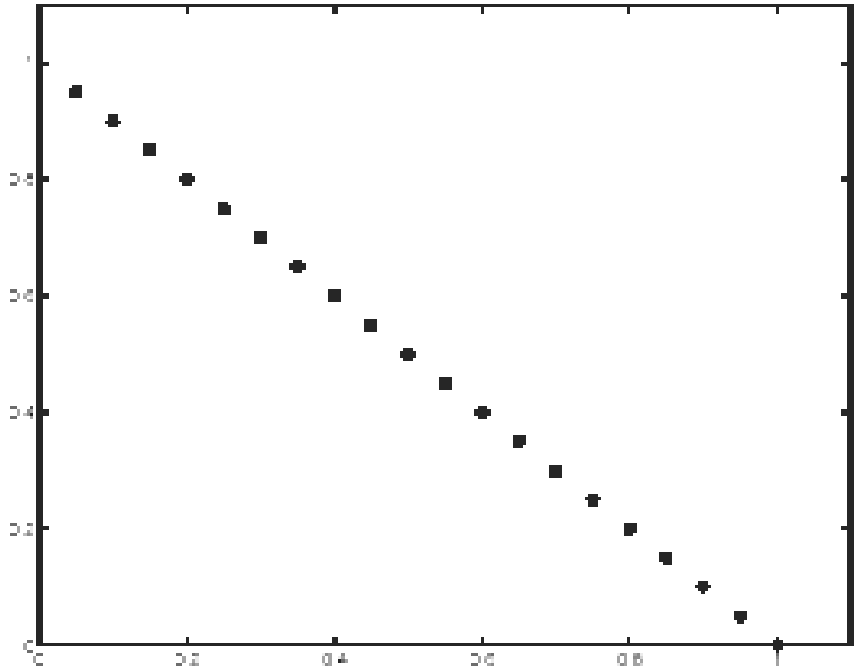
   for $\theta = 0$ to 180 // some quantization

   $d = x\cos\theta + y\sin\theta$

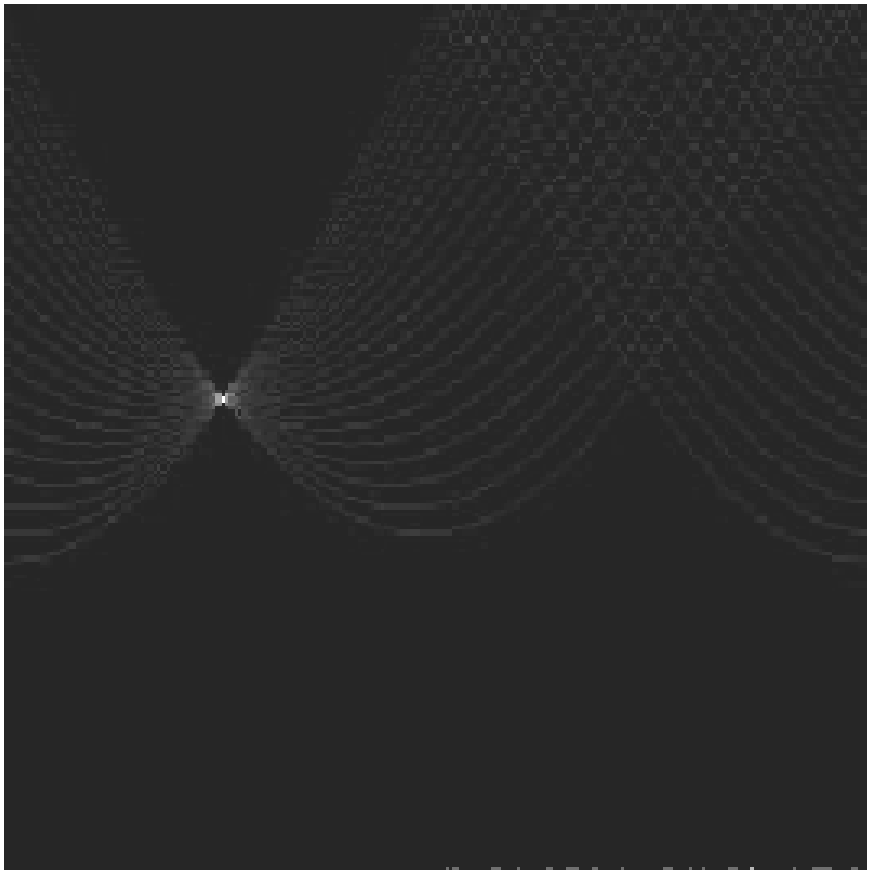
   H[d, θ] += 1
3. Find the value(s) of (d,θ) where $H[d,\theta]$ is maximal.
4. The detected line in the image is given by $d = x\cos\theta + y\sin\theta$

- Time complexity (in terms of number of votes)?

Hough transform - experiments



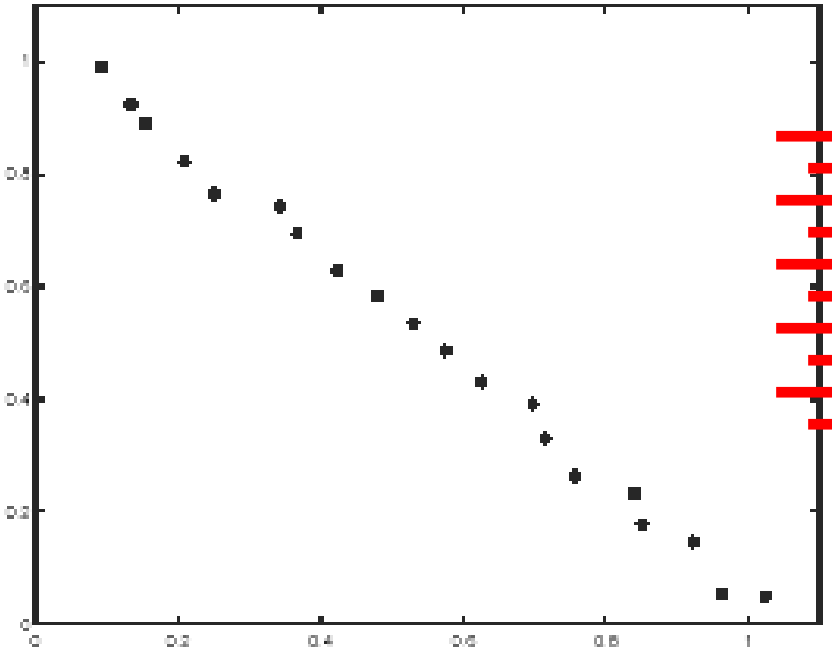
features



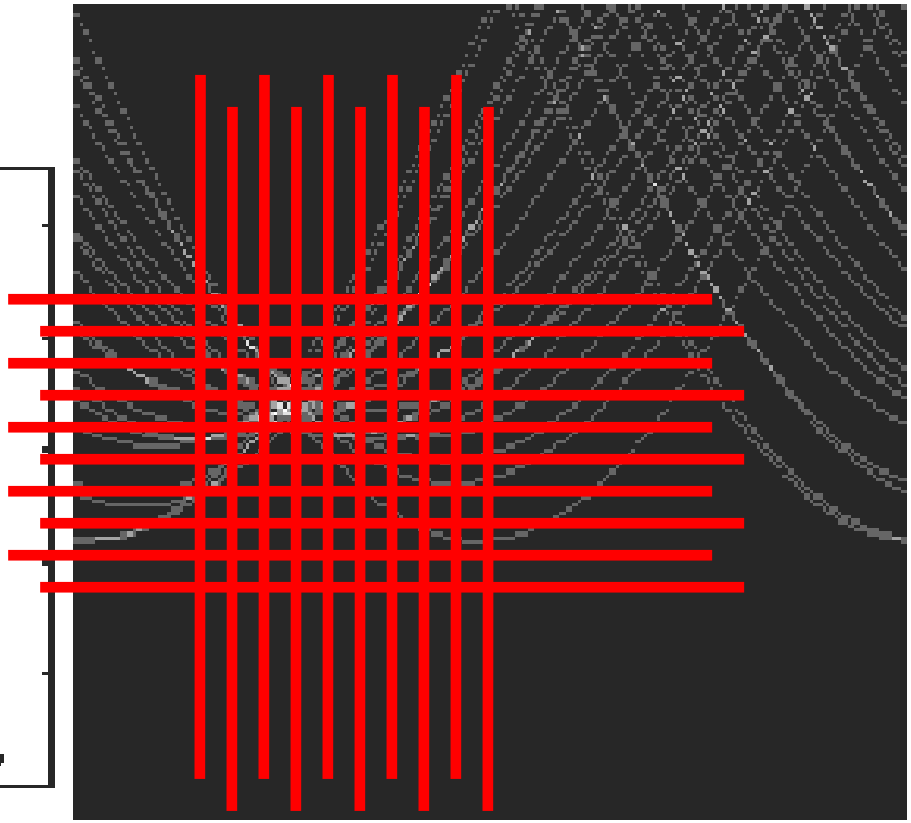
votes

Hough transform - experiments

Noisy data



features

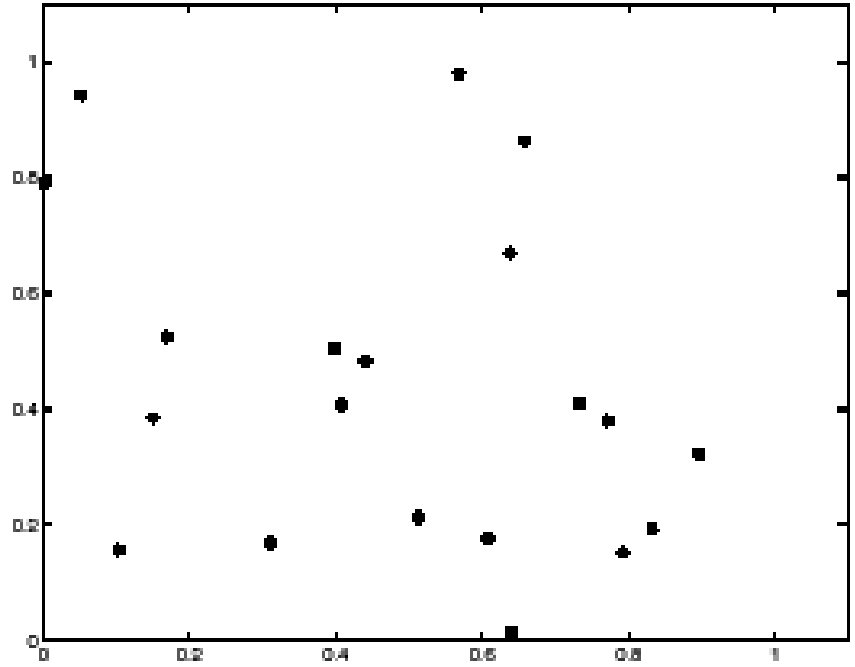


votes

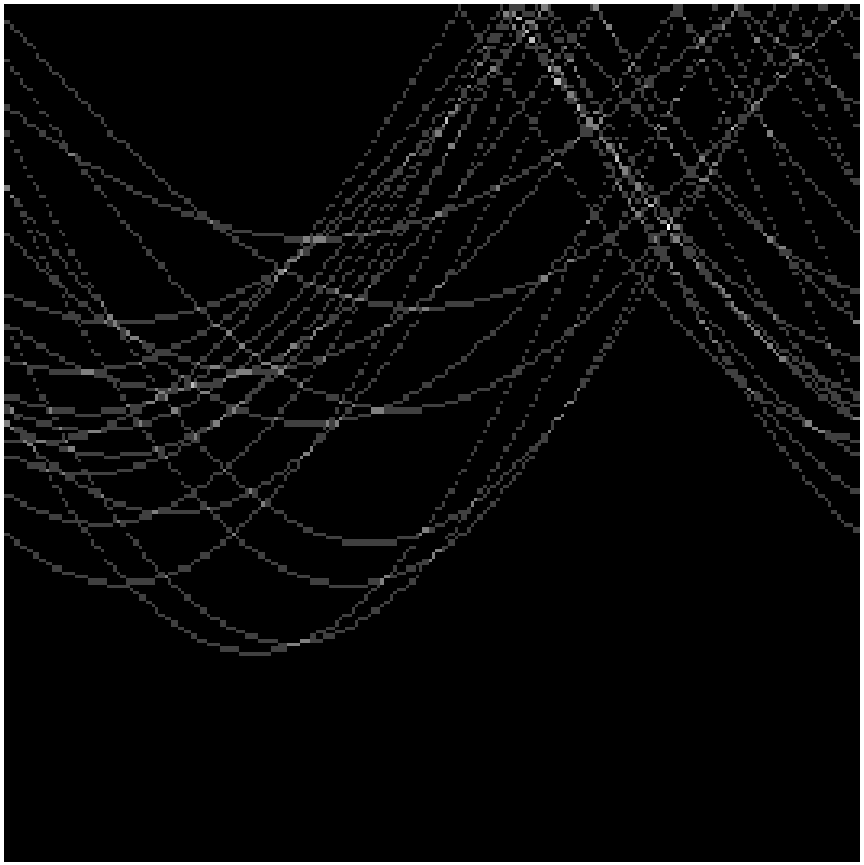
Need to adjust grid size or smooth

Slide from S. Savarese

Hough transform - experiments



features



votes

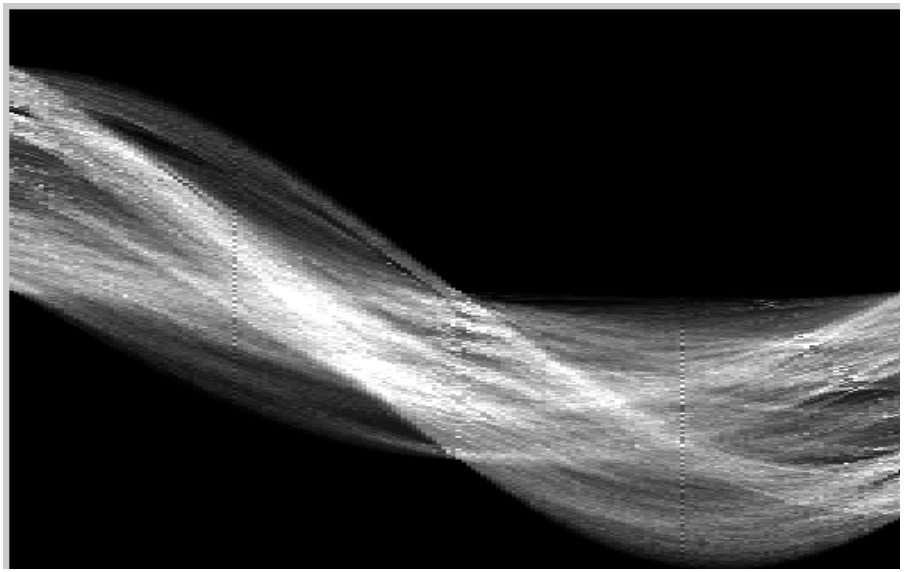
Issue: spurious peaks due to uniform noise

1. Image → Canny

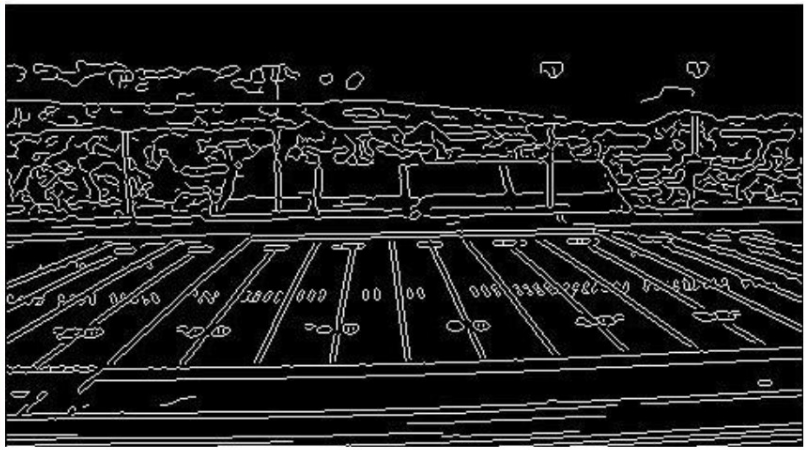
2. Canny → Hough votes

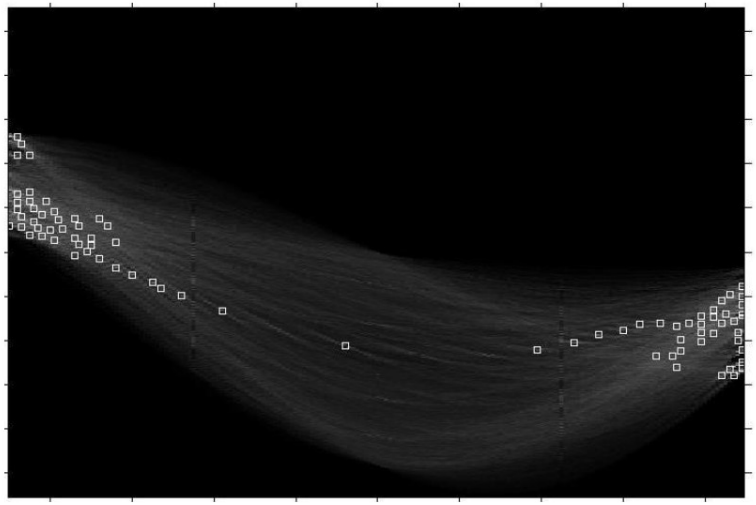
3. Hough votes → Edges

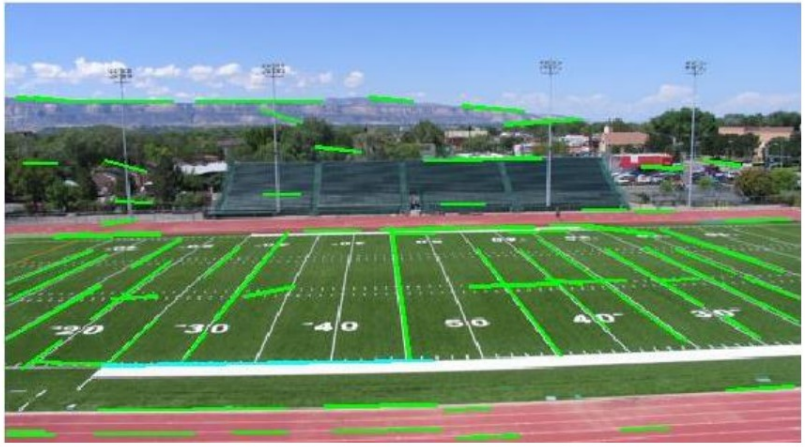
Find peaks and post-process



Real-World Example







Showing longest segments found

Hough transform: pros and cons

Pros

- All points are processed independently, so can cope with occlusion, gaps
- Some robustness to noise: noise points unlikely to contribute *consistently* to any single bin
- Can detect multiple instances of a model in a single pass

Cons

- Complexity of search time increases exponentially with the number of model parameters
- Non-target shapes can produce spurious peaks in parameter space
- Quantization: can be tricky to pick a good grid size

Hough transform conclusions

Common applications

- Line fitting (also circles, ellipses, etc.)
- Object instance recognition (parameters are position/scale/orientation)
- Object category recognition (parameters are position/scale)

RANSAC

- RANdom Sample Consensus

- **Approach**: we want to avoid the impact of outliers, so let's look for "inliers", and use those only.

- **Intuition**: if an outlier is chosen to compute the current fit, then the resulting line (transformation) won't have much support from rest of the points (matches).

RANSAC

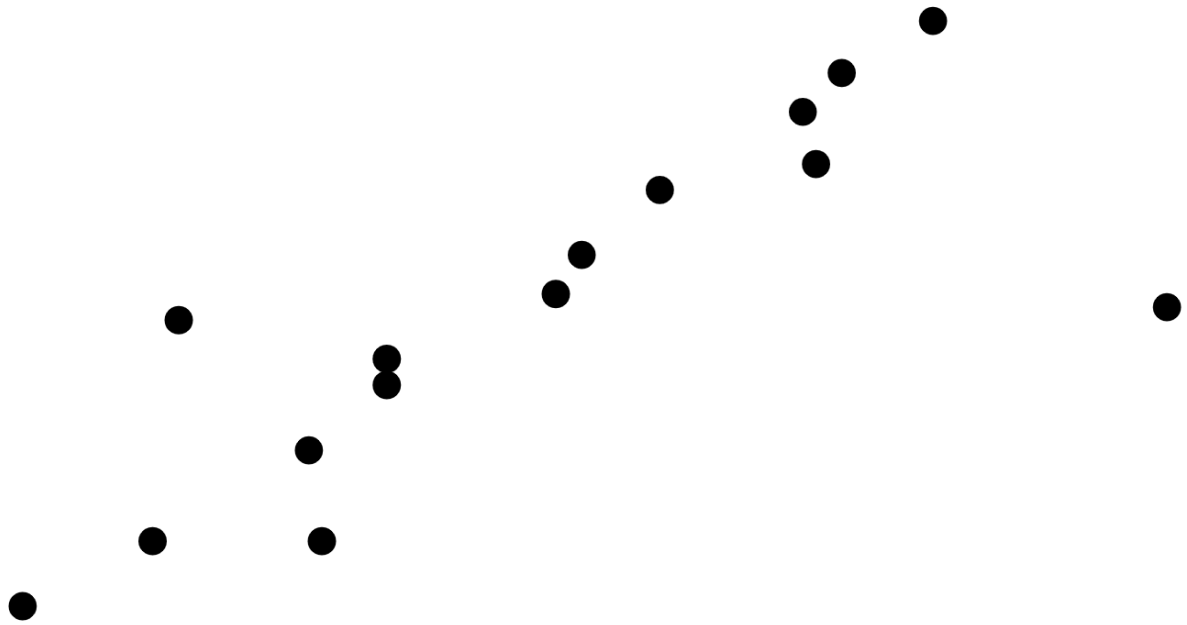
RANSAC loop:

1. Randomly select a *seed group* of points on which to base transformation estimate (e.g., a group of matches)
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers

- Keep the transformation with the largest number of inliers

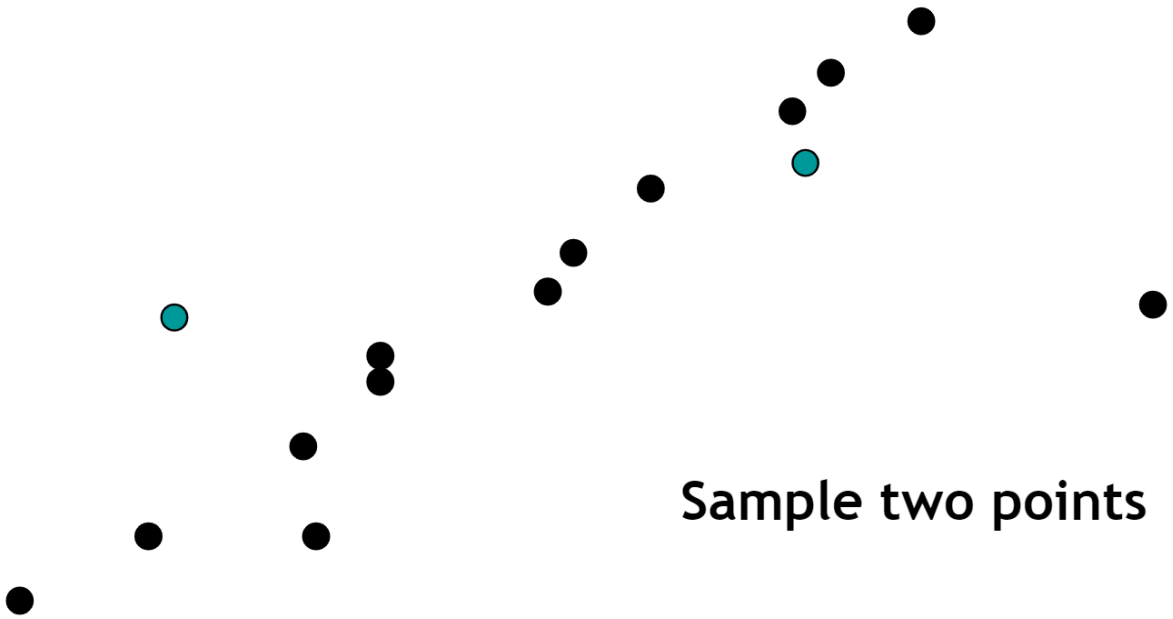
RANSAC Line Fitting Example

- **Task: Estimate the best line**
  - ***How many points do we need to estimate the line?***



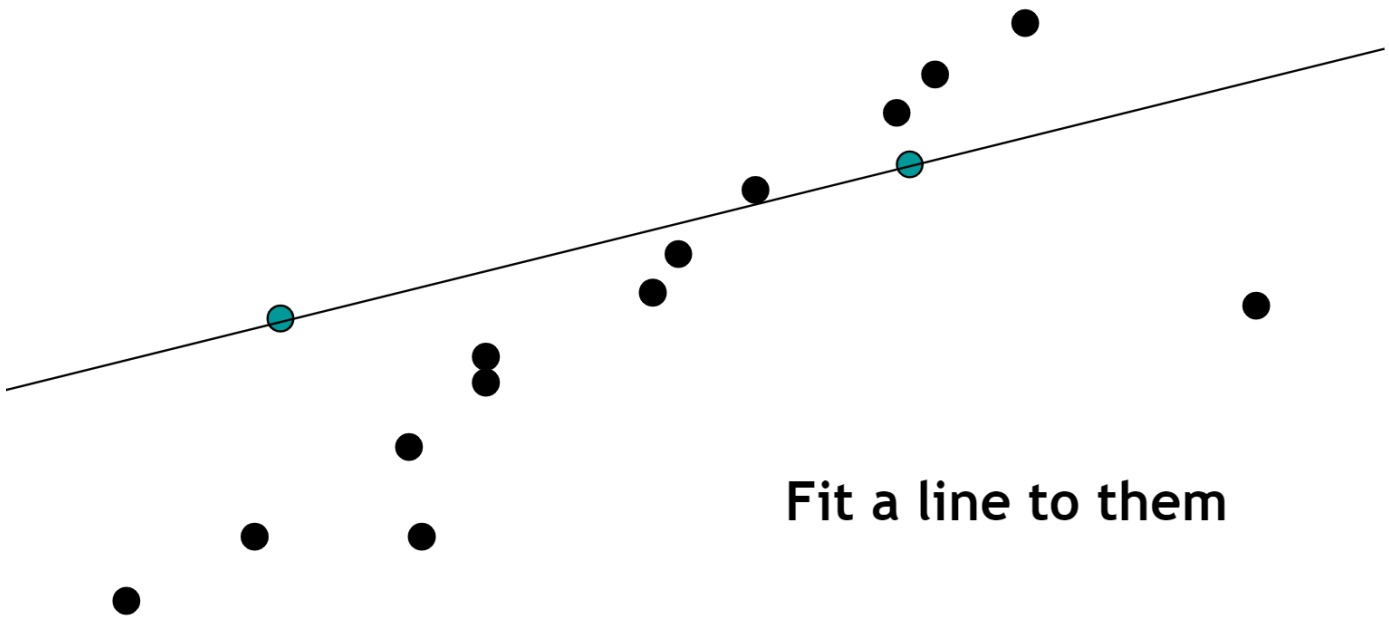
RANSAC Line Fitting Example

- **Task: Estimate the best line**



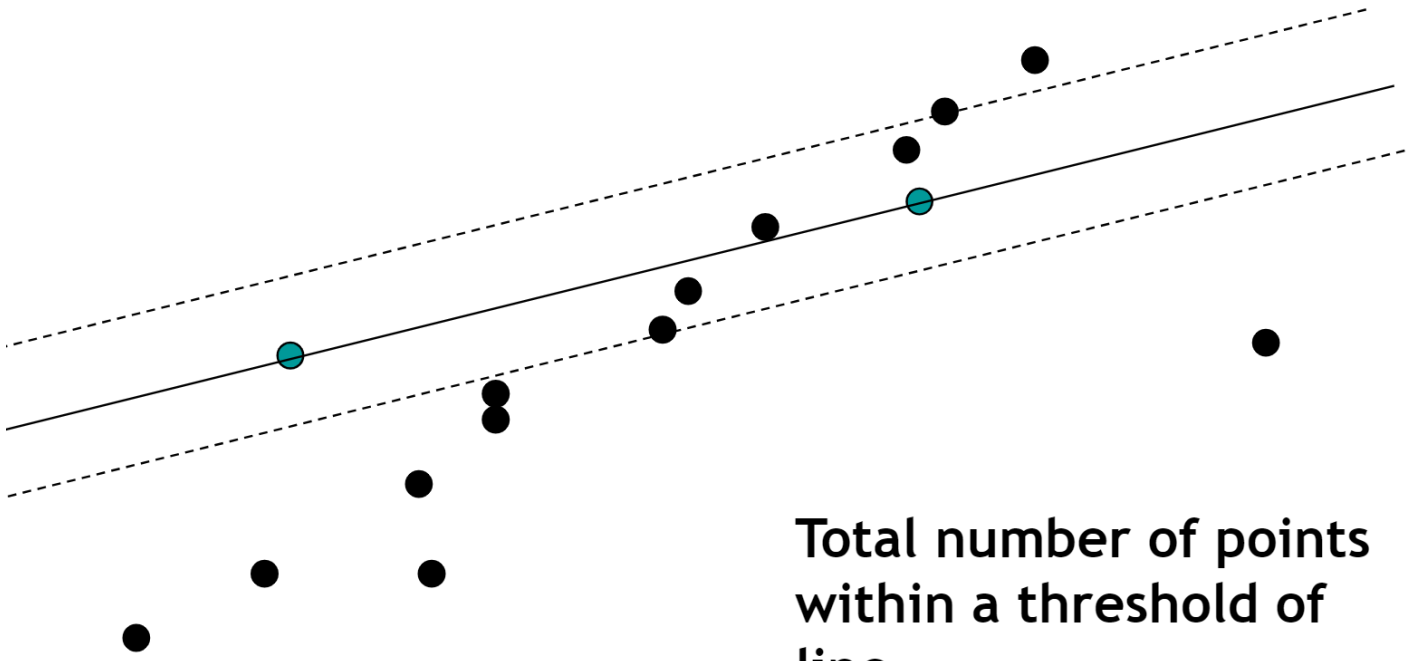
RANSAC Line Fitting Example

- **Task: Estimate the best line**



RANSAC Line Fitting Example

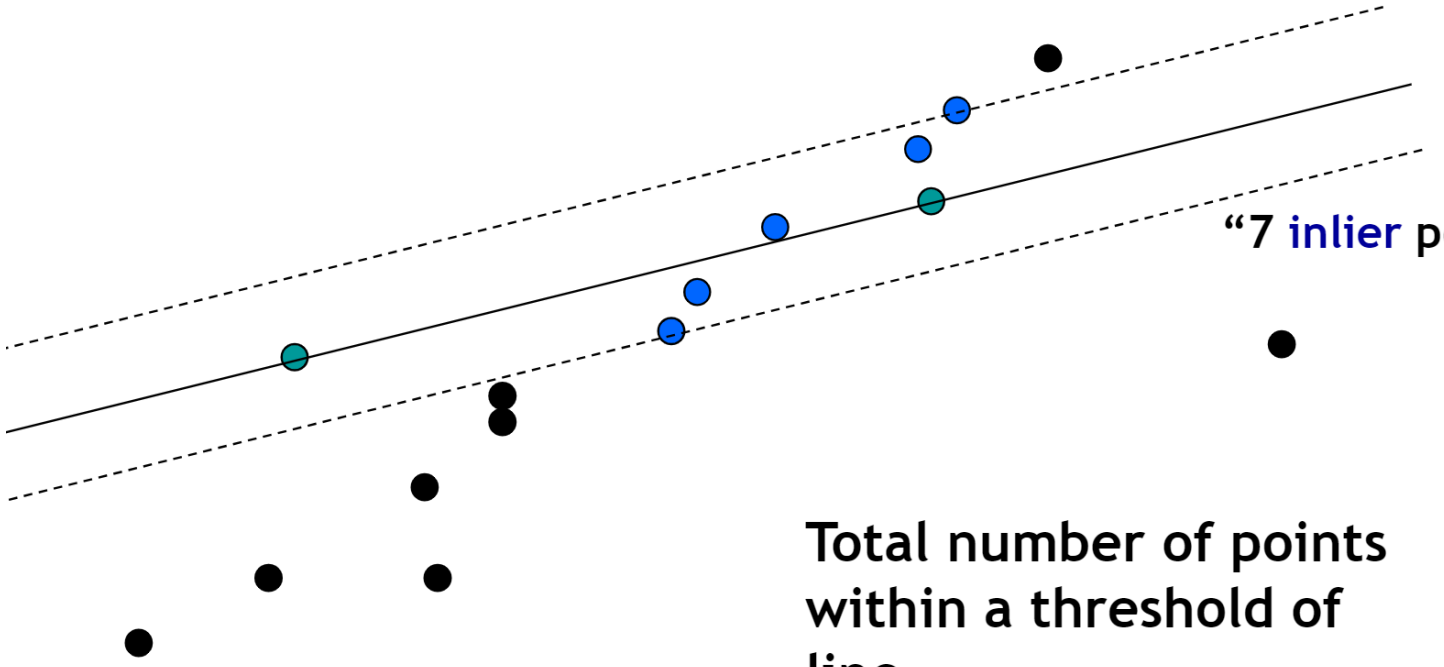
- **Task: Estimate the best line**



Total number of points within a threshold of line.

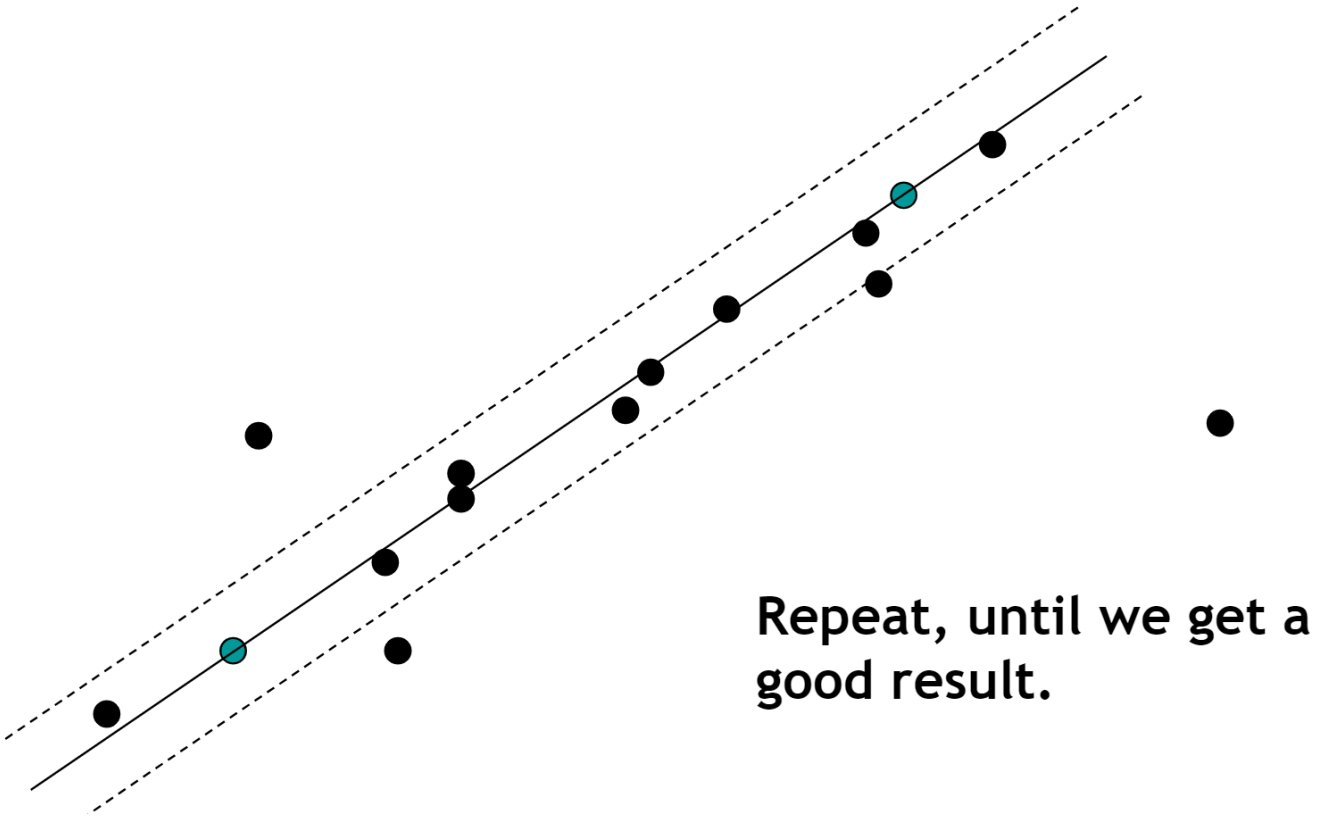
RANSAC Line Fitting Example

- **Task: Estimate the best line**



RANSAC Line Fitting Example

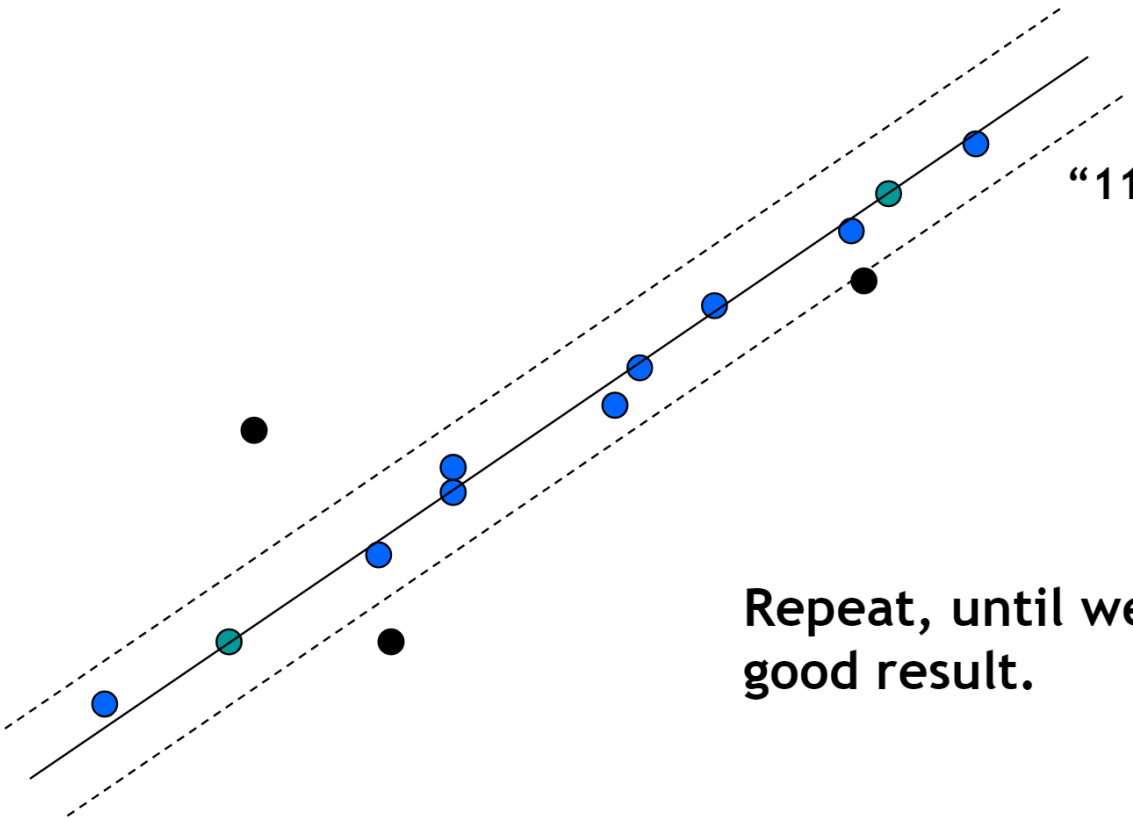
- **Task: Estimate the best line**



Repeat, until we get a good result.

RANSAC Line Fitting Example

- **Task: Estimate the best line**



"11 inlier points"

Repeat, until we get a good result.

RANSAC: How many samples?

- How many samples are needed?
  - Suppose w is fraction of inliers (points from line).
  - n points needed to define hypothesis (2 for lines)
  - k samples chosen.
- Prob. that a single sample of n points is correct: w^n
- Prob. that all k samples fail is: $(1-w^n)^k$

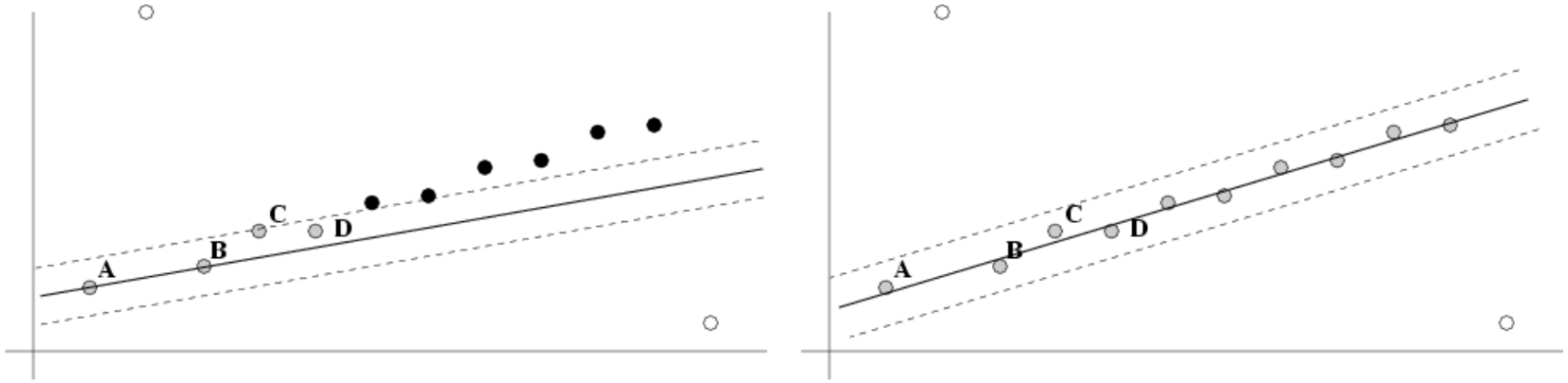
$\Rightarrow$ Choose k high enough to keep this below desired failure rate.

RANSAC: Computed k (p=0.99)

Sample size n	Proportion of outliers						
	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

After RANSAC

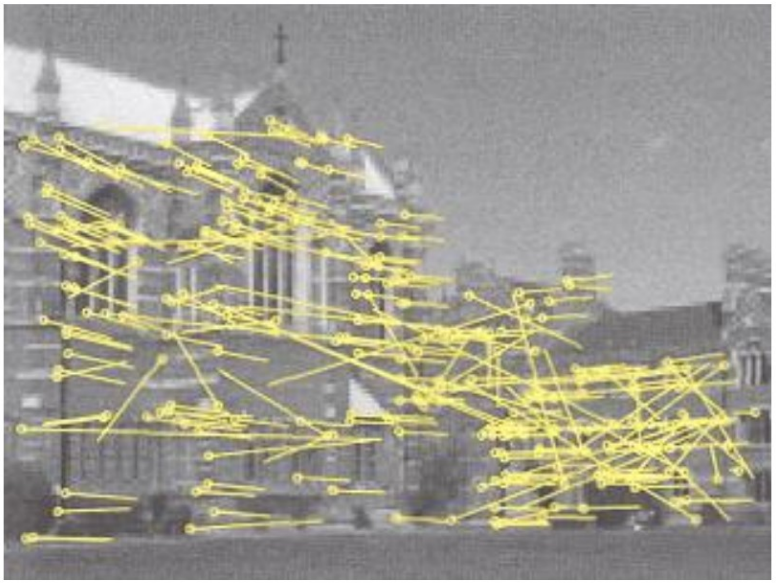
- RANSAC divides data into inliers and outliers and yields estimate computed from minimal set of inliers.
- Improve this initial estimate with estimation over all inliers (e.g. with standard least-squares minimization).
- But this may change inliers, so alternate fitting with re-classification as inlier/outlier.



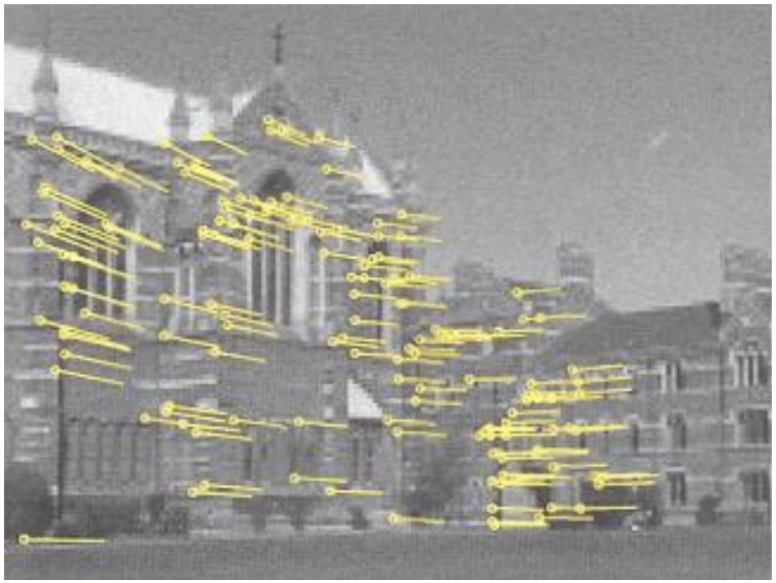
RANSAC Example

- Find best stereo match within a square search window (here 300 pixels^2)
- Global transformation model: epipolar geometry

before RANSAC



after RANSAC



Images from Hartley & Zisserman

RANSAC conclusions

Good

- Robust to outliers
- Applicable for larger number of objective function parameters than Hough transform
- Optimization parameters are easier to choose than Hough transform

Bad

- Computational time grows quickly with fraction of outliers and number of parameters
- Not as good for getting multiple fits (though one solution is to remove inliers after each fit and repeat)

Common applications

- **Computing a homography (e.g., image stitching)**
- Estimating fundamental matrix (relating two views)

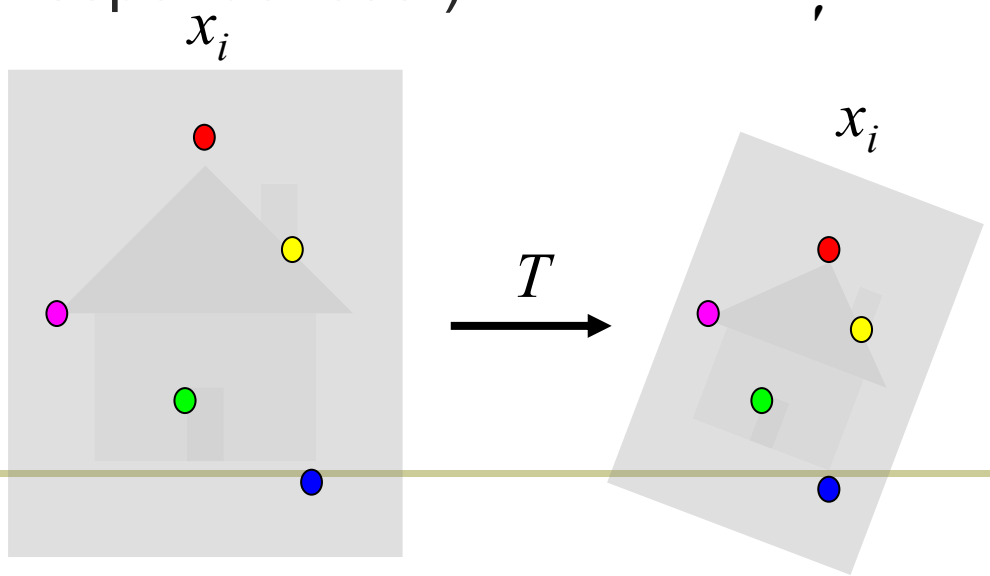
Today's class

- Introduction to alignment
- Alignment methods
  - Global methods
  - Hypothesize and test
- Image Transformation
  - **Common image transformations**
  - Examples of solving image alignment
- Homework: Mosaics

Alignment problem

- We have previously considered how to **fit a model to image evidence**
  - e.g., a line to edge points

- In alignment, we will fit the parameters of some **transformation** according to a set of matching feature pairs (“correspondences”).



Common transformations

original

Transformed

translation

rotation

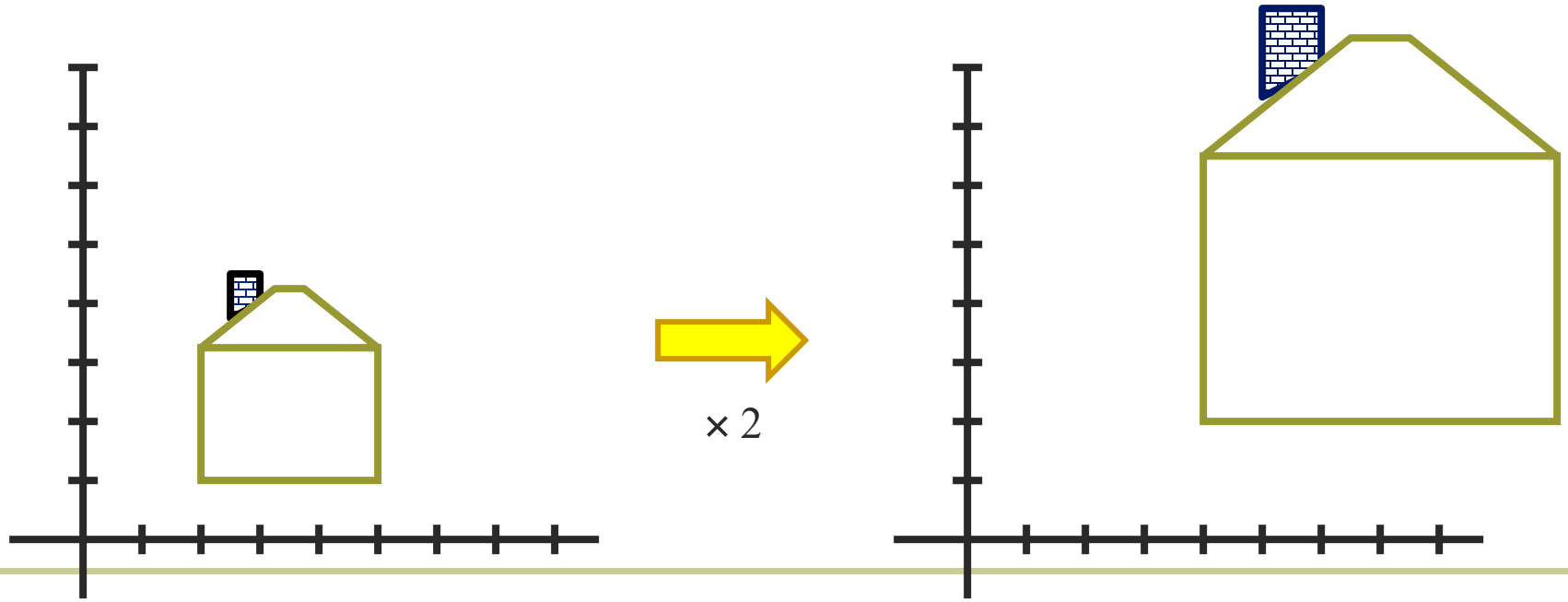
aspect

affine

perspective

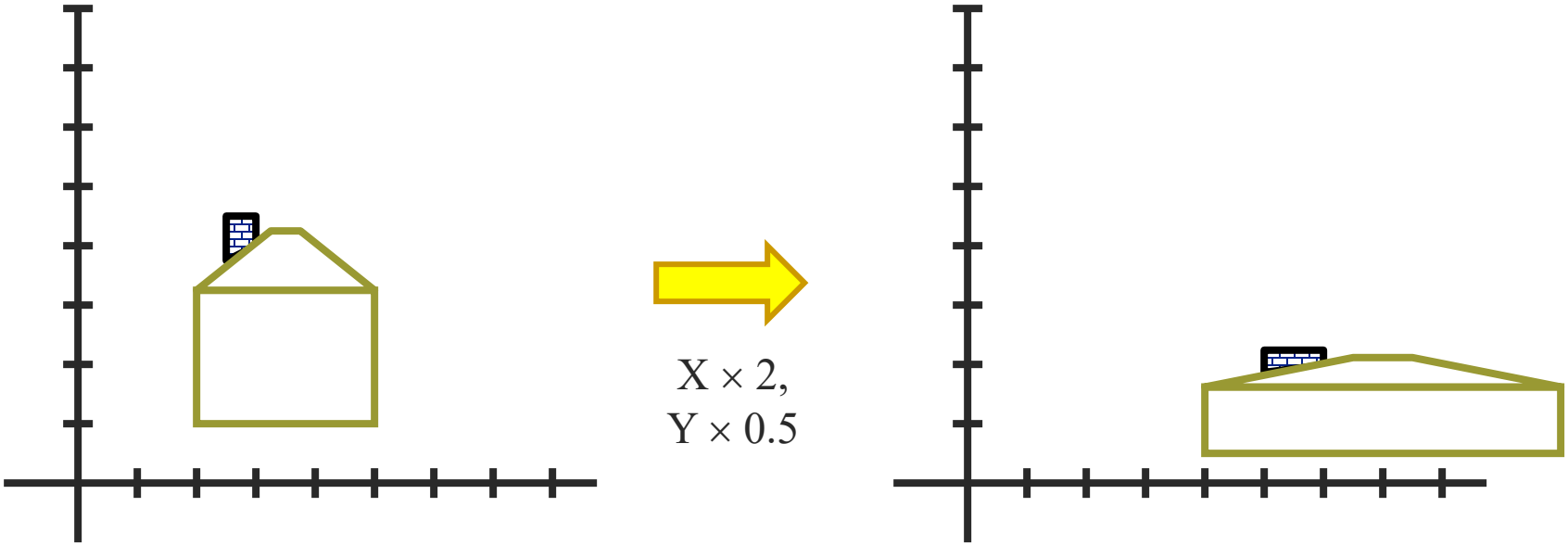
Scaling

- *Scaling* a coordinate means multiplying each of its components by a scalar
- *Uniform scaling* means this scalar is the same for all components:



Scaling

- *Non-uniform scaling*: different scalars per component:



Scaling

- Scaling operation:

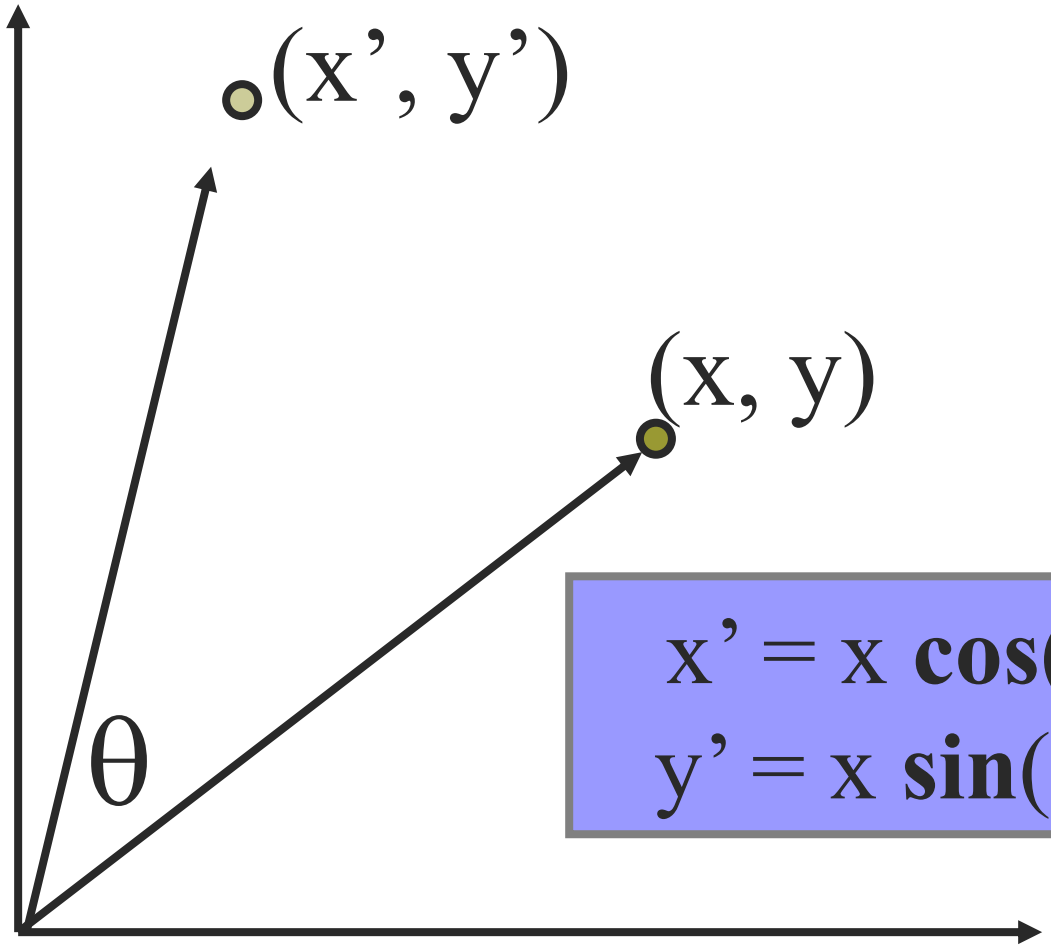
$$x' = ax$$

$$y' = by$$

- Or, in matrix form:

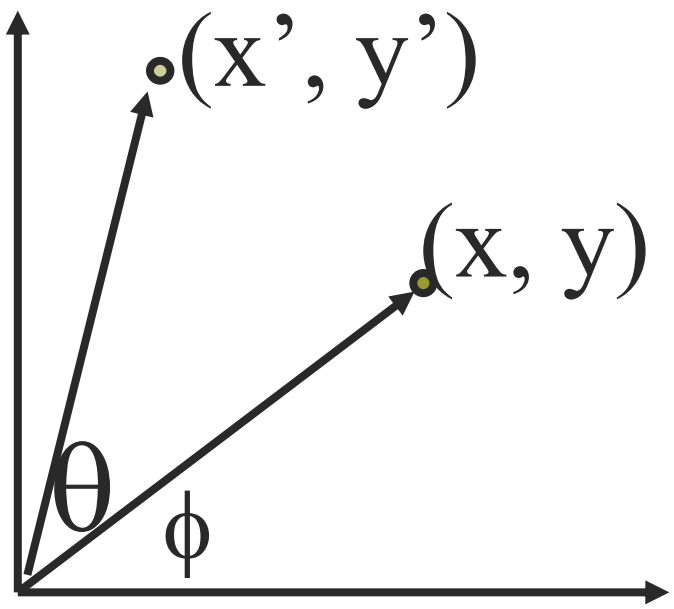
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\text{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

2-D Rotation



$$x' = x\cos(\theta) - y\sin(\theta)$$
$$y' = x\sin(\theta) + y\cos(\theta)$$

2-D Rotation



Polar coordinates…

$x = r \cos(\phi)$

$y = r \sin(\phi)$

$x' = r \cos(\phi + \theta)$

$y' = r \sin(\phi + \theta)$

Trig Identity…

$x' = r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta)$

$y' = r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta)$

Substitute…

$x' = x \cos(\theta) - y \sin(\theta)$

$y' = x \sin(\theta) + y \cos(\theta)$

2-D Rotation

This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

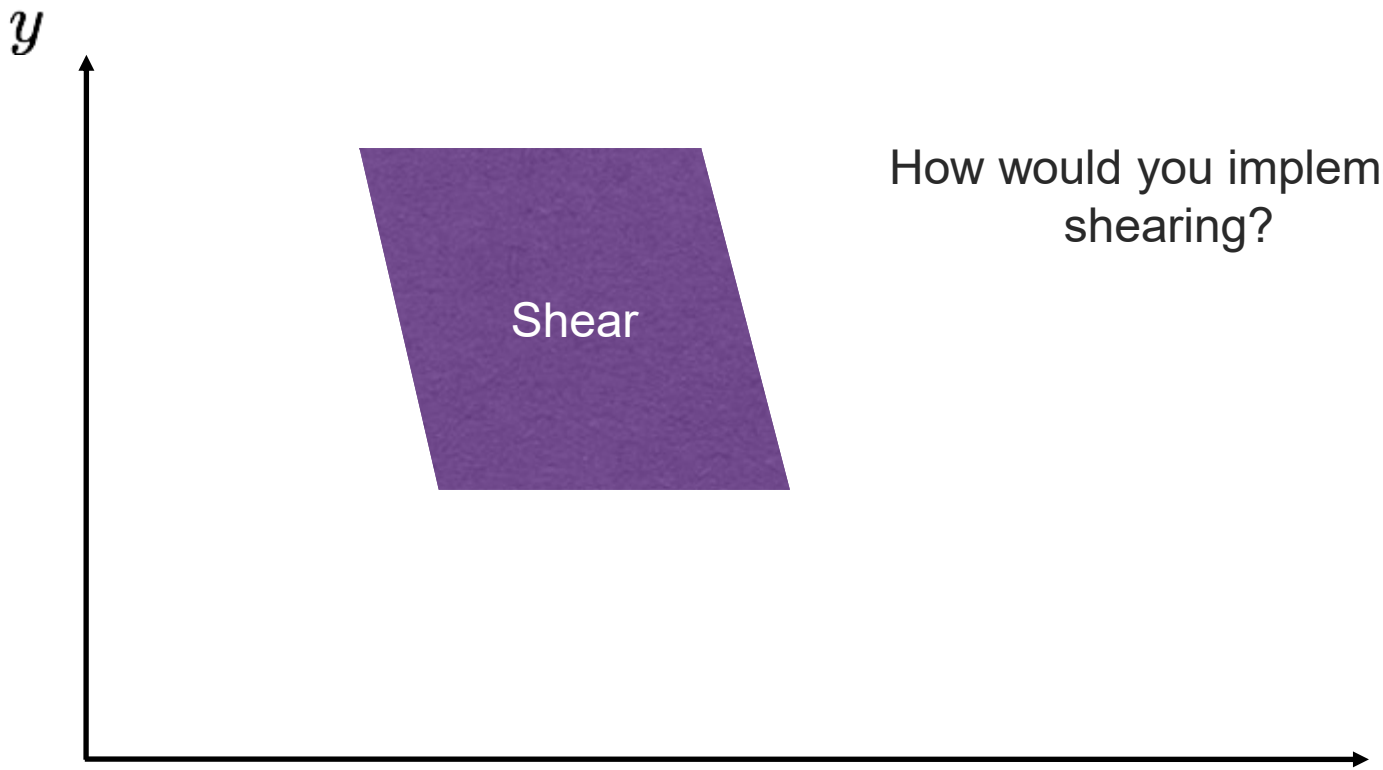
Even though sin(θ) and cos(θ) are nonlinear functions of θ,

- ***x' is a linear combination of x and y***
- ***y' is a linear combination of x and y***

What is the inverse transformation?

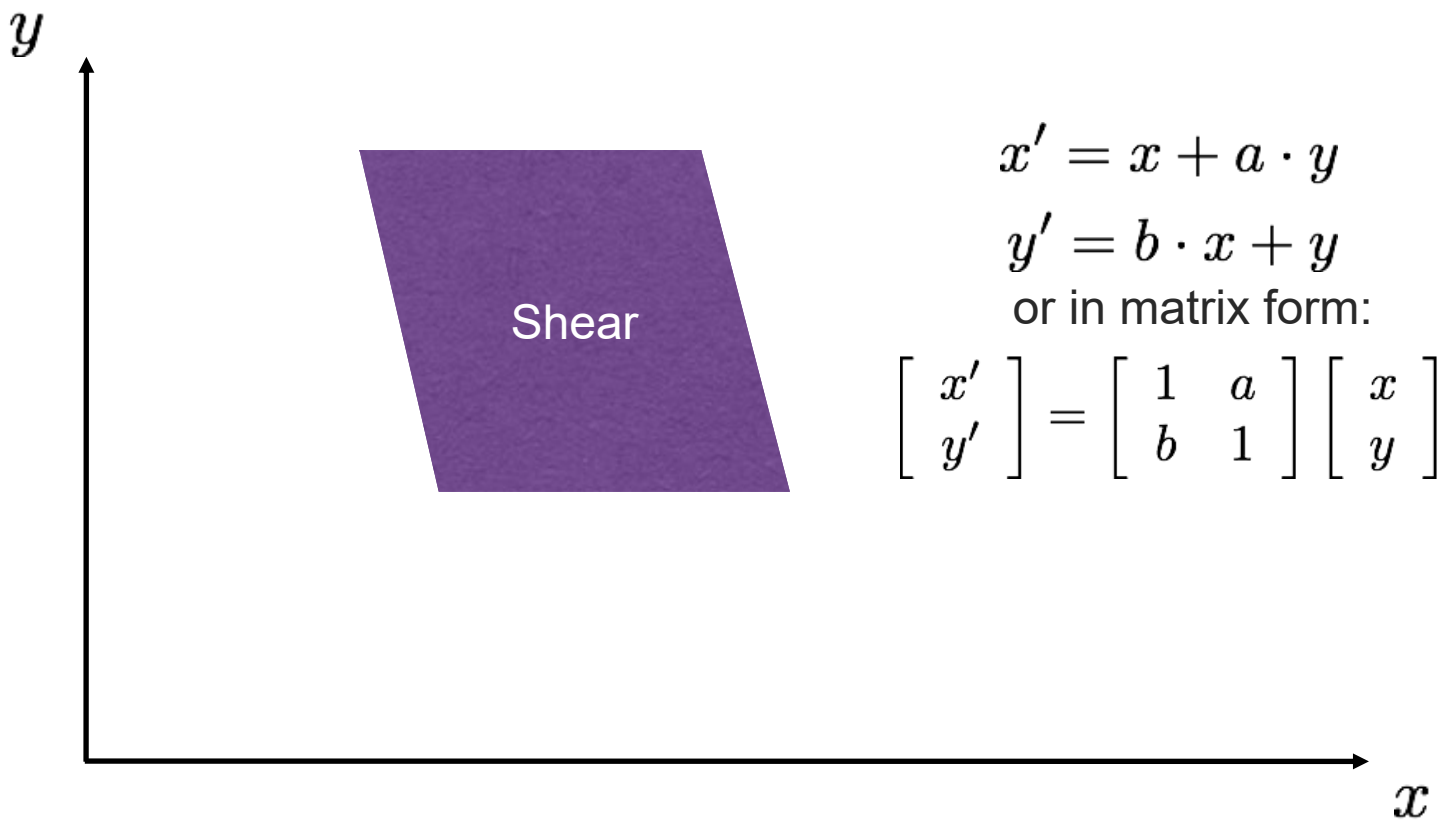
- Rotation by –θ
- For rotation matrices $\mathbf{R}^{-1} = \mathbf{R}^{T}$

2-D Shearing



How would you implement shearing?

2-D Shearing



$$x' = x + a \cdot y$$

$$y' = b \cdot x + y$$

or in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

What transformations can be represented with a 2x2 matrix?

2D Scaling?

$$x' = s_x * x$$

$$y' = s_y * y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Rotate around (0,0)?

$$x' = \cos\Theta * x - \sin\Theta * y$$
$$y' = \sin\Theta * x + \cos\Theta * y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Shear?

$$x' = x + sh_x * y$$

$$y' = sh_y * x + y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

What transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

$$x' = -x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$x' = -x$$
$$y' = -y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Translation?

$$\boldsymbol{x}' = \boldsymbol{x} + \boldsymbol{t}_x$$
$$\boldsymbol{y}' = \boldsymbol{y} + \boldsymbol{t}_y$$

NO!

Source: Alyosha E

2D Linear Transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Only linear 2D transformations can be represented with a 2x2 matrix.
- Linear transformations are combinations of …
  - Scale,
  - Rotation,
  - Shear, and
  - Mirror

Source: Alyosha Ef

Homogeneous coordinates

To convert to homogeneous coordinates:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Homogeneous Coordinates

- Q: How can we represent 2d translation as a 3x3 matrix using homogeneous coordinates?

$$x' = x + t_x$$

$$y' = y + t_y$$

- A: Using the rightmost column:

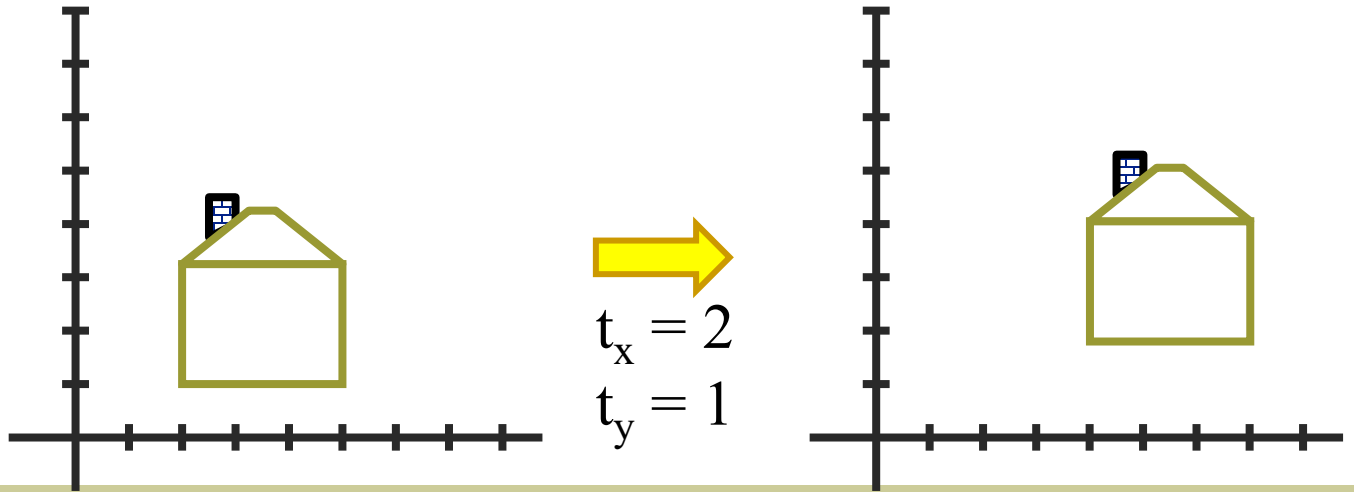
$$\mathbf{T}\boldsymbol{ranslation} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Source: Alyosha E

Translation

Homogeneous Coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$



Source: Alyosha E

Basic 2D Transformations

- Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_x & 0 & 0 \\ 0 & \boldsymbol{s}_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

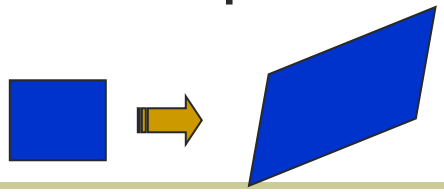
$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh}_x & 0 \\ \boldsymbol{sh}_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

2D Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Affine transformations are combinations of …
  - Linear transformations, and
  - Translations
- Parallel lines remain parallel



Affine Transformations

Affine transformations are combinations of

- Linear transformations, and
- Translations

Properties of affine transformations:

- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

or

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Projective Transformations

Projective transformations are combos of

- Affine transformations, and
- Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of projective transformations:

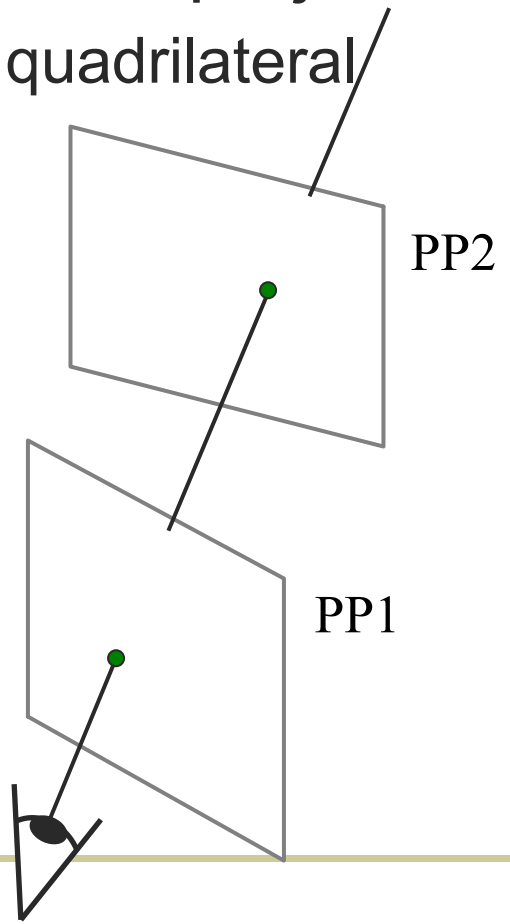
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition
- Models change of basis
- Projective matrix is defined up to a scale (8 DOF)

Image reprojection: Homography

- A projective transform is a mapping between any two PPs with the same center of projection
  - rectangle should map to arbitrary quadrilateral
  - parallel lines aren't
  - but must preserve straight lines
- called **Homography**

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{p'} \qquad \mathbf{H} \qquad \mathbf{p}$$



Homography

(x, y)

$\left(wx'/w, wy'/w\right) = (x', y')$

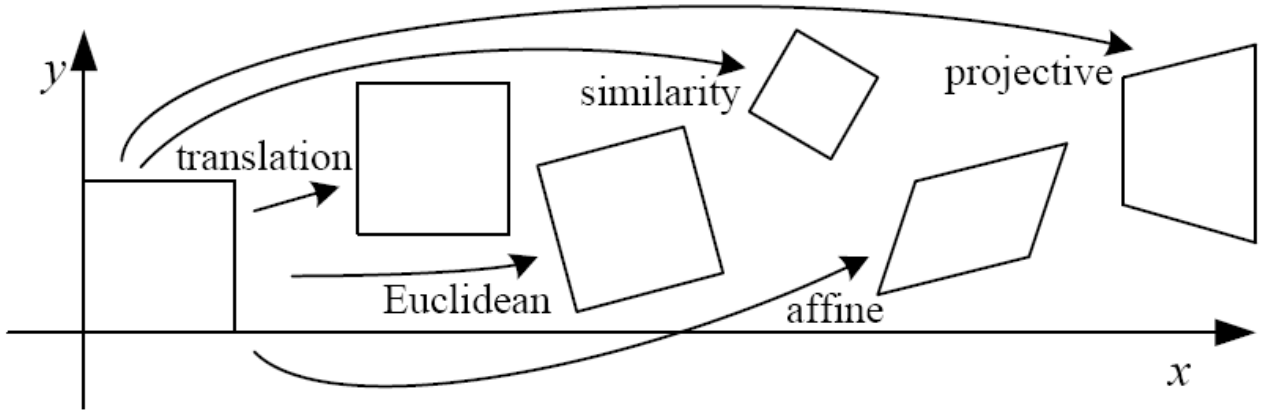
To **apply** a given homography **H**

- Compute **p'** = **Hp** (regular matrix multiply)
- Convert **p'** from homogeneous to image coordinates

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

p' **H** **p**

2D image transformations

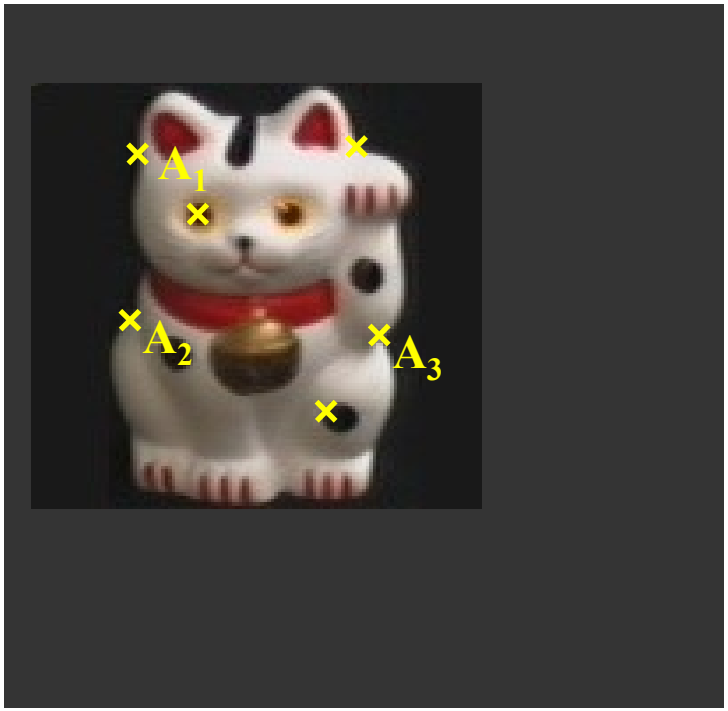


Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$	2	orientation $+\cdots$	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$	3	lengths $+\cdots$	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$	4	angles $+\cdots$	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times3}$	6	parallelism $+\cdots$	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times3}$	8	straight lines	

Today's class

- Introduction to alignment
- Alignment methods
  - Global methods
  - Hypothesize and test
- Image Transformation
  - Common image transformations
  - **Examples of solving image alignment**
- Homework: Mosaics

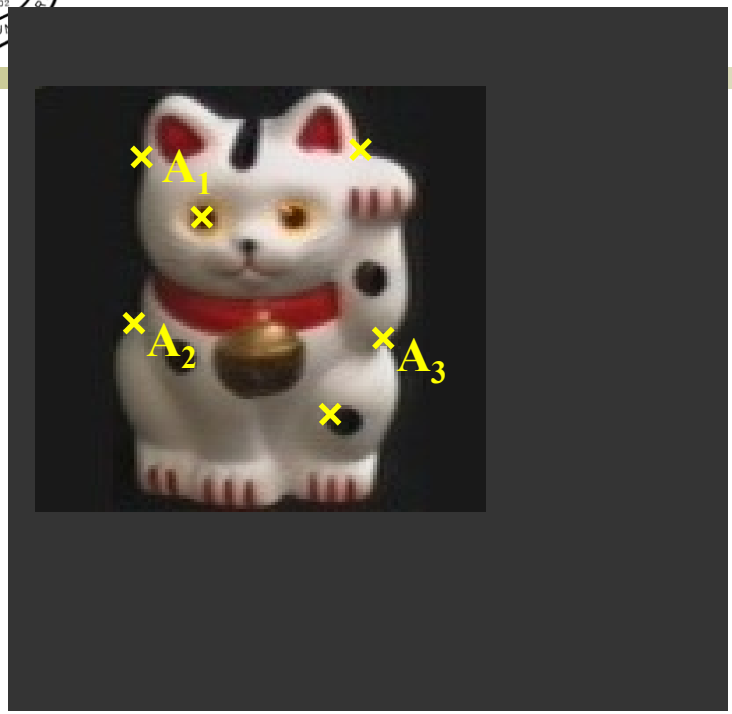
Example: solving for translation



Given matched points in {A} and {B}, estimate the translation of the object

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation



(t_x, t_y)

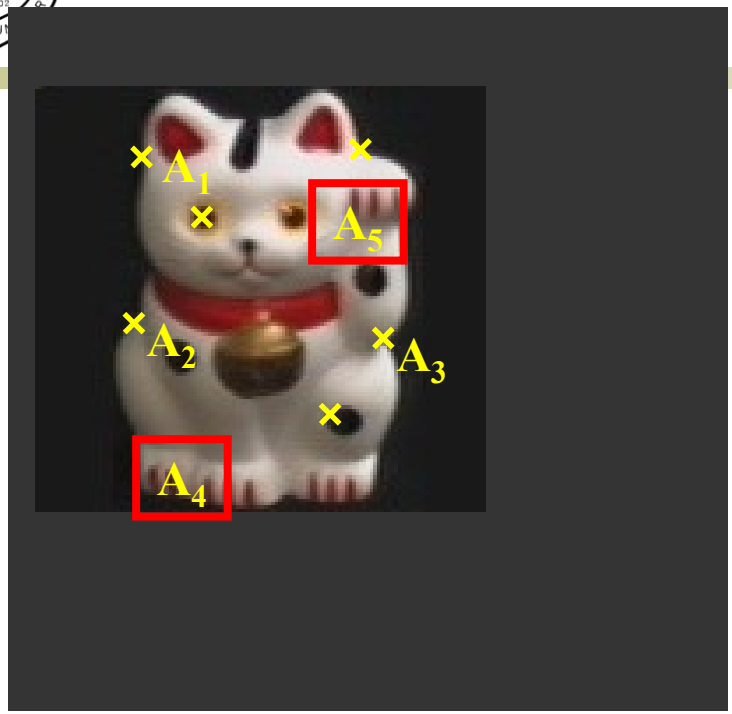
Least squares solution

1. Write down objective function
2. Derived solution
   a) Compute derivative
   b) Compute solution
3. Computational solution
   a) Write in form Ax=b
   b) Solve using pseudo-inverse or eigenvalue decomposition

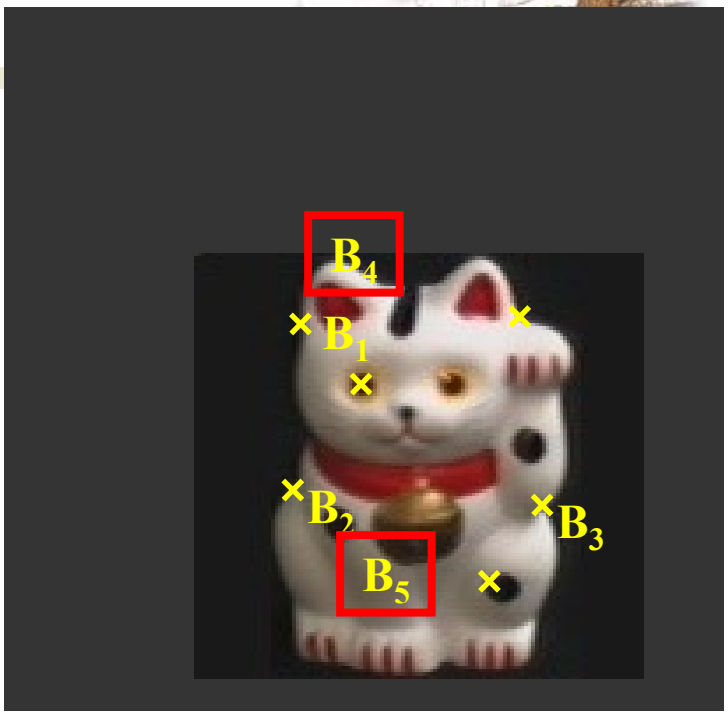
$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} x_1^B - x_1^A \\ y_1^B - y_1^A \\ \vdots \\ x_n^B - x_n^A \\ y_n^B - y_n^A \end{bmatrix}$$

Example: solving for translation



(t_x, t_y)



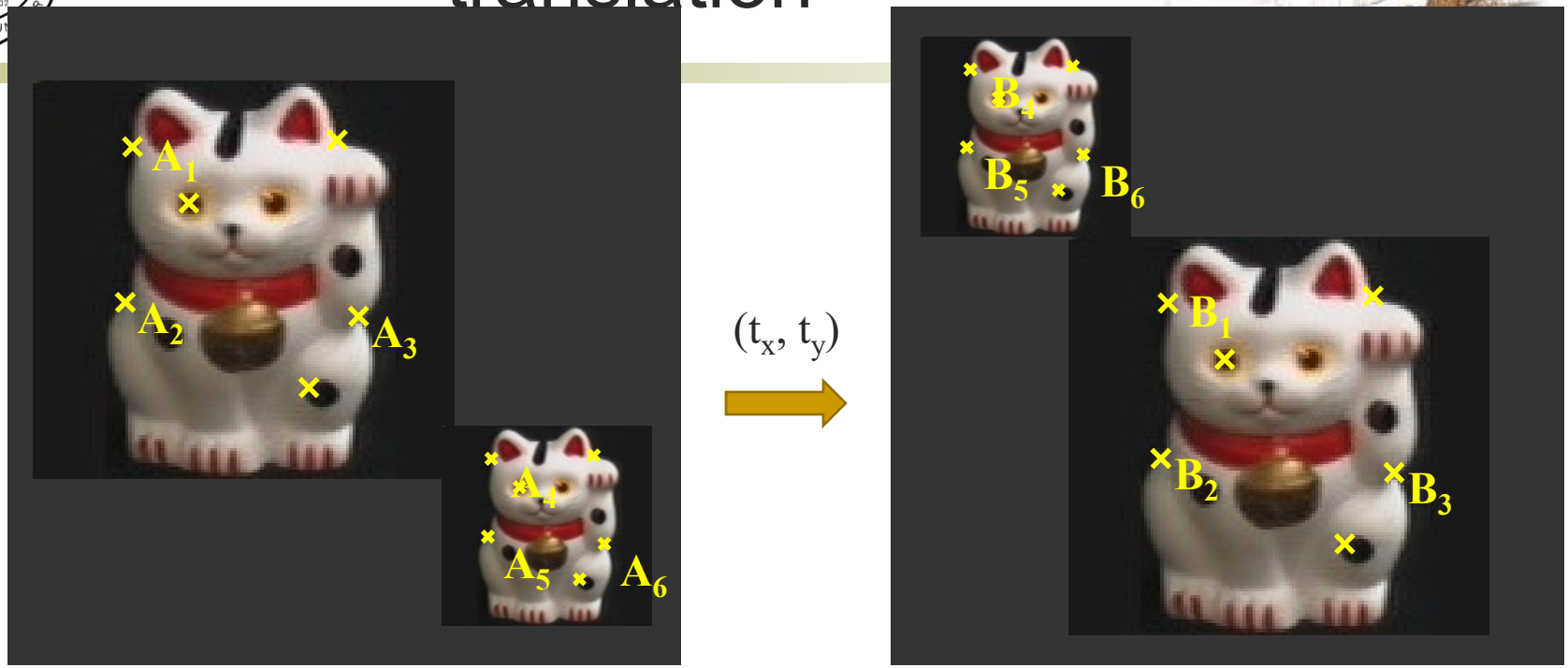
Problem: outliers

RANSAC solution

1. Sample a set of matching points (1 pair)
2. Solve for transformation parameters
3. Score parameters with number of inliers
4. Repeat steps 1-3 N times

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation



Problem: outliers, multiple objects, and/or many-to-one matches

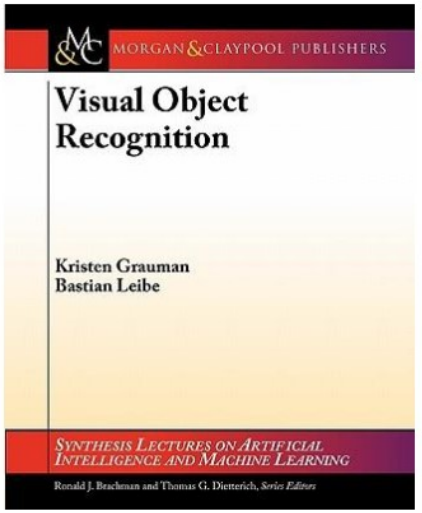
Hough transform solution

1. Initialize a grid of parameter values
2. Each matched pair casts a vote for consistent values
3. Find the parameters with the most votes
4. Solve using least squares with inliers

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

References

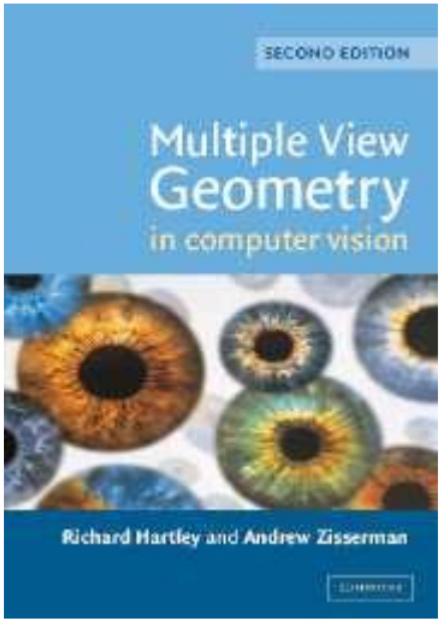
- We've created a script… for the part of the lecture on object recognition & categorization
  - K. Grauman, B. Leibe
    Visual Object Recognition
    Morgan & Claypool publishers, 2011



- Chapter 3: Local Feature Extraction (Last lecture)
- Chapter 5: Geometric Verification (Today)

References

- More details on homography estimation can be found in Chapter 4.7 of
  - R. Hartley, A. Zisserman
    Multiple View Geometry in Computer Vision
    2nd Ed., Cambridge Univ. Press, 2004



- Details about the DoG detector and the SIFT descriptor can be found in
  - D. Lowe, Distinctive image features from scale-invariant keypoints, *IJCV* 60(2), pp. 91-110, 2004

- Try the available local feature detectors and descriptors
  - http://www.robots.ox.ac.uk/~vgg/research/affine/detectors.html#binaries

Today's class

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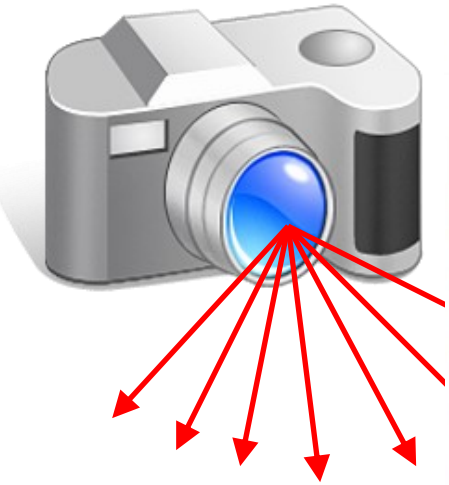
Homework: Mosaics

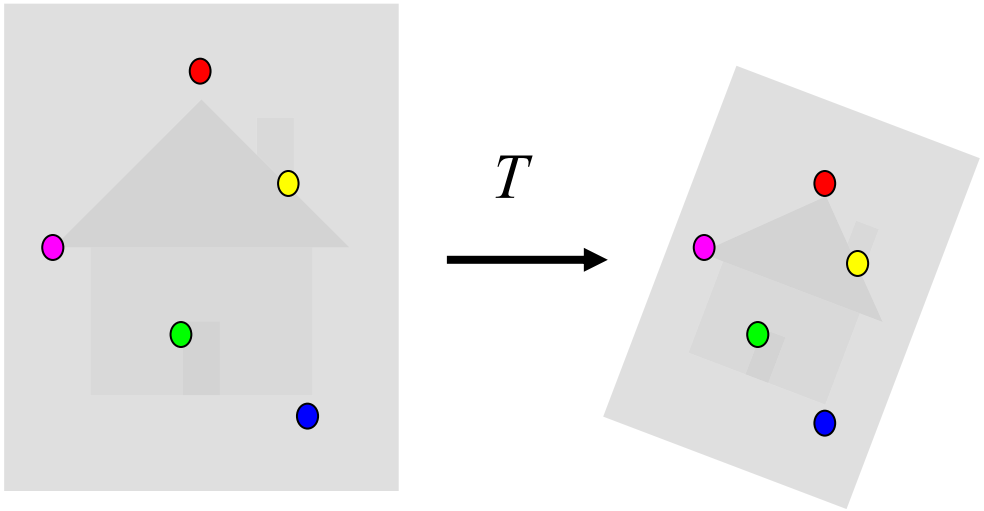


. . .

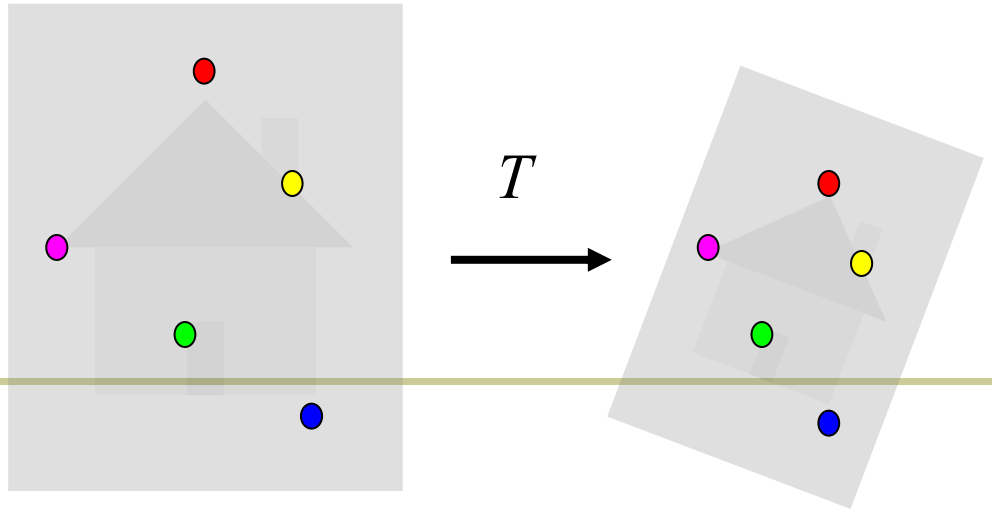
image from S. Seitz

Obtain a wider angle view by combining multiple images.

Main questions



Alignment: Given two images, what is the transformation between them?



Warping: Given a source image and a transformation, what does the transformed output look like?

How to stitch together a panorama (a.k.a. mosaic)?

- Basic Procedure
  - Take a sequence of images from the same position
    - Rotate the camera about its optical center
  - Compute transformation (homography) between second image and first using corresponding points.
  - Transform the second image to overlap with the first.
  - Blend the two together to create a mosaic.
  - (If there are more images, repeat)

本科生大作业

- 综述性研究报告：针对一个专门话题，进行深入调研分析，形成一篇高质量的综述。
- 培养：查阅文献，阅读文献，撰写文献的能力，可能启发对某个方向深入研究的兴趣。
- 提交格式：CVPR论文格式，建议英文撰写。
- 调研范围：顶级会议和顶级期刊论文，以及最新的arXiv论文，
- **不要仅仅去看一些公众号、知乎、中文博客**
- **提交时间：6月20号之前（毕业班）**

话题

- Low-level vision: denoise, super resolution etc.
- Edge detection
- Grouping and segmentation
- Local descriptor and image matching
- Deep learning for vision
- Tracking, video analysis
- Vision and language
- Weakly/Self supervised learning
- Etc.